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The effect of surface tension upon the flow of fluids through packed beds

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THE EFFECT OF SURFACE TENSION UPON THE
FLOW OF FLUIDS THROUGH PACKED BEDS

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THE EFFECT OF SURFACE TENSION UPON THE FLOW OF FLUIDS THROUGH PACKED BEDS

SUMMARY

Flow tests through beds of unconsolidated sand, glass beads and 1/4 inch Berl saddles showed that wide variations in the surface tension of a fluid had no significant effect on its flowrate through the bed. Viscosity and density were the only fluid properties affecting such flow. Excellent comparative results were observed between the flowrates through sand beds and those predicted by the Leva equation.

The complex Brownell and Katz equation for predicting the residual saturation of a packed bed was found to apply for fluids having a much wider range of surface tension than those originally used to derive their equation.

A new relationship of greater utility than the Brownell and Katz equation was derived for predicting the residual saturation of a packed bed. It permits the direct calculation of residual saturation without the need of secondary or auxiliary graphs. The new equation is as follows:

$$S_r = 1/\,30.2\left(\frac{D^2\phi^2 X^3 \Delta P}{(1-X)^2 \gamma L}\right)^{-0.365}$$

Operating holdup of a packed column was found to depend solely upon the flowrate and was not a function of fluid surface tension as contended by Jesser and Elgin.

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THE EFFECT OF SURFACE TENSION UPON THE FLOW OF FLUIDS THROUGH PACKED BEDS

INTRODUCTION

While the flow of fluids through conduits has been well established, the complete understanding of flow through packed columns or even through beds of consolidated or unconsolidated granular materials has not been as well defined. An understanding of the mechanism involved in this type of flow is of considerable importance in a wide variety of engineering fields; for example, it is of obvious concern to those industries which employ packed towers for absorption, distillation or catalytic work. A less obvious field is the process of filtration, which is in effect, an adaptation of the principles of fluid flow through packed beds. Furthermore, the mechanisms of fluids percolating through granular beds or rock strata are of vital importance to the petroleum industry especially in those locations where secondary methods of crude recovery by water flooding are extensively practiced.(1)

It is therefore easy to understand why, especially in the recent years, a considerable effort has been made by many engineers to further the general knowledge on this type of fluid flow. However, there has been a considerable variance between authors and their data. In fact, in many cases there has been a definite lack of correlation between their results. For the most part the investigators have assumed that flow through porous media closely resembles flow through pipes, and consequently they have used theoretical approaches analagous to those proposed by Fanning for relating the friction factor to the Reynold's number. It has likewise been generally accepted that various assumptions or modifications were required in this type of analysis, since the area which was available for the flowing fluid in a packed bed was in some way defined by the size and shape of the particle and the porosity of the bed.

In their analysis of the factors involved in flow through packed beds, the majority of the authors in the literature have further assumed that the only fluid properties which affect flow or fluid behavior are density and viscosity. Only a few authors have apparently considered the possible effect of the fluid surface tension upon either single phase or two phase flow.

In the initial studies of flow through packed beds, Chilton and Colburn(2) were one of the first authors to successfully correlate their data by the use of a modified Fanning equation; however, Perry(3) has suggested that for beds of abnormal voids content (beyond the range of 35-45%) a more reliable estimate of the flow conditions may be obtained from the Carman equation in which was incorporated a consideration of the surface area and shape of the particles. In 1933, Fancher and Lewis(4) made extensive studies on the flow of oil and water through various sand beds, and their efforts resulted in an average curve which described the flow through beds of unconsolidated sands.

Within the more recent years, Leva(5)(6) investigated the flow through packed beds, and in 1949 he formulated general equations for both viscous and turbulent flow which, in general, corresponded to those given by Carman. Ergun(7) has now further developed equations which differentiate between the viscous and kinetic energy losses, and thereby determined their proportionate roles in the total pressure drop.

In the relationships which have been discussed so far, the fluid properties which have been used in describing the flow mechanism through packed beds have included only the fluid viscosity and the density. The literature contains data from only a few authors who have also considered the surface tension of the fluid and its effect upon the flow characteristics through porous media. One of the first to study the effect of surface tension were Wyckoff and Botset(8) who studied the flow of water-oil and gas-oil mixtures through porous sand beds. They found that moderate variations in liquid viscosity and surface tension had negligible effects upon the permeability-saturation properties of sand beds.

Perhaps the most recent consideration of the effect of surface tension upon the behavior of a fluid in a packed bed has been given by Brownell and Katz(9) in their first series of articles. From a wide variety and range of published data they derived theoretical equations in which the amount of liquid retained by the bed was accounted for, and the pore volume thus removed from the total bed voidage was assumed as unavailable for gaseous flow. They termed this maximum pore space which was removed from flow by capillary forces as the "residual saturation," and they derived the following equation to define this quantity:

$$S_r = 1/86.3 \left[\frac{D^2 X^{n-m} \Delta P}{32L \gamma \cos \theta} \right]^{-0.264}$$

where: γ = is the surface tension or interfacial tension, #/ft.
S_r = residual saturation = fraction of total porosity filled with capillary-held liquid
D = particle diameter, ft.
X = porosity of bed
ΔP = pressure drop, #/sq.ft.
L = bed thickness, ft.
θ - is the contact angle (taken as 180° for all cases)
n-m are empirical exponents

Several questionable points are immediately forthcoming upon a close analysis of this equation. Firstly, the values of the exponents n and m are highly significant especially since they are used as the powers to which the porosity is raised. Lapple severely questions the validity of these exponents and illustrates how they result in widely divergent values as compared with the Carman equation. Furthermore, the use of the residual saturation equation is completely unworkable unless the user has on hand the graphs relating the exponents, n and m, to the ratio of the sphericity of the particle and the pososity of the bed.

Since these exponents, n and m, are used by Brownell and Katz in their theoretical flow equations, it is reasonable to assume that some simpler relationship might exist between them and the porosity exponents suggested by other authors. In other words, it is possible that the residual saturation of a porous bed might be expressed in terms of easily calculated properties of the bed and fluid, and not be dependent upon cumbersome graphical analyses.

It has also been suggested that surface tension may influence flooding velocity and liquid holdup in packed columns. Several authors(10,11,12,13) have examined the variables which affect column holdup and, in general, it has been established that the holdup is independent of the gas velocity in two phase flow, and instead is solely dependent upon the rate of liquid flowing. Furnas and Bellinger(14) showed that the holdup was proportional to the liquor rate raised to a variable power which was dependent upon the type of packing.

The effect of surface tension upon liquid holdup, however, has only been investigated by Jesser and Elgin(15) who carefully distinguished between "static" holdup, which was the fluid required to wet the bed, and the "operating" holdup. These authors claimed that the "static" holdup was independent of the liquor rate while the true "operating" holdup varied exponentially as described by Furnas. Their test data showed that surface tension has little effect upon the operating holdup of a packed tower at low liquor rates up to 1200 #/hr.sq.ft., but that at higher rates the holdup was markedly greater for fluids of lower surface tension. Actually their test data also illustrated that at the lower flow rates there was a reversal of their general conclusion.

Some question is raised in the interpretation of Jesser and Elgin's definition of static holdup and the method they employed in measuring operating holdup. For example, if static holdup is merely the liquid required to wet the bed, then the operating holdup is the <u>total</u> liquid which would completely drain from the column since the static holdup would not leave the bed except by evaporation. However, Jesser and Elgin actually differentiated between static and operating holdup by determining the point at which the rate of drainage became constant. They claimed that at this point all of the operating holdup had left the column and that all the remaining liquid was merely static holdup. Obviously this is not consistent with their original definition of static holdup. Furthermore, all of their holdup values were measured at a constant drainage time (10 minutes for a 6 inch diameter column packed to a height of 50½ inches with ½ inch carbon rings) irrespective of the flowrate or the total holdup.

Since there was no definite conclusion in the literature, it was the purpose of this study to investigate the effect of surface tension upon the phenomena of flow through porous beds.

Specifically, in these experiments the fluid surface tension was varied over a much wider range than has been given in the reference literature, and the effect of this change was actually studied from four different viewpoints and conditions; namely:

1. Continuous flow through completely saturated sand beds.
2. Continuous flow through partially saturated beds of glass beads.
3. The effect upon the residual saturation of a porous bed, and the development of a simplified and easily workable equation for estimating this quantity.
4. The effect upon the operating holdup of a packed column.

EQUIPMENT AND MATERIALS

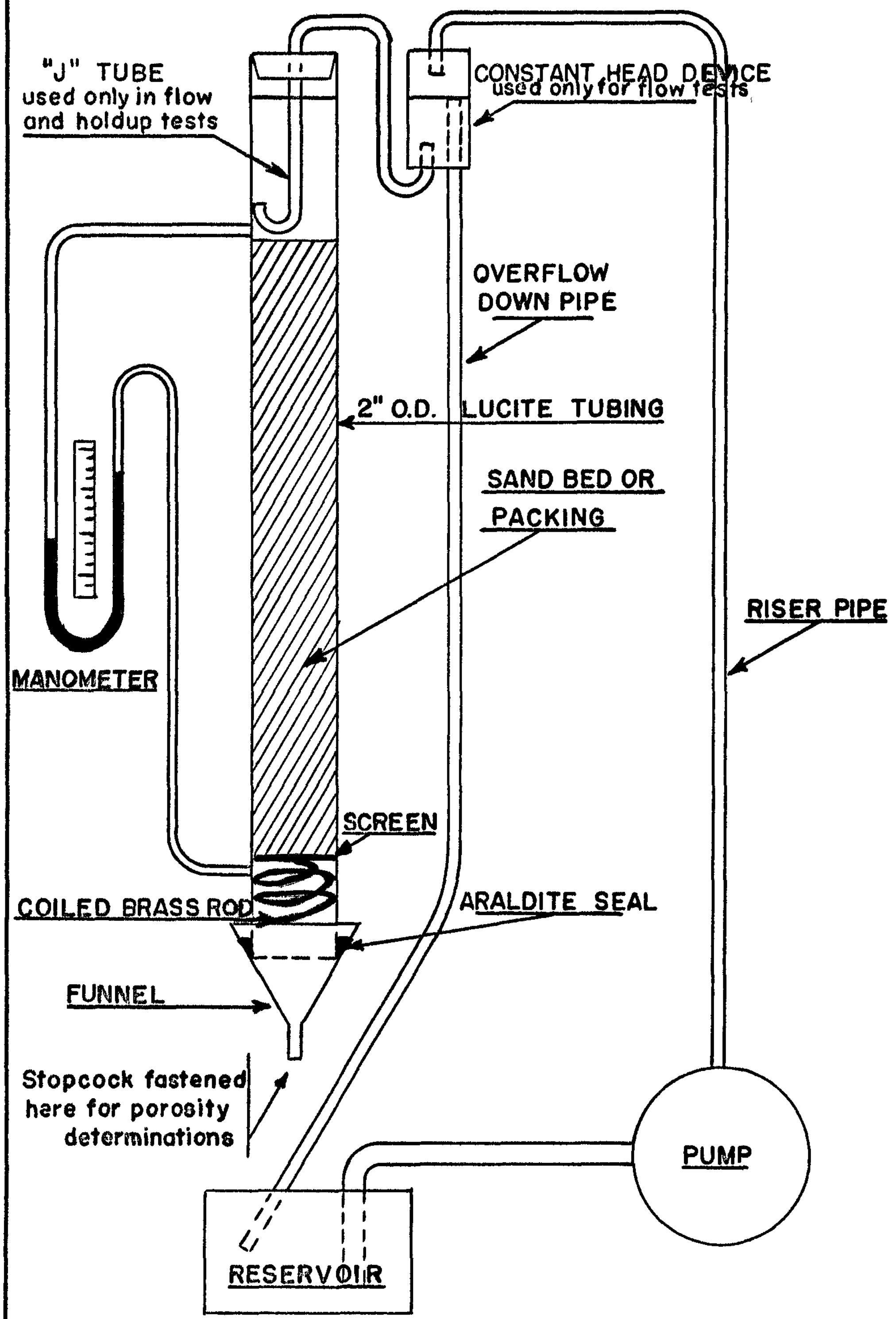
The column which was used in these tests is shown diagrammatically in Figure 1. It was made from standard two inch diameter Lucite tubing which had a 1.611 inch inside diameter. A spiral coil of brass rod was used as the screen and bed support and was merely press-fitted or snapped into the lower end of the column. A 35M screen was used on all the work on sand beds while a 10M screen was used with the glass beads and Berl saddles. Pressure taps were drilled into the side of the Lucite tubing at points below the screen support and above the packing bed, and the pressure tap leads were connected to a simple U-tube mercury manometer. The lower end of the column was fitted into a 60° glass funnel and then cemented in place with Araldite resin. It was then possible to use a rubber tubing connection to fasten a stopcock fitting on to the end of the funnel and thereby carefully meter the fluid as it filled the bed during the porosity determinations. A constant head device was fashioned from large glass tubing and, as is shown in Figure 1, regulated the head of fluid above the packed bed.

The packing materials which were used in these tests were as follows:

1. -20 + 30M Ottawa sand which was generously donated for these tests by the Ottawa Silica Company of Ottawa, Illinois.
2. 5mm diameter glass beads.
3. Standard 1/4 inch Maurice Knight Berl Saddles.

FIG. I

EQUIPMENT USED IN FLOW STUDIES



Fluids of widely differing surface tension were employed in these tests. Tap water served as the reference basis and the surface tension was lowered by additions of surface active agents such as Aerosol OT, AquaRex D, and Tergitol #7. Aerosol OT is a dioctyl ester of sodium sulfo succinic acid, Tergitol #7 is a sodium sulfate derivitive of 3,9 diethyl tridecanol-6, while AquaRex D is a sodium sulfate monoester of a mixture of higher fatty alcohols consisting chiefly of the lauryl and myristyl derivitives. These solutions did not differ significantly from each other or from water in either density or viscosity, and thereby permitted a good means of investigating solely the effect of fluid surface tension. Other fluids were also examined, however, and these included a 4M/ Kg H_2O solution of $CaCl_2$ and an 86% glycerine-water solution. These latter solutions not only differed from water with respect to surface tension, but also in density and viscosity.

The surface tensions of these various solutions varied from approximately 1.70 to 5.55×10^{-3} #/ft, and it should be noted that this range considerably exceeds those used by Brownell and Katz (2.33 to 4.98×10^{-3}) in the derivation of their residual saturation equation.

Surface tensions were determined using a DuNouy ring apparatus and the measurements were made on the fluid after it had been recycled through the packed bed. Densities were determined by pycnometer, and viscosities by the Hoeppler falling ball technique. All of the test properties checked well with those given in the literature although, of course, some variations in the surface tension of the tap water were observed as compared with those given for distilled water.

TEST PROCEDURES

Bed Porosity

The porosity of the packed bed was measured in two ways. First, it was calculated from the weight of dry packing, the density of the packing material, and the volume of the packed bed. Second, it was actually measured by determining the quantity of fluid required to fill the pores of the bed. In this second method, the stopcock at the end of the column was closed and the column was evacuated through a stopper which was tightly fitted in the upper end of the column. With the system under vacuum, water was admitted through the lower stopcock until the entire bed was filled. During the filling process the column was constantly and vigorously tapped to expell any entrapped air. To determine the porosity, the quantity of water required to fill the funnel up to the bottom of the screen was subtracted from the total quantity required to fill the entire column up to the top of the packing. This method yielded very reproducible results as long as the bed was carefully and slowly filled. Porosity was then expressed as the volume of voids divided by the total bed volume.

Residual Saturation

By definition, the residual saturation of a porous bed is the fraction of the total voids in the bed which are filled with fluid held by capillary forces and thereby not available as effective areas of flow. Similarly it is also the amount of fluid required to wet the packing. Residual saturation was determined in these tests merely by allowing the sand columns to drain for 12 hours after the porosity measurements had been made. Packed beds of glass beads and Berls required only one hour drainage time. The difference between the total quantity of fluid admitted to the column and the quantity withdrawn represented the amount of fluid remaining in the bed; i.e., the fluid held by capillary forces. The residual saturation was then calculated as the volume of fluid retained in the bed divided by the total volume of voids.

Flow Through Completely Saturated Sand Bed

The column of sand was completely filled with the fluid in the same manner as in the porosity measurements, but additional fluid was admitted to the bed so that the fluid level was approximately eight or nine inches above the top of the bed. The lower stopcock then was closed and the upper end of the column fitted with the constant head device as shown in Figure 1. The pump was started and the leads from the constant head reservoir were filled with fluid before the lower stopcock was removed. With the fluid flowing through the bed, the level of the fluid above the sand bed was regulated to the desired height and the system was permitted to come to equilibrium. In all of the sand bed tests the funnel was completely filled with the fluid flowing through the bed. After the system had reached equilibrium for the given height, several flow rate measurements were made by collecting the effluent and carefully weighing it. Pressure drop - flow rate data were obtained for each fluid at several different fluid heights.

Operating Holdup Tests

For these tests the lower funnel and the constant head device were removed from the column, and the fluid was pumped directly to the column through the glass J tube. Since this column was considerably smaller than the one used by Jesser and Elgin in their studies, it was necessary to use much shorter time intervals in determining the operating holdup. Hence, the holdup was determined for draining time intervals of 5, 15, and 30 seconds after the pump had been shut off. There was little or no significant time lag between shutting down the circulating pump and catching the effluent. Draining cycles of longer duration were not used since after 30 seconds the effluent had reached the slow drop-wise stage.

Several check tests were made both of the flow rate and holdup measurements (approximately 12 flow rate and 6 holdup measurements per test), and the average of the readings were taken as representative of the operating conditions.

DISCUSSION OF TEST DATA (Part 1)

Flow Through Packed Beds

Using the equipment pictured in Figure 1, a series of tests were made to investigate the effect of surface tension upon the flow rate through a packed bed. The surface tension of water was lowered from 69.1 to 25.2 dynes per centimeter through the use of surface active agents, and the flow rates were observed for various pressure drops across beds of -20+30M Ottawa sand and glass beads. The effect of these surface active agents upon the viscosity of the water was negligible, hence they offered a good means of evaluating solely the effect of surface tension. A concentrated solution of calcium chloride was also used as representative of a fluid having both high surface tension (81.0 dynes/cm) and relatively high viscosity (2.68 centipoise), while a glycerine-water solution represented a high viscosity (73.1 centipoise) but similar surface tension fluid as compared with water.

All of the sand bed flow tests were made in the viscous flow region, and the test data observed during these tests are given in Table 1. In the case of the flow tests through the beds of glass beads there was excessive foaming when solutions of low surface tension were used. It was obvious from visual examination that the column was rapidly becoming plugged with minute air bubbles, and accurate or reliable flow rates could not be obtained. Similar difficulties were encountered in flow tests using Berl saddles, but no trouble was found during the sand bed tests. This was probably due to the less vigorous action during the viscous flow of the fluid through the sand beds as compared with the more turbulent flow through the larger sized packings.

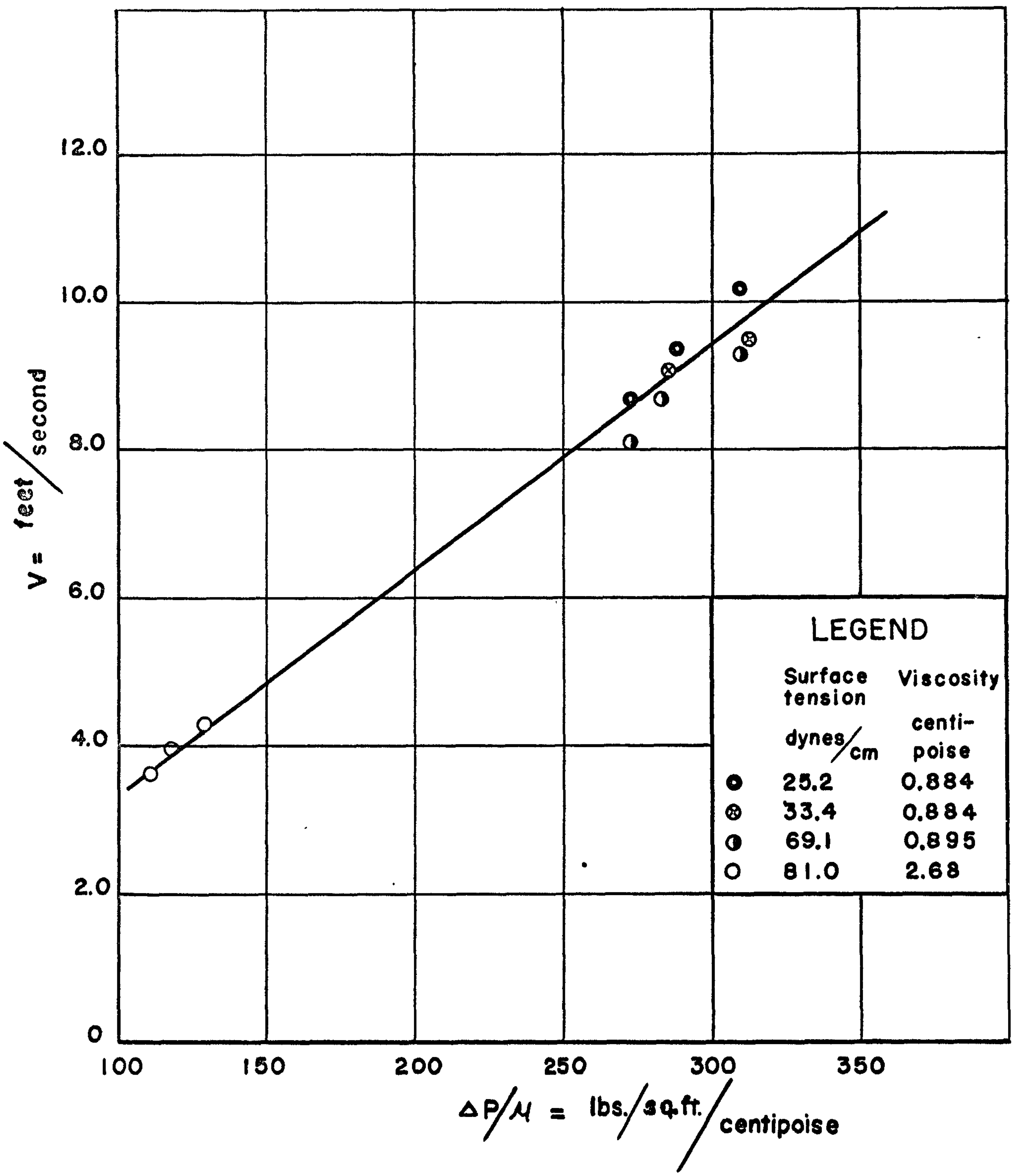
The test data which were observed under these conditions showed that there was no significant change in flow rate through the sand beds due to the wide variations in surface tension. Whatever changes occurred could be related to the corresponding changes in fluid viscosity. This is illustrated in Graph A. It is further illustrated in Table 2 where the ratio $V\mu/P$ has been tabulated for the different fluids having varying surface tension. Statistically there are no significant differences between these ratios.

It was also of interest to compare the observed flow rates through the various beds with those calculated by the different equations given in the literature. The velocity of flow has been calculated for the different fluids using the equations of Fancher and Lewis, Leva, Carman, Ergun, Brownell and Katz, and Chilton. The comparison of the observed flows with the calculated values are given in Table 3.

Excellent comparative results were obtained between the observed data and those predicted by Leva's equation. On the average, the deviation from the observed flow rates through the sand beds was approximately 3%.

Graph A

EFFECT OF SURFACE TENSION ON FLOW THROUGH SAND BEDS



The poorest correlation of calculated and observed data was noted with the Brownell and Katz equation and the Chilton-Colburn equation. As has been previously stated, this latter equation is not recommended for use on packed beds outside of the range of 35-45%. These words of caution are substantiated in these tests where a poor correlation is observed on beds of approximately 32% voids and a somewhat better correlation on beds of 38% voids. It was interesting to note that the Brownell and Katz equation appeared to have a similar restriction. It is believed that the source of variation lies in their porosity exponents, n and m. Lapple has pointed out that serious variations exist between the Carman equation and the Brownell equation which are attributed to these exponents. In his criticism, Lapple also illustrated that the equations approach equal results around a bed porosity of 40% and then deviate sharply as the bed porosity becomes either larger or smaller. It would appear from the present data that Lapple's criticism has merit.

TABLE I

EFFECT OF VARYING SURFACE TENSION ON FLOW THROUGH PACKED BEDS

Type of Packing	Fluid or Solution	Temp °C	Fluid Properties: Density #/cu ft	Viscosity Centipoise	Sur. Ten. dynes/cm	Bed Properties: Height cm	Porosity Dry	Porosity Wet	ΔP #/sq ft	Average Flow Rate g/min	ft/sec
Ottawa Sand	Water	24.0	62.3	0.895	69.1	84.1	31.7	32.3	244.0	194.8	0.00811
-20+30M	"	"	"	"	"	"	"	"	252.0	208.5	0.00871
"	"	"	"	"	"	"	"	"	277.5	222.5	0.00930
"	0.1% Aerosol	24.5	"	0.884	25.2	84.8	32.3	31.9	242.0	209.6	0.00874
"	"	"	"	"	"	"	"	"	252.0	224.5	0.00938
"	"	"	"	"	"	"	"	"	273.0	244.6	0.01021
"	0.1% Aqua Rex D	24.0	"	0.884	33.4	84.6	32.2	32.1	252.0	218.0	0.00910
"	"	"	"	"	"	"	"	"	277.5	227.1	0.00947
"	$CaCl_2$* Solution	24.5	78.6	2.68	81.0	84.2	31.8	31.6	297.0	109.7	0.00362
"	"	"	"	"	"	"	"	"	317.0	120.6	0.00398
"	"	"	"	"	"	"	"	"	347.5	129.6	0.00428
"	Glycerine & Water	30.0	75.7	73.1	65.0	84.7	32.2	32.9	322.0	4.92	0.000169
5mm Glass Beads	Water	28.5	62.3	0.813	68.5	90.8	38.2	38.0	13.3	1182.0	0.0494
"	"	"	"	"	"	"	"	"	11.0	1140.0	0.0476
"	0.1% Aerosol Sol.	"	"	0.801	24.8	"	"	"	13.7	905.0	0.0378‡
"	"	"	"	"	"	"	"	"	15.9	1000.0	0.0419‡
"	"	"	"	"	"	"	"	"	17.2	985.0	0.0410‡

* $CaCl_2$ solution: 4M/kg H_2O.
‡ Excessive foaming, values unreliable.

TABLE 2

FLOW RATIO, $v\mu/P$, FOR SOLUTIONS OF VARYING SURFACE TENSIONS

Surface Tension Dynes/cm	Ratio $v\mu/P$			Average Ratio $v\mu/P$
81.0	0.0326	0.0336	0.0330	0.0331
69.1	0.0298	0.0310	0.0300	0.0303
65.0	0.0386	--	--	0.0386
33.4	0.0319	0.0302	--	0.0311
25.2	0.0318	0.0329	0.0330	0.0329

TABLE 3

COMPARISON OF OBSERVED FLOW RATE VS CALCULATED VALUES

Bed of Material	Fluid Properties: Sur.Ten. dynes/cm	Fluid Properties: Viscosity Centipoise	Flow Rate Observed ft/sec x 10^3	Calculated Flow Rate ft/sec x 10^3: Fancher & Lewis	Leva	Carman	Ergun	Brownell & Katz	Chilton & Colburn
-20+30M Ottawa Sand	69.1	0.895	8.11	7.66	8.49	9.42	8.75	15.2	15.3
"	"	"	8.71	7.91	8.76	9.74	9.00	15.7	15.9
"	"	"	9.30	8.70	9.60	10.69	9.70	17.3	17.2
"	25.2	0.884	8.74	7.66	8.47	9.41	8.80	15.0	15.3
"	"	"	9.38	7.98	8.83	9.80	9.05	15.6	15.9
"	"	"	10.21	8.65	9.56	10.61	9.65	16.9	17.3
"	33.4	0.884	9.10	7.96	8.75	9.71	8.80	14.4	15.9
"	"	"	9.47	8.76	9.63	10.70	9.70	15.8	17.6
"	81.0	2.68	3.62	3.12	3.28	3.64	4.65	5.62	6.24
"	"	"	3.98	3.34	3.49	3.88	5.03	6.00	6.65
"	"	"	4.28	3.66	3.82	4.25	5.35	6.57	7.30
"	65.1	73.1	0.17	0.12	0.14	0.16	0.20	0.23	0.25
5mm Glass Beads	68.5	0.813	49.4	*	55.4	61.5	34.0	56.5	45.9
"	"	"	47.6	*	45.8	50.9	30.0	46.6	38.0
"	24.8	0.801	37.8ǂ	*	57.6	64.1	34.0	58.4	48.0
"	"	"	41.9ǂ	*	67.5	75.0	38.0	68.0	56.0
"	"	"	41.0ǂ	*	72.5	80.5	40.0	73.1	60.1

* Fancher and Lewis equation applicable only on unconsolidated sand beds.
ǂ Excessive foaming, observed flow rates doubtful.

DISCUSSION OF TEST DATA (Part 2)

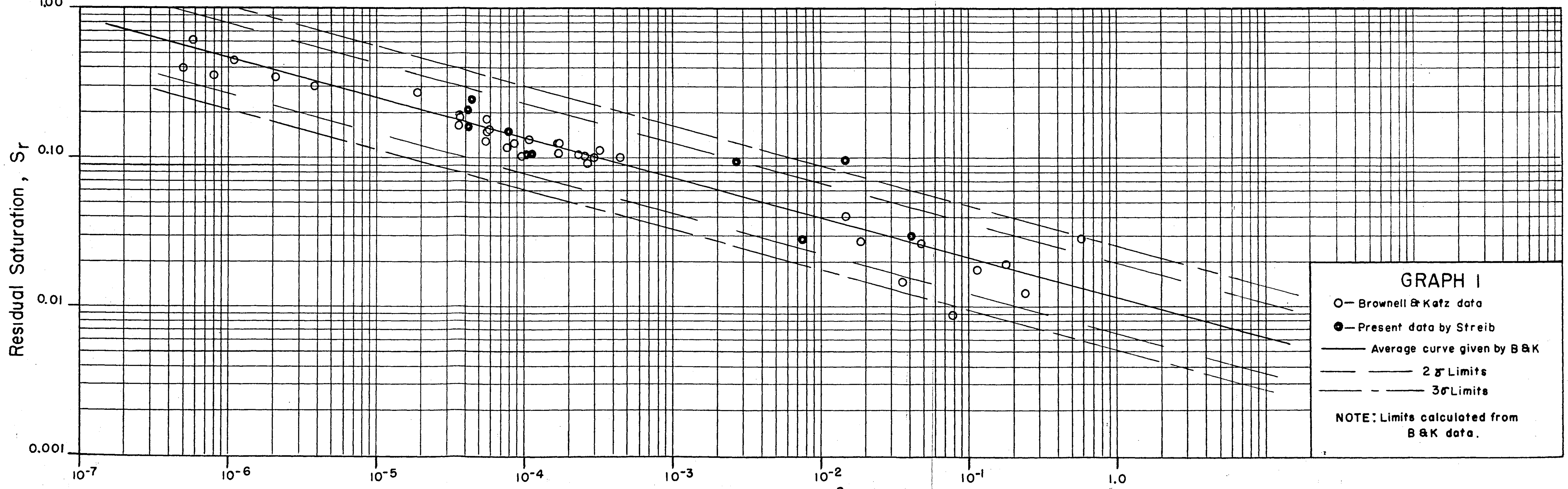
Residual Saturation of Packed Beds

The range of surface tensions investigated in these tests far exceeded those used by Brownell and Katz. As can be seen from Table 4, the fluids used in these studies ranged from 1.70 to 5.55×10^{-3} pounds per foot, while the Brownell and Katz equation was based upon fluids ranging from 2.74 to 4.98×10^{-3} pounds per foot. Residual saturation values were determined for beds of Ottawa sand, glass beads, and Berl saddles; consequently these tests also covered a wide range of bed porosities and particle sphericities.

The test data are given in Table 4 and it can be seen that, in general, they check well with the values predicted from the Brownell and Katz equation. It must be concluded that the equation as given by these authors may be considered satisfactory for fluids whose surface tensions exceed the range of values used in their derivation.

However, the variations that did exist between the presently observed and calculated values prompted a closer examination of the Brownell and Katz data to determine whether these variations exceeded the limits of the straight line given by them. From a cusory examination of their graph it would appear that there had been excellent agreement of their data and their proposed equation. However, a statistical analysis of the data indicated that while there was a high degree of correlation (0.968) between log S_r and log Cap. No., the

RESIDUAL SATURATION VS. CAPILLARY



$$\text{Capillary \# } = K = \frac{D^2 X^{n-m} \Delta P}{32 L \sigma \cos\theta}$$

confidence limits (σ_{sy} = 0.114 in log analysis) were such that all but one of the presently observed data fell within the two sigma limits. For illustration purposes the Brownell and Katz data have been plotted in Graph 1 together with the confidence limits of the equation. The data given in this work have also been plotted in the same graph. Obviously the equation presented by these authors has rather wide acceptance limits when applied to beds of all porosities and particle dimensions. Similarly it should be noted that their equation is far less reliable for the higher porosity beds of packings such as Berl saddles than it is for beds of unconsolidated sand.

It has already been observed that the use of the Brownell and Katz equation is unwieldy and restricted due to the need of a second graph to determine the porosity exponents, n and m. Hence, an attempt was made to derive a new equation which would be based solely upon the physical characteristics of the packing particles, the dimensions of the packed bed, and the properties of the fluid.

Since Brownell and Katz utilize the exponents n and m in their flow equations, it was possible to equate their value for the fluid velocity through the bed to the value given by the Leva equation. By doing so it was then possible to solve for the relation of X^{n-m} in terms of the particle sphericity (ϕ) and the porosity of the bed (X). This calculation was as follows:

1. $$V = \frac{X^{n-m} \Delta P g D^2}{32 \mu L}$$ (Brownell and Katz)

2. $$V = \frac{\Delta P \; D^2 g \phi^2 X^3}{200 L \mu (1-X)^2} = \frac{\Delta P D^2 g}{L \mu} \left[\frac{\phi^2 \; X^3}{200(1-X)^2} \right]$$ (Leva)

where: $\phi = 4.87 v^{2/3}/A$

and v = volume of particle
A = surface area of particle

Hence

$$\frac{X^{n-m}}{32} = \frac{\phi^2 \; X^3}{200(1-X)^2}$$

or

$$X^{n-m} = C \; \frac{\phi^2 \; X^3}{(1-X)^2}$$

where C = a constant

Therefore, the proposed method of plotting then becomes:

$$\log \frac{D^2 \phi^2 \; X^3 \Delta P}{(1-X)^2 \gamma L} \text{ vs } \log S_r$$

Except for the consolidated sands which had no particle diameter given, all of the values used by Brownell and Katz in their original derivation were used in this new relationship. The derived values are given in Table 5, together with the corresponding residual saturation values. Theoretically, the log-log plot of these points should yield a straight line, and from Graph 2 it can be seen that this was the case. A statistical analysis of this new relationship of residual saturation indicated a high degree of correlation (0.911) but a somewhat higher degree of dispersion ($\sigma_{sy} = 0.176$) than was observed on the original Brownell and Katz equation. The equation for this newly derived relation is as follows:

$$S_r = 1/30.2 \left[\frac{D^2 \phi^2 \; X^3 \Delta P}{(1-X)^2 \gamma L} \right]^{-0.365}$$

or: $$\text{Log } S_r = 0.365 \text{ Log } \frac{D^2 \phi^2 \; X^3 \Delta P}{(1-X)^2 \gamma L} - 1.4797$$

All but one of the residual saturation values observed in this thesis presentation fell within the two sigma limits for this equation.

Hence, despite its slightly higher degree of uncertainty this new relationship has greatly improved utility. It now permits the direct calculation of residual saturation values of packed beds without the need of secondary or auxiliary graphs.

GRAPH 2

NEW RELATIONSHIP FOR DETERMINING RESIDUAL SATURATION OF PACKED BEDS

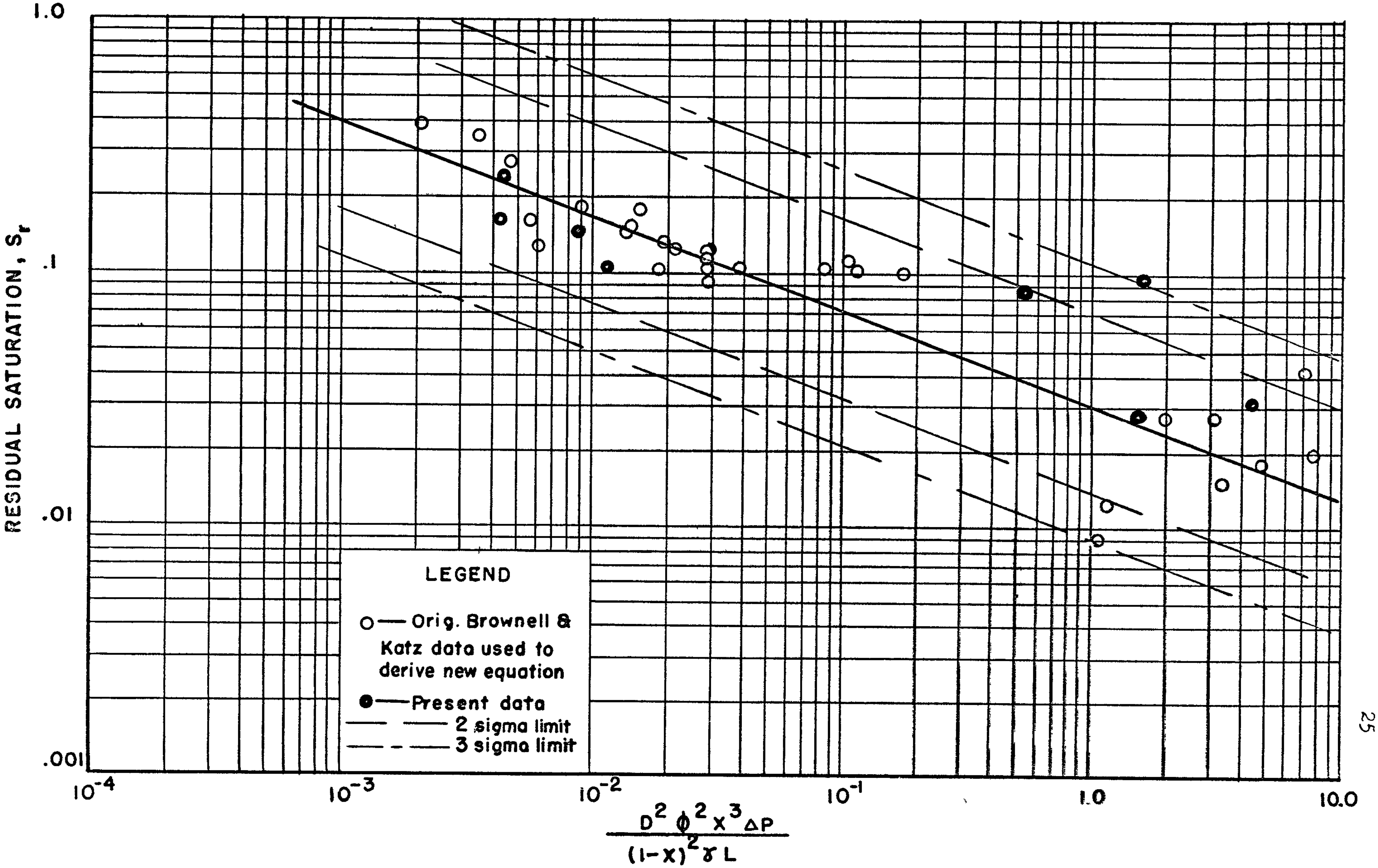


TABLE 4

RESIDUAL SATURATION OF VARIOUS PACKED BEDS

Porous Media	Fluid or Solution	Particle Diameter ft x 10^3	Bed Porosity (% Voids) Det. Wet	Det. Dry	Avg.	ΔP/L	Inter-facial or Sur. Ten. #/ft x 10^3	Sphericity Φ *	Porosity‡ Exponents n	m	Cap. #‡‡ x 10^6	Residual Saturation Observed	Calculated‡‡
-20+30M Ottawa Sand	Water	2.35	31.7	32.3	32.0	62.4	4.74	0.91**	6.7	3.2	42.1	0.162	0.166
"	0.1% Aqua Rex D	"	32.1	32.2	32.2	"	2.29	"	6.8	3.2	80.0	0.150	0.140
"	0.1% Aerosol OT	"	31.9	32.3	32.1	"	1.73	"	6.7	3.2	115.4	0.106	0.127
"	0.1% Tergitol	"	31.1	31.7	31.4	"	1.79	"	6.6	3.1	104.4	0.108	0.131
"	$CaCl_2$ 4M/Kg H_2O	"	32.7	31.8	32.2	78.5	5.55	"	6.8	3.2	41.7	0.212	0.167
		"	31.6	31.8	31.7	78.5	5.55	"	6.6	3.1	44.2	0.248	0.164
5mm Glass Beads	Water	17.2	38.0	38.1	38.1	62.4	4.74	1.00	7.6	3.7	2820.0	0.0956	0.0546
"	01% Aerosol	"	37.8	38.1	38.0	"	1.70	"	7.6	3.7	7794.0	0.0283	0.0416
1/4" Berl Saddles	Water	20.8	58.6	--	58.6	"	4.74	0.480	10.4	5.8	15230.0	0.0990	0.0354
"	0.1% Aerosol	"	"	--	"	"	1.70	"	10.4	5.8	42470.0	0.0310	0.0267

* Defined as ratio of area of sphere having same volume as particle divided by area of particle.

** Value suggested by Leva for angular particles similar to sand.

‡ As given by Brownell and Katz.

‡‡ Calculated from Brownell and Katz equation.

TABLE 5

DATA FROM BROWNELL AND KATZ USED TO DERIVE NEW RESIDUAL SATURATION EQUATION

Material	D in ft.	ϕ	X	1-X	$\Delta P/L$	γ in #/ft.	$\frac{D^2\phi^2 X^3 \Delta P}{(1-X)^2 \gamma L}$	S_r
Sands: Mixture I	0.00268	0.91	.328	.672	644.9	.00274	.1091	0.1109
-20+30M Ottawa	0.00233	"	.322	.678	"	"	.1189	0.1029
"	"	"	.322	.678	215.2	"	.0396	0.1039
"	"	"	.322	.678	1030.4	.00288	.1809	0.1011
Monterez 28-35	0.00165	"	.393	.607	644.9	.00274	.0868	0.1066
"	"	"	"	.607	215.2	"	.0290	0.1193
Mixture II	0.00104	"	.338	.662	644.9	"	.01859	0.1075
	"	"	"	"	215.2	"	.00617	0.1304
	"	"	"	"	1030.4	"	.02960	0.0931
	"	"	"	"	1030.4	.00288	.02820	0.1015
Mills 48-65	0.000825	"	.381	.619	644.9	.00274	.0191	0.1340
"	"	"	"	"	215.2	"	.00572	0.1690
"	"	"	"	"	1030.4	"	.0305	0.1259
"	"	"	"	"	1030.4	.00288	.0290	0.1259
Mills 65-100	0.000583	"	.429	.571	644.9	.00274	.01591	0.1841
"	"	"	"	"	215.2	"	.00477	0.2800
"	"	"	"	"	1030.4	.00288	.0242	0.1281
Mills 100-150	0.000412	"	.442	.558	644.9	.00274	.00918	0.1875
"	"	"	"	"	1030.4	"	.01468	0.1570
"	"	"	"	"	1030.4	.00288	.01395	0.1495
Mixture III	0.000327	"	.354	.646	644.9	.00274	.002215	0.3960
"	"	"	"	"	1030.4	"	.003530	0.3590
Raschig Rings	0.0833	0.435	.72	.28	62.4	.00498	7.80	0.0197
"	"	"	.829	.171	"	"	3.19	0.0280
Berl Saddles	"	0.336*	.725	.275	"	"	4.95	0.0180
Porcelain Saddles	0.0394	0.377	.757	.243	"	"	2.03	0.0277
"	0.0820	0.239*	.839	.161	"	"	1.09	0.009
Glass Rings	0.0208	0.438*	.75*	.25*	"	"	7.02	0.042
"	0.0417	0.306*	.818	.182	"	"	3.39	0.0152
Porcelain Rings	0.0833	0.343*	.79	.21	"	"	1.14	0.0127

(Continued)

* Calculated from test data given in original articles quoted by Brownell and Katz.

TABLE 5 (Continued)

DATA FROM BROWNELL AND KATZ USED TO DERIVE NEW RESIDUAL SATURATION EQUATION

Material	D in ft.	ϕ	X	1-X	$\Delta P/L$	γ in #/ft.	$\frac{D^2\phi^2X^3\Delta P}{(1-X)^2\gamma L}$	S_r
Data Observed in Present Tests (Not used in correlation but plotted in Graph 2)								
-20+30M Ottawa Sand	0.00235	0.91	.321	.679	62.4	.00474	.00433	0.162
"	"	"	"	"	"	.00229	.00895	0.150
"	"	"	"	"	"	.00173	.01181	0.106
"	"	"	.317	.683	78.5	.00555	.00443	0.248
Glass Beads	0.0172	1.00	.380	.620	62.4	.00474	.555	0.0956
"	"	"	"	"	"	.00170	1.545	0.0283
Berl Saddles	0.0208	0.480	.586	.414	"	.00474	1.541	0.0990
"	"	"	"	"	"	.00170	4.420	0.0310

DISCUSSION OF TEST DATA (Part 3)

Effect of Surface Tension On Operating Holdup

An attempt was made to investigate the Jesser and Elgin contention that operating holdup increased as the surface tension of the liquid decreased. The same column was used in these tests as has been described in previous sections of this thesis. Since these authors used a much larger column and since they based their conclusion on the belief that the operating holdup had left the column when the drainage rate became constant, it was necessary to run a series of tests to determine the corresponding drainage cycle required for the smaller column used in these tests.

At the end of a thirty second drainage interval the effluent from beds of 1/4 inch Berl saddles and 5mm glass beads had reached a slow dropwise rate. Visual examination of the column showed that at this time the main body of the liquid had left the packed bed. Therefore, three time intervals were investigated; namely, 5, 15 and 30 seconds, and the test data which were observed are given in Table 6.

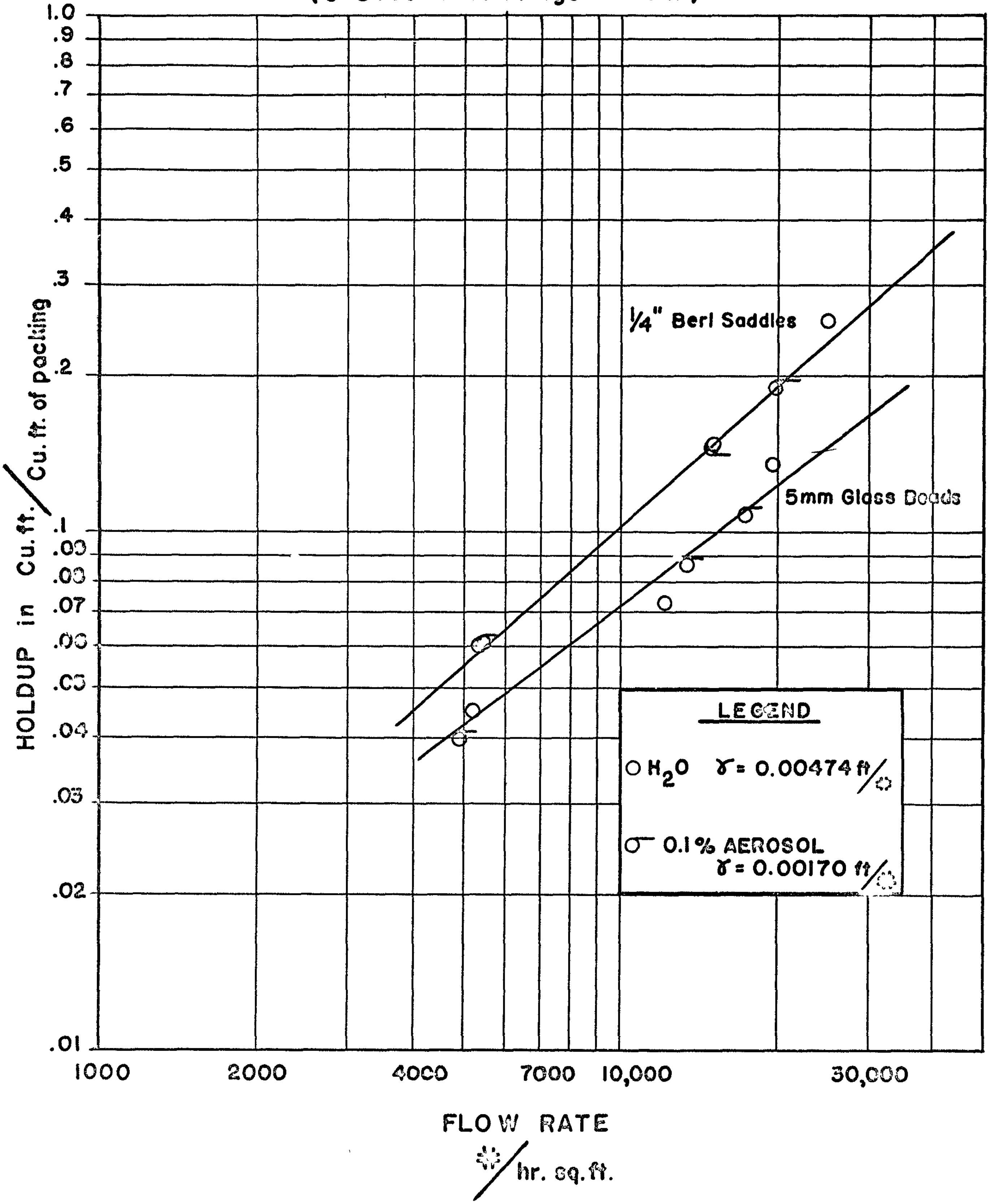
Plots of log holdup vs log flow rate are given in Graphs 3, 4 and 5. The data fell in fairly good straight lines thereby validating the conclusion of Furnas that holdup is some exponential function of the flowrate through the column.

However, for similar flow rates there was no significant difference between the operating holdup for water and the corresponding holdup for the lower surface tension solution of Aerosol OT. This was the case for both beds of Berl saddles and beds of glass beads. Furthermore, these data also showed that there was no significant change in flow rate due to the lowered surface tension which was in agreement with the results noted in Part 1.

Obviously the present data do not confirm the findings of Jesser and Elgin despite the fact that the range of surface tensions investigated in these tests exceeded the values used in their work. It is also interesting to note that the holdup observed for water in the column of 1/4 inch Berl saddles (15 second drainage time) agreed very well with other Jesser and Elgin data for the same type of packing in a larger column. Therefore, it can be inferred that the technique used in these tests did not vary too greatly from their methods.

GRAPH 3

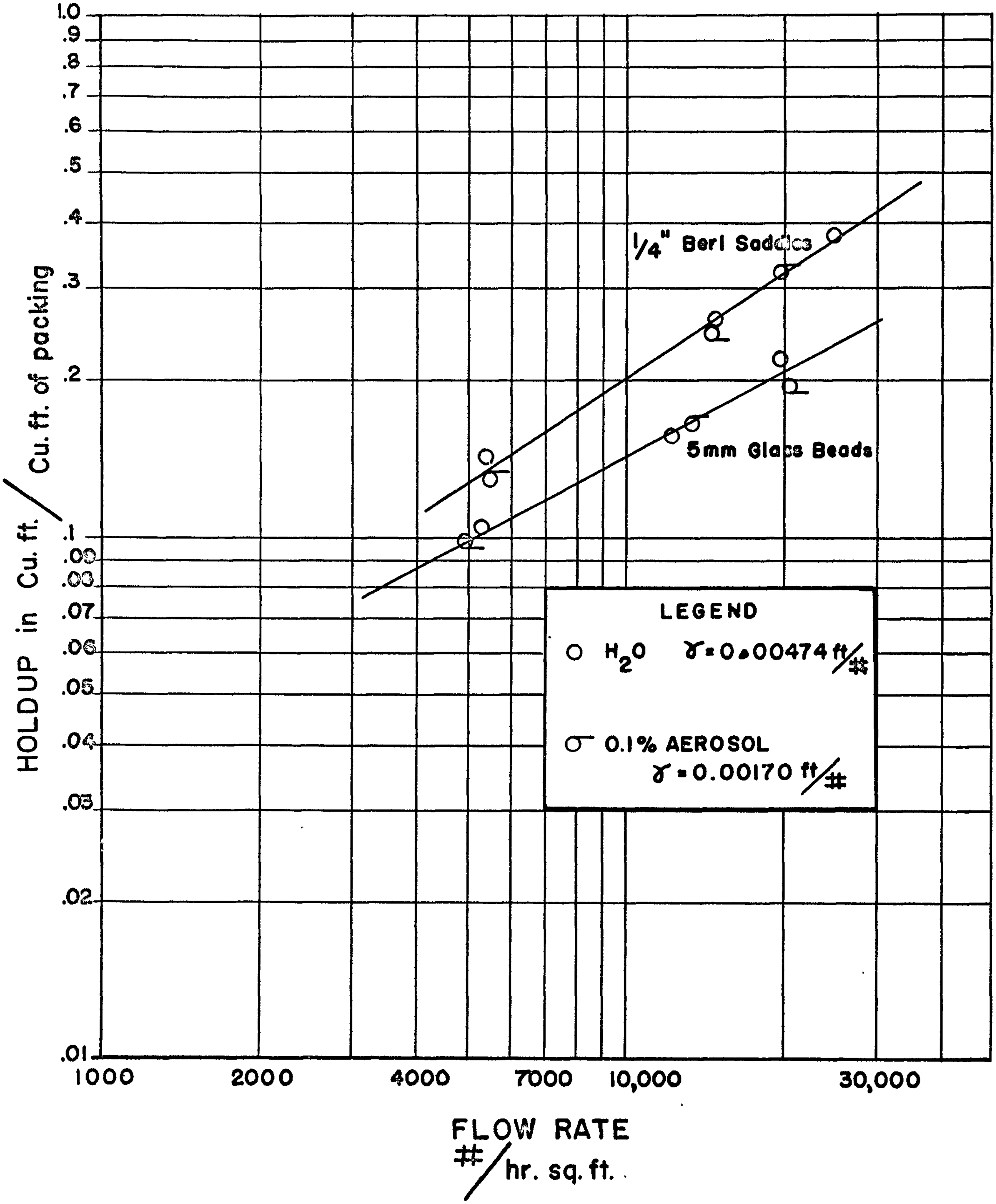
OPERATING HOLDUP vs. FLOWRATE
(5 Second drainage interval)



GRAPH 4

OPERATING HOLDUP vs. FLOW RATE

(15 Second drainage interval)



GRAPH 5

OPERATING HOLDUP vs. FLOW RATE

(30 Second drainage interval)

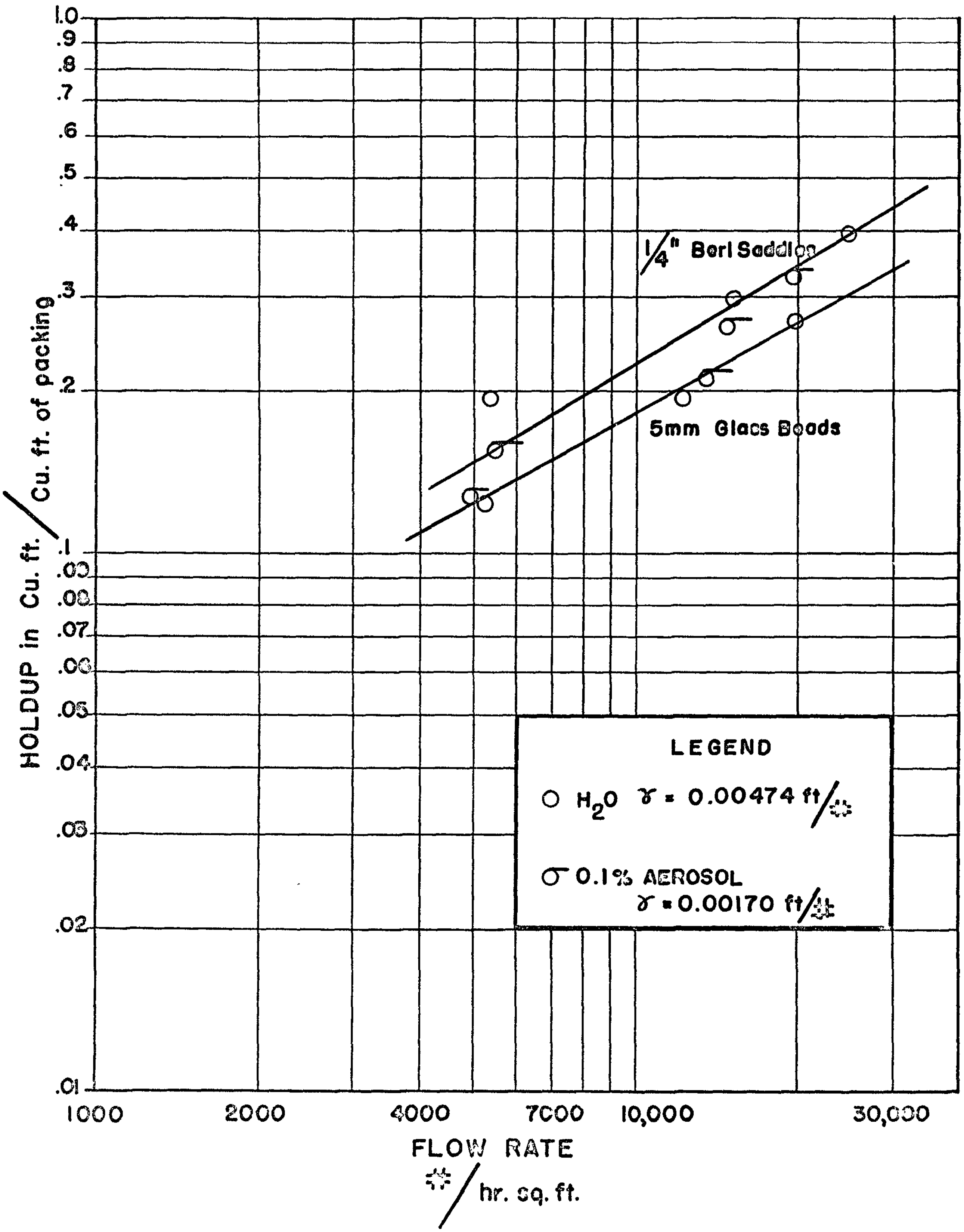


TABLE 6

SURFACE TENSION VS OPERATING HOLDUP

Type Of Packing	Height of Packing cm.	Solution	Surface Tension #/ft	Flow Rate #/hr sq ft	Operating Holdup cu ft/cu ft Packing @5 sec	@15 sec	@30 sec	
5mm Glass Beads	90.8	H_2O	0.00474	5330	0.0456	0.1052	0.1241	
"	"	H_2O	"	12230	0.0739	0.1569	0.1950	
"	"	H_2O	"	19750	0.1360	0.2210	0.2715	
"	"	0.1% Aerosol Solution	0.00170	4980	0.0406	0.0985	0.1279	
"	"	"	"	13520	0.0868	0.1656	0.2138	
"	"	"	"	17420	0.1085	--	--	
"	"	"	"	20609	--	0.1950	--	
¼" Berl Saddles	65.3	H_2O	0.00474	5370	0.0600	0.1429	0.1970	.150*
"	"	"	"	15080	0.1479	0.2640	0.3000	.269*
"	"	"	"	24990	0.2563	0.3800	0.3960	.350*
"	"	0.1% Aerosol	0.00170	5490	0.0614	0.1299	0.1564	
"	"	"	"	14960	0.1460	0.2445	0.2665	
"	"	"	"	19771	0.1901	0.3138	0.3283	

* Jesser and Elgin data for 1/4" Berls in a 6" column at equivalent flow rates.

CONCLUSIONS

1. Little or no mention has been made in the literature as to the effect of fluid surface tension upon its flow through packed beds. The present tests have shown that in the viscous region, flow rates through beds of unconsolidated sand were not affected by changes in the surface tension of the flowing fluid. It was further shown that the fluid viscosity and density were the only fluid properties affecting such flow.

2. Excellent comparative results were obtained between the observed flowrates and those predicted by Leva's equation for viscous flow through packed beds. The poorest correlation was noted between the observed values and the Brownell-Katz and Chilton-Colburn equations. It is believed that these equations lead to considerable error when the bed porosity ranges beyond the limits of 35-45%.

3. Fluids which represented a much wider range of surface tension than those employed by Brownell and Katz were used in a series of residual saturation tests on beds of widely varying particle shapes and sizes. The observed residual saturation values were found to agree well with the values predicted by the complex Brownell and Katz equation.

4. A new equation was developed to relate the residual saturation of packed beds with the diameter and sphericity of the packing particle, the porosity of the packed bed, the pressure drop per unit length of bed, and the surface tension of the liquid. This new equation has more

direct utility than the original Brownell and Katz equation in that it permits the direct calculation of residual saturation without requiring the use of any secondary or auxiliary graphs.

5. Operating holdup data observed on beds of 1/4 inch Berl saddles validated the conclusion of Furnas that holdup is some exponential function of the flowrate through the column. For similar flowrates, however, there was no significant difference between the operating holdup for water and the holdup for the lower surface tension solutions of Aerosol. Despite the fact that excellent comparative holdup values were observed for water, these present observations are not in agreement with those noted by Jesser and Elgin who claimed that operating holdup increased with decreasing surface tension.

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