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The analogy between heat and mass transfer in packed beds

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THE ANALOGY BETWEEN HEAT AND MASS TRANSFER IN PACKED BEDS

BY

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ABSTRACT

An operation which recurs in many branches of chemical
engineering is that in which a fluid - gas or liquid - is passed
through a bed of granular solids for the purpose of removing
from, or adding something to the fluid (or both). In the oil and
sugar industries, oils or syrups are passed through beds of
adsorbents to remove impurities causing color and other undesireable
effects. In the recovery of volatile solvents, air charged with
solvent vapor is passed over solid adsorbents. In leaching,
liquids are passed through beds of solids to remove some
constituent of the solids. In heat recovery in regenerators, air
or another gas is passed through checker work in order to transfer
heat first from one gas to the checker work and then to another
gas.

There is a generally accepted term for this broad class of
operations, the term "transfer" seems convenient and suitable.
In the last quarter century material for a theory of the unit
operation has been accumulating, and progress has been especially
rapid in the last fisteen years.

The object of this paper is not to make additions to the
theory, but to direct attention to the scattered literature and
to summarize the results so far obtained, without reproducing the
derivations and proofs. It does not cover experimental work
(although calculations and some data are illustrated). A review
at this time seems especially desireable since a considerable
amount of work has been repeated, later investigators being

unacquainted with what has been done earlier.

It is understood that transfer is here considered a chemical engineering operation. There is a large amount of literature on the physical chemistry of absorption, some of which furnishes the necessary background as applied to absorption. There is a great deal of literature of heat transmission and transfer also. Something more is desired than accumulations of data in empirical equations. This something more is chemical engineering theory on an analogy between heat and mass transfer.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

INTRODUCTION

Information has long been sought on the transfer properties
of fluids flowing through granular solids. It is especially
important to have such data on the transfer properties of the
flowing gas phase independent of the properties of the liquids
and solids which may be present and participating in the transfer.

Transfer data is of fundamental value in such operations as
the adsorption of gases by solids, the adsorption of gases in
granular beds, the transfer of heat and mass in packed columns,
the drying of gases by solid dessicants, and the drying of solids
by through circulation of air. Such data are also needed in
chemical processes, such as the reaction between a gas and a
granular solid or in chemical reactions of gases which are
catalyzed by granular surfaces. A calculation of the temperature
drop from a gas to a particle of granular catalysts has been sought
and is essential to the rational development of the theory of
heterogenous reactions. These concepts of transfer are important
in processes and process design and their correlation is vital
to the development of chemical engineering thoery.

FUNDAMENTAL UNITS OF TRANSFER IN FLUIDS.

For expressing the transfer rates of heat and mass in fluid films, three different concepts have been developed, namely, transfer coefficients designated as h for heat transfer and k_g for mass transfer; the j transfer factors, designated as j_h for heat transfer and j_d for mass transfer; and the height of the transfer unit, designated as $(HTU)_h$ for heat and $(HTU)_d$ for mass. The heat transfer coefficient, h, is defined by the equation

$$q = h \; a \; V \Delta t_m \tag{1}$$

where q = rate of heat transfer

a = effective area of heat transfer per unit volume of bed

V = volume

Δt_m = mean temperature difference from gas stream to
the interface

The mass transfer coefficient, k_g, is defined by the equation

$$w = k_g \; a \; V \Delta p_m \tag{2}$$

where w = rate of mass transfer

Δp_m = mean partial pressure difference of gas transferred,
measured from main gas stream to the interface.

The transfer coefficients h and k_g have the advantage of simplicity in expressing the rate equations, but have the disadvantage of not being dimensionless and requiring many units for definition; furthermore, these coefficients are not simply related to the properties of the gas stream.

The transfer properties as factors were developed by Colburn (2)

and are defined by the following equations: for heat transfer

$$j_h = \frac{h}{c_p G} \left[\frac{c_p \mu}{k} \right]_f^{2/3} \tag{3}$$

where c_p = heat capacity

   G = mass velocity

   μ = absolute viscosity of gas film

   k = thermal conductivity of the gas film

The subscript f refers to the properties of the gas film.
For mass transfer

$$j_d = \frac{k_g \, p_{gf} \, M_m}{G} \left[\frac{\mu}{\rho \, D_v} \right]_f^{2/3} \tag{4}$$

where p_{gf} = log mean partial pressure of the non-transferred

   gases in the gas film.

   M_m = mean molecular weight of gas stream

   ρ = density of gas in the film

   D_v = diffusivity of gas in the film.

The j factors were introduced to improve the correlation
of experimental data. In transfer experiments mass velocity is
often the most significant variable, and the correlation with
velocity is usually obtained by a plot against the modified
Reynolds number $D_p G/\mu$ where D_p is the average particle
size in a granular bed. Colburn (2) has shown that the correlation
of transfer coefficients against the Reynolds number is
equivalent to plotting a variable against itself, whereas the
misleading predicament is avoided by plotting the j factors
against the Reynolds number. The j factors have the additional
advantages of being dimensionless and expressible in terms of

two dimensionless groups for each j factor.

The height of the transfer unit, HTU, was developed by Chilton and Colburn (4) and defined as follows for gases:

For Heat Transfer:

$$a(HTU)_h = \frac{G\, c_p}{h} = \frac{a\, L}{\int_{t_1}^{t_2} \frac{dt}{\Delta t}} \tag{5}$$

where L = height of transfer zone

t_1 = entrance temperature of gas

t_2 = exit temperature of gas

dt = temperature change of gas in direction of gas flow

For Mass Transfer:

$$a(HTU)_d = \frac{G}{k_g\, p_{gf}\, M_m} = \frac{aL}{\int_{p_1}^{p_2} \frac{dp}{\Delta p}\, \frac{p_{gf}}{p_g}} \tag{6}$$

where p_1 = entrance partial pressure of gas transferred

p2 = exit partial pressure of gas transferred

p_g = partial pressure of inert gas in the main gas stream

dp = partial pressure change in direction of gas flow

The term a(HTU) is also dimensionless, whereas HTU is expressed simply as a unit of length.

The transfer of momentum in a fluid stream is expressed in terms of pressure drop in the direction of flow. This drop in pressure is expressed by the Fanning equation for turbulent flow, thus,

$$\Delta p = \frac{2\, f\, L\, G^2}{g_c\, D_p\, \rho} \tag{7}$$

where $\triangle p$ = pressure drop in the direction of flow

D_p = particle diameter

L = depth of bed

ρ = density of fluid

g_c = gravitational constant

The term f is the so-called friction factor for pressure drop and is related to the modified Reynolds number, $D_p G/\mu$, for the flow of gases through granular beds. The friction factor depends upon the ratio of particle size to vessel diameter and to the amount of liquid on the surface of the solid particles. Chilton and Colburn (5) correlated the existing data for the friction factor in granular beds against the modified Reynolds number. See Figure 1. Where this type of correlation is employed, the Fanning equation (7) may be used for any type of flow, laminar, turbulent or intermediate.

The nomenclature for all symbols used hereafter appear at the end of this paper.

THE CORRELATION OF GAMSON, THODOS AND HOUGEN

An investigation conducted by Gamson, Thodos and Hougen [20] embraced the simultaneous studies of the rates of transfer of energy, mass and momentum in gases flowing through beds of granular solids. The scheme of approach was developed from the exploratory studies on through-circulation drying by Marshall and Hougen [19] wherein it was found that a prolonged constant rate period existed, during which the gas phase only contributed resistance to the transfer of heat and mass. Accordingly, experiments in the vaporization of water from various catalyst carriers into a stream of air during the constant rate period of drying offered a means of solving this complex problem.

Gamson, Thodos and Hougen found the Prandtl number, $c_p \mu/k$, and the Schmidt number, $\mu/\rho D_v$, were nearly independent of temperature, pressure and humidity; $c_p \mu/k$ varied from 0.72 to 0.75 and $\mu/\rho D_v$ from 0.61 to 0.62 for air-water vapor mixtures.

For heat transfer the logarithmic mean of the terminal temperature differences, dry-bulb minus wet-bulb, is corrected for the constant-rate period. For mass transfer, the following equation for the mean partial pressure differences during the constant rate period was derived:

$$\Delta p_m = \frac{\dfrac{p_2}{P - p_2} - \dfrac{p_1}{P - p_1}}{\dfrac{p}{P - p_w}\left(\dfrac{1}{P - p_1} - \dfrac{1}{P - p_2}\right) + \dfrac{1}{P - p_w}\log_n \dfrac{(p_w - p_1)(P - p_2)}{(p_w - p_2)(P - p_1)}} \tag{8}$$

Where p_w, p_1 and p_2 are small compared to P, Equation (8)

simplifies to the logarithmic mean of the terminal differences;

Over the range of partial pressures encountered in the Gamson, Thodos

and Hougen experiment, the logarithmic mean was never in error

more than 2 per cent.

Interpretation and Correlation of Data

In the correlation of Gamson, Thodos and Hougen, experimental

data was obtained by plotting the heat transfer factor, j_h,

against the modified Reynolds number, $D_p G/\mu$, in Figures 2

and 3. Despite wide range in mass velocity, particle sizes,

density, humidity and temperature all runs are on the same line

with average deviation of only $\pm$ 3 1/2 per cent. Cylinder

shape was converted to same spherical surface area by:

$$D_p = \sqrt{D_c H_c + \frac{D_c^2}{2}} \qquad (9)$$

where D_c = actual diameter of cylinder

H_c = height

From an examination of Figure 2, it will be observed that

straight line plots appear for values of $D_p G/\mu$ above 350

and below 40. Values of $D_p G/\mu$ above 350 correspond to

conditions of turbulent flow and values below 40 to laminar flow;

the intermediate range from 40 to 350 corresponds to a region of

transition.

Points in the low velocity range are sparse, but reliable

experimental data in this region are difficult to obtain because

temperature differences approach zero and errors are magnified in

basing calculations upon the small differences of large numbers.

A similar correlation of data was obtained by Gamson, Thodos and Hougen by plotting values of the transfer factor, j_d, against the modified Reynolds number on a logarithmic plot. See Figures 2 and 3. Again it will be observed that the results of all runs appeared on a single line with an average deviation of only ± 4 per cent for spheres or cylinders despite the wide range in mass velocity, particle size and shape, density, humidity and temperature. Again the transition range from laminar to turbulent flow appeared over the same interval as for heat transfer, namely 40 to 350. The resultant equations for mass are similar to heat transfer. Further reliable equations and data for the laminar region were not obtained because values of $\triangle p$ used in the calculations approached zero in this region.

Figure 2 give us the following equations:

$$j_h = 1.064 \left[\frac{D_p G}{\mu}\right]^{-0.41} \quad \text{for} \quad \left[\frac{DpG}{\mu}\right] > 350 \tag{10}$$

$$j_h = 18.1 \left[\frac{D_p G}{\mu}\right]^{-1.0} \quad \text{for} \quad \left[\frac{D_p G}{\mu}\right] < 40 \tag{11}$$

$$j_d = 0.989 \left[\frac{D_p G}{\mu}\right]^{-0.41} \quad \text{for} \quad \left[\frac{D_p G}{\mu}\right] > 350 \tag{12}$$

$$j_d = 16.8 \left[\frac{D_p G}{\mu}\right]^{-1.0} \quad \text{for} \quad \left[\frac{D_p G}{\mu}\right] < 40 \tag{13}$$

It will be observed that for all conditions of flow, turbulent, laminar or transition, the ratio of j_h to j_d

remains constant, that is

$$\frac{j_h}{j_d} = 1.076 \qquad (14)$$

Thus, if either transfer factor is known, the other can be at once calculated.

It is of interest to note that the ratio of j_h/j_d can also be obtained directly from single experimental runs from the following derivations for processes where mass transfer and heat transfer accompany each other.

The rate of heat transfer, q, can be related to the rate of mass transfer in case of vaporization process by the relation that $q = \lambda w$ where λ is the molal heat of vaporization, or in case of a chemical reaction taking place at the surface, $q = w \Delta H$, where ΔH is the molal heat of reaction. By combining equation (1) with (3) and equation (2) with (4), there results

$$\frac{j_h}{j_d} = \frac{\Delta H \, \Delta p_m}{c_p \, M_m \, \Delta t_m \, p_{gf}} \left[\frac{c_p \, \mu}{k}\right]_f^{2/3} \left[\frac{\rho D_v}{\mu}\right]_f^{2/3} \qquad (15)$$

a(HTU) Values

In commercial design it is convenient to use HTU's or transfer coefficients, h or k_g, instead of transfer factors, j_h and j_d. These relationships can now be at once established. Combining equation (5) with equation (10) and also with (11) results in the following:

For turbulent flow, Re $>$ 350

$$a(HTU)_h = 0.939 \left[\frac{D_p G}{\mu}\right]^{0.41} \left[\frac{c_p \mu}{k}\right]_f^{2/3} \tag{16}$$

For laminar flow, Re $<$ 40

$$a(HTU)_h = 0.0552 \left[\frac{D_p G}{\mu}\right]^{1} \left[\frac{c_p \mu}{k}\right]_f^{2/3} \tag{17}$$

The equation for the height of transfer unit for mass transfer was obtained by combining equation (6) with (12) and also with (13). This results in the following:

For turbulent flow, Re $>$ 350

$$a(HTU)_d = 1.011 \left[\frac{D_p G}{\mu}\right]^{0.41} \left[\frac{\mu}{\rho D_v}\right]_f^{2/3} \tag{18}$$

For laminar flow, Re $<$ 40

$$a(HTU)_d = 0.0595 \left[\frac{D_p G}{\mu}\right]^{1} \left[\frac{\mu}{\rho D_v}\right]_f^{2/3} \tag{19}$$

These equations for j_d, j_h, $a(HTU)_h$ and $a(HTU)_d$ are shown graphically in Figure 3.

Heat and Mass Transfer Coefficients.

Equations for heat transfer coefficients may now be obtained by combining equations (3) with equation (10) for turbulent flow and equation (11) for laminar flow, thus,

For turbulent flow, Re $>$ 350

$$h = 1.064 \, c_p G \left[\frac{D_p G}{\mu}\right]^{-0.41} \left[\frac{c_p \mu}{k}\right]_f^{-2/3} \tag{20}$$

For laminar flow, Re $<$ 40

$$h = 18.1 \, c_p G \left[\frac{D_p G}{\mu}\right]^{-1} \left[\frac{c_p \mu}{k}\right]_f^{-2/3} \tag{21}$$

The equation for laminar flow may be simplified, taking values of viscosity the same in both moduli, to the following:

$$\left[\frac{hD_p}{k}\right] = 18.1 \left[\frac{c_p \mu}{k}\right]^{1/3} \tag{22}$$

For most gases $\left[\frac{c_p \mu}{k}\right]^{1/3} = 0.90$, hence in laminar flow

$$\left[\frac{h\,D_p}{k}\right] = 16.3 \tag{23}$$

It will be thus observed that in the laminar flow region the heat transfer coefficient is independent of mass velocity in agreement with theory.

For a single particle covered by a fluid of indefinite thickness in laminar flow the limiting value of $\left[\frac{h\,D_p}{k}\right] = 2.0$

Equations for mass transfer coefficients are obtained by combining equation (4) with equation (12) for turbulent flow and equation (13) for laminar flow.

For turbulent flow, Re $>$ 350

$$k_g = \frac{0.989\,G}{p_{gf}\,M_m} \left[\frac{D_p G}{\mu}\right]^{-0.41} \left[\frac{\mu}{\rho D_v}\right]_f^{-2/3} \tag{24}$$

For laminar flow, Re $<$ 40

$$k_g = \frac{16.8\,G}{p_{gf}\,M_m} \left[\frac{D_p G}{\mu}\right]^{-1} \left[\frac{\mu}{\rho D_v}\right]_f^{-2/3} \tag{25}$$

The equation for mass transfer in laminar flow may be simplified, taking values of viscosity the same in both moduli to give,

$$k_g = \frac{16.8}{p_{gf}\,M_m\,D_p} \left(\mu \rho^2 D_v^2\right)^{1/3} \tag{26}$$

Thus in the laminar flow region the mass transfer coefficients is independent of mass velocity in agreement with theory.

Friction Factor For Pressure Drop

The friction factor, f, for the pressure drop in the flow of a gas through a granular bed is defined by equation (7); and the existing experimental data, corrected for wall effect, correlated by Chilton and Colburn (5) against the modified Reynolds number $D_p G/\mu$ are shown in Figure 1. The friction factor, f, was found to be dependent also upon the ratio of particle diameter to the diameter of vessel and increased by the flow of liquid over the surface of the particles.

Thus $\quad f = f' \ f''$ (27)

where f' = friction factor corrected for wall effect

$\quad$ f'' = wall effect factor (evaluated from the work of

$\qquad$ Furnas, (94) and (95))

$\quad$ f = friction factor for combined particles and walls.

An additional correction factor is reported for the effect of liquid flowing over the particles.

In the Gamson, Thodos and Hougen investigation f and f' were measured for both wetted and dry surfaces. f vs $D_p G/\mu$ are shown in Figure 1 to show experimental results on a summarized plot of Chilton and Colburn.

The correlation is not good. The wide deviation of results should be compared with the small deviations obtained for the j factors for the same experiments. Also it should be noted that different curves result for spheres and cylinders and for dry and wetted particles as compared with j factors where all experimental results fall on the same lines. The friction factors

thus depend upon the shape of the particle, whereas the j

factors for heat and mass transfer are independent of shape when

the proper value of particle size is used. Thus, separate curves

are required for expressing the friction factors for different

shapes; whereas a single curve suffices for j_d or j_h. No

correlation was found between the friction factor for pressure

drop and the various coefficients and factors for heat and mass

transfer for gases flowing through granular beds. For flow

through pipes there is a correlation between heat transfer

and pressure drop, $f = 2 j$, whereas in granular beds the

values of f are one hundred-fold those of j and without correlation.

Another point is of interest. The friction factors for the dry

pellets are 11 per cent higher than for the wetted pellets.

It is reasonable that the wetted surface is smoother and hence

offers less resistance to flow. This effect should not be

confused with the results of Chilton and Colburn, who showed

that the friction factor was increased by a flow of liquid

over the surface of the solid. In this latter case the void space

in the bed was reduced by the flow of liquid, whereas in the

Gamson, Thodos and Hougen experiments the bed was drained before

testing, no liquid was flowing and no reduction in voids occured.

In the Gamson, Thodos and Hougen experiments the water served

to lubricate the surface of the solid.

The frictionfactors, f', for cylinders are in fair agreement

with the average results compiled by Chilton and Colburn, however

the friction factor for spheres is far out of line. The friction

factor for the short cylinders is about 90 per cent higher than

the spheres. This is in agreement with the fact that granular
particles are usually rough and with edges and corners, and not
approaching spheres in smoothness of surface.

The poor correlation of data on the friction factor, f, is
also due to variations in randomness and channeling which are not
reflected in mass and heat transfer factors. The frictional drop
through a granular bed depends upon the manner and rate in which
the grains are poured or packed into the vessel. The interstitial
space freezes into position as the grains are poured in and
further shaking or agitation are unsatisfactory in producing
uniform or reproducible results. Mass and heat transfer factors
were not affected at all, since these latter are dependent upon
surface rather than configuration of void space.

SHERWOOD

Sherwood made the following observation in the article by Gamson, Thodos and Hougen (20). He noted that by a mathematical derivation:

$$\frac{j_h}{j_d} = \frac{w \wedge \Delta p}{w \, \Delta t \, c_p \, p_{gf} \, M_m} \left[\frac{c_p \, \mu}{k} \right]^{2/3} \left[\frac{\rho \, D_v}{\mu} \right]^{2/3} \tag{28}$$

$$c_p \, (t_a - t_w) = \frac{\wedge}{M_a} (H_w - H_a) = \frac{\wedge}{M_b} \left[\frac{p_w}{P - p_w} - \frac{p_a}{P - p_a} \right] \tag{29}$$

$$c_p \, (t_a - t_w) = \frac{\Delta p}{\Delta t} = \frac{c_p \, p_{gf} \, M_m}{\wedge} \tag{30}$$

Combining (28) and (30)

$$\frac{j_h}{j_d} = \left[\frac{c_p \, \mu}{k} \right]^{2/3} \left[\frac{\rho \, D_v}{\mu} \right]^{2/3} \tag{31}$$

j_h / j_d gives a constant of 1.1

An average value of 1.076 is really an average of values calculated from a large number of readings from a particular humidity chart.

WILKE AND HOUGEN

Wilke and Hougen (26) added additional data for the laminar region of flow. They suggested the following equations for values of the Reynolds numbers, $D_p G/\mu$, below 350 to replace the equation reported by Gamson, Thodos and Hougen (20).

$$j_d = \frac{k_g M_m P_{gf}}{G} \left[\frac{\mu}{\rho D_v} \right]_f^{2/3} \tag{32}$$

Figure 4 shows a plot of their results for j_d vs $D_p G/\mu$

$$j_d = 1.82 \left[\frac{D_p G}{\mu} \right]^{-0.51} \quad \text{for values of } \frac{D_p G}{\mu} < 350 \tag{33}$$

Similarly

$$a_v H_d = 0.055 \left[\frac{D_p G}{\mu} \right]^{0.51} \left[\frac{\mu}{\rho D_v} \right]_f^{2/3} \quad \text{for } \frac{D_p G}{\mu} < 350 \tag{34}$$

Combining equation (32) with (33) gives

$$k_g = \frac{1.82\, G}{P_{gf} M_m} \left[\frac{D_p G}{\mu} \right]^{-0.51} \left[\frac{\mu}{\rho D_v} \right]^{-2/3} \quad \text{for } \frac{D_p G}{\mu} < 350 \tag{35}$$

For values of $D_p G/\mu$ above 350 the same equation is recommended as previously reported by Gamson, Thodos and Hougen, namely,

$$j_d = 0.989 \left[\frac{D_p G}{\mu} \right]^{-0.41} \quad \text{for values of } \frac{D_p G}{\mu} > 350 \tag{36}$$

Equation (36) for the region of high Reynolds numbers was verified by 45 additional runs with 7 sizes of cylindrical pellets and found to be in excellent agreement with the previous 180 experiments by Gamson, Thodos and Hougen made on 7 sizes of both spheres and cylinders and several different materials.

HOBSON AND THODOS

Hobson and Thodos (47) in their article in the flow of liquids through beds of granular solids added to the information advanced by Gamson, Thodos and Hougen (20). The basic work on mass transfer factors for gases through granular solids represented the first significant contribution toward the estimation of mass transfer coefficients for the gas filled film from the physical properties of the system. These studies utilized the vaporization of water from the surface of spheres and cylinders to an air stream flowing through a bed of these media. In the further development of this work, Wilke and Hougen (26), using the same system, extended these studies in the low Reynolds number ranges. See Figure 4 for the Hobson and Thodos plot. The work of Hurt (22) and the experiments of White and Resnick (51) deal with the transfer of naphthalene vapors from naphthalene granular beds to a flowing air stream. All of these studies are concerned exclusively with the properties of the gas stream. Careful restriction had been made to eliminate completely the presence of a liquid film by the choice of systems for investigation.

A complete and comprehensive development of the field in mass transfer necessitates the extension of knowledge to include the variables associated with and governing the mass transfer coefficients for the liquid film. Present knowledge on mass transfer coefficients for the liquid film and their estimation are limited by the scarcity of reliable experimental data. Ordinarily, experimental data on mass transfer coefficients, such as might be obtained from gas absorption studies, do not

lend themselves to the direct determination of the factors influencing the nature and magnitude of the liquid film resistance. The work of Hobson and Thodos (47) attempts to evaluate the factors influencing mass transfer through the liquid film and thus fills the existing gap in this field. Such information proves invaluable in implementing theory on gas absorption, liquid-liquid extraction and the kinetics of liquid phase catalytic reactions carried out in the presence of granular catalysts.

For liquids:

$$(j_d)_l = \frac{k_l \, c_{1f} \, M_m}{L} \left[\frac{\mu}{\rho \, D_L} \right]_f^{2/3} \tag{37}$$

$(j_d)_l$ = mass transfer factor for liquids

k_l = mass transfer factor for the liquid film

c_{1f} = mean concentration of non-transferable component in the liquid film

M_m = mean molecular weight of flowing liquid

L = superficial mass velocity of flowing liquid

μ = absolute viscosity of film

ρ = density of film

D_L = diffusivity of transferable liquid in the film

Height of transfer unit for the liquid film - by direct analogy:

$$a(HTU) = \frac{L}{k_l \, c_{1f} \, M_m} = \frac{1}{(j_d)_l} \left[\frac{\mu}{\rho \, D_l} \right]_f^{2/3} \tag{38}$$

Hobson and Thodos have then generalized a mass transfer factor for fluids:

$$j_d = \frac{k\, E_f\, M_m}{F} \left[\frac{\mu}{\rho\, D} \right]_f^{2/3} \qquad (39)$$

where

j_d = mass transfer factor

k = mass transfer coefficient for fluid film

lbmoles/ sq ft unit driving potential

M_m = mean molecular weight of flowing medium lb/lbmole

F = superficial mass velocity of flowing fluid, lb/hr.sq.ft.

E_f = mean value of the inert constituent expressed in units

similar to the driving potential across the fluid film

Hobson and Thodos used isobutyl alcohol-water and methyl ethyl ketone-water systems for their experiments. Their results are shown plotted in Figure 5, and they are compared to the ranges of experimentation by Hougen, Gamson and Thodos as well as with Hougen and Wilke.

CHILTON AND COLBURN

Chilton and Colburn (6) were concerned with a means of estimating the rate of transfer of a diffusing component per unit of size of the apparatus considered. They decided that since relatively few experiments had been run for the various types of equipment and conditions encountered, and owing to difficulties incident to such studies, the reliability of the results were open to question. They sought a convenient method of applying well-substantiated correlations from the analagous processes of fluid friction and heat transfer to test the diffusion data available and to permit predictions to be made where there was no applicable data. Their paper on Mass Transfer Coefficients (6) compared representative experimental data on diffusional processes with results of fluid friction and heat transfer studies.

The Chilton and Colburn method had, as its basis, the Reynolds ahalogy between heat transfer and fluid fluid friction. This analogy postulates that

"the ratio of the momentum lost by skin friction between two sections a differential apart to the total momentum of the fluid will be the same as the ratio of the heat actually supplied by the surface to that which would have been supplied if the whole of the fluid had been carried up to the surface."

The equations were developed by Colburn (2) and his equation as it applies to heat transfer reads:

$$\frac{\Delta p \, g}{\varsigma \, u^2} \left[\frac{S}{A} \right] = \frac{t_1 - t_2}{\Delta t_m} \left[\frac{S}{A} \right] \left[\frac{c \, \mu}{k} \right]^{2/3} \tag{40}$$

The quantity on the left of the equation has long been found to be a function of the Reynold's number, $D \, u \, \varsigma / \mu$, and has often been represented by the symbol, $Re = 1/2 \, f$. This function can be expressed not only in terms of overall pressure drop and ratio of cross section to surface area, but also in terms of skin friction per unit surface area, as:

$$\frac{\Delta p \, g}{\varsigma \, u^2} \left[\frac{S}{A} \right] = \frac{R}{\varsigma \, u^2} = 1/2 \, f \tag{41}$$

Similarly, for heat transfer the ratios can be expressed not only in terms of the temperature change but also in terms of the film coefficient of heat transfer per unit of surface area:

$$\frac{(t_1 - t_2)}{\Delta t_m} \left[\frac{S}{A} \right] \left[\frac{c \, \mu}{k} \right]^{2/3} = j = \frac{h}{CG} \left[\frac{c \mu}{k} \right]^{2/3} \tag{42}$$

The function of the dimensionless group, $c \, \mu / k$, was not included in the original Reynolds analogy, although it was recognized by Reynolds himself that some function of the ratio of viscosity to thermal conductivity should be introduced. The power function employed in the equations given is derived from numerous correlations of data on heat transfer in turbulent flow, where it serves to relate these factors as single-valued functions of the Reynolds number, independent of the properties of the fluid/

As shown in the paper by Colburn (6), Equation (40) (the modified Reynolds number analogy) holds for fully turbulent

flow inside tubes, and for flow parallel to plane surfaces, but does not apply to streamline tubes or flow across tubes and tube banks. Hence, as indicated in Equations (41) and (42), different symbols have been used to represent the friction and heat transfer factors.

Processes in which material is transferred by diffusion are closely related to heat transfer, since the latter can be considered merely as the diffusion of hot molecules into a region of cold ones and a corresponding diffusion of cold molecules in the reverse direction. Since the mechanism is so similar, it would be expected that a relationship could be obtained for diffusional processes entirely analagous to that for heat transfer. The diffusional process most nearly similar is rectification, in which the total number of moles of material passing through the apparatus remains constant, and diffusion occurs in both directions. For this process the rate of material transfer, w, can be expressed either in terms of the change in partial pressure of one of the diffusing components or in terms of a mass transfer coefficient, k_g, in moles per unit time per unit area per unit partial pressure difference as follows:

$$w = \frac{(p_2 - p_1)\, G\, S}{P\, M_m} = k_g\, \Delta p_m\, A \tag{43}$$

Rearrangement of the terms of this equation leads to an expression involving the ratio of the change in partial pressures to the mean difference in partial pressures between the gas mixture and the surface, analagous to the relating temperature change and temperature difference. An extension of the Reynolds

analogy leads to the expectation that this ratio

$$\frac{(p_2 - p_1)}{\Delta p_m} \left[\frac{S}{A}\right] \left[\frac{\mu}{\rho k_d}\right]^{2/3} = \frac{k_g \, P}{(G/M_m)} \left[\frac{\mu}{\rho \, k_d}\right]^{2/3} = j \quad (44)$$

will be the same function of Reynolds number as the corresponding

heat transfer factor, and will equal the friction factor under

the same conditions as it does. It is therefore designated by

the same symbol, j, as the heat transfer factor defined by

Equation (42).

In equation (43) a function of the ratio of viscosity

to diffusivity has been inserted exactly analagous to that employed

on the $(c \mu / k)$ group in defining the heat transfer factor. The

latter function has been shown by Colburn (6) to give a satisfactory

correlation of heat transfer data over a range of $(c \mu / k)$ values

from 0.7 to 1000; and since it has been shown further that the

Prandtl equation, involving a different function of $(c \mu/k)$, is

not so satisfactory for high values of this group, the power

function of $(\mu/\rho k_d)$ included in the above equations is now

preferred to the theoretical equation proposed several years

before by Colburn (2) for correlating diffusional data. It is

possible that the correct value of the exponent may not be the

same as on the $(c \mu/k)$ group for heat transfer, but it will be

necessary to have data covering a wide range of $(\mu /\rho k_d)$

values to justify any considerable change in this function.

In other processes, such as absorption, stripping, evaporation

of a liquid into a gas, or condensation of a vapor from a

mixture with inert gas, the total number of moles does not change as the gas mixture passes through the apparatus, and also the diffusion is chiefly, if not wholly, in one direction. The differential rate of material transfer can then be expressed:

$$dw = d\left[\frac{p\,G\,S}{M_m\,P}\right] = k_g\,\Delta p\,dA \qquad (45)$$

Carrying out the differentiation, considering S and P constant and making use of the relation $G/M_m = G_i/M_i\,(P/P-p)$, leads to the equation:

$$dw = d\left[\frac{p\,G\,S}{p_g M_m}\right] = k_g\,\Delta p\,dA \qquad (46)$$

where $p_g = P - p$ and (G_i/M_i) = molar mass velocity of the inert gas . The corresponding mass transfer factor for this case then becomes:

$$\left[\frac{dp}{\Delta p}\right]\left[\frac{p_{gf}}{p_g}\right]\left[\frac{S}{dA}\right]\left[\frac{\mu}{\rho k_d}\right]^{2/3} = \frac{k_g\,p_{gf}}{(G/M_m)}\left[\frac{\mu}{\rho k_d}\right]^{2/3} = j \qquad (47)$$

The quantity p_{gf} has been included on both sides of the equation, since k_g varies inversely with p_{gf}, as follows from the Stefan diffusion equation; and the same function of $(\mu/\rho k_d)$ is included as in equation (44).

When the diffusing vapor is relatively dilute throughout the apparatus, and in some other cases, average values may be used for p_{gf}, p_g, k_g, and G, and equation (47) may be integrated to give:

$$\left[\frac{p_1 - p_2}{\Delta p_m}\right]\left[\frac{p_{gf}}{p_g}\right]\left[\frac{S}{A}\right]\left[\frac{\mu}{\rho k_d}\right]^{2/3} = \frac{k_g\,p_{gf}}{G/M_m}\left(\frac{\mu}{\rho k_d}\right)^{2/3} = j \qquad (48)$$

When the diffusing component changes greatly in partial

pressure through the apparatus, p_{gf}, will change considerably, and also G, M_m, and sometimes $(\mu/\rho k_d)^{2/3}$, so that k_g is not a constant. Furthermore, the true mean driving force, Δp_m, is not in general equal to the logarithmic mean of the terminal driving forces. In such cases, k_g, Δp, and w must be computed at several intermediate values of composition; then from a plot of $1/(k_g \Delta p)$ vs w, the required surface area is obtained by a graphical integration, according to the equation:

$$A = \int_0^w \frac{dw}{k_g \Delta p} = \frac{S}{j} \int_{p_1}^{p_2} \frac{p_{gf}}{p_g} \frac{dp}{\Delta p} \tag{49}$$

Or instead of calculating values of k_g, the integration can almost as well be made in terms of partial pressures, as shown by the last term in Equation (49), since j generally varies so slightly with velocity that an average value can be used satisfactorily.

When the diffusing component is so dilute that p_{gf}, G and k_g can be considered substantially constant, and when the solute follows Henry's law, or the solution exerts a negligible vapor pressure over the working range, then the driving force is equal to the logarithmic mean of the terminal values, and the required surface area is simply:

$$A = \frac{w}{\Delta p_m k_g} = \frac{(p_1 - p_2)}{\Delta p_m} \frac{p_{gf}}{p_g} \frac{S}{j} \tag{50}$$

It should be emphasized that these equations apply only to diffusion rate into or out of the fluid undergoing relative motion, and do not allow for any liquor film resistance.

Friction and heat transfer factors were determined and compared from heat transfer data and pressure drop correlations and shown in plots for flow inside of conduits. See Figure 6 for the plot. The j factors could be used to predict mass transfer coefficients by employing equations (44) and (48) which are analagous to equation (42) the heat transfer equation.

TAECKER AND HOUGEN

Taecker and Hougen (41) continued the experiments of Gamson, Thodos and Hougen (20), Wilke and Hougen (26), Hurt (22), and van Krevelen and Hoftijzer (33, 42, 44). The investigations which obtained single curves for the transfer factors of heat and mass, j_h and j_d, when plotted against a modified Reynolds number regardless of the size, shape, porosity, density and composition of the solid provided the surface was maintained wetted at a constant temperature regardless of temperature, humidity, pressure and velocity of the air stream was continued with Raschig rings and Berl saddles. Runs were repeated with similar packings to determine agreement with previous investigators and continued to determine the effects of packing arrangement and of entrance disturbances.

The van Krevelen and Hoftijzer (33, 42, 44) experiments set up correlations of the mass transfer coefficients of gas films in packed towers where the liquid flowed concurrently to the gas stream. With no liquid flowing over the packing their equation became:

$$\frac{k_g\, d}{D_v} = 0.0125 \left[\frac{G}{a\,\mu}\right]^{0.8} \left[\frac{\mu}{\rho\, D_v}\right]^{1/3} \tag{51}$$

The Gamson and Thodos (20) equation was:

$$k_g = \frac{1.25\, G}{p_f\, M_m} \left[\frac{\mu}{\rho\, D_v}\right]^{-2/3} \left[\frac{G\sqrt{A_p}}{\mu}\right]^{-0.41} \tag{52}$$

where A_p = external area of a single particle and $\dfrac{G\sqrt{A_p}}{\mu} > 620$

For a sphere

$$d = D_p \sqrt{\frac{A_p}{\pi}} = 0.567 \sqrt{A_p} \tag{53}$$

and

$$a_v = \frac{6 (1-F_e)}{d} \tag{54}$$

where

F_e = external void fraction of packing

a_v = surface area per unit volume

or

$$a_v = \frac{6(1 - F_e)}{0.567 \sqrt{A_p}} = \frac{10.6 (1 - F_e)}{\sqrt{A_p}} \tag{55}$$

Equation (52) is then expanded to read:

$$\frac{k_g d}{D_v} = \left[\frac{G}{a_v \mu} \right]^{0.59} \left(\frac{\mu}{\rho D_v} \right)^{1/3} \frac{2.86 (1 - F_e)^{0.59} \rho}{p_f M_m} \tag{56}$$

For dilute gases $p_f M_m$ is equal to $R T \rho$, where R is the universal gas constant, at $t = 100^o$ F, $F_e = 0.5$, $R = 0.729$ and Equation (56) becomes:

$$\frac{k_g d}{D_v} = 0.00465 \left[\frac{G}{a_v \mu} \right]^{0.59} \left(\frac{\mu}{\rho D} \right)^{1/3} \tag{57}$$

The van Krevelen and Hoftijzer equation is less satisfactory than equation (52) in that the exponent 0.8 is too high and the use of a nominal diameter for tower packing makes no allowance for the shape and hollowness of the packing.

A summary of the experimental results of Taecker and Hougen are plotted in Figure 7. Properties of the tower packings are listed in Table 8. Sample Calculations are

made for Table 9 Which tabulate the laboratory data and the calculated results of Taecker and Hougen. The D_p in the modified Reynolds number, $D_p G/\mu$, is the effective diameter of the particle equivalent to the diameter of a sphere having the same surface area as the particle and is $= \sqrt{A_p/\pi}$; where A_p is the area of the particle or $D_p = 0.567\sqrt{A_p}$. Taecker and Hougen feel that the confusion resulting from trying to visualize the significance of D_p for rings and saddles is eliminated by the use of the modified Reynolds number modulus

$\dfrac{G\sqrt{A_p}}{\mu}$ instead of $\dfrac{D_p G}{\mu}$.

Therefore
$$\frac{D_p G}{\mu} = 0.567 \ \frac{G\sqrt{A_p}}{\mu} \tag{58}$$

Taecker and Hougen calculated j_d from j_h by the ratio

$j_h / j_d = 1.076$.

Entrance effects had no effect on values of j_h also no difference was obtained in comparing random to staggered arrangements. Difference in j_h for rings as compared to spheres and cylinders is accounted for since the inside area of packing is less accessible compared to the outside area. In calculating values of A_p both inside and outside areas of the rings were included, whereas with solid spheres and cylinders there were no inside areas.

Summarizing Taecker and Hougen values of j_h and j_d:

$$j_h = 1.148 \left[\frac{G\sqrt{A_p}}{\mu}\right]^{-0.41} \qquad \text{from 100 to 20,000} \qquad (59)$$

$$j_d = 1.070 \left(\frac{G\sqrt{A_p}}{\mu}\right)^{-0.41} \qquad \text{for Raschig rings and}$$

partition rings $\qquad (60)$

$$j_h = 0.920 \left(\frac{G\sqrt{A_p}}{\mu}\right)^{-0.34} \qquad (61)$$

for Berl Saddles

$$j_d = 0.855 \left(\frac{G\sqrt{A_p}}{\mu}\right)^{-0.34} \qquad \text{from 70 to 3000} \qquad (62)$$

$$j_h = 1.346 \left(\frac{G\sqrt{A_p}}{\mu}\right)^{-0.41} \qquad \text{from 620 and up} \qquad (63)$$

$$j_d = 1.251 \left(\frac{G\sqrt{A_p}}{\mu}\right)^{-0.41} \qquad \text{for solid spheres}$$

and cylinders $\qquad (64)$

$$j_h = 2.63 \left(\frac{G\sqrt{A_p}}{\mu}\right)^{-0.51} \qquad \text{for solid spheres} \qquad (65)$$

and cylinders

$$j_d = 2.44 \left(\frac{G\sqrt{A_p}}{\mu}\right)^{-0.51} \qquad \text{below 620} \qquad (66)$$

Transfer factors of j_h and j_d are related to the corresponding transfer coefficients and corresponding heights of transfer units by the following relationships:

$$j_h = \frac{h}{c_p G}\left(\frac{c_p \mu}{k}\right)^{2/3}_f \qquad (67)$$

$$j_d = \frac{k_g P_f M_m}{G}\left(\frac{\mu}{\rho D_v}\right)^{2/3}_f \qquad (68)$$

$$a_v H_h = \frac{G c_p}{h} \qquad (69)$$

$$a_v H_d = \frac{G}{k_g P_f M_m} \qquad (70)$$

EVANS AND GERALD

Evans and Gerald (87) confined their study in the range of Reynolds numbers for 1 to 1000. They accepted this phase of the mass transfer problem because they felt that the data of Hurt (22) and Resnick and White (51) had divergent results. The particular system they used was naphthalene - gas. Data for particles in smaller size ranges were presented, and the use of irregular granules gave qualitative results on the effect of particle shape.

Evans and Gerald also found, as did all investigators, that $\log j_d$ varied linearly with the log of a modified Reynolds number.

The fixed bed points all fit the line:

$$j_d = 1.48 \left(\frac{D_p G}{\mu}\right)^{-0.52} \tag{71}$$

For the dilute solutions used:

$$j_d = \frac{k_f}{G} \left(\frac{\mu}{\rho D}\right)^{2/3} \tag{72}$$

This correlation for fixed bed data is nearly identical with the relation for liquid-phase fixed-bed mass-transfer discovered by McCune and Wilhelm (50) for this range of Reynolds numbers, and essentially the same as proposed by Wilke and Hougen (26) from gas-phase data. It is in agreement with some other investigators, but in disagreement with the correlation for low Reynolds numbers proposed by Hobson and Thodos (47). The McCune-Wilhelm and Hobson-Thodos correlations are shown dashed on Figure 10.

Correlations with Gamson is shown in Figure 11.

An effective area factor shape factor, such as proposed
by Gamson (67), would vary not only with modified Reynolds
number, but also with frastion voids if it made all such data
fall on the line or same correlating curve. The difference
in slope compared to Gamson's line for spheres emphasizes the
difficulties in obtaining a general correlation by this means.

The use of the transfer analogy, according to Evans and Gerald,
is to relate mass transfer with fluid friction, has been useful in
correlating and explaining mass-transfer rates for flow in tubes
and around various shapes. The application of the analogy to
flow of fluids in beds of particles is not nearly as straight forward
because of the unknown effects of curvature, expansion and
contractkon effects, and ratio of skin friction to total pressure
loss. In spite of these complications, the j_d factor and friction
factor should be interrelated.

The friction factor in fixed and fluidized beds has been
variously defined and the proper definition is still open to
question. A friction factor in fixed and fluidized beds has
an equation definition similar to Carman's (8,9), based on
the particle-surface area, would seem to have some theoretical
justification, because of the importance of surface to skin
friction and mass transfer. Carman's friction factor also
appears to represent more exactly the actual flow conditions
of velocity and hydraulic radius in the bed. The curvature
of flow and form drag effects are less dependent on surface

than on particle size and particle separation, measured at least in part by D_p, so that the usual modified Reynolds number and friction factor is

$$f = \frac{g_c D_p \rho \Delta P}{2 G^2 L} \tag{73}$$

may be chosen to express the friction loss. Both relations were tested by Evans and Gerald (87).

DRYDEN, STRANG AND WITHROW

Dryden, Strang and Withrow (89) noted that mass transfer at a solid-fluid interface in packed beds had been actively investigated. Study of the liquid phase were done only by McCune and Wilhelm (50), Hobson and Thodos (66) and Gaffney and Drew (63). In all cases Reynolds numbers below the region of one were not investigated. Therefore Dryden, Strang and Withrow covered the rate of solution of 2-naphthol and benzoic acid in water. The entire resistance of the solid spheres to mass transfer can then be attributed to the liquid phase.

Results are based on calculations involving the following concepts and assumptions previously outlined by McCune and Wilhelm (50):

1. The effective driving force is a concentration gradient accross a diffusional boundary layer surrounding the particles.

2. Solute concentrations of solid-liquid interface is saturation value of mean temperature of system.

3. Low concentrations are involved at all points in system and simplifying assumptions are made accordingly.

In addition, the following points are important in considering calculations in viscous-flow region:

1. No axial diffusion exists.

2. No free convection exists.

Dryden, Strang and ithrow note that the Reynolds number is defined by

$$Re = \frac{D_p G}{\mu \epsilon} = \frac{6 G}{S \mu \epsilon} \tag{74}$$

By dimensionless analysis, they derived the correlation equation for mass transfer for dilute solutions as shown also by Chilton and Colburn (6) for turbulent conditions:

$$j_\epsilon = \frac{k_\epsilon}{G} \left(\frac{\mu}{\rho D}\right)^q = \phi \left[\frac{D_p G}{\mu \epsilon}\right]^m \tag{75}$$

Generally the exponent q on the Schmidt number is accepted to be 2/3. Gaffney and Drew (63) found q = 0.58 for Schmidt numbers ranging from 100 to 1000 in liquid sytems. The exponent m is a function of the Reynolds number.

One of the difficulties in obtaining data at low mass velocities in packed beds is to maintain a flow pattern through a finite depth of actively dissolving solids, and yet prevent saturation within the bed. This condition is alleviated by the proper choice of experimental conditions and materials.

1. large particles.

2. solute which can be assayed quite accurately.

3. a system having a high Schmidt number,

4. a minimum depth of active solids.

Dryden, Strang and Withrow used a colorimetric analysis for 2-naphthol and a volumetric analysis for benzoic acid. Their correlation is shown in Figure 12 (for an exponent of 0.58). Their correlation is shown in Figure 13 (for an exponent of 2/3).

JU CHIN CHU, KALIL, AND WETTEROTH

Ju Chin Chu, Kalil and Wetteroth (86) found that the mass-transfer data for aggregative-type (gas-solid) fluidization are in agreement with other mass-transfer data in fixed beds as well as with the data for particulate (liquid-solid) type fluidization. All the mass-transfer data for both types of fluidization, as well as for stationary beds, for widely varying systems can be correlated on a plot of mass-transfer factor j_d vs a modified Reynolds number $D_p G / \mu (1-\epsilon)$. See Figure 14.

The varying of bed voidage ϵ, can be incorporated into the modified Reynolds number. During the course of the investigation by Chu, Kalil and Wetteroth, the bed voidage was varied from approximately 0.25 to 0.97.

Mass-Transfer and pressure drop data for granular beds are related by the simple equation $j_d = f/10$.

Mass-Transfer factors, like friction factors, are independent of bed height. For the runs made, the quiescent bed height was varied from 0.1 to 3.6 inches.

Mass transfer factors in a fluidized bed are independent of particle density, except that the particle density is an important factor in determining the initial fluidization velocity. The particles used varied in density from about 1 to 9 grams per cubic centimeters.

The effect of particle size can be correlated by the use of the modified Reynolds number. The particle size was varied from 0.03 to 0.5 inches.

The mass-transfer data in granular beds as plotted in Figure 14 can be correlated by:

$$j_d = 1.77 \left(\frac{D_p G}{\mu (1 - \epsilon)} \right)^{-0.44} \quad \text{for 30 to 10,000 Modified Reynolds No.} \quad (76)$$

$$j_d = 5.7 \left(\frac{D_p G}{\mu (1 - \epsilon)} \right)^{-0.78} \quad \text{for 1 to 30 Modified Reynolds No.} \quad (77)$$

The voidage in the fluidized bed in the turbulent region can be predicted for use in the preceding equation by means of a nomograph developed from the extension of the Carman-Kozeny equation to the fluidized bed (65, 73).

SUMMARY OF CORRELATIONS.

Since numerous industrial processes, such as coke or
gasification of coal, blast furnace operation, catalyst regeneration,
adsorption, drying, solution, and many exchange processes
involve interaction and mass and heat transfer between solid
particles in fixed beds and the gas or liquid streams, then
the importance of evaluating heat and mass transfer rates
in such fixed beds to their design and successful operation
is generally recognized and much discussed.

The problem has been treated mostly on empirical bases
and no correlation applicable to all types of systems has been
found. The purpose of this paper was

a) to review the existing methods of empirical correlations,

b) to point out their implications,

c) to review their theoretical developments,

d) to see the developed equations on theoretical grounds, and

e) to check some of these equations with published data.

The packed systems exhibit complexities, e.g., variations
in temperature and pressure, changes in cross-section, flow
rates, solid surface areas, etc. Any general equation must
necessarily be a differential one so as to permit integration
over ranges of variables encountered. Available data on the
subject, however meager, are fortunately for simple cases
of constant temperatures and substantially constant flow rates.
Thus the validity of the equations developed were tested
in a simple manner.

Factors, such as k_g, j_d, j_h, HTU, D_pG/μ, etc. can be calculated from the data taken from a column in operation. The problem lies in predicting any of these factors for another system to be designed, or for the same system under different flow conditions. The j_d and j_h factors were introduced for this purpose. Unfortunately the relations obtained by various investigators differed in many respects. They usually failed to agree with one another.

Gamson, Thodos and Hougen (20) made psychrometric measurements on water evaporation from cylindrical and spherical pellets into as air stream. Plots of their j factors against Reynolds numbers gave:

$$j_d = 0.989 \left[\frac{D_p G}{\mu} \right]^{-0.41} \quad \text{for} \quad \frac{D_p G}{\mu} > 350$$

$$j_d = 16.8 \left(\frac{D_p G}{\mu} \right)^{-1.0} \quad \text{for} \quad \frac{D_p G}{\mu} < 40$$

Wilke and Hougen (26) further investigated Reynolds numbers in the region of lower gas rate. They modified the equation of Gamson, Thodos and Hougen to:

$$j_d = 1.82 \left(\frac{D_p G}{\mu} \right)^{-0.51} \quad \text{from 1 to 100 for} \quad \frac{D_p G}{\mu}$$

Hurt (22) investigated the adsorption of water vapor from moist air by particles of silica gel and by particles coated with phosphorous pentoxide, adiabatic humidification of air over silica gel and wetted with water, evaporation of naphthalene from naphthalene flakes into air and hydrogen streams. Fractional

Resnick and White (51) worked with rates at which naphthalene was evaporated into air, hydrogen and carbon dioxide streams from fixed and fluidized beds. Differences in particle sizes were noted in the scattered results. Their correlation resulted in:

$$\frac{j_d}{D_p^{1.5}} = .15 \left(\frac{D_p G}{\mu} \right)^{-0.273} \qquad \text{for} \quad \frac{D_p G}{\mu} < 25$$

Taecker and Hougen (41) found their results quite similar to Gamson, Thodos and Hougen as well as with Wilke and Hougen. Taecker and Hougen used Raschig rings, partition rings and Berl saddles instead of spheres and cylinders. Their equations were:

$$j_d = 1.251 \left(\frac{D_p G}{\mu} \right)^{-0.41} \qquad \text{for} \quad \frac{D_p G}{\mu} > 620$$

$$j_d = 2.24 \left(\frac{D_p G}{\mu} \right)^{-0.51} \qquad \text{for} \quad \frac{D_p G}{\mu} < 620$$

Hobson and Thodos (47) passed water through cellite spheres and measured concentrations effluent with time. Some question as to accuracy in extrapolating their curves as well as how saturated were the pellets initially and the effect on the surface area. The Hobson and Thodos equation was:

$$\log \ j_d = 0.7683 - 0.9175 \log \left(\frac{D_p G}{\mu} \right) + 0.0817 \log \left(\frac{D_p G}{\mu} \right)^2$$

Gaffney and Drew (63) used a solid-liquid stream. Their work was similar to McCune and Wilhelm (50). Fractional void volumes were considered. The systems investigated were benzene-salicylic acid, acetone-succinic acid, and n-butanol-succinic acid. Their Schmidt number was raised to the 0.58 instead of the 2/3 power. Their equations read:

$$j_d \quad 1.97 \left(\frac{D_p G}{\mu} \right)^{-0.613} \quad \text{for} \quad \frac{D_p G}{\mu \epsilon} < 200$$

$$j_d = 0.290 \left(\frac{D_p G}{\mu} \right)^{-0.254} \quad \text{for} \quad \frac{D_p G}{\mu \epsilon} > 200$$

McCune and Wilhelm (50) measured mass transfer rates from 2-naphthol pellets into water streams. Uni-form sized pellets to estimate accurately the surface area exposed to flow and the interfacial area for mass transfer and accuracy as to voidage was their forte. Their equations were:

$$j_d = 1.625 \left(\frac{D_p G}{\mu} \right)^{-0.507} \quad \text{for} \quad \frac{D_p G}{\mu} < 120$$

$$j_d = 0.687 \left(\frac{D_p G}{\mu} \right)^{-0.327} \quad \text{for} \quad \frac{D_p G}{\mu} > 120$$

Evans and Gerald (87) utilized a naphthalene-gas system and got results agreeing closely with McCune and Wilhelm and also with Wilke and Hougen. Friction was considered important and was checked to see variation with flow curvature and form drag effects with particle size. They also discussed particle separation.

The Evans and Gerald equation was found to be:

$$j_d = 1.48 \left(\frac{D_p G}{\mu} \right)^{-0.52}$$

Dryden, Strang and Withrow (89) worked with a 2-naphthol and benzoic acid water system. Working with all liquid and solid spheres, they tried to control the flow pattern in order to get good bed wetting for the greatest accuracy. They derived the general equation by dimensionless analysis and found Schmidt numbers to be raised to the 0.58 instead of the 2/3 power. Their general equation was:

$$j_\epsilon = \frac{k_\epsilon}{G} \left(\frac{\mu}{\rho D} \right)^q = \phi \left(\frac{D_p G}{\mu \epsilon} \right)^m$$

Ju Chin Chu, Kalil and Wetteroth (86) varied bed-voidage and included this factor in their correlation. They found the resulting correlations:

$$j_d = 1.77 \left(\frac{D_p G}{\mu (1-\epsilon)} \right)^{-0.44} \quad \text{for a range of 30 to 10,000}$$

$$j_d = 5.7 \left(\frac{D_p G}{\mu (1-\epsilon)} \right)^{-0.78} \quad \text{for a range of 1 to 30.}$$

Other investigators are listed in the bibliography. A great deal of work has been done in this field. Many investigators seemed to be unaware of work done by their colleagues. Only the above were selected for this study since their work tied in fairly closely with each other.

DISCUSSION

The work of the investigators listed in the preceding report as well as others listed in the bibliography showed a lack of agreement in their correlations. Perhaps a further analysis of the problem based on more fundamental considerations is necessary before relationships that could be valid for all systems considered should be tried to achieve a correlation for the mass and heat transfer analogy in packed beds.

The application of the Reynolds analogy as proposed by Chilton and Colburn (6) to the data for gas flow in tubes has been quite successful. However its applicability to flow in packed columns has only been fair and needs quite a bit more work by many more investigators.

A correlation between pressure drop and mass transfer has been sought and the specific functions of fractional void volume, height of bed, particle size and fluid viscosity should be included in a correlation. It should be possible to obtain theoretical equations which will formally relate heat and mass transfer rates to pressure drop in packed columns.

Unfortunately, also, most workers in this field have not reported the pressure drop with the mass transfer data. The effects of mixing is still a problem to be evaluated. Application of the analogy for the liquid streams to the reported data for gas streams in packed beds has not been successful largely because of the deficiency and uncertainty of the data. Not much work has been done for the liquid phase problem, since most investigators veered away from

the solid-liquid systems by judicial selection of materials
for solid-gas systems which they preferred to investigate
to simplify their work.

The absence of generally valid relationships in the literature
for heat and mass transfer in packed systems can be attributed
to the lack of knowledge of fundamental relationships in
their original derivations. Also they may have perhaps
misinterpreted the Reynolds arguments. Therefore the first
approach should be to develop fundamental equations with
other factors which seemed unnecessary to the first workers
and later to be found to have profound effects in the resulting
correlations.

In the various plots of the j transfer factors versus
the modified Reynolds numbers it is easily observed that
the various investigators were in disagreement. A composite
of some work was illustrated in Figure 16. It is seen that
although all correlations follow the same general pattern,
the curves veer from each other for some explainable reasons.

The correlations of the many who did preliminary research
show they developed their equations with different criteria
as a basis. Particle size, shape, interstitial space in the
packed bed, the method of packing the bed, voidage, and many
other factors are involved. Some factors were deemed unnecessary
by some and fairly important by others as to make their entire
research depend upon one or two factors alone, i.e., bed
voidage.

Suspicion that some of the investigators did not

work too accurately, on the fact that surface area may not

have been completely wetted, can be seen in the results of their

plots. If the bed is partially dry due to channeling or working

down in the low flow rates would mean that all the surface area

was not completely available for mass and heat transfer.

This fact is borne out in the Composite graph Figure 16 which

reveals the greatest discrepancies exist in the low range

of modified Reynolds numbers.

The modified Reynolds numbers were all based upon the

particle size and the D_p or A_p or $\sqrt{A_p}$ factors were calculated

by the various investigators on different concepts of how best

to relate the area of surface available for transfer in the

modified Reynolds numbers. This area factor was also subject

to some error in that friction varied more with sharp edged

cylinders and other shapes as compared with the smoother shaped

spheres. It is understandable that form drag effects become quite

important where sharp corners and edges are available as against

a wetted sphere which has a comparatively frictionless and

smooth surface. Also the fact that the solid sphere has an area

which has quite a different effect on transfer surface as compared

to a hollow cylinder, ring or saddle which may have an equivalent

surface area but may have different characteristics for heat

and mass transfer. The varying results are illustrated in the

plots.

A good deal of the experimentation only concerned itself with either making the gas film resistance only or the liquid film only the controlling resistance. Some unit operations can be easily visualized which may have a combined liquid and gas film resistance controlling at the same time. The existing correlations do not cover this possibility.

The value of the exponent in the analogy correlations has not been determined with exact certainty. Almost all authors have accepted the 2/3 power without question. One author evaluated an exponent of 0.58 as being the more accurate. Further experimentation will in all probability determine this value with greater exactness.

The general opinion was that randomness and channeling were not reflected in the mass and heat transfer factors. The method of pouring in and packing of the pellets gave no uniform or reproducible results. The configuration of the void spaces may very well affect the mass and heat transfer factors.

Size, density, shape, porosity and composition of solid packing was deemed to have little effect provided the surface was maintained wetted and at constant temperatures. Also humidity, temperature, and pressure as well as fluid velocity were not too important in considering the overall result.

The unknown effects of curvature, expansion and contraction and the ratio of skin friction to the total pressure loss were not seen in the correlations. Entrance effects seemed to make little difference in comparing the packed arrangements.

APPLICATIONS

The analogy and the correlations have been utilized to
some extent by some experts in the field. Colburn and Hougen (7)
have already shown the Design of a Cooler-Condenser for mixtures
of vapors with non-condensing gases; utilizing the Reynolds
analogy. Bras (90, 91, 92) has also applied the analogy to the
design of a Cooler-Condenser for vapor-gas mixtures. Here by
a point-to-point calculation and using a vapor pressure versus
temperature plot, Bras gives shortcuts on cooler-condenser
design as well as examples on gas-cooling towers. It can
be shown that packed towers can be solved with the analogy.

NOMENCLATURE

a = Effective surface area of grains or pellets per unit volume, sq. ft./ cu. ft.

c_p = Specific Heat at constant pressure, Btu/lb ^{o}F

D = Diameter, ft.

D_p = Effective particle diameter based upon total surface area, equivalent to the diameter of a sphere having the same surface area as the particle, ft.

D_v = Diffusivity of gas, sq. ft./hr.

f = Friction factor for pressure drop in Fanning's equation.

f' = Friction factor corrected for wall effect

f'' = Wall effect factor.

g_c = Acceleration constant.

G = Mass velocity, lb./hr. sq.ft.

h = Heat transfer coefficient for gas film, Btu/hr. sq.ft.^{o}F.

Δh = Pressure drop, inches of water.

H = Absolute humidity, lb. of water/ lb. of dry air.

H_c = Height of cylinder.

ΔH = Molal heat of reaction, Btu/lb.mole.

HTU = Height of transfer unit, ft.

j = Transfer factor, dimensionless, j_d for mass, j_h for heat transfer.

k = Thermal conductivity, Btu/hr.sq.ft.^{o}F/ft.

k_G = Mass transfer coefficient of gas film, lb.moles/hr.sq.ft.atm.

L = Length of transfer zone, ft. also Depth of bed, ft.

M = Molecular weight, lb./lb.mole

p $\quad$ = Partial pressure, atm.

dp $\quad$ = Pressure differential in the direction of flow, atm.

Δp = Partial pressure driving force across gas film, atm.

$\qquad$ also pressure drop in direction of flow, lb./sq.ft.

P $\quad$ = Total pressure, atm.

q $\quad$ = Rate of heat flow, Btu/hr.

t $\quad$ = Temperature, ^{o}F.

dt $\quad$ = Temperature differential in the direction of flow, ^{o}F.

Δt = Temperature driving force across gas film, ^{o}F.

V $\quad$ = Volume, cu. ft.

w $\quad$ = Rate of mass transfer, lb.moles/hr.

L $\quad$ = superficial mass velocity of flowing liquid

F $\quad$ = Superficial mass velocity of flowing fluid, lb/hr.sq.ft.

E_f = mean value of the inert constituent expressed in units

$\qquad$ similar to the driving potential across the fluid film.

S $\quad$ = cross sectional area, ft^2

A $\quad$ = surface area of total pellets, sq.ft.

G_i/M_i = molar mass velocity of the inert gas

A_p $\quad$ = external area of a single particle.

Fe $\quad$ = external void fraction of packing.

R $\quad$ = Universal gas constant = 0.729

$$\frac{c_p \mu}{k} = \text{Prandtl Number}$$

$$Re = \frac{D_p G}{\mu} = \text{Reynolds Number}$$

$$\frac{\mu}{\rho D_v} = \text{Schmidt number}$$

GREEK

Δ = Symbol for difference.

θ = Time, hr.

λ = Molal heat of vaporization, Btu/lb.mole.

μ = Absolute viscosity, lb./hr.ft.

ρ = Dendity, lb./cu.ft.

ϵ = Fractional void volume in packed bed.

SUBSCRIPTS

$()_c$ = cylinder

$()_d$ = mass transfer

$()_{da}$ = Dry air

$()_f$ = Gas film

$()_l$ = Liquid film

$()_{gf}$ = Inert gas in gas film

$()_h$ = Heat transfer

$()_{lm}$ = Log mean value

$()_m$ = Mean value

$()_w$ = Wet bulb conditions

$()_1$ = Entering

$()_2$ = Leaving

BIBLIOGRAPHY

1. Blake, <u>Tr. A.I.Ch.E.</u>, <u>14</u>, p.415, 1922

2. Colburn, <u>Tr.A.I.Ch.E.</u>, <u>29</u>, p. 174, 1933

3. Kozeny, <u>Ber. Wien. Akad.</u>, <u>136a</u>, p.271, 1927.

4. Chilton and Colburn, <u>I.E.C.</u>, <u>27</u>, p.255, 1935.

5. Chilton and Colburn, <u>Tr.A.I.Ch.E. 23</u>, p. 166, 1931.

6. Chilton and Colburn, <u>I.E.C.</u>, <u>26</u>, p. 1183, 1934.

7. Colburn and Hougen, <u>I.E.C.</u>, <u>26</u>, p. 1178, 1934.

8. Carman, <u>Tr.I.Ch.E. (London)</u>, <u>15</u>, p.150, 1937.

9. Carman, <u>Soc. Chem. Ind.</u>, <u>57</u>, p. 225, 1938.

10. Sherwood, <u>Tr.A.I.Ch.E.</u>, <u>36</u>, p.817, 1940.

11. Krischer, <u>Z.Ver.deut.Ing.Bisjaft.Folge</u>, p.17, Jan. 1940

12. Saunders and Ford, <u>J.Iron and Stl.Inst.(London)</u>, 26 pp, May 1940

13. Kirschbaum and Kangle, <u>Chem. Fabrik.</u> p.171, 1941

14. Matz, <u>Z.Ver.deut.Ing.Ver.fahrenstechnik</u>, p.171, June 1940.

15. Boelter, Martinelli and Jonassin, <u>Tr.A.I.Mech.E.</u>, <u>63</u>, p.447, 1941

16. Colburn, <u>I.E.C.</u>, <u>33</u>, p.459, April 1941.

17. Duncan, Koffolt and Withrow, <u>Tr.A.I.Ch.E.</u>, <u>38</u>, p.259, 1942.

18. Molstad, Abbey, Thompson, & McKinney, <u>Tr.A.I.Ch.E.</u>, <u>38</u>, p.387, 1942

19. Marshall and Hougen, <u>Tr.A.I.Ch.E.</u>, <u>38</u>, p.91, 1942.

20. Gamson, Thodos & Hougen, <u>Tr.A.I.Ch.E. 39</u>, p.1, 1943.

21. Krischer, <u>Forsch.Gebiete Ingenieur</u>, <u>415</u>, 22 pp., April 1942.

22. Hurt, <u>I.E.C.</u>, <u>35</u>, p.522, May 1943.

23. Molstad, McKinney and Abbey, <u>Tr.A.I.Ch.E.</u>, <u>39</u>, p.605, 1943.

24. Matz, <u>Z.Ver.deut.Ing.</u>, <u>85</u>, p. 517, 1941.

25. Waddams, Chemistry and Industry, p. 206, 1944.

26. Wilke and Hougen, Tr.A.I.Ch.E., 41, p. 445, 1945.

27. Waddam, J.Soc.Chem.Ind. 63, p. 337, 1944.

28. Thiele, I.E.C., 38, p. 646, 1946.

29. Zhavoronkoz and Furmer, Khimicheskaya Prom., 12, p.7, 1944.

30. Grossman, Tr.A.I.Ch.E., 42, p. 535, 1946.

31. Van Krevelan and Hoftijzer, Rec.trav.chim. 66, p.48, 1947.

32. Leva, I.E.C., 39, p.857, 1947.

33. Van Krevelan, Hoftijzer, & van Hooran, Rec.trav.chim.,66,p.513,1947.

34. Klinkenberg and Mooy, Chem.Eng.Prog.,44, Tr.A.I.Ch.E.,p.17,Jan.1948.

35. Glof and Hawley, I.E.C., 40, p.1061, 1948.

36. Brinn, Friedman,Gluckert and Pigford, I.E.C.,40,p.1050,1948.

37. Wilhelm and Kwauk, Ch.Eng.Pr., 44, p.201, 1948.

38. Leva, Weintraub,Grummer & Pollchik, CEP,44,p.55,&619,1948.

39. Boreskov, Khim.Prom., 8, p. 10, and 9, p.5, 1947.

40. Thuston, J.Soc.Chem.Ind.(London), 67, p.289, 1948.

41. Taecker and Hougen, C.E.P., 45, p.188, 1949.

42. Van Krevelen and Krekels, Rec.trav.chim.,67, p.512, 1948.

43. Leroy, Hutchings, Stutzman, & Koch, C.E.P., 45, p.253, 1949.

44. Van Krevelen and Hoftijzer, C.E.P., 44, p.529, 1948.

45. McAdams, Pohleng, and St.John, C.E.P., 45, p.241, 1949.

46. Hobler, Przeglad chem., 6, p.1, 1948.

47. Hobson and Thodos, C.E.P., 45, p.517, 1949.

48. Klinkenberg, I.E.C., 40, p.1992, 1948.

49. Hansen, Angew Chem., B20, p. 177, 1948.

50. McCune and Wilhelm, I.E.C., 41, p.1124, 1949.

51. Resnick and White, C.E.P., 45, p.377, 1949.

52. Leva, Weintraub, Grummer and Pollchik, I.E.C., 41, p.1206, 1949.

53. Miller, Tr.A.I.Mech.E., 71, p. 357, 1949.

54. Morse, I.E.C., 41, p.1117, 1949.

55. Leva, C.E.P., 45, p. 563, 1949.

56. Levenspeil and Walton, Tr.A.I.Mech.E., p.139, 1949.

57. Hickley and Trilling, I.E.C., 41, p.1135, 1949.

58. Bedingfield, Drew, I.E.C., 42, p. 1164, 1950.

59. Singer and Wilhelm, C.E.P., 46, p. 343, 1950.

60. Berman, der warmetech Inst., 16, 8, p.11, 1947.

61. Weisman and Bonilla, I.E.C., 42, p.1099, 1950.

62. Linton and Sherwood, C.E.P., 46, p. 258, 1950

63. Gaffney and Drew, I.E.C., 42, p. 1120, 1950.

64. Kettenring, Monderfield and Smith, C.E.P., 46, p.139, 1950.

65. Kalil, M. Ch. E. Thesis, Brooklyn Polytech, 1950.

66. Hobson and Thodos, C.E.P., 47, p. 370, 1951.

67. Gamson, C.E.P., 47, p. 19, Jan. 1951

68. Shapatina and Kalyuzhuyi, Doklady Akad Nauk SSSR, 72, p.503, 1950.

69. Norman, Tr.I.Ch.E., 26, p. 81, 1948.

70. Migushina and Nakajina, Chem.Eng.(Japan), 15, p.30, 1951.

71. Yoshida and Tanaka, I.E.C., 43, p.1467, 1951.

72. Sherwood, I.E.C., 42, p.2077, 1950.

73. Kalil, D. of Chem. Eng. Thesis, Brooklyn Polytech., 1951.

74. Baron, C.E.P., 48, p.118, 1952.

75. Holtan, J. Chem. Phys., 20, p.526, 1952.

76. Brotz, _Chem. Ing. Tech._, _23_, p.408, 1951.

77. Wicke, _Chimia (Switz.)_, _5_, p. 173, 1951.

78. Havail and Traybal, _C.E.P._, _48_, p.362, 1952.

79. Pratt, _Tr.A.I.Ch.E. (London)_, _28_, p.177, 1950.

80. Ciborowski and Lesniewicz, _Przemysl Chem._, 7, p.621, 1951.

81. Gerald, _I.E.C._, _44_, p.233, 1952.

82. Nesselmann, _Chimia (Switz.)_, 6, p.106, 1952.

83. Schuler, Stallings & Smith, _C.E.P._, _48_, p.19, April 1952.

84. Kaston, Lapidus and Amundson, _J.Phys.Chem._, _56_, p.683, 1952.

85. Van Heerden, Nobel and van Krevelen, _I.E.C._, 45, p.1237, 1953.

86. Ju Chin Chu, Kalil, & Wetteroth, _C.E.P._, _49_, p.141, 1953.

87. Evans and Gerald, _C.E.P._, _49_, p. 135, 1953.

88. Lewis and White, _I.E.C._, _45_, p. 486, 1953.

89. Dryden, Strang and Withrow, _C.E.P._, _49_, p. 191, 1953.

90. Bras, _Chem. Eng._, _60_, p. 223, April 1953.

91. Bras, _Chem. Eng._, _60_, p. 238, May 1953.

92. Bras, _Chem. Eng._, _62_, p. 195, Jan.1955.

93. Kayser, _I.E.C._, p. 2634, Dec. 1953.

94. Furnas, _U.S.Bur.Mines Bull. 307_, 1929.

95. Furnas, _I.E.C._, _22_, p.721, 1930.

96. Ergun, _C.E.P._, _48_, p.227, 1952.

97. Perry's _Chemical Eng. Handbook_, 3 ed., McGraw-Hill, NY, p.546, 1950.

98. Hougen and Watson, _Chem. Proc. Principles_, Wiley, NY, 1948.

APPENDIX

MODIFIED FRICTION FACTOR, f', FOR GASES FLOWING IN GRANULAR SOLIDS.

$$f' = \frac{\Delta P_g \, D_P}{2 N \rho \mu^2 f''}$$

CHILTON & COLBURN CURVE (5)

COMPLETELY DRY CYLINDERS
WETTED CYLINDERS

COMPLETELY DRY SPHERES
WETTED SPHERES

CYLINDERS AND SPHERES FROM GAMSON, THODOS & HOUGEN (20)

Re
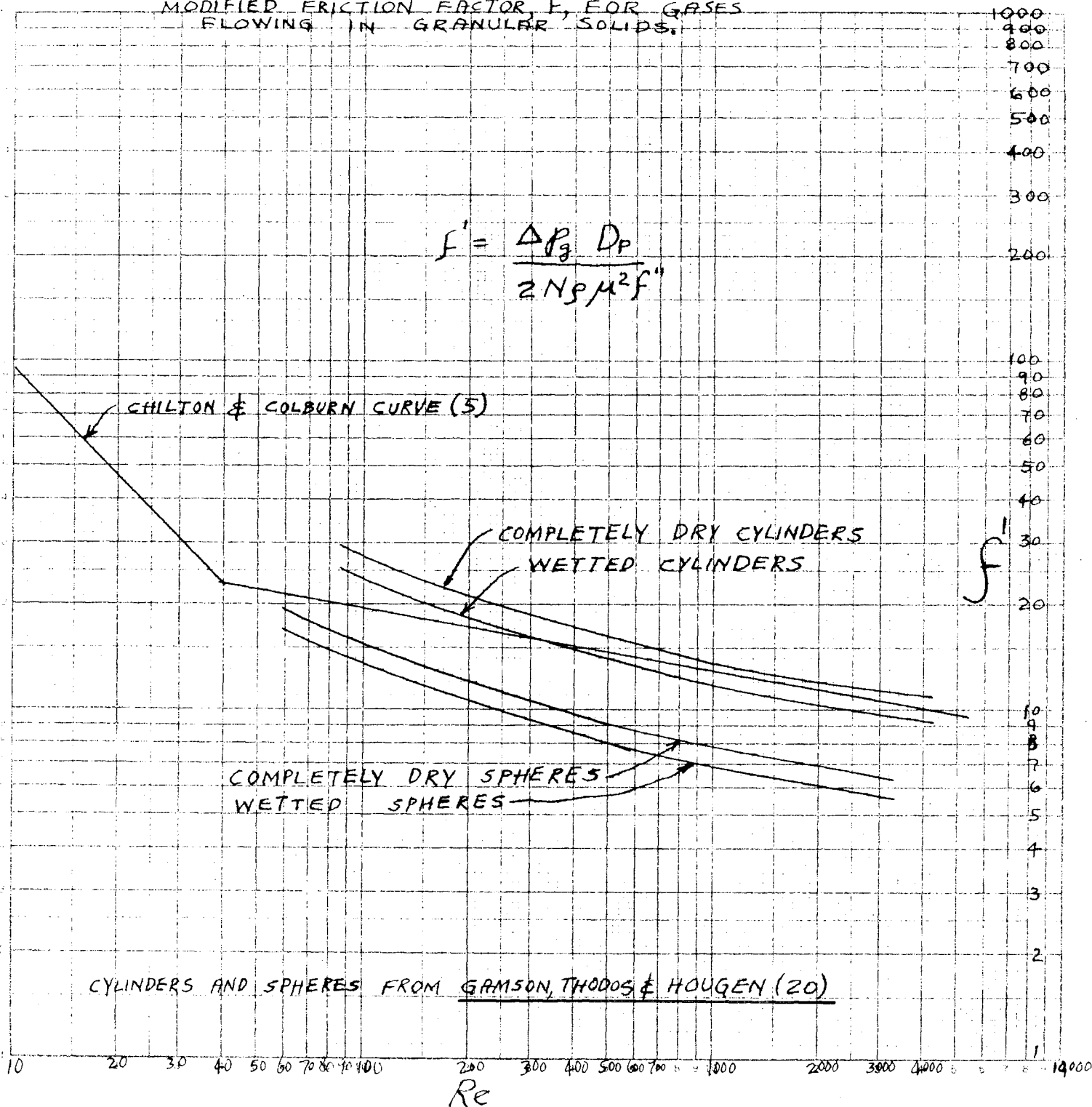
FIGURE #1

j_d and j_h vs $\left[\dfrac{D_p G}{\mu}\right]$

j_d — NUMBER FOR MASS TRANSFER IN GRANULAR SOLIDS

j_h — NUMBER FOR HEAT TRANSFER IN GRANULAR SOLIDS

$$j_d = \frac{k_g\, a\, M_m\, P_{gf}}{G a}\left[\frac{\mu}{\rho D_v}\right]_f^{2/3}$$

$$j_d = 0.989\left[\frac{D_p G}{\mu}\right]^{-0.41} \quad \text{FOR} \left[\frac{D_p G}{\mu}\right] > 350 \quad (12)$$

$$j_d = 16.8\left[\frac{D_p G}{\mu}\right]^{-1.0} \quad \text{FOR} \left[\frac{D_p G}{\mu}\right] < 40 \quad (13)$$

$$j_h = \frac{h}{c_p G}\left[\frac{c_p \mu}{k}\right]_f^{2/3}$$

$$j_h = 1.064\left[\frac{D_p G}{\mu}\right]^{-0.41} \quad \text{FOR} \left[\frac{D_p G}{\mu}\right] > 350 \quad (10)$$

$$j_h = 18.1\left[\frac{D_p G}{\mu}\right]^{-1.0} \quad \text{FOR} \left[\frac{D_p G}{\mu}\right] < 40 \quad (11)$$

j_h

j_d

j_h
OR
j_d

40

350

FROM GAMSON, THODOS & HOUGEN (20)

MODIFIED REYNOLDS NUMBER
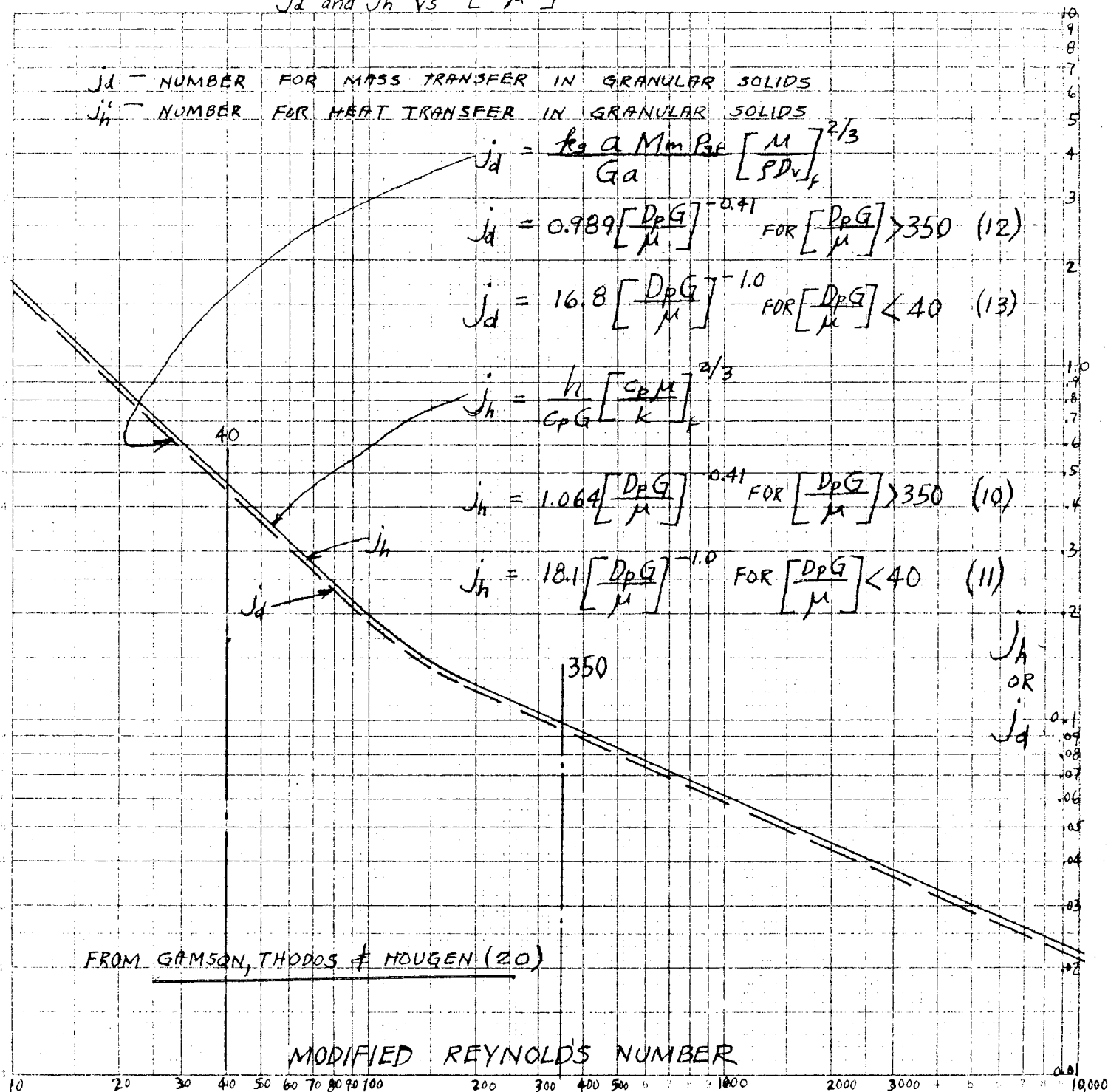
FIGURE #2

j AND a (HTU) NUMBERS FOR MASS AND HEAT TRANSFER

GASES FLOWING THROUGH GRANULAR BEDS

$$j_h = \frac{h}{c_p G} \left(\frac{c_p \mu}{k}\right)_f^{2/3}$$

$$j_h = 1.064 \left(\frac{D_p G}{\mu}\right)^{-0.41} \text{ FOR } \frac{D_p G}{\mu} > 350$$

$$j_h = 18.1 \left(\frac{D_p G}{\mu}\right)^{-1.0} \text{ FOR } \frac{D_p G}{\mu} < 40$$

$$j_d = \frac{k_g M_m p_{gf}}{G} \left(\frac{\mu}{\rho D_v}\right)_f^{2/3}$$

$$j_d = 0.989 \left(\frac{D_p G}{\mu}\right)^{-0.41} \text{ FOR } \frac{D_p G}{\mu} > 350$$

$$j_d = 16.8 \left(\frac{D_p G}{\mu}\right)^{-1.0} \text{ FOR } \frac{D_p G}{\mu} < 40$$

FROM – GAMSON, THODOS AND HOUGEN (20)

$\left[\frac{\mu}{\rho D_v}\right] = 2.5$

2.0

1.5

1.0

0.8

0.6

0.4

$\left[\frac{c_p \mu}{k}\right] = 0.80$

$= 0.74$

$$a(HTU)_h = \frac{1}{j_h} \left(\frac{c_p \mu}{k}\right)_f^{2/3}$$

$$a(HTU)_d = \frac{1}{j_d} \left(\frac{\mu}{\rho D_v}\right)_f^{2/3}$$

MODIFIED REYNOLDS NUMBER $\frac{D_p G}{\mu}$

FIGURE #3

j_d NUMBERS FOR MASS TRANSFER IN GASES FLOWING THROUGH GRANULAR SOLIDS

PREVIOUS EXTRAPOLATION
DATA OF GAMSON, THODOS & HOUGEN (20)

j_d

DATA OF
WILKE & HOUGEN (26)

$j_d = 1.82 \left(\dfrac{D_p G}{\mu}\right)^{-0.51}$ FOR $\dfrac{D_p G}{\mu} < 350$

$j_d = 0.989 \left(\dfrac{D_p G}{\mu}\right)^{-0.41}$

FOR $\dfrac{D_p G}{\mu} > 350$

FROM WILKE & HOUGEN, TR. AIChE, 41, p.445, 1945 (26)

REYNOLDS NUMBER $\dfrac{D_p G}{\mu}$

FIGURE #4

FIGURE #5

TRANSFER PROCESSES IN CONDUITS

FROM — CHILTON & COLBURN (6)

HEAT TRANSFER $\frac{L}{d}\left/\left(\frac{c\mu}{k}\right)^{2/3}\right.=1$

FRICTION $\frac{L}{d}\left/\left(\frac{c\mu}{k}\right)^{2/3}\right.=10$

$\frac{L}{d}\left/\left(\frac{c\mu}{k}\right)^{2/3}\right.=100$

$\frac{L}{d}\left/\left(\frac{c\mu}{k}\right)^{2/3}\right.=1000$

$\frac{1}{2}f, j$

.5

.5

2

$\frac{\mu_o}{\mu_f}=0.5$

1

$\frac{\mu_o}{\mu_f}=1$

2

$\frac{\mu_o}{\mu_f}=2$

HEAT TRANSFER AND FRICTION

HEAT TRANSFER: $j=\frac{h}{cG}\left(\frac{c\mu}{k}\right)^{2/3}=\frac{t_2-t_1}{\Delta t_m}\frac{d}{4L}\left(\frac{c\mu}{k}\right)^{2/3}$

MASS TRANSFER: $j=\frac{K P_{gf}}{\left(\frac{G}{M_m}\right)}\left(\frac{\mu}{\rho k_d}\right)^{2/3}=\frac{P_2-P_1}{\Delta P_m}\frac{P_{gf}}{P_g}\frac{d}{4L}\left(\frac{\mu}{\rho k_d}\right)^{2/3}$

FRICTION: $\frac{1}{2}f=\frac{R_g}{G^2}=\frac{\Delta P_{gg}}{G^2}\frac{d}{4L}$

$\frac{dG}{\mu_f}=$ MOD. R$_e$ No.

FIGURE # 6

FACTOR j_h AND j_d FOR HEAT AND MASS TRANSFER IN GASES FLOWING THROUGH TOWER PACKINGS

$$j_h = \frac{h}{c_P G}\left[\frac{c_P \mu}{K}\right]_f^{2/3}$$

$$j_d = \frac{k_g M_m P_{\mathit{ff}}}{G}\left[\frac{\mu}{\rho D_v}\right]_f^{2/3}$$

FOR BERL SADDLES

$$j_h = 0.920\left[\frac{G\sqrt{A_P}}{\mu}\right]^{-0.34}$$

$$j_d = 0.855\left[\frac{G\sqrt{A_P}}{\mu}\right]^{-0.34}$$

FOR RASCHIG & PARTITION RINGS

$$j_h = 1.148\left[\frac{G\sqrt{A_P}}{\mu}\right]^{-0.41}$$

$$j_d = 1.070\left[\frac{G\sqrt{A_P}}{\mu}\right]^{-0.41}$$

FOR SPHERES & CYLINDERS

$$j_h = 1.346\left[\frac{G\sqrt{A_P}}{\mu}\right]^{-0.41}$$

$$j_d = 1.251\left[\frac{G\sqrt{A_P}}{\mu}\right]^{-0.41}$$

WHERE $\left[\frac{G\sqrt{A_P}}{\mu}\right] > 620$
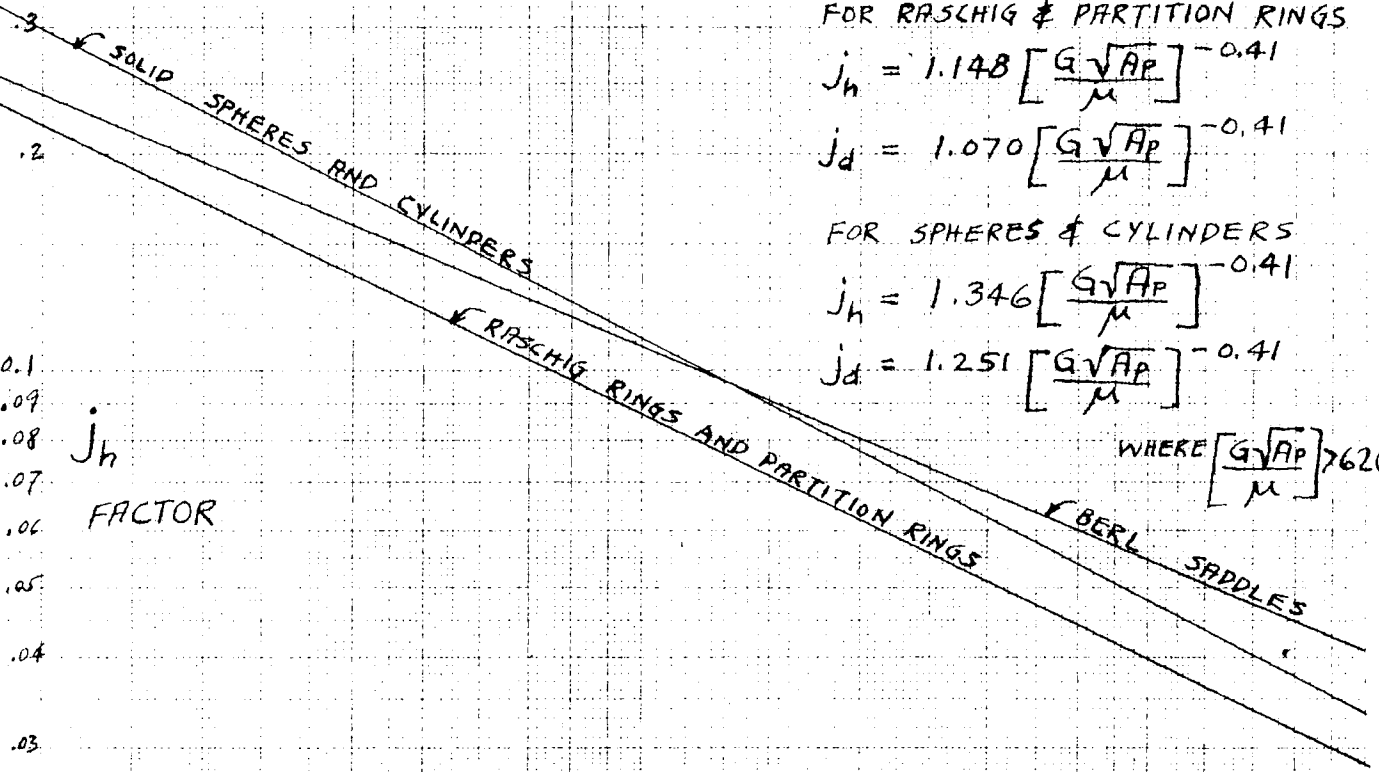
j_h FACTOR

SOLID SPHERES AND CYLINDERS

RASCHIG RINGS AND PARTITION RINGS

BERL SADDLES

FROM TRECKER & HOUGEN (41)

MODIFIED REYNOLDS NUMBER $\frac{G\sqrt{A_P}}{\mu}$

FIGURE #7

TABLE 8

PROPERTIES OF TOWER PACKINGS.

Packing	Nominal Size inches	Diameter Outside inches	Diameter Inside inches	Height inches	$\sqrt{A_p}$ feet	a_v sq.ft./ cu.ft.	Number per cu.ft.
Raschig rings	1/2	0.499	0.310	0.505	0.103	111	10,500
Raschig rings	1	1.017	0.756	1.012	0.210	58	1,350
Raschig rings	2	2.028	1.499	2.022	0.419	29	165
Partition rings (4 partitions 0.228 in.) random	2	1.990	1.491	2.054	0.490	36	150
staggered	2	1.990	1.491	2.054	0.490	--	---
Berl Saddles	1/4	------	------	------	0.0493	274	113,000
Berl Saddles	1/2	------	------	------	0.0968	155	17,600

From:

Taecker and Hougen (41)

TABLE (9)

From: Taecker and Hougen   (41)

Run No.	p	R_c	w	t_1-t_2	t_1	t_w	$a_v V$	h	G_1	$\sqrt{A_p}$	j_h	$\dfrac{G\sqrt{A_p}}{\mu}$
1	735	0.272	58.8	20.2	105.62	72.90	6.68	2.03	86.7	0.419	0.0783	792
2	735	0.206	43.1	20.8	105.48	73.85	6.68	1.67	63.6	0.419	0.0877	578
3	727	0.531	129	18.0	105.60	69.64	6.68	5.23	190	0.419	0.0568	1730
4	734	0.599	207	11.6	106.51	79.00	6.78	4.12	315	0.419	0.0450	2760
5	734	0.838	329	11.1	103.84	73.00	6.68	5.34	484	0.419	0.0368	4390
6	739	1.51	659	10.1	101.73	65.30	6.68	7.69	970	0.419	0.0265	8800
7	739	2.11	1100	8.40	103.64	68.86	6.68	10.9	1620	0.419	0.0225	14600
8	734	1.25	620	8.83	100.98	70.20	6.68	7.54	913	0.419	0.0276	8300
9	735	1.97	1040	8.25	103.39	70.72	6.68	10.9	1540	0.419	0.0233	13900
10	737	0.500	133	16.5	103.35	70.65	6.68	3.35	196	0.419	0.0572	1730
11	737	0.542	132	18.0	99.81	65.88	4.26	5.65	195	0.103	0.0970	443
12	737	0.378	79.4	20.9	100.04	66.79	4.26	4.42	117	0.103	0.126	226
13	723	0.210	35.3	26.0	101.66	68.99	4.26	3.18	52.2	0.103	0.203	119
14	734	0.126	18.4	29.9	102.06	69.35	4.26	2.54	27.2	0.103	0.312	62.3
15	745	0.599	177	14.9	100.11	69.04	4.26	6.80	260	0.103	0.0875	590
16	738	0.574	175	14.5	100.58	66.92	4.89	4.81	257	0.210	0.0626	1190
17	739	0.956	339	12.4	103.73	68.54	4.89	7.22	499	0.210	0.0484	2300
18	734	0.353	92.2	16.8	100.56	68.67	4.89	3.39	136	0.210	0.0836	629
19	734	0.227	45.9	21.7	103.87	70.30	4.89	2.34	67.6	0.210	0.116	313
20	740	0.068	12.2	24.2	101.39	74.12	4.89	1.32	18.0	0.210	0.245	83.7
21	736	0.726	29.3	10.9	100.90	68.22	4.86	5.85	431	0.210	0.0453	1990
22	737	1.33	705	8.26	100.93	68.67	4.86	10.3	1040	0.210	0.0551	4750
23	737	1.22	623	8.61	100.02	66.74	4.86	9.16	916	0.210	0.0354	4250

TABLE (9) (continued)

Run No.	p	R_c	w	t_1-t_2	t_1	t_w	a_vV	h	G_1	$\sqrt{A_p}$	j_h	$\dfrac{G\sqrt{A_p}}{\mu}$
24	739	1.69	1030	7.19	96.69	65.30	4.86	13.2	1520	0.210	0.0291	7040
25	738	2.07	1030	8.85	94.94	62.96	8.15	9.79	1520	0.490	0.0216	16400
26	729	1.98	8.33	9.90	97.25	64.45	8.15	9.32	1300	0.490	0.0240	14000
27	728	1.59	668	10.5	95.56	62.91	8.15	7.55	983	0.490	0.0250	10600
28	728	1.16	401	12.7	99.20	63.09	8.15	5.12	591	0.490	0.0290	6840
29	737	0.437	96.9	20.0	95.35	59.68	8.15	2.34	143	0.490	0.0550	1560
30	724	0.0192	31.9	26.6	101.52	63.07	8.15	1.10	47.0	0.490	0.0781	511
31	724	0.130	19.4	29.5	100.90	63.82	8.15	0.90	28.6	0.490	0.105	311
32	740	0.559	92.6	26.6	114.35	67.19	8.15	2.26	137	0.490	0.0553	1460
33	732	1.19	336	15.7	104.76	63.32	8.15	4.69	495	0.490	0.0317	5320
34	741	0.926	208	19.0	105.7	73.0	4.75	9.37	306	0.0968	0.101	652
35	741	0.706	157	19.4	105.4	72.5	4.75	7.05	231	0.0968	0.101	491
36	741	1.37	330	18.1	104.7	70.7	4.75	12.75	485	0.0968	0.0871	1030
37	741	1.28	294	18.8	105.2	70.1	4.75	11.41	433	0.0968	0.0874	921
38	735	0.252	1000	18.8	103.4	78.0	4.75	28.6	1470	.0968	0.0633	3150
39	740	0.195	676	10.8	98.8	70.9	4.75	20.4	995	0.0968	0.0679	2180
40	739	1.76	681	10.6	99.7	73.0	4.74	19.6	1000	.0968	0.0678	2140
41	739	1.56	467	12.5	102.5	73.0	4.74	16.1	701	0.0968	0.0756	1510
42	738	0.343	79.2	11.5	103.8	77.2	4.74	4.96	114	0.0968	0.145	240
43	738	0.176	36.3	14.4	102.3	77.2	4.05	2.24	53.3	.0968	0.201	114
44	735	0.236	61.3	18.6	102.9	77.6	4.05	3.98	90.1	.0968	0.147	193
45	735	0.181	45.5	20.7	102.7	78.9	4.05	3.36	67.2	.0968	0.167	144
46	735	0.110	26.6	16.8	101.2	76.6	4.05	2.42	39.0	.0968	0.0209	33.4
49	754	0.266	79.3	17.8	100.6	77.0	3.62	4.00	117	0.0968	0.159	250

TABLE (9) (continued)

Run No.	p	R_c	w	t_1-t_2	t_1	t_w	$a_v V$	h	G_1	$\sqrt{A_p}$	j_h	$\dfrac{G\sqrt{A_p}}{\mu}$
50	734	0.171	44.8	17.7	99.8	77.2	3.63	3.87	66.0	0.0968	0.195	141
51	734	0.102	27.9	12.4	88.1	77.2	3.62	2.81	41.2	0.0968	0.226	87.8
52	742	0.945	526	14.2	107.2	78.6	4.07	11.4	481	0.0968	0.0784	1030
53	742	0.895	291	16.3	107.9	79.3	4.07	10.8	428	0.0968	0.0834	907
54	741	1.19	325	12.9	99.9	62.0	4.29	10.0	480	0.103	0.0691	1090
55	741	0.972	265	16.3	98.6	62.6	4.29	8.80	390	0.103	0.0751	880
56	745	0.338	81.5	15.8	104.8	72.6	4.29	3.80	120	0.103	0.106	271
57	739	0.268	82.5	14.0	105.6	81.0	4.29	3.95	122	0.103	0.108	275
58	745	0.244	52.0	16.1	103.6	72.7	4.29	3.19	76.6	0.103	0.139	174
59	745	0.151	31.4	21.2	101.3	72.8	4.29	2.39	45.7	0.103	0.174	104
60	739	0.122	31.4	16.7	103.0	81.0	4.29	2.54	46.1	0.103	0.183	105
61	739	0.351	81.5	18.5	107.4	81.4	4.05	6.10	120	0.0493	0.169	130
62	742	0.220	45.6	21.0	100,7	74.8	4.05	4.55	67.2	0.0493	0.286	73.0
63	742	0.172	31.7	23.6	102.0	75.6	4.05	4.26	46.8	0.0493	0.304	50.9
64	738	0.690	215	14.1	88.7	64.5	4.05	11.1	316	0.0493	0.118	344

SAMPLE CALCULATIONS

For Table 9.

For 2 in. Raschig rings, 2.028 in. OD, 1.499 in. ID, 2.022 in. high

$$A_p = \frac{\pi(2.028)2.022 + \pi(1.499)2.022}{144 \text{ in}^2 / \text{sq.ft.}} = 0.174 \text{ sq.ft. surface area of one particle}$$

$$\sqrt{A_p} = \sqrt{0.174} = 0.419 \text{ ft.}$$

The effective diameter of the particle equivalent to the diameter of

a sphere having the same surface area as the particle is:

$$d = 0.567\sqrt{A_p} = 0.567 (0.419) = 0.237 \text{ ft.}$$

$$V = 97.9 \text{ sq. in.} \frac{\text{sq.ft.}}{144 \text{ sq.in.}} \quad 4 \text{ in. high bed} \frac{\text{ft.}}{12 \text{ in.}} = 0.23 \text{ cu.ft. of bed volume}$$

$$a_v = \frac{0.174 \text{ sq.ft.}}{\text{particle}} \frac{165 \text{ particles}}{\text{cu.ft.}} = 29 \frac{\text{sq.ft. of surface}}{\text{cu.ft. of volume}}$$

$$a_v V = 29 (0.23) = 6.68 \text{ sq. ft.}$$

Perry's Pg. 371 gives 0.019 centipoises for air at 105 ^{o}F

$$0.019 \text{ centipoises} \frac{(2.42 \text{ lb./hr.ft.})}{\text{centipoise}} = 0.046$$

$$G = \frac{w}{\text{area}} = \frac{58.8 \text{ lbs/hr}}{97.9 \text{ sq.in.}} \frac{144 \text{ sq.in.}}{\text{sq.ft.}} = 86.7 \frac{\text{lb}}{\text{hr.sq.ft.}}$$

$$\frac{G\sqrt{A_p}}{\mu} = \frac{86.7 (0.419)}{0.046} = 792$$

c_p for air at 85 ^{o}F (29 ^{o}C) from Perry's Pg. 229 = 0.26

K for air at 85 ^{o}F from Perry's Pg. 461 = 0.014

$$j_h = \frac{h}{c_p G}\left(\frac{c_p \mu}{k}\right)^{2/3} = \frac{2.03}{0.26(86.7)}\left(\frac{0.26(0.046)}{0.014}\right)^{0.67} = 0.0783$$

Hougen and Watson humidity chart Pg. 101, for $t_w = 72.9$ & $t_1 = 105.62$

gives humidity of 0.0135 lb.moles water/ lb.moles of dry air

$$0.0135 \frac{\text{lb.moles water}}{\text{lb.moles air}} \frac{18 \text{ lb/lb.moles water}}{29 \text{ lb/lb.moles dry air}} = 0.0084 \frac{\text{lb.water}}{\text{lb. dry air}}$$

$$w = 1.0084 \frac{\text{lb. water}}{\text{lb.wet air}} \frac{86.3 \text{ lbs.wet air}}{\text{hr.sq.ft.}} 0.68 \text{ sq.ft.} = 58.8 \frac{\text{lb.water}}{\text{hr.}}$$

$p = 735$ mm Hg = 735/760 = 0.97 atm, $R_c = 0.272$ lb water evap/hr

$t_1 = 105.62$ ^{o}F, $t_2 = 35.42$ ^{o}F

BY CORRENTI

j FACTOR FOR FIXED AND FLUIDIZED BEDS

EVANS AND GERALD (87)

MODIFIED REYNOLDS NUMBER $\frac{D_p G}{\mu}$

j_d

HOBSON AND THODOS

McCUNE AND WILHELM

24-28 MESH BENZOIC ACID-FLUIDIZED

20-24 MESH BENZOIC ACID-FLUIDIZED

12-14 MESH BENZOIC ACID-FLUIDIZED

8-10 MESH BENZOIC ACID-FLUIDIZED

FIXED BEDS — ALL MESHES OF BENZOIC ACID

FIGURE 10

Vertical axis label: $\dfrac{j_d}{(1-\epsilon)^{0.2}}$

Horizontal axis label: $\dfrac{6G}{S\mu(1-\epsilon)}$

ALL MESHES OF BENZOIC ACID IN FIXED BEDS

ALL MESHES OF BENZOIC ACID IN FLUIDIZED BEDS

GAMSON CORRELATION FOR SPHERES

EVANS AND GERALD'S (87)
COMPARISON WITH GAMSON'S (67)
CORRELATION FOR FIXED AND
FLUIDIZED BED MASS TRANSFER

BY CORRENTI

FIGURE #11

CORRELATION OF MASS TRANSFER DATA IN GRANULAR BEDS OF PACKED SOLIDS USING SCHMIDT NUMBERS RAISED TO THE 0.58 POWER.

FIGURE #12

CORRELATION OF MASS TRANSFER DATA FROM GRANULAR SOLIDS
USING SCHMIDT NUMBERS RAISED TO THE 2/3 POWER

FIGURE #13

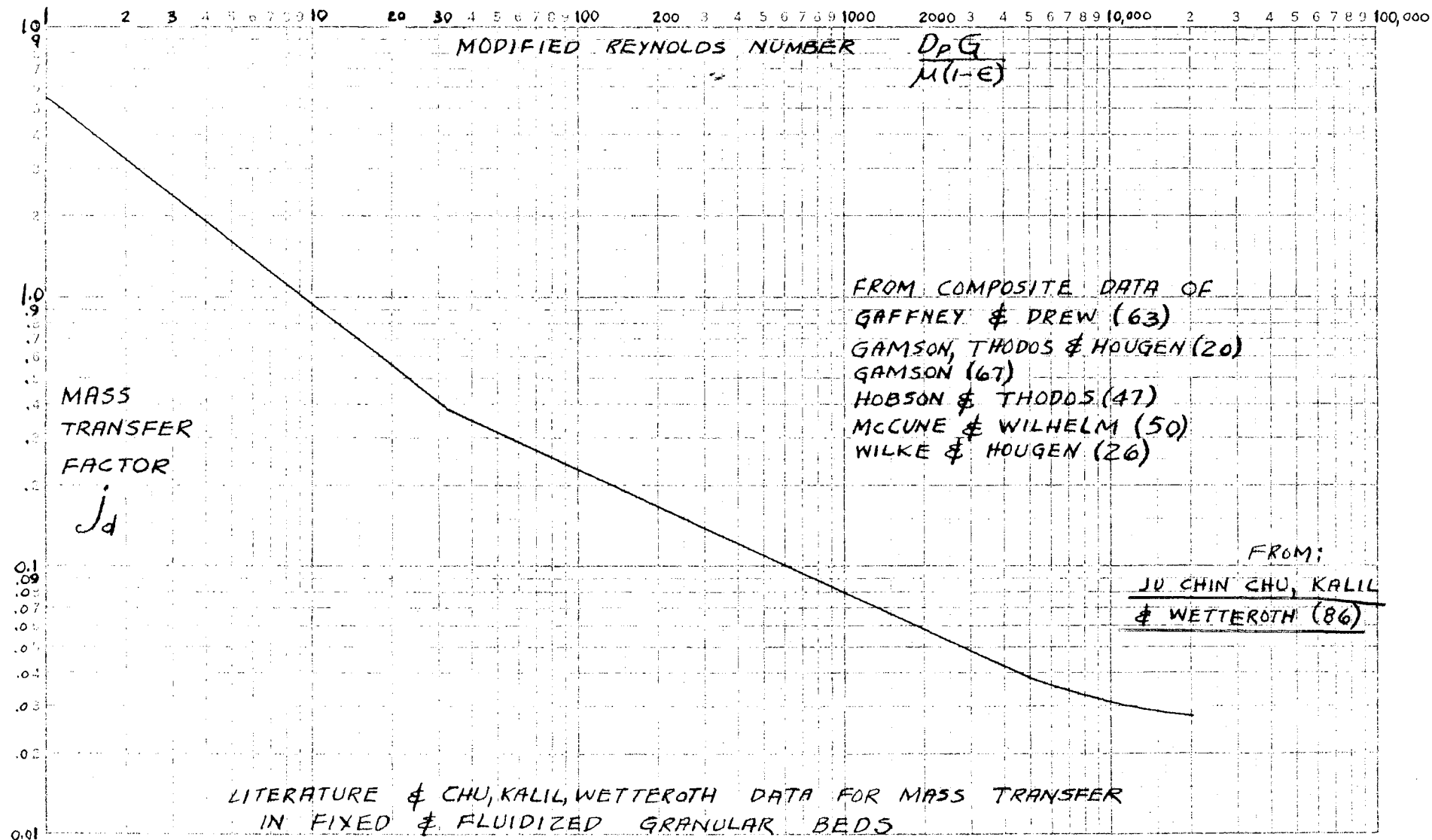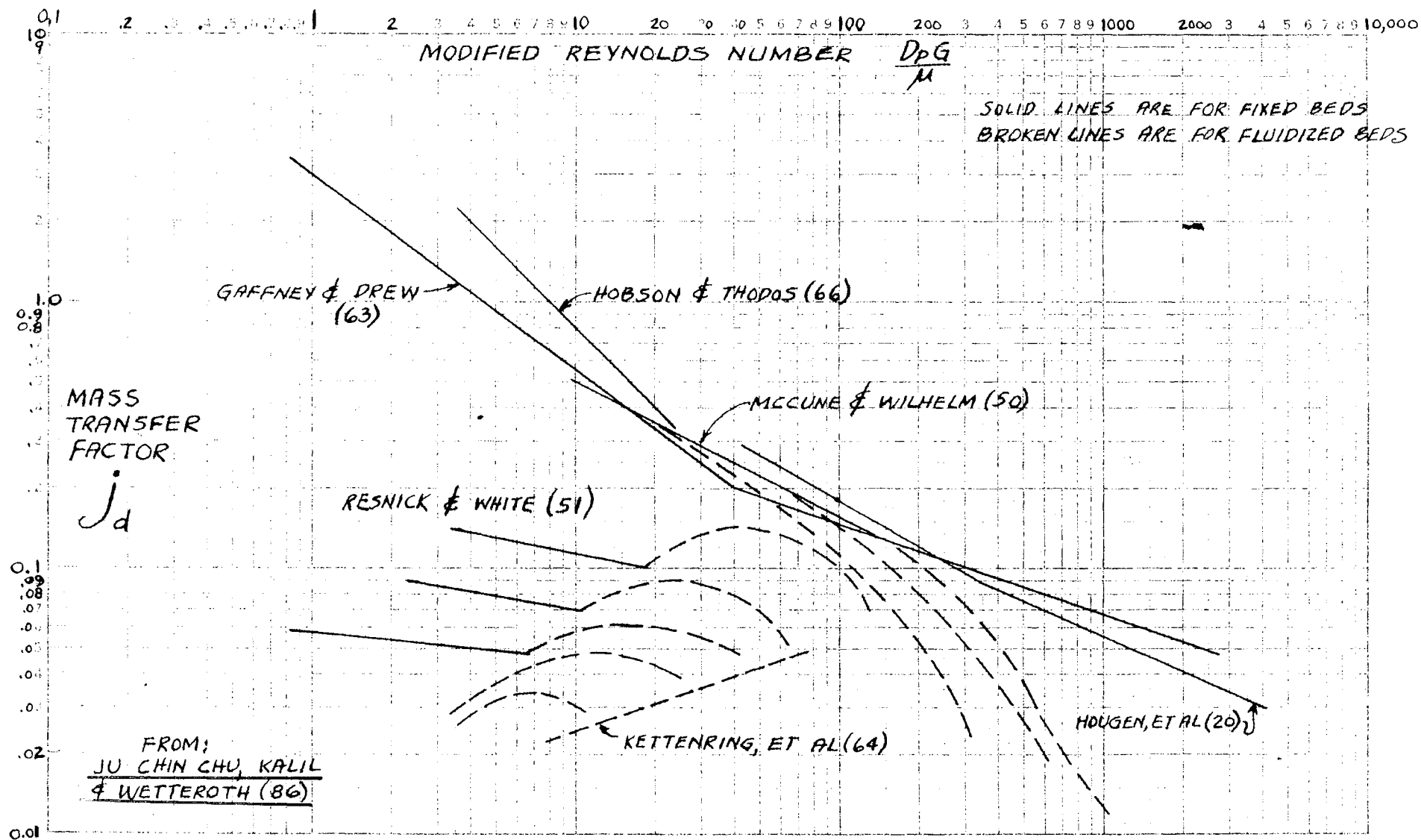MODIFIED REYNOLDS NUMBER $\dfrac{D_p G}{\mu (1-\epsilon)}$

MASS TRANSFER FACTOR j_d

FROM COMPOSITE DATA OF
GAFFNEY & DREW (63)
GAMSON, THODOS & HOUGEN (20)
GAMSON (67)
HOBSON & THODOS (47)
McCUNE & WILHELM (50)
WILKE & HOUGEN (26)

FROM:
JU CHIN CHU, KALIL
& WETTEROTH (86)

LITERATURE & CHU, KALIL, WETTEROTH DATA FOR MASS TRANSFER
IN FIXED & FLUIDIZED GRANULAR BEDS

FIGURE #14

MODIFIED REYNOLDS NUMBER $\dfrac{D_p G}{\mu}$

SOLID LINES ARE FOR FIXED BEDS
BROKEN LINES ARE FOR FLUIDIZED BEDS

1.0
0.9
0.8

MASS
TRANSFER
FACTOR

j_d

GAFFNEY & DREW
(63)

HOBSON & THODOS (66)

McCUNE & WILHELM (50)

RESNICK & WHITE (51)

0.1
.09
.08
.07
.06
.05
.04

.03

.02

FROM:
JU CHIN CHU, KALIL
& WETTEROTH (86)

KETTENRING, ET AL (64)

HOUGEN, ET AL (20)

0.01

COMPARISONS OF VARIOUS INVESTIGATIONS IN FIXED & FLUIDIZED GRANULAR BEDS

FIGURE #15

COMPOSITE OF PLOTS

BY CORRENTI
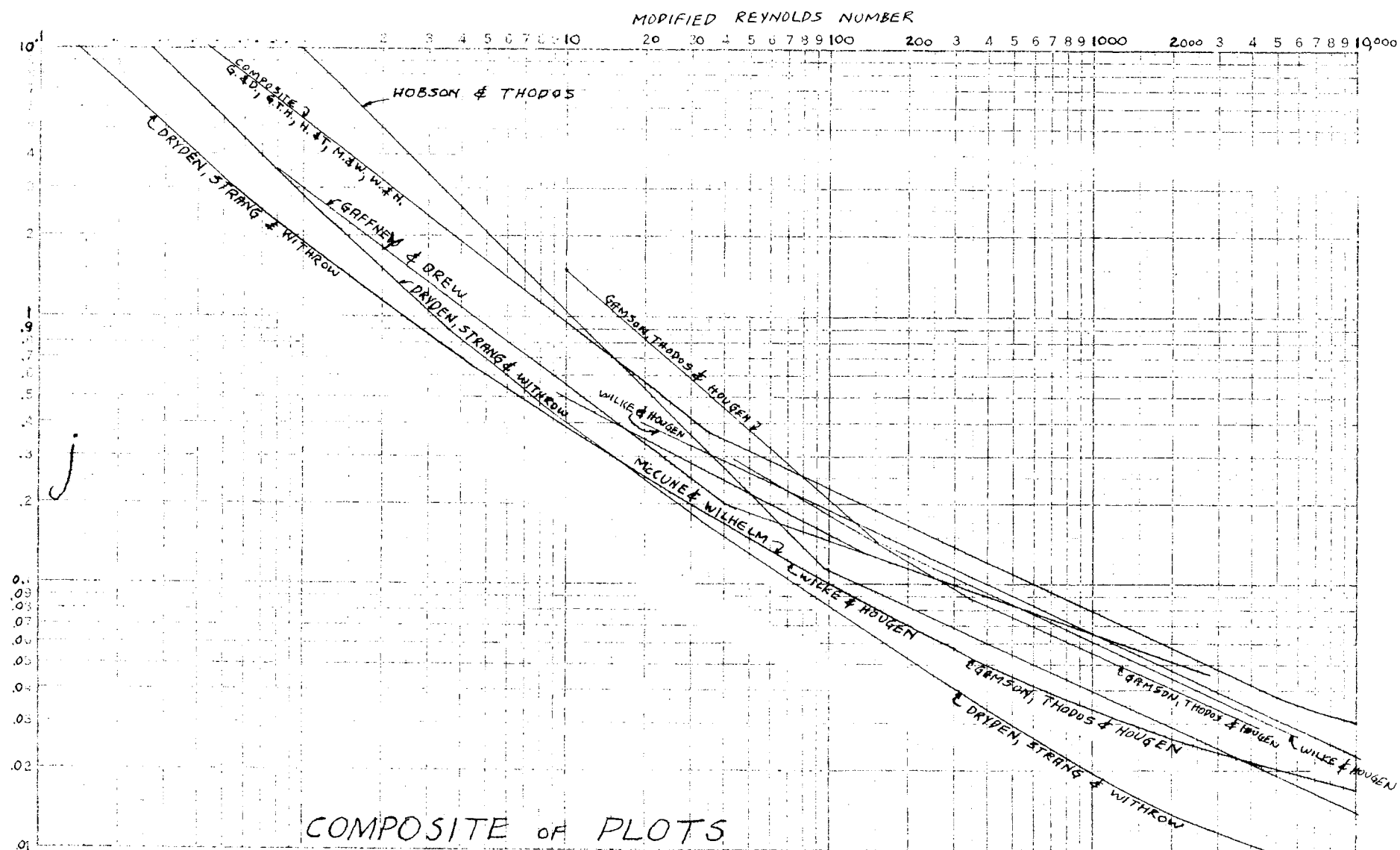
FIGURE #16

PROBLEM

Air is to be dried by contact with Sulfuric acid in a tower packed with one inch Raschig rings; 10,000 cubic feet of humid air per hour enters at $100^{\circ}F$ and a humidity of 0.015 lb water / lb dry air. It is to be dried to a humidity of 0.003 lb water / lb dry air by means of acid containing 70% Sulfuric acid by weight. At the mass velocity of 65 lb of dry air per hour per sq. ft. The overall mass transfer coefficient is 12 lb / hr.cu.ft. for each lb solute/lb dry air of driving force. The following are experimental values of equilibrium data.

mols of water/mols of H_2SO_4	mols H_2O/mols dry air
2	0.0010
3	0.0036
4	0.0070
5	0.0104
6	0.0134
7	0.0160
8	0.0178
10	0.0198

The exit acid is assumed to contain 40% H_2SO_4 by weight

Find:

a) The diameter of the tower in feet

b) The number of lbs of strong acid required perhour

c) The height of an overall HTU

d) The number of overall Transfer units

e) The height of the tower

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PROBLEM

$H_2O = 0.003 \dfrac{lb\,water}{lb\,DA}$ $H_2SO_4 = 70\%$
$H_2O = 30\%$ $X_2 = 2.33$

$Y_2 = 0.00483$

1"Raschig Rings

$G = \dfrac{V}{S} = 65\,lb\,DA/hr\,sq\,ft$

$K_y a = \dfrac{12\,lb\,water}{hr\,cuft\,lb\,solute} \; lbs\,DA\,driving\,force$

$Y_1 = 0.0241$ $8.16 X_1 = H_2SO_4 = 40\%$
$H_2O = 60\%$

a) Y_2exit $= 0.003 \dfrac{\#water}{\#DA} \dfrac{29\#air / \#mol\,air}{18\#water / \#mol\,water} = \dfrac{0.000167\,mol\,water}{0.0345\,mol\,DA} = 0.00483 \dfrac{mol\,water}{mol\,DA}$

Y_1entering $= 0.015 \dfrac{\#water}{\#DA} \dfrac{29\#air / \#mol\,DA}{18\#water / \#mol\,water} = \dfrac{0.00083\,mol\,water}{0.0345\,mol\,DA} = 0.0241 \dfrac{mol\,water}{mol\,DA}$

$\dfrac{1\;lb\;DA + 0.015\;lb\;water}{1\;lb\;DA} = \dfrac{\dfrac{1\,mol\,DA}{29} + \dfrac{0.015\,mol\,water}{18}}{1\;lb\;DA} = 0.0353 \dfrac{mol\,wet\,air}{lb\,DA}$

Humid volume $= \dfrac{359\,cuft\,wet\,air}{mol\,wet\,air} \dfrac{560}{492} \dfrac{0.0353\,mol\,wet\,air}{lb\,DA} = 14.43 \dfrac{cuft\,wet\,air}{lb\,DA}$

$\dfrac{wet\,air}{hr} = 10{,}000 \dfrac{cu\,ft\,wet\,air}{hr} \dfrac{lb\,DA}{14.43\,cuft\,wet\,air} = 693 \dfrac{lb\,DA}{hr}$

Area $= 693 \dfrac{lb\,DA}{hr} \dfrac{hr\,sq\,ft}{65\,lb\,DA} = 10.62$ sq ft

$\dfrac{\pi D^2}{4} = 10.67$ sq ft

$D = \sqrt{\dfrac{4(10.67)}{\pi}} = 3.69$ ft in diameter

b) acid

$\dfrac{30}{70} = 0.429 \dfrac{lb\,water\,entering}{lbs\,H_2SO_4}$

$\dfrac{60}{40} = 1.500 \dfrac{lb\,water\,leaving}{lb\,H_2SO_4}$

$\dfrac{lbs\,pure\,acid}{hr}\left(1.500 - 0.429 \dfrac{lbs\,water}{lbs\,acid}\right) = 693 \dfrac{lbsDA}{hr}\left(0.015 - 0.003 \dfrac{lbs\,water}{lbs\,DA}\right)$

acid = 11.1 lbs of strong acid required.

c) HTU $= 65 \dfrac{lbs\,DA}{hr\,sq\,ft} \dfrac{hr\,cu\,ft\,lbs\,water}{12\,lbs\,water\;lbs\,DA} = 5.42$ ft

Y VS X

$Y = \dfrac{\text{mols. WATER}}{\text{mols. D.A.}}$

$\begin{cases} X_1 = 8.16 \\ Y_1 = 0.0242 \end{cases}$

OPERATING LINE

EQUILIBRIUM LINE

F_1

$\begin{cases} X_2 = 2.34 \\ Y_2 = 0.0048 \end{cases}$

F_2

= 5.286 STEPS

$X = \dfrac{\text{MOLS WATER}}{\text{MOLS ACID}}$
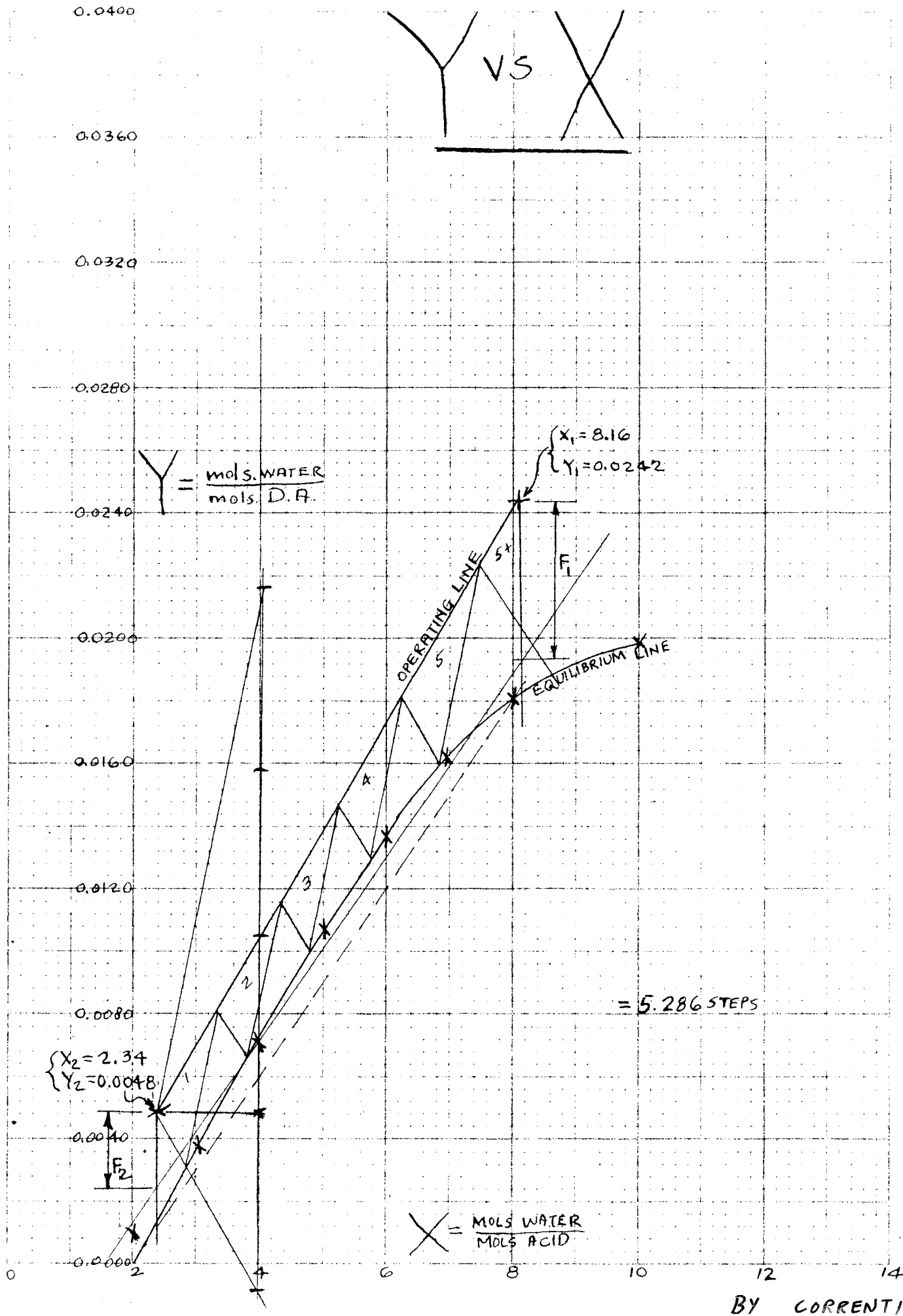
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$$\frac{1}{Y - Y^*} \quad \text{VS} \quad Y$$

AREA UNDER CURVE
$= N_{OY} = 275 \times 0.0194$
$= 5.30$ FT

BY CORRENTI

d)

$$X_1 = 1.500 \frac{\text{lbs water}}{\text{lbs acid}} \cdot 98 \frac{\text{lbs acid}}{\text{mols acid}} \cdot \frac{\text{mol water}}{18 \text{ lbs water}} = 8.16 \frac{\text{mols water}}{\text{mols acid}}$$

$$X_2 = 0.0429 \frac{\text{lbs water}}{\text{lbs acid}} \cdot 98 \frac{\text{lbs acid}}{\text{mols acid}} \cdot \frac{\text{mols water}}{18 \text{ lbs water}} = 2.34 \frac{\text{mols water}}{\text{mols acid}}$$

and $Y_1 = 0.0242 \dfrac{\text{mol water}}{\text{mol DA}}$

$Y_2 = 0.0048 \dfrac{\text{mol water}}{\text{mol DA}}$

X	Y	Y*	Y-Y*	1/Y-Y*	
2.34(X_2)	0.0048	0.0017	0.0031	323	
3	0.0080	0.0036	0.0034	294	
4	0.0120	0.0070	0.0033	303	Picking values
5	0.0160	0.0104	0.0033	303	directly from
6	0.0180	0.0134	0.0036	278	graph
7	0.0200	0.0160	0.0043	233	
8	0.0220	0.0178	0.0058	173	
8.16(X_1)	0.0242	0.0180	0.0062	161	

e) $F_1 = Y_1 - Y_1^* = 0.0048$ from graph

$F_2 = Y_2 - Y_2^* = 0.0028$ from graph

$$F_{1m} = \frac{F_1 - F_2}{\log_n \frac{F_1}{F_2}} = \frac{0.0048 - 0.0028}{\log_n \frac{0.0048}{0.0028}} = \frac{0.0020}{\log_n 1.71} = 0.0037$$

$$N_{oy} = \frac{Y_1 - Y_2}{F_{1m}} = \frac{0.0242 - 0.0048}{0.0037} = \frac{0.0194}{0.0037} = 5.2 \text{ checks graph}$$

f) $Z = (HTU)(N) = 5.42(5.30) = 28.7$ ft high tower

SOLUTION BY J_h

1" Raschig ring, 1.017" OD, 0.756 " ID, 1.012" High

$$A_p = \frac{\pi(1.017)1.012 + \pi(0.756)1.012}{144} = \frac{3.23+2.43}{144} = 0.044 \frac{ft^2 \text{ surface}}{\text{for one particle}}$$

$$\sqrt{A_p} = \sqrt{0.044} = 0.210 \text{ ft}$$

effective diameter of particle is equivalent to the diameter of a sphere having the same surface area as the particle.

$$d = 0.567\sqrt{A_p} = 0.567(0.210) = 0.119 \text{ ft}$$

$$a_v = 0.044 \frac{ft^2}{\text{particle}} \left(1350 \frac{\text{particles}}{\text{cuft}}\right) = 58 \frac{\text{sq ft of surface}}{\text{cu ft of volume}}$$

$$\mu = 0.019 \text{ centipoises } (2.42) = 0.046 \frac{lb}{hr \ ft}$$

from Perry's Pg. 371, μ for air at 100°F

$$\frac{G\sqrt{A_p}}{\mu} = \frac{65(0.210)}{0.046} = 297 \text{ for the modified Reynolds number}$$

$$j_h = 1.148\left[\frac{G\sqrt{A_p}}{\mu}\right]^{-0.41} = \frac{1.148}{(297)^{0.41}} = \frac{1.148}{10.4} = 0.1105$$

$$j_d = 1.070\left[\frac{G\sqrt{A_p}}{\mu}\right]^{-0.41} = \frac{1.070}{(297)^{0.41}} = \frac{1.070}{10.4} = 0.1030$$

c_p for air at 100° F = 0.26 from Perry's Pg. 229

k for air at 100°F = 0.014 from Perry's Pg. 461

$$\frac{h}{c_p G} = j = 0.1105 \quad \text{and} \quad \frac{c_p}{h} = \frac{1}{0.1105} = 9.05$$

h = 0.1105 (0.26) 65 = 1.865

$$a_v H_h = \frac{1}{0.1105} = 9.05$$

$$H_h = \frac{9.05}{58} = 0.156$$

$$\left[\frac{c_p \mu}{k}\right]^{2/3} = \left[\frac{(0.26)0.046}{0.014}\right]^{2/3} = (0.855)^{0.67} = 0.90$$

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$$j_d = \frac{k_g \; p_{gf} \; M_m}{G} \left[\frac{\mu}{\rho \; D_v} \right]^{2/3}$$

$$\frac{j_d}{\left[\frac{\mu}{\rho \; D_v} \right]^{2/3}} = \frac{k_g \; p_{gf} \; M_m}{G} = \frac{1}{a_v \; H_d}$$

D_v from Perry's Pg. 539

$$D_v = 0.256 \; \frac{cm^2}{sec} \left[\frac{in}{2.54 \; cm} \right]^2 \; \frac{ft^2}{144 \; sq \; in} \; \frac{3600 \; sec}{hr} = 0.993 \; \frac{ft^2}{hr}$$

$\rho = 0.0808$ lbs/cu ft for air from Perry's Pg. 176

$$\left[\frac{\mu}{\rho \; D_v} \right]^{2/3} = \left[\frac{0.046}{0.0808(0.993)} \right]^{2/3} = (0.574)^{0.67} = 0.69$$

$$\frac{j_d}{\left[\frac{\mu}{\rho \; D_v} \right]^{2/3}} = \frac{1}{a_v \; H_d}$$

$$\frac{0.1030}{0.69} = \frac{1}{58 \; H_d}$$

$$H_d = \frac{0.69}{58(0.1030)} = 0.116$$

j_d form graph for $\left[\frac{G \; A_p}{\mu} \right] = 297$ is 0.155

DISCUSSION OF PROBLEM

The solutions to the previous problem will show a discrepancy between the classical method of graphical integration and the j_d method. Integration gives the graphical basic absorption theoretical method and is academically correct. The j_d method does not agree with the first method for a perhaps obvious reason that the problem as stated is mythical. All data for the equilibrium conditions, flow rates, humidity, concentrations, entrance and exit conditions and coefficients have been assumed.

The only reason for problem illustration is to show the two methods of solution. The simplicity of solution by the j_d method will seem clear when knowing certain data such as the characteristics of the packing material, handbook information, a plot of modified Reynolds number versus j_d factors and the basic j_d equations can predict a design of a tower filled with packing material.

The analogy of heat and mass transfer can be therefore quite useful to the practical chemical engineer in his designs for industrial purposes. The analogy of course can be extended to heat exchangers and condensers and other types of unit operations.