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Graphical correlation of pressure drop and temperature drop to friction and velocity for adiabatic flow of compressible fluids

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GRAPHICAL CORRELATION OF PRESSURE DROP AND TEMPERATURE DROP TO FRICTION AND VELOCITY FOR ADIABATIC FLOW OF COMPRESSIBLE FLUIDS

BY

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TABLE OF CONTENTS

ABSTRACT

Problems involving adiabatic flow of compressible fluids in horizontal conduits of constant cross-section usually require tedious trial and error solutions. Although several somewhat simplified methods have been developed, most of these methods are not ideally suited for problems involving flow between two sections of a conduit.

The graphical solutions presented in this paper, relating pressure and temperature ratios to friction and initial Mach numbers, have been developed specifically for flow between sections of a conduit. Use of the graphs requires only the knowledge of initial conditions and determination of the friction factor.

Initial velocities from 30% to 70% of the initial acoustic velocity have been presented in this paper. Graphs for initial velocities of 5% to 30% of the initial acoustic velocity have previously been presented by Straub.(7) Combination of both papers provides a continous range of initial velocities from 5% to 70% of the initial acoustic velocity.

INTRODUCTION

Flow equations for compressible fluids in conduits of constant cross-section have been developed for both isothermal and adiabatic conditions. For isothermal conditions both algebraic and graphical methods of calculation have been developed and presented in many texts. Equations for adiabatic flow have been presented by Dodge and Thompson[3] and summarized by Perry.[6] These equations, containing specific heat ratios, pressure ratios, velocity ratios, Mach numbers, and friction, require the use of trial and error methods for most solutions. Because of the complexity

$$fL/R_H = -2.3\left[(K+1)/K\right]\left[\log\,(V_2/V_1)\right]$$
$$+(1/K)\left[(1/M_1)^2+(K-1)/2\right]\left[1-(V_1/V_2)^2\right]$$
$$P_2/P_1 = (V_1/V_2)\left(1+\left[(K-1)/2\right](M_1)^2\left[1-(V_2/V_1)^2\right]\right)$$
$$T_2/T_1 = (P_2/P_1)(V_2/V_1)$$

of these equations, several investigators have presented various types of graphical solutions.

Dodge and Thompson, besides presenting the flow equations, illustrated graphically the complex relationships between velocity, friction, and pressure drop for gases with a specific heat ratio of $K = 1.32$. This graph, however, cannot be read with any degree of accuracy, and the presentation of only one specific heat ratio makes interpolation for other specific heat ratios impossible. A similar graph, presented by Binder[1] for a specific heat ratio of $K = 1.4$, involves the same accuracy and interpolation limitations.

A graphical solution employing conditions in a stagnant reservoir has been developed by Lapple.[5] Since velocity in a stagnant reservoir is negligible, theoretical isothermal flow through a frictionless nozzle was employed as a reference flow rate. For flow from a stagnant reservoir into a pipeline, the Lapple chart provides a rapid method of calculation. For flow between two sections of a conduit, however, a lengthly trial and error calculation is required to determine conditions in the theoretical reservoir before a solution can be obtained.

A nomograph developed by Thompson,[8] relating velocity and friction to pressure drop for isothermal flow can be adapted for adiabatic flow.[5] Although the use of the nomograph for isothermal flow requires very little preliminary calculation, several rather tedious calculations are necessary for adiabatic flow.

A method of calculation, based on the thermodynamic properties of the fluid, employing the "Fanno lines" is also available. This method, although quite accurate, requires the use of a special set of rather lengthly tables.

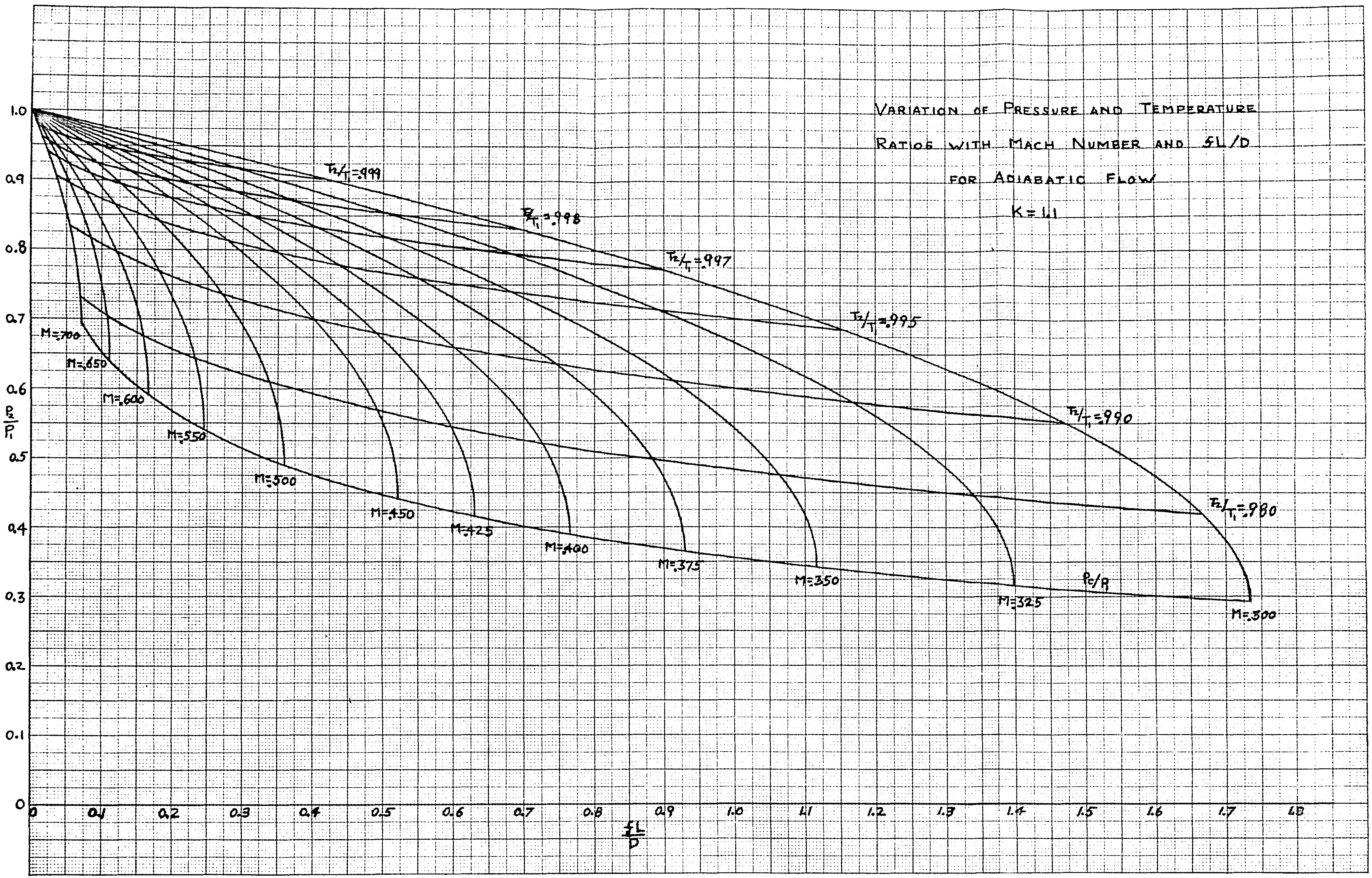
Because of the difficulties involved when using the above methods of calculation, Straub(7) presented an accurate graphical solution similar to those illustrated in Dodge and Thompson, and Binder. Additional parametric curves relating temperature ratios to friction and initial Mach numbers were included to increase the utility of the graphs. Mach numbers from 0.050 to 0.300 were employed, to cover as wide a range of conditions as possible without sacrificing accuracy.

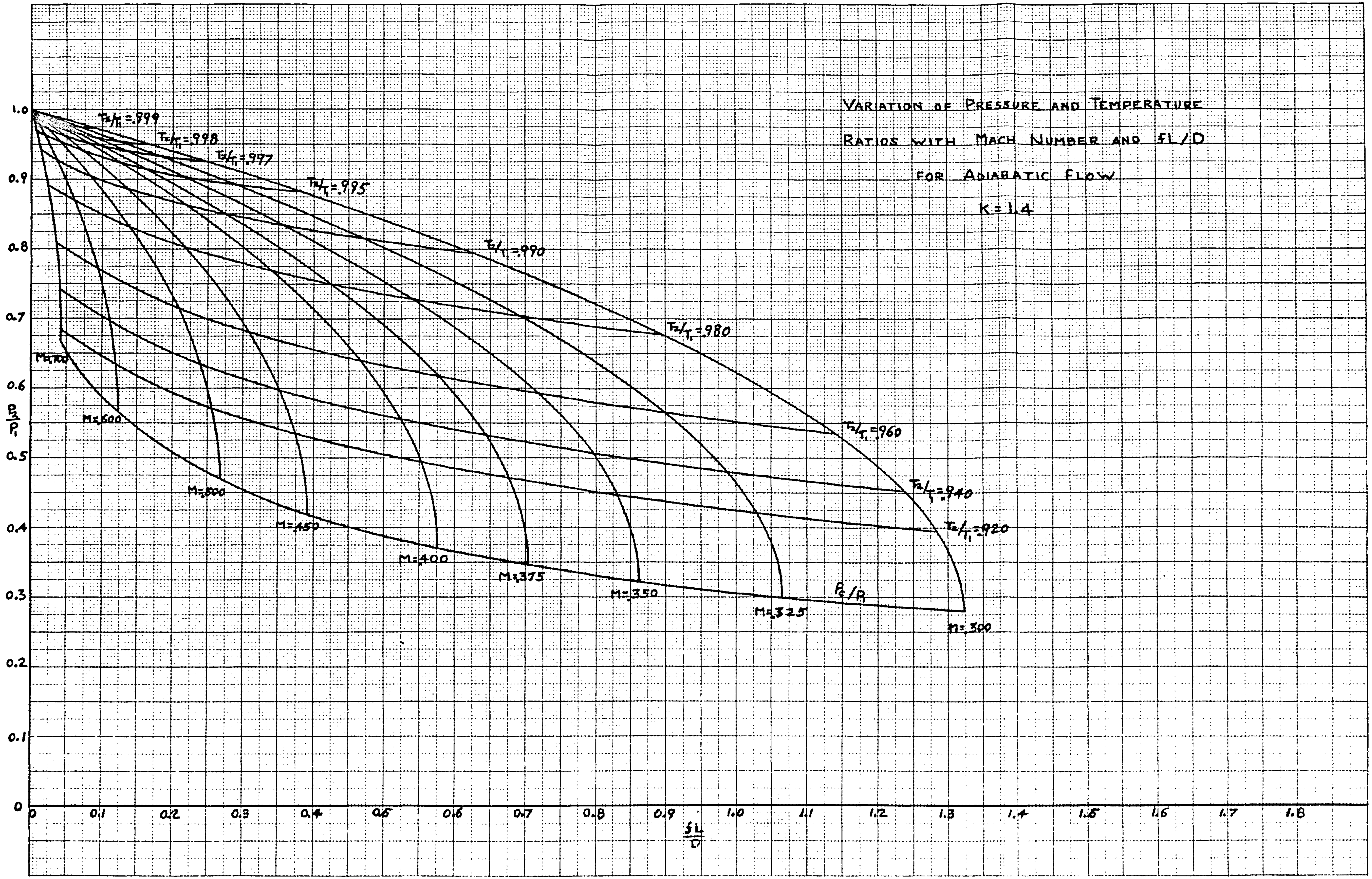
Since many flow problems involve inlet velocities greater than M = 0.300, it was felt that construction of similar graphs with Mach numbers of 0.300 to 0.700 would provide a valuable contribution to the work of Straub. This paper has therefore been prepared as an extension of Straub's work. A combination of both papers provides a continous range of Mach numbers from 0.050 to 0.700.

RESULTS

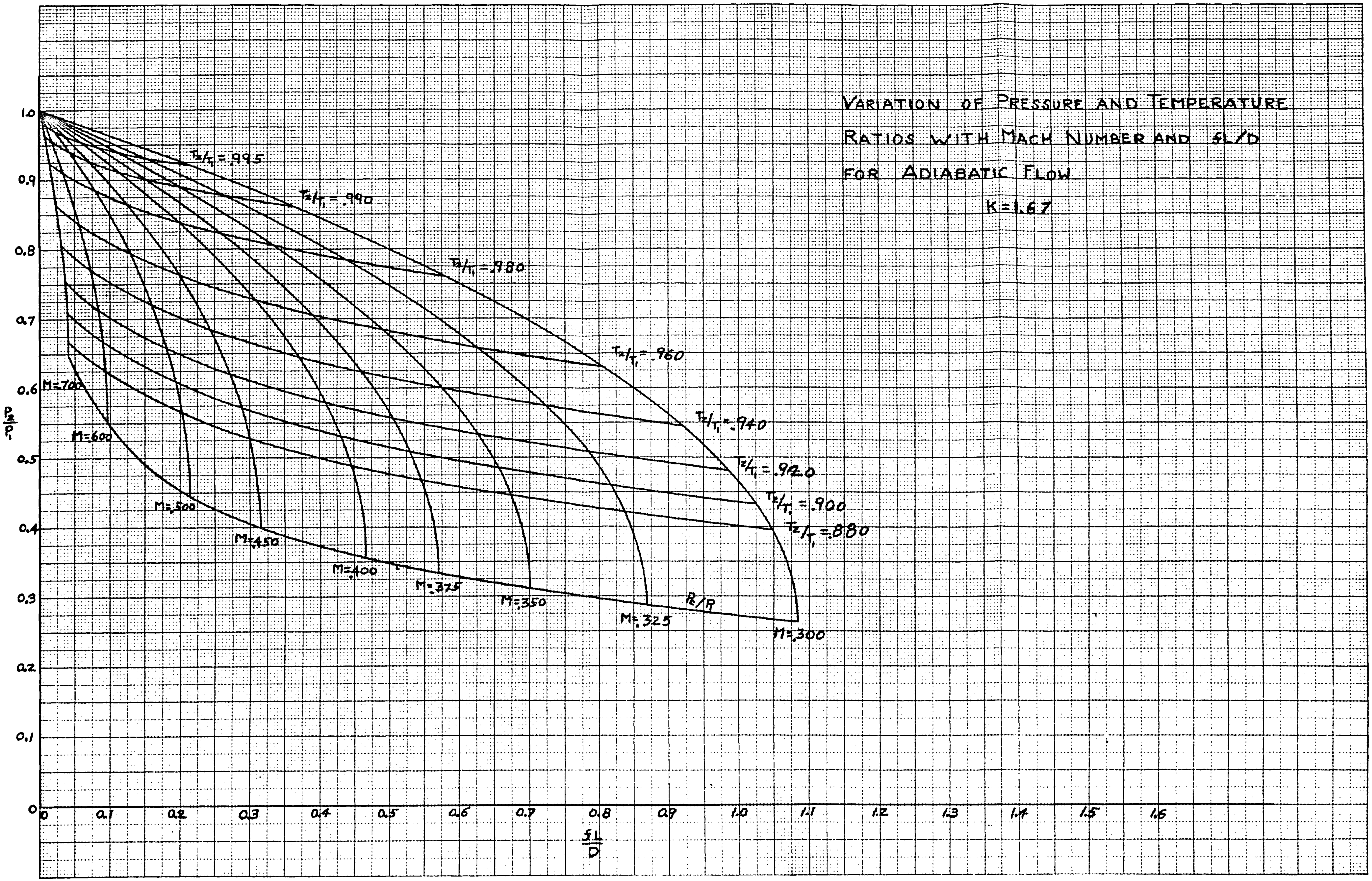
The graphs presented on the following pages represent the relationship of pressure drop and temperature drop to initial Mach number and friction for perfect gases flowing adiabatically through conduits of constant cross-section. With these graphs and the graphs presented by Straub[7] it is possible, knowing the inlet conditions and the friction factor, f, to determine the conditions of flow at any point along the conduit for initial Mach numbers up to 0.700.

The curve on each graph marked, P_c/P_1 represents the maximum possible pressure drop attainable in the conduit from the given inlet conditions. At this point the velocity at the conduit outlet equals the acoustic velocity, thereby producing maximum flow conditions.


VARIATION OF PRESSURE AND TEMPERATURE
RATIOS WITH MACH NUMBER AND fL/D
FOR ADIABATIC FLOW
K = 1.1
T2/T1 = .999
T2/T1 = .998
T2/T1 = .997
T2/T1 = .995
T2/T1 = .990
T2/T1 = .980
M = .700
M = .650
M = .600
M = .550
M = .500
M = .450
M = .425
M = .400
M = .375
M = .350
M = .325
M = .300
Pc/P1
P2/P1
fL/D
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8



VARIATION OF PRESSURE AND TEMPERATURE
RATIOS WITH MACH NUMBER AND fL/D
FOR ADIABATIC FLOW
K = 1.4
T2/T1 = .999
T2/T1 = .998
T2/T1 = .997
T2/T1 = .995
T2/T1 = .990
T2/T1 = .980
T2/T1 = .960
T2/T1 = .940
T2/T1 = .920
M=.700
M=.600
M=.500
M=.450
M=.400
M=.375
M=.350
M=.325
M=.300
Pc/P1
P2/P1
fL/D
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8



VARIATION OF PRESSURE AND TEMPERATURE RATIOS WITH MACH NUMBER AND fL/D FOR ADIABATIC FLOW
K=1.67
T2/T1 = .995
T2/T1 = .990
T2/T1 = .980
T2/T1 = .960
T2/T1 = .940
T2/T1 = .920
T2/T1 = .900
T2/T1 = .880
M=.700
M=.600
M=.500
M=.450
M=.400
M=.375
M=.350
M=.325
M=.300
P2/P1
fL/D
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6

CONCLUSIONS

The solution to many problems concerning adiabatic flow of compressible fluids in horizontal conduits of constant cross-section can be greatly simplified by the use of the graphical relationships presented in this thesis. These graphs, constructed for original Mach numbers of 0.300 to 0.700, represent an extension of the work of Straub.(7) By combining both sets of graphs, it is possible to solve flow problems with inlet velocities for air from a minimum of 55 ft./sec. to a maximum of 790 ft./sec. at 75°F.

Further work toward the construction of similar graphs for other specific heat ratios between 1.1 and 1.67 would simplify and increase the accuracy of interpolation between graphs.

Construction of graphs with initial Mach numbers above 0.700 would have only limited application. Maximum L/D ratios, for gases such as air, would be less than 10, requiring conduits of short lengths or large diameters.

DISCUSSION

The flow equations employed in this paper, summarized by Perry,[6] are as follows:

$$fL/R_H = -2.3 \left[(K+1)/K\right]\left[\log (V_2/V_1)\right] + (1/K) \left[(C_1/V_1)^2 + (K-1)/2\right]\left[1-(V_1/V_2)^2\right] \quad (1)$$

$$P_2/P_1 = (V_1/V_2)\left[1+ \left((K-1)/2\right) (M_1)^2 \left(1-(V_2/V_1)^2\right)\right] \quad (2)$$

$$T_2/T_1 = (P_2/P_1)(V_2/V_1) \quad (3)$$

Mach number $M_1 = V_1/C_1$ $\qquad$ $R_H = D/4$, for circular conduits.

By selecting constant values of specific heat ratios, K, and Mach numbers, M, and substituting selected values for V_2/V_1 in equations 1 and 2, points on the parametric Mach number curves were determined. The resulting values of P_2/P_1 multiplied by V_2/V_1 gave T_2/T_1. The selected V_2/V_1 values together with the determined values of fL/D, P_2/P_1 and T_2/T_1 for specific values of K and M, are included in the appendix as Tabulated Results.

Since the work of Straub[7] covered the range of Mach numbers from 0.050 to 0.300, this paper has been presented for Mach numbers beginning at 0.300. Maximum Mach numbers of 0.700 were considered sufficient to maintain maximum accuracy in the graphs. It was naturally necessary to employ the same specific heat ratios as those of Straub, K=1.1, 1.4 and 1.67.

The curves of T_2/T_1 not only increase the utility of the graphs, but also increase the range of application. For instance, without the T_2/T_1 curves it is necessary to know inlet pressure and velocities for direct use of the graphs. In many practical problems, flow rates represent the condition to be determined. In this case knowledge of the upstream and downstream temperatures and pressures give sufficient information to determine P_2/P_1 and T_2/T_1. These values are all that are required to determine the point on the graph necessary to give the initial velocity.

The limiting pressure ratio, represented by the curve P_c/P_1, indicates the point where the discharge velocity has reached the acoustic velocity. A further increase in pressure drop cannot be accomplished without first changing inlet conditions. Limiting pressure ratio curves were determined from the following formula appearing in Dodge and Thompson.[3]

$$P_c/P_1 = M_1^2 \sqrt{\left[(K-1)/(K+1)\right] \left[1 + \left(2/(K-1)M_1^2\right)\right]}$$

Three assumptions were used in the development of the flow equations 1, 2, and 3 and were therefore employed in the construction of the graphs presented in this problem. These assumptions are as follows:

1. The friction factor remains constant throughout the conduit.

2. Velocity distribution across the conduit cross-section is uniform.

3. All gases are perfect, requiring no correction for compressibility.

For certain design problems some correction will be necessary for these assumptions.

In problems involving large velocity changes it might be necessary to first solve the problem using the initial friction factor, f. Then from the downstream conditions, the final friction factor could be determined, and the solution repeated using the arithmetic average of the initial and final friction factors.

Velocity distribution across conduits is discussed in many texts. If necessary, corrections for actual frictional velocities are easily applied.

The compressibility correction for gases is negligible under normal pressure and temperature conditions, but under extreme conditions of pressure and temperature these corrections become significant.

CONSTRUCTION OF GRAPHS

The computation will be carried out to determine a point on the M_1 = .400 curve for the graph of K = 1.4.

Arbitrarily selecting a velocity ratio, $V_2/V_1 = 2.0$ and employing the Mach number and specific heat ratio, M and K, as noted above, the following equations can then be solved for fL/D.

$$fL/R_H = \left[-2.3(K+1)/K\right]\left[\log\ (V_2/V_1)\right]$$
$$+(1/K)\ \left[(1/M_1)^2+(K-1)/2\right]\left[1-(V_1/V_2)^2\right]$$
$$fL/D = fL/4R_H$$
$$fL/R_H = \left[-2.3(1.4+1)/1.4\right]\left[\log\ 2.0\right]$$
$$+(1/1.4)\ \left[(1/.40)^2+(1.4-1)/2\right]\left[1-(1/2.0)^2\right]$$
$$fL/R_H = 2.2672$$
$$fL/D = 0.5668$$

The pressure ratio is determined from the following equation.

$$P_2/P_1 = (V_1/V_2)\ \left[1+\ \left((K-1)/2\right)\ (M_1)^2\ \left(1-(V_2/V_1)^2\right)\right]$$
$$P_2/P_1 = (1/2.0)\ \left[1+\ \left((1.4-1)/2\right)\ (.400)^2\ \left(1-(2.0)^2\right)\right]$$
$$P_2/P_1 = 0.4520$$

The temperature ratio is then easily determined from the following expression.

$$T_2/T_1 = (P_2/P_1)(V_2/V_1)$$

$$T_2/T_1 = (.4520)(2.0)$$

$$T_2/T_1 = .90400$$

SAMPLE PROBLEM

To compare the several methods for solving a problem of this type, a sample problem is presented.

Data:

Air enters a standard 4 inch steel line at 3000 ft.^{3}/min. at a pressure of 14.0 psia and a temperature of 75°F. Determine the discharge conditions 20 feet from the inlet.

A. Method of Trial and Error.

ρ_1 = (.0808)(14.0/14.7)(492/535) = 0.0707 lb.ft.^{3}

T_1 = 460+75 = 535°R.

μ_1 = 0.0178 cp.(1/1488) = 1.196x10^{-5} lb. mass/ft. sec.

D = 4.026/12 = 0.335 ft.

V_1 = 3000/60(144/12.73) = 567 ft./sec.

Re = $DV\rho/\mu$ = (.335)(567)(.0707)/1.196x10^{-5} = 1.12x10^6

From the Reynold's number curve in the Chemical Engineer's Handbook.(6)

f = 0.0043

For air the specific heat ratio, K = 1.4

$C_1 = \sqrt{Kg_cRT_1/m} = \sqrt{1.4x32.2x1546x535/29}$ = 1133 ft./sec.

$M_1 = V_1/C_1$ = 567/1133 = .500

fL/D = (.0043)(20)/.335 = 0.2565

fL/R_H = 4fL/D = 4(.2565) = 1.026

From the Chemical Engineer's Handbook.[6]

$$fL/R_H = \left[-2.3(K+1)/K\right]\left[\log\,(V_2/V_1)\right]$$
$$+(1/K)\left[(C_1/V_1)^2+(K-1)/2\right]\left[1-(V_1/V_2)^2\right]$$

$$P_2/P_1 = (V_1/V_2)\left[1+\ ((K-1)/2)\ (M_1)^2\ \left(1-(V_2/V_1)^2\right)\right]$$

Assuming $V_2/V_1 = 1.50$

$$fL/R_H = \left[-2.3(1.4+1)/1.4\right]\left[\log 1.50\right]$$
$$+(1/1.4)\left[(1/.500)^2+(1.4-1)/2\right]\left[1-(1/1.50)^2\right]$$

$fL/R_H = 0.957$, which does not check the correct value of $fL/R_H = 1.026$.

Adjusting $V_2/V_1 = 1.60$ and repeating the above calculations gives $fL/R_H = 1.026$.

Therefore P_2/P_1

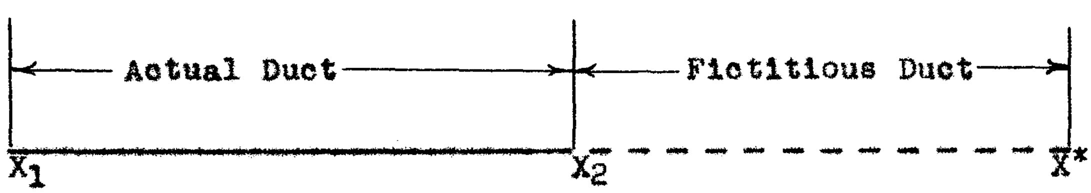
$$= (1/1.60)\left[1+\ ((1.4-1)/2)\ (.500)^2\ \left(1-(1.60)^2\right)\right] = .576$$

$$P_2 = P_1(P_2/P_1) = (14.0)(.576) = 8.06 \text{ psia.}$$

$$T_2 = T_1(P_2/P_1)(V_2/V_1) = (535)(.576)(1.60) = 493°R.$$

B. Method Using Fanno Tables.

When using the Fanno Tables[4] a length of fictitious duct must be added to the actual duct so that the exit velocity equals the acoustic velocity.



The friction factor used in the Fanno Tables is equal to 4f

therefore $f' = 4f = 4(.0043) = .0172.$

From the Fanno Tables with $K = 1.4$ and $M_1 = .500$

$f'L/D = f'(X^* - X_1)/D = 1.069$ $\quad T_1/T^* = 1.143$ $\quad P_1/P^* = 2.138$

From the original data $L = X_2 - X_1 = 20$

therefore $f'L/D = f'(X_2 - X_1)/D = 1.026$

$f'(X^* - X_2)/D = \left[f'(X^* - X_1)/D\right] - \left[f'(X_2 - X_1)/D\right]$

$= 1.069 - 1.026 = 0.043$

From the Fanno Tables with $K = 1.4$ and $f'L/D = 0.043$

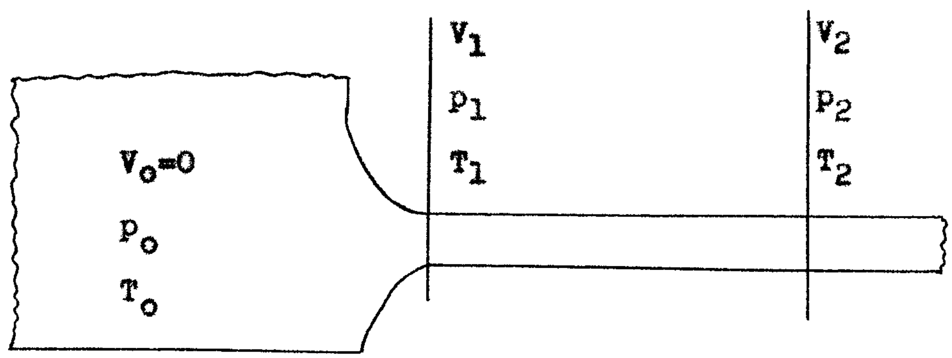
$T_2/T^* = 1.052$ $\qquad P_2/P^* = 1.224$

therefore $T_2 = T_1(T_2/T^*)(T^*/T_1) = 535(1.052)(1/1.143) = 492°R.$

and $P_2 = P_1(P_2/P^*)(P\ /P_1^*) = 14.0(1.224)(1/2.138) = 8.02$ psia.

C. Lapple Method.

When using the Lapple charts[2] it must be assumed that the air enters the pipe through a frictionless nozzle from a reservoir of stagnant air. Therefore conditions in the theoretical reservoir must be determined.



$G_1 = V_1 \rho_1 = (567)(.0707) = 40.1$ lb./ft.^{2} sec.

$p_1 = 14.0(144) = 2015$ lb./ft.^{2}

$N = fL/R_H = 1.026$

Assuming $p_1/p_o = 0.900$, from the Lapple chart of $K = 1.4$ with $N = 0$

$G/G_{cni} = 0.688$ and $T_1/T_o = 0.972$

then $T_o = T_1/.972 = 535/.972 = 550°R.$

$G_{cni} = G/.688 = 40.1/.688 = 58.2$ lb./ft.^{2} sec.

$$G_{cni} = p_o \sqrt{g_c m/eRT_o}$$

$$p_o = 58.2/\sqrt{(32.2)(29)/(2.718)(1546)(550)}$$

$p_o = 3890$ lb./ft.^{2}

$p_1/p_o = 2015/2890 = 0.698$, which does not check the assumed value of $p_1/p_o = .900$

Adjusting $p_1/p_o = .841$ gives $G/G_{cni} = 0.840$ and $T_1/T_o = 0.952$ from the Lapple chart.

Repeating the previous calculations gives $T_o = 562°R$, $G_{cni} = 47.7$ and $p_o = 2395$

therefore $p_1/p_o = 2015/2395 = 0.841$. This checks the assumed value.

Then for $N = 1.026$, $K = 1.4$ and $G/G_{cni} = 0.840$ the Lapple chart gives $p_2/p_o = 0.495$ and $T_2/T_o = .872$

$$P_2 = P_1(p_2/p_o)(p_o/p_1) = 14.0(.495)(1/.841) = 8.23 \text{ psia}$$

$$T_2 = T_1(T_2/T_o)(T_o/T_1) = 535(.872)(1/.952) = 490°R.$$

D. Method Using Thompson's Nomograph.(8)(5)

The use of the nomograph first requires the solution for Y and Z in the following equations:

$$Y = (K+1)M_1^{\,2}/\left[2+(K-1)M_1^{\,2}\right]$$

$$Y = (1.4+1)(0.5)^2/\left[2+(1.4-1)(0.5)^2\right]$$

$$Y = 0.286$$

$$1-Z = NM_1^{\,2}K/\left[2+(K-1)M_1^{\,2}\right]$$

$$N = f'L/D = 1.026$$

$$1-Z = (1.026)(0.5)^2(1.4)/\left[2+(1.4-1)(0.5)^2\right]$$

$$1-Z = 0.171 \qquad\qquad Z = 0.829$$

Connecting Y and Z with a straight line on the nomograph gives two values of X, $X = .470$ and $X = .620$

The value of X closest to Z is the correct value to be used, therefore $X = .620$

The P_2/P_1 and T_2/T_1 ratios can then be calculated from the following equations:

$$P_2/P_1 = X\left(1+\left[(K-1)M_1^{\,2}/2\right]\left[1-(1/X^2)\right]\right)$$

$$= .620\left(1+\left[(1.4-1)(0.5)^2/2\right]\left[1-(1/.620^2)\right]\right)$$

$$P_2/P_1 = .570$$

$$T_2/T_1 = (P_2/P_1)/X = .570/.620 = .920$$

$$P_2 = P_1\ (P_2/P_1) = 14.0(.570) = 7.98 \text{ psia}$$

$$T_2 = T_1\ (T_2/T_1) = 535(.920) = 492°R.$$

E. Method Using the Graphs Developed in This Thesis.

Using the K = 1.4 chart, with M = .500 and fL/D = .2565

$P_2/P_1 = .573$ and $T_2/T_1 = .920$

$$P_2 = P_1(P_2/P_1) = 14.0(.573) = 8.02 \text{ psia}$$

$$T_2 = T_1(T_2/T_1) = 535(.920) = 492°R.$$

Summary of Results

Method	P_2	T_2
Trial and Error	8.06 psia	493°R
Fanno Tables	8.02	492
Lapple Charts	8.23	490
Thompson's Nomograph	7.98	492
Author's Graph	8.02	492

TABLE OF NOMENCLATURE

ρ = density of fluid, lbs. mass/ft.^{3}

T = absolute temperature, °R.

μ = viscosity of fluid, lbs. mass/ft. sec.

D = diameter of pipe, ft.

V = velocity of fluid, ft./sec.

P = pressure of fluid, lbs./in.^{2} abs.

Re = Reynolds number, no units.

f = friction factor, no units.

f' = 4f = friction factor, no units.

K = C_p/C_v = specific heat ratio for fluid, no units.

C = acoustic velocity in fluid, ft./sec.

g_c = conversion factor, (lbs. mass.)ft./(lbs. force) sec.^{2}

R = gas constant, (lbs. force)ft./(lb. mole)°R.

m = molecular weight of fluid, lbs. mass/lb. mole.

L = length, ft.

R_H = hydraulic radius, ft. = D/4 for circular ducts.

X = distance from zero reference point, ft. (Fanno method)

X^* = distance from outlet of fictitious duct to zero reference point, ft. (Fanno method)

G = mass velocity of fluid, lbs. mass/ft.^{2} sec.

G_{cni} = maximum mass velocity for isothermal, frictionless flow in a nozzle, lbs. mass/ft.^{2} sec. (Lapple method)

M = Mach number, ratio of velocity to acoustic velocity in fluid, no units.

p = pressure of fluid, lbs./ft.^{2} abs. (Lapple method)

sub 1 = conditions at pipe inlet.

sub 2 = conditions at pipe outlet.

sub 0 = conditions in stagnant reservoir. (Lapple method)

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APPENDIX

TABULATED RESULTS

K = 1.1 M = 0.300

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.2068	0.9519	0.99954
1.1	0.3949	0.9082	0.99905
1.2	0.6882	0.8317	0.99802
1.3	0.9105	0.7668	0.99689
1.4	1.0818	0.7112	0.99568
1.5	1.2158	0.6629	0.99437
1.6	1.3215	0.6206	0.99298
1.7	1.4057	0.5832	0.99149
1.8	1.4733	0.5500	0.98992
1.9	1.5276	0.5201	0.98825
2.0	1.5716	0.4933	0.98650
2.1	1.6072	0.4689	0.98465
2.2	1.6362	0.4467	0.98272
2.3	1.6596	0.4264	0.98069
2.4	1.6784	0.4077	0.97858
2.5	1.6934	0.3905	0.97637
2.6	1.7054	0.3746	0.97408
2.7	1.7145	0.3599	0.97169
2.8	1.7215	0.3462	0.96922

$K = 1.1$ $M = 0.300$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
2.9	1.7268	0.3333	0.96665
3.0	1.7304	0.3213	0.96400
3.1	1.7325	0.3103	0.96125
3.2	1.7336	0.2995	0.95842
3.3	1.7339	0.2895	0.95549
		$0.2934 = P_c/P_1$	

TABULATED RESULTS

$K = 1.1$ $M = 0.325$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.1729	0.9519	0.99946
1.1	0.3300	0.9081	0.99889
1.2	0.5740	0.8314	0.99768
1.3	0.7580	0.7664	0.99636
1.4	0.8989	0.7107	0.99493
1.5	1.0083	0.6623	0.99340
1.6	1.0939	0.6199	0.99176
1.7	1.1614	0.5824	0.99002
1.8	1.2150	0.5490	0.98817
1.9	1.2576	0.5191	0.98622
2.0	1.2915	0.4921	0.98416
2.1	1.3184	0.4676	0.98199
2.2	1.3399	0.4453	0.97972
2.3	1.3567	0.4249	0.97734
2.4	1.3697	0.4062	0.97486
2.5	1.3797	0.3889	0.97227
2.6	1.3871	0.3729	0.96958
2.7	1.3923	0.3581	0.96678
2.8	1.3957	0.3442	0.96388

$K = 1.1$ $M = 0.325$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
2.9	1.3977	0.3313	0.96083
3.0	1.3984	0.3193	0.95775
3.1	1.3980	0.3079	0.95453
		$0.3180 = P_c/P_1$	

TABULATED RESULTS

$K = 1.1$ $\qquad$ $M = 0.350$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.1440	0.9518	0.99939
1.1	0.2746	0.9079	0.99871
1.2	0.4765	0.8311	0.99730
1.3	0.6276	0.7660	0.99577
1.4	0.7425	0.7101	0.99412
1.5	0.8310	0.6616	0.99234
1.6	0.8993	0.6190	0.99044
1.7	0.9527	0.5814	0.98842
1.8	0.9943	0.5479	0.98628
1.9	1.0268	0.5179	0.98401
2.0	1.0521	0.4908	0.98162
2.1	1.0716	0.4662	0.97911
2.2	1.0866	0.4439	0.97648
2.3	1.0979	0.4234	0.97372
2.4	1.1060	0.4045	0.97085
2.5	1.1115	0.3871	0.96784
2.6	1.1151	0.3710	0.96472
2.7	1.1168	0.3561	0.96147
2.8	1.1172	0.3422	0.95811
		$0.3426 = P_c/P_1$	

TABULATED RESULTS

$K = 1.1$ $\qquad$ $M = 0.375$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.1243	0.9517	0.99928
1.1	0.2370	0.9077	0.99852
1.2	0.4103	0.8308	0.99691
1.3	0.5393	0.7655	0.99515
1.4	0.6366	0.7095	0.99325
1.5	0.7107	0.6608	0.99121
1.6	0.7675	0.6181	0.98903
1.7	0.8111	0.5804	0.98671
1.8	0.8447	0.5468	0.98425
1.9	0.8703	0.5167	0.98165
2.0	0.8898	0.4895	0.97891
2.1	0.9043	0.4648	0.97602
2.2	0.9149	0.4423	0.97300
2.3	0.9224	0.4217	0.96984
2.4	0.9271	0.4027	0.96653
2.5	0.9298	0.3852	0.96309
2.6	0.9307	0.3690	0.95950
2.7	0.9301	0.3540	0.95577
		$0.3673 = P_c/P_1$	

TABULATED RESULTS

K = 1.1 M = 0.400

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.1066	0.9515	0.99912
1.1	0.2031	0.9076	0.99836
1.2	0.3506	0.8304	0.99648
1.3	0.4594	0.7650	0.99448
1.4	0.5407	0.7088	0.99232
1.5	0.6020	0.6600	0.99000
1.6	0.6482	0.6172	0.98752
1.7	0.6831	0.5793	0.98488
1.8	0.7094	0.5456	0.98208
1.9	0.7288	0.5153	0.97912
2.0	0.7430	0.4880	0.97600
2.1	0.7530	0.4632	0.97272
2.2	0.7597	0.4406	0.96928
2.3	0.7637	0.4199	0.96568
2.4	0.7654	0.4008	0.96192
2.5	0.7654	0.3832	0.95800
		$0.3919 = P_c/P_1$	

TABULATED RESULTS

$K = 1.1$ $M = 0.425$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0919	0.9515	0.99907
1.1	0.1749	0.9074	0.99810
1.2	0.3010	0.8300	0.99603
1.3	0.3932	0.7644	0.99377
1.4	0.4613	0.7081	0.99133
1.5	0.5119	0.6591	0.98871
1.6	0.5494	0.6162	0.98591
1.7	0.5771	0.5782	0.98293
1.8	0.5973	0.5443	0.97977
1.9	0.6116	0.5139	0.97643
2.0	0.6214	0.4865	0.97291
2.1	0.6275	0.4615	0.96920
2.2	0.6310	0.4388	0.96532
2.3	0.6321	0.4179	0.96126
2.4	0.6314	0.3988	0.95701
		$0.4166 = P_c/P_1$	

TABULATED RESULTS

K = 1.1 M = 0.450

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0795	0.9514	0.99896
1.1	0.1513	0.9072	0.99787
1.2	0.2594	0.8296	0.99554
1.3	0.3377	0.7639	0.99301
1.4	0.3947	0.7073	0.99028
1.5	0.4364	0.6582	0.98734
1.6	0.4666	0.6151	0.98420
1.7	0.4882	0.5770	0.98086
1.8	0.5033	0.5430	0.97732
1.9	0.5133	0.5124	0.97357
2.0	0.5195	0.4848	0.96962
2.1	0.5225	0.4597	0.96547
2.2	0.5232	0.4369	0.96112
		$0.4414 = P_o/P_1$	

TABULATED RESULTS

$K = 1.1$ $M = 0.500$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0602	0.9512	0.99872
1.1	0.1143	0.9067	0.99738
1.2	0.1943	0.8288	0.99450
1.3	0.2506	0.7626	0.99137
1.4	0.2903	0.7057	0.98800
1.5	0.3179	0.6562	0.98437
1.6	0.3366	0.6128	0.98050
1.7	0.3488	0.5743	0.97637
1.8	0.3559	0.5400	0.97200
1.9	0.3592	0.5091	0.96737
2.0	0.3596	0.4813	0.96250
		$0.4910 = P_o/P_1$	

TABULATED RESULTS

$K = 1.1$ $\qquad$ $M = 0.550$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0459	0.9509	0.99845
1.1	0.0869	0.9062	0.99682
1.2	0.1461	0.8278	0.99334
1.3	0.1862	0.7612	0.98954
1.4	0.2130	0.7039	0.98548
1.5	0.2302	0.6541	0.98109
1.6	0.2405	0.6103	0.97640
1.7	0.2455	0.5714	0.97141
1.8	0.2468	0.5367	0.96612
		$0.5408 = P_c/P_1$	

TABULATED RESULTS

K = 1.1 M = 0.600

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0350	0.9506	0.99815
1.1	0.0661	0.9057	0.99622
1.2	0.1094	0.8267	0.99208
1.3	0.1372	0.7597	0.98758
1.4	0.1542	0.7019	0.98272
1.5	0.1636	0.6517	0.97750
1.6	0.1673	0.6075	0.97192
1.7	0.1671	0.5682	0.96598
		$0.5907 = P_c/P_1$	

TABULATED RESULTS

$K = 1.1$ $M = 0.650$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0265	0.9503	0.99783
1.1	0.0499	0.9051	0.99556
1.2	0.0809	0.8256	0.99070
1.3	0.0991	0.7580	0.98542
1.4	0.1085	0.6998	0.97972
1.5	0.1117	0.6491	0.97359
1.6	0.1104	0.6044	0.96704
		$0.6410 = P_o/P_1$	

TABULATED RESULTS

K = 1.1 M = 0.700

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0198	0.9500	0.99749
1.1	0.0370	0.9044	0.99485
1.2	0.0582	0.8244	0.98922
1.3	0.0688	0.7562	0.98309
1.4	0.0722	0.6975	0.97648
1.5	0.0705	0.6462	0.96937
		0.6914 = P_o/P_1	

TABULATED RESULTS

K = 1.4 M = 0.300

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.1623	0.9506	0.99815
1.1	0.3098	0.9057	0.99622
1.2	0.5391	0.8267	0.99208
1.3	0.7122	0.7597	0.98758
1.4	0.8451	0.7019	0.98272
1.5	0.9484	0.6517	0.97750
1.6	1.0294	0.6075	0.97192
1.7	1.0935	0.5682	0.96592
1.8	1.1446	0.5332	0.95968
1.9	1.1853	0.5016	0.95302
2.0	1.2178	0.4730	0.94600
2.1	1.2437	0.4470	0.93862
2.2	1.2646	0.4231	0.93088
2.3	1.2811	0.4012	0.92278
2.4	1.2940	0.3810	0.91432
2.5	1.3039	0.3622	0.90550
2.6	1.3116	0.3447	0.89632
2.7	1.3170	0.3284	0.88678
2.8	1.3208	0.3132	0.87688

$K = 1.4$ $M = 0.300$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
2.9	1.3234	0.2988	0.86662
3.0	1.3246	0.2853	0.85600
3.1	1.3235	0.2726	0.84502
		$0.2763 = P_o/P_1$	

TABULATED RESULTS

$K = 1.4$ $M = 0.325$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.1357	0.9503	0.99783
1.1	0.2589	0.9051	0.99556
1.2	0.4494	0.8256	0.99070
1.3	0.5924	0.7580	0.98542
1.4	0.7013	0.6998	0.97972
1.5	0.7854	0.6491	0.97359
1.6	0.8506	0.6044	0.96704
1.7	0.9016	0.5647	0.96007
1.8	0.9417	0.5293	0.95268
1.9	0.9730	0.4973	0.94486
2.0	0.9977	0.4683	0.93662
2.1	1.0168	0.4419	0.92796
2.2	1.0318	0.4177	0.91888
2.3	1.0431	0.3954	0.90934
2.4	1.0514	0.3748	0.89944
2.5	1.0574	0.3556	0.88909
2.6	1.0615	0.3378	0.87832
2.7	1.0638	0.3212	0.86712
2.8	1.0648	0.3055	0.85550
2.9	1.0647	0.2908	0.84346
		$0.2998 = P_c/P_1$	

TABULATED RESULTS

K = 1.4 M = 0.350

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.1145	0.9500	0.99749
1.1	0.2184	0.9044	0.99485
1.2	0.3783	0.8244	0.98922
1.3	0.4973	0.7562	0.98309
1.4	0.5873	0.6975	0.97648
1.5	0.6560	0.6462	0.96937
1.6	0.7087	0.6011	0.96178
1.7	0.7493	0.5610	0.95369
1.8	0.7806	0.5251	0.94512
1.9	0.8047	0.4927	0.93605
2.0	0.8230	0.4633	0.92650
2.1	0.8367	0.4364	0.91645
2.2	0.8470	0.4118	0.90592
2.3	0.8542	0.3891	0.89489
2.4	0.8590	0.3681	0.88338
2.5	0.8618	0.3485	0.87137
2.6	0.8630	0.3303	0.85888
2.7	0.8628	0.3133	0.84589
		0.3234 = P_o/P_1	

TABULATED RESULTS

$K = 1.4$ $M = 0.375$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0975	0.9496	0.99712
1.1	0.1858	0.9037	0.99409
1.2	0.3209	0.8230	0.98762
1.3	0.4206	0.7543	0.98059
1.4	0.4953	0.6950	0.97300
1.5	0.5516	0.6432	0.96484
1.6	0.5942	0.5976	0.95612
1.7	0.6265	0.5570	0.94684
1.8	0.6508	0.5206	0.93700
1.9	0.6689	0.4877	0.92659
2.0	0.6821	0.4578	0.91562
2.1	0.6915	0.4305	0.90409
2.2	0.6980	0.4055	0.89200
2.3	0.7019	0.3823	0.87934
2.4	0.7038	0.3609	0.86612
2.5	0.7040	0.3409	0.85234
		$0.3471 = P_o/P_1$	

TABULATED RESULTS

$K = 1.4$ $M = 0.400$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0836	0.9493	0.99672
1.1	0.1591	0.9030	0.99328
1.2	0.2739	0.8216	0.98592
1.3	0.3578	0.7522	0.97792
1.4	0.4200	0.6923	0.96928
1.5	0.4662	0.6400	0.96000
1.6	0.5005	0.5988	0.95008
1.7	0.5259	0.5527	0.93952
1.8	0.5445	0.5157	0.92832
1.9	0.5577	0.4824	0.91648
2.0	0.5668	0.4520	0.90400
2.1	0.5726	0.4242	0.89088
2.2	0.5759	0.3987	0.87712
2.3	0.5772	0.3751	0.86272
2.4	0.5767	0.3532	0.84768
		$0.3710 = P_c/P_1$	

TABULATED RESULTS

$K = 1.4$ $M = 0.450$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0623	0.9484	0.99585
1.1	0.1185	0.9014	0.99149
1.2	0.2023	0.8185	0.98218
1.3	0.2622	0.7477	0.97205
1.4	0.3052	0.6865	0.96112
1.5	0.3360	0.6329	0.94937
1.6	0.3577	0.5855	0.93682
1.7	0.3727	0.5432	0.92345
1.8	0.3825	0.5052	0.90928
1.9	0.3883	0.4707	0.89429
2.0	0.3911	0.4393	0.87850
2.1	0.3915	0.4104	0.86189
		$0.4190 = P_c/P_1$	

TABULATED RESULTS

$K = 1.4$ $M = 0.500$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0471	0.9475	0.99487
1.1	0.0894	0.8995	0.98950
1.2	0.1511	0.8150	0.97800
1.3	0.1938	0.7427	0.96550
1.4	0.2232	0.6800	0.95200
1.5	0.2429	0.6250	0.93750
1.6	0.2556	0.5763	0.92200
1.7	0.2631	0.5326	0.90550
1.8	0.2667	0.4933	0.88800
1.9	0.2672	0.4576	0.86950
		$0.4677 = P_c/P_1$	

TABULATED RESULTS

$K = 1.4$ $M = 0.600$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0273	0.9454	0.99262
1.1	0.0515	0.8953	0.98488
1.2	0.0844	0.8069	0.96832
1.3	0.1047	0.7310	0.95032
1.4	0.1163	0.6649	0.93088
1.5	0.1217	0.6067	0.91000
1.6	0.1227	0.5548	0.88768
		$0.5671 = P_o/P_1$	

TABULATED RESULTS

$K = 1.4$ $M = 0.700$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0131	0.9428	0.98995
1.1	0.0243	0.8904	0.97942
1.2	0.0365	0.7974	0.95688
1.3	0.0407	0.7172	0.93238
1.4	0.0395	0.6471	0.90592
		$0.6696 = P_c/P_1$	

TABULATED RESULTS

$K = 1.67$ $M = 0.300$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.1359	0.9494	0.99691
1.1	0.2594	0.9033	0.99367
1.2	0.4508	0.8223	0.98673
1.3	0.5948	0.7532	0.97920
1.4	0.7048	0.6936	0.97106
1.5	0.7900	0.6415	0.96231
1.6	0.8564	0.5956	0.95297
1.7	0.9085	0.5547	0.94302
1.8	0.9498	0.5180	0.93246
1.9	0.9823	0.4849	0.92131
2.0	1.0081	0.4548	0.90955
2.1	1.0283	0.4272	0.89719
2.2	1.0443	0.4019	0.88422
2.3	1.0567	0.3785	0.87066
2.4	1.0661	0.3569	0.85649
2.5	1.0731	0.3367	0.84171
2.6	1.0782	0.3178	0.82634
2.7	1.0814	0.3001	0.81036
2.8	1.0833	0.2835	0.79377

$K = 1.67$ $\qquad$ $M = 0.300$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
2.9	1.0842	0.2678	0.77659
3.0	1.0840	0.2529	0.75880
		$0.2635 = P_c/P_1$	

TABULATED RESULTS

$K = 1.67$ $M = 0.325$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.1136	0.9489	0.99637
1.1	0.2167	0.9023	0.99257
1.2	0.3756	0.8204	0.98443
1.3	0.4943	0.7505	0.97559
1.4	0.5842	0.6900	0.96603
1.5	0.6532	0.6372	0.95577
1.6	0.7064	0.5905	0.94480
1.7	0.7476	0.5489	0.93312
1.8	0.7796	0.5115	0.92074
1.9	0.8044	0.4777	0.90765
2.0	0.8235	0.4469	0.89385
2.1	0.8380	0.4187	0.87934
2.2	0.8491	0.3928	0.86413
2.3	0.8571	0.3688	0.84820
2.4	0.8627	0.3465	0.83157
2.5	0.8664	0.3257	0.81423
2.6	0.8685	0.3062	0.79619
2.7	0.8691	0.2879	0.77743
2.8	0.8686	0.2707	0.75797
		$0.2862 = P_o/P_1$	

TABULATED RESULTS

$K = 1.67$ $M = 0.350$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0959	0.9484	0.99579
1.1	0.1828	0.9013	0.99138
1.2	0.3159	0.8183	0.98194
1.3	0.4146	0.7474	0.97168
1.4	0.4886	0.6861	0.96060
1.5	0.5448	0.6325	0.94870
1.6	0.5874	0.5850	0.93598
1.7	0.6199	0.5426	0.92243
1.8	0.6447	0.5045	0.90807
1.9	0.6633	0.4699	0.89289
2.0	0.6771	0.4384	0.87688
2.1	0.6871	0.4095	0.86005
2.2	0.6942	0.3829	0.84241
2.3	0.6988	0.3582	0.82394
2.4	0.7014	0.3353	0.80465
2.5	0.7024	0.3138	0.78454
2.6	0.7021	0.2937	0.76362
		$0.3091 = P_c/P_1$	

TABULATED RESULTS

$K = 1.67$ $M = 0.375$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0816	0.9478	0.99517
1.1	0.1554	0.9001	0.99011
1.2	0.2678	0.8161	0.97927
1.3	0.3503	0.7442	0.96749
1.4	0.4115	0.6820	0.95477
1.5	0.4573	0.6274	0.94111
1.6	0.4914	0.5791	0.92651
1.7	0.5169	0.5359	0.91096
1.8	0.5358	0.4969	0.89448
1.9	0.5494	0.4616	0.87704
2.0	0.5590	0.4293	0.85867
2.1	0.5653	0.3997	0.83935
2.2	0.5693	0.3723	0.81910
2.3	0.5711	0.3469	0.79790
2.4	0.5712	0.3232	0.77576
		$0.3321 = P_c/P_1$	

TABULATED RESULTS

$K = 1.67$ $M = 0.400$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0699	0.9472	0.99451
1.1	0.1330	0.8989	0.98874
1.2	0.2284	0.8137	0.97642
1.3	0.2976	0.7408	0.96302
1.4	0.3483	0.6775	0.94854
1.5	0.3857	0.6220	0.93300
1.6	0.4129	0.5727	0.91638
1.7	0.4326	0.5286	0.89870
1.8	0.4466	0.4889	0.87994
1.9	0.4562	0.4527	0.86010
2.0	0.4623	0.4196	0.83920
2.1	0.4657	0.3892	0.81722
2.2	0.4670	0.3610	0.79418
2.3	0.4666	0.3348	0.77006
		$0.3553 = P_o/P_1$	

TABULATED RESULTS

$K = 1.67$ $M = 0.450$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0521	0.9458	0.99305
1.1	0.0990	0.8961	0.98575
1.2	0.1684	0.8085	0.97015
1.3	0.2175	0.7332	0.95319
1.4	0.2522	0.6678	0.93488
1.5	0.2765	0.6101	0.91520
1.6	0.2932	0.5589	0.89417
1.7	0.3042	0.5128	0.87179
1.8	0.3109	0.4711	0.84804
1.9	0.3142	0.4331	0.82294
2.0	0.3150	0.3982	0.79649
		$0.4025 = P_c/P_1$	

TABULATED RESULTS

$K = 1.67$ $M = 0.500$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0394	0.9442	0.99141
1.1	0.0746	0.8931	0.98241
1.2	0.1255	0.8026	0.96315
1.3	0.1601	0.7248	0.94221
1.4	0.1834	0.6569	0.91960
1.5	0.1985	0.5969	0.89531
1.6	0.2076	0.5433	0.86935
1.7	0.2124	0.4951	0.84171
1.8	0.2138	0.4513	0.81240
1.9	0.2127	0.4113	0.78141
		$0.4453 = P_o/P_1$	

TABULATED RESULTS

$K = 1.67$ $M = 0.600$

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0228	0.9406	0.98764
1.1	0.0428	0.8861	0.97467
1.2	0.0695	0.7891	0.94694
1.3	0.0854	0.7052	0.91679
1.4	0.0938	0.6316	0.88422
1.5	0.0969	0.5662	0.84925
1.6	0.0961	0.5074	0.81186
		$0.5497 = P_c/P_1$	

TABULATED RESULTS

K = 1.67 M = 0.700

V_2/V_1	fL/D	P_2/P_1	T_2/T_1
1.05	0.0128	0.9364	0.98317
1.1	0.0236	0.8778	0.96553
1.2	0.0358	0.7731	0.92777
1.3	0.0403	0.6821	0.88674
1.4	0.0397	0.6017	0.84242
		0.6537 = P_o/P_1	