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## The slow settling of a sphere in a viscous fluid in the proximity of a corner

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THE SLOW SETTLING OF A SPHERE IN A VISCOUS  
FLUID IN THE PROXIMITY OF A CORNER  
BY  
JOSEPH KISUTCZA

A THESIS  
PRESENTED IN PARTIAL FULFILLMENT OF  
THE REQUIREMENTS FOR THE DEGREE  
OF  
MASTER OF SCIENCE IN CHEMICAL ENGINEERING  
AT  
NEW JERSEY INSTITUTE OF TECHNOLOGY

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Newark, New Jersey  
1978

APPROVAL OF THESIS  
THE SLOW SETTLING OF A SPHERE IN A VISCOUS  
FLUID IN THE PROXIMITY OF A CORNER

BY

JOSEPH KISUTCZA

FOR

DEPARTMENT OF CHEMICAL ENGINEERING  
NEW JERSEY INSTITUTE OF TECHNOLOGY

BY

FACULTY COMMITTEE

APPROVED: \_\_\_\_\_  
\_\_\_\_\_  
\_\_\_\_\_

NEWARK, NEW JERSEY

MAY, 1978

### ABSTRACT

Experimental settling velocities for three different sizes of Delrin spheres in Ucon lubricant were determined at 20.2 degrees Celsius in order to confirm the validity of a theoretically derived equation for the settling of a sphere in the proximity of a corner. The experiments were conducted in a wedge shaped column with a circular sector base, filled with the viscous fluid, where the angle of the wedge was varied for experimental purposes. The distance from the wedge apex to the particle was also changed for the different runs.

The experimental data gave a good approximation of the values evaluated by the basic equation utilizing the drag force considering the wedge walls only. A modified form of the basic equation considering the additional drag from the vessel wall showed an improved agreement with the experimental data.

### ACKNOWLEDGEMENTS

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## CHAPTER I

### INTRODUCTION

#### Scope and Purpose of Investigation

The topic of experimentally verifying the theoretical solutions for a sphere settling in the proximity of a corner in a viscous medium first come up in a discussion of its merits with E. Bart, author of a presently unpublished manuscript on the subject. In his work, Bart<sup>4</sup> derived expressions which mathematically evaluated the effects of two planes of arbitrary angles on a sphere settling parallel to their line of intersection. Investigation of the available literature showed that other authors<sup>18,19</sup> arrived at similar theoretical results, but experimental confirming data on the subject was non-existent.

These equations by Bart were derived for use in a medium bounded by an infinite wedge. Therefore a program was needed to evaluate the desired data where the theoretical conditions were approximated by actual equipment.

Aside from generating the experimental data, this author hoped that a modified form of Bart's equations for actual equipment might be established empirically if there was a considerable difference between the results of the experimental work and the calculated values from the basic equation. It should be noted that no rigorous solution applicable to a real container can be obtained,

but by piecing extant solutions together, a fairly accurate representation can be obtained.

It was anticipated that in order to achieve a good data fit, the effects of the vessel wall on the settling particle would have to be accounted for. Instead of deriving a new expression this author elected to augment Bart's work by a modified eccentricity and wall effect correction factor from the study of Greenstein and Happel<sup>10</sup>.

The basic program for the investigation comprised the following:

- 1, to design and build an experimental system capable of simulating theoretical conditions and variables,
- 2, to gather experimental data on sphere settling in various wedge shaped domains filled with a viscous liquid,
- 3, to compare theoretical and experimental results and to evaluate the modified equations.

### Literature Survey

The slow settling of particles in the presence of stationary surfaces has been of interest for many years. As early as 1896, Lorentz<sup>15</sup> treated the problem of a sphere slowly settling parallel to a plane wall by means of reflections. In 1907, Ladenburg<sup>14</sup> used the same technique to treat the settling of a sphere along the axis of an infinitely long cylinder at low Reynolds' number to a



first approximation. Since then, a multitude of solutions have appeared in the literature using the method of reflections to treat various configurations.

The problems concerning the slow motion of a sphere in the proximity of some stationary surface or surfaces can generally be broken down into three categories:

- 1, a particle moving parallel to a surface,
- 2, a particle moving either toward or away from a surface,
- 3, a combination of the two.

A solution presented by Sonshine, Cox and Brenner<sup>20</sup> for a sphere settling in a cylinder filled to a finite depth is an example of the last case. Any problem in which the sphere moves toward or away from a stationary surface must be of an unsteady nature. The literature is teeming with such solutions. Happel and Brenner<sup>12</sup> have reviewed many of the existing solutions. Since Happel and Brenner's overview was published, Sono and Hasimoto<sup>18,19</sup> have studied both categories 1 and 3 intensively.

Of greater concern here is the motion of a particle parallel to a flat surface or surfaces. These solutions can be of an unsteady nature if the particle is assumed to accelerate from rest. The more usual cases treat the steady motion of a particle parallel to a surface or surfaces. The aforementioned solutions of Lorentz<sup>15</sup>

and Ladenburg<sup>14</sup> fall into this category. Faxen<sup>6,7,8</sup> investigated several problems in this category. He corrected Ladenburg's work for the cylinder and verified the solution of Lorentz for the sphere and flat plate. He has further extended Lorentz's problem by obtaining a torque solution and by obtaining higher ordered corrections. In addition, he solved the problem of a sphere settling parallel to and between parallel plates, of which the sphere and the flat plate problem is a special case. More recently Happel and Bart<sup>11</sup> treated a related problem of a sphere falling parallel to four walls, that is, the settling of a sphere along the axis of an infinitely long square duct. The solutions presented for a sphere settling in the proximity of a corner by Bart<sup>4</sup>, which formed the theoretical basis for this thesis appears identical to those derived by Sano and Hasimoto<sup>19</sup>. Although the presentation of the derivation was slightly different, these authors independently using the reflection method, arrived at the same end results. The expressions obtained by Bart seem to have a slight advantage over the work by Sano and Hasimoto since the former has obtained some higher ordered corrections for the translating particle and derived expressions for the rotational effects, while the latter considered only the first order effects without rotation.

Several additional investigators have examined var-

ious aspects of the problem of a sphere in a cylinder. Most notable, in terms of the present work, are those works concerning the settling of a sphere parallel to the axis at some distance. Happel and Brenner<sup>12</sup> presented a discussion showing how these solutions reduce in limiting cases to Lorentz's problem of a sphere and a flat plate.

Brenner and Happel<sup>5</sup> developed expressions and coefficients for the drag and torque for the translation of a single spherical particle in an infinitely long cylinder where the particle is kept from rotating. Later Greenstein and Happel<sup>10</sup> extended the problem treated by Brenner and Happel where the sphere may rotate and developed corrected values for the coefficients.

## CHAPTER II

### THEORY

#### Translation of a Sphere in a Corner

Consider a sphere oriented upon the midplane of a space formed by the intersection of two planes at some arbitrary dihedral angle. The wedge-shaped space thus formed is filled with an incompressible viscous liquid. The sphere is assumed to settle under the influence of gravity in a direction parallel to the apex of the wedge as shown in Figure 1. The angle of the wedge is arbitrary but must be sufficiently large so that the walls do not touch the sphere. Instead of using the wedge angle, it is more convenient to work with the half wedge angle,  $\phi_0$ , since solutions symmetrical about the plane containing the sphere center are used. Thus, if  $\phi_0$  is less than  $\pi/2$  the sphere is falling within a corner. If  $\phi_0$  is  $\pi/2$ , the sphere is falling parallel to a flat plane, which should yield values confirming the solutions of Lorentz<sup>15</sup> and Faxen<sup>6,7,8</sup>. When  $\phi_0$  is larger than  $\pi/2$ , the sphere is external to the corner and when  $\phi_0$  is equal to zero, a degenerate case occurs in which the wedge is a plane whose sharp edge faces the sphere. In the last case, the sphere settles in an otherwise unbounded fluid parallel to the sharp edge of an infinitely thin plate. The fluid velocity is zero upon both surfaces of this thin plate.

FIGURE 1  
SPHERE SETTLING IN A CORNER

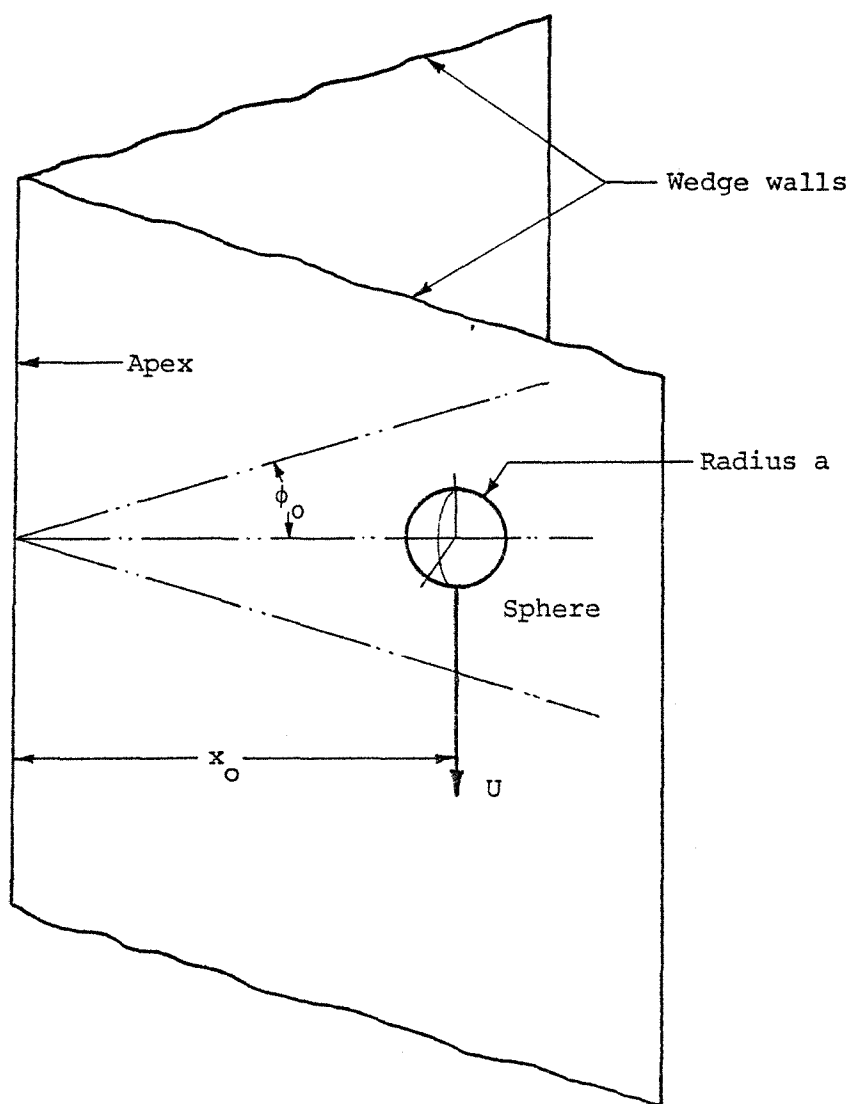


Figure 2 depicts the sphere-wall geometries described on the preceding page.

The equations to be solved are the creeping motion equation

$$\mu \nabla^2 \bar{\mathbf{v}} = \nabla p \quad (2.1)$$

and the equation of continuity

$$\nabla \cdot \bar{\mathbf{v}} = 0 \quad (2.2)$$

The boundary conditions which define the fluid velocity are that

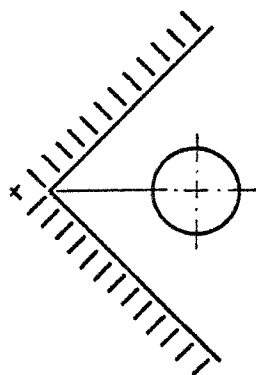
- 1, at the fluid-solid interface there is no relative motion;
- 2, the velocity at the sphere surface is the settling speed of the sphere.

The boundary value problem can be solved by a technique of successive approximations known as the method of reflections. For a comprehensive description of the method, see the treatise on the subject by Happel and Brenner<sup>12</sup>. The solutions for fluid velocity, drag force, and torque may be obtained by summing the contributions of the individual fields.

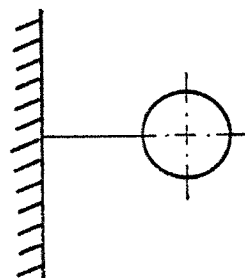
$$\bar{\mathbf{v}} = \sum_{i=1}^{\infty} \bar{\mathbf{v}}^{(i)}. \quad (2.3)$$

$$\bar{\mathbf{F}} = \sum_{i=-1}^{\infty} \bar{\mathbf{F}}^{(i+2)}. \quad (2.4)$$

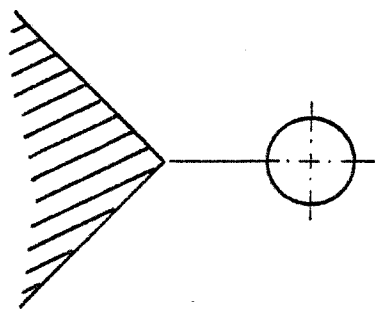
FIGURE 2  
SPHERE-WALL GEOMETRY



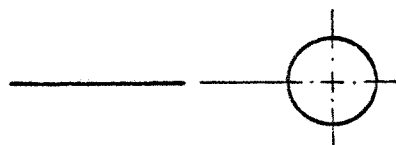
a,  $\phi_0 = \pi/4$



b,  $\phi_0 = \pi/2$



c,  $\phi_0 = 3\pi/4$



d,  $\phi_0 = \pi$

$$\bar{T} = \sum_{i=-1}^{\infty} \bar{T}^{(i+2)}. \quad (2.5)$$

To insure that the alternate velocity solutions are independent, the odd numbered solutions are unbounded at the sphere center and are zero infinitely far from the sphere, whereas the even numbered ones are finite at the sphere center and also vanish at a distance infinitely far removed from the sphere. This will insure that at large distances from the disturbing influence of the sphere the fluid velocity becomes zero. Because of this, only the odd numbered fields make contributions to the final drag and torque solutions.

When the results of the reflection solutions are summed in accordance with equations (2.4) and (2.5), the following expressions for the drag force and torque are obtained

$$\begin{aligned} \bar{F} = 6\pi\mu Ua\bar{k}\{ & 1+f_1(\phi_0)(a/x_0)+f_1(\phi_0)^2(a/x_0)^2 \\ & +[f_1(\phi_0)^3+f_2(\phi_0)](a/x_0)^3\} \end{aligned} \quad (2.6)$$

and

$$\begin{aligned} \bar{T} = 4\pi\mu Ua^2\bar{j}\{ & g_1(\phi_0)(a/x_0)^2+f_1(\phi_0)g_1(\phi_0)(a/x_0)^3 \\ & +[f_1(\phi_0)^2g_1(\phi_0)+g_2(\phi_0)](a/x_0)^4\}. \end{aligned} \quad (2.7)$$

The power series may be replaced by the sum of a geometric progression, using techniques presented in Happel and Brenner<sup>12</sup>, to produce a still better approximations of the results for the drag force.



$$\bar{F} = \frac{6\pi\mu Ua\bar{k}}{1 - f_1(\phi_0)(a/x_0) - f_2(\phi_0)(a/x_0)^3}. \quad (2.8)$$

A similar representation of the power series for the torque in powers of  $f_1(\phi_0)(a/x_0)$  would yield

$$\bar{T} = 4\pi\mu Ua^2 \bar{j} \frac{g_1(\phi_0)(a/x_0)^2 + g_2(\phi_0)(a/x_0)^4}{1 - f_1(\phi_0)(a/x_0)}. \quad (2.9)$$

The settling particles ability to achieve free rotation will introduce some uncertainty in the solution for drag and torque. The coefficients of the powers of  $(a/x_0)$  are dependent of whether or not rotation is possible. This effect will not be obvious until  $(a/x_0)^4$  is reached in the drag solution and not until  $(a/x_0)^5$  is reached in the torque solution. Therefore, equations (2.6) and (2.7) are correct for both cases as far as the approximations are concerned.

#### Evaluation of Terminal Settling and Angular Velocities in a Corner

The terminal velocity of a sphere settling in a corner may be evaluated from the drag obtained from either equation (2.6) or (2.8), depending on the degree of approximation desired and from Stokes law:

$$\bar{F} = 6\pi\mu U_s a \bar{k} \quad (2.10)$$

where

$$U_s = \frac{2(\rho_p - \rho_l)ga^2}{9\mu}. \quad (2.11)$$

Equating equation (2.10) and (2.6), it is apparent that

$$\begin{aligned} U_s/U = 1 + f_1(\phi_0)(a/x_0) + f_1^2(\phi_0)(a/x_0)^2 \\ + [f_1^3(\phi_0) + f_2(\phi_0)](a/x_0)^3. \end{aligned} \quad (2.12)$$

Combining equation (2.8), using the geometric series approximation of higher ordered terms, with equation (2.10) yields

$$U/U_s = 1 - f_1(\phi_0)(a/x_0) - f_2(\phi_0)(a/x_0)^3. \quad (2.13)$$

The angular velocity of the sphere as it falls will depend upon the spherical isotropy of the falling sphere. For a sphere where the centroid of the particle is not at the sphere center, the angular velocity must be zero. However, for a perfectly spherical freely rotating sphere, the torque necessary to prevent rotation must be

$$\bar{T} = 8\pi\mu a^3 \omega \bar{j}. \quad (2.14)$$

Combining this with either equation (2.7) or (2.9), yields, respectively

$$\begin{aligned} \omega = U/2a\{g_1(\phi_0)(a/x_0)^2 + f_1(\phi_0)g_1(\phi_0)(a/x_0)^3 \\ + [f_1^2(\phi_0)g_1(\phi_0) + g_2(\phi_0)](a/x_0)^4\} \end{aligned} \quad (2.15)$$

and

$$\omega = \frac{U/2a[g_1(\phi_0)(a/x_0)^2 + g_2(\phi_0)(a/x_0)^4]}{1 - f_1(\phi_0)(a/x_0)}. \quad (2.16)$$

The functions  $f_1(\phi_0)$ ,  $f_2(\phi_0)$ ,  $g_1(\phi_0)$  and  $g_2(\phi_0)$  have been evaluated numerically by Bart<sup>4</sup> for various values of

the parameter  $\phi_0$  and the results are tabulated in Tables 1 and 2.

### Translation of a Sphere in a Cylindrical Tube

The inclusion of the sections on a sphere settling in a cylinder was necessitated by the fact that there are no infinite wedges in the real world. Therefore, to properly evaluate the experimental data, the wall effects must be evaluated and included in the final equation. The derivation of the equations dealing with the wall effects are shown in this and the following sections.

Consider the translation and rotation of a sphere moving with an arbitrary constant velocity through a viscous fluid in an infinitely long cylindrical tube. The sphere moves with a constant velocity parallel to the cylinder axis, displaced from the axis by some distance, as shown on Figure 3.

The fluid motion is governed by the creeping motion and continuity equations, (2.1) and (2.2), respectively. To solve these equations, the boundary conditions required are that

- 1, at the fluid-solid interface there is no relative motion,
- 2, at large distances from the disturbance caused by the moving sphere the velocity distribution becomes Poiseuillian.

The solution for the above problem makes use of the reflec-

TABLE 1

VALUES OF  $f_1(\phi_0)$  AND  $f_2(\phi_0)$  FOR VARIOUS VALUES OF  $\phi_0$ 

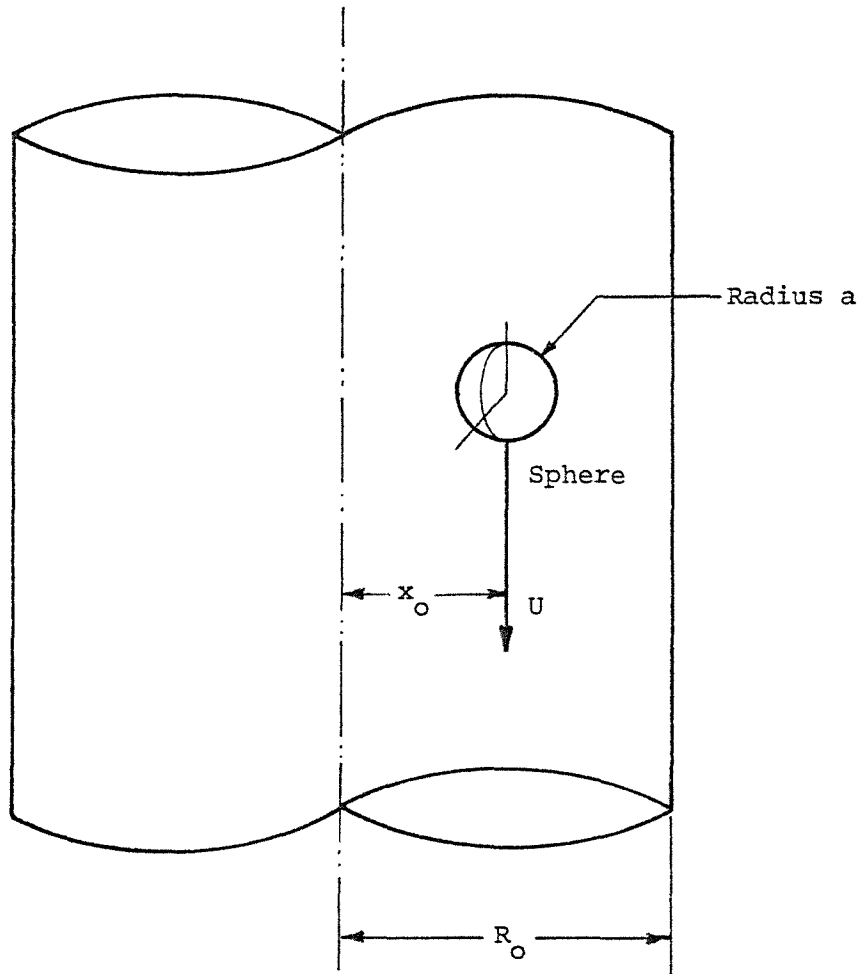
<u><math>\phi_0</math></u>	<u><math>f_1(\phi_0)</math></u>	<u><math>f_2(\phi_0)</math></u>
$\pi$	0.4775	-0.05305
$\pi/2$	0.5625	-0.125
$\pi/4$	1.1584	-0.8416
$\pi/6$	1.7891	-2.7820

TABLE 2

VALUES OF  $\sigma_1(\phi_0)$  AND  $\sigma_2(\phi_0)$  FOR VARIOUS VALUES OF  $\phi_0$ 

<u><math>\phi_0</math></u>	<u><math>\sigma_1(\phi_0)</math></u>	<u><math>\sigma_2(\phi_0)</math></u>
$\pi$	0.3581	-0.08952
$\pi/2$	0.0	0.1875
$\pi/4$	0.4354	1.1490
$\pi/6$	3.9386	3.6930

FIGURE 3  
SPHERE SETTLING IN A CYLINDRICAL TUBE



tion method as previously described for a sphere settling in a corner.

The frictional force and the torque can be evaluated by adding the contributions of each field.

$$\bar{F} = \sum_{i=0}^{\infty} \bar{F}^{(i)}. \quad (2.17)$$

$$\bar{T} = \sum_{i=0}^{\infty} \bar{T}^{(i)}. \quad (2.18)$$

The final result for the frictional force for a sphere settling in a quiescent fluid where we set  $\beta = x_0/R_0$  is as follows:

$$\bar{F} = 6\pi\mu Ua\bar{k}[1 + f(\beta)(a/R_0) + f^2(\beta)(a/R_0)^2] \quad (2.19)$$

and, for a freely rotating sphere, the torque is

$$\bar{T} = 8\pi\mu Ua^2\bar{j}\{g(\beta)(a/R_0)^2[1 + g(\beta)(a/R_0)]\}. \quad (2.20)$$

#### Evaluation of Terminal Settling and Angular Velocities in a Cylinder

The terminal velocity of a sphere settling in a cylinder, offset from the cylinder axis, may be derived by combining equations (2.19) and (2.10):

$$U/U_s = 1 - f(\beta)(a/R_0). \quad (2.21)$$

Similarly, for the angular velocity, equating (2.14) and (2.20) yields:

$$\omega = U/a\{g(\beta)(a/R_0)^2[1+g(\beta)(a/R_0)]\}. \quad (2.22)$$

The functions  $f(\beta)$  and  $g(\beta)$  have been previously defined and reported in Happel and Brenner<sup>12</sup>, but an expanded and corrected set of values are presented in the work by Greenstein and Happel<sup>10</sup>. These latter values are listed in Tables 3 and 4.

#### Combined Equation for Wedge Contained in a Vessel

The equation (2.13) derived by Bart<sup>4</sup> accounts for the effect of the wedge on the settling particle, while the equation (2.21) presented by Greenstein and Happel<sup>10</sup> describes the wall effects. The mode of their derivation and the format, in which they are presented suggest that these equations may be combined to form an expression to estimate the settling velocities for a sphere in a column of viscous liquid, where the base of the column may be described as a sector of a circle with finite dimensions. In a column of a large diameter where the wall effects are small, as in the experimental vessel, this treatment should yield reasonably accurate results.

The proposed equation takes the form of Bart's equation (2.13) augmented by a modified term from equation (2.21). The modification consisted of reducing the calculated wall effect by a fraction which is the available circle segment divided by circle circumference. The final combined equation is as shown on the following page.



TABLE 3

TABULATION OF  $f(\beta)$  FOR VARIOUS VALUES OF  $\beta$ 

<u><math>\beta</math></u>	<u><math>f(\beta)</math></u>	<u><math>\beta</math></u>	<u><math>f(\beta)</math></u>
0.00	2.10444	0.40	2.04388
0.01	2.10433	0.41	2.04391
0.02	2.10415	0.43	2.04522
0.03	2.10381	0.45	2.04819
0.05	2.10270	0.50	2.06557
0.10	2.09758	0.55	2.10274
0.15	2.08962	0.60	2.16980
0.20	2.07937	0.65	2.28060
0.25	2.06801	0.70	2.45850
0.30	2.05687	0.75	2.742
0.35	2.04800	0.80	3.20
0.37	2.04561	0.85	3.96
0.39	2.04419	0.90	5.30

TABLE 4

TABULATION OF  $g(\beta)$  FOR VARIOUS VALUES OF  $\beta$ 

<u><math>\beta</math></u>	<u><math>g(\beta)</math></u>	<u><math>\beta</math></u>	<u><math>g(\beta)</math></u>
0.00	0.0	0.32	0.393691
0.01	0.0129614	0.33	0.404624
0.02	0.0259183	0.35	0.426101
0.03	0.0388690	0.40	0.477443
0.04	0.0518074	0.45	0.525110
0.05	0.0647301	0.50	0.568742
0.08	0.1033672	0.55	0.60823
0.10	0.128974	0.60	0.64376
0.15	0.192253	0.65	0.67574
0.20	0.254081	0.70	0.7059
0.25	0.313972	0.75	0.7378
0.27	0.337270	0.80	0.7802
0.29	0.360192	0.85	0.857
0.30	0.371474	0.90	1.03
0.31	0.382645		

$$\begin{aligned}
 U/U_s = 1 - f_1(\phi_0)(a/x_0) - f_2(\phi_0)(a/x_0)^3 \\
 - f(\beta)(\phi_0/\pi)(a/R_0),
 \end{aligned}
 \tag{2.23}$$

where the coefficients  $f_1(\phi_0)$ ,  $f_2(\phi_0)$  and  $f(\beta)$  are as listed in Tables 1 and 3.

## CHAPTER III

### PHYSICAL PROPERTIES DETERMINATIONS

#### Fluid Medium Description

The fluid utilized in this experimental work was Ucon lubricant, type 50 HB-5100 from Union Carbide. It is a water soluble polyalkylene glycol type heat transfer and lubricating agent. Its stability under conditions encountered during testing, its density range, and its temperature-viscosity properties made it an excellent candidate for measuring slow settling velocities. Although the manufacturer's publication lists some of the desired physical properties, they were redetermined for the expected operating range.

#### Fluid Density Measurements

The density of the Ucon lubricant was determined by a modified version of ASTM Standard Test D-891 Method C<sup>3</sup>. A brief description of the modified test procedure follows.

The bath was preset to the desired temperature and the calibrated 25 ml Gay-Lussac specific gravity bottle filled with Ucon lubricant was suspended in the bath. A second bottle equipped with a thermometer, also filled with the lubricant, was suspended next to the first bottle to check when thermal equilibrium was reached (usually 5-10 minutes). At the correct temperature the cover

was placed on the specific gravity bottle and the volume was adjusted. Upon removal from the bath the bottle was dried and weighed on a Satorius 3482/Electronic analytical balance and the data was recorded. The procedure was repeated for each desired temperature.

The experimentally generated data points were regressed linearly. The regression coefficients were calculated using the Curve Fitting Program SD-03A<sup>13</sup> on a Hewlett-Packard 97 calculator. The correlation yielded the following equation:

$$\rho = 1.076666667 - 0.000758889T, \quad (3.1)$$

where  $\rho$  is the density in gms/cm<sup>3</sup> and T is the temperature in °C.

The equation correlated to the data points appears to fit very closely, since the regression yielded a correlation coefficient of 0.999912117. The experimentally acquired fluid densities were plotted in Figure 4 as the function of the temperature.

The accuracy of equation (3.1) was checked by comparing the values generated by this equation to those listed by the manufacturer<sup>9</sup>. The agreement was excellent as shown in Table 5.

#### Fluid Viscosity Measurements

The Ucon lubricant viscosity was evaluated according to ASTM Standard Test D-445<sup>1</sup>. The experimental measure-

FIGURE 4  
EXPERIMENTAL FLUID DENSITIES FOR UCON LUBRICANT

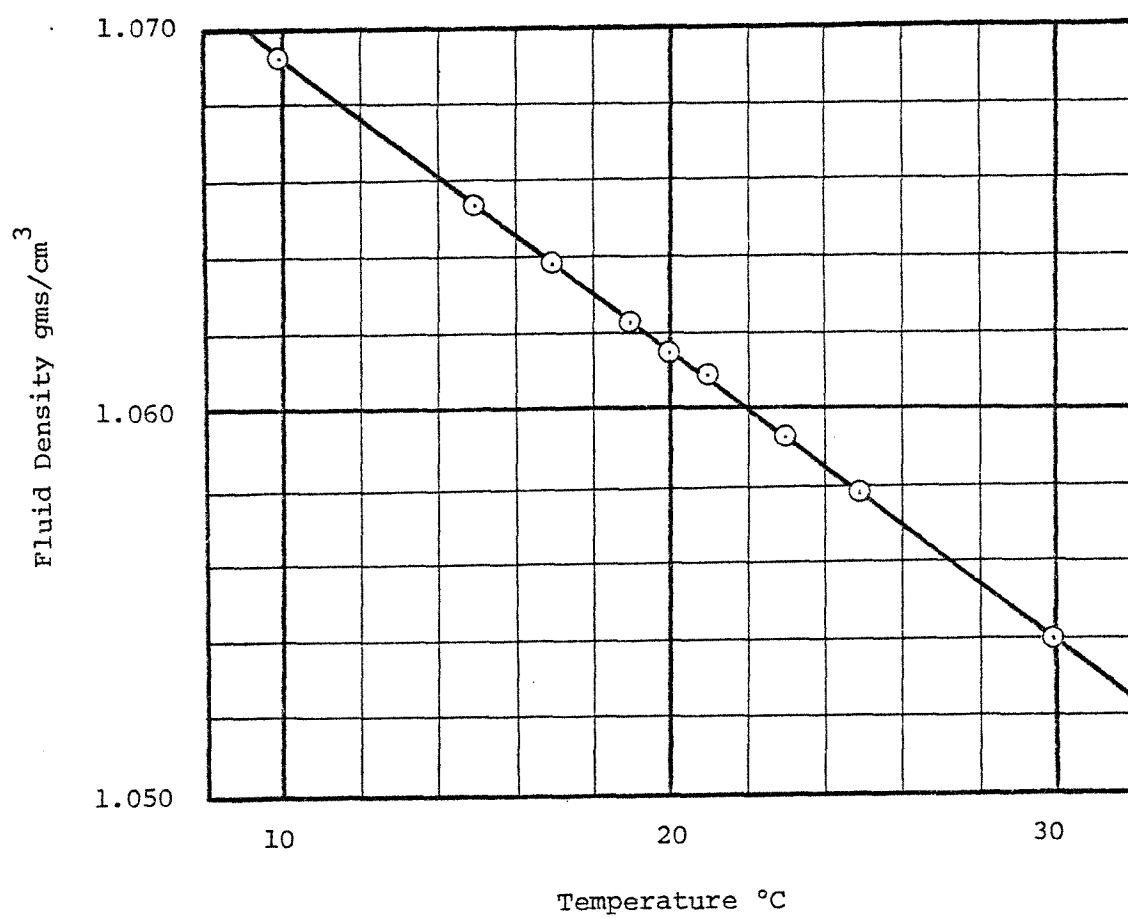


TABLE 5

COMPARISON OF LIQUID DENSITIES FROM MANUFACTURER'S  
LITERATURE<sup>9</sup> WITH EQUATION (3.1) FOR UCON LUBRICANT

Temperature °C	Liquid Density from Manufacturer's Literature <sup>9</sup> . gms/cm <sup>3</sup>	Liquid Density from Equation (3.1). gms/cm <sup>3</sup>
98.8	1.003	1.0017
37.8	1.048	1.0480
15.6	1.065	1.0648

ments were carried out with a size 500 Cannon-Fenske viscometer. The viscometer was calibrated with water for an earlier unrelated experiment by the author. The viscometer calibration constant vs. temperature curve from the earlier work is reproduced in Figure 5.

The experimental viscosity data fitted to the type of equation developed by Watson, Wein and Murphy<sup>21</sup>. The regression coefficients were calculated on Hewlett-Packard 97 calculator using the Curve Fitting Program SD-03A<sup>13</sup>. A variety of modifications to the basic equation was tried. The best results were achieved using the logarithmic curve fit which yielded the following:

$$\mu = e^{\{5.138244744 - 0.561074273 \ln(1.8T - 132)\} - 1.7}, \quad (3.2)$$

where  $\mu$  is the kinematic viscosity in centistokes and  $T$  is the temperature in  $^{\circ}\text{C}$ .

The fitting of the equation to the data was very successful, since the regression produced a correlation coefficient of 0.999987224. Figure 6 displays the plot of experimental fluid viscosities vs. temperature.

The fluid viscosities derived by equation (3.2) were compared to the manufacturer's data<sup>9</sup>. This comparison is shown in Table 6. The agreement was very good, since the slight positive deviation may be explained by the low moisture levels (0.26%) in the lubricant. This phenomenon



FIGURE 5  
VISCOMETER CALIBRATION CONSTANT  
FOR CANNON-FENSKE VISCOMETER

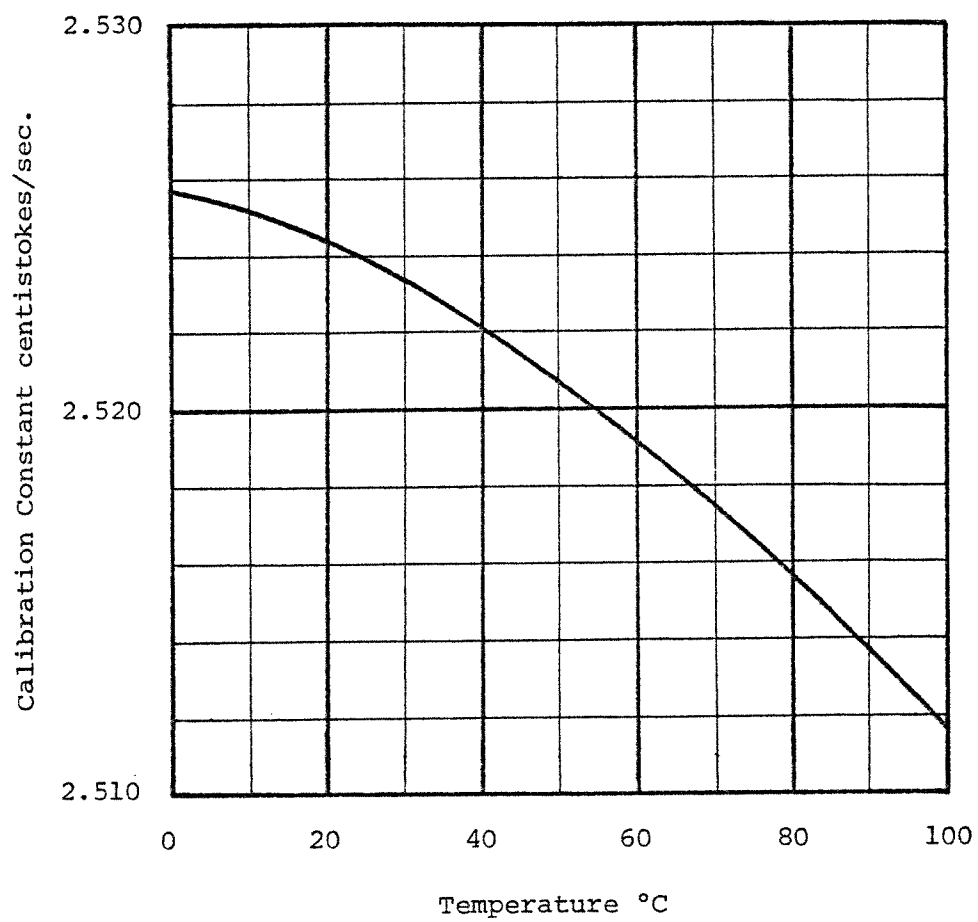


FIGURE 6  
EXPERIMENTAL KINEMATIC VISCOSITIES  
FOR UCON LUBRICANT

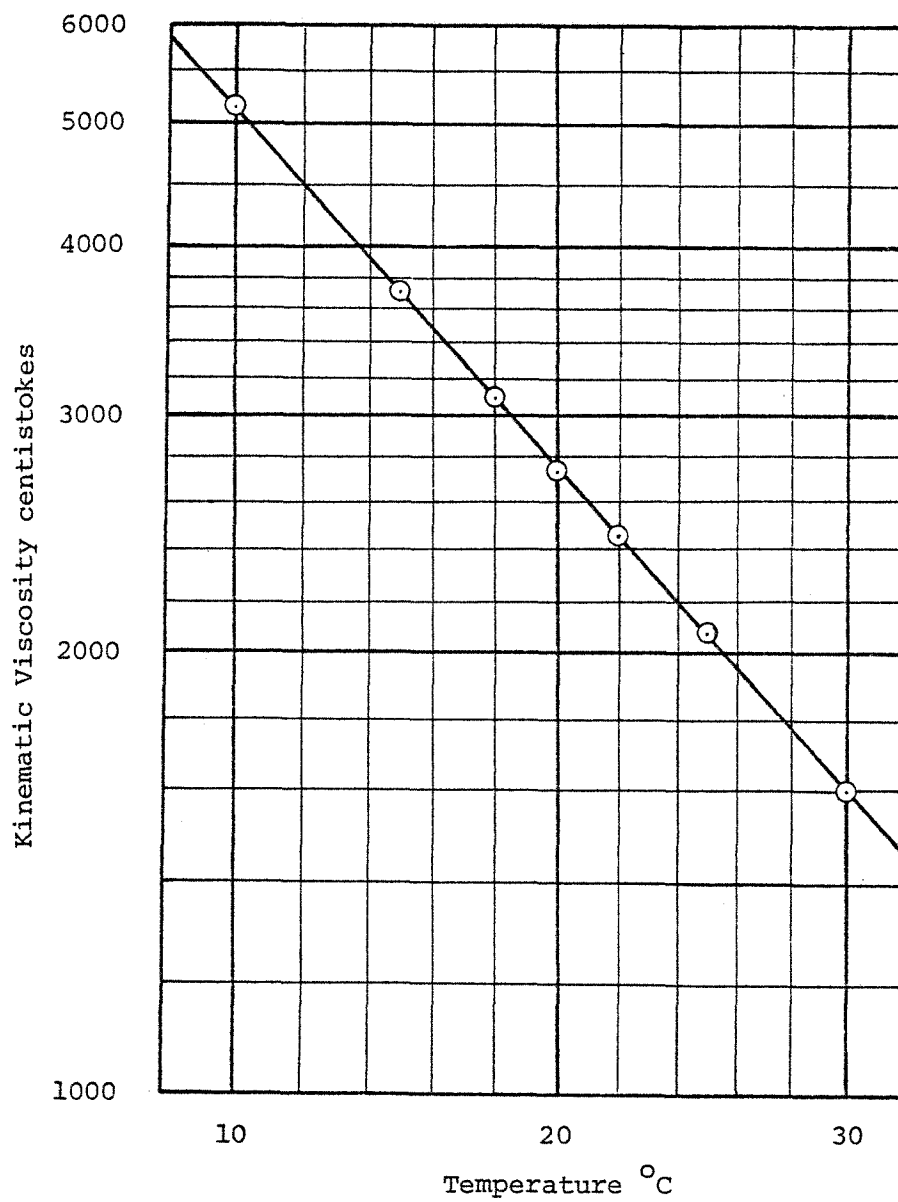


TABLE 6

COMPARISON OF LIQUID VISCOSITIES FROM MANUFACTURER'S  
LITERATURE<sup>9</sup> WITH EQUATION (3.2) FOR UCON LUBRICANT

Temperature °C	Liquid Viscosity from Manufacturer's Literature <sup>9</sup> . centistokes	Liquid Viscosity from Equation (3.2). centistokes
58.9	168	167
37.8	1104	1118
-17.8	~ 70000	70517

of a small increase in viscosity of fluids of this type at low levels of contained water is documented in the manufacturer's literature<sup>9</sup>.

#### Sphere Description

The spheres used for the experimental work were Delrin spheres of Grade 200 from Ultraspherics. The polymer used in manufacturing the spheres was developed by E.I. DuPont. Delrin is an opaque white, acetal type polymer. These spheres are normally used in highly critical bearing applications and they are highly polished. These spheres were selected for their stability under normal experimental conditions and for their relative density to the fluid medium. The sphere sizes acquired were 5/32, 1/4 and 11/32 inch nominal diameters.

#### Sphere Selection Process

The applications for which these spheres were designed, required that their basic diameter tolerance be very low: therefore, sizewise, they are nearly identical. However, spotchecking revealed that there was a considerable variation in densities for the same sizes and even a larger difference was found between the different ones.

A procedure was instituted to select a number of spheres of each size with uniform densities. An abbreviated account of the procedure is listed below.

A solution of 350 gms of Tetrachloromethane (MCB

Spectroquality, S.G.=1.5940) and 1,2-Dichloroethane (MCB Spectroquality, S.G.=1.2351) was placed in a 6 inch diameter glass cylinder and 100 each of the spheres of 5/32, 1/4 and 11/32 inch nominal diameter were placed in the solution. All the spheres sunk to the bottom of the cylinder. Tetrachloromethane was added to the solution at 1.25 ml increments and the resulting solution was stirred. After stirring the solution was allowed to come to rest. Prior to each addition all spheres that have risen from the bottom were collected and segregated by size and approximate density. When large segments of the spheres of each size were collected, the density of the solution was also determined.

For each nominal diameter, the group with the largest number of spheres with the same approximate density was selected. Each sphere from these groups was weighed individually and was subjected to a multiple point determination of its diameter. Of the ones which appeared identical, six were selected at random for determination of their exact densities.

#### Sphere Density Determination

The exact densities for the spheres were determined by a modified version of ASTM Standard Test D-167<sup>2</sup>. A brief description of the procedure used follows.

A 25 ml Walker type specific gravity bottle was calib-

rated. The six spheres were weighed collectively and placed in the bottle. The bottle was filled with 1,2-Dichloroethane (MCB Spectroquality) and placed in the constant temperature bath. When the solvent reached thermal equilibrium at 20.0 degrees Celsius, the volume was adjusted. From the resulting measured volumes, densities for the spheres were calculated. As a check on the measured diameters, the sphere volumes were also used to obtain calculated diameters.

The resultant properties for the spheres are tabulated in Table 7.

TABLE 7

## SELECTED DELRIN SPHERE DIAMETERS AND DENSITIES

Sphere Diameter Nominal inches	Sphere Diameter Average of Multipoint Measurement inches	Sphere Density from Experimentally Determined Volume gms/cm <sup>3</sup>	Sphere Diameter from Experimentally Determined Volume inches
5/32	0.1562	1.3883	0.1562
1/4	0.2497	1.3774	0.2497
11/32	0.3435	1.4001	0.3435

## CHAPTER IV

### EXPERIMENTAL SYSTEM

#### Design Considerations

The following considerations influenced the overall design of the experimental system:

- 1, a need for the largest possible diameter vessel to minimize the wall effects, but where the contained liquid is still transparent to allow a clear view of the settling particle;
- 2, a need for a stable platform to provide support for the sphere release mechanism and for the wedges suspended in the liquid;
- 3, a need for all internal parts to be constructed from translucent materials;
- 4, a need for a means to recover the spheres from the bottom of the tank without greatly disturbing the system;
- 5, a need for a constant temperature environment to minimize the temperature fluctuations in the liquid;

#### Experimental Equipment

Since none of the available equipment fitted the above mentioned considerations, a decision was made to design and build the equipment for use specifically in this

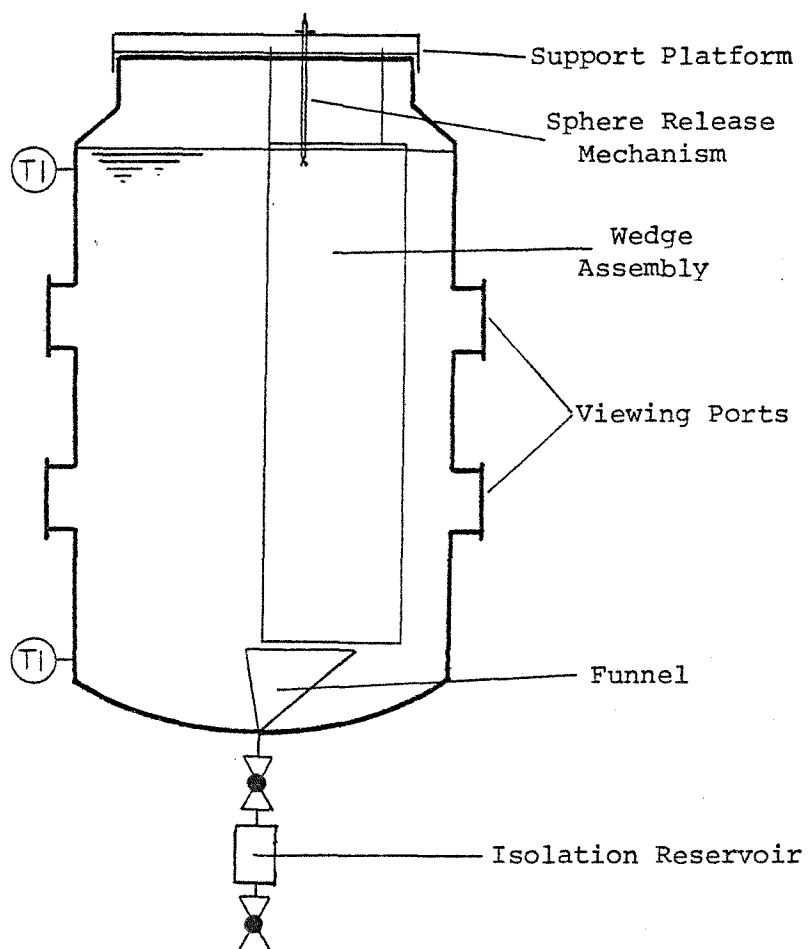


study. Although the overall design is unique, the design took advantage of commercially available pieces of equipment wherever possible. These pieces, with minor modifications, became part of the overall design. Sections, which were radically different from existing equipment, were designed for ease of use and with minimum expenditure of materials. The equipment, as designed, consisted of a large container with a dual purpose support and alignment platform, a sphere release mechanism and four wedges of various angles. The schematic of the equipment as built is shown on Figure 7.

#### Ucon Lubricant Container

The preliminary experiments for the determination of the maximum width of the liquid which does not impair the observation of the settling particle showed that when the viewing path exceeds 30 inches the observation becomes difficult. The optimum width of the viewing path through the liquid was found to be between 25 and 30 inches, where the Ucon lubricant takes on a deep green hue but stays transparent, therefore a 24.0 inch (I.D.) by 36.0 inch (T.L. to T.L.) by 3/32 inch (wall thickness) vessel was selected as the basic container for the sphere settling experiments. The container, prior to modifications, was an open top, dished bottom head feedtank of 316 S.S. construction with a 1 inch bottom drain. The container

FIGURE 7  
SCHEMATIC OF EXPERIMENTAL EQUIPMENT



was supported on 3 tubular legs with casters and a bolt type levelling assembly on each leg.

To comply with the desired design basis, various modifications were installed on the basic container. The description of these alterations are listed below.

1, Four custom made viewing ports were constructed from 4 inch I.D. 316 S.S. tube stub ends by placing a 4.75 inch diameter by 1/4 inch port glass, protected on both sides with CRT envelope gaskets, between a retaining ring and the flat of the stub end. The retaining ring and the flat of the stub end were drilled out in four places and bolted together. The container had four 4 inch holes (2 on each side) cut on 12 and 24 inch centers from the bottom tangent line and each of the assembled viewing ports were seal welded to the container. The internal weldseam and any other protrusions were ground to a mill finish.

2, A 5.5 inch high by 6 inch top radius half round powder funnel of 316 S.S. construction was force fitted into the bottom drain coupling with the round part toward one set of viewing ports. The segment of the tank which contained the circular portion of the funnel was designated as the front.

3, Two 1/8 inch compression fittings were attached to the vessel to act as the thermocouple connections. They were located 1 inch below the top tangent line

and 1 inch above the bottom tangent line on the same vertical as the rear viewing ports.

4, An isolation reservoir, similar to the one employed by Matyas<sup>16</sup> in his experimental work, was constructed from two 1 inch ball valves and a 1.5 inch I.D. by 3.5 inch long sight glass. The pieces were connected together with 1 inch minimum length pipe nipples. An identical nipple was used to join the completed isolation reservoir to the bottom coupling.

Figure 8 is a sketch of the Ucon lubricant container as used in the experiments, showing some of its critical dimensions.

#### Wedge Support and Alignment Platform with Sphere Release Mechanism Alignment Assembly

A platform, as shown on Figure 9, was designed for dual purpose and was constructed from 3/4 inch by 3/16 inch 316 S.S. barstock. Five of the arms are single layer construction and each of the arms had a hole drilled 1 inch from the outer end. These arms supported and aligned the wedges in the liquid. The three other arms of double construction had a second bar attached 0.6 inches above the lower ones. The double arms had 5 holes drilled through both bars at 1 inch intervals from the center. These double arms supplied the vertical and radial alignment for the sphere release mechanism. The platform was also

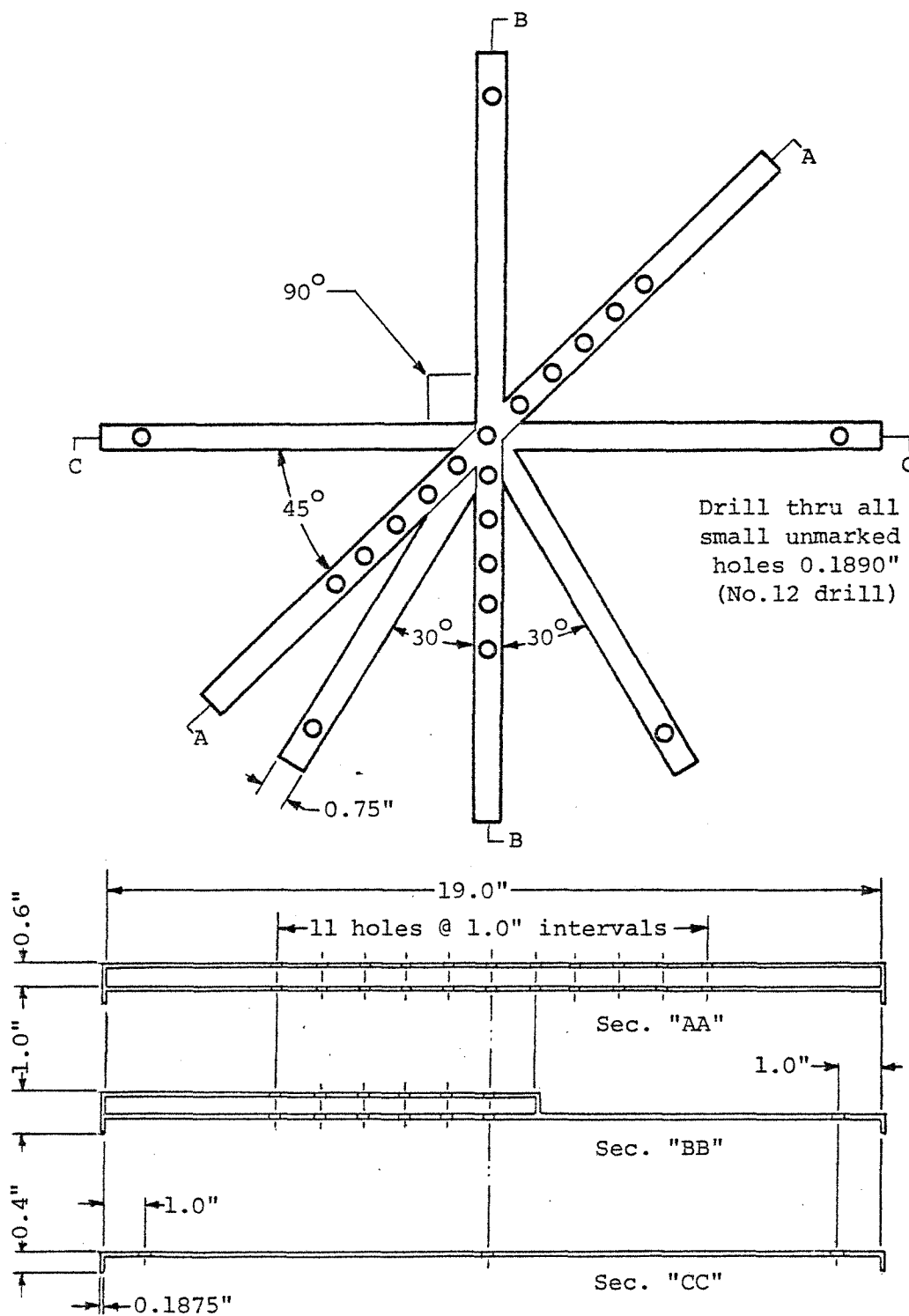
Technical drawing of a mechanical assembly, likely a pump or motor housing, showing dimensions in inches. The drawing includes a side view and a cross-sectional view.

**Dimensions (inches):**

- Overall width: 36.0"
- Overall height: 19.0"
- Top flange width: 1.0"
- Top flange thickness: 2.5"
- Bottom flange width: 12.0" (left section), 12.0" (right section)
- Bottom flange thickness: 2.0"
- Internal vertical distance: 6.8" (total), 6.0" (inner), 5.5" (outer)
- Horizontal distance from left wall to internal feature: 3.5"
- Horizontal distance from internal feature to right wall: 3.5"
- Horizontal distance from left wall to bottom flange: 30.0"
- Horizontal distance from bottom flange to right wall: 36.0"
- Horizontal distance from left wall to top flange: 2.5"
- Horizontal distance from top flange to right wall: 1.0"
- Horizontal distance from left wall to internal feature: 3.5"
- Horizontal distance from internal feature to right wall: 3.5"
- Horizontal distance from left wall to bottom flange: 3.5"
- Horizontal distance from bottom flange to right wall: 3.5"

FIGURE 9

WEDGE SUPPORT AND ALIGNMENT PLATFORM WITH  
SPHERE RELEASE MECHANISM ALIGNMENT ASSEMBLY



drilled out at its center for the central wedge support. All holes on the platform were drilled with a No. 12 drill bit (0.1890 inch I.D.).

#### Sphere Release Mechanism

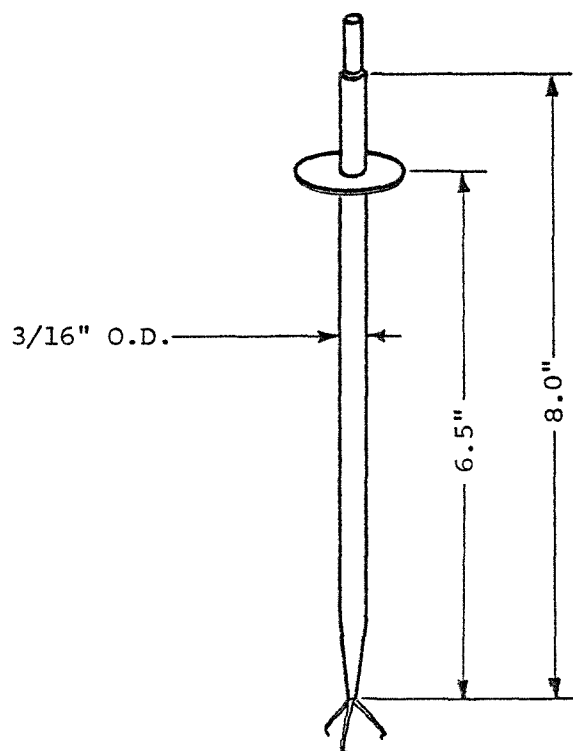
A Triceps type forceps Model T8 was modified to handle the positioning and release of spheres in the liquid. The modification consisted of attaching a 3/4 inch washer to the forceps with epoxy cement 1.5 inches from the top to act as a stop for its vertical travel. This unit inserted through the proper hole on the double arm gives a stable and reproducible starting point for the sphere during the experimental runs.

Figure 10 shows the sphere release mechanism.

#### Wedge Sections

The wedge sections were fabricated from Plexiglas brand 1/8 inch thick acrylic sheet (ANSI Z97.1-1966/72 079U) from Rohm & Haas. Four wedges were produced, each forming a different angle (60, 90 which also doubled as the 270, 180 and 360). Each wedge was 34 inches high with the sides having a radial distance of 12 inches. The support rod and the brackets were formed from 316 S.S. 10-24 threaded rods and 5/8 inch by 1/8 inch 316 S.S. channels respectively. The threaded rods were spot welded to the top of the channel. For the angled brackets the channels were cut and welded together to form the correct

FIGURE 10  
SPHERE RELEASE MECHANISM





angle before the threaded rod was attached. The completed brackets were placed at predetermined locations on the top edge of the wedge and each had two 1/8 inch holes drilled through both the channel and the acrylic sheet. The brackets were attached to the wedges through the pre-drilled holes with short 1/8 inch sheet metal screws.

The 180 degree wedge was formed from two 34 inch by 12 inch sheets, which were connected together with small hinges near the top and bottom edges. The hinging allowed the insertion of the wedge past the restriction on the top of the lubricant container. The central support on the 180 degree wedge was connected in place after the insertion into the container.

Figures 11 thru 14 are the assembly drawings of the wedge sections for the various angles.

#### Auxiliary Equipment

The temperature of the lubricant was constantly monitored during the experiments at 1 inch below the liquid surface and at the bottom of the container. The temperature measurement was accomplished by the use of calibrated 1/16 inch Chromel-Alumel thermocouples. Each of the thermocouples were connected to a CONDEC digital indicator, which provided continuous readout of the temperatures. The range of the instrument was 999.9 degrees Celsius with 0.1 degree accuracy.

FIGURE 11  
ASSEMBLY DRAWING FOR THE 60° WEDGE SECTION

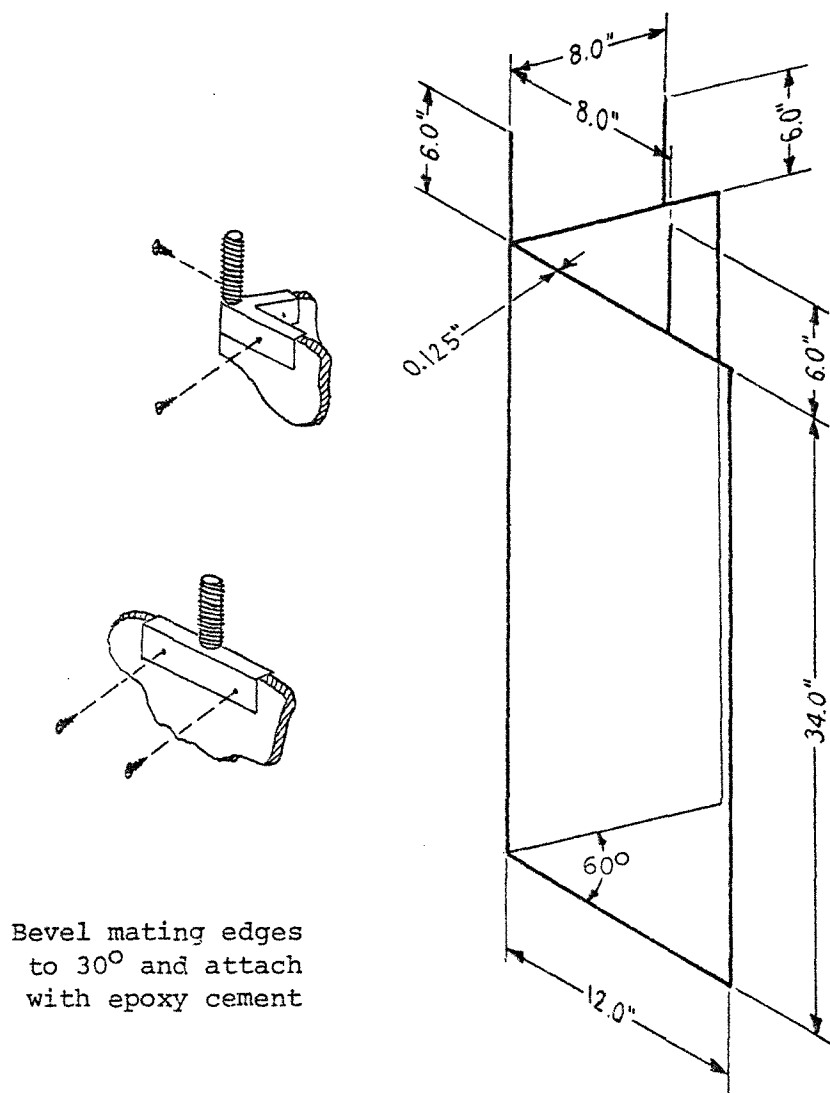


FIGURE 12  
ASSEMBLY DRAWING FOR THE 90° WEDGE SECTION

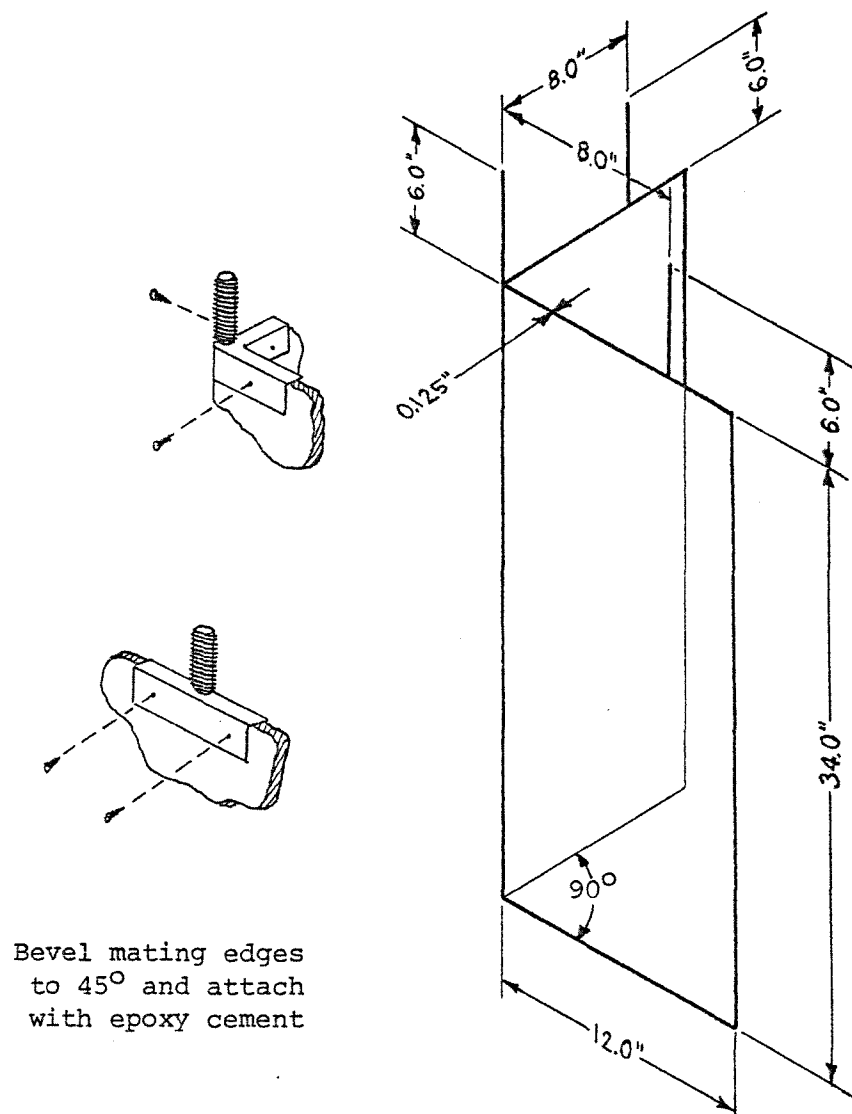


FIGURE 13  
ASSEMBLY DRAWING FOR THE 180° WEDGE SECTION

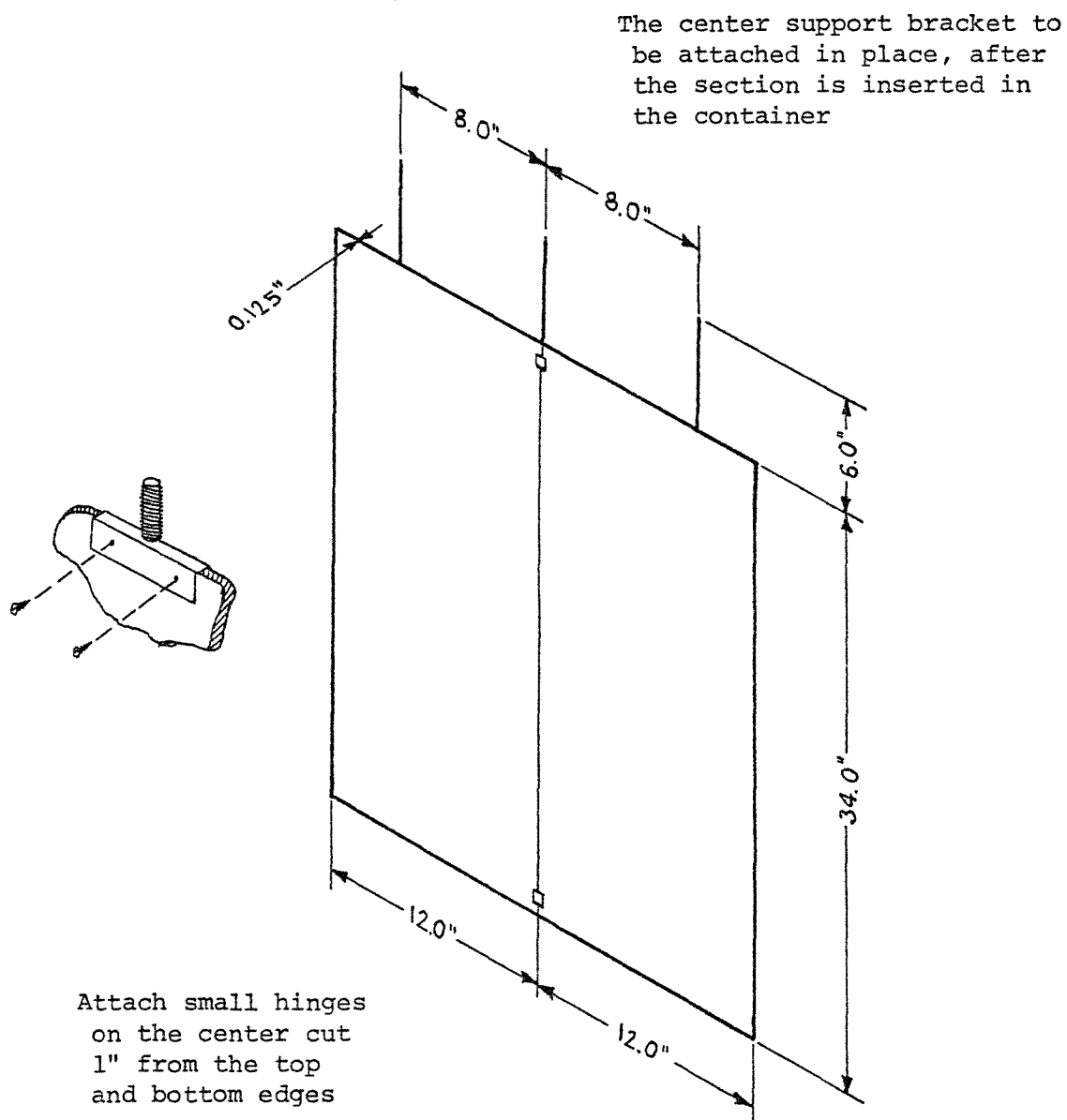
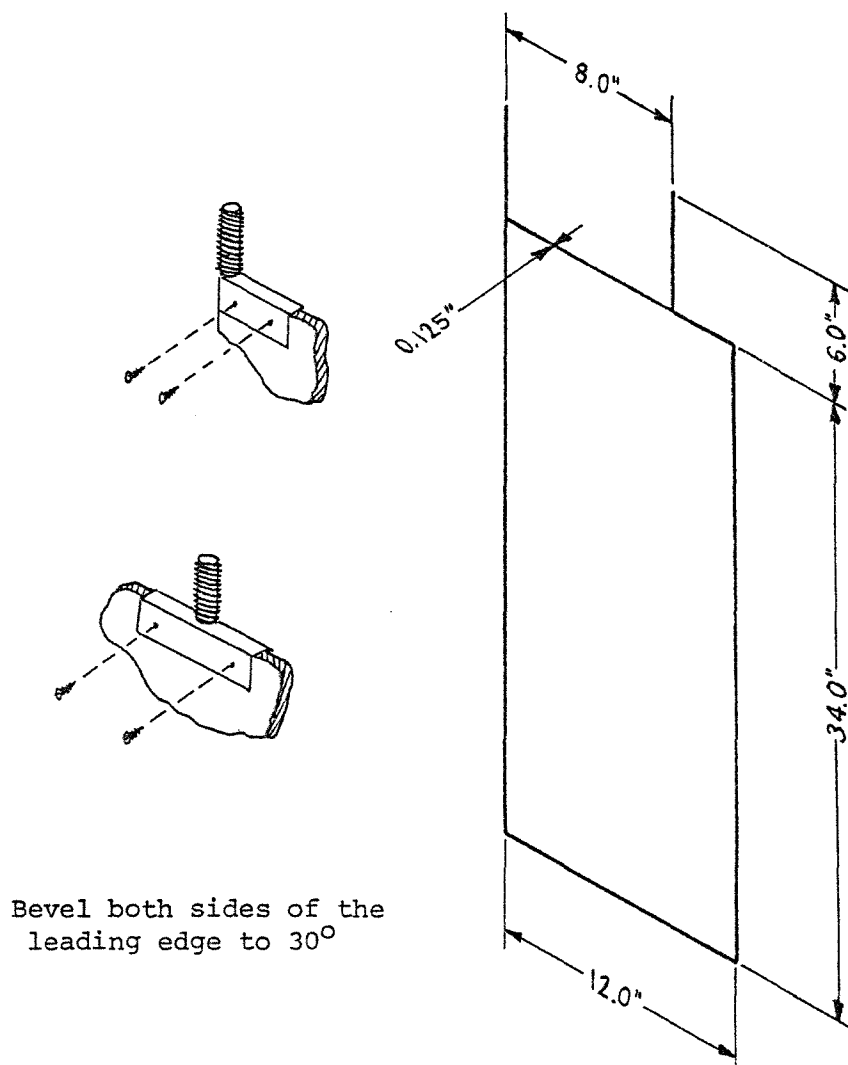


FIGURE 14  
ASSEMBLY DRAWING FOR THE 360° WEDGE SECTION



For the determination of the settling times, a Faehr brand digital stopwatch was utilized. The timer has a 5 digit display capability with 0.1 second accuracy.

The background lighting for the container was provided by a 2 feet long 32 watt fluorescent light, which was placed approximately 15 inches beyond the rear viewing ports.

### Testing Environment

To overcome the temperature fluctuation which plagued earlier experimental work<sup>16</sup>, the experimental equipment was placed in an 8 feet high by 7 feet by 6 feet Geldback temperature controlled enclosure, where all the sphere-dropping experiments were conducted.

To insure that the system reached thermal equilibrium, the control unit on the enclosure was set to 68 degrees Fahrenheit three days prior to the start of the experiments. During these days the lubricant and the enclosure air temperature was monitored.

Within 3 hours the lubricant temperature reached equilibrium at 20.2 degrees Celsius and remained there without changing. The equilibrium air temperature inside the enclosure measured 68.4 degrees Fahrenheit. Opening the door on the enclosure changed the temperature less than 1 degree Fahrenheit and after the door was closed the temperature returned to equilibrium within 3 min-

utes. Short openings of the door did not effect the lubricant temperature.

The enclosure was located within a larger room where the temperature was kept between 66 and 72 degrees Fahrenheit, which also helped to stabilize the temperature fluctuation in the controlled enclosure. This location was also used to store all equipment not in use.

#### Test Conditions and Limitations

To test the validity of the derived equations, all combinations of variables used in the equations were tested, with the exception of those which were the functions of the temperature. The lubricant temperature was kept constant at 20.2 degrees Celsius. In all cases, sextuplicate runs were made to test reproducibility. Agreement between the six runs never varied more than 3 percent.

As was explained earlier, that the primary limitation was the size of the lubricant container, which further restricted some of the other variables. The experiment was designed so that the wedge apex to particle center distance should vary up to approximately 40 percent of the container radius to reduce the effects of the container wall on the setting particle.

The number of different particle sizes were limited by the availability of various size spheres of proper

grade and material. Since these precision spheres are normally custom manufactured, we were fortunate to acquire a good selection of each of 3 widely diverse sizes. The larger sizes were approximately 1.6 and 2.2 times the diameter of the smallest one.

The final variable, the wedge angle had limitations imposed on by the theoretical work on which this experiment was based. The coefficients used in the equations had been calculated for only a few selected angles; therefore, the comparison of experimental to calculated results would not have been possible even if there were more wedges built. Only for one angle was data gathered where there were no coefficients calculated since the derivation of coefficients for the 270 degree angle was underway when the experimentation began, although at the writing of this thesis, it is still not completed.

#### Testing Procedure

A detailed description of the experimental procedure is listed below.

For each wedge angle the listed procedure was followed:

- 1, The proper wedge was lowered into the liquid and the support rods were attached loosely to the wedge support platform.
- 2, The container walls were tested for verticality with a long bubble type carpenters level. If it was needed, the leveling was accomplished by adjusting



the bolts on the leveling assemblies on each leg.

3, The wedge support platform was rotated to the proper orientation, i.e., the sphere release mechanism alignment bar was turned to the viewing port to viewing port axis.

4, The top edge of the wedge was leveled by the aid of the bubble level. The adjustment was done by tightening or loosening the nuts on the support rods.

5, After all disturbance of the liquid ceased, the system was allowed to come to equilibrium for a minimum of a half hour.

For each distance from the wedge apex to the particle center the procedure was as follows:

1, The sphere release mechanism was inserted through both holes at appropriate locations on the alignment bar.

2, All spheres were wetted with Ucon lubricant.

3, The sphere release mechanism was raised and a sphere was placed in the clampa.

4, The release mechanism was lowered into the liquid until the stop on it impeded the downward travel.

5, The sphere was released by pressing down the plunger on the top of the release mechanism and holding it down for 10 seconds.

6, The timer was started when the sphere intersected the plane formed by the timing marks on the upper view-

ing ports and was stopped when the sphere reached the plane formed by the timing marks on the lower viewing ports.

For each sphere diameter and repetitions, Steps 3 thru 6 of this section were repeated.

7, After 18 spheres were dropped, the upper valve of the lock system was closed and the lower one opened, thereby draining out the Ucon lubricant and the spheres.

8, When all spheres were removed, the lower valve was closed and the upper one opened and the lock was allowed to fill with lubricant again. The removed spheres were readied for other runs by draining the excess lubricant back to the container.

9, After all disturbance of the liquid ceased, the system was allowed to come to equilibrium for a minimum of a half hour.

## CHAPTER V

### EXPERIMENTAL RESULTS

#### Analysis of the Results

The experimentally determined settling times were converted to experimental settling velocities by taking the reciprocal of the settling times. This simple conversion was made possible since the distance where the settling was measured was exactly one foot. For each combination of variables, six determinations were made; therefore a mean value for the settling velocity and a standard deviation for the set were calculated. To examine the data scatter a conversion of the calculated standard deviations was required. The format where the comparison gave meaningful results was arrived at by dividing the standard deviation for each set by the calculated mean settling velocity for the same set. This data,  $\sigma/U_{em}$ , was plotted in Figures 15, 16 and 17 for each sphere size as the function of the distance the particle is from the wedge apex to the vessel radius ratio ( $x_o/R_o$ ). To determine if the plotted data followed a trend, the maximum and minimum values for  $\sigma/U_{em}$  for each  $x_o/R_o$  was used to obtain two lines representing the approximate limits for the data scatter by regressing the above mentioned data by the least squares method. These lines are also displayed on Figures 15, 16 and 17.

FIGURE 15  
 DATA SCATTER VARIATION FOR THE 5/32" SPHERE  
 WITH THE DISTANCE FROM THE WEDGE APEX

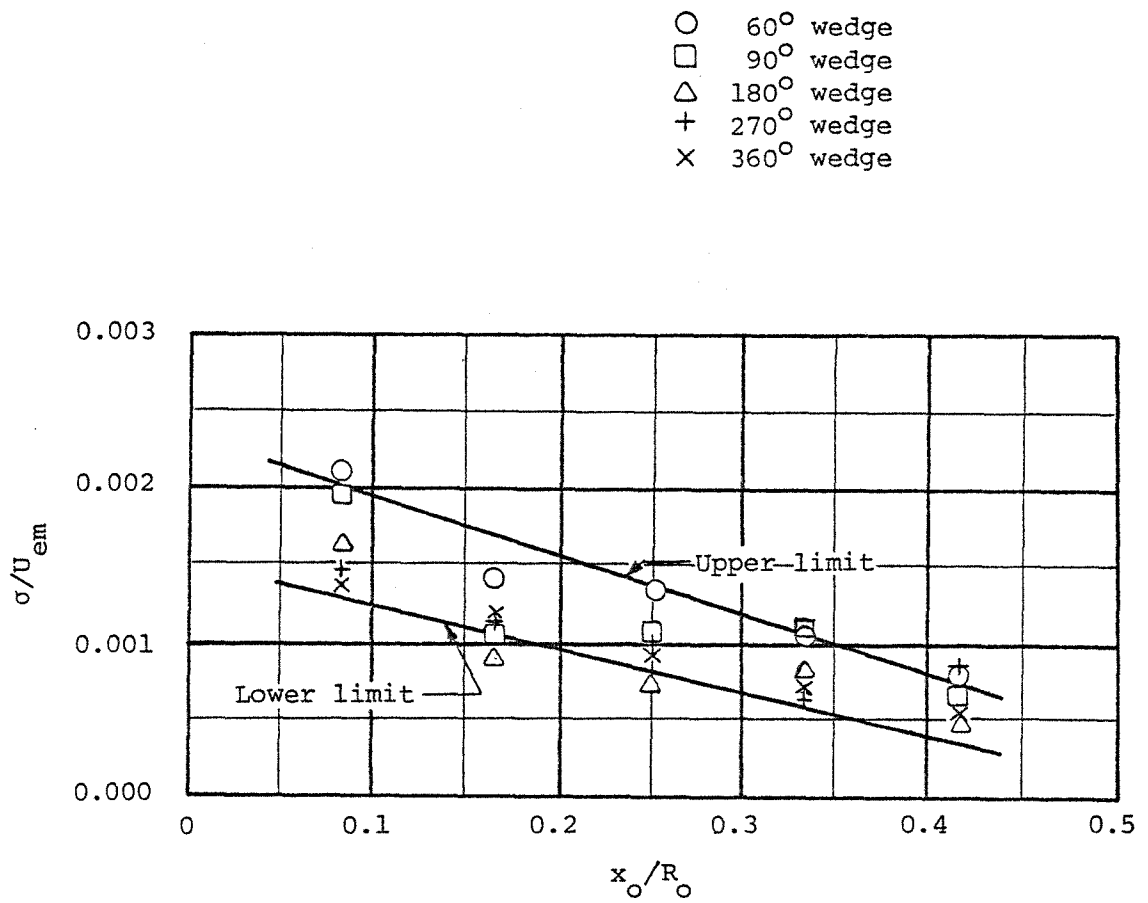


FIGURE 16  
 DATA SCATTER VARIATION FOR THE 1/4" SPHERE  
 WITH THE DISTANCE FROM THE WEDGE APEX

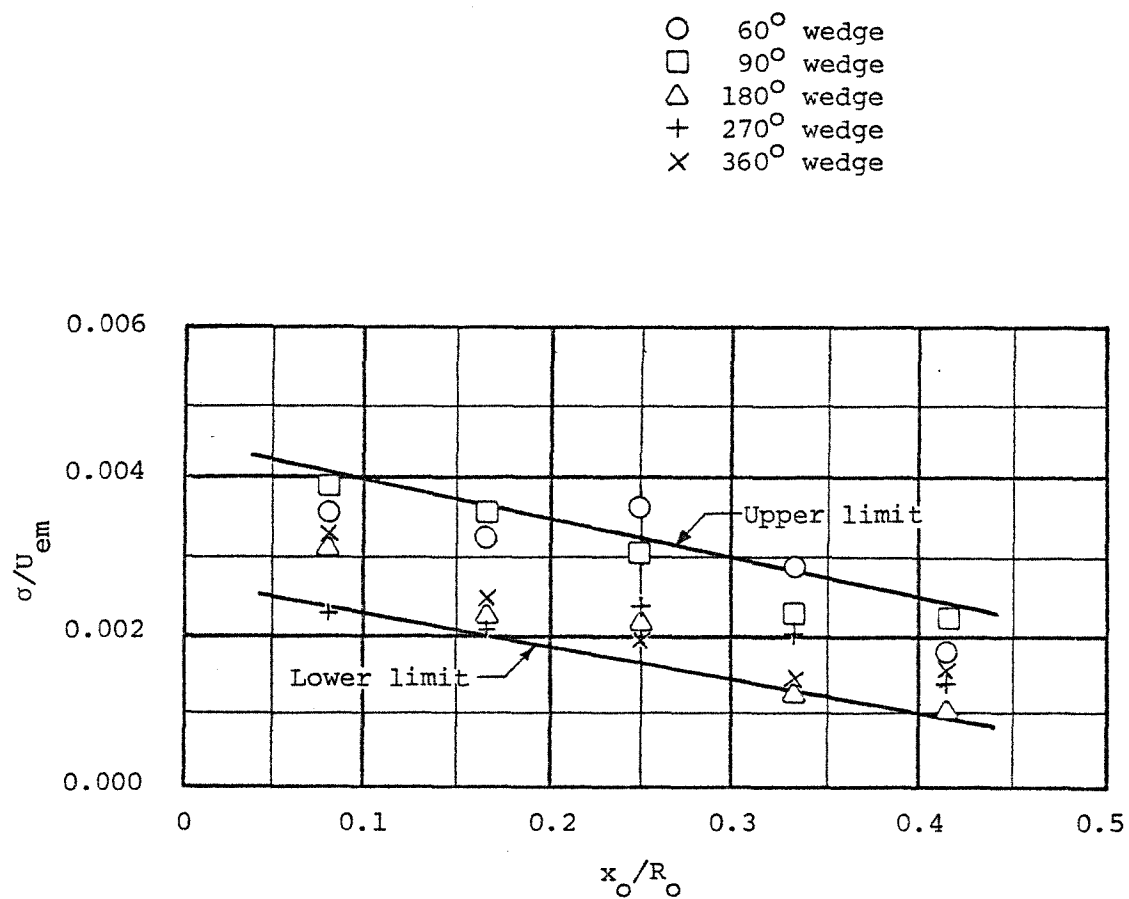
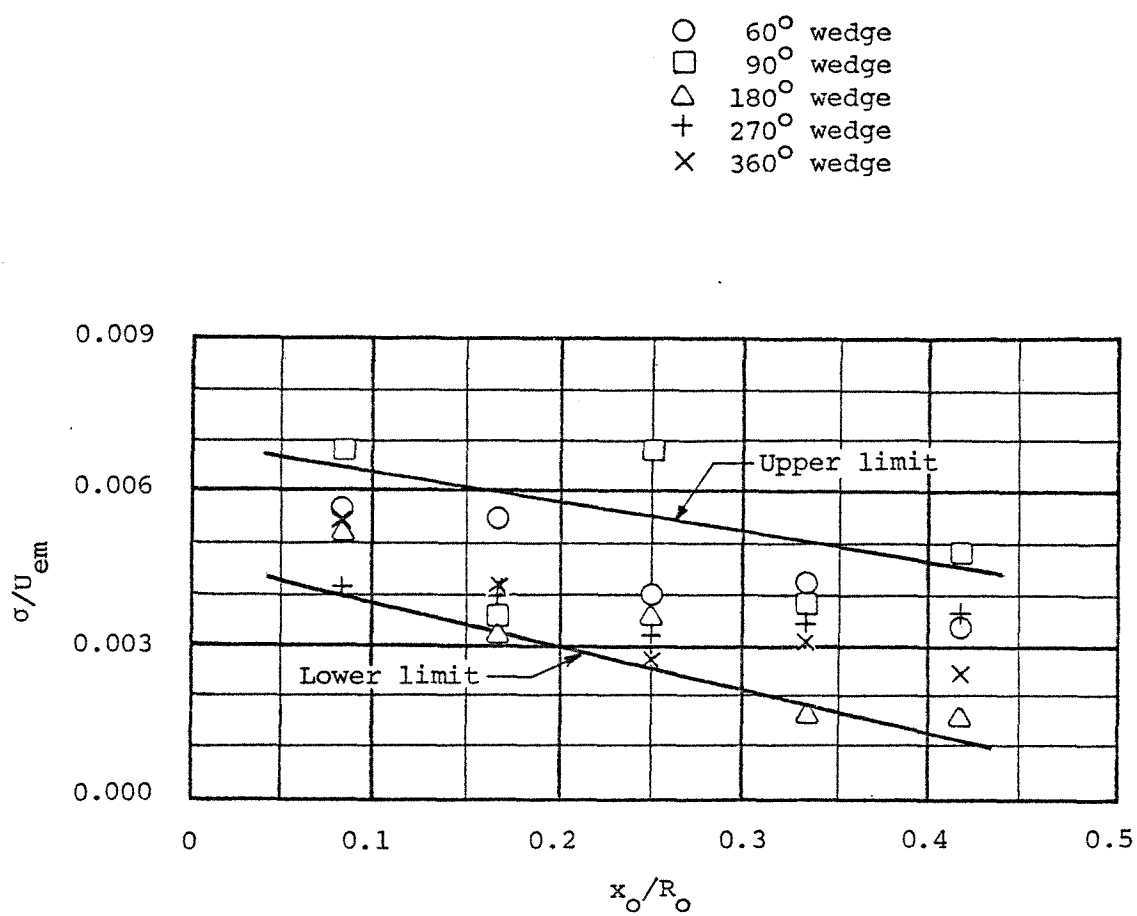


FIGURE 17  
 DATA SCATTER VARIATION FOR THE 11/32" SPHERE  
 WITH THE DISTANCE FROM THE WEDGE APEX



The slopes of the lines representing the approximate limits of the data indicate an inversely proportional relationship between  $\sigma/U_{em}$  and  $x_o/R_o$ . The reason for this effect could be traced back to the rotation of the particle imparted by the walls of the wedge. The effect of the rotation of the sphere was impossible to determine in the equipment used for this work; therefore its effect is not included in the final equations.

Although extreme care was exercised in the selection of the spheres, there was a definite possibility of the particles having non-uniform internal densities, i.e., the centroid of the particle is not at the sphere center. With the existence of spheres rotating with various angular velocities, the likelihood of a whole spectrum of solutions is possible.

#### Comparison of Theory [Equation (2.13)] and Experiment

A dimensionless form for the observed and predicted sphere settling velocities ( $U/U_g$ ) in Ucon lubricant at 20.2 degrees Celsius, as determined by the experiment and equation (2.13) for wedge angles of 60, 90, 180 and 360 degrees, are shown in graphical form as the function of  $x_o/a$  in Figures 18 thru 21, respectively. This equation predicted the settling velocities based on the effects of the wedge wall. In all cases, the experiment gave good agreement for angles less than 180 degrees. For the 360 degree angle, the experimental values appear to deviate

FIGURE 18

COMPARISON OF EXPERIMENTAL SETTLING VELOCITIES WITH THE THEORETICAL  
PREDICTION OF EQUATION (2.13) FOR THE  $60^\circ$  WEDGE ANGLE

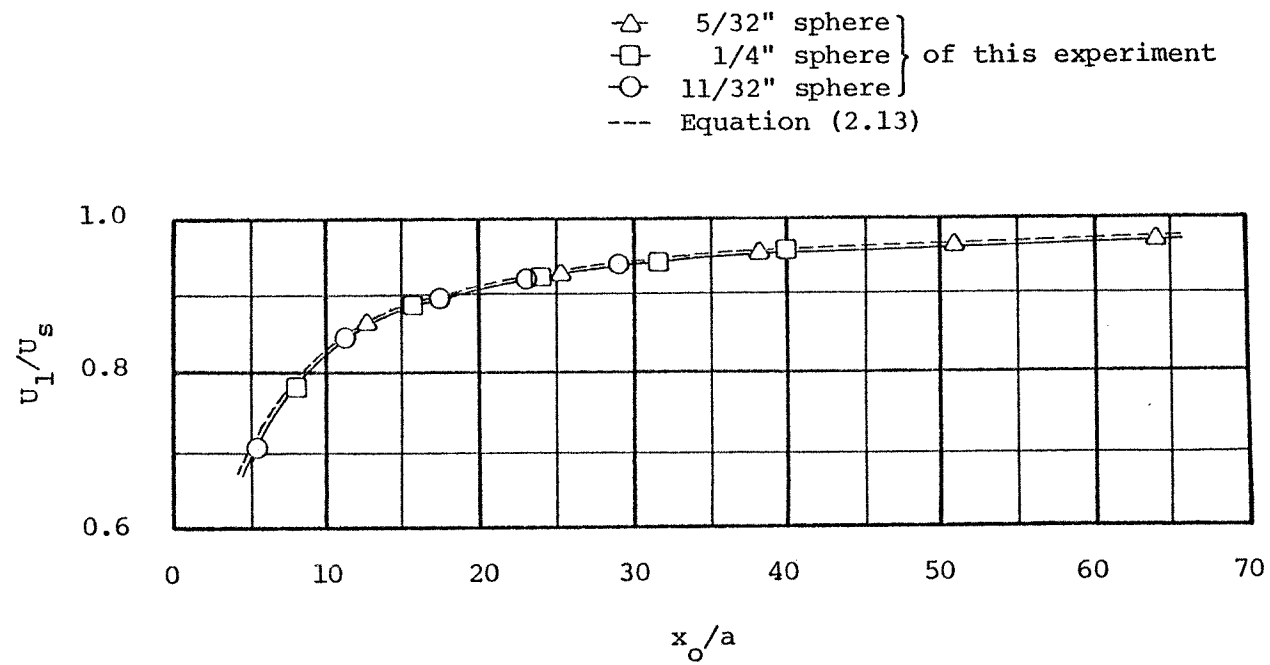




FIGURE 19

COMPARISON OF EXPERIMENTAL SETTLING VELOCITIES WITH THE THEORETICAL  
PREDICTION OF EQUATION (2.13) FOR THE  $90^\circ$  WEDGE ANGLE

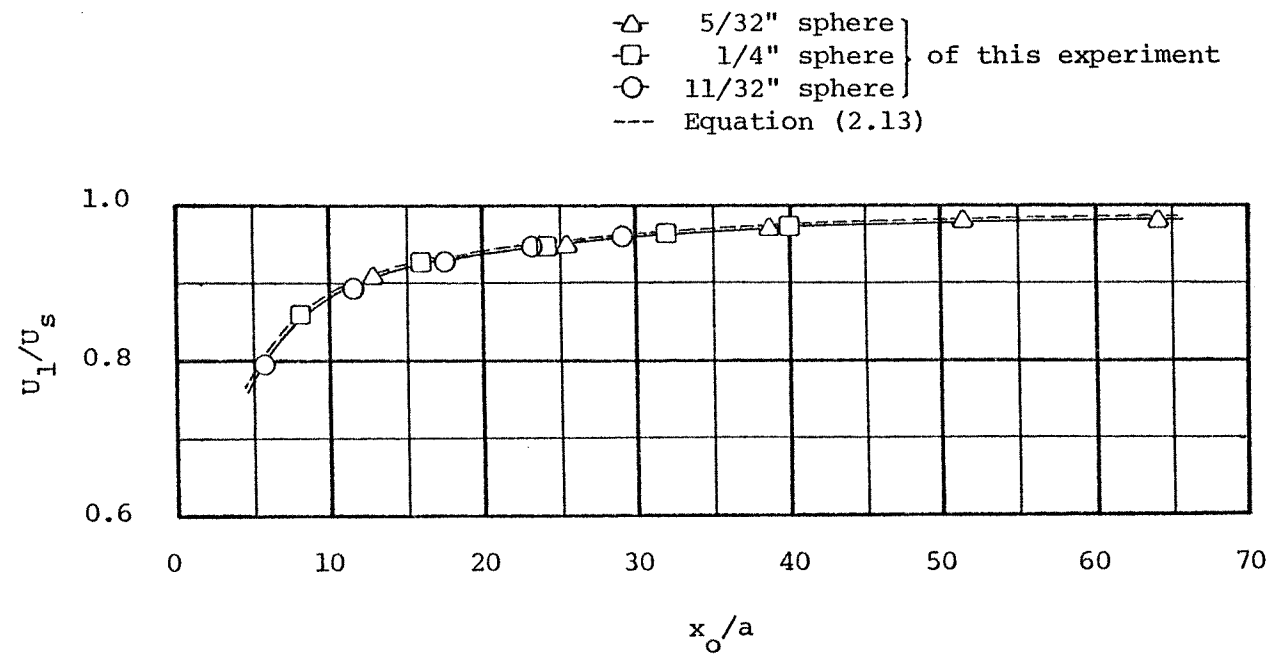


FIGURE 20

COMPARISON OF EXPERIMENTAL SETTLING VELOCITIES WITH THE THEORETICAL  
PREDICTION OF EQUATION (2.13) FOR THE  $180^\circ$  WEDGE ANGLE

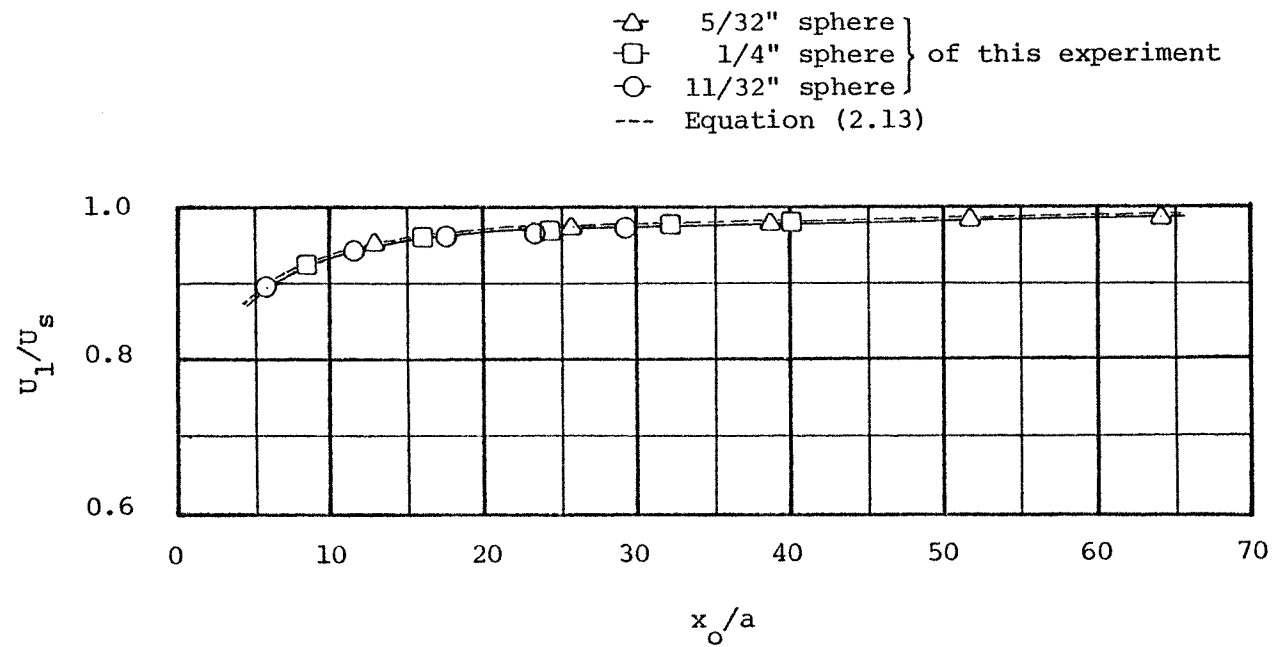
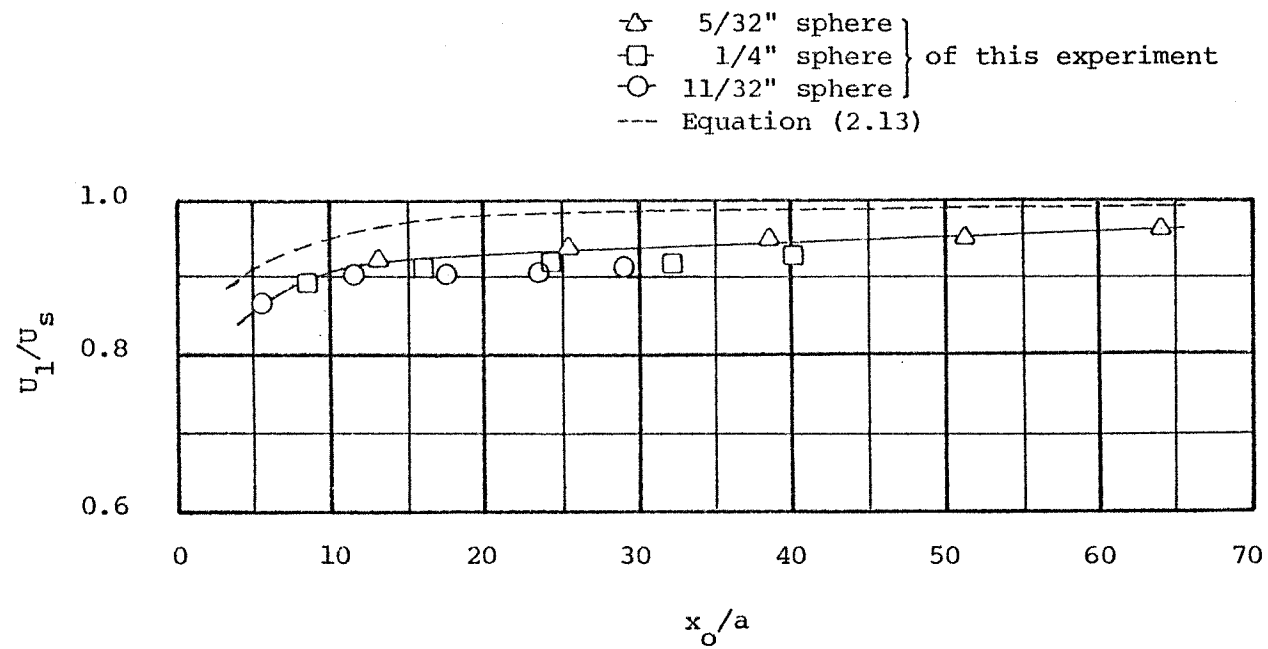


FIGURE 21

COMPARISON OF EXPERIMENTAL SETTLING VELOCITIES WITH THE THEORETICAL  
PREDICTION OF EQUATION (2.13) FOR THE  $360^\circ$  WEDGE ANGLE



from the theory by a small margin. This outcome may be diagnosed by reviewing the physical make-up of the equipment. The calculations in the theory were based on a sphere settling parallel to an infinitely thin plate, whereas in the actual test equipment a plate with 1/8 inch thickness was substituted. The additional drag from the edge could be the cause for the observed deviation.

#### Comparison of Theory [Equation (2.23)] and Experiment

The modified equation (2.23), accounting for both the wedge and vessel wall effects, was compared to the experimental data and shown graphically in Figures 22 thru 25 for the wedge angles 60, 90, 180 and 360 degrees. The agreement of the predicted dimensionless form of the settling velocities with the experimental data is excellent for all cases, showing a considerable improvement over equation (2.13). The largest deviation from the predicted values was observed for the 360 degree angle. The explanation offered for this deviation is identical to the one proposed in the preceding section for equation (2.13). The agreement for the experimental and calculated values are particularly striking for angles less than 180 degrees where the differences are less than 1 percent for most cases.

#### Empirical Determination of Coefficients $f_1(\phi_0)$ and $f_2(\phi_0)$

At the writing of this thesis the coefficients

FIGURE 22

COMPARISON OF EXPERIMENTAL SETTLING VELOCITIES WITH THE THEORETICAL  
PREDICTION OF EQUATION (2.23) FOR THE  $60^\circ$  WEDGE ANGLE

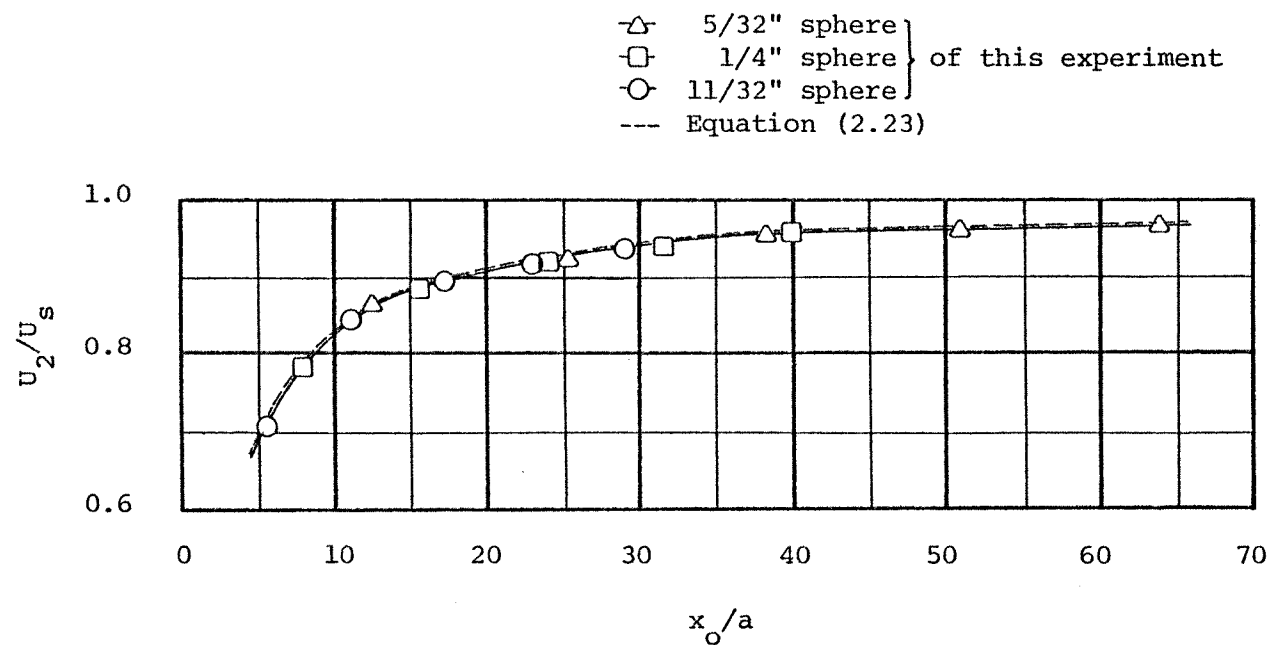


FIGURE 23

COMPARISON OF EXPERIMENTAL SETTLING VELOCITIES WITH THE THEORETICAL  
PREDICTION OF EQUATION (2.23) FOR THE  $90^\circ$  WEDGE ANGLE

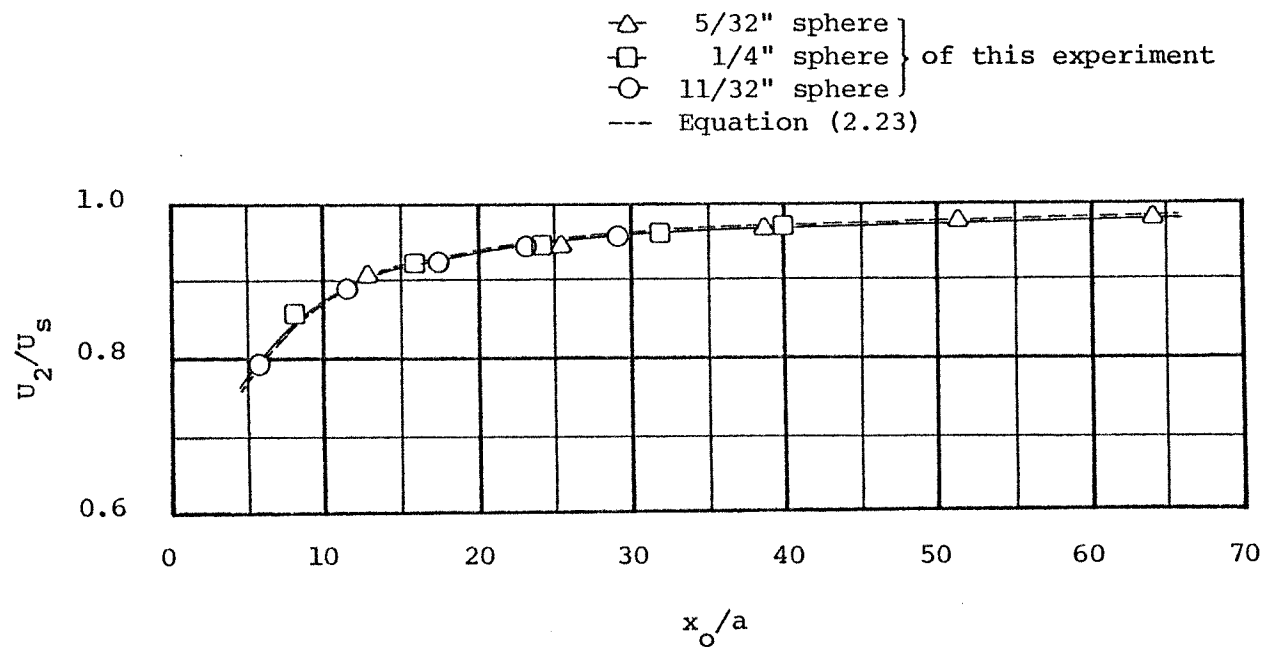


FIGURE 24

COMPARISON OF EXPERIMENTAL SETTLING VELOCITIES WITH THE THEORETICAL  
PREDICTION OF EQUATION (2.23) FOR THE  $180^\circ$  WEDGE ANGLE

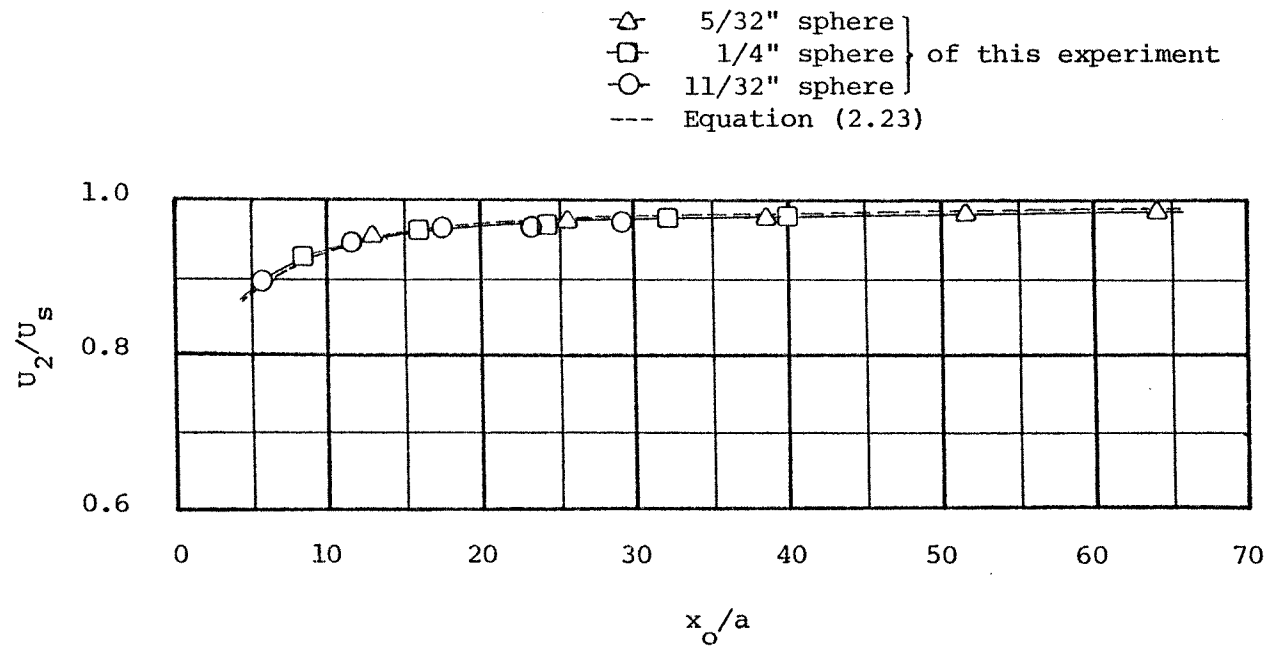
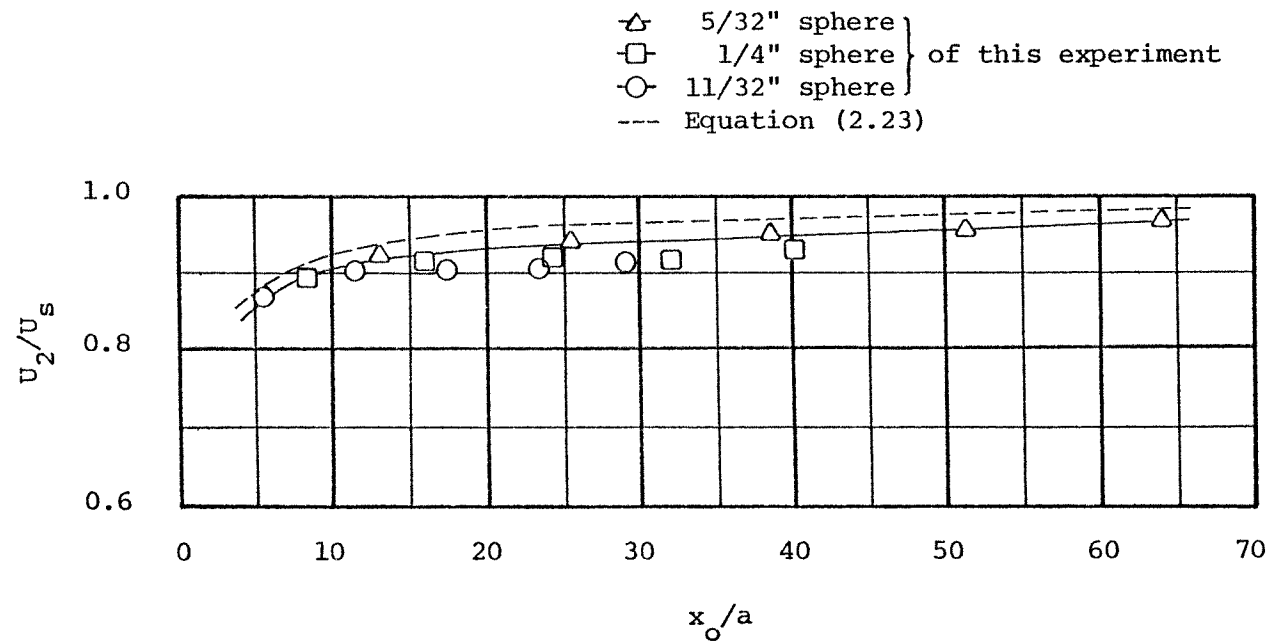


FIGURE 25

COMPARISON OF EXPERIMENTAL SETTLING VELOCITIES WITH THE THEORETICAL  
PREDICTION OF EQUATION (2.23) FOR THE  $360^\circ$  WEDGE ANGLE





$f_1(\phi_0)$  and  $f_2(\phi_0)$  for the 270 degree angle had still not been derived; therefore, the experimental settling velocities were used to obtain these coefficients. Equation (2.23) was converted to the following format:

$$-(a/x_0)f_1(\phi_0) - (a/x_0)^3f_2(\phi_0) = (U/U_s) + \{f(\beta)(\phi_0/\pi)(a/R_0)\} - 1, \quad (5.1)$$

in order to facilitate the calculation of the coefficients by determinants. Using Cramer's Rule, the following equations were derived for the coefficients:

$$f_1(\phi_0) = (b_i a_{j2} - b_j a_{i2}) / (a_{i1} a_{j2} - a_{j1} a_{i2}) \quad (5.2)$$

and

$$f_2(\phi_0) = (b_j a_{i1} - b_i a_{j1}) / (a_{i1} a_{j2} - a_{j1} a_{i2}), \quad (5.3)$$

where  $b_i$  and  $b_j$  is  $(U/U_s) + \{f(\beta)(\phi_0/\pi)(a/R_0)\} - 1$ ,

$a_{i1}$  and  $a_{j1}$  is  $-(a/x_0)$ , and  $a_{i2}$  and  $a_{j2}$  is  $-(a/x_0)^3$ ,

with  $i$  and  $j$  referring to two different linear equations for the same wedge angle.

All combinations of the available data were evaluated and an arithmetic mean and the deviation from the mean were determined.

The mean values for the coefficients determined by the above method are tabulated in Table 8.

The empirically determined coefficients were substituted into equations (2.13) and (2.23) and the calculated settling velocities, determined by this method,

TABLE 8

EMPIRICALLY DERIVED COEFFICIENTS FOR THE  
270 DEGREE WEDGE ANGLE

<u>Coefficient</u>	<u>Empirically derived mean value</u>	<u>Deviation from the empirical mean value</u>
$f_1(\phi_o)$	0.2274	$\pm 1.04$
$f_2(\phi_o)$	22.3129	$\pm 112.1$

compared to the experimental values. The results of this comparison are displayed on Figures 26 and 27.

FIGURE 26

COMPARISON OF EXPERIMENTAL SETTLING VELOCITIES WITH THE THEORETICAL  
PREDICTION OF EQUATION (2.13) USING THE EMPIRICALLY  
DERIVED COEFFICIENTS FOR THE  $270^\circ$  WEDGE ANGLE

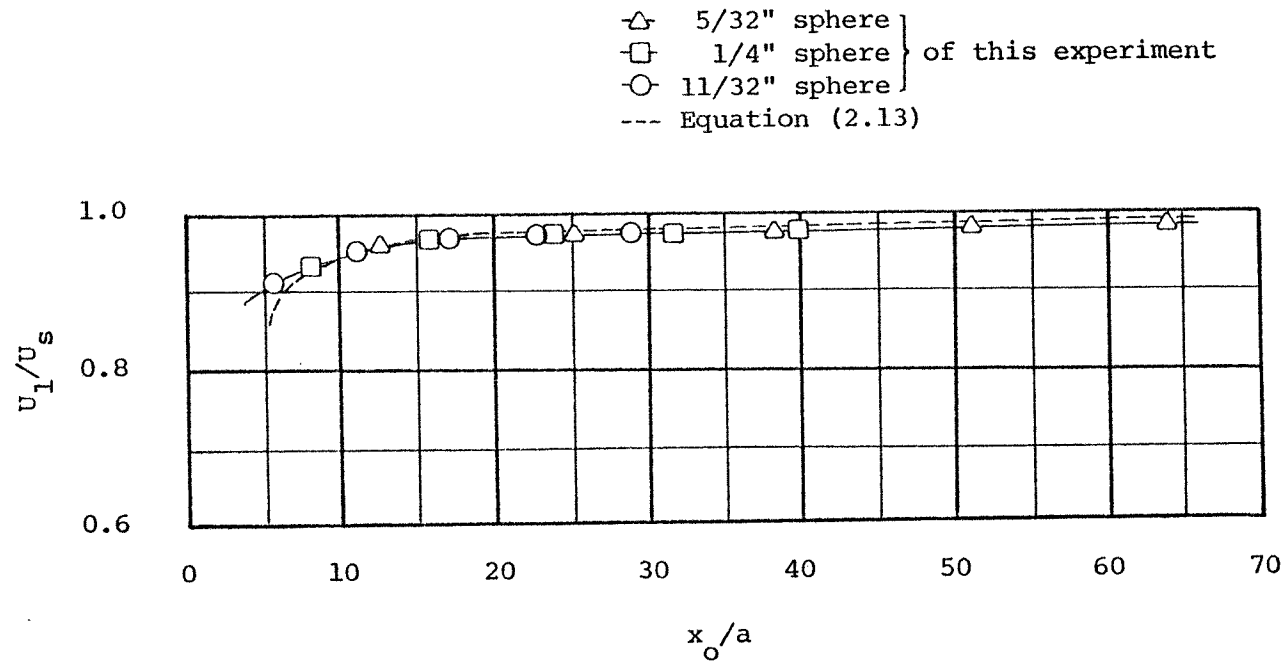
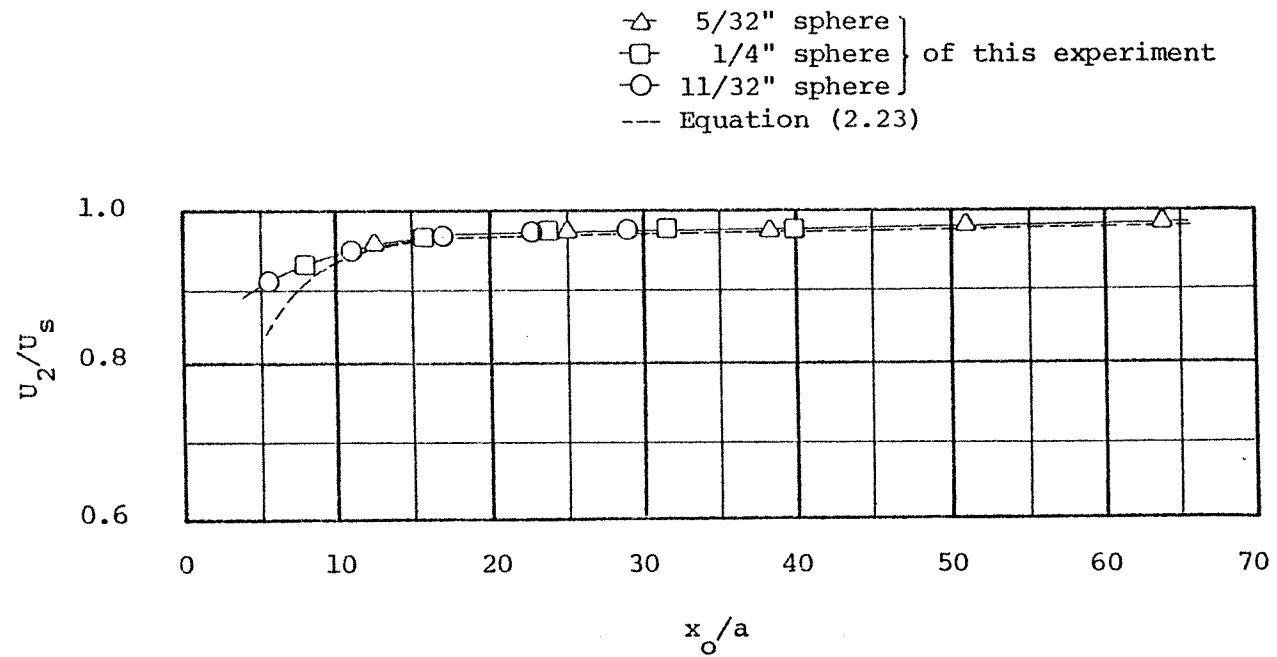


FIGURE 27

COMPARISON OF EXPERIMENTAL SETTLING VELOCITIES WITH THE THEORETICAL  
PREDICTION OF EQUATION (2.23) USING THE EMPIRICALLY  
DERIVED COEFFICIENTS FOR THE  $270^\circ$  WEDGE ANGLE



## CHAPTER VI

### CONCLUSIONS AND RECOMMENDATIONS

#### Conclusions

The results of the experiments conducted in the course of this thesis to determine the settling velocities of Delrin Spheres in Ucon lubricant may be summarized as follows:

- 1, Equation (2.13), using only the correction from the wedge wall effects, gives a reasonable approximation for the velocity of a particle settling in a wedge shaped domain of viscous liquid.
- 2, The modified equation (2.23), combining the vessel wall effects and the previously utilized wedge correction, accurately describes the settling of a sphere in a column of viscous fluid with a base of a circular sector.
- 3, The wedge walls impart a rotational effect of varying degrees on the particle settling near them. This induces a slight effect upon the translational velocity thereby increasing the scattering of the measured data. The unpredictable rotation is caused by the particles having non-homogeneous internal densities or by not being perfectly spherical.
- 4, The vessel wall contributes only a small portion

of the drag on the settling particle and it may be approximated by taking a fraction of equation (2.21).

- 5, The empirically derived coefficients for the 360 degree wedge angle differ only by a small amount from the theoretical values.

#### Recommendations

- 1, A study, effectively the continuation of this one, would be to determine the settling velocities near the vessel wall to test the validity of the derived equation for that region and expand its range. Experiments with other fluids and sphere materials could be used to further increase the confidence in the equation.
- 2, The rotation of the particle should be studied experimentally to quantify its effect on a settling particle, if spheres with uniform internal densities can be found.
- 3, A study not developed here would be the investigation of the effect the bottom of the vessel has on the particle settling toward it.
- 4, A study of particular importance would be that of the effect of the settling of multiple particles, which would extend the effectiveness of the equation for use in designing practical equipment.

## APPENDIX A



### SAMPLE CALCULATION

The physical properties determined prior to the start of the experimental runs and the settling times taken during a run were required to ascertain the following:

- 1, experimental settling velocities,
- 2, calculated settling velocities considering only the effects of the wedge on the settling particle,
- 3, calculated settling velocities based on the effects of both the wedge and the vessel wall.

A typical calculation for the above quantities is listed in this section. The data used for the sample calculation was collected for the run, where the wedge angle was 60 degrees and a 0.1562" diameter sphere was dropped 1 inch from the wedge apex.

#### Experimental Settling Velocities

The experimental settling velocities were calculated by taking the reciprocal of the recorded settling times, since the distance where the settling was measured was 1 foot.

$$U_{e(i)} = 1/\tau_{(i)}$$

where  $U_{e(i)}$  is the experimental settling velocity in ft/sec and  $\tau_{(i)}$  is the measured experimental settling time in seconds, with  $i$  referring to the replicate run for the same set.

Input data

$$\tau_1 = 362.0 \text{ sec.}$$

$$\tau_2 = 363.4 \text{ sec.}$$

$$\tau_3 = 364.5 \text{ sec.}$$

$$\tau_4 = 363.7 \text{ sec.}$$

$$\tau_5 = 362.6 \text{ sec.}$$

$$\tau_6 = 363.6 \text{ sec.}$$

Results

$$U_{e1} = 0.002762 \text{ ft/sec.}$$

$$U_{e2} = 0.002752 \text{ ft/sec.}$$

$$U_{e3} = 0.002743 \text{ ft/sec.}$$

$$U_{e4} = 0.002750 \text{ ft/sec.}$$

$$U_{e5} = 0.002758 \text{ ft/sec.}$$

$$U_{e6} = 0.002750 \text{ ft/sec.}$$

An arithmetic mean for the set was calculated. This value was compared to the theoretical predictions.

$$U_{em} = \left( \sum_{i=1}^n U_{e(i)} \right) / n$$

where  $U_{em}$  is the mean experimental settling velocity in ft/sec and  $n$  is the number of replications.

Input data

The above calculated experimental settling velocities.

Result

$$U_{em} = 0.002753 \text{ ft/sec.}$$

The standard deviation for the set was also determined to evaluate the data scatter.

$$\sigma = \left( \sum_{i=1}^n (U_{e(i)} - U_{em})^2 / n \right)^{1/2}$$

where  $\sigma$  is the standard deviation from the mean for the set.

Input data

The experimental and experimental mean settling velocities listed on the previous page.

Result

$$\sigma = 0.6096 \times 10^{-5}$$

#### Calculated Settling Velocities

Since the calculated settling velocities are based on the Stokes settling velocity, therefore a value for the latter was evaluated.

$$U_s = [2ga^2(\rho_p - \rho_l)] / 9\mu$$

where  $g$  is the acceleration of gravity in  $\text{ft/sec}^2$ ,  $a$  is the particle radius in  $\text{ft}$ ,  $\rho_p$  and  $\rho_l$  are the densities for the particle and liquid, respectively in  $\text{lb/ft}^3$ , and  $\mu$  is the liquid viscosity in  $\text{lb/ft}\cdot\text{sec}$ .

Input data

$$g = 32.2 \text{ ft/sec}^2.$$

$$a = 0.0781 \text{ inch}$$

$$= (0.0781 \text{ inch}) (0.0833 \text{ ft/inch})$$

$$= 0.0065 \text{ ft.}$$

$$\rho_p = 1.3883 \text{ gms/cm}^3 \text{ from Table 7.}$$

$$= (1.3883 \text{ gms/cm}^3) (62.4264 \text{ cm}^3 \cdot \text{lb/gms} \cdot \text{ft}^3)$$

$$= 86.6666 \text{ lb/ft}^3.$$

$$\rho_1 = 1.0613 \text{ gms/cm}^3 \text{ from Equation (3.1) for } 20.2^\circ\text{C.}$$

$$= (1.0613 \text{ gms/cm}^3) (62.4264 \text{ cm}^3 \cdot \text{lb/gms} \cdot \text{ft}^3)$$

$$= 66.2531 \text{ lb/ft}^3.$$

$$\mu = 2706.57 \text{ centistokes from Equation (3.2) for } 20.2^\circ\text{C.}$$

$$= (2706.57 \text{ centistokes}) (1.0764 \times 10^{-5} \text{ ft}^2/\text{centi-} \\ \text{stokes} \cdot \text{sec}) (66.2531 \text{ lb/ft}^3)$$

$$= 1.9302 \text{ lb/ft} \cdot \text{sec.}$$

#### Result

$$U_s = 0.003206 \text{ ft/sec.}$$

The calculated settling velocity considering only the wedge effects on the settling particle was evaluated.

$$U_1 = U_s [1 - (a/x_o) f_1(\phi_o) - (a/x_o)^3 f_2(\phi_o)]$$

where  $U_1$  is the calculated settling velocity in ft/sec,  $x_o$  is the distance from the wedge apex to the particle center in ft, and  $f_1(\phi_o)$  and  $f_2(\phi_o)$  are the wedge angle coefficients for translating particles.

#### Input data

$$a = 0.0065 \text{ ft.}$$

$$x_o = 0.0833 \text{ ft.}$$

$$f_1(\phi_o) = 1.7891$$

$$f_2(\phi_o) = -2.7820$$

$$U_s = 0.003206 \text{ ft/sec.}$$

Result

$$U_1 = 0.002761 \text{ ft/sec.}$$

Similarly, the calculated settling velocity considering the wedge and vessel wall effects on the settling particle was evaluated.

$$U_2 = U_s [1 - (a/x_o) f_1(\phi_o) - (a/x_o)^3 f_2(\phi_o) - (a/R_o) (\phi/180) f(\beta)]$$

where  $U_2$  is the calculated settling velocity in ft/sec,  $R_o$  is the fluid container radius in ft,  $\phi$  is half of the wedge angle in degrees, and  $f(\beta)$  is the eccentricity coefficient ( $x_o/R_o$ ).

Input data

$$a = 0.0065 \text{ ft.}$$

$$x_o = 0.0833 \text{ ft.}$$

$$f_1(\phi_o) = 1.7891$$

$$f_2(\phi_o) = -2.7820$$

$$R_o = 1.0 \text{ ft.}$$

$$\phi = 30 \text{ degrees}$$

An interpolated value was used for  $f(\beta)$ . From Table 4 the following was acquired:  $f(\beta)=2.10270$  for  $\beta=0.05$  and  $f(\beta)=2.09758$  for  $\beta=0.10$ . The interpolation

yielded  $f(\beta)=2.0993$  for  $\beta=0.0833$ .

$$f(\beta) = 2.0993$$

Result

$$U_2 = 0.002754 \text{ ft/sec.}$$

## APPENDIX B

TABLE 9

PHYSICAL PROPERTIES FOR UCON LUBRICANT 50-HB-5100<sup>9</sup>

<u>Property</u>	<u>Temperature</u>	<u>Units</u>	<u>Value</u>
Density	98.9°C	gms/cm <sup>3</sup>	1.003
	37.8°C		1.048
	15.6°C		1.065
Specific Gravity	20.0/20.0°C		1.063
Viscosity	98.9°C	centistokes	168
	37.8°C		1104
	-17.8°C		~ 70000
Viscosity Index			281
Refractive Index	20.0°C	N <sub>D</sub> <sup>20</sup>	1.462
Surface Tension	15.6°C	dynes/cm	35-40
Vapor Pressure	20.0°C	Torr	<0.001
Water Content		% by wt.	<0.25
Pour Point		°C	-28.9



## APPENDIX C

TABLE 10

PHYSICAL PROPERTIES FOR ACETAL (DELRIN) SPHERES<sup>17</sup>

<u>Property</u>	<u>Units</u>	<u>Value</u>
Specific Gravity		1.425
Water Absorption 24 hrs.	%	0.4
Rockwell Hardness	R scale	94-120
Tensile Strength	psi x 10 <sup>3</sup>	10
Flexural Strength	psi x 10 <sup>3</sup>	14
Clarity		opaque

## APPENDIX D

SOURCE LISTING FOR COMPUTER CALCULATIONS

The repetitive calculations to process the accumulated data were executed on Univac Series 70 computer in the N. J. I. T. Computer Center.

The source listing for the calculation and printing routine is included in this section.

The explanation of the special nomenclature used in the program is included in the source listing.

```

1      INTEGER TBL1(5),TBL2(5,5),TBL3(5)
2      REAL TEMP(5,3,5),RHOL(5,3,5),MUKL(5,3,5),US(5,3,5),
3      1MUAL(5,3,5),U1(5,3,5),U2(5,3,5),FBETA(5,3,5),TAU
4      2(5,3,5,6),UEXP(5,3,5,6),UEXPAV(5,3,5),SIGMA(5,3,5),
5      3PHIX2(5)
6      REAL*4 DIAP(3)/0.1562,0.2497,0.3435/,RHOP(3)/1.3883,
7      11.3774,1.4001/,FPHI1(5)/1.7891,1.1584,0.5625,'N/A',
8      20.4775/,FPHI2(5)/-2.7820,-0.8416,-0.125,'N/A',
9      3-0.05305/,XO(5)/1.0,2.0,3.0,4.0,5.0/,PHI(5)/30.0,
10     445.0,90.0,135.0,180.0/,RO/12.0/,G/32.2/,TEST/'N/A'/
11 C
12 C DIAP IS THE CALCULATED AND MEASURED DIAMETER OF THE
13 C PARTICLE IN INCHES
14 C RHOP IS THE CALCULATED DENSITY OF THE PARTICLE IN GMS/CC
15 C FPHI1 IS THE FIRST COEFFICIENT IN THE WEDGE CORRECTION
16 C EQUATION DIMENSIONLESS
17 C FPHI2 IS THE SECOND COEFFICIENT IN THE WEDGE CORRECTION
18 C EQUATION DIMENSIONLESS
19 C XO IS THE DISTANCE FROM THE WEDGE APEX TO THE PARTICLE
20 C CENTER IN INCHES
21 C PHI IS 1/2 OF THE WEDGE ANGLE IN DEGREES
22 C RO IS THE TANK RADIUS IN INCHES
23 C G IS THE ACCELERATION OF GRAVITY IN FT/SEC**2
24 C TEST CHECKS FOR THE AVAILABILITY OF COEFFICIENTS IN THE
25 C EQUATIONS
26 C
27     REAL*4 BETAL(26)/0.0,0.01,0.02,0.03,0.05,0.10,0.15,
28     10.20,0.25,0.30,0.35,0.37,0.39,0.40,0.41,0.43,0.45,
29     20.50,0.55,0.60,0.65,0.70,0.75,0.80,0.85,0.90/
30 C
31 C BETAL IS THE RATIO OF THE DISTANCE FROM WEDGE APEX TO THE
32 C PARTICLE CENTER OVER THE TANK RADIUS DIMENSIONLESS
33 C
34     REAL*4 FBETAL(26)/2.10444,2.10433,2.10415,2.10381,
35     12.10270,2.09758,2.08962,2.07937,2.06801,2.05687,
36     22.04800,2.04561,2.04419,2.04388,2.04391,2.04522,
37     32.04819,2.06557,2.10274,2.16980,2.28060,2.45850,
38     42.742,3.20,3.96,5.30/
39 C
40 C FBETAL IS THE LITERATURE VALUE FOR THE ECCENTRICITY
41 C CORRECTION FACTOR IN THE WALL CORRECTION EQUATION
42 C DIMENSIONLESS
43 C
44     DO 101 I=1,5
45     DO 101 J=1,3
46     READ(5,100) TEMP(I,J,1),TEMP(I,J,2),TEMP(I,J,3),
47     1TEMP(I,J,4),TEMP(I,J,5)
48 100  FORMAT(5F10.1)
49 101  CONTINUE
50 C

```

```

51 C TEMP IS THE MEASURED TEMPERATURE OF THE LIQUID IN DEG.C
52 C
53     DO 103 I=1,5
54     DO 103 J=1,3
55     DO 103 K=1,5
56     READ(5,102) TAU(I,J,K,1),TAU(I,J,K,2),TAU(I,J,K,3),
57     1TAU(I,J,K,4),TAU(I,J,K,5),TAU(I,J,K,6)
58 102  FORMAT(6F10.1)
59 103  CONTINUE
60 C
61 C TAU IS THE EXPERIMENTAL SETTLING TIME/FOOT OF DISTANCE
62 C   IN SEC/FT
63 C
64     READ(5,104) TBL1(1),TBL1(2),TBL1(3),TBL1(4),TBL1(5)
65 104  FORMAT(5I3)
66     DO 106 J=1,5
67     READ(5,105) TBL2(J,1),TBL2(J,2),TBL2(J,3),TBL2(J,4),
68     1TBL2(J,5)
69 105  FORMAT(5I3)
70 106  CONTINUE
71     READ(5,107) TBL3(1),TBL3(2),TBL3(3),TBL3(4),TBL3(5)
72 107  FORMAT(5I3)
73 C
74 C TBL1, TBL2 & TBL3 ARE TABLE DESIGNATIONS IN THE OUTPUT
75 C
76     DO 1000 I=1,5
77     PHIX2(I)=2.0*PHI(I)
78 C
79 C PHIX2 IS THE WEDGE ANGLE
80 C
81 1000 CONTINUE
82     DO 1006 I=1,5
83     WRITE(6,1001) TBL1(I)
84 1001 FORMAT('1'//',',30X,'TABLE ',I2//',',3X,'EXPERI'
85     1'MENTAL SETTLING TIMES FOR DELRIN SPHERES IN UCON '
86     2'LUBRICANT'//)
87     WRITE(6,1002) PHIX2(I)
88 1002 FORMAT('1'//',',10X,'WEDGE ANGLE= ',F5.1,' DEGREES')
89     DO 1006 J=1,3
90     WRITE(6,1003) DIAP(J)
91 1003 FORMAT('1'//',',10X,'SPHERE DIAMETER= ',F6.4,' INCH'
92     1'ES')
93     WRITE(6,1004)
94 1004 FORMAT('1'//',',5X,'DISTANCE FROM WEDGE',8X,'EXPERIMENTAL'
95     1' SETTLING TIMES'//',',6X,'APEX TO PARTICLE',12X,'IN'
96     2' SEC/FT OF DISTANCE'//',',6X,'CENTER IN INCHES',6X,
97     3'RUN 1 RUN 2 RUN 3 RUN 4 RUN 5 RUN 6'//)
98     DO 1006 K=1,5
99     WRITE(6,1005) X0(K),(TAU(I,J,K,L),L=1,6)
100 1005 FORMAT('1'//',',13X,F3.1,11X,6F6.1)

```

```

101 1006 CONTINUE
102      DO 1009 I=1,5
103      DO 1009 J=1,3
104      DO 1009 K=1,5
105      SUM=0.0
106      DO 1007 L=1,6
107      UEXP(I,J,K,L)=1.0/TAU(I,J,K,L)
108 C
109 C UEXP IS THE EXPERIMENTAL SETTLING VELOCITY FOR EACH RUN
110 C   IN A SET
111 C
112      SUM=SUM+UEXP(I,J,K,L)
113 1007 CONTINUE
114      UEXPAV(I,J,K)=SUM/6.0
115 C
116 C UEXPAV IS THE MEAN VALUE FOR THE EXPERIMENTAL VELOCITIES
117 C   IN EACH SET
118 C
119      SUM=0.0
120      DO 1008 L=1,6
121      SUM=SUM+(UEXP(I,J,K,L)-UEXPAV(I,J,K))**2
122 1008 CONTINUE
123      SIGMA(I,J,K)=SQRT(SUM/6.0)
124 C
125 C SIGMA IS THE STANDARD DEVIATION FOR EACH SET
126 C
127 1009 CONTINUE
128      DO 1017 I=1,5
129      DO 1017 K=1,5
130      WRITE(6,1010) TBL2(I,K)
131 1010 FORMAT('1'// ' ',30X,'TABLE ',I2// ' ',1X,'EXPERIMENT'
132 1'AL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON'
133 2' LUBRICANT'/)
134      WRITE(6,1011) PHIX2(I),XC(K)
135 1011 FORMAT('0',10X,'WEDGE ANGLE= ',F5.1,' DEGREES'// ' ',
136 110X,'DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= ',
137 2F3.1,' INCHES')
138      DO 1017 J=1,3
139      WRITE(6,1012) DIAP(J)
140 1012 FORMAT('- ',10X,'SPHERE DIAMETER= ',F6.4,' INCHES')
141      WRITE(6,1013)
142 1013 FORMAT('- ',9X,'RUN NUMBER',19X,'EXPERIMENTAL PARTIC'
143 1'LE'// ' ',11X,'IN SET',17X,'SETTLING VELOCITIES IN '
144 2'FT/SEC'//)
145      DO 1015 L=1,6
146      WRITE(6,1014) L,UEXP(I,J,K,L)
147 1014 FORMAT(' ',13X,I1,29X,F11.9)
148 1015 CONTINUE
149      WRITE(6,1016) UEXPAV(I,J,K),SIGMA(I,J,K)
150 1016 FORMAT(' ',4X,'MEAN VALUE OF SET IS',19X,F11.9// ' ',

```

```

151      12X,'WITH STANDARD DEVIATION OF',15X,F11.9)
152 1017 CONTINUE
153      DO 1020 I=1,5
154      DO 1020 J=1,3
155      DO 1020 K=1,5
156      IF(FPHI1(I).EQ.TEST.OR.FPHI2(I).EQ.TEST) GO TO 1020
157      RHOL(I,J,K)=1.076667-0.75889E-03*TEMP(I,J,K)
158 C
159 C RHOL IS THE CALCULATED DENSITY OF THE LIQUID IN GMS/CC
160 C
161      MUKL(I,J,K)=EXP(EXP(5.138245-0.561074*(ALOG(1.0*TEMP
162      1(I,J,K)+132.0)))-1.7)
163 C
164 C MUKL IS THE CALCULATED KINEMATIC VISCOSITY OF THE LIQUID
165 C IN CENTISTOKES
166 C
167      MUAL(I,J,K)=MUKL(I,J,K)*1.076391E-05*RHOL(I,J,K)*
168      162.42642
169 C
170 C MUAL IS THE CALCULATED ABSOLUTE VISCOSITY OF THE LIQUID
171 C IN LB/FT-SEC
172 C
173      US(I,J,K)=G*(((DIAP(J)/12.0)**2)*(RHOP(J)-RHOL(I,J,K)
174      1)*62.42642/(18.0*MUAL(I,J,K)))
175 C
176 C US IS THE STOKES SETTLING VELOCITY OF THE PARTICLE IN
177 C FT/SEC
178 C
179      U1(I,J,K)=US(I,J,K)*((1.0-DIAP(J)/2.0*FPHI1(I)/XO(K)-
180      1)(((DIAP(J)/2.0)/XO(K))**3)*FPHI2(I))
181 C
182 C U1 IS THE CALCULATED SETTLING VELOCITY OF THE PARTICLE
183 C USING THE WEDGE CORRECTION IN FT/SEC
184 C
185      DO 1018 II=2,26
186      IF(XO(K)/RO.LE.BETAL(II).AND.XO(K)/RO.GE.BETAL(II-1
187      1)) GO TO 1019
188 1018 CONTINUE
189 1019 BETA1=BETAL(II-1)
190      BETA2=BETAL(II)
191      FBETA1=FBETAL(II-1)
192      FBETA2=FBETAL(II)
193      FBETA(I,J,K)=(((XO(K)/RO-BETA1)/(BETA2-BETA1))*
194      1(FBETA2-FBETA1))+FBETA1
195 C
196 C FBETA IS THE INTERPOLATED ECCENTRICITY CORRECTION FACTOR
197 C IN THE WALL CORRECTION EQUATION DIMENSIONLESS
198 C
199      U2(I,J,K)=US(I,J,K)*((1.0-DIAP(J)/2.0*FPHI1(I)/XO(K)-
200      1)(((DIAP(J)/2.0)/XO(K))**3)*FPHI2(I)-DIAP(J)/(2.0*RO)

```



```

201      2*(PHI(I)/180.0)*FBETA(I,J,K))
202 C
203 C U2 IS THE CALCULATED SETTLING VELOCITY OF THE PARTICLE
204 C USING THE WEDGE AND MODIFIED WALL CORRECTION IN FT/SEC
205 C
206 1020 CONTINUE
207      DO 1028 I=1,5
208          WRITE(6,1021) TBL3(I)
209 1021 FORMAT('1'//',',30X,'TABLE ',I2//',',2X,'CALCULATED'
210 1' SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON '
211 2'LUBRICANT'//)
212          WRITE(6,1022) PHIX2(I)
213 1022 FORMAT('0',10X,'WEDGE ANGLE= ',F5.1,' DEGREES')
214          DO 1028 J=1,3
215          WRITE(6,1023) DIAP(J)
216 1023 FORMAT(' '//'-',10X,'SPHERE DIAMETER= ',F6.4,' INCH'
217 1'ES')
218          WRITE(6,1024)
219 1024 FORMAT('-',3X,'DISTANCE FROM WEDGE',7X,'CALCULATED'
220 1' PARTICLE SETTLING VELOCITIES'//',',4X,'APEX TO '
221 2'PARTICLE',9X,'IN FT/SEC. CORRECTED FOR THE EFFECTS'
222 3' OF'//',',4X,'CENTER IN INCHES',14X,'WEDGE ONLY',2X,
223 4'IIII',2X,'WEDGE & WALL'//)
224          DO 1028 K=1,5
225          IF(FPHI1(I).EQ.TEST.OR.FPHI2(I).EQ.TEST) GO TO 1026
226          WRITE(6,1025) X0(K),U1(I,J,K),U2(I,J,K)
227 1025 FORMAT(' ',11X,F3.1,19X,F11.9,9X,F11.9)
228          GO TO 1028
229 1026 WRITE(6,1027) X0(K)
230 1027 FORMAT(' ',11X,F3.1,18X,'NOT AVAILABLE',7X,'NOT AVA'
231 1'ILABLE')
232 1028 CONTINUE
233          WRITE(6,1029)
234 1029 FORMAT('1')
235      STOP
236      END

```

## APPENDIX E

TABLE 11  
EXPERIMENTAL SETTLING TIMES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 60.0 DEGREES

SPHERE DIAMETER= 0.1562 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	362.0	363.4	364.5	363.7	362.6	363.6
2.0	336.3	336.5	336.6	337.1	335.5	336.2
3.0	328.4	328.7	327.7	328.2	328.4	327.4
4.0	324.0	324.6	323.9	323.5	324.2	323.8
5.0	321.3	322.1	321.6	321.4	321.6	321.8

SPHERE DIAMETER= 0.2497 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	162.4	161.8	161.6	163.3	162.8	162.5
2.0	143.2	143.5	142.5	143.3	142.2	142.7
3.0	137.0	136.4	137.3	137.7	137.8	136.7
4.0	134.7	134.0	133.8	134.9	134.4	134.6
5.0	132.8	132.6	132.7	133.1	132.4	132.8

SPHERE DIAMETER= 0.3435 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	88.0	88.9	88.8	89.2	89.6	89.0
2.0	73.5	73.8	74.6	74.4	74.5	74.0
3.0	70.3	69.8	70.0	70.0	69.4	69.7
4.0	68.4	67.5	67.9	67.7	68.1	68.0
5.0	66.8	66.8	67.0	66.8	66.3	66.9

TABLE 12  
EXPERIMENTAL SETTLING TIMES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 90.0 DEGREES

SPHERE DIAMETER= 0.1562 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	343.7	343.8	343.0	344.2	344.9	344.9
2.0	327.6	328.2	327.5	327.0	327.6	327.7
3.0	322.5	322.0	323.1	322.7	322.8	322.4
4.0	320.4	320.1	320.7	319.8	319.7	320.4
5.0	318.3	318.7	319.0	318.6	318.5	318.5

SPHERE DIAMETER= 0.2497 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	148.5	147.2	148.1	149.1	148.5	148.4
2.0	136.1	136.3	137.0	137.3	136.9	137.4
3.0	133.1	133.3	134.1	133.5	133.2	132.8
4.0	131.6	131.3	131.6	132.1	132.1	131.5
5.0	130.1	131.0	130.2	130.5	130.5	130.6

SPHERE DIAMETER= 0.3435 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	77.6	77.1	78.0	78.8	78.1	78.3
2.0	69.7	70.0	70.0	69.6	70.3	70.2
3.0	66.6	66.8	67.4	67.8	67.8	67.4
4.0	65.6	66.1	66.5	66.0	66.1	66.0
5.0	65.6	64.8	65.3	65.5	65.5	65.8

TABLE 13  
EXPERIMENTAL SETTLING TIMES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 180.0 DEGREES

SPHERE DIAMETER= 0.1562 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	328.3	326.9	327.1	327.1	327.9	328.0
2.0	320.4	319.9	319.9	320.5	320.4	320.7
3.0	317.7	317.4	318.2	317.9	317.8	317.7
4.0	316.1	316.4	316.5	316.4	317.0	316.5
5.0	315.9	316.1	315.8	315.6	315.9	315.9

SPHERE DIAMETER= 0.2497 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	136.9	136.8	137.1	136.8	135.8	136.9
2.0	131.3	132.0	131.4	131.5	132.0	131.9
3.0	129.6	130.0	130.5	129.9	130.2	130.0
4.0	129.0	129.4	129.4	129.5	129.4	129.4
5.0	128.8	129.0	128.7	128.6	128.8	128.9

SPHERE DIAMETER= 0.3435 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	69.0	69.6	69.7	69.8	70.1	69.2
2.0	66.1	66.0	66.0	66.2	66.4	65.7
3.0	64.9	64.9	64.8	65.0	64.5	65.3
4.0	64.3	64.2	64.5	64.3	64.2	64.4
5.0	63.8	64.0	64.1	64.0	64.1	64.0

TABLE 14  
EXPERIMENTAL SETTLING TIMES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 270.0 DEGREES

SPHERE DIAMETER= 0.1562 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	325.7	326.7	326.0	325.9	325.0	326.2
2.0	320.2	319.3	319.7	319.0	319.7	319.8
3.0	318.2	317.4	317.9	317.3	317.1	317.6
4.0	316.4	316.4	317.0	316.5	316.4	316.5
5.0	315.9	316.3	316.1	316.4	316.1	315.6

SPHERE DIAMETER= 0.2497 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	135.4	135.3	136.0	135.6	135.6	135.0
2.0	131.6	131.0	130.7	131.2	131.2	131.1
3.0	129.8	129.4	129.6	129.8	130.0	130.3
4.0	129.1	128.9	129.6	129.0	129.1	129.5
5.0	129.0	129.2	128.9	129.1	128.6	129.9

SPHERE DIAMETER= 0.3435 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	68.3	68.5	68.5	69.0	68.1	68.2
2.0	65.6	65.9	65.4	65.7	65.3	65.1
3.0	65.0	64.6	64.5	64.3	64.6	64.6
4.0	63.8	64.5	64.0	64.2	64.2	64.0
5.0	64.2	63.9	64.2	64.0	64.0	63.5

TABLE 15  
EXPERIMENTAL SETTLING TIMES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 360.0 DEGREES

SPHERE DIAMETER= 0.1562 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	334.6	334.0	334.9	335.6	334.7	335.1
2.0	328.8	329.0	329.8	329.7	329.6	329.3
3.0	327.0	328.0	327.4	327.0	327.3	327.1
4.0	326.8	326.4	326.2	326.0	326.5	326.4
5.0	326.1	326.2	325.9	326.0	325.8	325.7

SPHERE DIAMETER= 0.2497 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	142.0	141.9	141.6	141.1	140.8	141.1
2.0	137.9	138.0	137.6	137.6	137.9	137.0
3.0	136.1	136.4	136.2	136.9	136.4	136.2
4.0	136.0	135.9	136.0	136.0	136.3	136.4
5.0	136.0	136.0	135.9	135.5	135.6	135.5

SPHERE DIAMETER= 0.3435 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	EXPERIMENTAL SETTLING TIMES IN SEC/FT OF DISTANCE					
	RUN 1	RUN 2	RUN 3	RUN 4	RUN 5	RUN 6
1.0	70.8	72.0	71.0	71.4	71.1	71.2
2.0	69.6	68.9	69.0	68.8	69.1	69.1
3.0	68.9	68.5	68.9	68.5	68.6	68.4
4.0	68.1	68.3	68.0	67.6	67.9	68.1
5.0	68.1	67.8	67.7	67.6	67.9	67.8

## APPENDIX F



TABLE 16  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 60.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 1.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.002762431
2	0.002751789
3	0.002743484
4	0.002749519
5	0.002757859
6	0.002750274
MEAN VALUE OF SET IS	0.002752559
WITH STANDARD DEVIATION OF	0.000006096

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.006157633
2	0.006180469
3	0.006188117
4	0.006123696
5	0.006142505
6	0.006153844
MEAN VALUE OF SET IS	0.006157707
WITH STANDARD DEVIATION OF	0.000021778

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.011363633
2	0.011248592
3	0.011261258
4	0.011210762
5	0.011160713
6	0.011235952
MEAN VALUE OF SET IS	0.011246808
WITH STANDARD DEVIATION OF	0.000061495

TABLE 17  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 60.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 2.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.002973535
2	0.002971768
3	0.002970884
4	0.002966478
5	0.002980626
6	0.002974420
MEAN VALUE OF SET IS	0.002972950
WITH STANDARD DEVIATION OF	0.000004262

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.006983239
2	0.006968640
3	0.007017542
4	0.006978367
5	0.007032346
6	0.007007707
MEAN VALUE OF SET IS	0.006997973
WITH STANDARD DEVIATION OF	0.000022808

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.013605442
2	0.013550133
3	0.013404824
4	0.013440859
5	0.013422817
6	0.013513513
MEAN VALUE OF SET IS	0.013489593
WITH STANDARD DEVIATION OF	0.000072661

TABLE 18  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 60.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 3.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003045068
2	0.003042288
3	0.003051572
4	0.003046923
5	0.003045068
6	0.003054369
MEAN VALUE OF SET IS	0.003047547
WITH STANDARD DEVIATION OF	0.000004145

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007299267
2	0.007331375
3	0.007283319
4	0.007262163
5	0.007256892
6	0.007315286
MEAN VALUE OF SET IS	0.007291380
WITH STANDARD DEVIATION OF	0.000026899

SPHERE DIAMETER= 0.3433 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.014224749
2	0.014326647
3	0.014285713
4	0.014285713
5	0.014409222
6	0.014347199
MEAN VALUE OF SET IS	0.014313199
WITH STANDARD DEVIATION OF	0.000057578

TABLE 19  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 60.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 4.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003086420
2	0.003080714
3	0.003087373
4	0.003091190
5	0.003084516
6	0.003088325
MEAN VALUE OF SET IS	0.003086422
WITH STANDARD DEVIATION OF	0.000003253

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007423904
2	0.007462684
3	0.007473841
4	0.007412896
5	0.007440474
6	0.007429417
MEAN VALUE OF SET IS	0.007440533
WITH STANDARD DEVIATION OF	0.000021463

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.014619883
2	0.014814813
3	0.014727540
4	0.014771048
5	0.014684286
6	0.014705881
MEAN VALUE OF SET IS	0.014720559
WITH STANDARD DEVIATION OF	0.000062113

TABLE 20  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 60.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 5.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003112355
2	0.003104625
3	0.003109452
4	0.003111389
5	0.003109452
6	0.003107520
MEAN VALUE OF SET IS	0.003109131
WITH STANDARD DEVIATION OF	0.000002536

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007530119
2	0.007541478
3	0.007535793
4	0.007513147
5	0.007552870
6	0.007530119
MEAN VALUE OF SET IS	0.007533919
WITH STANDARD DEVIATION OF	0.000012111

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.014970057
2	0.014970057
3	0.014925372
4	0.014970057
5	0.015082955
6	0.014947683
MEAN VALUE OF SET IS	0.014977682
WITH STANDARD DEVIATION OF	0.000049822

TABLE 21  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 90.0 DEGREES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 1.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.002909514
2	0.002908667
3	0.002915452
4	0.002905288
5	0.002899392
6	0.002899392
MEAN VALUE OF SET IS	0.002906283
WITH STANDARD DEVIATION OF	0.000005718

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.006734006
2	0.006793477
3	0.006752193
4	0.006706905
5	0.006734006
6	0.006738544
MEAN VALUE OF SET IS	0.006743185
WITH STANDARD DEVIATION OF	0.000026194

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.012886595
2	0.012970164
3	0.012820512
4	0.012690354
5	0.012804095
6	0.012771396
MEAN VALUE OF SET IS	0.012823839
WITH STANDARD DEVIATION OF	0.000087863

TABLE 22  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 90.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 2.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003052502
2	0.003046923
3	0.003053435
4	0.003058104
5	0.003052502
6	0.003051572
MEAN VALUE OF SET IS	0.003052505
WITH STANDARD DEVIATION OF	0.000003272

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007347535
2	0.007336754
3	0.007299267
4	0.007283319
5	0.007304601
6	0.007278018
MEAN VALUE OF SET IS	0.007308248
WITH STANDARD DEVIATION OF	0.000025773

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.014347199
2	0.014285713
3	0.014285713
4	0.014367811
5	0.014224749
6	0.014245015
MEAN VALUE OF SET IS	0.014292687
WITH STANDARD DEVIATION OF	0.000050991

TABLE 23  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 90.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 3.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003100775
2	0.003105590
3	0.003095016
4	0.003098854
5	0.003097893
6	0.003101738
MEAN VALUE OF SET IS	0.003099977
WITH STANDARD DEVIATION OF	0.000003303

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007513147
2	0.007501874
3	0.007457118
4	0.007490635
5	0.007507507
6	0.007530119
MEAN VALUE OF SET IS	0.007500064
WITH STANDARD DEVIATION OF	0.000022607

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.015015014
2	0.014970057
3	0.014836796
4	0.014749259
5	0.014749259
6	0.014836796
MEAN VALUE OF SET IS	0.014859527
WITH STANDARD DEVIATION OF	0.000101443



TABLE 24  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 90.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 4.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003121099
2	0.003124023
3	0.003118179
4	0.003126954
5	0.003127933
6	0.003121099
MEAN VALUE OF SET IS	0.003123213
WITH STANDARD DEVIATION OF	0.000003445

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007598784
2	0.007616144
3	0.007598784
4	0.007570021
5	0.007570021
6	0.007604562
MEAN VALUE OF SET IS	0.007593051
WITH STANDARD DEVIATION OF	0.000017283

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.015243899
2	0.015128590
3	0.015037593
4	0.015151512
5	0.015128590
6	0.015151512
MEAN VALUE OF SET IS	0.015140273
WITH STANDARD DEVIATION OF	0.000060287

TABLE 25  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 90.0 DEGREES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 5.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
----------------------	--

1	0.003141690
2	0.003137748
3	0.003134796
4	0.003138731
5	0.003139717
6	0.003139717

MEAN VALUE OF SET IS  
WITH STANDARD DEVIATION OF

0.003138731  
0.000002127

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
----------------------	--

1	0.007686391
2	0.007633585
3	0.007680491
4	0.007662833
5	0.007662833
6	0.007656965

MEAN VALUE OF SET IS  
WITH STANDARD DEVIATION OF

0.007663850  
0.000017080

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
----------------------	--

1	0.015243899
2	0.015432097
3	0.015313935
4	0.015267175
5	0.015267175
6	0.015197568

MEAN VALUE OF SET IS  
WITH STANDARD DEVIATION OF

0.015286960  
0.000073508

TABLE 26  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 180.0 DEGREES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 1.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003045994
2	0.003059040
3	0.003057168
4	0.003057168
5	0.003049711
6	0.003048780
MEAN VALUE OF SET IS	0.003052975
WITH STANDARD DEVIATION OF	0.000004982

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007304601
2	0.007309940
3	0.007293943
4	0.007309940
5	0.007363766
6	0.007304601
MEAN VALUE OF SET IS	0.007314462
WITH STANDARD DEVIATION OF	0.000022684

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.014492750
2	0.014367811
3	0.014347199
4	0.014326647
5	0.014265332
6	0.014450867
MEAN VALUE OF SET IS	0.014375091
WITH STANDARD DEVIATION OF	0.000076169

TABLE 27  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 180.0 DEGREES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 2.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003121099
2	0.003125978
3	0.003125978
4	0.003120125
5	0.003121099
6	0.003118179
MEAN VALUE OF SET IS	0.003122075
WITH STANDARD DEVIATION OF	0.000002925

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007616144
2	0.007575754
3	0.007610347
4	0.007604562
5	0.007575754
6	0.007581498
MEAN VALUE OF SET IS	0.007594008
WITH STANDARD DEVIATION OF	0.000016789

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.015128590
2	0.015151512
3	0.015151512
4	0.015105739
5	0.015060242
6	0.015220698
MEAN VALUE OF SET IS	0.015136369
WITH STANDARD DEVIATION OF	0.000048928

TABLE 28  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 180.0 DEGREES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 3.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003147624
2	0.003150600
3	0.003142678
4	0.003145644
5	0.003146633
6	0.003147624
MEAN VALUE OF SET IS	0.003146799
WITH STANDARD DEVIATION OF	0.000002386

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007716049
2	0.007692307
3	0.007662833
4	0.007698227
5	0.007680491
6	0.007692307
MEAN VALUE OF SET IS	0.007690366
WITH STANDARD DEVIATION OF	0.000016251

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.015408318
2	0.015408318
3	0.015432097
4	0.015384614
5	0.015503876
6	0.015313935
MEAN VALUE OF SET IS	0.015408516
WITH STANDARD DEVIATION OF	0.000056519

TABLE 29  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 180.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 4.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003163555
2	0.003160557
3	0.003159557
4	0.003160557
5	0.003154574
6	0.003159557
MEAN VALUE OF SET IS	0.003159725
WITH STANDARD DEVIATION OF	0.000002666

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007751938
2	0.007727973
3	0.007727973
4	0.007722005
5	0.007727973
6	0.007727973
MEAN VALUE OF SET IS	0.007730972
WITH STANDARD DEVIATION OF	0.000009626

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.015552096
2	0.015576322
3	0.015503876
4	0.015552096
5	0.015576322
6	0.015527949
MEAN VALUE OF SET IS	0.015548099
WITH STANDARD DEVIATION OF	0.000025777

TABLE 30  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 180.0 DEGREES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 5.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003165560
2	0.003163555
3	0.003166561
4	0.003168567
5	0.003165560
6	0.003165560
MEAN VALUE OF SET IS	0.003165892
WITH STANDARD DEVIATION OF	0.000001494

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007763974
2	0.007751938
3	0.007770006
4	0.007776048
5	0.007763974
6	0.007757951
MEAN VALUE OF SET IS	0.007763982
WITH STANDARD DEVIATION OF	0.000007781

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.015673980
2	0.015625000
3	0.015600622
4	0.015625000
5	0.015600622
6	0.015625000
MEAN VALUE OF SET IS	0.015625030
WITH STANDARD DEVIATION OF	0.000024453

TABLE 31  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 270.0 DEGREES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 1.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003070310
2	0.003060912
3	0.003067485
4	0.003068427
5	0.003076923
6	0.003065604
MEAN VALUE OF SET IS	0.003068275
WITH STANDARD DEVIATION OF	0.000004846

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007385522
2	0.007390980
3	0.007352941
4	0.007374629
5	0.007374629
6	0.007407404
MEAN VALUE OF SET IS	0.007381015
WITH STANDARD DEVIATION OF	0.000016762

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.014641285
2	0.014598537
3	0.014598537
4	0.014492750
5	0.014684286
6	0.014662754
MEAN VALUE OF SET IS	0.014613021
WITH STANDARD DEVIATION OF	0.000062230



TABLE 32  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 270.0 DEGREES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 2.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003123048
2	0.003131850
3	0.003127933
4	0.003134796
5	0.003127933
6	0.003126954
MEAN VALUE OF SET IS	0.003128751
WITH STANDARD DEVIATION OF	0.000003725

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007598784
2	0.007633585
3	0.007651109
4	0.007621948
5	0.007621948
6	0.007627763
MEAN VALUE OF SET IS	0.007625856
WITH STANDARD DEVIATION OF	0.000015623

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.015243899
2	0.015174508
3	0.015290521
4	0.015220698
5	0.015313935
6	0.015360981
MEAN VALUE OF SET IS	0.015267409
WITH STANDARD DEVIATION OF	0.000061671

TABLE 33  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 270.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 3.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003142678
2	0.003150600
3	0.003145644
4	0.003151591
5	0.003153578
6	0.003148613
MEAN VALUE OF SET IS	0.003148782
WITH STANDARD DEVIATION OF	0.000003682

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007704157
2	0.007727973
3	0.007716049
4	0.007704157
5	0.007692307
6	0.007674593
MEAN VALUE OF SET IS	0.007703204
WITH STANDARD DEVIATION OF	0.000016919

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.015384614
2	0.015479874
3	0.015503876
4	0.015552096
5	0.015479874
6	0.015479874
MEAN VALUE OF SET IS	0.015480030
WITH STANDARD DEVIATION OF	0.000049777

TABLE 34  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 270.0 DEGREES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 4.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003160557
2	0.003160557
3	0.003154574
4	0.003159557
5	0.003160557
6	0.003159557
MEAN VALUE OF SET IS	0.003159225
WITH STANDARD DEVIATION OF	0.000002128

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007745933
2	0.007757951
3	0.007716049
4	0.007751938
5	0.007745933
6	0.007722005
MEAN VALUE OF SET IS	0.007739965
WITH STANDARD DEVIATION OF	0.000015452

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.015673985
2	0.015503876
3	0.015625000
4	0.015576322
5	0.015576322
6	0.015625000
MEAN VALUE OF SET IS	0.015596747
WITH STANDARD DEVIATION OF	0.000053262

TABLE 35  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 270.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 5.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003165560
2	0.003161555
3	0.003163555
4	0.003160557
5	0.003163555
6	0.003168567
MEAN VALUE OF SET IS	0.003163890
WITH STANDARD DEVIATION OF	0.000002628

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007751938
2	0.007739935
3	0.007757951
4	0.007745933
5	0.007776048
6	0.007757951
MEAN VALUE OF SET IS	0.007754959
WITH STANDARD DEVIATION OF	0.000011394

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.015576322
2	0.015649453
3	0.015576322
4	0.015625000
5	0.015625000
6	0.015748031
MEAN VALUE OF SET IS	0.015633345
WITH STANDARD DEVIATION OF	0.000057814

TABLE 36  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 360.0 DEGREES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 1.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.002988642
2	0.002994012
3	0.002985967
4	0.002979737
5	0.002987751
6	0.002984183
MEAN VALUE OF SET IS	0.002986714
WITH STANDARD DEVIATION OF	0.000004353

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007042252
2	0.007047214
3	0.007062145
4	0.007087171
5	0.007102270
6	0.007087171
MEAN VALUE OF SET IS	0.007071368
WITH STANDARD DEVIATION OF	0.000022251

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.014124293
2	0.013888888
3	0.014084507
4	0.014005601
5	0.014064696
6	0.014044944
MEAN VALUE OF SET IS	0.014035482
WITH STANDARD DEVIATION OF	0.000074851

TABLE 37  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 360.0 DEGREES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 2.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003041362
2	0.003039513
3	0.003032140
4	0.003033061
5	0.003033980
6	0.003036744
MEAN VALUE OF SET IS	0.003036131
WITH STANDARD DEVIATION OF	0.000003395

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007251631
2	0.007246375
3	0.007267438
4	0.007267438
5	0.007251631
6	0.007299267
MEAN VALUE OF SET IS	0.007263962
WITH STANDARD DEVIATION OF	0.000017720

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.014367811
2	0.014513787
3	0.014492750
4	0.014534879
5	0.014471777
6	0.014471777
MEAN VALUE OF SET IS	0.014475454
WITH STANDARD DEVIATION OF	0.000053094

TABLE 38  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 360.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 3.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003058104
2	0.003048780
3	0.003054369
4	0.003058104
5	0.003055300
6	0.003057168
MEAN VALUE OF SET IS	0.003055302
WITH STANDARD DEVIATION OF	0.000003230

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007347535
2	0.007331375
3	0.007342141
4	0.007304601
5	0.007331375
6	0.007342141
MEAN VALUE OF SET IS	0.007333193
WITH STANDARD DEVIATION OF	0.000014083

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.014513787
2	0.014598537
3	0.014513787
4	0.014598537
5	0.014577255
6	0.014619883
MEAN VALUE OF SET IS	0.014570285
WITH STANDARD DEVIATION OF	0.000041811

TABLE 39  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 360.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 4.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003059975
2	0.003063726
3	0.003065604
4	0.003067485
5	0.003062787
6	0.003063726
MEAN VALUE OF SET IS	0.003063882
WITH STANDARD DEVIATION OF	0.000002326

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007352941
2	0.007358350
3	0.007352941
4	0.007352941
5	0.007336754
6	0.007331375
MEAN VALUE OF SET IS	0.007347550
WITH STANDARD DEVIATION OF	0.000009849

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.014684286
2	0.014641285
3	0.014705881
4	0.014792897
5	0.014727540
6	0.014684286
MEAN VALUE OF SET IS	0.014706016
WITH STANDARD DEVIATION OF	0.000046808



TABLE 40  
EXPERIMENTAL SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 360.0 DEGREES  
DISTANCE FROM WEDGE APEX TO PARTICLE CENTER= 5.0 INCHES

SPHERE DIAMETER= 0.1562 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.003066543
2	0.003065604
3	0.003068427
4	0.003067485
5	0.003069367
6	0.003070310
MEAN VALUE OF SET IS	0.003067954
WITH STANDARD DEVIATION OF	0.000001608

SPHERE DIAMETER= 0.2497 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.007352941
2	0.007352941
3	0.007358350
4	0.007380072
5	0.007374629
6	0.007380072
MEAN VALUE OF SET IS	0.007366501
WITH STANDARD DEVIATION OF	0.000012032

SPHERE DIAMETER= 0.3435 INCHES

RUN NUMBER IN SET	EXPERIMENTAL PARTICLE SETTLING VELOCITIES IN FT/SEC
1	0.014684286
2	0.014749259
3	0.014771048
4	0.014792897
5	0.014727540
6	0.014749259
MEAN VALUE OF SET IS	0.014745701
WITH STANDARD DEVIATION OF	0.000034149

## APPENDIX G

TABLE 41  
CALCULATED SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 60.0 DEGREES

SPHERE DIAMETER= 0.1562 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF WEDGE ONLY            WEDGE & WALL	
1.0	0.002761486	0.002754187
2.0	0.002981690	0.002974437
3.0	0.003055956	0.003048767
4.0	0.003093186	0.003086055
5.0	0.003115545	0.003108438

SPHERE DIAMETER= 0.2497 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF WEDGE ONLY            WEDGE & WALL	
1.0	0.006191861	0.006163038
2.0	0.007038619	0.007009976
3.0	0.007329602	0.007301211
4.0	0.007476062	0.007447906
5.0	0.007564161	0.007536095

SPHERE DIAMETER= 0.3435 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF WEDGE ONLY            WEDGE & WALL	
1.0	0.011351008	0.011270586
2.0	0.013620295	0.013540376
3.0	0.014422830	0.014343608
4.0	0.014829207	0.014750637
5.0	0.015074216	0.014995899

TABLE 42  
CALCULATED SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 90.0 DEGREES

SPHERE DIAMETER= 0.1562 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF WEDGE ONLY         WEDGE & WALL	
---	---	--

1.0	0.002916398	0.002905450
2.0	0.003060257	0.003049378
3.0	0.003108472	0.003097687
4.0	0.003132608	0.003121913
5.0	0.003147097	0.003136435

SPHERE DIAMETER= 0.2497 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF WEDGE ONLY         WEDGE & WALL	
---	---	--

1.0	0.006785411	0.006742179
2.0	0.007346604	0.007303640
3.0	0.007536311	0.007493723
4.0	0.007631458	0.007589221
5.0	0.007688612	0.007646512

SPHERE DIAMETER= 0.3435 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF WEDGE ONLY         WEDGE & WALL	
---	---	--

1.0	0.012932725	0.012812097
2.0	0.014470357	0.014350478
3.0	0.014996849	0.014878016
4.0	0.015261639	0.015143786
5.0	0.015420873	0.015303399

TABLE 43  
CALCULATED SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 180.0 DEGREES

SPHERE DIAMETER= 0.1562 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF WEDGE ONLY            WEDGE & WALL	
1.0	0.003064468	0.003042572
2.0	0.003134702	0.003112943
3.0	0.003158152	0.003136583
4.0	0.003169882	0.003148491
5.0	0.003176921	0.003155598

SPHERE DIAMETER= 0.2497 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF WEDGE ONLY            WEDGE & WALL	
1.0	0.007363420	0.007276952
2.0	0.007639751	0.007553223
3.0	0.007732254	0.007647075
4.0	0.007778548	0.007694073
5.0	0.007806335	0.007722132

SPHERE DIAMETER= 0.3435 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF WEDGE ONLY            WEDGE & WALL	
1.0	0.014518026	0.014276769
2.0	0.015284870	0.015045114
3.0	0.015542556	0.015304890
4.0	0.015671629	0.015435923
5.0	0.015749127	0.015514180

TABLE 44  
CALCULATED SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 270.0 DEGREES

SPHERE DIAMETER= 0.1562 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF	
	WEDGE ONLY	WEDGE & WALL
1.0	NOT AVAILABLE	NOT AVAILABLE
2.0	NOT AVAILABLE	NOT AVAILABLE
3.0	NOT AVAILABLE	NOT AVAILABLE
4.0	NOT AVAILABLE	NOT AVAILABLE
5.0	NOT AVAILABLE	NOT AVAILABLE

SPHERE DIAMETER= 0.2497 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF	
	WEDGE ONLY	WEDGE & WALL
1.0	NOT AVAILABLE	NOT AVAILABLE
2.0	NOT AVAILABLE	NOT AVAILABLE
3.0	NOT AVAILABLE	NOT AVAILABLE
4.0	NOT AVAILABLE	NOT AVAILABLE
5.0	NOT AVAILABLE	NOT AVAILABLE

SPHERE DIAMETER= 0.3435 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF	
	WEDGE ONLY	WEDGE & WALL
1.0	NOT AVAILABLE	NOT AVAILABLE
2.0	NOT AVAILABLE	NOT AVAILABLE
3.0	NOT AVAILABLE	NOT AVAILABLE
4.0	NOT AVAILABLE	NOT AVAILABLE
5.0	NOT AVAILABLE	NOT AVAILABLE

TABLE 45  
CALCULATED SETTLING VELOCITIES FOR DELRIN SPHERES IN UCON LUBRICANT

WEDGE ANGLE= 360.0 DEGREES

SPHERE DIAMETER= 0.1562 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF WEDGE ONLY            WEDGE & WALL	
1.0	0.003085634	0.003041844
2.0	0.003145327	0.003101809
3.0	0.003165241	0.003122102
4.0	0.003175199	0.003132417
5.0	0.003181175	0.003138531

SPHERE DIAMETER= 0.2497 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF WEDGE ONLY            WEDGE & WALL	
1.0	0.007446334	0.007273402
2.0	0.007681623	0.007509772
3.0	0.007760219	0.007589865
4.0	0.007799536	0.007630587
5.0	0.007823128	0.007654727

SPHERE DIAMETER= 0.3435 INCHES

DISTANCE FROM WEDGE APEX TO PARTICLE CENTER IN INCHES	CALCULATED PARTICLE SETTLING VELOCITIES IN FT/SEC. CORRECTED FOR THE EFFECTS OF WEDGE ONLY            WEDGE & WALL	
1.0	0.014746621	0.014264099
2.0	0.015401360	0.014921848
3.0	0.015620489	0.015145157
4.0	0.015730150	0.015258737
5.0	0.015795968	0.015326075

## NOMENCLATURE

$a$	= particle radius, ft
$a_1, a_2$	= coefficients used in the determinants, dimensionless
$b$	= coefficient used in the determinants, dimensionless
$\bar{F}$	= drag force exerted on the particle, $\text{lb}_f$
$f(\beta)$	= eccentricity coefficient, dimensionless
$f_1(\phi_0), f_2(\phi_0)$	= wedge angle coefficients for particle translation, dimensionless
$g$	= gravitational acceleration, $\text{ft}/\text{sec}^2$
$g_1(\phi_0), g_2(\phi_0)$	= wedge angle coefficients for particle rotation, dimensionless
$R_0$	= fluid container radius, ft
$\bar{T}$	= torque exerted on the particle, $\text{ft} \cdot \text{lb}_f$
$U, U_e, U_{em}, U_s$	= particle settling velocities, $\text{ft}/\text{sec}$
$\bar{v}$	= fluid velocity, $\text{ft}/\text{sec}$
$x_0$	= distance from wedge apex to particle center, ft



## Greek letters

- $\beta$  = eccentricity ratio ( $x_o/R_o$ ), dimensionless  
 $\mu$  = viscosity, lb/ft·sec  
 $\rho_1, \rho_p$  = density, lb/ft<sup>3</sup>  
 $\sigma$  = standard deviation for experimental settling velocities  
 $\tau$  = experimental particle settling time, sec  
 $\phi_o$  = half of the wedge angle, degrees  
 $\omega$  = angular velocity, rad/sec

## Subscripts

- $e$  = experimental  
 $em$  = experimental mean  
 $l$  = liquid  
 $o$  = center  
 $p$  = particle  
 $s$  = Stokes  
 $1,2$  = reference subscripts for constants

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