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Design optimization of laminated fiber composites

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ABSTRACT

Title of Thesis : Design Optimization of Laminated Fiber Composites.

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Laminated Fiber Composites are finding a wide range of applications in structural design especially for light weight structures that have stringent stiffness and strength requirements Finding an efficient composite structural design that meets the requirements of a certain application can be achieved not only by sizing the cross sectional areas and member thicknesses but also by global or local tailoring of the material properties through selective use of orientation. number and stacking sequence of the laminae that make up the composite laminate

The work presented here treats the design optimization problem involving minimum weight design of fiber composite laminates subject to inplane loading conditions which takes into account membrane stiffness and strength constraints The problem is a non linear mathematical programming problem in which the thicknesses of the material placed at preassigned orientation angles are treated as the only design variables Computational efficiency is achieved by using constraint deletion techniques in conjunction with Taylor series approximation for the constraints retained The optimization algorithm used employs a sequence of linear programs to converge to the optimum solution

The method presented offers an efficient and practical optimum design procedure.

DESIGN OPTIMIZATION OF LAMINATED FIBER COMPOSITES

BY

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Chapter 1

Introduction

1.1 Introduction

The design of laminated Fiber Composites has become a challenge to the designer. This is because of the wide range of parameters that can be varied and the complex behaviour of these structures that require sophisticated analysis techniques. Due to the large number of design variables involved, the designer has more control to fine tune his structure to meet the requirements of a design situation, if only the designer can find out how to select these variables. The possibility of achieving a design that meets multiple requirements efficiently coupled with the difficulty in selecting the values of a large set of design variables makes structural optimization an obvious tool for the design of laminated composite structures.

1.2 Definitions

Optimization is concerned with achieving the best outcome of a given objective while satisfying certain restrictions.

Optimal design can be defined as the best feasible design according to a preselected quantitative measure of effectiveness.

The notion of improving or optimizing a structure implicitly presupposes some freedom to change the structure. The potential for change is typically expressed in terms of ranges of permissible changes of a group of parameters. These parameters are called design variables.

The notion of optimization also implies that there are some merit functions or functions that can be improved. These functions are called objective functions.

The solution process consists of starting with an initial design and proceeding in small steps in order to improve the value of the objective function or degree of compliance with the constraints or both. The search is terminated when no progress can be made in improving the objective function without violating some of the constraints. The search can also be terminated when progress in improving the objective function becomes very slow.

1.3 Previous work

The minimum weight optimum design of laminates for strength and membrane stiffness was studied extensively by Foye. Multiple inplane loading conditions were considered and a random search method was used to find ply orientation angles such that the strength and stiffness requirements would be satisfied with the smallest number of plies.

Another procedure for the optimum design of laminates was given by Waddoups. Minimum weight designs were obtained considering strength requirements under multiple distinct loading conditions. Either Tsai Hill or maximum strain criteria was used and all the laminae were assumed to behave linearly up to failure. The search method employed was a systematic 'try them all' procedure.

Both these studies deal with discrete number of plies and they treat ply thicknesses as well as their orientation as design variables.

Verette has extended the laminate optimization procedure to include buckling based on stability analysis.

In the work presented here attention has been focused on developing a laminate optimization capability in which thickness of the material placed at specified orientation angles are treated as the only design variables

1.4 Problems in laminate design

The laminate stiffness matrices can be manipulated by changing either the number of layers or orientation. Using these quantities as design variables it is possible to change the material properties of the laminate as well as the thickness

In order to limit the size of the design problem, limitations are imposed on the stacking sequences The analysis of laminate with bending extension coupling is difficult because the out of plane deformation associated with inplane loads may be large and require non linear analysis capability. For symmetric laminates the bending and extensional responses are decoupled resulting in simpler analysis procedure The number of design variables are halved for the symmetric laminates It is also desirable to eliminate shear extension coupling by using negative angle plies for every positive angle ply used in the laminate Such laminates are called balanced laminates

In the work presented here. only balanced symmetric laminates have been considered for analysis

1.5 Applications

Some commercial applications of design optimization of fiber composites and related computer codes used are given below

Stiffened plate design: Laminated plates stiffened by longitudinal and transverse members are one of the most common structural components. Computer codes used for this purpose are VIPASA, CONMIN, PASCO and VICON [4]

VIPASA is the computer program for the design procedure of a stiffened panel CONMIN is the mathematical programming code based on the method of feasible directions algorithm. VICON is a combination of VIPASA (VI) and CONMIN (CON) Aeroelastic tailoring This is a major area of application of design optimization This concept is utilized in aircraft wing structures which involve aeroelastic constraints Aeroelastic tailoring involves the use of structural deformation to improve structural and aerodynamic characteristics of a lifting surface. The computer codes developed for this purpose are the TSO program, the finite element based FASTOP program and ASTROS [4]. TSO was one of the early efforts in introducing structural optimization into aeroelastic tailoring. This software was developed by General Dynamics ASTROS is an acronym for Automated structural optimization system developed by Northrop

Chapter 2

Laminate Analysis

2.1 Introduction

The word composites in composite materials signifies that two or more materials are combined on a macroscopic scale to form a useful material. The advantage of composites is that they usually exhibit the best qualities of their constituents and some qualities that neither constituents possess.

Laminated composites consists of at least two different materials that are bonded together. The properties that can be emphasized by laminates are strength, stiffness and low weight. The layers of the fiber reinforced laminates are built up with the fiber directions of each layer typically oriented in different directions. Thus the strengths and stiffnesses of the fiber reinforced composites can be designed to the specific requirements of the structural element.

2.2 Classical lamination theory

This theory embodies a collection of stress and deformation hypothesis which is useful in proceeding from the basic building block the lamina, to the structural laminate.

The stress strain relations in the principal material co-ordinates [fig. 2.1] for a lamina of an orthotropic material under plane stress are [1]

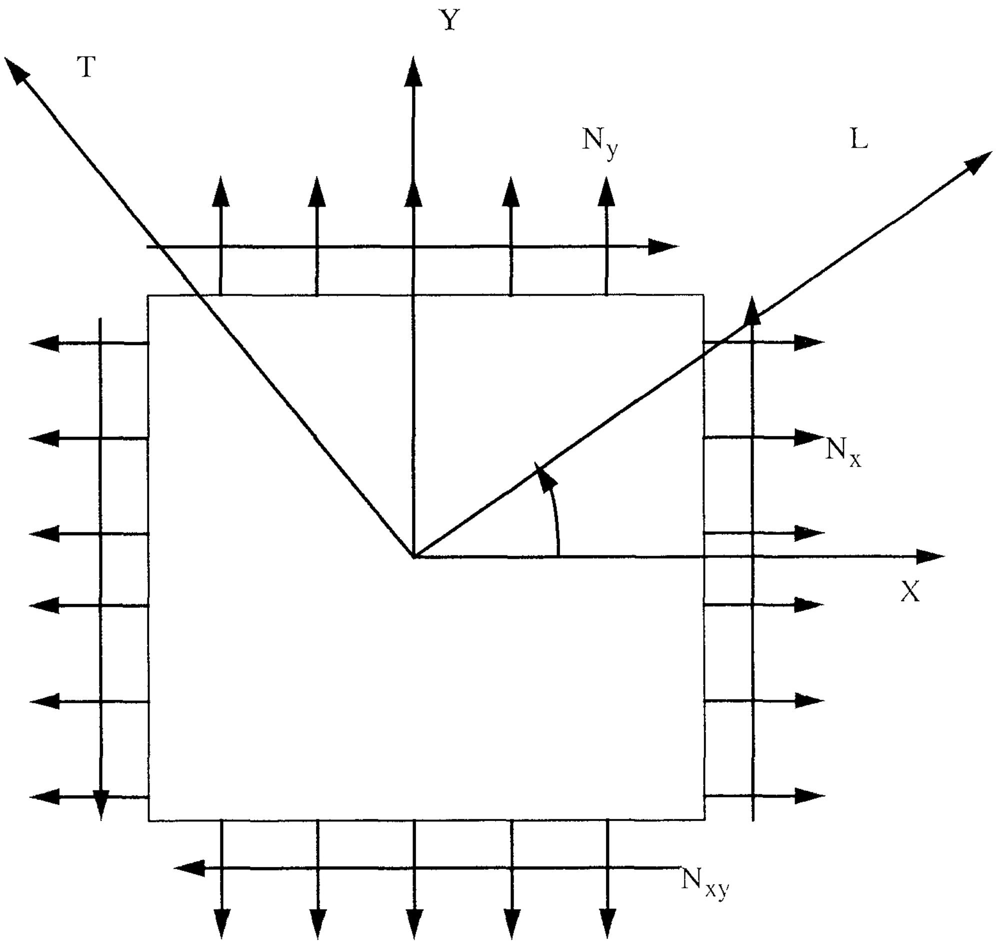


Fig 2.1 SYMMETRIC LAMINATE UNDER IN-PLANE LOADING

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \gamma_{12} \end{Bmatrix} \tag{2.1}$$

Q_{ij} are the reduced stiffnesses.

In any other co-ordinate system in the plane of the lamina, the stress strain relations are

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix} \begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{2.2}$$

$\overline{Q}_{ij}$ are the transformed reduced stiffnesses [Appendix I]

2.3 Assumptions in classical lamination theory

a. The laminate is assumed to consist of perfectly bonded laminae and that the bonds are non shear deformable.

b. The laminate acts as a single layer with very special properties

c. The displacements are continuous across lamina boundaries, so that no lamina can slip relative to each other.

By the Kirchoff hypothesis the laminate strains are given by

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix} \tag{2.3}$$

$\begin{Bmatrix} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix}$ is the vector of midplane strains

$\begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}$ is the vector of midplane curvatures.

z is the distance of each lamina from the midplane

The stresses in any layer (say k^{th}) of the laminate can be expressed as

$$\left\{ \begin{array}{c} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{array} \right\} = \left[\begin{array}{ccc} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{array} \right] \left\{ \begin{array}{c} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{array} \right\} \quad (2.4)$$

The resultant forces and moments acting on a laminate are obtained by the integration of stresses in each lamina through the laminate thickness

$$\left\{ \begin{array}{c} N_x \\ N_y \\ N_{xy} \end{array} \right\} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \left\{ \begin{array}{c} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{array} \right\} dz \quad (2.5)$$

$$\left\{ \begin{array}{c} M_x \\ M_y \\ M_{xy} \end{array} \right\} = \sum_{k=1}^{N} \int_{z_{k-1}}^{z_k} \left\{ \begin{array}{c} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{array} \right\} z\,dz \quad (2.6)$$

The Force and Moment vectors [fig. 2.2] can be expressed as [2]

$$\left\{ \begin{array}{c} N_x \\ N_y \\ N_{xy} \end{array} \right\} = \left[\begin{array}{ccc} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{array} \right] \left\{ \begin{array}{c} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{array} \right\} + \left[\begin{array}{ccc} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{array} \right] \left\{ \begin{array}{c} k_x \\ k_y \\ k_{xy} \end{array} \right\} \quad (2.7)$$

$$\left\{ \begin{array}{c} M_x \\ M_y \\ M_{xy} \end{array} \right\} = \left[\begin{array}{ccc} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{array} \right] \left\{ \begin{array}{c} \epsilon_x^0 \\ \epsilon_y^0 \\ \gamma_{xy}^0 \end{array} \right\} + \left[\begin{array}{ccc} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{array} \right] \left\{ \begin{array}{c} k_x \\ k_y \\ k_{xy} \end{array} \right\} \quad (2.8)$$

$$A_{ij} = \sum_{k=1}^{N} Q_{ij}(z_k - z_{k-1}) \quad (2.9)$$

A_{ij} [fig. 2.3] is the Extensional Stiffness matrix.

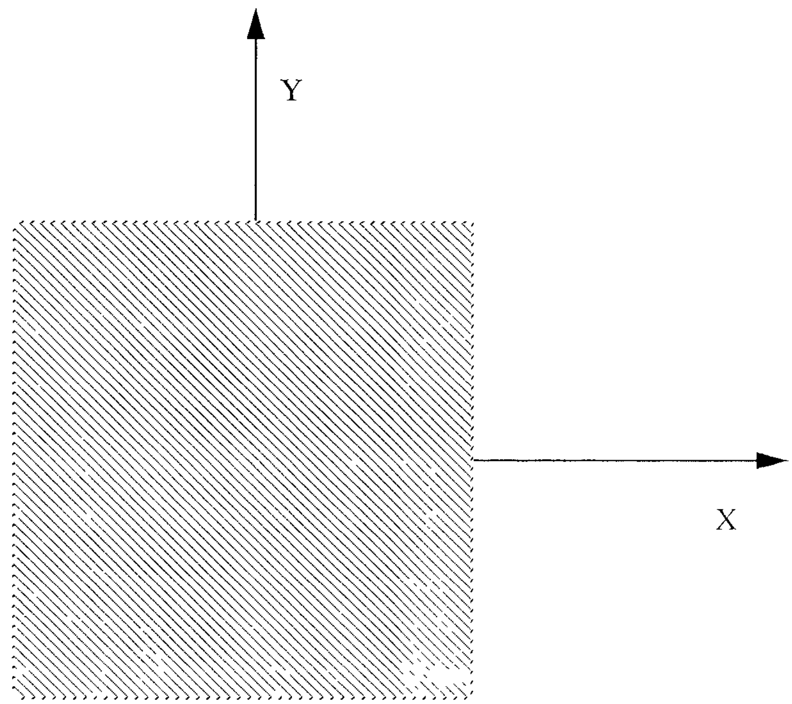
$$B_{ij} = 1/2 \sum_{k=1}^{N} Q_{ij}(z_k^2 - z_{k-1}^2) \quad (2.10)$$

B_{ij} is the Coupling Stiffness matrix.

$$D_{ij} = 1/3 \sum_{k=1}^{N} Q_{ij}(z_k^3 - z_{k-1}^3) \quad (2.11)$$

D_{ij} is the Bending Stiffness matrix.

The Coupling stiffness matrix causes coupling between bending and extension



LAMINA ORIENTED AT ANGLE THETA DEG.

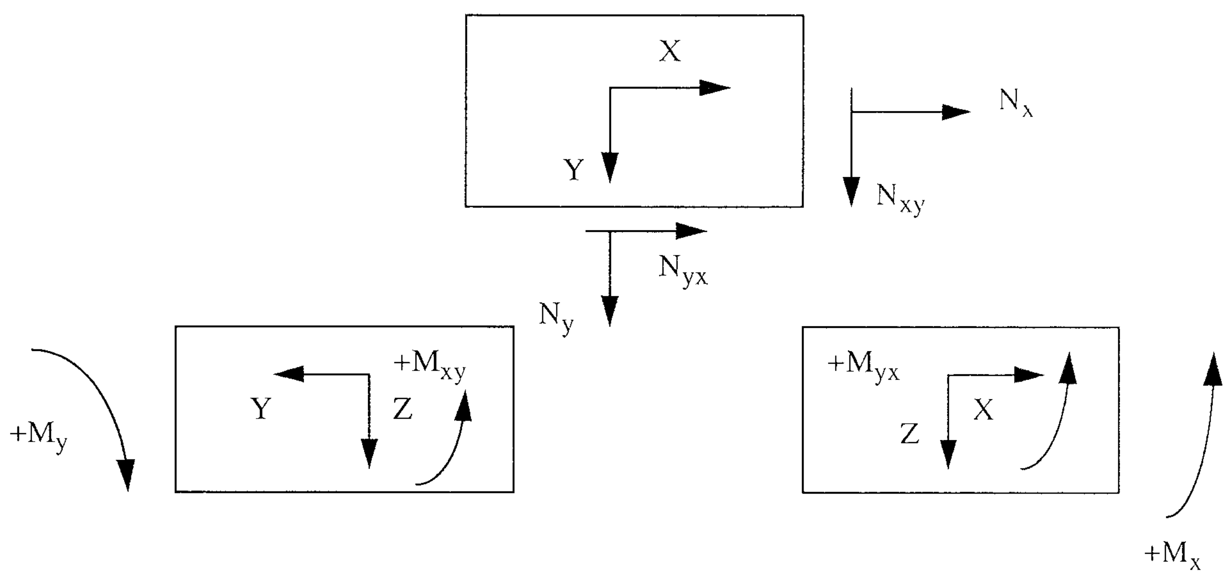


Fig 2.2 POSITIVE FORCES AND MOMENTS

The Extensional stiffness matrix relates the resultant forces to the midplane strains and the Bending stiffness matrix relates the resultant moments to the plate curvatures.

The Global stiffness matrix for the laminate is

$$\begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} \\ & & & & & \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{bmatrix} \tag{2.12}$$

The constitutive equation for the laminated plate can be written as

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \epsilon^0 \\ k \end{Bmatrix} \tag{2.13}$$

The Global stiffness matrix is inverted to get the midplane strains and curvatures.

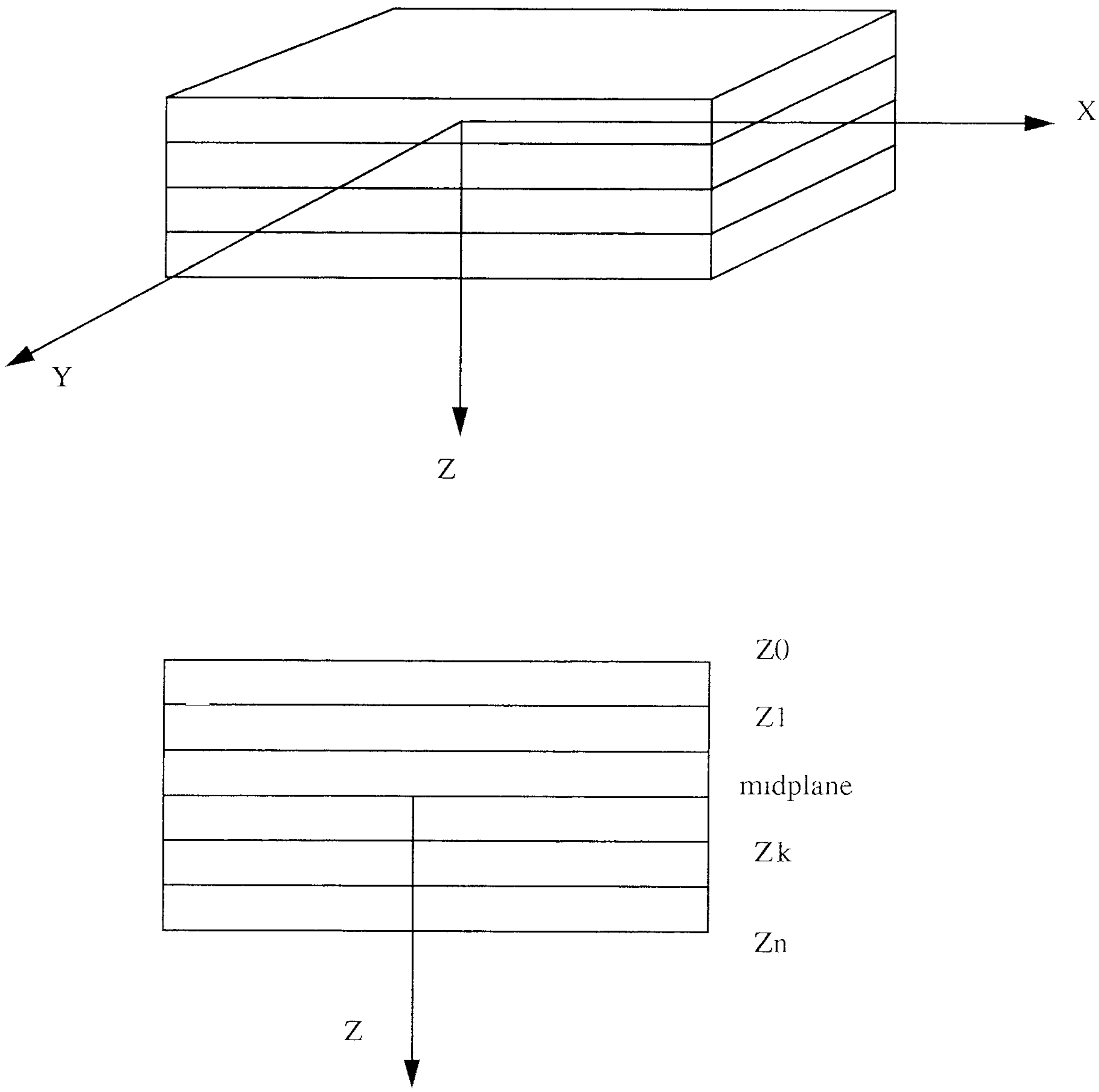


Fig 2.3 NUMBER AND COORDINATES OF THE LAMINAE IN A LAMINATE

Chapter 3

Optimization problem

3.1 Problem statement

The objective is to find the minimum weight design subject to strength and membrane stiffness requirements. The material properties and the available orientation angles of the fibers are known quantities. The thicknesses of the plies at each orientation angle are the only design variables which are to be optimized

The optimization problem can be stated as

$$W = \sum_{i=1}^{I} \rho_i t_i - - - > Min \tag{3.1}$$

W is the weight objective function, which is linear in the thickness design variables t_i.

subject to the following constraints

1.

$$A_j^i \varepsilon_{1i} + B_j^i \varepsilon_{2i} + C_j^i \gamma_{12i} \leq 1 \tag{3.2}$$

2

$$A_{11}^l \leq A_{11} \; A_{22}^l \leq A_{22} \; A_{66}^l \leq A_{66} \tag{3.3}$$

3

$$t_i \geq 0 \tag{3.4}$$

3.2 Description of the constraints

Equation (2) represents the strength criterion. The strains appearing in this constraint depend upon the design variables t_i in a non linear and implicit manner. The failure envelope is represented by a set of J planar facets in the $\varepsilon_1, \varepsilon_2, \gamma_{12}$ strain space (J = 6)

The coefficients [Appendix I] A_j^i, B_j^i and C_j^i are given in the following table.

j	$A_j^{(i)}$	$B_j^{(i)}$	$C_j^{(i)}$
1	$1/\varepsilon_{L_i}^t$	0	0
2	$1/\varepsilon_{L_i}^c$	0	0
3	0	$1/\varepsilon_{T_i}^t$	0
4	0	$1/\varepsilon_{T_i}^c$	0
5	0	0	$1/\gamma_{LT}^+$
6	0	0	$1/\gamma_{LT}^-$

Table 3.1 Values of coefficients for Max.Strain failure criterion

Equation (3) represents the stiffness criterion. The laminate membrane stiffnesses are linearly dependent on the design variable t_i

$$A_{ij} = \sum_{i=1}^{I} (Q_{ij}) t_i \quad i, j = 1, 2, 6 \tag{3.5}$$

Equation (4) represents the non-negativity constraint which requires that the thicknesses of the plies be positive always

It is seen that the objective function, the stiffness constraint and the non-negativity constraints are linear functions of the design variables. However the inequality constraint representing the strength criterion is non linear in the design variables.

Chapter 4

Optimization Procedure

4.1 Algorithm

The optimization procedure employed transforms the nonlinear programming problem into a sequence of linear problems that can be solved by using a simplex algorithm. The method adapted tends to generate a sequence of designs that are non critical ie the sequence of designs tend to funnel down the middle of the acceptable region. A constraint deletion technique is employed which retains only those constraints which are potentially critical at each stage of the optimization process. The inequality constraints ignored at each stage are automatically satisfied if critical and near critical constraints are satisfied.

Three important aspects are to be considered while applying the optimization algorithm to the laminate design problem. They are

1. A method to automatically generate an acceptable initial design.
2. A decision as to which of the inequality constraints are to be retained
3. A method to obtain the partial derivative expressions so that linearized representations of the constraints retained can be constructed

The Optimization procedure is shown in fig. 4.1.

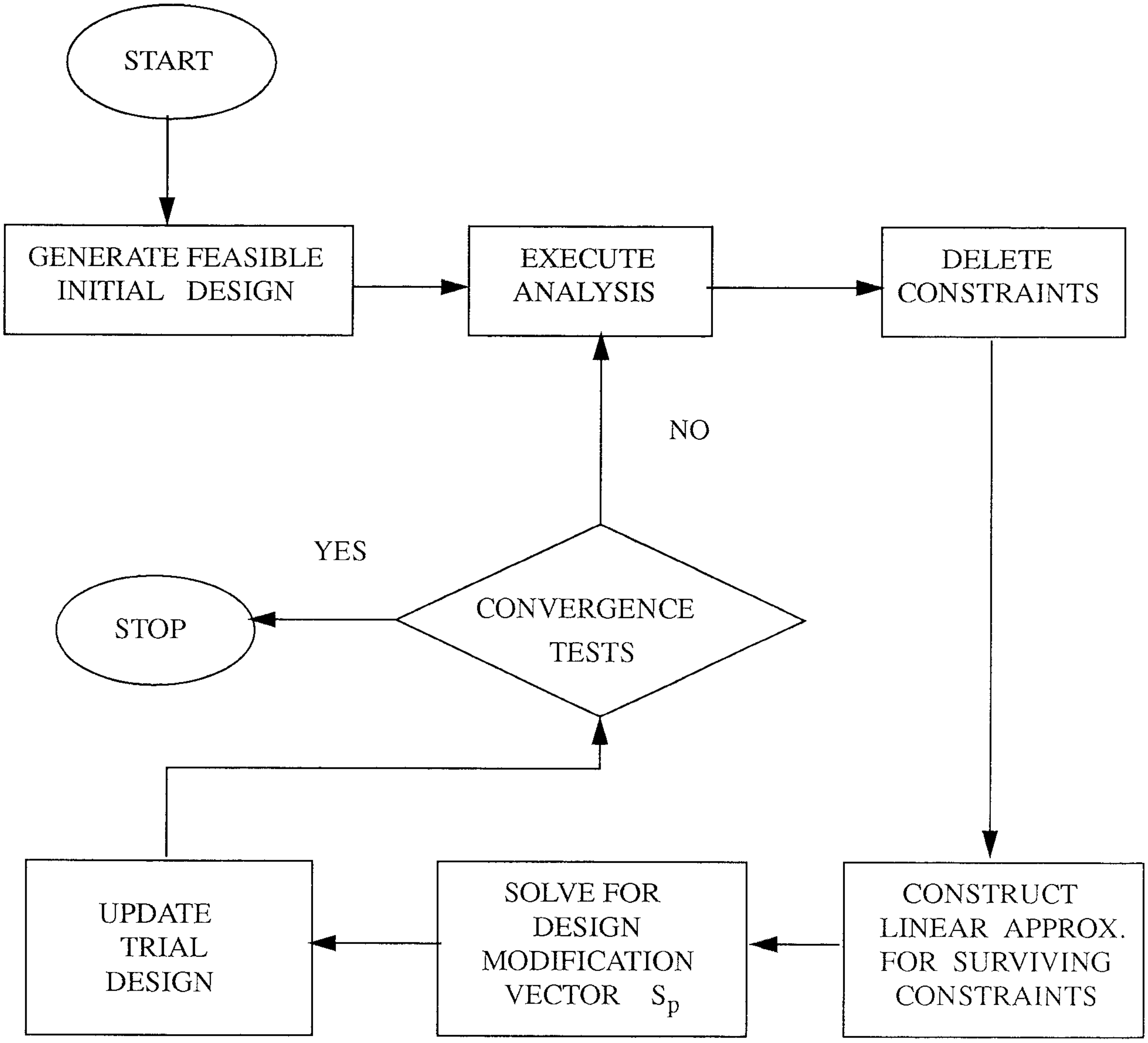


Fig 4.1 FLOW CHART FOR THE DESIGN OPTIMIZATION PROCEDURE

4.2 Initial Design

The basic thickness of all the plies in the laminate is assumed to be 0.005m. The thickness of the plies are determined so as to satisfy the stiffness requirements. The stiffness requirements are satisfied if the membrane stiffnesses A_{11}, A_{22} and A_{66} exceed the specified lower limits A^l_{11}, A^l_{22} and A^l_{66} respectively by a given starting point factor of safety.

It is also possible to determine the starting values of thickness of the plies by satisfying the strength requirements with a specified factor of safety.

The initial thickness of the plies is the larger of the two values got from the stiffness and strength criteria.

4.3 Constraint Deletion

The purpose of the constraint deletion process is to drastically reduce the number of inequality constraints used to represent the stiffness and strength constraints.

A compromise has to be made between the values of the control parameters (ACR & SCR) and the safety of the design. The larger the specified values of the control parameters the larger the number of inequality constraints retained and hence the risk of finding an unacceptable design is lower. On the other hand if the specified values of control parameters are smaller, the number of inequality constraints retained is smaller and consequently the risk of finding an unacceptable design is higher.

The inequality constraints on the membrane stiffnesses are expressed as

$$A^l_{11} \leq A_{11}\ A^l_{22} \leq A_{22}\ A^l_{66} \leq A_{66} \quad (4.1)$$

A control parameter ACR, is specified such that if

$$A_{ii}/A^l_{ii} \geq ACR,\ i = 1, 2, 6 \quad (4.2)$$

then the corresponding constraint $A_{rr} \leq A^l_{rr}$ is ignored

The effect of this procedure is to delete a stiffness constraint if the ratio of the of A_{rr}/Arr^l for the current design over the corresponding lower limit value equals or exceeds the specified value of the control parameter

The strength constraint can be expressed as

$$Q_{ji} = A^i_j \varepsilon_{1i} + B^i_j \varepsilon_{2i} + C^i_j \gamma_{12i} \leq 1 \tag{4.3}$$

If $Q_{ji} \leq 0$ then the corresponding constraint $Q_{ji} \leq 1$ is ignored. Also if $Q_{ji} \gg 0$ and $1/Q_{ji} \gg$ SCR then the corresponding constraint Q_{ji} is deleted. SCR is another control parameter.

4.4 Partial derivatives for non linear constraints

Of all the constraints, the inequality constraints representing the strength criterion is non linear in the design variables t_i. The partial derivatives of Q_{ji} with respect to the design variable t_i are needed to construct the linearized representation of the non linear constraint

The non linear constraint

$$Q_{ji} = A^i_j \varepsilon_{1i} + B^i_j \varepsilon_{2i} + C^i_j \gamma_{12i} \leq 1 \tag{4.4}$$

can be expressed as

$$h^p_q(t) = Q_{ji_{(t)}} - 1 \leq 0, \; i = 1, \ldots I, \; j = 1, \ldots J \tag{4.5}$$

The linearized approximation of these constraints based on a Taylor series expansion about the design point t_p with the components t_{i_p}, is

$$h^p_t = Q_{ji}(t_p) - 1 + \sum_{i=1}^{I} (t_i - t^p_i) \frac{\partial Q_{ji}}{\partial t_i}(t_p) \tag{4.6}$$

where

$$\frac{\partial Q_{ji}}{\partial t_i}(t_p) = A_j^i \frac{\partial \varepsilon_{1i}}{\partial t_i} + B_j^i \frac{\partial \varepsilon_{2i}}{\partial t_i} + C_j^i \frac{\partial \gamma_{12i}}{\partial t_i} \tag{4.7}$$

evaluated at the current design tp.

The relation between the membrane forces and the strains in the X-Y frame of reference can be expressed in matrix form as

$$\left\{ N \right\} = \left[A \right] \left\{ \varepsilon \right\} \tag{4.8}$$

Differentiating this expression with respect to t_i

$$\left\{ \frac{\partial N}{\partial t_i} \right\} = \left[\frac{\partial A}{\partial t_i} \right] \varepsilon_k + \left[A \right] \left\{ \frac{\partial \varepsilon_k}{\partial t_i} \right\} = 0 \tag{4.9}$$

substituting for $\left[\frac{\partial A}{\partial t_i} \right]$ in the above equation and solving it for $\frac{\partial \varepsilon}{\partial t_i}$

$$\frac{\partial \varepsilon}{\partial t_i} = - \left[A \right]^{-1} \left[\overline{Q}_{ij} \right]_i \varepsilon \tag{4.10}$$

$$A_{ij} = \sum_{i=1}^{I} \left[\overline{Q}_{ij} \right]_i t_i \tag{4.11}$$

The strains in the principal material direction and the strains in the X-Y frame of reference are related by the transformation matrix [Appendix I]

$$\left\{ \varepsilon_i \right\} = \left[R_i \right] \left\{ \varepsilon \right\} \tag{4.12}$$

$$\frac{\partial \varepsilon_i}{\partial t_i} = \left[R_i \right] \frac{\partial \varepsilon}{\partial t_i} \tag{4.13}$$

substituting for $\frac{\partial \varepsilon}{\partial t_i}$ in the above equation

$$\frac{\partial \varepsilon_i}{\partial t_i} = \left[R_i \right] - \left[A \right]^{-1} \left[\overline{Q}_{ij} \right]_i \varepsilon \tag{4.14}$$

From this equation the partial derivatives of the strength constraints with respect to the design variable can be found, from which linearized approximation of the strength constraints can be constructed.

4.5 Simplex tableau

The design modification vector $\left\{ S_p \right\}$ is determined by solving the optimization problem formulated above using a Simplex procedure.

The components s_i of the vector $\left\{ S_p \right\}$ are expressed as the difference of two non negative variables s_i' and s_i'' such that

$$s_i = s_i' - s_i'' \tag{4.15}$$

The constraint equations in the Simplex table are as follows

Q constraints

$\alpha_{11}s_1' - \alpha_{11}s_1'' + \ . \ + \alpha_{1I}s_I' - \alpha_{1I}s_I'' = \psi_1$

$\alpha_{21}s_1' - \alpha_{21}s_1'' + .. \ + \alpha_{2I}s_I' - \alpha_{2I}s_I'' = \psi_2$

. . .

. .

.

$\alpha_{Q1}s_1' - \alpha_{Q1}s_1'' + \ ... + \alpha_{QI}s_I' - \alpha_{QI}s_I'' = \psi_Q$

$Q + 1_{th}$ constraint

$\delta_1 s_1' - \delta_1 s_1'' + \delta_I s_I' - \delta_I s_I'' = 0$

Linking constraint

$s_i' - s_i'' - s_{i+1}' + s_{i+1}'' = 0$

Total number of linking constraints $= (I-2)/2$, assuming I even

The coefficients of the constraint equations in the simplex table are as follows

$$\alpha_{qi} = \frac{\partial h_q / \partial t_i}{|\nabla h_q|}_{t=t_p} \quad i = 1, 2 \ \ I \ q = 1, 2 \ \ Q \tag{4.16}$$

$$\delta_i = \frac{\partial W / \partial t_i}{|\nabla W|}_{t=t_p} \tag{4.17}$$

$$\psi_q = -\frac{h_q(t_p)}{|\nabla h_q(t_p)|} \tag{4.18}$$

The design is updated as

$$t_{p+1} = t_p + S_p \tag{4.19}$$

The iterative design procedure is continued to convergence. The convergence criteria is based on the diminishing returns with repect to the weight reduction after successive iterations. An option to terminate the iterative procedure after a prespecified number of stages is provided

Chapter 5

Program Organization

5.1 Introduction

The Program has basically two modules. They are

1. Laminate stress analysis module

2. Laminate Design optimization module

These modules can be run seperately or together. The computer code has been developed in the 'C' language on the SUN/SPARC Workstation. The software package GAMS (General algebraic modelling system) has been used to solve the Simplex problem in the optimization module

The sample input files and the corresponding output files for each module is given in the appendix II

5.2 Stress analysis module

The input to this module can be either given interactively or from an input file. If the interactive mode is selected, the user is prompted for various input parameters (material properties, loading conditions etc) which are to be typed in through the keyboard. A file containing the user given input (input.dat) is created by default. On the other hand if the input from a file is opted for, then the user is prompted for an input file name. This file should contain all the requisite input quantities. The program will read the input from this file and begin execution.

The 'laminate' function can be thought of as the heart of the stress analysis module. To begin with the program calculates the stiffness matrix of the laminate in the principal material direction. The stiffness matrices of each lamina oriented at predefined angles is found. The Extensional stiffness matrix, Bending stiffness matrix and the Coupling matrix is calculated from the stiffness matrices of the individual laminae. The Global stiffness matrix of the laminate is then assembled from the Extensional, Bending and Coupling stiffness matrices.

The Global stiffness matrix is inverted to get the Global compliance matrix. The inverse function uses a L-U decomposition technique along with back substitution.

The laminate midplane strains and curvatures are got by multiplying the compliance matrix with the load vector. The strain in the midplane of each lamina is calculated from which the corresponding stresses are got. The stresses and strains in the principal material direction is got by using a transformation matrix.

The last part in the 'laminate' function is the failure analysis. The laminate is tested for failure depending on the load condition and the type of failure criterion selected. The user has the choice of selecting the failure criterion based on the following theories.

1. Maximum stress theory. This theory states that failure will occur if any of the stresses in the principal material direction exceeds the corresponding allowable stress. The following equations have to be satisfied

$$\sigma_L \ll \sigma_{LU}$$
$$\sigma_T \ll \sigma_{TU} \qquad (5.1)$$
$$\tau_{LT} \ll \tau_{LTU}$$

2 Maximum strain theory. This theory states that failure will occur if any of the strains in the principal material direction exceeds the corresponding allowable strain. The following equations have to be satisfied

$$\begin{aligned} \varepsilon_L &\ll \varepsilon_{LU} \\ \varepsilon_T &\ll \varepsilon_{TU} \\ \gamma_{LT} &\ll \gamma_{LTU} \end{aligned} \qquad (5\ 2)$$

3. Maximum work (Tsai Hill) theory. This theory states that in the plane stress states, failure is initiated if the following inequality is violated

$$(\sigma_L/\sigma_{LU})^2 - (\sigma_L/\sigma_{LU})(\sigma_T/\sigma_{TU}) + (\sigma_T/\sigma_{TU})^2 + (\tau_{LT}/\tau_{LTU})^2 \leq 1 \qquad (5\ 3)$$

4. Tsai Wu Tensor theory According to this theory a failure surface exists in the stress space in the form

$$F_i\sigma_i + F_{ij}\sigma_i\sigma_j = 1 \quad i, j = 1, \quad 6 \qquad (5\ 4)$$

F_i and F_{ij} are strength tensors of the second and fourth rank respectively

For an orthotropic lamina under plane stress condition the failure criterion can be stated as

$$F_1\sigma_1 + F_2\sigma_2 + F_6\sigma_6 + F_1\sigma_1{}^2 + F_2\sigma_2^2 + F_6\sigma_6^2 + 2F_{12}\sigma_1\sigma_2 = 1 \qquad (5\ 5)$$

Each lamina is tested for failure based on the failure criterion selected. If all the laminae fail. it is assumed that the laminate failure has occured

All parameters that are calculated in the laminate function are displayed on the screen and also written to an output file 'des.out'

5.3 Optimization module

As in the stress analysis module the input can be given either interactively or from an input file.

The Optimization module is made up of four main routines They are

1. Initial design
2. Constraint deletion.
3. Simplex formulation.
4. GAMS.

5.3.1 Initial design

The thicknesses of the laminae required to satisfy the stiffness requirements are determined. The stiffness criteria can be stated as

$$A_{11} \geq SPFS * A_{11l}$$

$$A_{22} \geq SPFS * A_{22l} \qquad (5.6)$$

$$A_{66} \geq SPFS * A_{66l}$$

SPFS is the startring point factor of safety.

The thicknesses of the laminae to satisfy the strength constraint is also determined.

The strength constraint is

$$Q_{ji} = A_j^i \varepsilon_{1i} + B_j^i \varepsilon_{2i} + C_j^i \gamma_{12i} \leq 1 \qquad (5.7)$$

The strains in the principal material directions used in the above equation are got from the 'laminate' function.

The initial design thicknesses of all the laminae are taken to be the higher of the two sets of thicknesses got from the stiffness and the strength criteria

5.3.2 Constraint deletion

Of the three stiffness constraints only those that are critical are retained to be used in the simplex formulation

The total number of strength constraints before deletion is equal to the product of the number of strain facets and the number of layers in the laminate The simplex formulation is simplified by deleting the non critical strength constraints

5.3.3 Simplex formulation

This part of the program calculates the coeffecients needed for the constraint equations in the simplex table.

The coeffecients for the strength constraint equations are

1.

$$a_{qi} = \left. \frac{(\partial h_q / \partial t_i)}{|\nabla h_q|} \right|_{t=t_p} \quad i = 1, 2 \ldots I \quad q = 1, 2 \ldots Q \tag{5.8}$$

Q is the number of undeleted strength constraints

h_q is the non-linear constraint

$h_q^p(t) = Q_{ji(t)} - 1 \le 0 \quad i = 1, \ldots I, \; j = 1, \ldots J$

as given by eqn 4.5

The derivative of h_q with respect to the thickness t_i is $\frac{\partial h_q}{\partial t_i} = \frac{\partial Q_{ji}}{\partial t_i}$

The expression for $\frac{\partial Q_{ji}}{\partial t_i}$ is

$\frac{\partial Q_{ji}}{\partial t_i}(t_p) = A_j^i \frac{\partial \varepsilon_{1i}}{\partial t_i} + B_j^i \frac{\partial \varepsilon_{2i}}{\partial t_i} + C_j^i \frac{\partial \gamma_{12i}}{\partial t_i}$

as given by eqn 4.6.

The gradient of h_q. ∇h_q is a vector given by

$$\nabla h_q = \frac{\partial h_q}{\partial t_1} i + \frac{\partial h_q}{\partial t_2} j + \ldots + \frac{\partial h_q}{\partial t_I} n \tag{5.9}$$

The magnitude of the gradient is

$$|\nabla h_q| = \sqrt{\frac{\partial h_q}{\partial t_1}^2 + \frac{\partial h_q}{\partial t_2}^2 + \ldots + \frac{\partial h_q}{\partial t_I}^2} \tag{5.10}$$

2. The coeffecient on the right hand side of the strength equation is

$$\psi_q = -\frac{h_q(t_p)}{|\nabla h_q(t_p)|} \tag{5.11}$$

3. The coeffecients of the $Q + 1_{th}$ constraint equation is

$$\delta_i = \frac{\partial W / \partial t_i}{|\nabla W|}\bigg|_{t=t_p} \tag{5.12}$$

W is the weight per unit surface area of the laminate given by $W = t_1 + t_2 + t_3 + \ldots + t_i$

The derivative of W with respect to any of the thicknesses is always 1

The gradient of the weight function is expressed as

$$\nabla W = \frac{\partial W}{\partial t_1} i + \frac{\partial W}{\partial t_2} j + \ldots + \frac{\partial W}{\partial t_I} n \tag{5.13}$$

The magnitude of the gradient of W is the square root of the total number of layers in the laminate.

The Linking constraint is included if a balanced laminate is desired. The number of linking constraints depends on the number of layers and is equal to $(I-2)/2$. This constraint requires that if any two of the layers have orientation θ_i and θ_j and if $\theta_i = \theta_j$ then the design modification vectors S_i and S_j must be equal i.e. $S_i = S_j$. This will keep the thickness of the layers same. $(t_i = t_j)$

The stiffness constraints, the non-negativity constraints and the requirement that the next design be lighter than the current design are appended at each stage of the simplex formulation.

The constraint equations and the objective function are written on into the file 'pr.gms' which is the input file for the GAMS package. The linear programming problem is solved by GAMS for the design modification vector S_p. The design is then updated as $t_{p+1} = t_p + S_p$

5.3.4 GAMS

GAMS [20] is an acronym for General Algebraic Modelling System. This has been used to construct and solve the optimization problem for the design modification vector. GAMS has been developed based on ideas drawn from the relational database theory and mathematical programming. Relational databases provide a structured frame work for developing general data organization and transformation capabilities. Mathematical programming provides a way of describing a problem and a variety of methods for solving it.

A shell tool is opened for GAMS to execute. The output from the GAMS which consists of the components of the design modification vector S is directed to a file named 'update'. The optimization module reads the modification vector from the file 'update' and appends it to the current design vector to get the new design vector. This cycle is repeated until there is no further improvement in the design vector.

An output file 'optdes.out' is got at the end of the program execution.

Chapter 6

Conclusions

6.1 Results

A practical and efficient method for the stress analysis and minimum weight optimum design of symmetric laminates taking into account the stiffness and strength limitations has been presented. Attention was focused on developing a laminate design capability in which the thickness of the material at specified orientation angles θ_i are the only design variables. The laminate optimization task was formulated as a non linear mathematical programming problem. This is transformed into a sequence of linear problems These linear problems have been solved using a standard simplex algorithm. The constraint deletion technique adopted enhances the efficiency of the method significantly by temporarily ignoring constraints that are not even near critical

The laminate was loaded with a shear load N_{xy} of 3000 lbs and the optimum thickness was found to be 0 07649 inch. The same laminate was loaded with a normal load N_x of 3000 lbs and the optimum thickness was found to be 0 0905 inch When the laminate was loaded with both shear and normal loads together the optimum thickness was found to be 0.127 inch This value is almost equal to the sum of the optimum thickness values, got when the laminate was loaded seperately with normal and shear loads.

It is seen from the graph of shear load v/s optimum thickness that the optimum

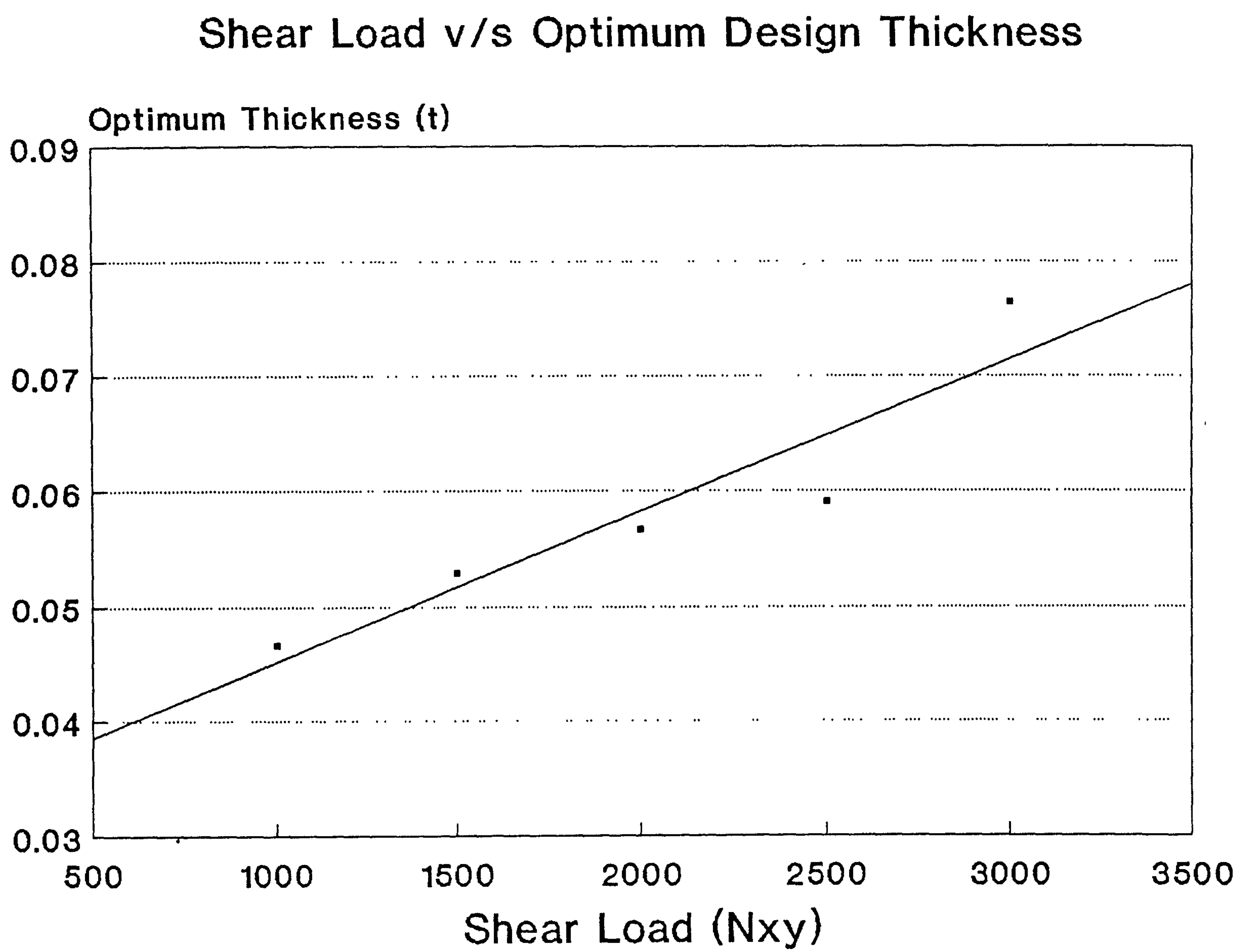

Shear Load v/s Optimum Design Thickness
Optimum Thickness (t)
0.09
0.08
0.07
0.06
0.05
0.04
0.03
500
1000
1500
2000
2500
3000
3500
Shear Load (Nxy)

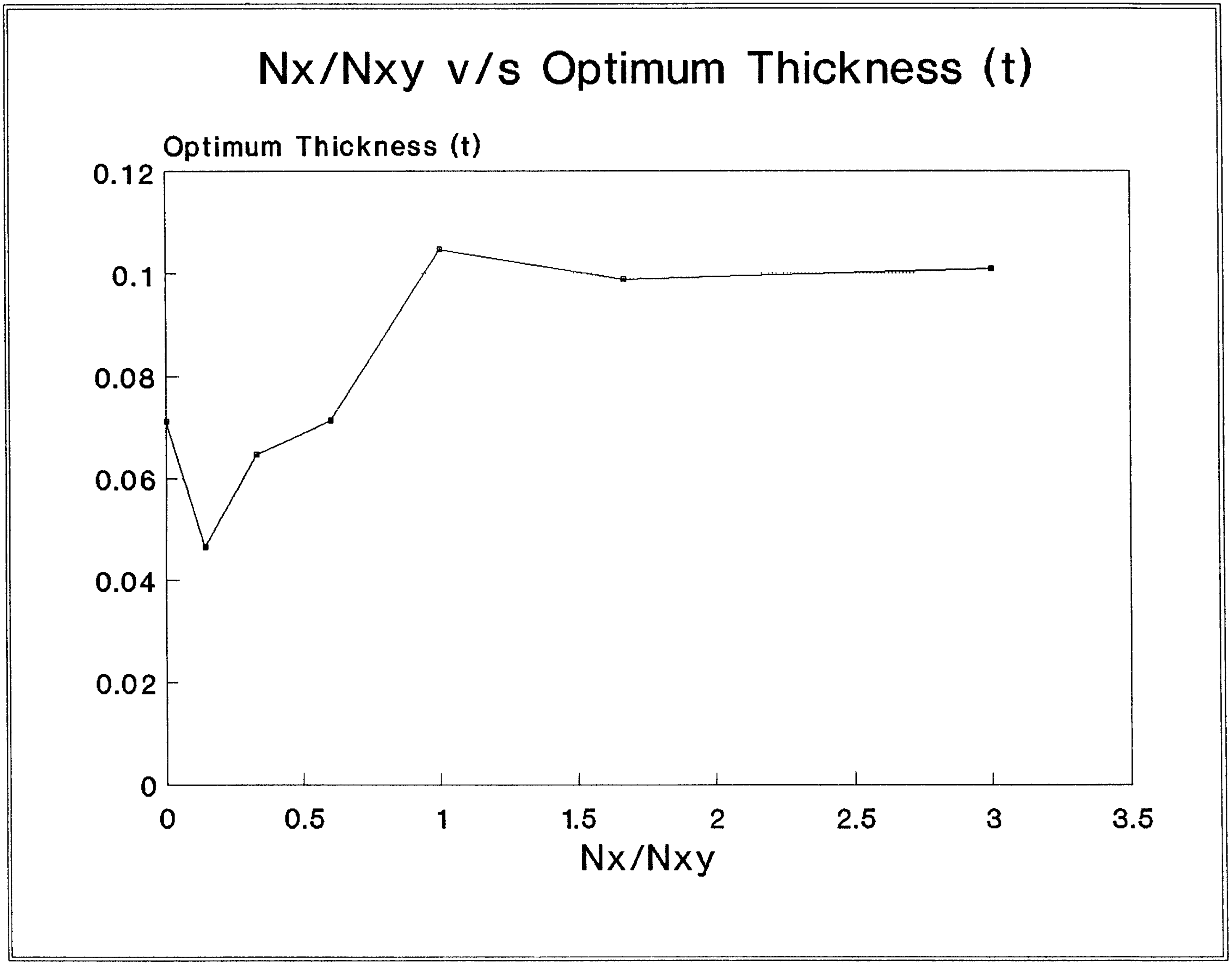
Fig. 6.1

thickness of the laminae increases with increase in load

It is observed from the graph of N_x/N_{xy} v/s optimum design thickness that the maximum thickness is got when $N_x = N_{xy}$.

6.2 Further Scope

In the method presented herein, the GAMS package has been used to formulate and solve the simplex problem. A computer code developed specifically to formulate and solve the simplex problem in the laminate optimization problem would probably give better results. In the work presented the minimum weight optimum design of laminates is achieved subject to strength and stiffness constraints It would be logical to extend this work to take care of buckling loads This would involve elastic stability constraints and would give rise to an eigenvalue problem


Nx/Nxy v/s Optimum Thickness (t)
Optimum Thickness (t)
0.12
0.1
0.08
0.06
0.04
0.02
0
0
0.5
1
1.5
2
2.5
3
3.5
Nx/Nxy

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Appendix I

${A_{11}}^l, {A_{22}}^l, {A_{66}}^l$ = Specified lower limits for the diagonal elements of the laminate membrane stiffness matrix

${A_j}', {B_j}', {C_j}'$ = Constants defining the j^{th} planar facet of the strength envelope for the material oriented at θ_i degrees.

A_{rs} = Membrane stiffnesses of the laminate with respect to the reference co-ordinate system (x,y). r,s = 1, 2, 6.

E_{Li} = Modulus of elasticity in the direction of fibers for material oriented at θ_i degrees

E_{Ti} = In-plane modulus of elasticity transverse to the direction of fibers for material oriented at θ_i degrees

G_{LTi} = In-plane shear modulus with respect to axes of orthotropy (1,2) for material oriented at θ_i degrees

I = Number of available orientation angles

J = Number of planar facets in the strength envelope

N_x, N_y, N_{xy} = Applied membrane force resultants in the laminate reference co-ordinate system

M_x, M_y, M_{xy} = Applied moments in the laminate reference co-ordinate system

W = weight per unit surface area of the laminate

i = Index identifying available orientation angle

j = Index identifying j_{th} planar facet of the strength envelope

t_i = Thickness of material oriented at angle θ_i

γ_{12i} = In plane shear strain for the material oriented at θ_i degrees

${\gamma_{LTi}}^+ = {\gamma_{LTi}}^-$ = In-plane shear limiting strain in the fiber composite material placed at an orientation of θ_i

γ_{xy} = Laminate shear strain with respect to the reference axes

ϵ_{1i} = Normal strain in the direction of fibers in the material oriented at θ_i degrees.

ϵ_{2i} = Normal in-plane strain transverse to the direction of fibers in the material

${\epsilon_{Li}}^c$ = Longitudinal compressive limiting strain in the fiber composite material at an orientation θ_i.

${\epsilon_{Li}}^t$ = Longitudinal tension limiting strain in the fiber composite material at an orientation θ_i.

${\epsilon_{Ti}}^c$ = Transverse in-plane compressive limiting strain in the fiber composite material at an orientation θ_i.

${\epsilon_{Ti}}^t$ = Transverse in-plane tension limiting strain in the fiber composite material at an orientation θ_i

ϵ_x = Laminate normal strain in the x direction.

ϵ_y = Laminate normal strain in the y direction

θ_i = Angular orientation of fibers with respect to the X reference axis

ν_{LTi} = Poisson's ratio relating contraction in the in-plane transverse direction due to extension in the longitudinal direction

ν_{TLi} = Poisson's ratio relating contraction in the longitudinal direction due to extension in the in-plane transverse direction.

ρ_i = Weight density of the material oriented at θ_i

σ_{1i} = Normal stress in the direction of fibers in the material oriented at θ_i degrees

σ_{2i} = Normal in-plane stress transverse to the direction of fibers in the material oriented at θ_i degrees.

τ_{12i} = In-plane shear stress with respect of orthotropy for material oriented at θ_i degrees.

The elements of the stiffness matrix $[q]_{LT}$, of the laminate are given by

$$[q_{LT}]_{11} = E_L/(1 - \nu_{LT}\nu_{TL}) \tag{1.1}$$

$$[q_{LT}]_{12} = \nu_{LT}E_{Li}/(1 - \nu_{LTi}\nu_{TLi}) \tag{1.2}$$

$$[q_{LT}]_{22} = E_{Ti}/(1 - \nu_{LTi}\nu_{TLi}) \tag{1.3}$$

$$[q_{LT}]_{66} = G_{LT} \tag{1.4}$$

The elements of the reduced stiffness matrix depends upon the orientation angles θ_i and the elastic constants of the material as follows

$$[qq_{11}]_i = [q_{LT}]_{11}l_i^4 + 2[q_{LT}]_{12}l_i^2m_i^2 + [q_{LT}]_{22}m_i^4 + 4[q_{LT}]_{66}l_i^2m_i^2 \tag{1.5}$$

$$[qq_{12}]_i = [q_{LT}]_{11}l_i^2m_i^2 + [q_{LT}]_{12}(l_i^4 + m_i^4) + [q_{LT}]_{22}l_i^2m_i^2 - 4[q_{LT}]_{66}l_i^2m_i^2 \tag{1.6}$$

$$[qq_{16}]_i = [q_{LT}]_{11}l_i^3m_i + [q_{LT}]_{13}(m_i^3l_i - l_i^3m_i) - [q_{LT}]_{22}m_i^3l_i + 2[q_{LT}]_{66}(m_i^3l_i - l_i^3m_i) \tag{1.7}$$

$$[qq_{22}]_i = [q_{LT}]_{11}m_i^4 + 2[q_{LT}]_{12}l_i^2m_i^2 + [q_{LT}]_{22}m_i^4 + 4[q_{LT}]_{66}l_i^2m_i^2 \tag{1.8}$$

$$[qq_{26}]_i = [q_{LT}]_{11}m_i^3l_i + [q_{LT}]_{12}(l_i^3m_i - m_i^3l_i) - [q_{LT}]_{22}m_il_i^3 + 2[q_{LT}]_{66}(l_i^3m_i - m_i^3l_i) \tag{1.9}$$

$$[qq_{66}]_i = [q_{LT}]_{11} {l_i}^2 {m_i}^2 - 2[q_{LT}]_{12} {l_i}^2 {m_i}^2 + [q_{LT}]_{22} {l_i}^2 {m_i}^2 + [q_{LT}]_{66} ({l_i}^2 - {m_i}^2)^2 \quad (1.10)$$

where $l_i = cos\theta_i$ and $m_i = sin\theta_i$

The transformation matrix used to transform the stress and strain components form the principal material directions to any other reference co-ordinate system is as follows

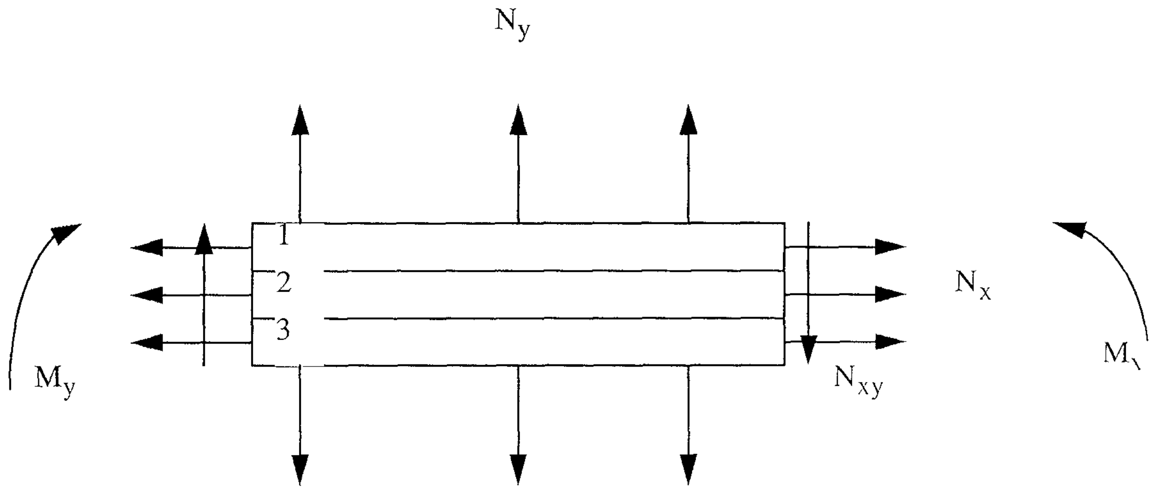
$$\begin{bmatrix} cos^2\theta_i & sin^2\theta_i & sin\theta_i cos\theta_i \\ sin^2\theta_i & cos^2\theta_i & -sin\theta_i cos\theta_i \\ -2sin\theta_i cos\theta_i & 2sin\theta_i cos\theta_i & (cos^2\theta_i - sin^2\theta_i) \end{bmatrix} \quad (1.11)$$

Appendix II

Example problems to test Stress analysis module and Optimization module

1. Stress analysis example.

2. Design Optimization example I



NUMBER OF LAYERS = 3

Layer	Orientation	Thickness
1	0	0.1
2	30	0.2
3	60	0.1

Fig A2.1 EXAMPLE I

This is the input file for the Stress analysis example problem.

```
1.200000e+04
1.000000e+03
7.000000e+02
2.500000e-02
3.000000e-01
3
1.000000e-01
0.000000e+00
2.000000e-01
3.000000e+01
1.000000e-01
6.000000e+01
1.000000e+01
1.000000e+01
1.000000e+01
1.000000e+01
1.000000e+01
1.000000e+01
1
1
1.000000e+02
8.000000e+00
6.000000e+00
```

Output file for the Stress analysis example.

FIBER COMPOSITES - ANALYSIS

Longitudinal Young's Modulus of the material ...1.200000e+04

Transverse Young's Modulus of the material ...1.000000e+03

Rigidity Modulus of the material, GLT ...7.000000e+02

Major Poisson's ratio of the material ...2.500000e-02

Minor Poisson's ratio of the material ...3.000000e-01

Number of layers in laminate ...3

Thickness of the lamina # 1 ...0.100000 deg
Orientation of the lamina # 1...0.000000 deg

Thickness of the lamina # 2 ...0.200000 deg
Orientation of the lamina # 2...30.000000 deg

Thickness of the lamina # 3 ...0.100000 deg
Orientation of the lamina # 3...60.000000 deg

LAMINATE LOADING CONDITIONS

Memberane force in the X direction, NX...1.000000e+01

Memberane force in the Y direction, NY...1.000000e+01

Memberane force in shear, NY ...1.000000e+01

Resultant Moment , MX ...1.000000e+01

Resultant Moment, MY ...1.000000e+01

Resultant Moment, MXY ...1.000000e+01

Lamina Stiffness matrix in the material direction

1.209068e+04	3.022670e+02	0.000000e+00

3.022670e+02	1.007557e+03	0.000000e+00
0.000000e+00	0.000000e+00	7.000000e+02

Stiffness matrix of the lamina # 1 oriented at 0.000000

1.209068e+04	3.022670e+02	0.000000e+00
3.022670e+02	1.007557e+03	0.000000e+00
0.000000e+00	0.000000e+00	7.000000e+02

Stiffness matrix of the lamina # 2 oriented at 30.000000

7.502330e+03	2.119836e+03	3.448940e+03
2.119836e+03	1.960768e+03	1.350193e+03
3.448940e+03	1.350193e+03	2.517569e+03

Stiffness matrix of the lamina # 3 oriented at 60.000000

1.960768e+03	2.119836e+03	1.350192e+03
2.119836e+03	7.502331e+03	3.448940e+03
1.350192e+03	3.448940e+03	2.517569e+03

Extensional Stiffness matrix [A] for the laminate

2.905611e+03	6.661776e+02	8.248073e+02
6.661776e+02	1.243142e+03	6.149326e+02
8.248073e+02	6.149326e+02	8.252708e+02

Coupling Stiffness matrix [B] for the laminate

-1.519487e+02	2.726354e+01	2.025289e+01
2.726354e+01	9.742161e+01	5.173410e+01
2.025289e+01	5.173410e+01	2.726354e+01

Bending Stiffness matrix [D] for the laminate

3.778827e+01	7.064798e+00	5.449743e+00
7.064798e+00	2.116358e+01	8.947657e+00
5.449743e+00	8.947657e+00	9.186041e+00

GLOBAL STIffNESS matrix for the laminate

2.905611e+03 6.661776e+02 8.248073e+02 -1.519487e+02
2.726354e+01 2.025289e+01

6.661776e+02 1.243142e+03 6.149326e+02 2.726354e+01
9.742161e+01 5.173410e+01

8.248073e+02 6.149326e+02 8.252708e+02 2.025289e+01
5.173410e+01 2.726354e+01

-1.519487e+02 2.726354e+01 2.025289e+01 3.778827e+01
7.064798e+00 5.449743e+00

2.726354e+01 9.742161e+01 5.173410e+01 7.064798e+00
2.116358e+01 8.947657e+00

2.025289e+01 5.173410e+01 2.726354e+01 5.449743e+00
8.947657e+00 9.186041e+00

GLOBAL FLEXIBILITY {Compliance} matrix for the laminate

8.086837e-04 -1.090328e-04 -8.193665e-04 3.870558e-03
1.033482e-03 -2.039987e-03

-1.090329e-04 1.746227e-03 -8.045037e-04 1.305752e-07 -
4.904179e-03 -2.429494e-03

-8.193664e-04 -8.045040e-04 2.755811e-03 -4.219615e-03 -
1.443072e-03 2.067221e-03

3.870557e-03 1.310868e-07 -4.219617e-03 4.783145e-02 -
5.582175e-04 -2.384372e-02

1.033482e-03 -4.904178e-03 -1.443075e-03 -5.582169e-04

1.012337e-01 -6.865163e-02
-2.039988e-03 -2.429496e-03 2.067224e-03 -2.384373e-02 -
6.865162e-02 2.019213e-01

Mid plane strains for the laminate

2.744338e-02
-6.500852e-02
-2.463525e-02

Mid plane curvatures for the laminate

2.308058e-01
2.671008e-01
1.070237e+00

Mid plane Strains in the individual lamina of the laminate (X-Y)

Lamina #	Strain (X)	Strain (Y)	Shear Strain (XY)
1	-7.177497e-03	-1.050736e-01	-1.851708e-01
2	2.744338e-02	-6.500852e-02	-2.463525e-02
3	6.206425e-02	-2.494340e-02	1.359003e-01

Stesses in the individual lamina of the laminate (X-Y)

Lamina #	Stress (X)	Stress (Y)	ShearStress (XY)
1	-1.185411e+02	-1.080372e+02	-1.296196e+02
2	-1.688365e+01	-1.025535e+02	-5.514438e+01

3	2.523092e+02	4.131444e+02	3.399087e+02

Strains in the individual lamina of the laminate in the material direction (L-T)

Lamina #	Strain (L)	Strain (T)	Shear Strain (LT)
1	-7.177497e-03	-1.050736e-01	-1.851708e-01
2	-6.336972e-03	-3.122817e-02	-9.238331e-02
3	5.565506e-02	-1.853421e-02	-1.433010e-01

Stesses in the individual lamina of the laminate in the material direction (l-T)

Lamina #	Stress (L)	Stress (T)	ShearStress (LT)
1	-1.185411e+02	-1.080372e+02	-1.296196e+02
2	-6.217933e+01	-5.725782e+01	-1.017645e+02
3	5.201204e+02	1.453332e+02	-3.066701e+01

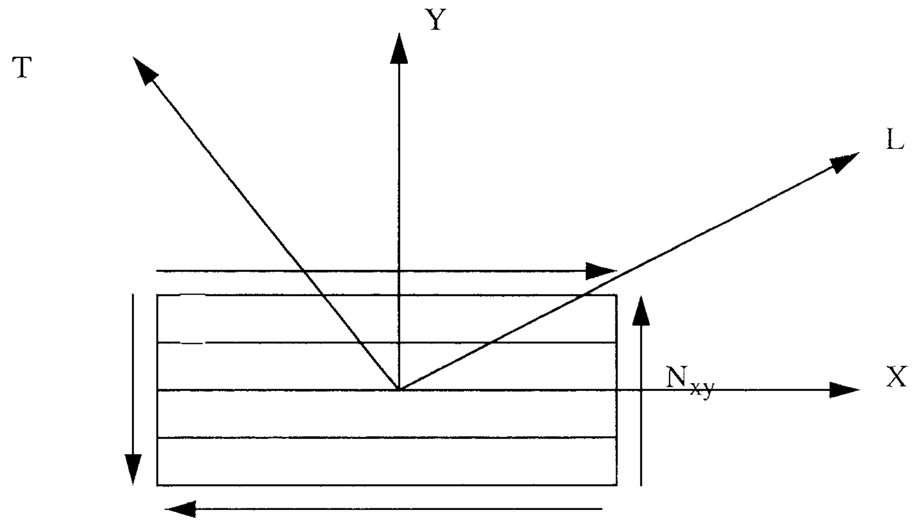
LAMINA FAILURE DATA

Lamina failure criterion based on Max.stress theory

Lamina # 1 is safe

Lamina # 2 is safe

Lamina # 3 fails



NUMBER OF LAYERS = 4

Layer	Orientation
1	0
2	+45
3	-45
4	+90

Fig A2.2 EXAMPLE II

```
This is the input file for the optimization example problem I


2.000000e+07
1.300000e+06
6.500000e+05
0.000000e+00
3.040000e-01
4
5.000000e-03
0.000000e+00
5.000000e-03
4.500000e+01
5.000000e-03
-4.500000e+01
5.000000e-03
9.000000e+01
0.0
0.0
3000.00
0.000000e+00
0.000000e+00
0.000000e+00
3.000000 3.000000
1.500000 1.500000
5.000000e+05
5.000000e+05
0.000000e+00
8.250000e-03
-5.750000e-03
6.150000e-03
-2.306000e-02
2.460000e-02
```

Output file for the Optmization example I

FIBER COMPOSITES - ANALYSIS

Longitudinal Young's Modulus of the material ...2.000000e+07

Transverse Young's Modulus of the material ...1.300000e+06

Rigidity Modulus of the material, GLT ...6.500000e+05

Major Poisson's ratio of the material ...1.976000e-02

Minor Poisson's ratio of the material ...3.040000e-01

Number of layers in laminate ...4

Thickness of the lamina # 1 ...0.002790 deg
Orientation of the lamina # 1...0.000000 deg

Thickness of the lamina # 2 ...0.035460 deg
Orientation of the lamina # 2...45.000000 deg

Thickness of the lamina # 3 ...0.035460 deg
Orientation of the lamina # 3...-45.000000 deg

Thickness of the lamina # 4 ...0.002790 deg
Orientation of the lamina # 4...90.000000 deg

LAMINATE LOADING CONDITIONS

Memberane force in the X direction, NX...0.000000e+00

Memberane force in the Y direction, NY...0.000000e+00

Memberane force in shear, NY ...3.000000e+03

Resultant Moment , MX ...0.000000e+00

Resultant Moment, MY ...0.000000e+00

Resultant Moment, MXY ...0.000000e+00

Lamina Stiffness matrix in the material direction

2.012087e+07	3.975883e+05	0.000000e+00
3.975883e+05	1.307856e+06	0.000000e+00
0.000000e+00	0.000000e+00	6.500000e+05

Stiffness matrix of the lamina # 1 oriented at 0.000000

2.012087e+07 -	3.975883e+05	0.000000e+00
3.975883e+05 -	1.307856e+06	0.000000e+00
0.000000e+00	0.000000e+00	6.500000e+05

Stiffness matrix of the lamina # 2 oriented at 45.000000

6.205974e+06	4.905975e+06	4.703252e+06
4.905975e+06	6.205976e+06	4.703252e+06
4.703252e+06	4.703252e+06	5.158386e+06

Stiffness matrix of the lamina # 3 oriented at -45.000000

6.205974e+06	4.905975e+06	0.000000e+00
4.905975e+06	6.205976e+06	0.000000e+00
0.000000e+00	0.000000e+00	5.158386e+06

Stiffness matrix of the lamina # 4 oriented at 90.000000

1.307856e+06	3.975883e+05	0.000000e+00
3.975883e+05	2.012087e+07	0.000000e+00
0.000000e+00	0.000000e+00	6.500000e+05

Extensional Stiffness matrix [A] for the laminate

4.999138e+05	3.501503e+05	1.667773e+05
3.501503e+05	4.999139e+05	1.667773e+05
1.667773e+05	1.667773e+05	3.694598e+05

Coupling Stiffness matrix [B] for the laminate

-1.934455e+03	5.531311e-04	-2.956961e+03
5.531311e-04	1.934454e+03	-2.956961e+03
-2.956961e+03	-2.956961e+03	7.629395e-04

Bending Stiffness matrix [D] for the laminate

2.657196e+02	1.488460e+02	6.990255e+01
1.488460e+02	2.657195e+02	6.990255e+01
6.990255e+01	6.990255e+01	1.582630e+02

GLOBAL STIFFNESS matrix for the laminate

```
 4.999138e+05   3.501503e+05   1.667773e+05   -1.934455e+03
5.531311e-04   -2.956961e+03
 3.501503e+05   4.999139e+05   1.667773e+05   5.531311e-04
1.934454e+03   -2.956961e+03
 1.667773e+05   1.667773e+05   3.694598e+05   -2.956961e+03   -
2.956961e+03   7.629395e-04
 -1.934455e+03   5.531311e-04   -2.956961e+03   2.657196e+02
1.488460e+02   6.990255e+01
 5.531311e-04   1.934454e+03   -2.956961e+03   1.488460e+02
2.657195e+02   6.990255e+01
 -2.956961e+03   -2.956961e+03   7.629395e-04   6.990255e+01
```

6.990255e+01 1.582630e+02

GLOBAL FLEXIBILITY {Compliance} matrix for the laminate

4.261883e-06 -2.607819e-06 -6.273755e-07 2.114067e-05 -
6.236851e-06 2.432143e-05
-2.607818e-06 5.325068e-06 -2.004645e-06 -2.611979e-05 -
7.109487e-05 9.370701e-05
-6.273767e-07 -2.004644e-06 4.748155e-06 3.153117e-05
7.509557e-05 -9.627179e-05
2.114070e-05 -2.611981e-05 3.153118e-05 6.066349e-03 -
2.407492e-03 -1.709095e-03
-6.236912e-06 -7.109482e-05 7.509557e-05 -2.407492e-03
7.428734e-03 -3.662666e-03
2.432149e-05 9.370699e-05 -9.627183e-05 -1.709095e-03 -
3.662666e-03 1.089645e-02

Mid plane strains for the laminate

-1.882127e-03
-6.013935e-03
1.424446e-02

Mid plane curvatures for the laminate

9.459354e-02
2.252867e-01
-2.888155e-01

Mid plane Strains in the individual lamina of the laminate (X-Y)

Lamina #	Strain (X)	Strain (Y)	Shear Strain (XY)
1	-5.368371e-03	-1.431688e-02	2.488876e-02
2	-3.559270e-03	-1.000827e-02	1.936516e-02

3	-2.049831e-04	-2.019601e-03	9.123766e-03
4	1.604118e-03	2.289006e-03	3.600170e-03

Stesses in the individual lamina of the laminate (X-Y)

Lamina #	Stress (X)	Stress (Y)	ShearStress (XY)
1	-1.020851e+06	-4.087252e+05	4.389525e+05
2	1.524321e+06	1.313756e+06	1.885579e+06
3	-5.546646e+05	-6.318321e+05	2.406995e+06
4	-1.516460e+04	3.145640e+05	2.426589e+05

Strains in the individual lamina of the laminate in the material direction (L-T)

Lamina #	Strain (L)	Strain (T)	Shear Strain (LT)
1	-5.368371e-03	-1.431688e-02	2.488876e-02
2	2.898812e-03	-1.646635e-02	-6.448999e-03
3	-5.674175e-03	3.449590e-03	1.814618e-03
4	2.289006e-03	1.604118e-03	-3.600170e-03

Stesses in the individual lamina of the laminate in the material direction (1-T)

Lamina #	Stress (L)	Stress (T)	Stress (LT)
1	-1.061643e+07	-5.999264e+06	6.167964e+06
2	3.279530e+07	6.535408e+06	-3.303516e+06
3	-2.749171e+07	7.538908e+06	1.352264e+06
4	1.390595e+06	-4.342796e+05	-3.556252e+06

LAMINA FAILURE DATA

Lamina # 1 is safe

Lamina # 2 is safe

Lamina # 3 is safe

Lamina # 4 is safe

****

OPTIMIZATION DATA

****

Initial design thickness of all the plies...3.400000e-02

Initial total design thickness of the laminate...1.360000e-01

*************ITERATION # 1*************

```
Number of undeleted Strength constraints =  4

Number of undeleted Stiffness constraints =  2

Undeleted Strength Constraints qo[1][2] =  3.535524e-01
Undeleted Strength Constraints qo[2][3] =  9.516579e-01
Undeleted Strength Constraints qo[3][3] =  4.954439e-01
Undeleted Strength Constraints qo[5][1] =  4.862203e-01


Undeleted Stiffness constraints ao[1][1] = 1.150583e+06
Undeleted Stiffness constraints ao[2][2] = 1.150583e+06
Undeleted Stiffness constraints ao[3][3] = 3.949703e+05


DESIGN MODIFICATION VECTOR  S[1] = -1.506000e-02
DESIGN MODIFICATION VECTOR  S[2] = -3.400000e-02
DESIGN MODIFICATION VECTOR  S[3] = -3.400000e-02
DESIGN MODIFICATION VECTOR  S[4] = 5.691000e-02

    LAMINA THICKNESSES AFTER ITERATION # 1

        |------------|--------------|
        | LAMINA # 1 | 1.894000e-02 |
        |------------|--------------|
        | LAMINA # 2 | 0.000000e+00 |
        |------------|--------------|
        | LAMINA # 3 | 0.000000e+00 |
        |------------|--------------|
        | LAMINA # 4 | 9.091000e-02 |
        |------------|--------------|

TOTAL THICKNESS OF THE LAMINATE AFTER ITERATION #  1 = 1.098500e-01

*************ITERATION # 2*************

Number of undeleted Strength constraints =  4

Number of undeleted Stiffness constraints =  1

Undeleted Strength Constraints qo[1][2] =  2.546383e+00
Undeleted Strength Constraints qo[2][3] =  3.653507e+00
Undeleted Strength Constraints qo[3][3] =  3.415880e+00
Undeleted Strength Constraints qo[5][1] =  1.707940e+00
```

```
Undeleted Stiffness constraints ao[1][1] = 4.999865e+05
Undeleted Stiffness constraints ao[2][2] = 1.853959e+06
Undeleted Stiffness constraints ao[3][3] = 7.140250e+04


 DESIGN MODIFICATION VECTOR  S[1] = -1.713000e-02
 DESIGN MODIFICATION VECTOR  S[2] = 3.735000e-02
 DESIGN MODIFICATION VECTOR  S[3] = 3.735000e-02
 DESIGN MODIFICATION VECTOR  S[4] = -9.091000e-02


     LAMINA THICKNESSES AFTER ITERATION # 2
```

LAMINA # 1	1.810001e-03
LAMINA # 2	3.735000e-02
LAMINA # 3	3.735000e-02
LAMINA # 4	0.000000e+00

```
 TOTAL THICKNESS OF THE LAMINATE AFTER ITERATION #  2 = 7.651000e-02

*************ITERATION # 3*************

 Number of undeleted Strength constraints =  4

 Number of undeleted Stiffness constraints =  2

 Undeleted Strength Constraints qo[1][2] =  3.384311e-01
 Undeleted Strength Constraints qo[2][3] =  8.891850e-01
 Undeleted Strength Constraints qo[3][3] =  5.506796e-01
 Undeleted Strength Constraints qo[5][1] =  1.093985e+00


Undeleted Stiffness constraints ao[1][1] = 5.000051e+05
Undeleted Stiffness constraints ao[2][2] = 4.659536e+05
Undeleted Stiffness constraints ao[3][3] = 3.865080e+05


 DESIGN MODIFICATION VECTOR  S[1] = 9.800000e-04
 DESIGN MODIFICATION VECTOR  S[2] = -1.890000e-03
```

DESIGN MODIFICATION VECTOR S[3] = -1.890000e-03
DESIGN MODIFICATION VECTOR S[4] = 2.790000e-03

LAMINA THICKNESSES AFTER ITERATION # 3

LAMINA # 1	2.790001e-03
LAMINA # 2	3.546000e-02
LAMINA # 3	3.546000e-02
LAMINA # 4	2.790000e-03

TOTAL THICKNESS OF THE LAMINATE AFTER ITERATION # 3 = 7.649999e-02

*************ITERATION # 4*************

Number of undeleted Strength constraints = 4

Number of undeleted Stiffness constraints = 2

Undeleted Strength Constraints qo[1][2] = 3.513711e-01
Undeleted Strength Constraints qo[2][3] = 9.868130e-01
Undeleted Strength Constraints qo[3][3] = 5.609090e-01
Undeleted Strength Constraints qo[5][1] = 1.011738e+00

Undeleted Stiffness constraints ao[1][1] = 4.999138e+05
Undeleted Stiffness constraints ao[2][2] = 4.999139e+05
Undeleted Stiffness constraints ao[3][3] = 3.694598e+05

DESIGN MODIFICATION VECTOR S[1] = 0.000000e+00
DESIGN MODIFICATION VECTOR S[2] = 0.000000e+00
DESIGN MODIFICATION VECTOR S[3] = 0.000000e+00
DESIGN MODIFICATION VECTOR S[4] = 0.000000e+00

LAMINA THICKNESSES AFTER ITERATION # 4

LAMINA # 1	2.790001e-03

LAMINA # 2	3.546000e-02
LAMINA # 3	3.546000e-02
LAMINA # 4	2.790000e-03

TOTAL THICKNESS OF THE LAMINATE AFTER ITERATION # 4 = 7.649999e-02