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A theoretical study of continuous parametric pumping at constant temperature

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A THEORETICAL STUDY OF
CONTINUOUS PARAMETRIC PUMPING
AT CONSTANT TEMPERATURE

BY

PHILIP F. WEINGARTNER

A THESIS
PRESENTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE
OF
MASTER OF SCIENCE IN CHEMICAL ENGINEERING
AT
NEWARK COLLEGE OF ENGINEERING

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Newark, New Jersey
1973

APPROVAL OF THESIS
A THEORETICAL STUDY OF
CONTINUOUS PARAMETRIC PUMPING
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PHILIP F. WEINGARTNER

FOR

DEPARTMENT OF CHEMICAL ENGINEERING
NEWARK COLLEGE OF ENGINEERING

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ABSTRACT

A continuous parametric pump, operating at constant temperature with pressure as the driving force, is analyzed by means of an equilibrium theory. A thorough explanation of the process and the resulting mathematical model reveal how the parameters effect pump performance. The experimental data cited agree well with the analytical results in the CO₂-He on silica gel system. Complete removal of the solute from one of the product streams can be achieved through the proper selection of penetration distances and pressure levels.

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INTRODUCTION

Wilhelm and Sweed (1) introduced parametric pumping in 1966. They experimented with a binary liquid mixture in a batch pump. Using temperature changes as the driving force, they separated the two components to a high degree. Their model, however, could not reproduce this phenomena quantitatively.

Pigford, Baker and Blum (2) refined the model for parametric pumping by assuming a linear equilibrium existed between the adsorbed and fluid phase. With this, they obtained good agreement between their model and experimental data.

Chen and Hill (3) made several contributions to the liquid phase system, driven by temperature changes. They refined the batch model to the point where they could accurately predict the compositions of the top and bottom reservoirs. A clear picture of the concentration front patterns entering the reservoirs made this possible, proving the maximum number of concentration fronts is three.

More significantly, they extended the batch pump to semi-continuous and continuous models. Semi-continuous operation is accomplished by introducing feed either at the top or bottom of the column. Product is drawn off the opposite end during only one of the half-cycles. By feeding

at the top and drawing the proper amount of product from the bottom, an essentially solute free stream emerges. Any other scheme results in a product stream which is not completely solute free. For continuous operation, a product stream, enriched in solute to a finite degree, is drawn off during downflow, while the bottom product is still drawn off during downflow, as in semi-continuous operation.

While the continuous and semi-continuous operations have the obvious advantages of a flow process, the batch process comes to steady state faster than both. As might be expected, the semi-continuous process is faster than the continuous in delivering a solute free product stream. Experimental work in subsequent studies, agreed remarkably well with their model (4,5).

Experimental work was also done with a gas phase system. Jenczowski and Meyers (6) examined a binary mixture of propane and ethane. Activated carbon was used as the adsorbent, having a greater affinity for propane than ethane. The batch apparatus consisting of a single jacketed column equipped with piston pumps for the reservoirs was almost identical to that used for liquid systems. Temperature changes were also used as the driving force in the separation.

They started their experimental runs with an equimolar mixture. Keeping the cycle time at 2 minutes, they ran

the column for 60 cycles. At the end, they analyzed the top and bottom reservoirs. The top composition was 36% ethane while the bottom yielded 67% ethane, for a typical run.

In this study, the authors proved that gases could be separated to a degree using temperature changes in a batch pump. Although they did not attempt to justify their relatively poor separation, there are obvious limiting factors. While temperature changes are sufficient for liquid separations, pressure may be a more efficient driving force for gases.

Shendalman and Mitchell (7) made a more significant contribution to gas phase parametric pumping by introducing a continuous flow system where the driving force comes from pressure changes. The pump consists of two packed columns which are interconnected to produce the desired parametric effect. In their model, they present the transient equations for the two product streams and define the region in which the apparatus should produce a solute free stream.

To confirm this model, a total of four runs were made. The product rates were varied while the feed rate and pressure levels remained constant. A dilute mixture of CO_2 in helium was to be separated using silica gel as the adsorbent.

Data were taken on the top product stream only. This makes the comparison between theory and experiment much more difficult, as there is no way to check the results. It could be seen from this data that the top product transient concentration does depend on the flow rates, while the theory cannot account for this. The theory suggests only one transient curve for the top product which depends on the value of the partition coefficient in the equilibrium relationship. Data on the bottom product would have made the selection of this value easier and more exact. As it is, the partition coefficient is just assumed to be very large, considering some isotherm data.

In this thesis, a clearer picture of gas phase parametric pumping will be presented, allowing a better understanding of what happens in this process. A highly simplified model will explain how the apparatus functions as a parametric pump. A more detailed model will be used to predict under what conditions total separation will be achieved

PROCESS DESCRIPTION

The system we are modeling consists of two packed columns, shown in Fig. 1a. The packing, an adsorbent, is stationary, high pressure feed enters column A at a flow rate F , volume per unit time. For a half-cycle this column is maintained at a pressure P_1 with an interstitial fluid velocity v_1 . Part of the flow out of column A goes through a check valve and out as high pressure product, still at pressure P_1 and at a flow rate Q . The rest of the flow out of column A is expanded through an orifice to a pressure P_2 . It enters column B and maintains it at pressure P_2 with a fluid velocity v_2 . This stream leaves the column as low pressure purge at flow rate W .

At the end of the half-cycle, the feed is switched away from column A by closing the three way solenoid valve at the bottom of A. The valve at the bottom of B changes to the appropriate position and allows high pressure feed to enter. As soon as the pressure in column B reaches that of column A, the solenoid valve at the bottom of A opens the column to the purge line and the pressure in column A is reduced to P_2 . The flow pattern set up is shown in Fig. 1b. The roles of the columns have been completely interchanged. The amount of time needed to switch the columns is small in comparison with the half-cycle time.

The principle involved in this model can be demonstrated by considering the following events. A mixture of species X and O has been flowing into column B at high pressure P_1 , purge flowing out the bottom of column A at low pressure P_2 . At the end of the half-cycle, the pressures in the two columns are changed. The effect of this pressure change is seen in Fig. 2. The high pressure in column A forces X to be adsorbed onto the stationary packing, shown as the area on the left in the column. The change to low pressure in column B does not effect the two molecules we are interested in. Fig. 3 shows the flow pattern set up during the subsequent half-cycle.

In Fig. 4, the pressure in the columns have been interchanged by switching the feed stream into column B and allowing column A to vent to the low pressure output line or purge. In column A, X will desorb off the packing but it will not return to the same segment of fluid from which it originated. In column B, a molecule of X is now adsorbed, similar to what had happened in column A during the previous half-cycle. After the system becomes pressurized fully, the flow pattern in Fig. 5 develops.

The result of this complete cycle is a net migration of X, the solute, toward the bottom of the column, while O travels to the top. In each column the flow pattern and

pressure are synchronously changed, setting up the conditions necessary for parametric pumping to take place.

The previous development gives information about operations during high and low pressure cycles and the relationship between high and low pressure operation. Additional information is needed, during the change over in pressure either from high to low, called blowdown, or from low to high, called repressurization, making the picture more complete.

At the end of a half-cycle, the valve feeding the high pressure column, which is column A, is shut, allowing no flow in or out of the bottom of this column. Simultaneously, the valve releasing purge from the bottom of the low pressure column, labeled B, is switched, letting in high pressure feed. This column is quickly pressurized by the feed stream and is now at the same pressure as column A, P_1 . It can safely be assumed that there has been no net transfer of material between the two columns since the feed was switched off column A. As soon as column A is allowed to purge, by allowing flow out the bottom of the column, its pressure will decrease to P_2 .

This whole operation has had the effect of shifting the concentration profile of column A downward by a distance ΔZ_1 , which went from high to low pressure. The concentra-

tion profile of column B, which went from low to high pressure, was shifted upward by a distance ΔZ_2 . Since no material flows out the top of either column into the other during this switching operation, the shifts in profile of columns A and B can be considered independent of each other.

MATHEMATICAL MODEL

We will consider the parametric pump represented schematically in Figs. 1a and 1b. The column is filled with adsorbent particles, and the voids in the column are filled with a two-component mixture, one component of which distributes between the two phases. At pump startup the distributing solute fluid phase concentration is equal to the feed concentration y_0 throughout the apparatus and is in equilibrium with the solute concentration on the adsorbent particles.

We will assume as did Shendalman and Mitchell (1971) local interphase equilibrium exists with a linear distribution law having a pressure-dependent distribution coefficient, blowdown and repressurization are instantaneous, there is negligible axial diffusion, plug flow exists, the column temperature is constant, and the ideal gas law can be applied. Based on these assumptions, the material balance for the solute over a differential volume of the column may be expressed as,

$$\epsilon \frac{\partial \rho y}{\partial t} + \epsilon \frac{\partial \rho y v}{\partial z} + (1-\epsilon) \rho_s \frac{\partial x}{\partial t} = 0 \quad (1)$$

and the total mass balance is

$$\epsilon \frac{\partial \rho}{\partial t} + \epsilon \frac{\partial \rho v}{\partial z} + (1-\epsilon) \rho_s \frac{\partial x}{\partial t} = 0 \quad (2)$$

Now let

$$x = m(T)Py \quad (3)$$

Substitute Eq. 3 into Eq. 1

$$\epsilon \frac{\partial \rho y}{\partial t} + \epsilon \frac{\partial \rho y v}{\partial z} + (1-\epsilon) \rho_s \frac{\partial m P y}{\partial t} = 0 \quad (4)$$

Expanding the first two terms on the left and noting m is not a function of time

$$\epsilon y \frac{\partial \rho}{\partial t} + \epsilon \rho \frac{\partial y}{\partial t} + \epsilon y \frac{\partial \rho v}{\partial z} + \epsilon \rho v \frac{\partial y}{\partial z} + (1-\epsilon) \rho_s m \frac{\partial P y}{\partial t} = 0 \quad (5)$$

Multiplying Eq. 2 by y , and combining with Eq. 3

$$\epsilon y \frac{\partial \rho}{\partial t} + \epsilon y \frac{\partial \rho v}{\partial z} + y(1-\epsilon) \rho_s m \frac{\partial y P}{\partial t} = 0 \quad (6)$$

Subtracting Eq. 6 from Eq. 5

$$\epsilon \rho \frac{\partial y}{\partial t} + \epsilon \rho v \frac{\partial y}{\partial z} + m(1-\epsilon)(1-y) \rho_s \frac{\partial y P}{\partial t} = 0 \quad (7)$$

By the ideal gas law,

$$\rho = P/RT \quad (8)$$

Substitute this into Eq. 7 and expand the last term on the left.

$$\frac{\epsilon P}{RT} \frac{\partial y}{\partial t} + \frac{\epsilon v P}{RT} \frac{\partial y}{\partial z} + y m (1-\epsilon) (1-y) \rho_s \frac{\partial P}{\partial t} + P m (1-y) (1-\epsilon) \rho_s \frac{\partial y}{\partial t} = 0 \quad (9)$$

To simplify this equation, let

$$M = m(1-\epsilon)RT\rho_s/\epsilon \quad (10)$$

Substituting Eq. 10 into Eq. 9

$$\frac{\epsilon P}{RT} \frac{\partial y}{\partial t} + \frac{\epsilon P v}{RT} \frac{\partial y}{\partial z} + \frac{\epsilon P(1-y)M}{RT} \frac{\partial y}{\partial t} + \frac{\epsilon M y(1-y)}{RT} \frac{\partial P}{\partial t} = 0 \quad (11)$$

Dividing by $\epsilon P/RT$ and rearranging

$$-y(1-y) \frac{M}{P} \frac{\partial P}{\partial t} = [1+M(1-y)] \frac{\partial y}{\partial t} + v \frac{\partial y}{\partial z} \quad (12)$$

Eq. 12 is a hyperbolic partial differential equation which can be solved by the method of Lagrange-Charpit commonly known as the method of characteristics. Since $y = f(t, z)$, the following total derivative can be written:

$$\left\{ \frac{dy}{dS} \right\}_{\phi} = \frac{\partial y}{\partial t} \left\{ \frac{dt}{dS} \right\}_{\phi} + \frac{\partial y}{\partial z} \left\{ \frac{dz}{dS} \right\}_{\phi} \quad (13)$$

Comparing Eq. 12 with Eq. 13

$$\begin{aligned} \left\{ \frac{dy}{dS} \right\}_{\phi} &= -y(1-y) \frac{M}{P} \frac{\partial P}{\partial t} \\ \left\{ \frac{dt}{dS} \right\}_{\phi} &= 1 + M(1-y) \\ \left\{ \frac{dz}{dS} \right\}_{\phi} &= v \end{aligned} \quad (14)$$

Thus,

$$\begin{aligned} dS &= \frac{-dy}{y(1-y) \frac{M}{P} \frac{\partial P}{\partial t}} && \text{For } \phi \text{ constant} \\ dS &= \frac{dt}{1 + M(1-y)} && \text{For } \phi \text{ constant} \end{aligned} \quad (15)$$

$$dS = \frac{dz}{v} \quad \text{For } \phi \text{ constant}$$

Which leads to the following equation:

$$\frac{-dy}{y(1-y)^M} \frac{\partial P}{\partial t} = \frac{dt}{1 + M(1-y)} = \frac{dz}{v} \quad (16)$$

From equation 16,

$$\frac{dz}{dt} = \frac{v}{1 + M(1-y)} \quad (17)$$

and

$$\frac{dy}{dt} = \frac{-y(1-y)^M}{1 + M(1-y)} \frac{\partial P}{\partial t} \quad (18)$$

Let us assume further that $y \ll 1$, then Eq. 17 and 18 become:

$$\frac{dz}{dt} = \frac{v}{1+M} \quad (19)$$

and

$$-\frac{dy}{y} = \frac{M}{1+M} \frac{dP}{P} \quad \text{or} \quad y(P)^{M/M+1} = \text{constant} \quad (20)$$

From Eqs. 19 and 20, one may see that characteristic curves are linear with slopes $v_1/1+M$ in upflow and $-v_2/1+M$ in downflow. Along these curves the fluid phase solute concentration remains constant while the column remains at constant pressure. This concentration changes by the factor $(P_1/P_2)^{M/1+M}$ when the pressure is decreased from its upper level to its lower level and by the factor $(P_2/P_1)^{M/1+M}$ for the opposite change.

The distance of the column penetration by a concentration entering the column is equal to the product of the slopes of the characteristic curves and the half-cycle duration, and is

$$L_1 = \frac{v_1}{1+M} \frac{\pi}{\omega} \quad (21)$$

in upflow, and

$$L_2 = \frac{v_2}{1+M} \frac{\pi}{\omega} \quad (22)$$

in downflow.

If equation 22 is divided by equation 21,

$$\frac{L_2}{L_1} = \frac{v_2}{v_1} = \frac{W}{F} = \frac{\text{purge volumetric flow rate}}{\text{feed volumetric flow rate}} = \gamma \quad (23)$$

Since we are assuming that blowdown and repressurization are instantaneous, the overall equation of continuity during that period is

$$-\frac{\partial \rho}{\partial t} = \frac{\partial \rho v}{\partial z} \quad (24)$$

By the ideal gas law, Eq. 24 becomes

$$\frac{-1}{RT} \frac{\partial P}{\partial t} = \frac{1}{RT} \frac{\partial P v}{\partial z} \quad (25)$$

or

$$-\frac{\partial P}{\partial t} = P \frac{\partial v}{\partial z} + v \frac{\partial P}{\partial z} \quad (26)$$

Assuming negligible pressure gradients in the column, Eq. 26 becomes

$$-\frac{\partial P}{\partial t} = P \frac{\partial v}{\partial z}$$

$$\text{or } -\frac{\partial P}{P} = \frac{\partial v}{\frac{\partial z}{\partial t}}$$

$$\text{Since } \frac{\partial z}{\partial t} = \frac{v}{1+M}$$

$$-\frac{\partial P}{P} = (1+M) \frac{\partial v}{v} = (1+M) \frac{\partial Z}{Z} \quad (27)$$

Where Z represents the position of a characteristic relative to the top of the bed during repressurization or blowdown.

If Z_2 is the distance from the top of the column at the end of the low pressure half-cycle, then at the end of repressurization, Z_2 is shifted to Z_2' . Here Z_2' may be obtained by Eq. 27,

$$Z_2' = (P_2/P_1)^{1/1+M} Z_2 \quad (28)$$

Similarly

$$Z_1' = (P_1/P_2)^{1/1+M} Z_1 \quad (29)$$

Where Z_1' and Z_1 are the distances from the top of the column at the end of the high pressure half-cycle and at the end of the blowdown, respectively.

The calculation of the pump performance depends on the relative magnitudes of $L_2(P_2/P_1)^{1/1+M}$, L_1 and h . Fig. 6 indicates three possible operating regimes.

When the pumps are operated in Region 1 we will find that at steady-state, solute removal from the top product stream will be complete. When the pumps are operated outside Region 1 solute removal from the top product stream will be incomplete. We will now examine Region 1 quantitatively.

As shown in Fig. 7, during the first half-cycle ($n=1$) the feed stream passes through the column without being changed. The two columns are pictured here on the same time axis. The graph for column B is inverted over column A, going from a height h to height 0 at the top of the graph.

At the end of the first half-cycle, the feed is switched from the bottom of column A to the bottom of column B. As the pressure rises in column B, its fluid concentration decreases by a factor $(P_2/P_1)^{M/M+1}$, while being compressed into the upper portion of the column, a space of height $h(P_2/P_1)^{1/1+M}$, having been displaced by feed. The top region is then pushed up a distance L_1 , the high pressure penetration distance. Some of the fluid in the top of the column will flow out the top as top product (TP) while the rest of the fluid being pushed out will pass through a pressure reducing valve and into the top of column A. This entering stream will form a concentration region to a distance L_2 , the low pressure penetration distance, from the top of the

column.

Column A is at low pressure now. During blowdown the pressure in the column was reduced by switching the feed stream to column B and opening the bottom of column A to the bottom product line. This change in pressure removes some of the contents of the column while at the same time increasing the concentration in the column by a factor of $(P_1/P_2)^{M/M+1}$.

One may see that there are no characteristic lines produced during the first half-cycle since there has been no pressure change to alter the concentrations. Prior to the next half-cycle, column B is repressurized by introducing feed at the bottom of the column. This changes the concentration to $y_0(P_2/P_1)^{M/M+1}$. Feed will penetrate the column to a distance L_1 , while some fluid will be drawn off the top as product. An additional amount will be drawn off at the same time and will pass through a reducing valve in to column A. Its concentration will be the same as that at the top of column B and will penetrate a distance L_2 . The rest of the column and the bottom product will have a concentration $y_0(P_1/P_2)^{M/M+1}$ due to the pressure change from high to low pressure.

At the end of the second half-cycle, column A is repressurized. The two regions in the column are compressed into

the top of the column. The two concentrations are reduced by the $(P_2/P_1)^{M/M+1}$ factor. The opposite is true in column B where the concentrations are increased by the $(P_1/P_2)^{M/M+1}$ factor. As soon as the columns are at their proper pressure levels, flow will occur between the two columns. Fluid from column A will flow into the top of column B to a distance from the top, L_2 . The other two concentration regions will move down the same distance.

As shown in Fig. 7, the pattern of characteristic lines in column B is the mirror image of column A, lagging behind by a half-cycle. It also shows that the pattern of characteristic lines does not change after n , the half-cycle number, reaches $2p_1+2$. Here p_1 is a positive integer or zero, and $0 \leq q_1 < 1$ are defined by

$$p_1 + q_1 = \frac{h - L_1(P_1/P_2)^{1/1+M}}{L_2 - L_1(P_1/P_2)^{1/1+M}} \quad (30)$$

From Fig. 7 it can easily be seen that the average top product concentration, $\langle y_{TP} \rangle_n$, is defined by

$$\langle y_{TP} \rangle_n = \psi^{n-1} y_0 \quad \text{where } \psi = (P_2/P_1)^{M/M+1} \quad (31)$$

and $n = 1, 2, 3, \dots$

For the first half-cycle, the average bottom product concentration equals the feed concentration and

$$\langle y_{BP} \rangle_1 = y_0 \quad (32)$$

Until the pattern of characteristic lines is fully developed, the average bottom product concentration for even and odd number half-cycles is given by

$$\frac{\langle y_{BP} \rangle_n}{y_0} = (P_1/P_2)^{M/M+1} \quad \text{For } n = 2, 4, \dots, 2p_1+1 \quad (33)$$

and

$$\frac{\langle y_{BP} \rangle_n}{y_0} = (\overline{ab}/\overline{ad})(P_1/P_2)^{M/M+1} + (\overline{bd}/\overline{ad}) \quad (34)$$

For $n = 3, 5, \dots, 2p_1+1$

The fractions $(\overline{ab}/\overline{ad})$ and $(\overline{bd}/\overline{ad})$ refer to line \overline{ad} of Fig. 7. From similar triangles they become

$$(\overline{ab}/\overline{ad}) = \overline{ae}/L = (L_1/L_2)(P_1/P_2)^{1/1+M}$$

$$(\overline{bd}/\overline{ad}) = 1 - (\overline{ab}/\overline{ad}) = 1 - (L_1/L_2)(P_1/P_2)^{1/1+M}$$

Substituting these two equations into Eq. 34 and simplifying

$$\frac{\langle y_{BP} \rangle_n}{y_0} = (L_1/L_2)(P_1/P_2) + 1 - (L_1/L_2)(P_1/P_2)^{1/1+M} \quad (35)$$

When the pattern is fully developed, $n > 2p_1+1$, the bottom product concentration is given by

$$\frac{\langle y_{BP} \rangle_n}{y_0} = [L_1/L_2][P_1/P_2] + [1 - (L_1/L_2)(P_1/P_2)^{1/1+M}] \times [q_1 \langle y_{TP} \rangle_{n-2p_1-2} + (1-q_1) \langle y_{TP} \rangle_{n-2p_1}] \quad (36)$$

The two top product concentrations appearing here are given by Eq. 31 which reduces Eq. 36, after simplification to

$$\frac{\langle y_{BP} \rangle_n}{y_0} = (L_1/L_2)(P_1/P_2) + [1 - (L_1/L_2)(P_1/P_2)^{1/1+M}]$$

$$[q_1 \psi^{n-2p_1} + (1-q_1) \psi^{n-2p_1-2}]$$

For $n > 2p_1 + 1$ (37)

As steady-state is reached ($n \rightarrow \infty$), the bottom product concentration is given by Eq. 37 with only the first term remaining. The other terms approaching zero.

$$\frac{\langle y_{BP} \rangle_n}{y_0} = \frac{L_1}{L_2} \frac{P_1}{P_2} = \frac{1}{\gamma} \frac{P_1}{P_2} \quad (38)$$

To check this, a solute material balance can be made at steady-state (In = Out), noting as $n \rightarrow \infty$, $\langle y_{TP} \rangle_\infty \rightarrow 0$.

$$\frac{y_0 P_1 F}{R T} = \frac{\langle y_{BP} \rangle_\infty P_2 W}{R T} \quad (39)$$

Rearranging and simplifying,

$$\frac{\langle y_{BP} \rangle_\infty}{y_0} = \frac{P_1 F}{P_2 W} = \frac{1}{\gamma} \frac{P_1}{P_2} \quad (40)$$

This checks with Eq. 38.

RESULTS AND DISCUSSION

For a dilute mixture, the maximum degree of solute removal will depend on the volumetric flow rates chosen for upflow and downflow. How soon the product streams come to specifications, depends on M and the ratio of the operating pressures during upflow and downflow. Once these conditions are specified, the remaining variables are set.

REGION 1

To achieve complete solute removal, the volumetric feed rate must be adjusted so that L_1 , the upflow penetration distance, is less than the column height. This assures us that the feed stream will not penetrate into the top product stream which is to be solute free. The purge volumetric flow rate must always be adjusted in order that L_2 times $(P_2/P_1)^{1/1+M}$ will be greater than L_1 . As long as γ is greater than $(P_1/P_2)^{1/1+M}$ this condition will be satisfied. The concentration fronts in the column will only originate in the top of the column, again keeping it from being contaminated with additional solute.

REGION 2

When γ is less than $(P_1/P_2)^{1/1+M}$, L_1 will be greater than $L_2(P_2/P_1)^{1/1+M}$. The top product stream will always be contaminated with solute by its migration up the column from the feed.

REGION 3

The top product stream in this region always contains solute. The flow rates in both upflow and downflow are high enough so that fluid from the feed reaches the top product stream during every half-cycle. This is characterized by having L_1 and L_2 greater than h .

PARAMETERS

Both M and the pressure ratio (P_1/P_2) control the time necessary to achieve whatever separation is desired in Region 1. High values of M coupled with a high pressure ratio allow the fastest separation.

COMPARISON WITH EXPERIMENT

Equations 31, 32, 33, 35 and 37 were used to calculate the product concentrations. P_1 , P_2 , γ , h , L_1 and M were needed for these calculations. The curves obtained from these computations appear on Fig. 8 along with experimental data from Krewiec (8). As seen here, the model predicts the experimental results fairly well. Toward the end, there is deviation in the top product. This is most likely due to diffusion effects since there are high concentration gradients present in the column after a large number of half-cycles.

The value of M used in the computations was .13. After fitting the best straight line through the top product

concentration on the semi-log plot, M was calculated from the slope according to Eq. 31.

Comparison with only one set of data is by no means conclusive. Krewiec's data was the only study available. No other information has been reported in the literature. This model may or may not agree with subsequent studies. The next phase of this investigation will be to test the model thoroughly and prove its validity. The model will be modified as indicated by this additional information.

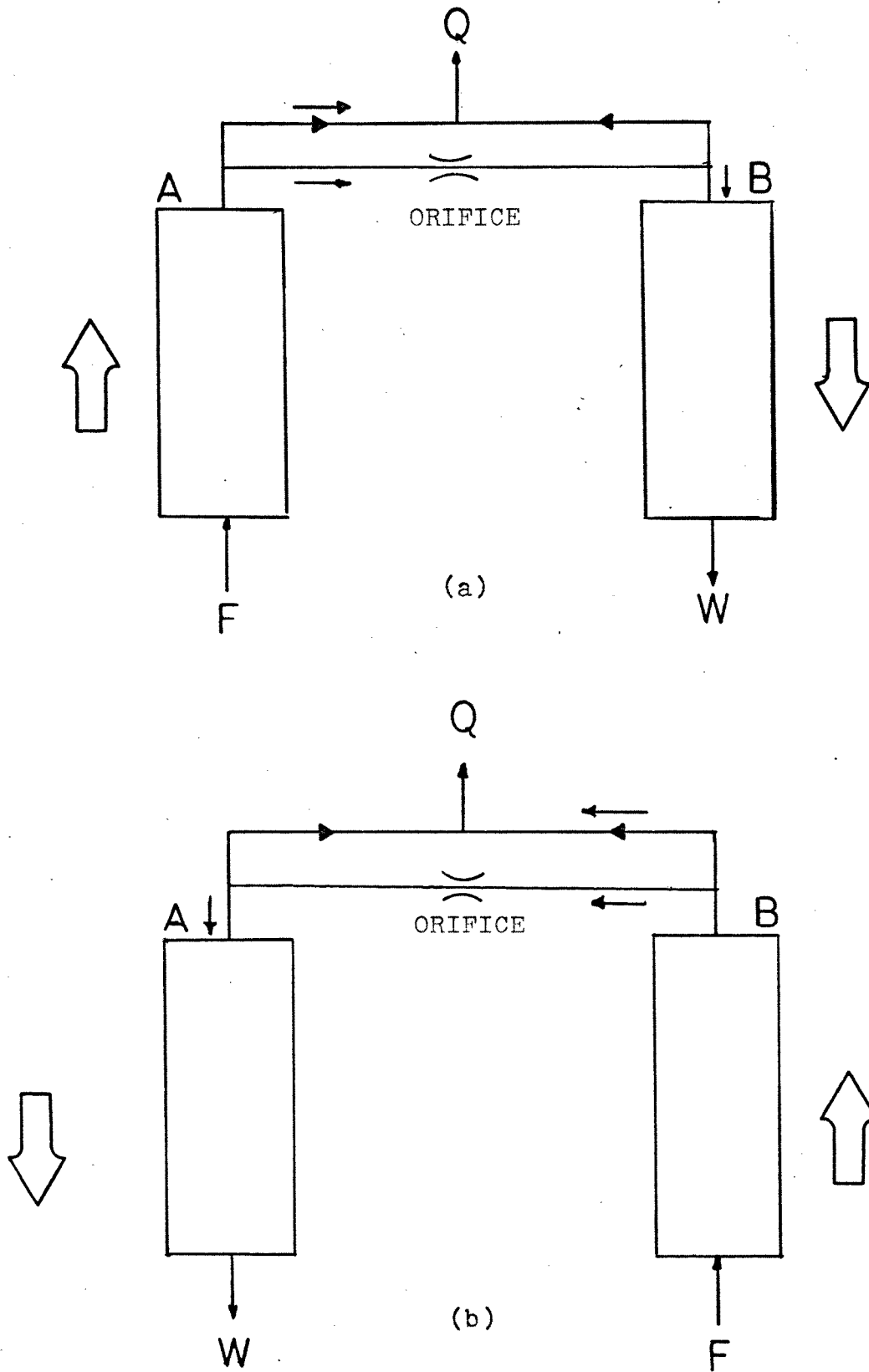


FIG. 1 PROCESS FLOW DIAGRAMS

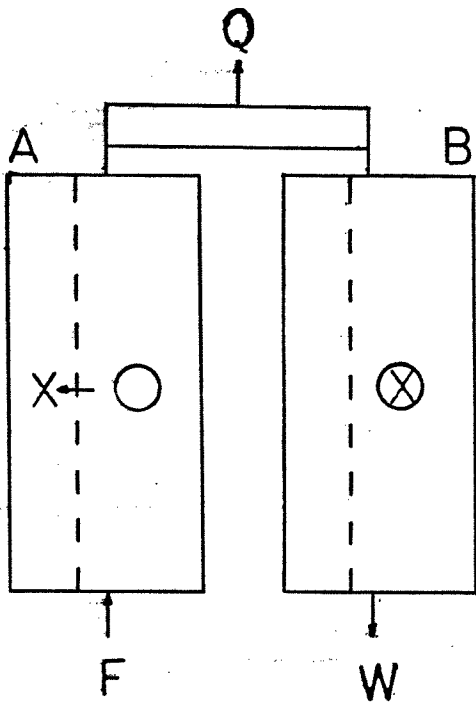


FIG. 2

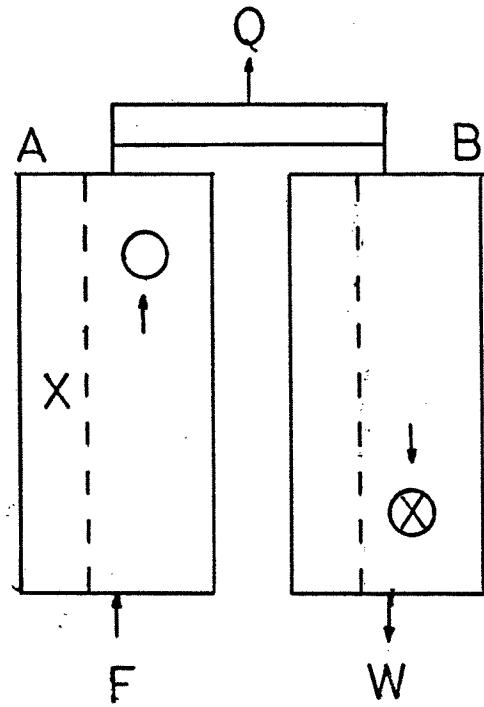


FIG. 3

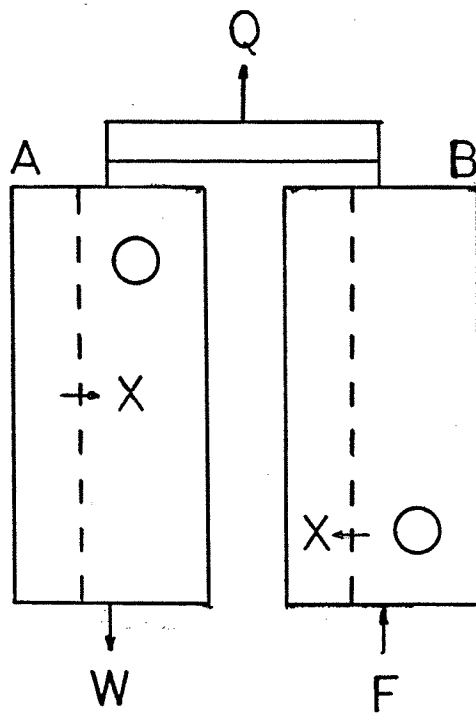


FIG. 4

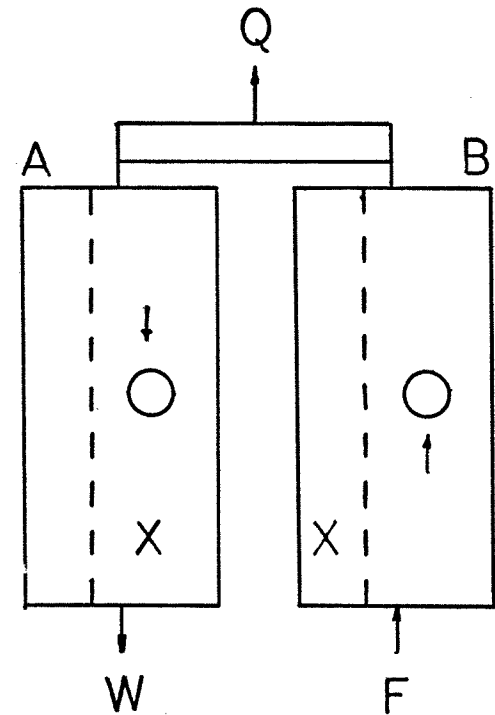


FIG. 5

FIGS. 2-5 PROCESS DIAGRAMS FOR SIMPLIFIED MODEL

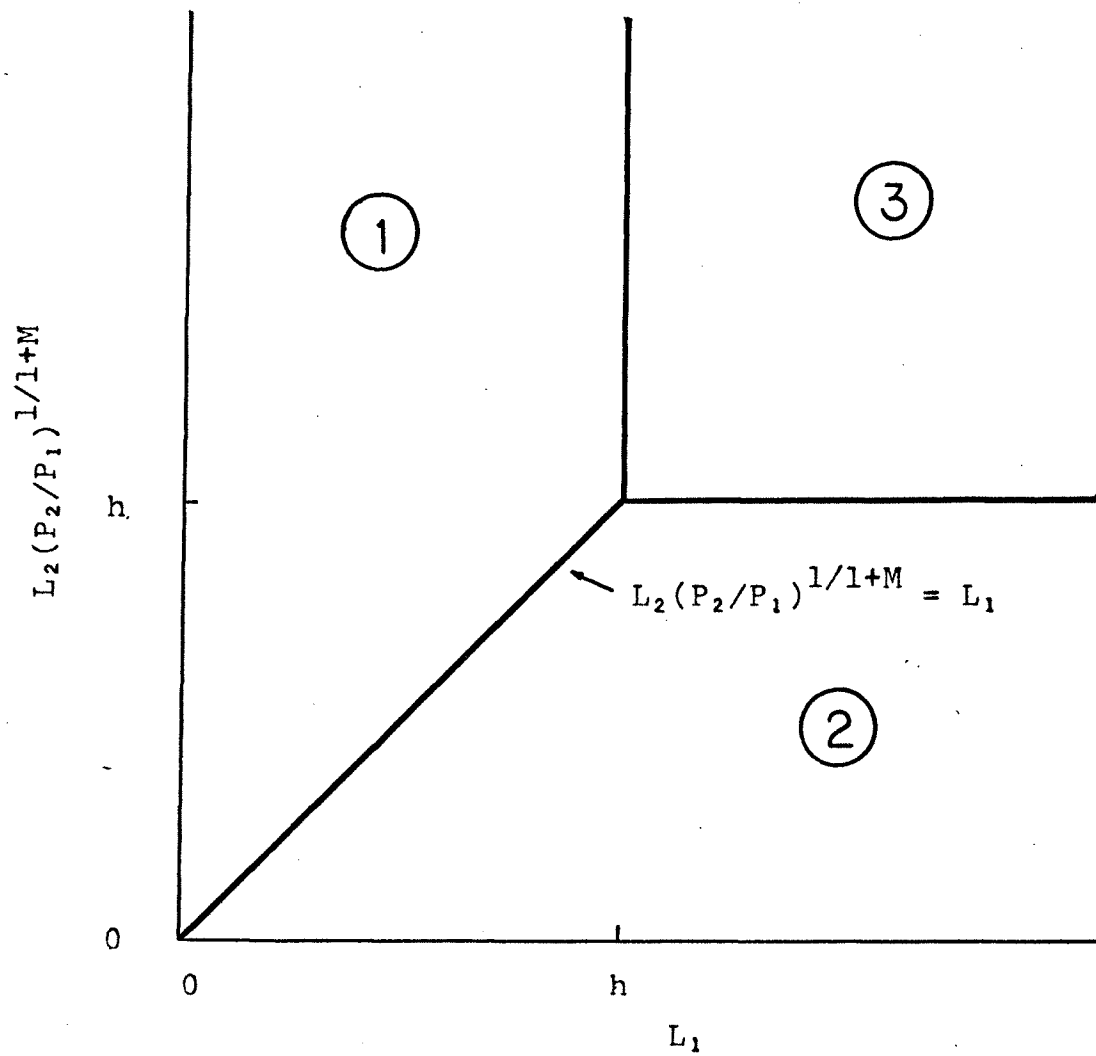


FIG. 6 REGIONS OF OPERATION FOR PARAMETRIC PUMPING

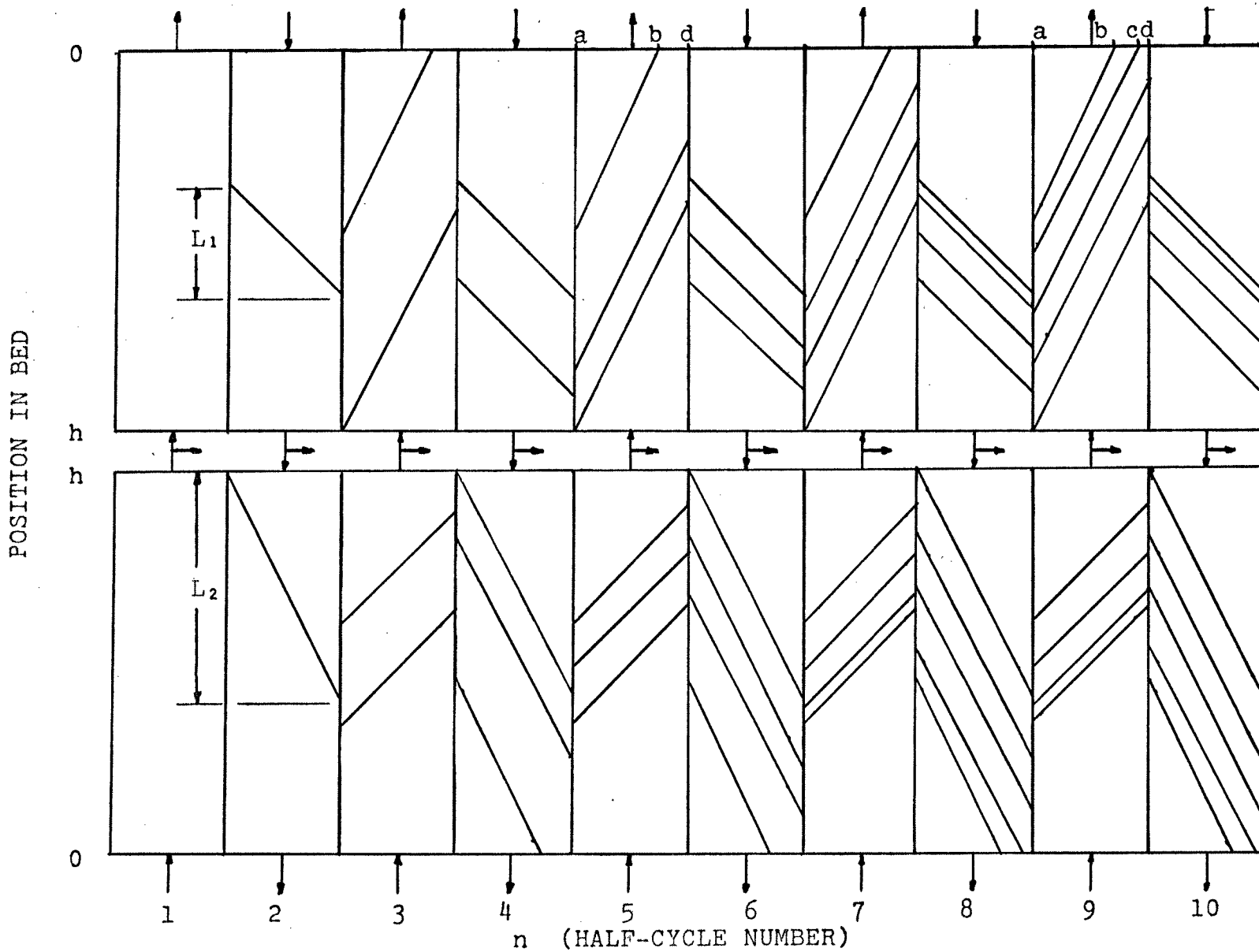


FIG. 7 CONCENTRATION ZONES FOUND IN REGION 1

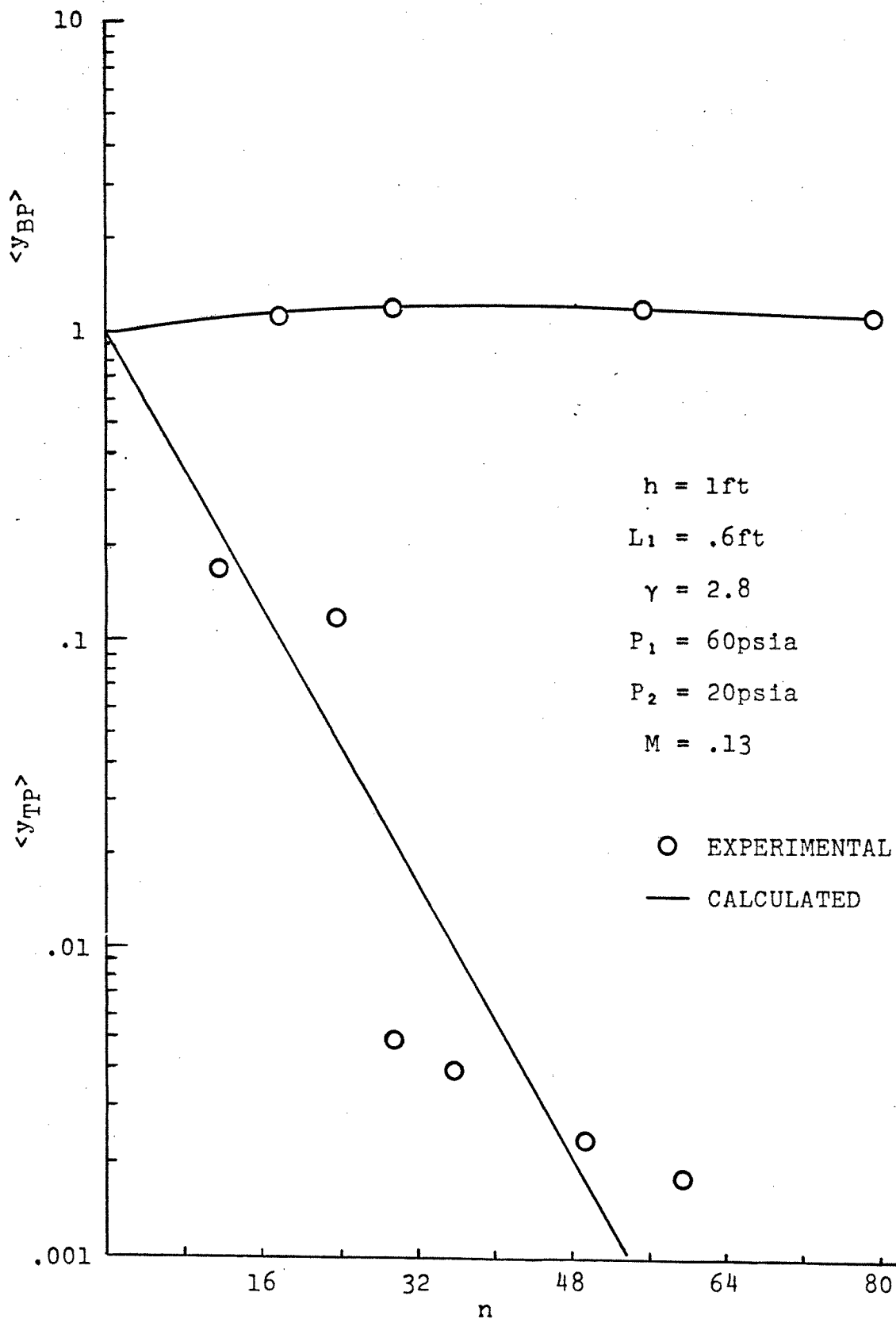


FIG. 8 TRANSIENT CONCENTRATIONS IN REGION 1

NOMENCLATURE

- F = Feed volumetric flow rate
h = Height of column
L = Penetration distance
m = Equilibrium constant
M = Dimensionless equilibrium constant
n = Number of half-cycles
p = Number of concentration fronts
P = Pressure
q = Fraction of a concentration front
R = Gas law constant
S = Intensive property
t = Time
T = Temperature
v = Interstitial fluid velocity
W = Purge volumetric flow rate
x = Moles solute/mass adsorbent
y = Moles solute/mole fluid
z = Axial distance relative to the bottom of the column
Z = Axial distance relative to the top of the column

GREEK SYMBOLS

- Δ = Difference between two values
 γ = Ratio of the purge to feed volumetric flow rates
 ϵ = Void fraction

ρ = Density
 ϕ = Intensive property
 ψ = Variable defined in Eq. 31
 ω = Frequency

SUBSCRIPTS

BP, Bottom product
n, Half-cycle number
s, Solid phase
TP, Top product
 \circ , Initial value
 $\subscript{1}$, Upflow - high pressure or Region 1
 $\subscript{2}$, Downflow - low pressure

SYMBOLS

< > denotes an average value
' denotes a distance after blowdown or repressurization

APPENDIXCALCULATION OF M

Starting with Eq. 31,

$$\langle y_{TP} \rangle_n = y_0 \psi^{n-1}$$

Substituting in for ψ and rearranging,

$$\langle y_{TP} \rangle_n / y_0 = (P_2/P_1)^{M(n-1)/M+1}$$

Since the only unknown is M, all other quantities are fixed or can be read off the straight line in Fig. 8, it will be solved for directly with $n=54$, $\langle y_{TP} \rangle_n / y_0 = .001$ and $(P_2/P_1) = .33$.

$$(P_1/P_2)^{M(n-1)/M+1} = y_0 / \langle y_{TP} \rangle_n$$

$$(3)^{M53/M+1} = 1000$$

$$\text{Log}[(3)^{M53/M+1}] = \text{Log}(1000)$$

$$\frac{M(53)}{1+M} \cdot 477 = 3$$

$$M = .13$$

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