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Digital FM-demodulation using digital signal processing techniques

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DIGITAL FM-DEMODULATION USING DIGITAL SIGNAL PROCESSING TECHNIQUES

by
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ABSTRACT

Title of Thesis: Digital FM-Demodulation Using Digital Signal Processing Techniques

Peter Wagner, Master of Science in Electrical Engineering, 1985

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ABSTRACT

This thesis presents the results of research in the area of digital demodulation of FM signals. The first part of the thesis describes the performance of digital Hilbert-transformers and Differentiators that are optimized in a determined bandwidth. These digital filters are used to build a digital Klapper-Kratt detector. According to this procedure, the coefficients of the digital Klapper-Kratt detectors up to the filter length N=25 are calculated and the resulting discriminator ripple computed and evaluated. One suggested algorithm gives the opportunity to balance the detector at a certain center-frequency. The major part of this thesis describes a new approach towards the digital FM-demodulation. This approach uses an FIR-linear filter as a frequency discriminator. The structure of the discriminator has no restrictions as to certain lengths or certain kinds of symmetry of coefficients. A practical approach to the FIR discriminator is given and the chosen approximation technique is described. The dependence of FIR-discriminator length, optimized bandwidth and resulting discriminator ripple is presented. Weighted and split weighting functions are described with discriminator examples. Changes in the desired weighting function can be used to shape the resulting discriminator ripple in a desired way. Discriminators with these additional design criteria are presented. Finally the results of the new design is discussed and a comparison and evaluation given.

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CHAPTER I

INTRODUCTION

1.1 Different Approaches for the Demodulation of FM Signals

Conventional FM demodulators can be classified into 4 families: Zero-crossing or cycle counting detectors exploit zero crossing of the carrier. FM modulation of the carrier causes displacement or density changes in regular crossings. This displacememt can be detected. Discriminators employing linear filters represent the second family. A linear filter converts frequency modulation into amplitude modulation and then AM demodulation techniques recover the baseband signal. Feedback demodulators comprise the third family of FM demodulators. The Klapper-Kratt detector [1] and the FM-detector described by Park [2] use linear filters to demodulate FM signals. The Klapper-Kratt detector simultaniously cancels the carrier. These approaches make up the fourth family of FM detectors.

The field of digital signal processing has grown enormously in the last decade. Application areas that have traditionally relied on analog signal processing have switched to digital signal processing techniques. This trend is also valid for the demodulation of FM-signals. Several authors describe the results of experimental detectors or simulations. Ray [3], El-Ghoroury and Gupta [4], Garodnick, Greco and Schilling [5] and Kelly and Gupta [6] present digital or modified techniques for the demodulation of FM-signals. The digital implementation of the Klapper-Kratt detector described by Kratt and Klapper [7] uses, similar to the analog version, the quadrature technique for the demodulation of FM-signals. Kammeyer [8] and Finck and Hoelzl [9] follow the same method, realizing the advantages in using quadrature signals.

1.2 Nomenclature

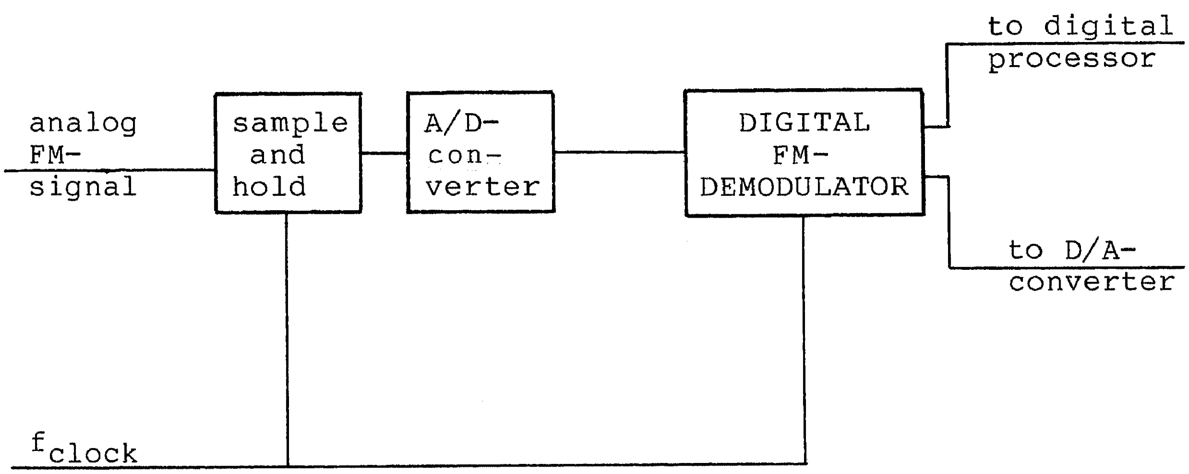

Fig. 1-1 Digital FM-demodulator

The digital FM demodulator processes digitized FM signals of the sampling frequency F_s. One characteristic of this system is the use of the frequency F_{clock} as the frequency to trigger the sample-and-hold-circuit and as the clock frequency for the digital demodulator. In this system, the sampling frequency F_s has to be at least twice as high as the highest frequency to be processed.

It is often desirable to express frequencies $F_{a'}$, $F_{b'}$ in terms of a unit frequency that involve the sampling frequeny F_s. For this reason all frequencies are normalized to the sampling frequency F_s.

$$F_a = \frac{F_a'}{F_s} \qquad (1\text{-}1)$$

One significant parameter for calculations of FM-discriminators is the linearity of the discriminator. Currently used optimization programs [10] give the maximum deviation in amplitude of a bandpass filter, Hilbert-transformers, differentiators or in this thesis described discriminators. The following equation is used to calculate the absolute discriminator ripple in dB with given optimized bandwidth F_{bw}, slope s of the discriminator and relative discriminator ripple dev.

$$r = 20 \log \left(\frac{dev}{s} \cdot \frac{2}{F_{bw}} \right) \qquad (1\text{-}2)$$

Figure 1-2 illustrates this:

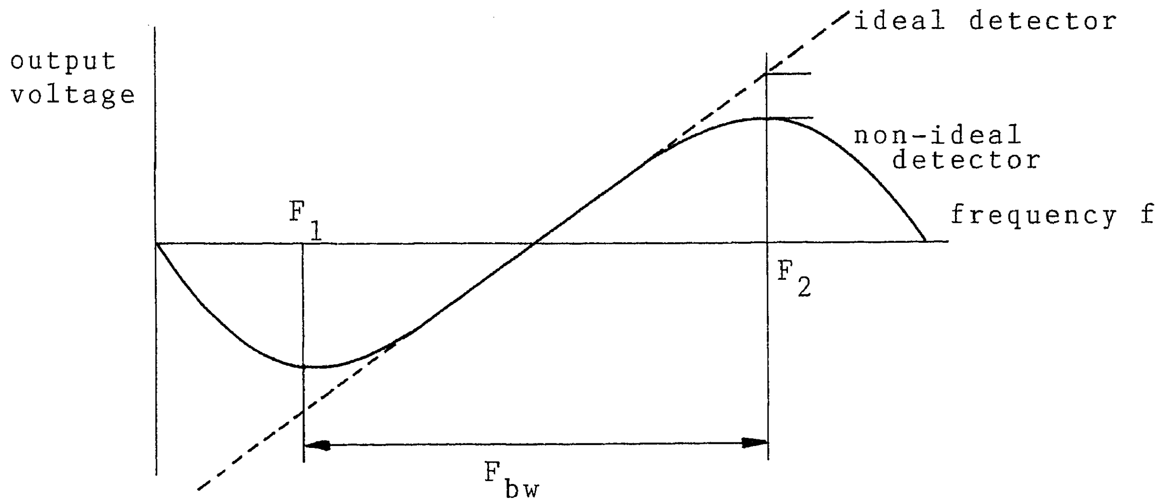

Fig. 1-2 Discriminator linearity

Table 1-1 gives some typical values for the absolute discrimminator ripple:

linearity in %	bandwidth 0.1	0.2	0.3	0.4
0.1	-86.02	-80.00	-76.47	-73.97
0.5	-72.04	-66.02	-62.49	-60.00
1.0	-66.02	-60.00	-56.47	-53.97
2.5	-58.06	-52.04	-48.51	-46.02
5.0	-52.04	-46.02	-42.49	-40.00

Table 1-1: Discriminator ripple in dB

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CHAPTER II

DIGITAL KLAPPER-KRATT DETECTOR

2.1 Functional Description

Klapper and Kratt [1] introduced a detector with extreme linearity, low deleay, and excellent sensitivity. Later, Kratt and Klapper [2] discussed a digital implementation of the detector. Fig. 2-1 shows the block diagram of one member of the family of Klapper-Kratt detectors:

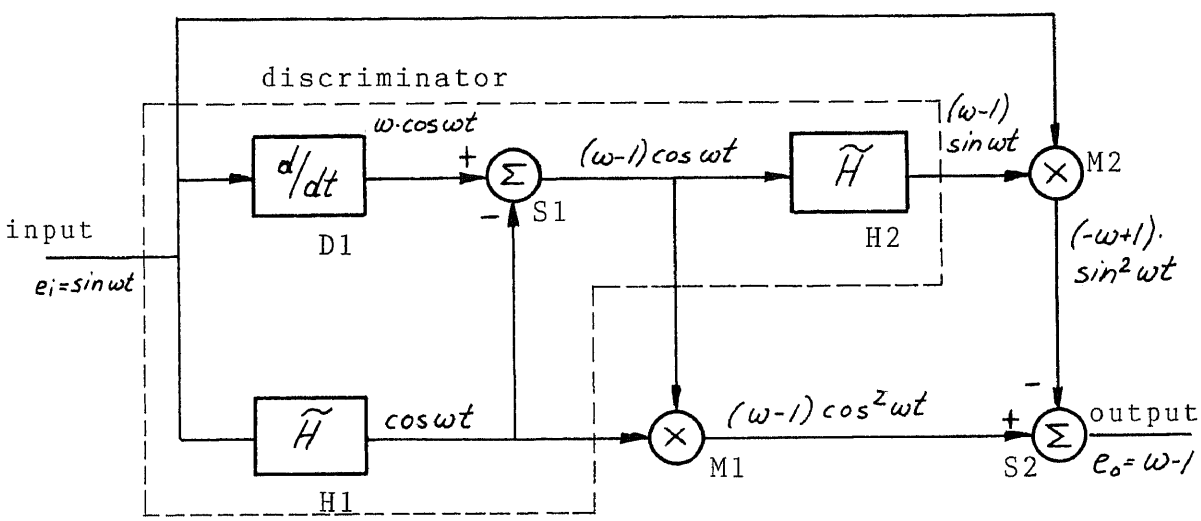

Fig 2-1 Block diagram of a Klapper-Kratt detector

This detector consists of one differentiator (Dl), two Hilbert-transformers (Hl, H2), two adders (Sl, S2) and two multipliers (Ml, M2). The components of the detector can easily be implemented by using digital signal processing techniques. The Hilbert-transformer and the differentiator can be built as Finite Impulse Response (FIR) filters. These are nonrecursive filters with linear phase. Additions and multiplications are simple binary arithmetic operations.

The detector shown in Fig. 2-1 is one member of the Klapper-Kratt detector family. Other configurations are possible and are described in references 1 and 2. All detectors have in common an excellent sensitivity and a good wideband capability.

The described detector consists of a frequency discriminator and a quasi-coherent detector. Fig. 2-1 shows the processing of an input signal $e_i(t) = \sin \omega t$. The processing of sine is shown at the output of each block of the detector. Thecancellation of the carrier is achieved by applying $\sin^2 \omega t + \cos^2 \omega t = 1$. The output of the detector shows the linear dependence $e_o(t) = \omega - 1$ of the detected signal on the radian frequency ω.

2.2 Theoretical Performance

For a causal FIR-system with the impulse response

$$c(n), \quad \text{for} \quad 0 \leqslant n \leqslant N - 1$$

we get the frequency response [Reference 3, p 20]

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} c(n)\, e^{-j\omega n} \qquad (2\text{-}1)$$

For odd filter length N and by applying anti-symmetrical filter coefficients, this formula reduces to [Reference 3, p 20]

$$H(e^{j\omega}) = a(n) \sum_{n=1}^{(N-1)/2} \sin(\omega n) \qquad (2\text{-}2)$$

This is purely a real function and can be used to compute the frequency response of an FIR filter. Kratt and Klapper analyzed the detector given in figure 2-1 and found the following output of the detector:

$$e_o(t) = 0.5 \sum_{m=1}^{N} \sum_{n=1}^{N} (c_{Dn} - c_{Hn}) \quad c_{Hm} \, [\cos\,(m - n)\; 2\pi f - \cos\,(N + 1 - n - m)\; 2\pi f] \qquad (2\text{-}3)$$

They made four assumptions, to arrive at this formula:

- the length of all filters N (components of the detector), is the same for all
- Hilbert-transformer H1 and H2 have the same coefficients
- the length of the filters is odd
- the symmetry of the coefficients is negative

2.3 Synthesis of the Digital Klapper-Kratt Detector

Using an odd filter length, all calculated values at the output of the Hilbert-transformers and differentiator occur exactly at the sampling instant. Even filter length creates calculated values with a timing that lies exactly between two sampling instances. The processing of these signals is impossible in the given structure of the block diagram of Fig. 2-1.

Filters with negative coefficient symmetry cause a filter output of zero at the frequency zero. This is one design criterion for the differentiators. McClellan, Parks, and Rabiner [4] present a computer program that is used to calculate the coefficients of the Hilbert-transformers and differentiators. This computer program applies the Remez-method [5] to optimize the design of filters in the Cheby-chev- or Minimax-sense. It optimizes the coefficients a(n) of equation 2-2 to get minimum error in a chosen frequency range.

2.3.1 Performance of Hilbert-Transformers

The frequency response of the ideal Hilbert-transformer [6] can be described by

$$H_h(e^{j\omega}) = -j \qquad \text{for} \quad 0 \leqslant \omega < \pi \qquad (2-4)$$

$$H_h(e^{j\omega}) = j \qquad \text{for} \quad \pi \leqslant \omega < 2\pi \qquad (2-5)$$

The impulse response of the ideal Hilbert-transformer can be described for odd filter length by:

$$h_h(n) = \frac{\sin^2[\frac{\pi}{2} (n - \tau)]}{n - \tau} \qquad \text{for } n \neq 0 \qquad (2-6)$$

$$h_h(n) = 0 \qquad \text{for } n = 0 \qquad (2-7)$$

For even filter length:

$$h_h(n) = \frac{1}{\pi (n + 0.5)} \qquad (2-8)$$

The following figures show the comparison between the ideal Hilbert-transformer and an optimized Hilbert-transformer with filter length N = 11. The Hilbert-transformer is optimized between the frequencies $F_1 = 0.15$ and $F_2 = 0.35$.

The next figures show an example of the frequency response and ripple of a non-ideal Hilbert-transformer with filterlength N = 11.

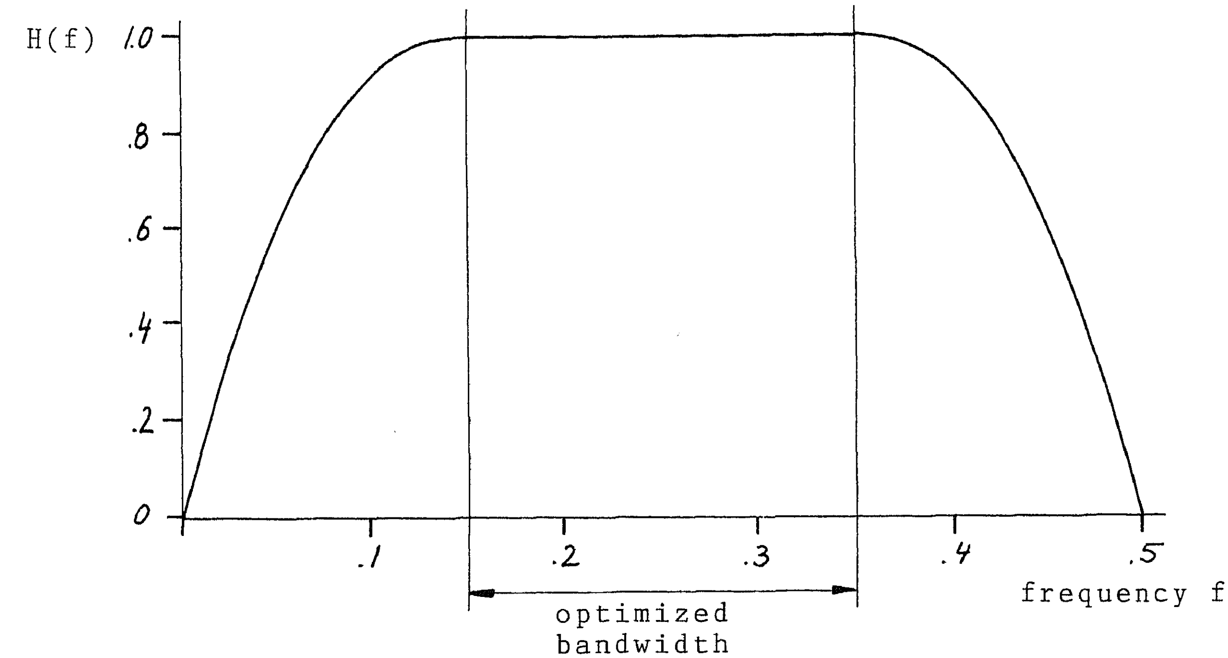

Fig. 2-2 Frequency response of ideal and non-ideal Hilbert-transformer, F_1=0.15, F_2=0.35, N=11

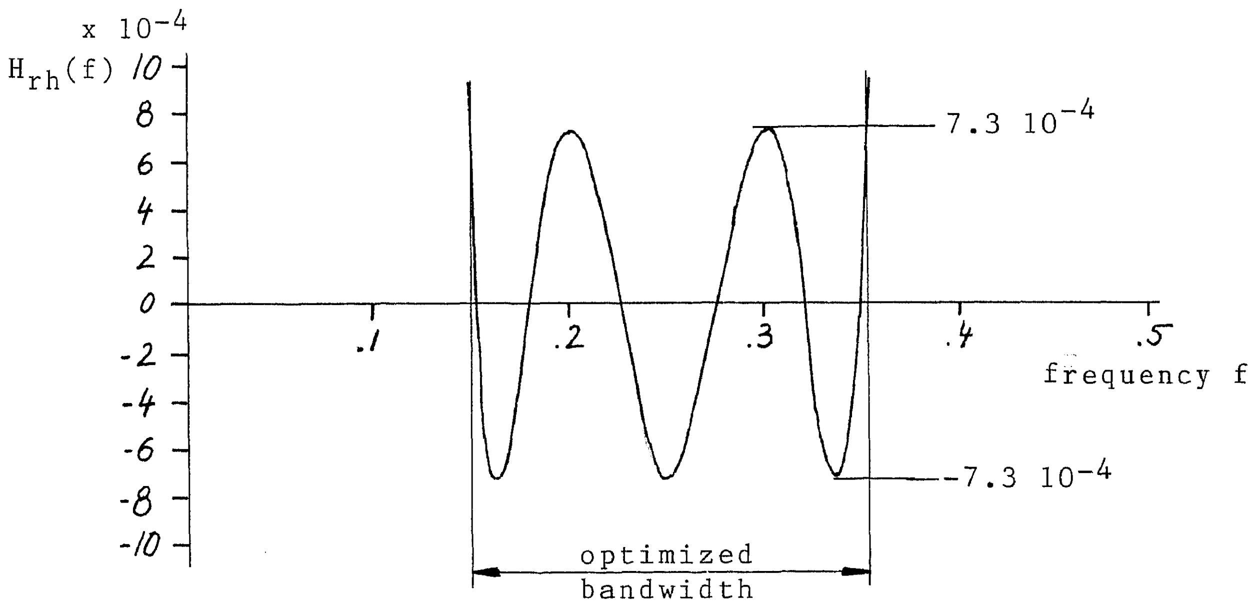

Fig. 2-3 Frequency ripple of the non-ideal Hilbert-transformer, F_1=0.15, F_2=0.35, N=11

The ripple in the frequency domain is the design criterion for the computer program. It is defined:

$$H_{rh} = 1 - H_{non-ideal} \qquad (2-9b)$$

The following table gives the coefficients of the Hilbert-transformer with filter length N = 11 and the first coefficients of the ideal Hilbert-transformer. The number of coefficients for the ideal Hilbert-transformer is infinite.

Coefficients of Hilbert-transformer	optimized	ideal
H(6)	0.0	0
H(5), -H(7)	0.59718	0.63662
H(4), -H(8)	0.00001	0
H(3), -H(9)	0.11623	0.21221
H(2), -H(10)	0.00001	0
H(1), -H(11)	0.01942	0.12732

Table 2-1 Coefficients of ideal and non-ideal Hilbert-transformers

The following tables and figures present the filter ripple r vs. odd and even filter length N:

Filter length	5	7	9	11	13
Ripple in dB	-19.6	-41.6	-41.6	-62.7	-62.7
Filter length	15	17	19	21	23
Ripple in dB	-83.3	-83.3	-104.0	-104.0	-126.9

Table 2-2 Ripple of Hilbert-transformer with odd filter length N, $F_1 = 0.15$, $F_2 = 0.35$

Filter length	4	6	8	10	12
Ripple in dB	-21.7	-33.1	-44.0	-54.6	-65.1
Filter length	14	16	18	20	22
Ripple in dB	-75.6	-86.0	-96.1	-106.3	-116.3

Table 2-3 Ripple of Hilbert-transformer with even filter length N,, $f_1 = 0.15$, $f_2 = 0.35$

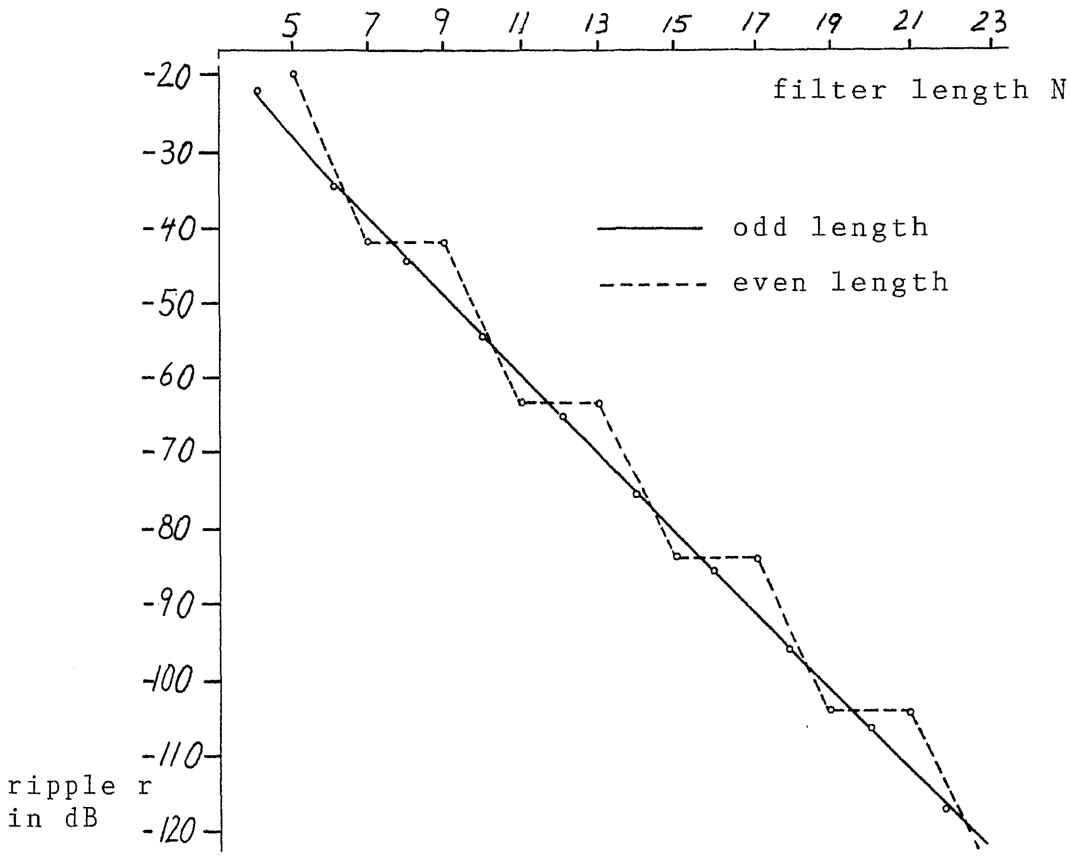

Fig. 2-4 Ripple of the Hilbert-transformer vs. length of the Hilbert-transformer N

The results of the calculations show that for N=N'+2, where N' =3,7,11,15,...,(4N-1); N = positive integer, there is no improvement in the resulting ripple of the discriminator.

2.3.2 Performance of Differentiators

The frequency response of the ideal differentiator [7] can be described by

$$H_d(e^{j\omega}) = j\omega\, e^{-j\omega} \tag{2-10}$$

$$H_d(e^{j\omega}) = j\,(\omega - 2\pi)\, e^{-j(\omega - 2\pi)} \tag{2-11}$$

The impulse response of the corresponding ideal differentiator can be described for odd filter length by:

$$h_d(n) = \frac{\cos(\pi n)}{n} \qquad \text{for } n \neq 0 \tag{2-12}$$

$$h_d(n) = 0 \qquad \text{for } n = 0 \tag{2-13}$$

For even filter length:

$$h_d(n) = \frac{-4}{\pi}\,\frac{\cos(\pi n)}{(2n+1)^2} \qquad \text{for } n \leq \frac{N}{2} \tag{2-14}$$

The following figures show a comparison between the ideal differentiator and an optimized differentiator with the filter length N = 11. The differentiator is optimized between the frequencies $F_1 = 0.15$ and $F_2 = 0.35$.

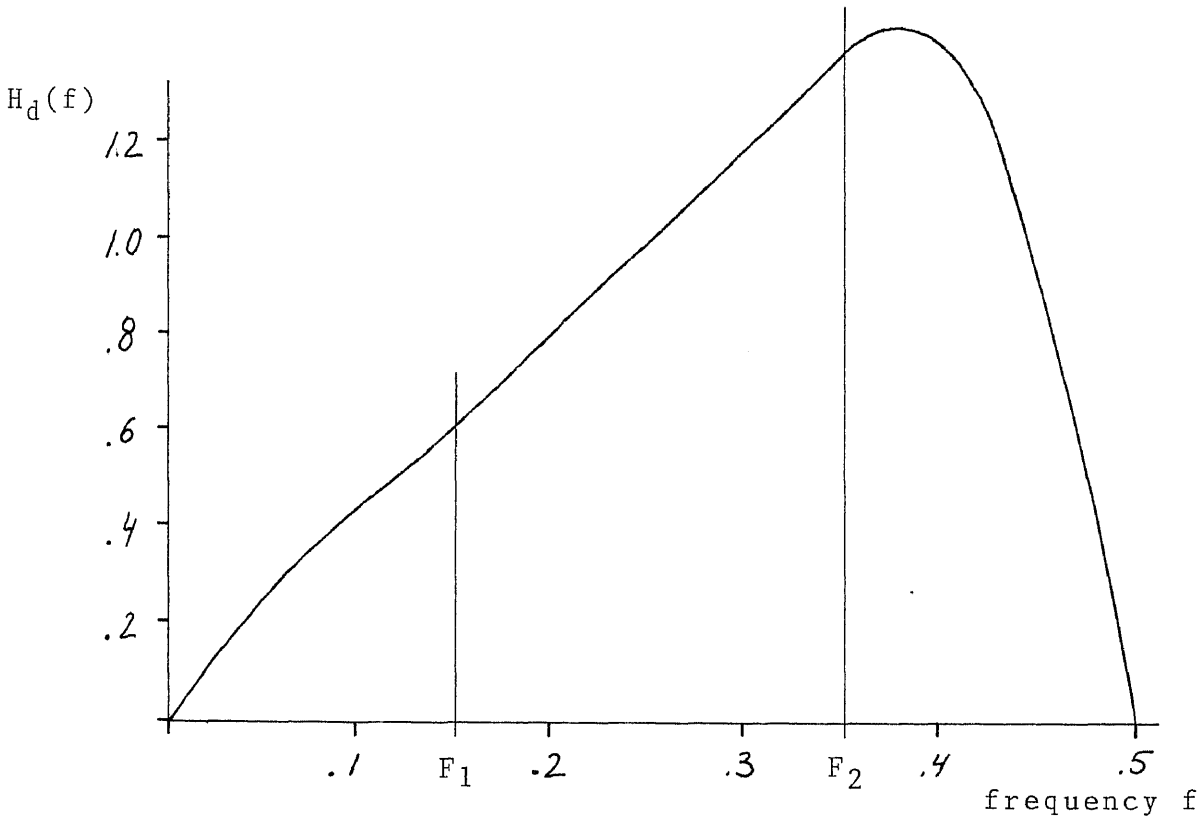

Fig. 2-5 Frequency response of a non-ideal differentiator, $F_1 = 0.15$, $F_2 = 0.35$, N=11

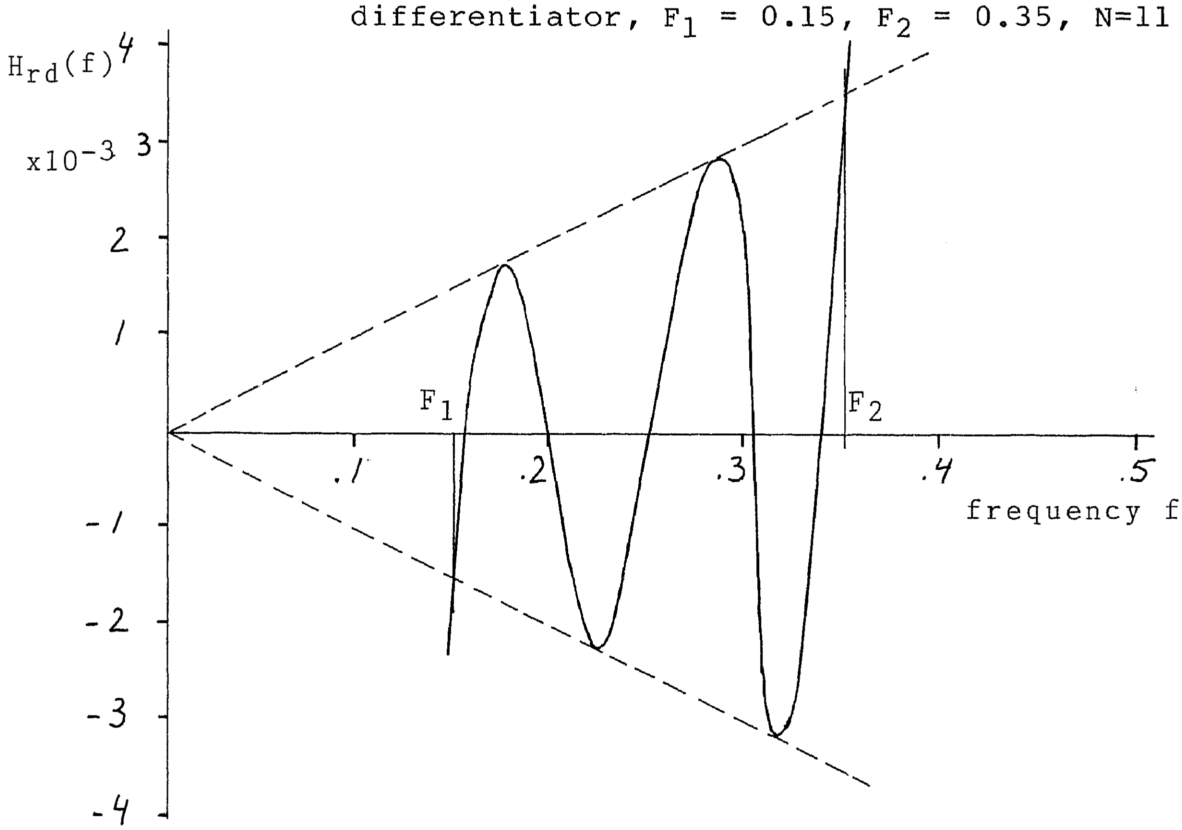

Fig. 2-6 Frequency ripple of an non-ideal differentiator, $F_1 = 0.15$, $F_2 = 0.35$, N=11

The ripple in the frequency domain is the design criterion for the computer program.

$$H_{rd} = f - H_{non-ideal} \qquad (2\text{-}9a)$$

It is important, due to the definition of the differentiator, that the error of the differentiator be permitted to increase with freququency. The following table gives the coefficients of the differentiator with filter length N = 11 and the first coefficients of the ideal differentiator. The number of the coefficients for the ideal differentiator is infinite.

Differentiator coefficient	optimized	ideal
D(6)	0.0	0
D(5), -D(7)	0.14917	0.159154
D(4), -D(8)	-0.05839	-0.079577
D(3), -D(9)	0.02881	0.053051
D(2), -D(10)	-0.00993	-0.039789
D(1), -D(11)	0.00471	0.031831

Table 2-4 Coefficients of ideal and non-ideal differentiator

The following tables and figures present filter ripple r vs. odd and even filter length N:

Filter length	5	7	9	11	13
ripple in dB	-19.5	-30.7	-41.5	-52.1	-62.6
Filter length	15	17	19	21	23
ripple in dB	-73.1	-83.3	-93.6	-103.8	-113.9

Table 2-5 Ripple of differentiator with odd filter length N, $F_1 = 0.15$, $F_2 = 0.35$

Filter length	4	6	8	10	12
Ripple in dB	-38.8	-53.5	-66.8	-79.3	-91.4
Filter length	14	16	18	20	22
Ripple in dB	-103.2	-114.8	-125.8	-138.0	-152.0

Table 2-6 Ripple of differentiator with even filter length N, $F_1 = 0.15$, $F_2 = 0.35$

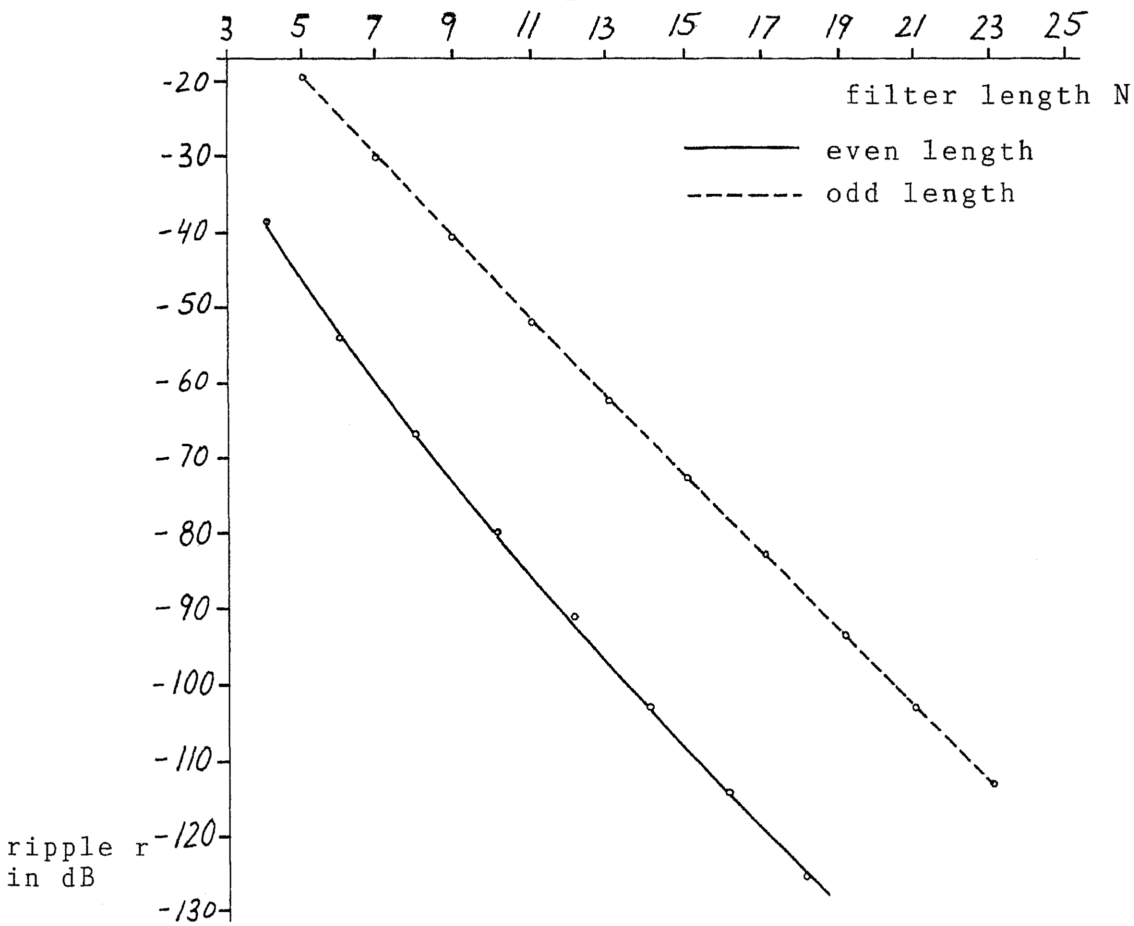

Fig. 2-7 Ripple of the differentiator vs. length of the differentiator N

The results presented in this part are similar to the results in [7]. The ripple of differentiators with even filter length is more than 26 dB smaller than differentiators with similar length but odd filter length. An increase in differentiator length always means an improvement in ripple.

2.3.3 Balancing Problem of the Klapper-Kratt Detector

The computer program by McClellan, Parks, and Rabiner [3] calculates the coefficients for the Hilbert-transformers and differentiators. The coefficients H(n) of the Hilbert-transformers can be used directly. To get the desired detector output, the slope of the discriminator has to be approximately the value 4. This allows, a desired zero detector-output at the frequency $F_c = 0.25$ to be achieved. The coefficients $D(n) = h_d(n)$ have to be corrected with the factor k.

$$D'(n)^{*} = k \cdot D(n) \qquad (2\text{-}15)$$

The correction factor k is dependent on the filter-length N and the coefficients of the Hilbert-transformer and differentiator.

$$N = 5 \qquad k = \frac{H(2)}{D(2)} \qquad (2\text{-}16)$$

$$N = 7 \qquad k = \frac{H(1)-H(3)}{D(1)-D(3)} \qquad (2\text{-}17)$$

$$N = 9 \qquad k = \frac{H(2)-H(4)}{D(2)-D(4)} \qquad (2\text{-}18)$$

$$N = 11 \qquad k = \frac{H(1)-H(3)+H(5)}{D(1)-D(3)+D(5)} \qquad (2\text{-}19)$$

$$N = 13 \qquad k = \frac{H(2)-H(4)+H(6)}{D(2)-D(4)+D(6)} \qquad (2\text{-}20)$$

$$N = 15 \qquad k = \frac{H(1)-H(3)+H(5)-H(7)}{D(1)-D(3)+D(5)-D(7)} \qquad (2\text{-}21)$$

These formulas can be proved by evaluating equation 2-3 at the frequency f = 0.25. Every other sample has no influence on balancing the frequency response at this frequency.

The next table shows the coefficients of the Hilbert-transformer and the coefficients of the differentiator with slope equal to 4 before and after correction. The filter length N is 7 and the calculated filter is optimized between the two frequencies $F_1 = 0.15$ and $F_2 = 0.35$.

Coefficient C(n)	Hilbert-transformer	Differentiator slope = 4	Differentiator corrected
C(4)	0	0	0
C(3), -C(5)	0.5805339	0.5795584	0.5773027
C(2), -C(6)	-0.0000329	-0.1949638	-0.1942053
C(1), -C(7)	0.0847717	0.0818591	0.0815405

Table 2-7 Correction of optimized coefficients

The correction coefficient used in this example has the value k = 3.9844315. The correction improved the suppression of the carrier at $F_c = 0.25$ from -48.9 dB to -245.4 dB. It isobvious that a big improvement of the cancellation of the carrier can be achieved by applying the suggested formulas. The suggested equations are not only valid for the design of Klapper-Kratt detectors according to this design procedure, but for all design procedures, if a cancellation of the carrier at F_c = 0.25 is desired. The observation that every other coefficient of the Hilbert-transformer and differentiator is almost zero can be used to simplify the FM detector.

2.3.4 Performance of the Klapper-Kratt Detector with Optimized Hilbert-Transformers and Differentiators

Optimized Hilbert-transformers and differentiators are used for the synthesis of the digital Klapper-Kratt detector. The mentioned computer program [4] calculates the coefficients. The coefficients of the digital Klapper-Kratt detector are also corrected according to the earlier described procedure. The next table presents as an example the coefficients of the detector with filter length N = 13.

Coefficient C(n)	Hilbert-transformer	Differentiator
C(7)	0	0
C(6), -C(8)	0.5971854	0.5971771
C(5), -C(9)	0.0000342	-0.2537362
C(4), -C(10)	0.1162392	0.1162203
C(3), -C(11)	0.0000370	-0.0611803
C(2), -C(12)	0.0194197	0.0194091
C(1), -C(13)	0.0000182	-0.0094088

Table 2-8 Optimized and corrected coefficients, N = 13, F_1 = 0.15, F_2=0.35

The following figures present the detector output and the detector ripple.

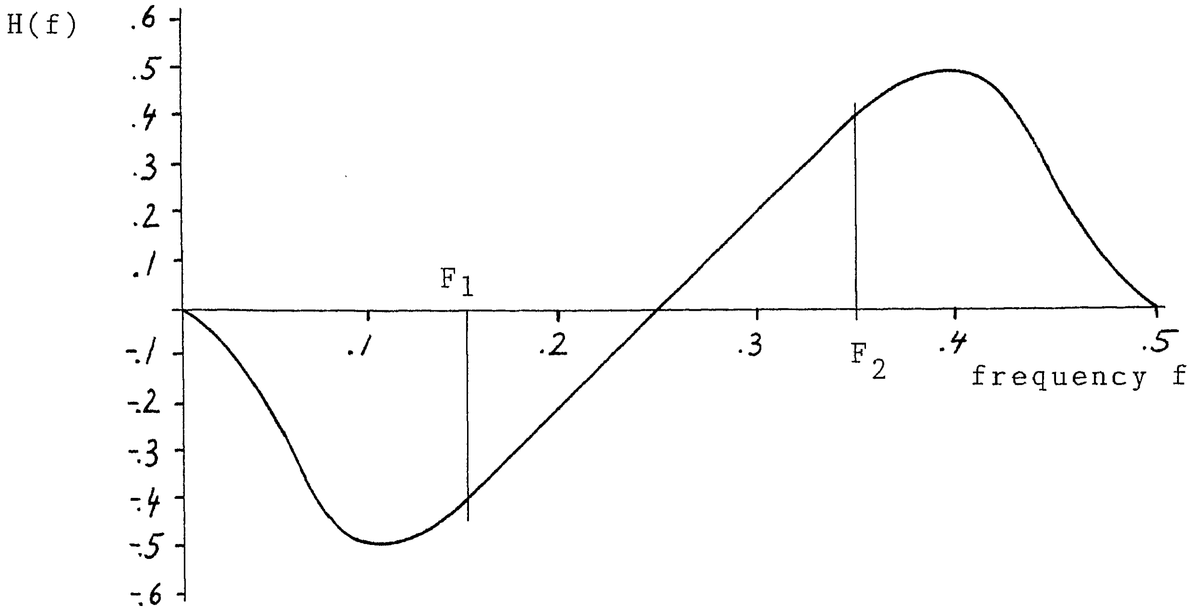

Fig 2-8 Detector output vs. frequency, N=13, F_1= 0.15, F_2= 0.35

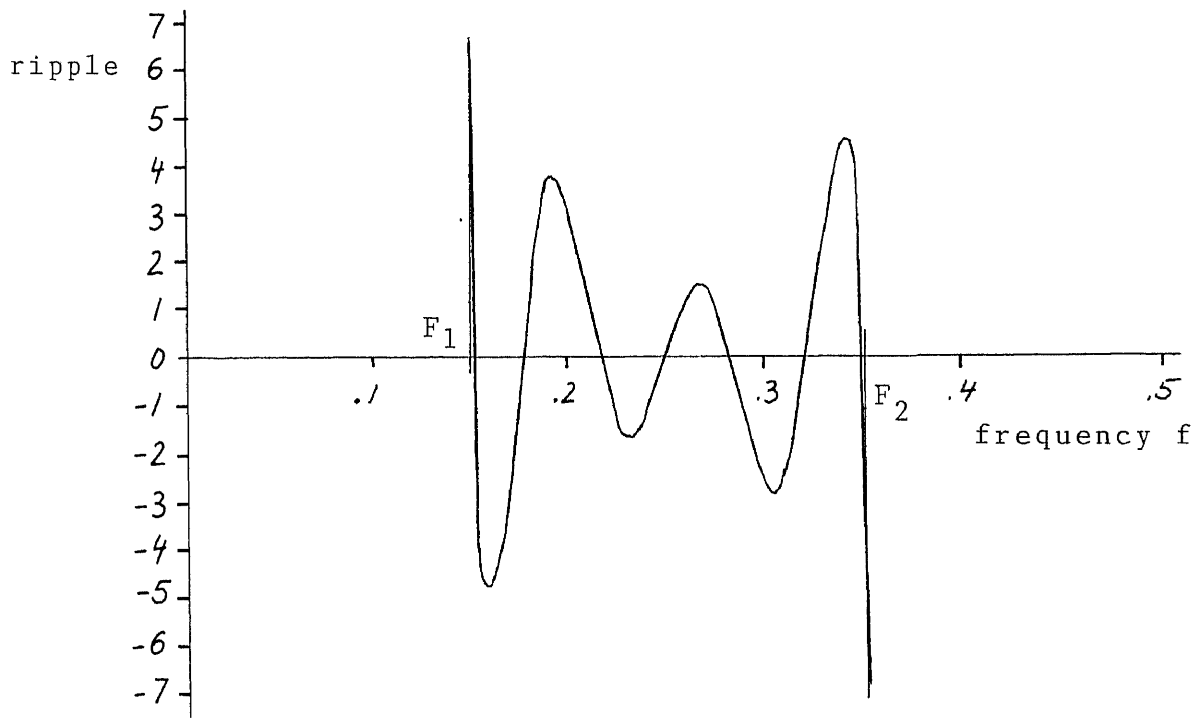

Fig 2-9 Detector ripple vs. frequency, N=13, F_1= 0.15, F_2= 0.35

It is obvious that the ripple of the synthesized Klapper-Kratt detector is no longer as smooth as the ripple of its components.

The following table and figure give the resulting ripple for the detectors synthesized out of optimized Hilbert-transformers and differentiators.

Detector length N	5	7	9	11
Detector ripple in dB	-21.9	-28.8	-43.3	-50.5
Detector length N	13	15	17	19
Detector ripple in dB	-64.6	-70.7	-77.7	-92.0

Table 2-9 Detector ripple of the Klapper-Kratt detector, N=13, F_1= 0.15, F_2= 0.35

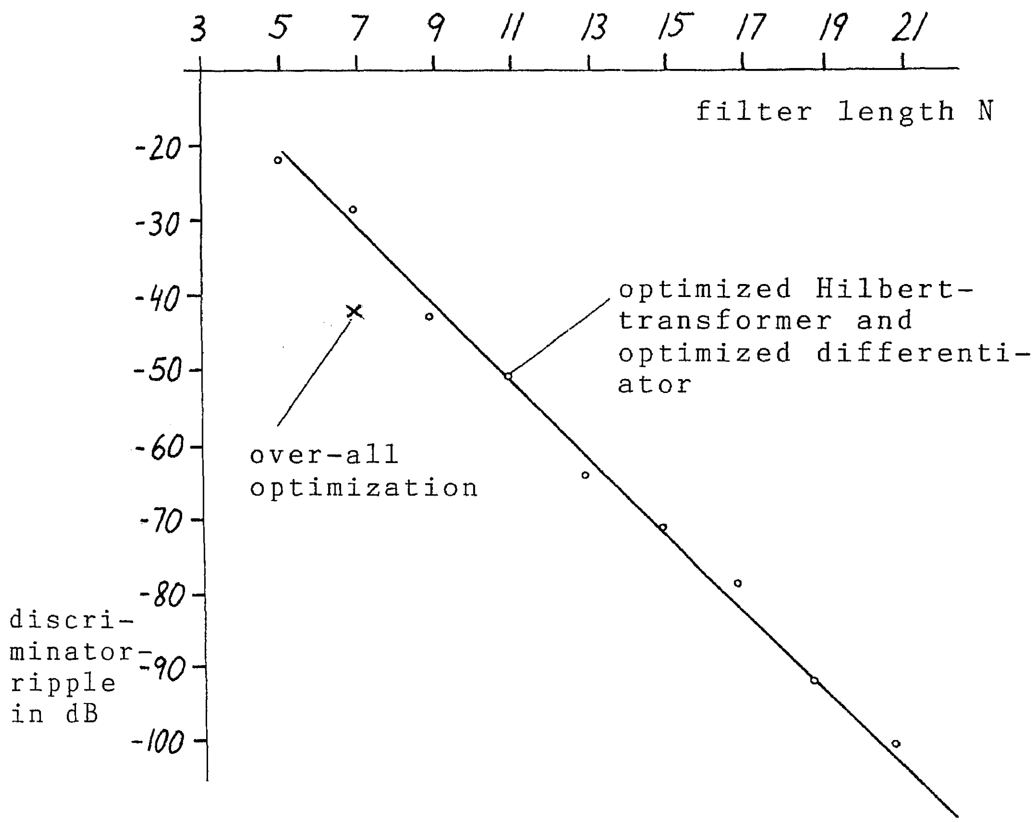

Fig. 2-10 Detector ripple of the Klapper-Kratt detector, N=13, F_1= 0.15, F_2= 0.35

The ripple of the detector can be closely calculated by the following formula:

$$d_r = - 4.97\ \text{dB} \cdot N + 3.49\ \text{dB} \qquad (2\text{-}21)$$

The ripple of a Klapper-Kratt detector with an over-all optimlization is approximately 6×10^{-3}. This is equivalent to -44.4 dB. Each filter of the Klapper-Kratt detector has a length of N=7. This detector is for the comparison also in figure 2-10.

2.3.5 Discussion of the Calculated Klapper-Kratt Detector

As shown earlier, the research shows a clear superiority of the over-all optimized detector over the detector with optimized components. The gain of the optimization for the whole detector is approximately 12 dB at the filter length N = 7. The results show also, that there is an advantage for certain filter structures and that the restriction to a certain filter structure puts restrictions on the performance of the filters.

REFERENCES - CHAPTER II

[1] Jacob Klapper and Edward J. A. Kratt,III, "A new family of low-delay FM detectors," IEEE Transactions on Communications, Vol. COM-27, pp 419-429, Feb. 1979.

[2] E. Kratt and J. Klapper, "A New Digital Detector for Frequency Modulated Waves," 27th Midwest Symposium on Circuits and Systems, Morgentown WVa., June 1984, pp 242-245.

[3] L. R. Rabiner and B. Gold, Theory and Application of Digital Signal Processing, Englewood Cliffs, NJ: Prentice-Hall, 1975.

[4] J. H. McClellan, T. W. Parks, and L. R. Rabiner, "A computer program for designing optimum FIR linear phase digital filters," IEEE Transactions on Audio and Electroacoustics, Vol. AU-21, pp. 506-525, Dec. 1972.

[5] John Ll. Morris, Computational Methods in Elementary Numerical Analysis, John Wiley, pp. 187-197, 1983.

[6] L. R. Rabiner and R. W. Schafer, "On the behavior of Minimax FIR digital Hilbert transformers," Bell System Technical Journal, Vol. 53, pp. 363-390, Feb. 1974.

[7] L. R. Rabiner and R. W. Schafer, "On the behavior of Minimax relative error FIR digital differentiators," Bell System Technical Journal, Vol. 53, pp. 333-361, Feb. 1974.

CHAPTER III

FINITE IMPULSE RESPONSE FILTER APPROACH FOR FREQUENCY DISCRIMINATORS

3.1 Introduction

This chapter presents a brief summary of possible FIR filters cases and their characteristics. As shown earlier, FIR filters can be used to build multiple bandpass/stopband filter, Hilbert-transformers and differentiators. The FIR discriminator is introduced and analyzed as a fourth type of FIR linear phase filter.

3.1 Minimax-Error FIR Discriminators

Rabiner and Gold [1] distinguish FIR filters by symmetry and filter length. They categorize the filters into the following 4 cases:

Case	Symmetry	Length
1	positive	odd
2	positive	even
3	negative	odd
4	negative	even

Table 3-1 Categorization of FIR-filters

Positive symmetry of the coefficients is defined by

$$h(k) = h(N - 1 - k) \qquad (3\text{-}1)$$

Negative symmetry of the coefficients is defined by

$$h(k) = -h(N - 1 - k) \qquad (3\text{-}2)$$

The following figures show typical impulse responses of FIR-filters

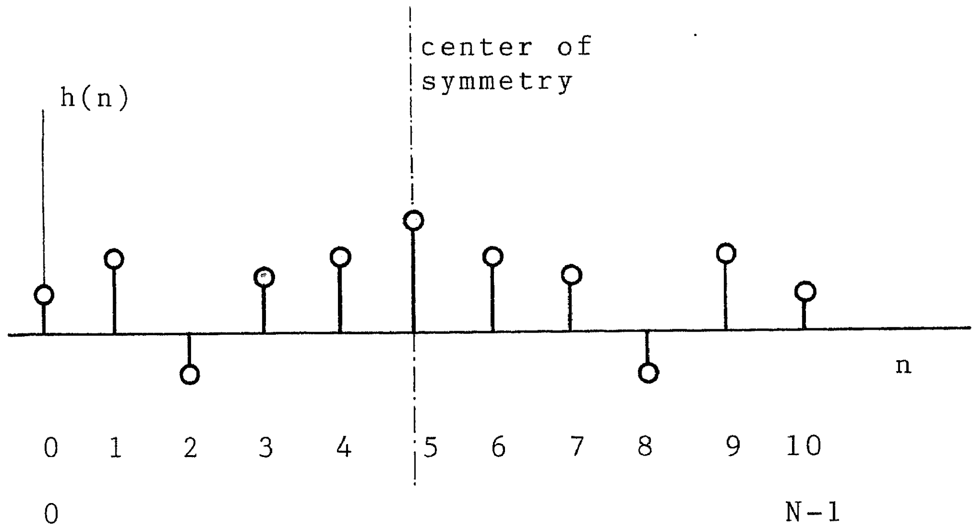

Fig. 3.1 Typical impulse response for case 1, N odd, positive symmetry, N=11

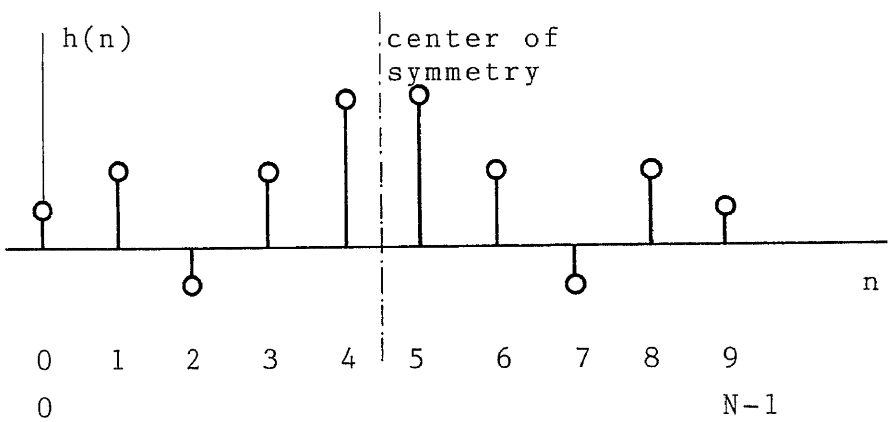

Fig. 3.2 Typical impulse response for case 2, N even, positive symmetry, N=10

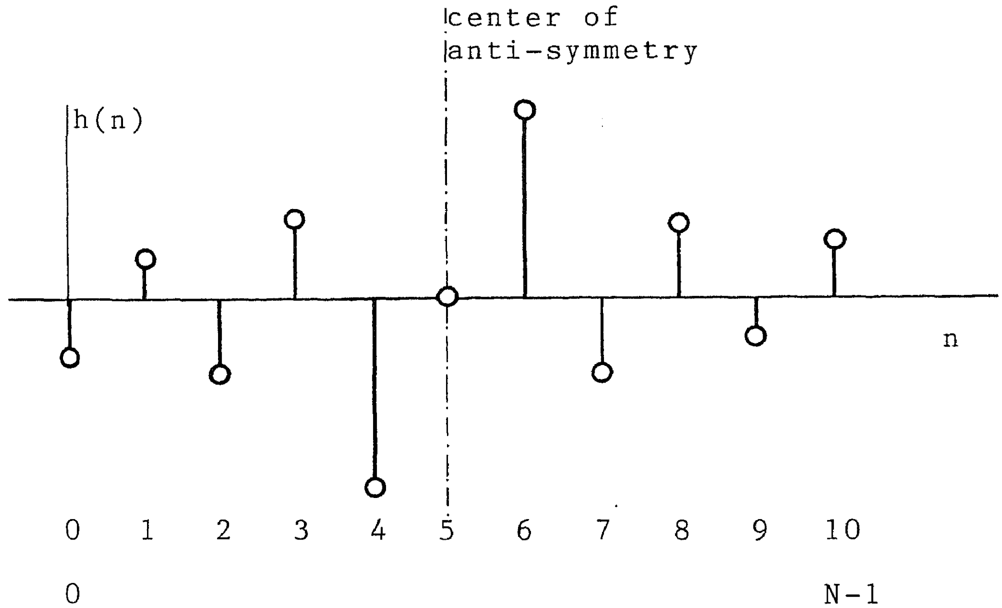

Fig. 3.3 Typical impulse response for case 3, N odd, negative symmetry, N=11

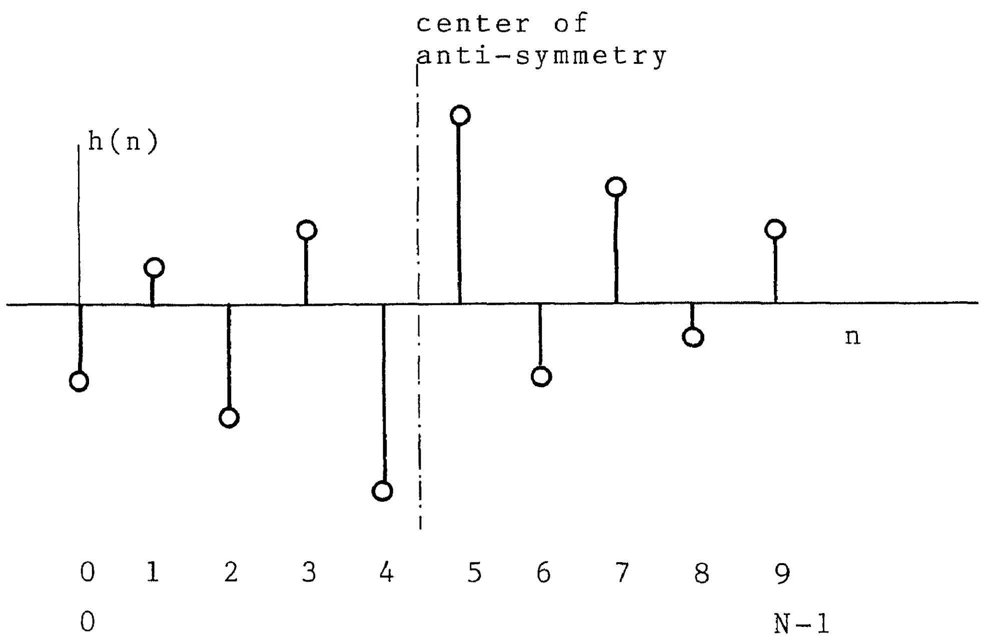

Fig. 3.4 Typical impulse response for case 4 N even, negative symmetry, N=10

Rabiner and Gold prove, that all four cases of FIR-filters imply exactly linear phase filters.

$$\Theta(\omega) = -\alpha \cdot \omega \qquad -\pi \leqslant \omega < \pi \qquad (3-3)$$

where is a constant phase delay in samples:

$$H(e^{j\omega}) = \sum_{n=0}^{N-1} h(n)\ e^{-j\omega n} = H(e^{j\omega}) \quad e^{-j\alpha} \qquad (3-4)$$

Applying symmetry, the frequency response of the filter reduces to:

Case 1: Symmetrical impulse response, N odd:

$$H(e^{j\omega}) = \sum_{n=0}^{(N-1)/2} a(n) \cos(2\pi f\ n) \qquad (3-5)$$

with $a(0) = h[(N-1)/2]$ (3-6) and

$a(n) = 2\ h[(N-1)/2 - n]$ (3-7)

Case 2: Symmetrical impulse response, N even:

$$H(e^{j\omega}) = \sum_{n=1}^{N/2} b(n) \cos[2\pi f(n - 0.5)] \qquad (3-8)$$

with $b(n) = 2\ h[(N/2 - n]$ (3-9)

Case 3: Antisymmetrical impulse response, N odd:

$$H(e^{j\omega}) = \sum_{n=1}^{(N-1)/2} c(n) \sin(2\pi f\ n) \qquad (3-10)$$

with $c(n) = 2\ h[(N-1)/2 - n]$ (3-11)

Case 4: Antisymmetrical impulse response, N even:

$$H(e^{j\omega}) = \sum_{n=1}^{(N-1)/2} d(n) \sin [2\pi f (n - 0.5) \qquad (3\text{-}12)$$

$$d(n) = 2 h[(N/2) - n] \qquad (3\text{-}13)$$

Depending on the case of FIR-filter, several restrictions apply. The following table gives an overview over these restrictions in the frequency-domain

Case #	Symmetry	Length	H at f = 0	H at f = 0.5
Case 1	positive	odd	X	X
Case 2	positive	even	X	0
Case 3	negative	odd	0	X
Case 4	negative	even	0	0

X = No restriction

Table 3-2 Frequency-restrictions for different cases of FIR filters

This table shows, that FIR filter of case 2 necessarily have to have an output of zero at frequency f = 0.5. Sometimes this is desired, in other cases (highpass filter) it is not desired.

One result up to now is, that for the optimization of certain filter types, filter cases should be chosen, whose restrictions in the frequency domain do not contradict the desired frequency response.

Rabiner, et al. [1,2] describe a method to rewrite the four cases in a common form. By this, the four cases are expressed as a summation of cosines.

$$D(e^{j\omega}) = Q(e^{j\omega})\ P(e^{j\omega}) \qquad (3\text{-}14)$$

	$Q(e^{j\omega})$	$P(e^{j\omega})$
Case 1	1	$\sum_{n=0}^{(N-1)/2} a(n) \cos(\omega n)$
Case 2	$\cos(\omega/2)$	$\sum_{n=0}^{(N/2-1)} b(n) \cos(\omega n)$
Case 3	$\sin(\omega)$	$\sum_{n=0}^{(N-3)/2} c(n) \cos(\omega n)$
Case 4	$\sin(\omega/2)$	$\sum_{n=0}^{(N/2-1)} d(n) \cos(\omega n)$

Table 3-3 Transformations for Remez-exchange algorithm

A central computation method, based on the Remez exchange algorithm [3] is used to calculate the best approximation. D(f) is the desired magnitude response.

Rabiner, Gold, McClellan, and Parks apply the Remez exchange algorithm to calculate bandpass filters, Hilbert-transformers and differentiators. The optimization of these filters is performed between two chosen frequencies F_1 and F_2.

To calculate optimized FIR discriminators, it is necessary, to define D(f) for the optimization.

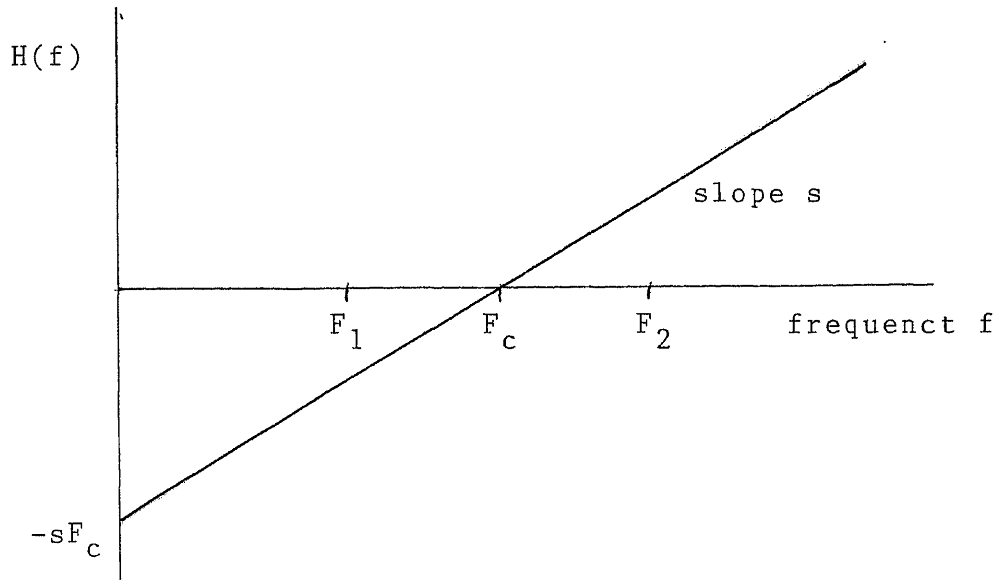

Fig. 3-5 Ideal frequency response D(f) of the FIR-discriminator, s = slope

The ideal frequency response of the differentiator can be expresses as:

$$D(f) = - s \cdot F_c + f \cdot s \qquad 0 \leq f \leq 0.5 \qquad (3\text{-}15)$$

The weighted error $E(e^j\ )$ of approximation is by definition:

$$E(e^{j\omega}) = W(e^{j\omega})\ [D(e^{j\omega}) - H(e^{j\omega})] \qquad (3\text{-}16)$$

W(f) is a modified weighting function. The Chebyshev approximation in the frequency range f between F_1 and F_2 for the discriminator may be stated as the minimization of the quantity

$$||E(f)|| = \max_{f \in F} W(f)\ |D(f) - P(f)| \qquad (3\text{-}17)$$

by choice of the coefficients of P(f). The original weighting function is W(f). McClellan, Parks and Rabiner suggest a filter design algorithm [2]. The following chart shows the original chart with changes for the calculation of FIR discriminators.

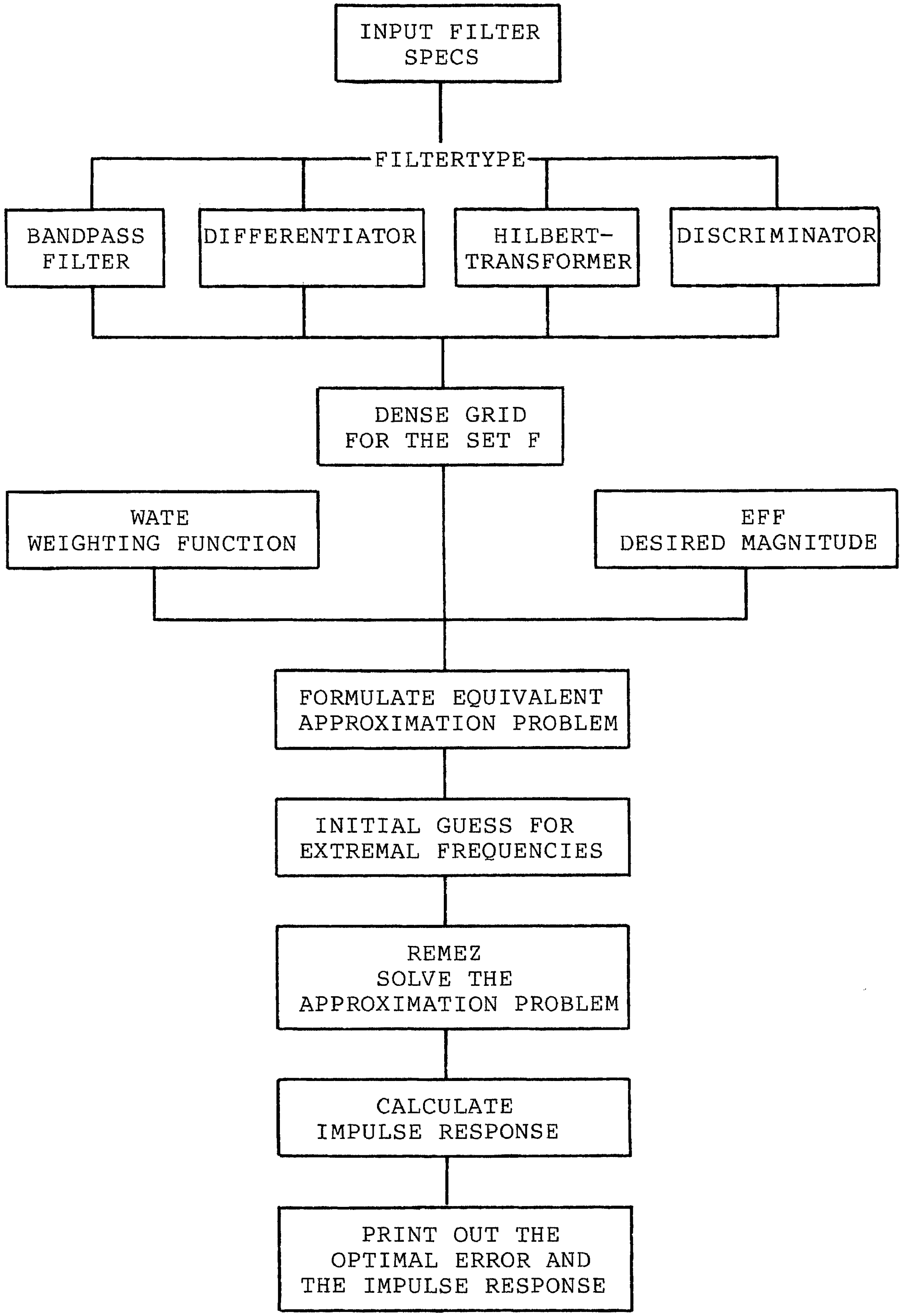

Fig. 3-6 Overall flow chart of filter design algorithm

Programmodifications were done in the following sections:

- input section
- output section
- weighting function (WATE)
- desired value function (EFF)

These changes are described in Appendices A, B and C. Filters calculated according to this approach, are presented later in this thesis.

3.3 A Practical Approach to FIR Discriminators

The following pages describe the initial approach towards the FIR discriminator. In the original computer program [3] of McClellan, Parks and Rabiner present the desired magnitude function (EFF). This function calculates the ideal magnitude of bandpass filters, differentiators and Hilbert-transformers. The weighting function (WATE) determines the desired tolerance scheme. This practical approach towards FIR discriminators, starts with the FIR differentiator and changes the WATE- and EFF-function to get the magnitude of a FIR discriminator. On the following pages, one example of the design of an FIR discriminator with this design algorithm is presented. This example illustrates the evolution of the FIR differentiator to an FIR discriminator with filter length N=7.

The first figures show the frequency response and the ripple of the original differentiator

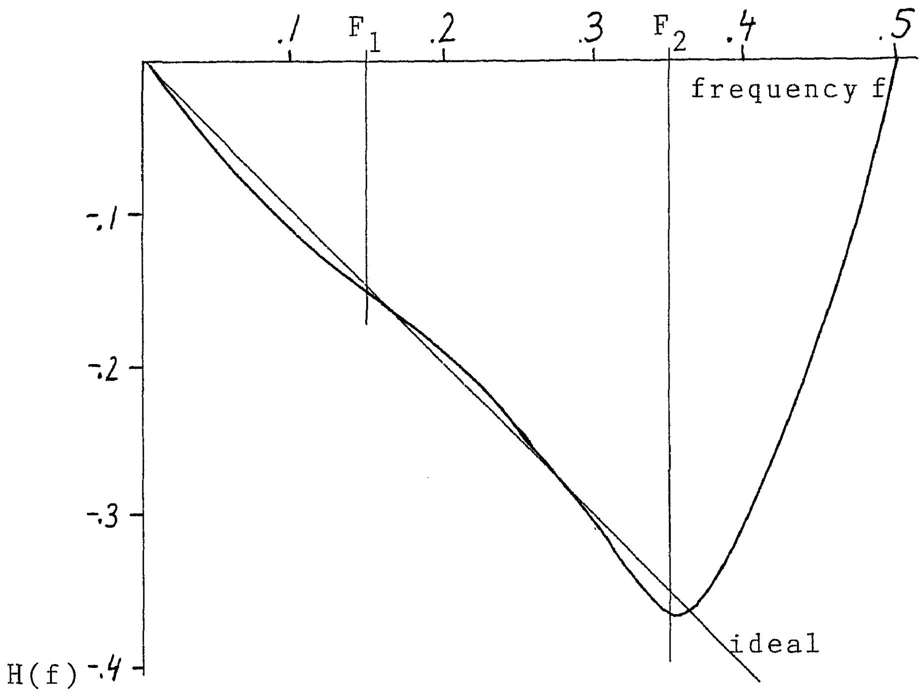

Fig. 3-7 Frequency response of the original differentiator, N=7, F_1=0.15, F_2=0.35

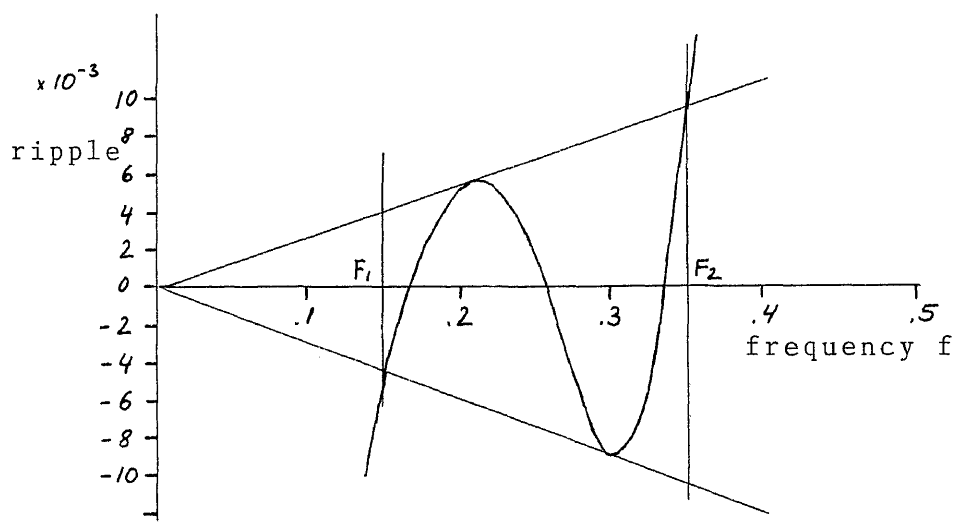

Fig. 3-8 Ripple of the original differentiator N=7, F_1=0.15, F_2=0.35

The figures show, that the output of the differentiator and its ripple are increasing with frequency.

The following frequency response is the result of a modified desired magnitude function EFF. The desired magnitude of the FIR filter is chosen to equal the magnitude of the ideal discriminator. The ripple function of the discriminator still shows the typical behavior of the ripple function of the differentiator.

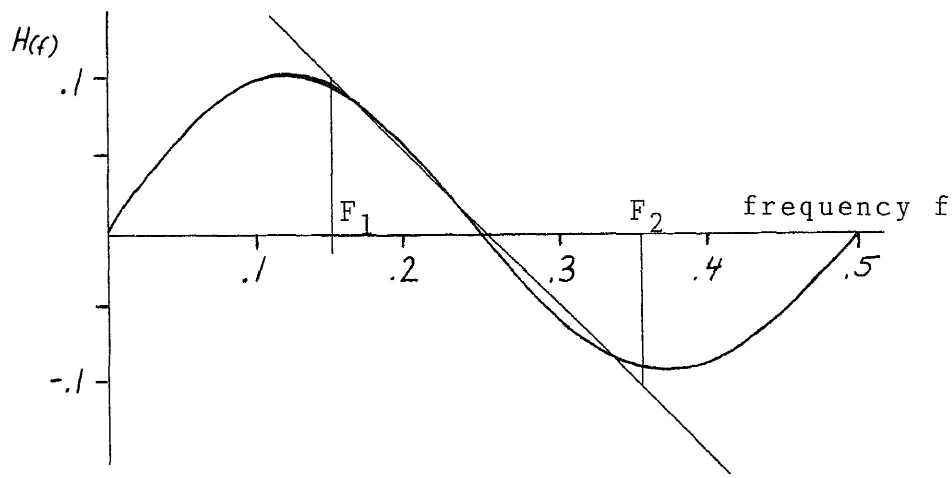

Fig. 3-9 Frequency response of discriminator A
$N=7$, $F_1=0.15$, $F_2=0.35$

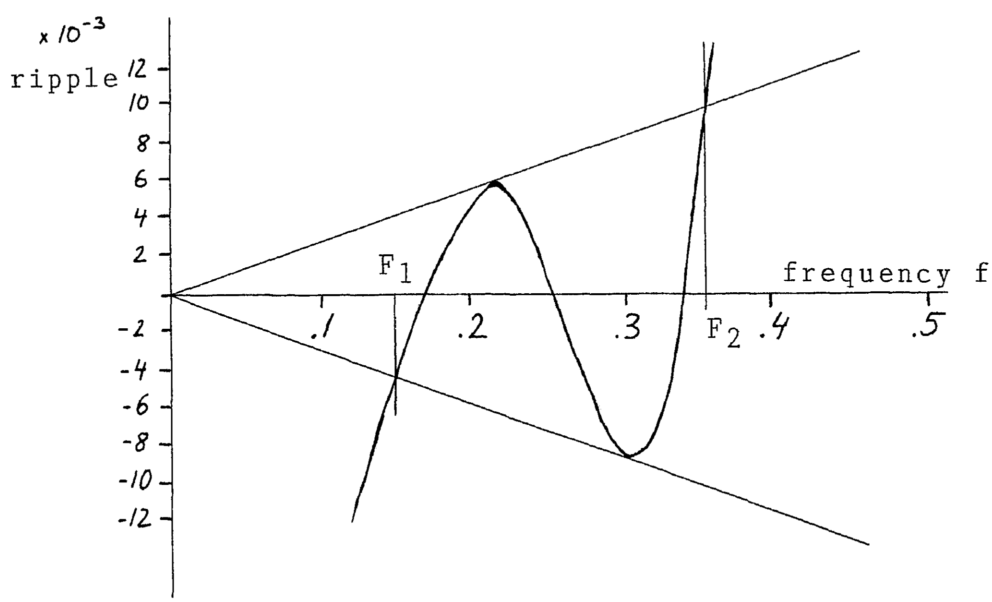

Fig. 3-10 Ripple of discriminator A
$N=7$, $F_1=0.15$, $F_2=0.35$

Modification of the desired magnitude function EFF and the weighting function WATE result in the final FIR discriminator.

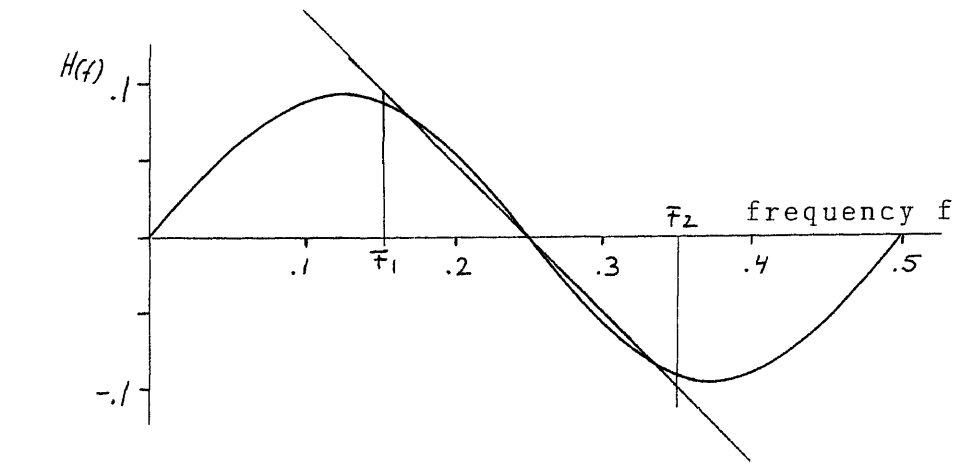

Fig. 3-11 Frequency response of the FIR discriminator, (Discr. B),

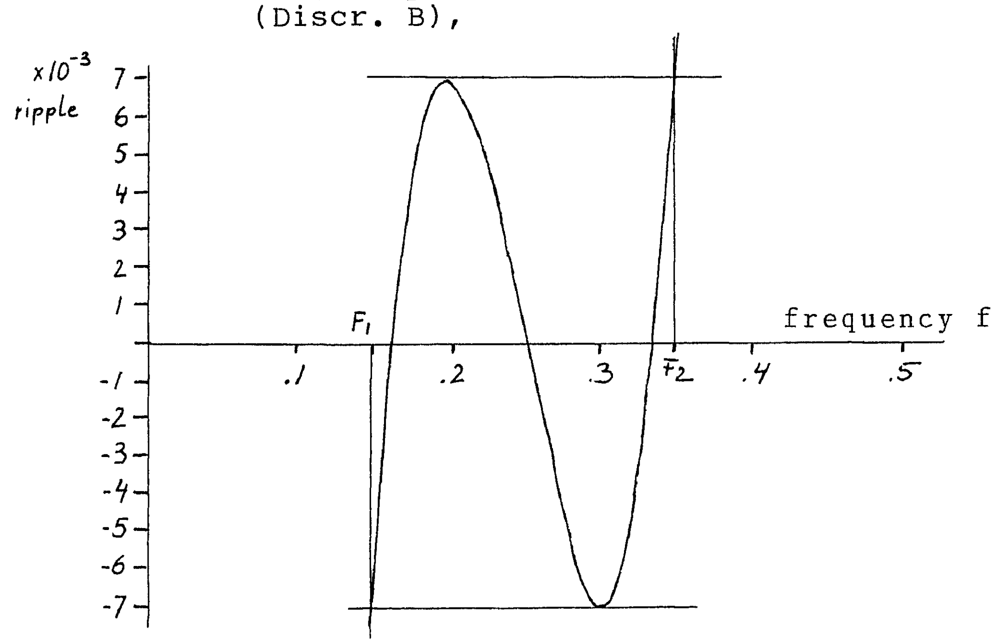

Fig. 3-12 Ripple of the FIR discriminator (Discr. B) N=7, F_1=0.15, F_2=0.35

The following table gives the coefficients of 3 filters. These coefficients are the coefficients of the original differentiator, the discriminator with an error function of the differentiator and the final FIR discriminator with equiripple error.

	Different.	Discr. A	Discr. B
D(4)	0	0	0
D(3) = -D(5)	0.14487990	-0.00088687	-0.00000003
D(2) = -D(6)	-0.04872001	-0.04853990	-0.04873794
D(1) = -D(7)	0.02046250	-0.00264300	-0.00000003

Different. = original differentiator
Discr. A = discriminator with unchanged error function
Discr. B = discriminator with modified error function

Table 3-4 Coefficients of differentiator, discriminator A and discriminator B

REFERENCES - CHAPTER III

[1] L. R. Rabiner and B. Gold, Theory and Application of Digital Signal Processing, Englewood Cliffs, NJ: Prentice-Hall, 1975.

[2] E. Kratt and J. Klapper, "A New Digital Detector for Frequency Modulated Waves," 27th Midwest Symposium on Circuits and Systems, Morgentown WVa., June 1984, pp 242-245.

[3] John Ll. Morris, Computational Methods in Elementary Numerical Analysis, John Wiley, pp. 187-197, 1983.

CHAPTER IV

CHARACTERISTICS OF FIR DISCRIMINATORS

4.1 Introduction

This chapter presents the results of research in the field of FIR discriminators. The previous chapter gives the theoretical background to this kind of FIR filters. The calculation and evaluation of approximately 500 FIR discriminators gives insight into a new application of digital FIR filters. Under research are such points as constant and weighted error function, shifted center frequencies of different classes of FIR filters and the detector sensitivity. The first part of this chapter presents results of FIR discriminators with constant weighting function. Weighting functions give the opportunity to shape the tolerance scheme of a desired magnitude.

Equation 3-16 shows that the error of the discriminator is the difference of the ideal and nonideal discriminator multiplied by a weighting function W(f). The weighting function can be independent of f

$$W(f) = k \qquad (4\text{-}1)$$

or dependent on f

$$W(f) = F(f) \qquad (4\text{-}2)$$

The next section presents the characteristics of FIR discriminator with constant weighting functions.

4.2 Constant Weighting Functions for FIR Discriminators

4.2.1 Dependence of FIR Discriminator Ripple on Filter Length

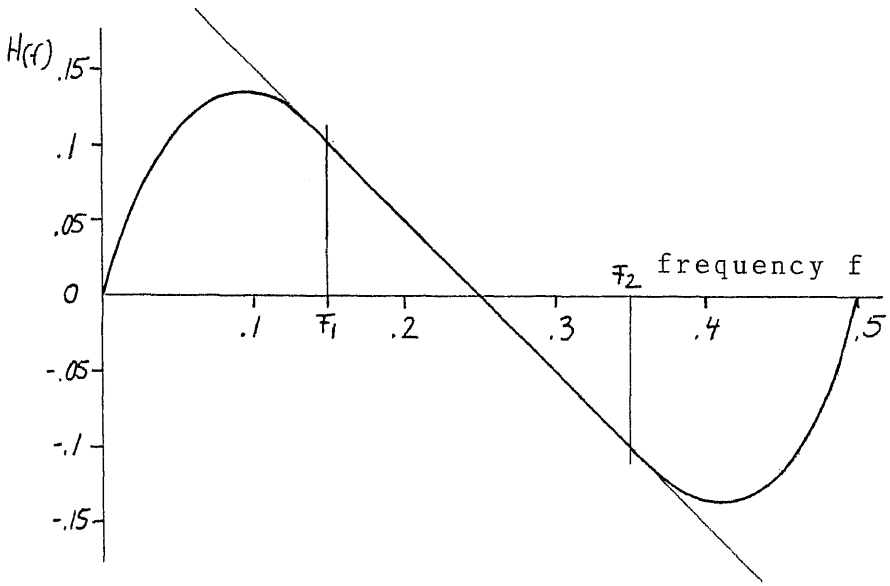

Fig. 4-1 Frequency response of typical FIR discriminator, N=15, F_1=0.15, F_2=0.35, W(f)=const., s=1

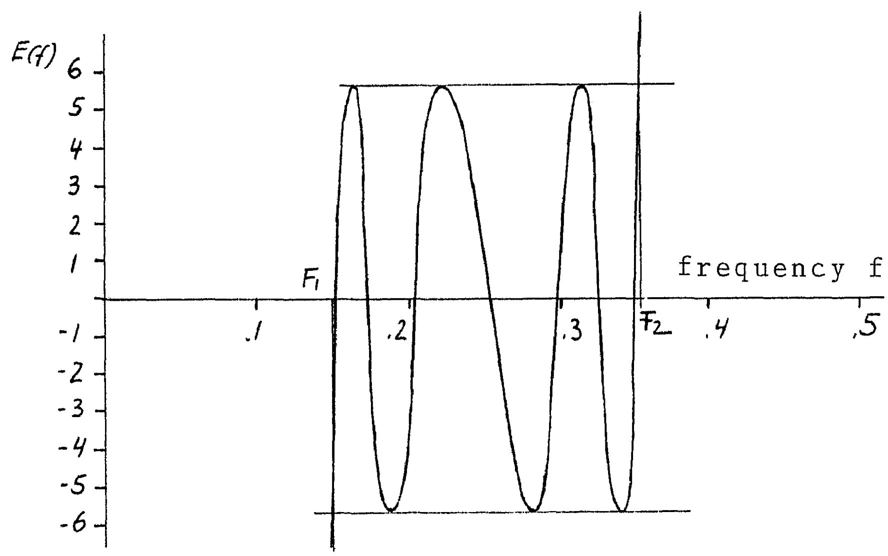

Fig. 4-2 Discriminator ripple of typical FIR discriminator, N=15, F_1=0.15, F_2=0.35, W(f)=const., s=1

The previous figures show a typical FIR discriminator (N=15) with constant weighting function. The following figure shows the absolute value of the discriminator ripple.

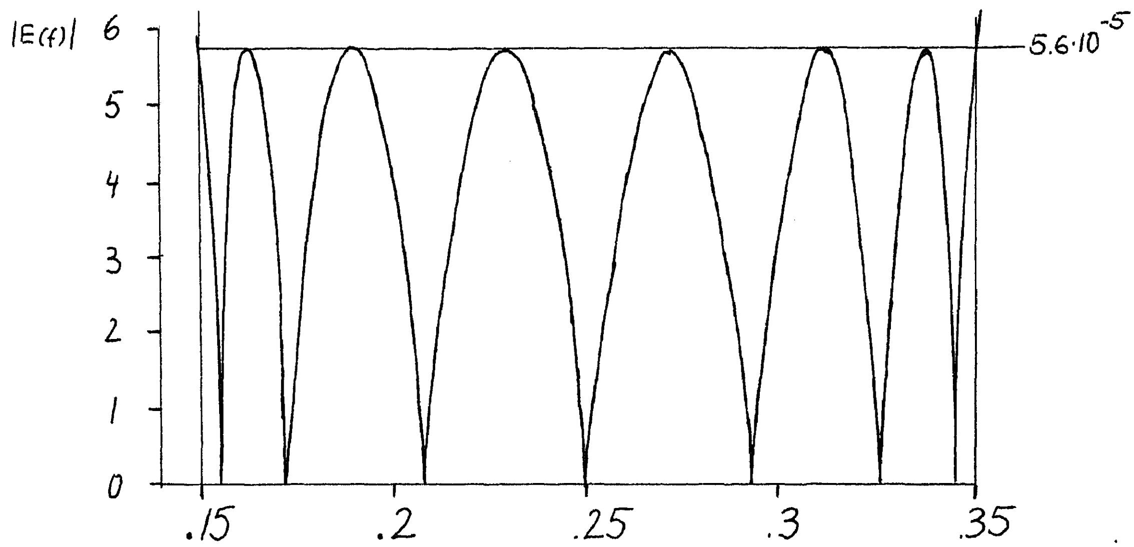

Fig. 4-3 Error magnitude for typical FIR discriminator, N=15, F_1=0.15, F_2=0.35, W(f)=const., s=1

The described filter has the following coefficients

D(n)	coefficient
D(8)	0
D(7) = -D(9)	0.00000070
D(6) = -D(10)	-0.06361061
D(5) = -D(11)	0.00000150
D(4) = -D(12)	-0.01547578
D(3) = -D(13)	0.00000113
D(2) = -D(14)	-0.00243084
D(1) = -D(15)	0.00000039

Table 4-1 Coefficients of a typical FIR discriminator

The following tables give the values of the filter ripple for all four filter cases (see Table 3-1) and with different filter bandwidths. The center frequencies in all cases is F_c=0.25.

filter length N	ripple case 2	ripple case 4	filter length N	ripple case 1	ripple case 3
-	-	-	3	-73.8	-
4	-47.9	-47.9	5	-73.8	-61.5
6	-63.3	-63.3	7	-113.2	-61.5
8	-82.0	-82.0	9	-112.7	-95.4
10	-97.7	-97.6	11	-149.5	-95.4
12	-115.6	-115.4	13	-149.6	-129.2
14	-131.6	-131.5	15	-198.8	-129.2
16	-148.4	-147.9	17	-198.2	-163.0
18	-167.2	-169.5	19	-200.0	-163.0
20	-187.1	-180.0	21	-200.0	-170.0
22	-200.0	-170.0	23	-200.0	-170.0

Table 4-2 FIR discriminator ripple vs filter length N for filter case 1-4, $F_1=0.2$, $F_2=0.3$

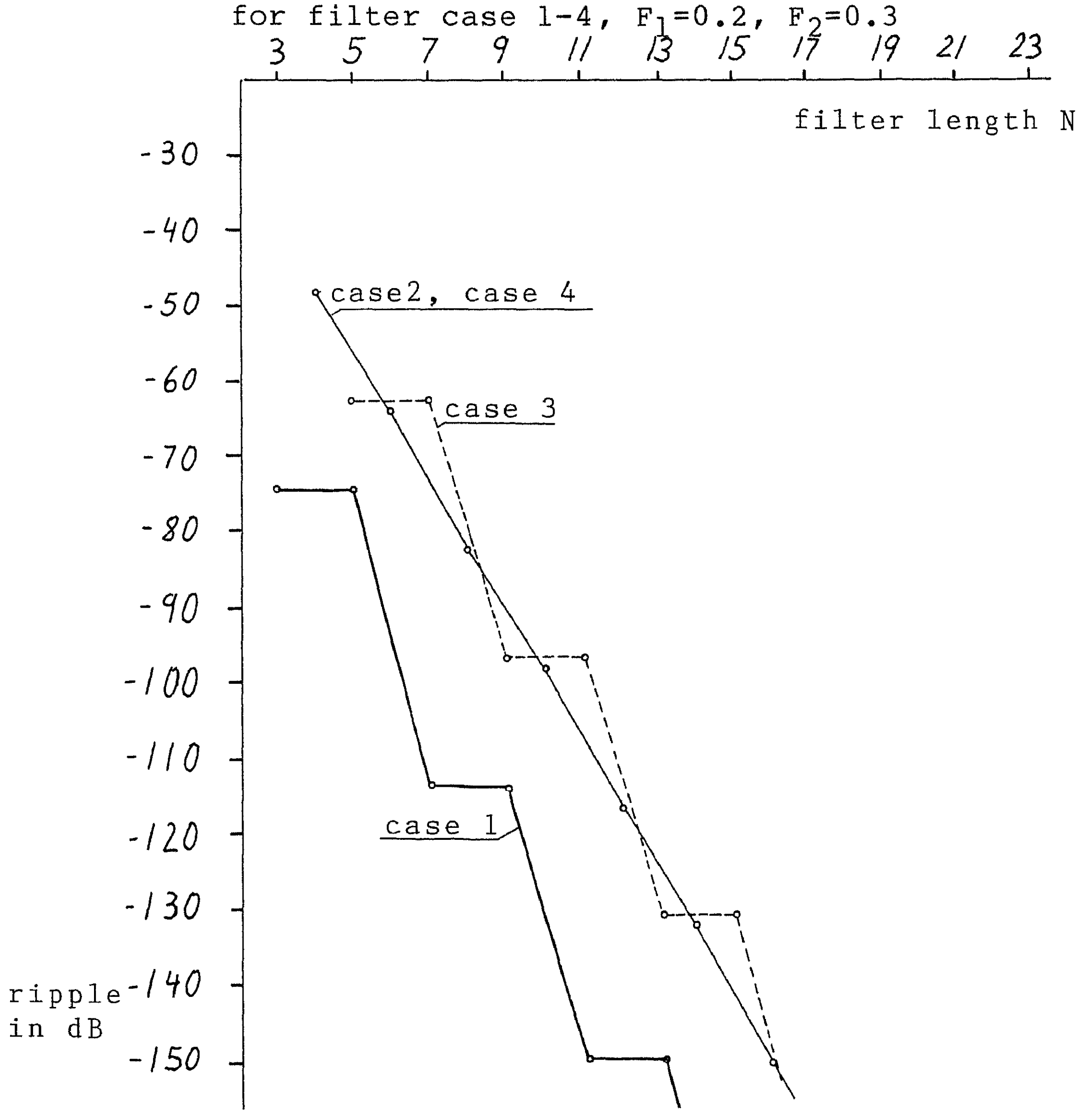

Fig. 4-4 FIR discriminator ripple vs filter length N for filter case 1-4, $F_1=0.2$, $F_2=0.3$

filter length N	ripple case 2	ripple case 4	filter length N	ripple case 1	ripple case 3
-	-	-	3	-55.5	-
4	-34.9	-34.9	5	-55.5	-42.7
6	-44.5	-44.5	7	-82.0	-42.7
8	-56.5	-56.5	9	-82.0	-64.2
10	-66.3	-66.3	11	-106.2	-64.2
12	-77.5	-77.5	13	-106.2	-85.1
14	-87.3	-87.3	15	-129.1	-85.1
16	-98.2	-98.2	17	-129.1	-105.7
18	-108.0	-108.0	19	-152.7	-105.7
20	-118.5	-118.5	21	-152.7	-126.0
22	-128.4	-128.4	23	-170.0	-126.0

Table 4-3 FIR discriminator ripple vs filter length N for filter case 1-4, F_1=0.15, F_2=0.35

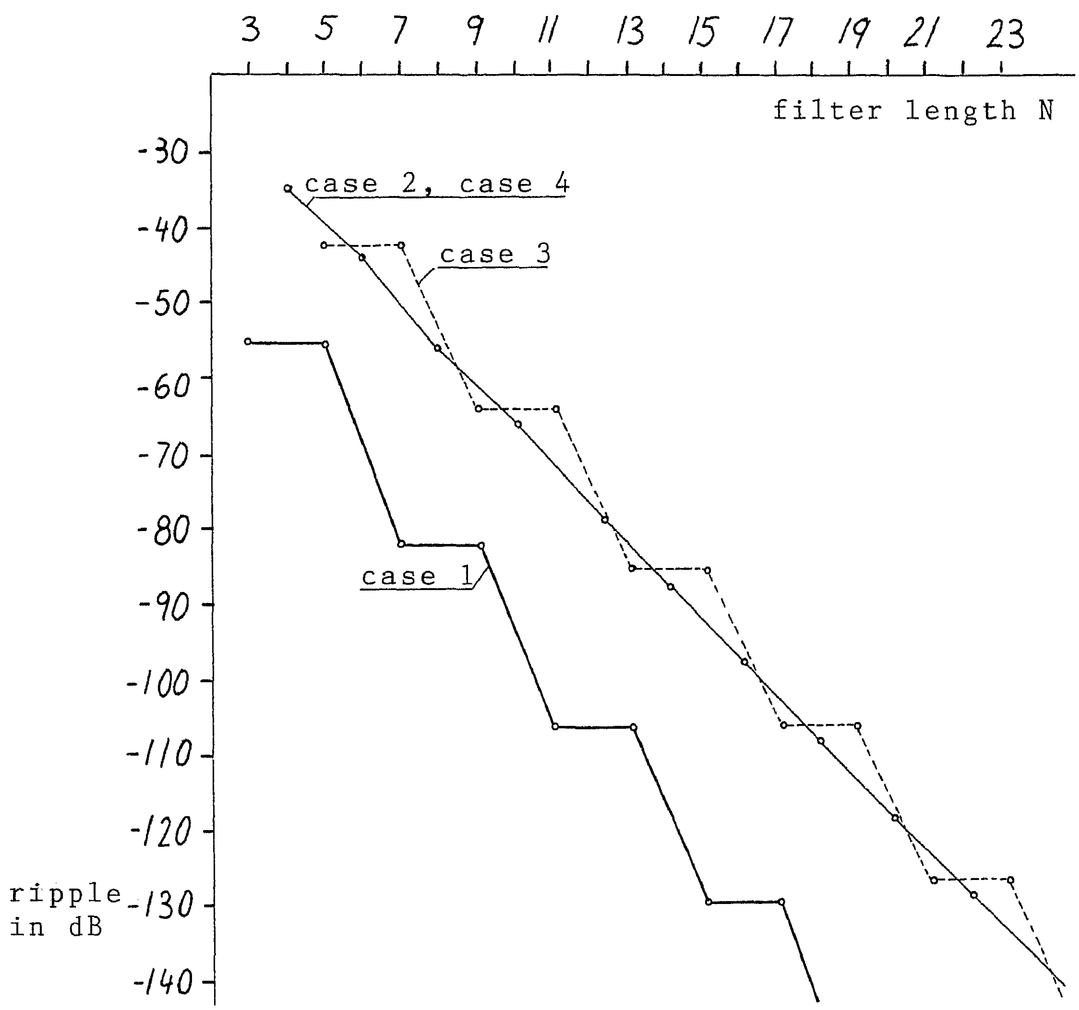

Fig. 4-5 FIR discriminator ripple vs filter length N for filter case 1-4, F_1=0.15, F_2=0.35

filter length N	ripple case 2	ripple case 4	filter length N	ripple case 1	ripple case 3
-	-	-	3	-44.6	-
4	-26.3	-26.3	5	-44.6	-30.9
6	-32.4	-32.4	7	-63.4	-30.9
8	-40.1	-40.1	9	-63.4	-44.5
10	-46.3	-46.3	11	-79.7	-44.5
12	-53.2	-53.3	13	-79.7	-57.7
14	-59.4	-59.4	15	-94.8	-57.5
16	-66.1	-66.1	17	-94.8	-70.3
18	-72.1	-72.1	19	-109.1	-70.3
20	-78.7	-78.7	21	-109.1	-82.8
22	-84.7	-84.7	23	-123.1	-82.8

Table 4-4 FIR discriminator ripple vs filter length N for filter case 1-4, F_1=0.1, F_2=0.4

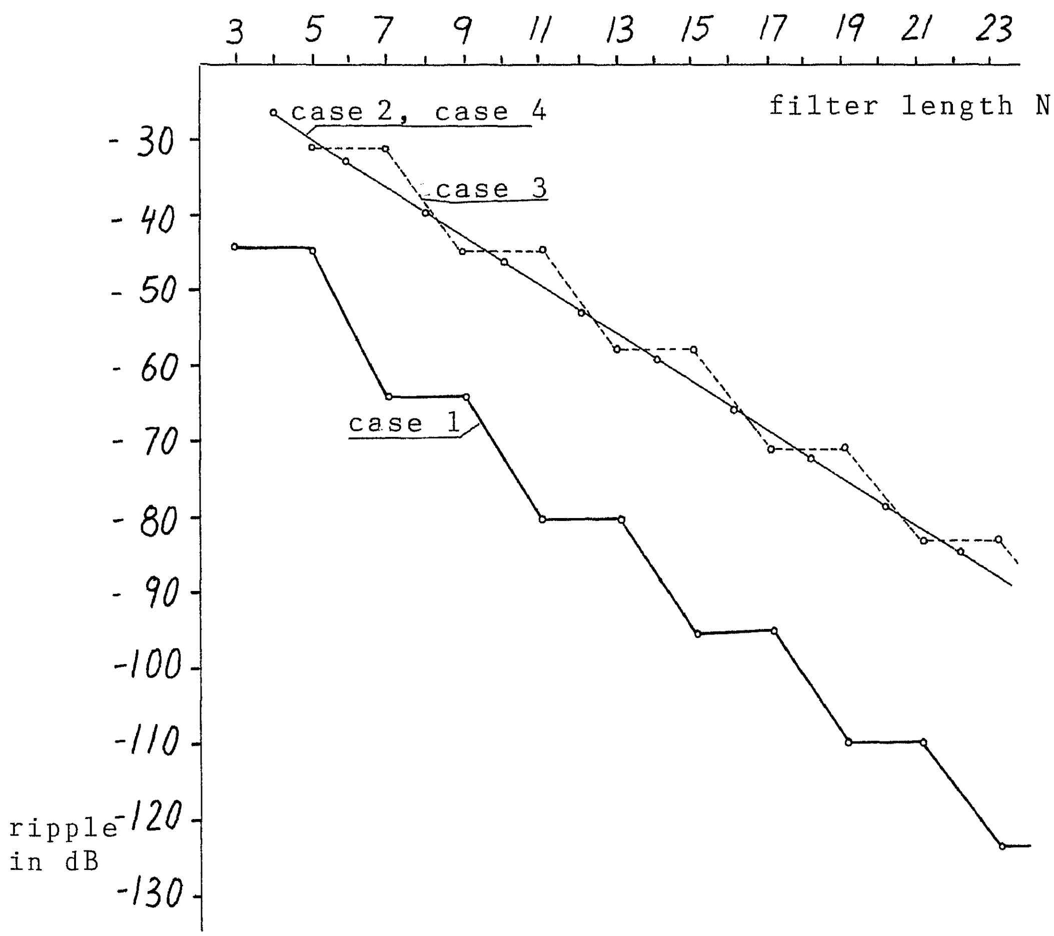

Fig. 4-6 FIR discriminator ripple vs filter length N for filter case 1-4, F_1=0.1, F_2=0.4

All the following results are drawn for the center frequency F_C=0.25. The calculations show the following results:

- the larger the bandwidth of the discriminator, the worse the resulting approximation and the larger the discriminator ripple, as expected.

- FIR discriminators with even length and positive (class 2) or negative coefficient symmetry (class 4) have the same ripple for the same filter length N.

- FIR discriminators with even length (class 1 and 3) do not show necessarily an improvement of discriminator ripple with two additional coefficients. Only every other couple of new coefficients result in an improvement in FIR discriminator ripple.

- FIR discriminators with odd length (class 2 and 4) show always an improvement of discriminator ripple with two additional coefficients.

- case 1 (odd filter length and positive symmetry of the coefficients) is the best approach for F_C=0.25 at different bandwidths.

4.2.2 Dependence of FIR Discriminator Ripple on Discriminator Bandwidth

To design FIR discriminators, it is useful to know the interdependence between FIR discriminator ripple and FIR discriminator bandwidth F_{bw}. The discriminator bandwidth is defined as the difference between the two limits of optimization F_1 and F_2.

$$F_{bw} = F_2 - F_1 \qquad (4\text{-}3)$$

The following evaluated filters have in common a center frequency of F_c=0.25. Assuming symmetry around F_c, F_1 and F_2 can be expressed by the discriminator bandwidth F_{bw}.

$$F_1 = F_c - \frac{F_{bw}}{2} \qquad (4\text{-}4)$$

$$F_2 = F_c + \frac{F_{bw}}{2} \qquad (4\text{-}5)$$

The next table gives the values of F_1 and F_2 for the discriminators with different discriminator bandwidths F_{bw}.

F_{bw}	F_1	F_2
0.025	0.2375	0.2625
0.05	0.225	0.275
0.1	0.2	0.3
0.15	0.175	0.325
0.2	0.15	0.35
0.25	0.125	0.375
0.3	0.1	0.4
0.35	0.075	0.425
0.4	0.05	0.45
0.45	0.025	0.475

Table 4-5 Discriminator bandwidth and limits of optimized frequencies

The following tables and graphs show the dependence between FIR discriminator bandwidth and FIR discriminator ripple. The chosen filter length for case 1 and case 3 are N=5, N=7, N=9, and N=11. For the other cases the chosen filter lengths are N=4, N=6, N=8, and N=10. All filters have the center frequency at F_c=0.25.

F_{bw}	filter length N 3	5	7	9
0.025	-100.0	-100.0	-150.0	-150.0
0.05	-91.8	-91.8	-143.4	-143.4
0.1	-73.7	-73.7	-113.2	-113.2
0.15	-63.0	-63.0	-94.9	-94.4
0.2	-55.5	-55.5	-82.0	-82.0
0.25	-49.5	-49.5	-71.9	-71.9
0.3	-44.6	-44.6	-63.4	-63.4
0.35	-40.4	-40.4	-56.0	-56.0
0.4	-36.7	-36.7	-49.3	-49.3
0.45	-33.4	-33.4	-43.2	-43.2
0.475	-31.8	-31.8	-40.2	-40.2

Table 4-6 FIR discriminator ripple in dB for different discriminator bandwidths for case 1 filter (N=odd, positive symmetry, F_c=0.25)

F_{bw}	filter length N 4	6	8	10
0.025	-72.2	-100.0	-125.0	-161.8
0.05	-60.1	-82.4	-106.4	-128.5
0.1	-47.9	-63.3	-82.0	-97.7
0.15	-40.4	-52.4	-67.3	-79.5
0.2	-34.9	-44.5	-56.5	-66.3
0.25	-30.3	-38.0	-47.8	-55.6
0.3	-26.3	-32.4	-40.1	-46.3
0.35	-22.6	-27.3	-33.1	-37.8
0.4	-19.1	-22.4	-26.4	-28.7
0.45	-15.6	-17.4	-19.6	-21.4
0.475	-14.5	-14.8	-15.9	-16.9

Table 4-7 FIR discriminator ripple in dB for different discriminator bandwidths for case 2 filter (N=even, positive symmetry, F_c=0.25)

F_{bw}	filter length N 5	7	9	11
0.025	-100.0	-100.0	-200.0	-200.0
0.05	-79.7	-79.9	-126.4	-126.4
0.1	-61.5	-61.5	-96.0	-96.0
0.15	-50.6	-50.6	-77.6	-77.6
0.2	-42.7	-42.7	-64.2	-64.2
0.25	-36.4	-36.4	-53.6	-53.6
0.3	-30.9	-30.9	-44.5	-44.5
0.35	-26.0	-26.0	-36.1	-36.1
0.4	-21.3	-21.3	-28.5	-28.5
0.45	-18.8	-16.8	-20.6	-20.6
0.475	-14.7	-14.7	-16.5	-16.5

Table 4-8 FIR discriminator ripple in dB for different discriminator bandwidths for case 3 filter (N=odd, negative symmetry, F_c=0.25)

F_{bw}	filter length N 4	6	8	10
0.025	-72.4	-100.6	-133.0	-150.0
0.05	-60.1	-82.4	-108.0	-128.5
0.1	-47.9	-63.3	-82.0	-97.7
0.15	-40.4	-52.4	-67.3	-79.5
0.2	-34.9	-44.5	-56.5	-66.3
0.25	-30.3	-38.0	-47.7	-55.6
0.3	-26.3	-32.4	-40.1	-46.3
0.35	-22.6	-27.3	-33.1	-37.8
0.4	-19.1	-22.4	-26.4	-29.7
0.45	-15.6	-17.4	-19.6	-21.4
0.475	-13.8	-14.8	-15.9	-16.9

Table 4-9 FIR discriminator ripple in dB for different discriminator bandwidths for case 1 filter (N=even, negative symmetry, F_c=0.25)

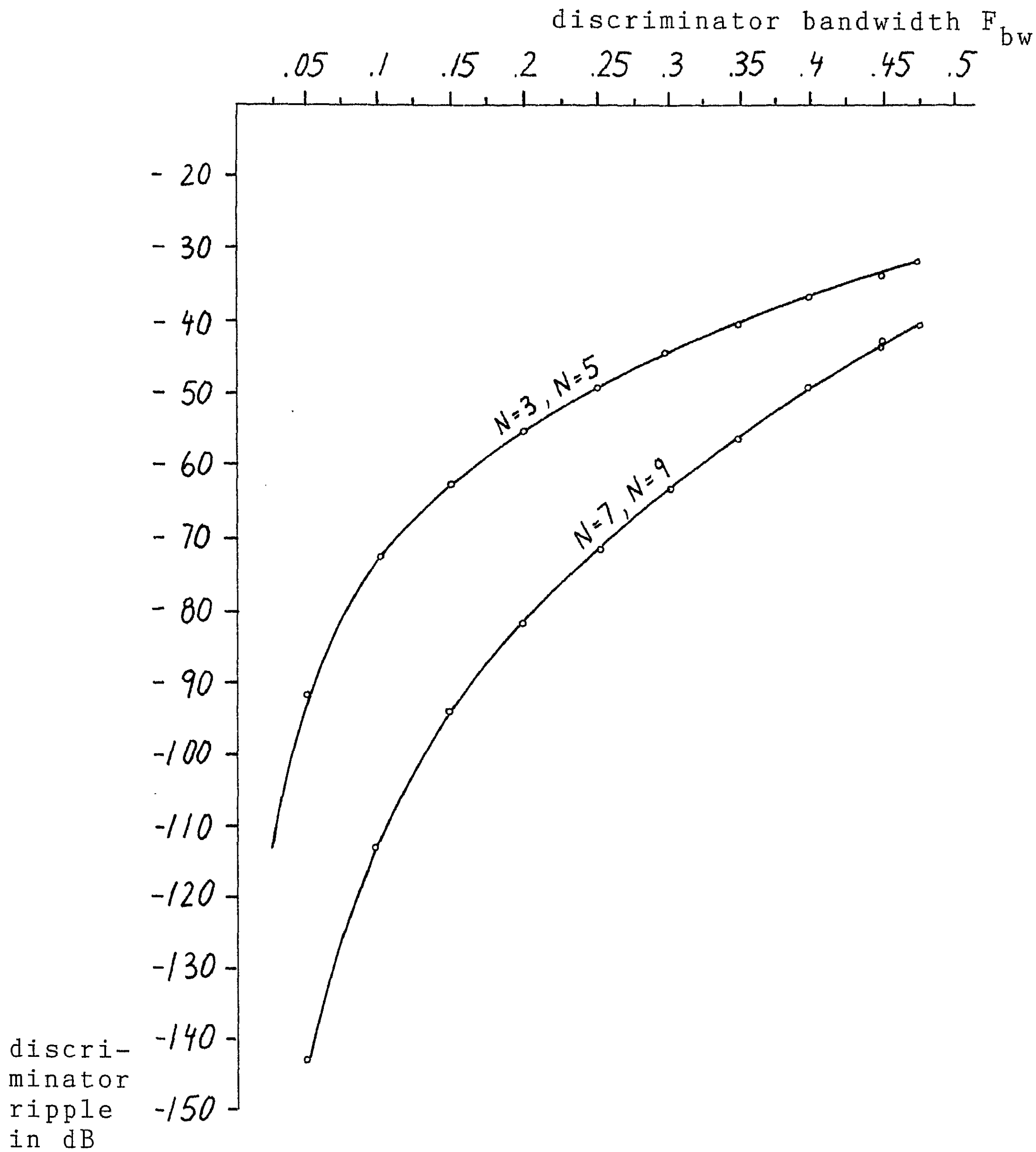

Fig. 4-7 FIR discriminator ripple for different discriminator bandwidths for case 1 filter (N=odd, positive symmetry, F_C=0.25)

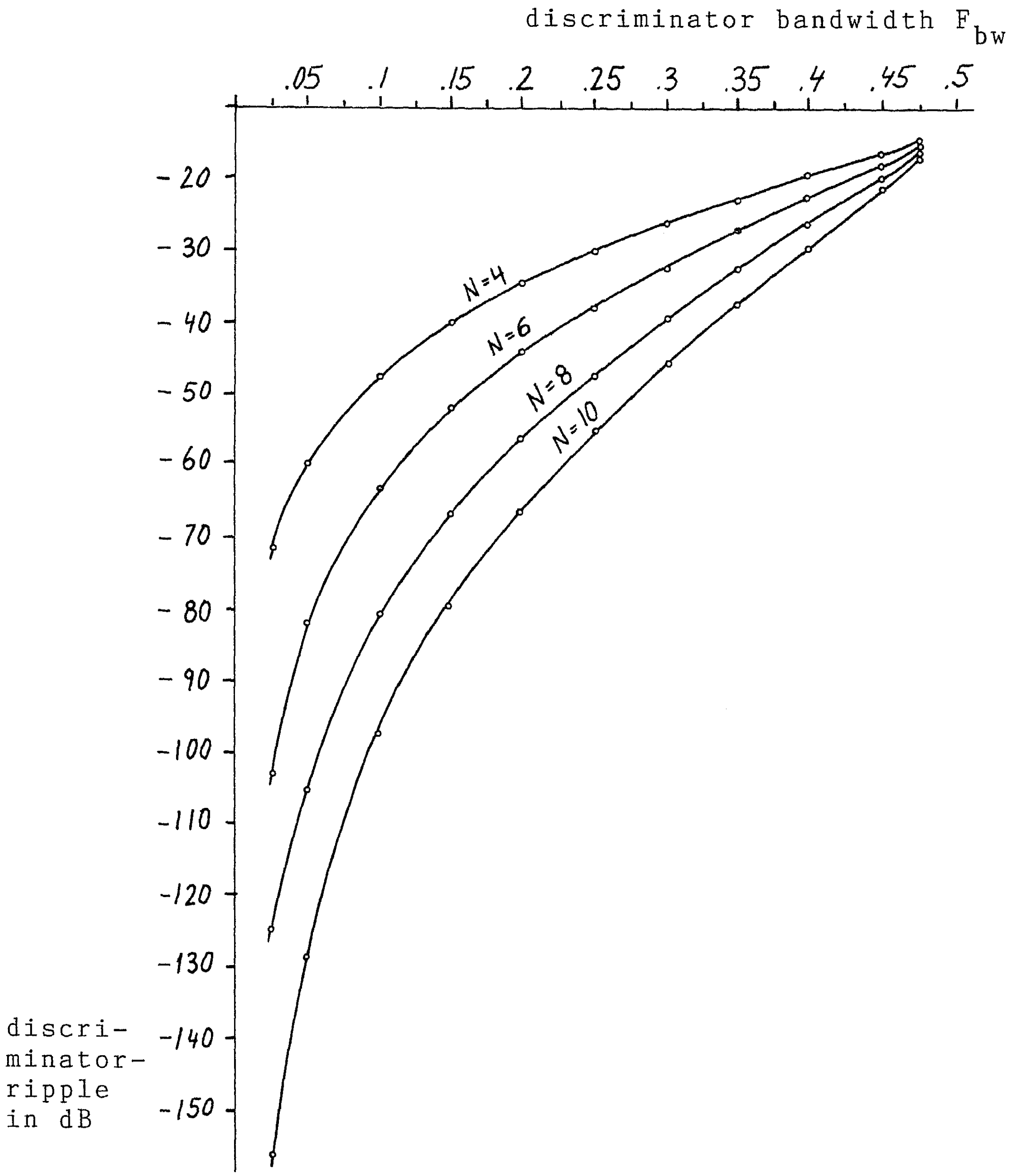

Fig. 4-8 FIR discriminator ripple for different discriminator bandwidths for case 2 filter (N=even, positive symmetry, F_c=0.25)

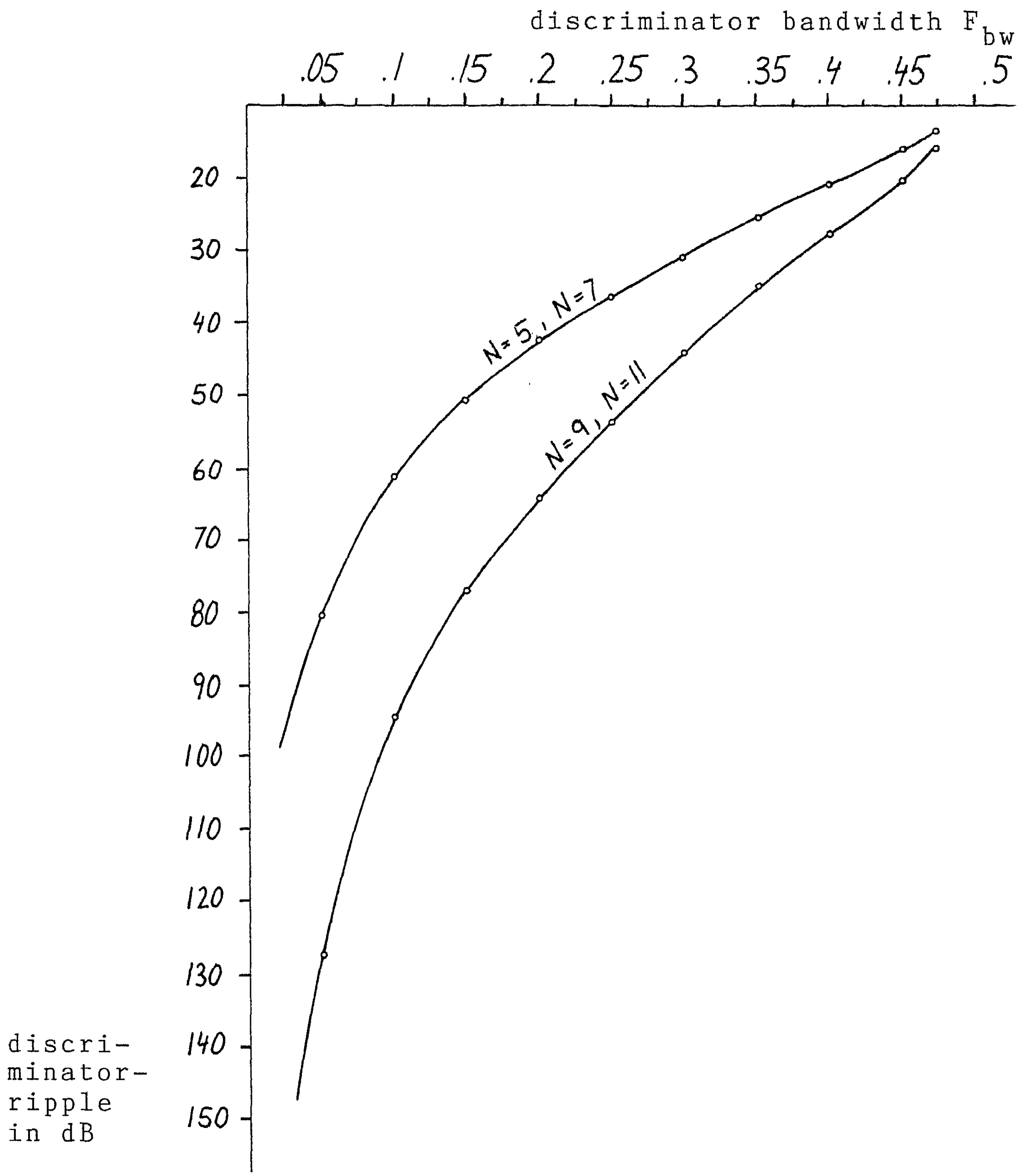

Fig. 4-9 FIR discriminator ripple for different discriminator bandwidths for case 3 filter (N=odd, negative symmetry, F_C=0.25)

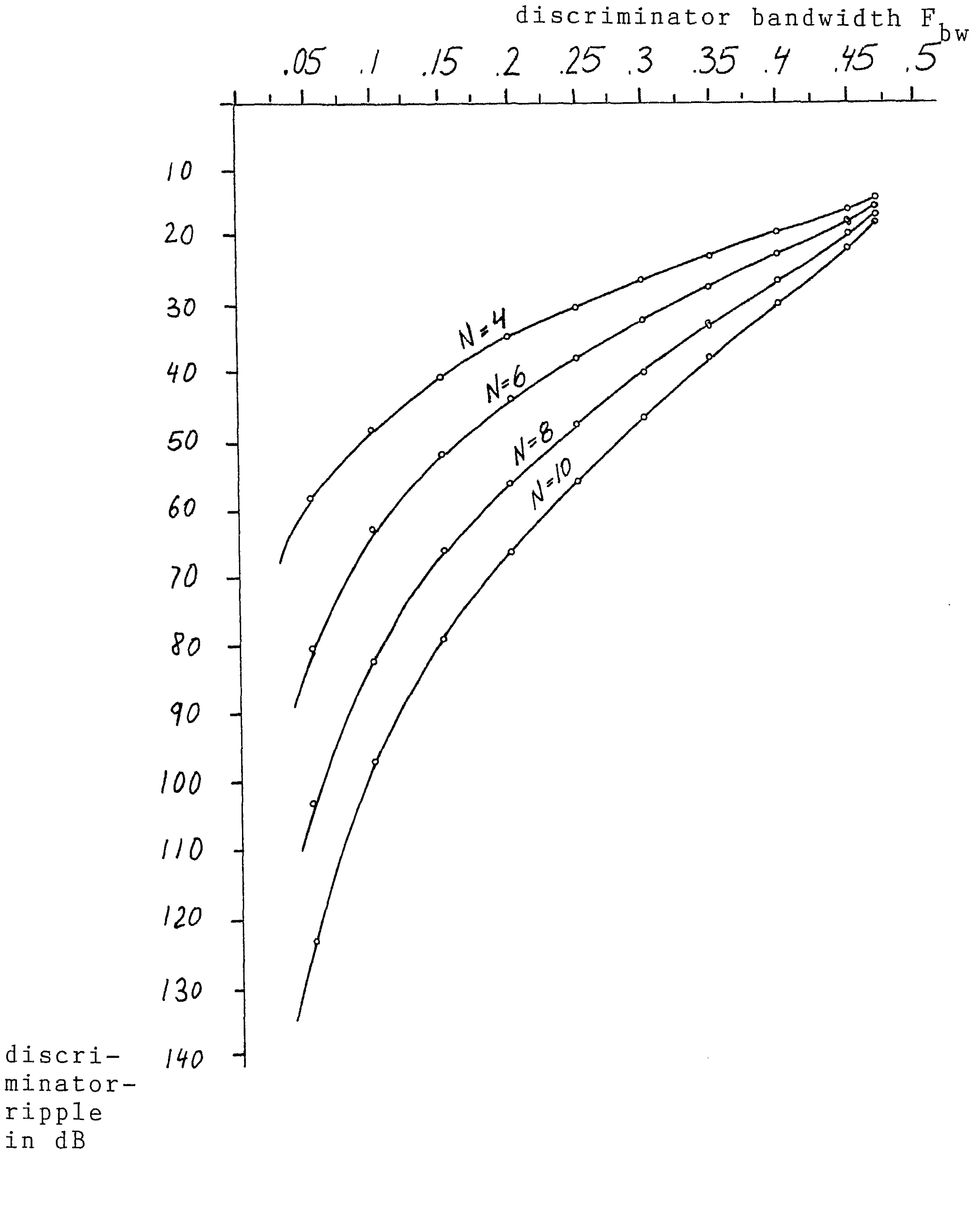

Fig. 4-10 FIR discriminator ripple for different discriminator bandwidths for case 4 filter (N=even, negative symmetry, F_C=0.25)

Preceding figures and tables show the interdependence between FIR discriminator bandwidth and FIR discriminator ripple for filters of different cases and discriminator lengths N. The charts allow the following conclusions for F_C=0.25 and a discriminator with a slope of s = 1:

- For a certain bandwidth, the resulting FIR disciminator ripple is dependent on the chosen filter case.

- The narrower the bandwidth, the better the approximation. This result was already derived from the charts discussed earlier. But these figures give values for the improvement of FIR discriminator ripple.

- identical characteristics are obtained for the following filters:
  filter class 1: N=3 identical with N=5
  filter class 1: N=5 identical with N=7
  filter class 3: N=5 identical with N=7
  filter class 3: N=9 identical with N=11

- conclusion: for odd length of N two additional coefficients do not mean automatically an improvement in filter ripple.

4.2.3 Shifted Center Frequency of the FIR Discriminator

All experiments up till now assumed a center frequency $F_c = 0.25$. In many practical cases this choice is favorable. The reason is the symmetry of frequency modulation relative to F_c and the requirement to sample the FM signal with frequency of at least two times the highest frequency component in the FM signal.

This part of the thesis deals with a shifted carrier frequency F_c. The folowing tables and figures are the result of computations of filters with a shifted carrier frequency. All calculated filters have in common a constant discriminator bandwidth F_{bw} of 0.2. The frequency shift F_{sh} is defined

$$F_{sh} = 0.25 - F_c \qquad (4\text{-}6)$$

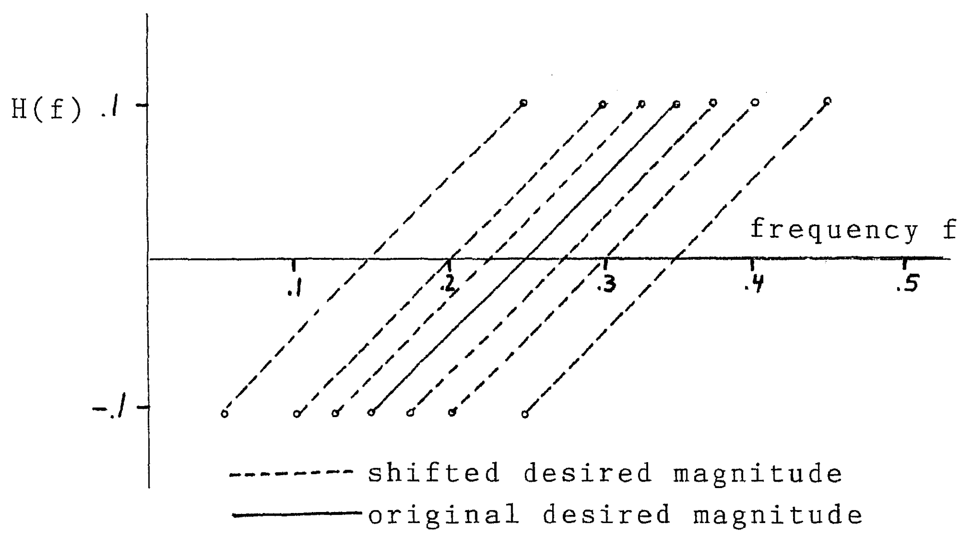

Fig. 4-11 Original and shifted desired magnitudes for FIR discriminators

Figure 4-11 shows the desired magnitude for original and shifted center frequencies between F_c=0.15 and F_c=0.35. The slope (s=1) and the bandwidth (F_{bw}=0.2) is constant for this set of calculated discriminators.

The objective of this experiment is to find the interdependence between shifted center frequency and ripple. The results are shown below. The tables and figures show the optimized frequency bands and the resulting discriminator ripple. The chosen filter in this case is a filter with odd filter length and negative symmetry of the coefficients.

shift	f_1	f_2	f_{bw}	f_c
0.0	0.15	0.35	0.2	0.25
0.025	0.125	0.325	0.2	0.225
0.05	0.1	0.3	0.2	0.2
0.1	0.05	0.25	0.2	0.15

Table 4.10 Shifted center frequencies

filter length N	F_{sh}			
	0.0	0.025	0.05	0.1
5	-42.7	-37.8	-33.6	-26.9
7	-42.7	-41.7	-39.2	-31.6
9	-64.2	-55.2	-48.6	-36.6
11	-64.2	-61.5	-55.7	-41.2
13	-85.1	-72.8	-63.8	-45.8
15	-85.1	-80.1	-71.2	-50.3
17	-105.7	-90.3	-78.8	-54.7
19	-105.7	-98.4	-86.2	-59.0
21	-126.0	-107.7	-93.7	-63.4
23	-126.0	-126.0	-116.0	-67.7

Table 4-11 Discriminator ripple of case 3 filters for different shifted centerfrequencies (s=1, F_{bw}=0.2)

This table and the following graph show, that any shift of the center frequency F_c=0.25 is equivalent to an increase of the filter ripple. The following chart shows the results of a shifted center frequency with discrete shifted frequencies F_{sh}. The results derived from figure 4-12 are only valid for filter class 3.

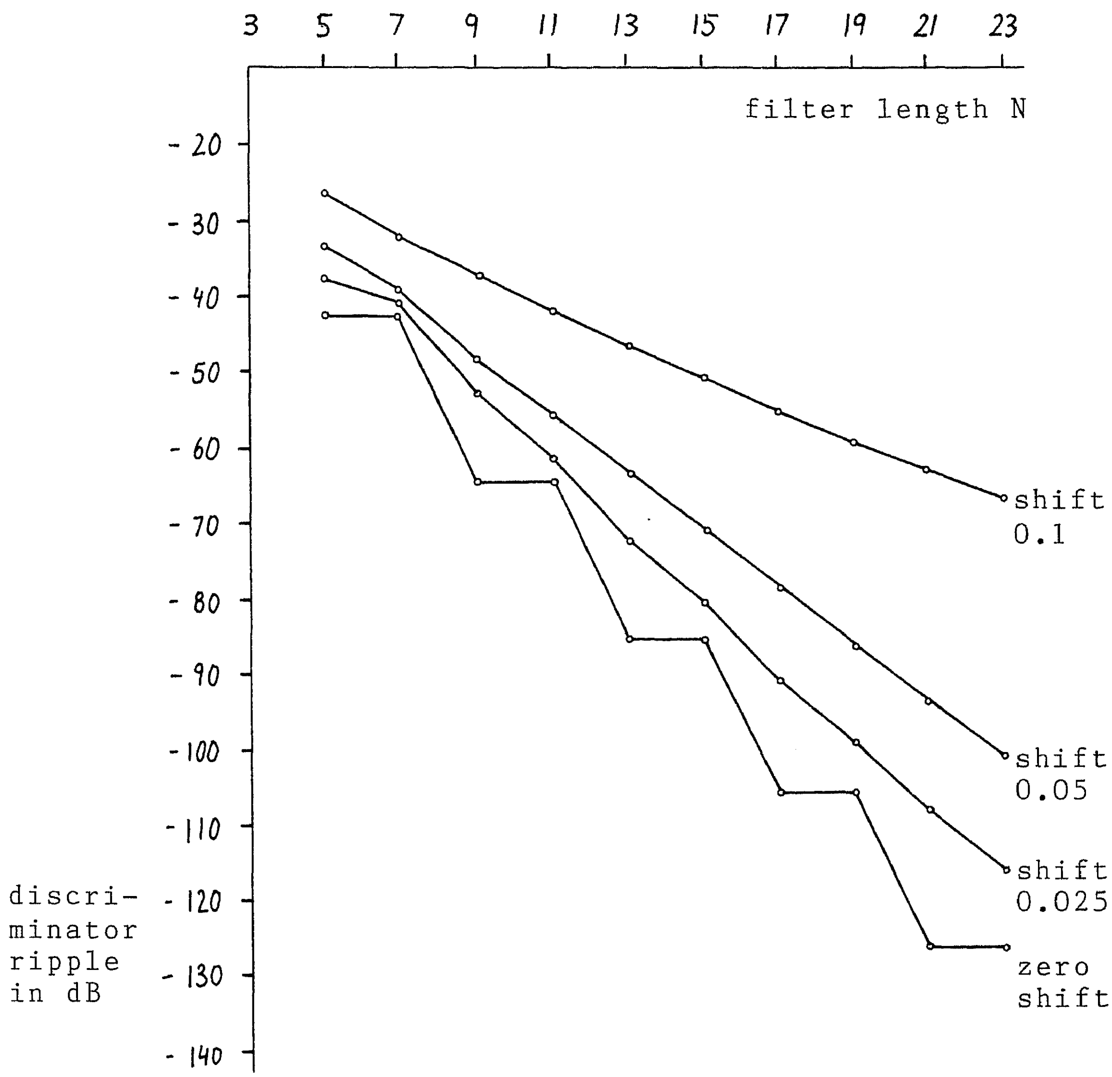

Fig. 4-12 Discriminator ripple vs. filter length N for original discriminator and shifted center frequencies (class 3 filter, s=1)

Calculations and evaluations of FIR discriminators with shifted center frequencies show that it is necessary to distinguish between all four filter cases. It is desirable to find the degredation or improvement of discriminator ripplefoe any center frequency F_C. This is the reason, why each filter case has to be calculated and evaluated seperately.

The following items are presented for eachfilter case:

- coefficients of selected filters
- frequency response of selected filters
- discriminator ripple of selected filters
- discriminator ripple for different filter-lengths and center frequencies.

FIR DISCRIMINATOR, CASE 1 (ODD N, POSITIVE SYMMETRY)

The chosen filter length N for this filter case is 9.

C	fc = 0.2	fc = 0.25	fc = 0.3
C5	0.03617845	-0.00000024	-0.03617878
C4 = C6	-0.08340978	-0.09172028	-0.08340996
C3 = C7	-0.00947005	-0.00000014	0.00946985
C2 = C8	-0.00100986	-0.00415503	-0.00100997
C1 = C9	-0.00259275	-0.00000005	0.00259271

Cn = C(n)

Table 4-12 Coefficients of selected filters for shifted center frequency (case 1)

The frequency responses of these discriminators are presented in Fig. 4-13. Fig 4-14 shows the magnitude of the discriminator ripple for the selected filters.

The table shows the filter ripple for case 1 filters of different filter length N and shifted center frequencies.

Fc	filter length N					
	3	5	7	9	11	13
0.1	-33.0	-38.0	-41.4	-43.8	-45.7	-47.2
0.125	-35.7	-42.7	-48.6	-53.4	-57.8	-61.7
0.15	-38.4	-46.7	-54.7	-61.4	-67.6	-73.5
0.175	-41.3	-50.1	-60.2	-68.4	-76.4	-83.9
0.2	-44.8	-52.9	-65.8	-74.7	-86.6	-93.3
0.225	-49.5	-54.7	-73.3	-79.8	-93.3	-101.8
0.25	-55.5	-55.5	-82.0	-82.0	-106.2	-106.1
0.275	-49.5	-54.7	-73.3	-79.8	-93.3	-101.8
0.3	-44.8	-52.9	-65.8	-74.7	-86.6	-93.3
0.325	-41.3	-50.1	-60.2	-68.4	-76.4	-83.9
0.35	-38.4	-46.7	-54.7	-61.4	-67.6	-73.5
0.375	-35.7	-42.7	-48.6	-53.4	-57.8	-61.7
0.4	-33.0	-38.0	-41.4	-43.8	-45.7	-47.2

Table 4-13 Discriminator ripple for center frequencies of the discriminator between 0.1 and 0.4 for case 1 filter, s = 1, F_{bw} = 0.2

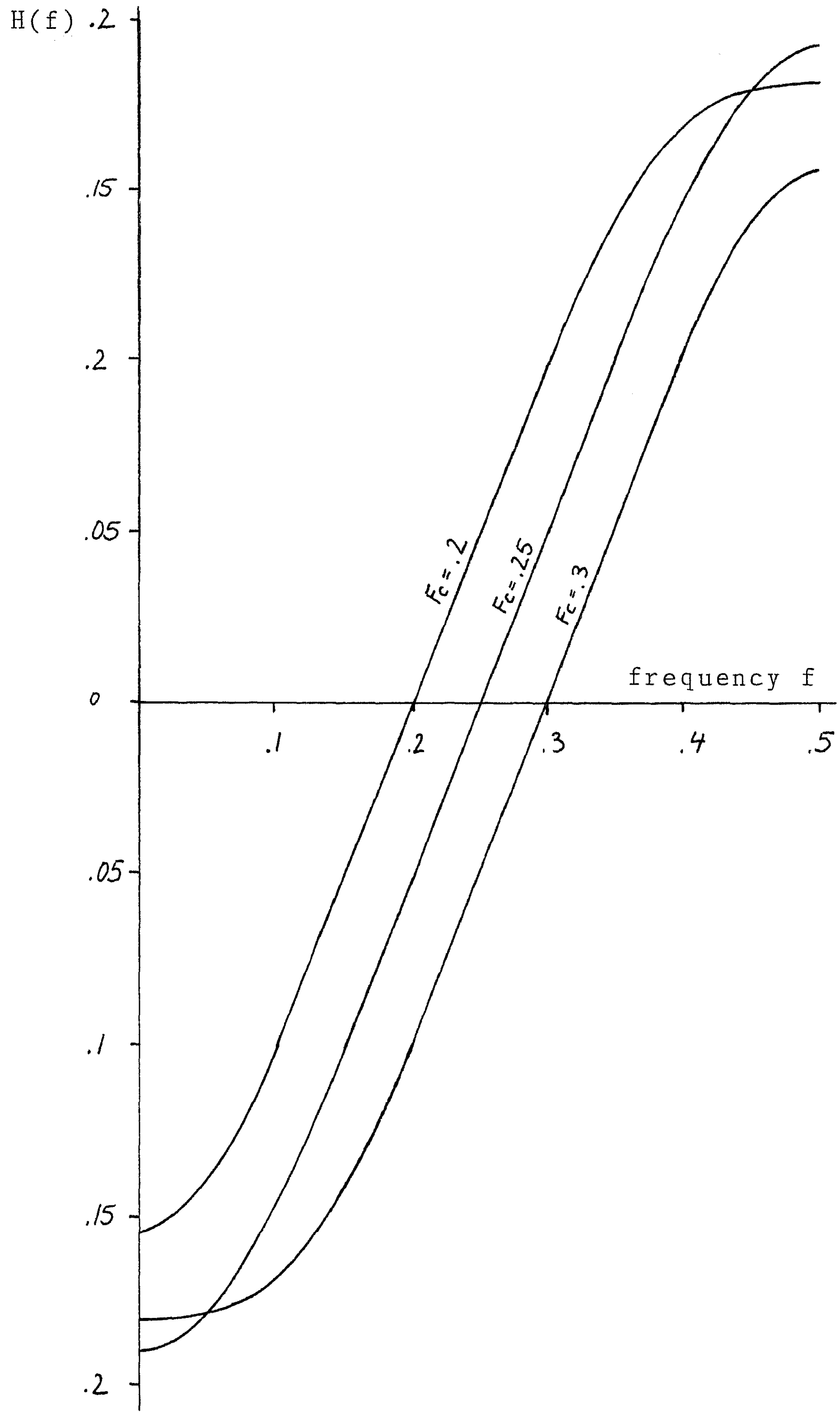

Fig 4-13 Frequency responses of selected filters (filter case 1)

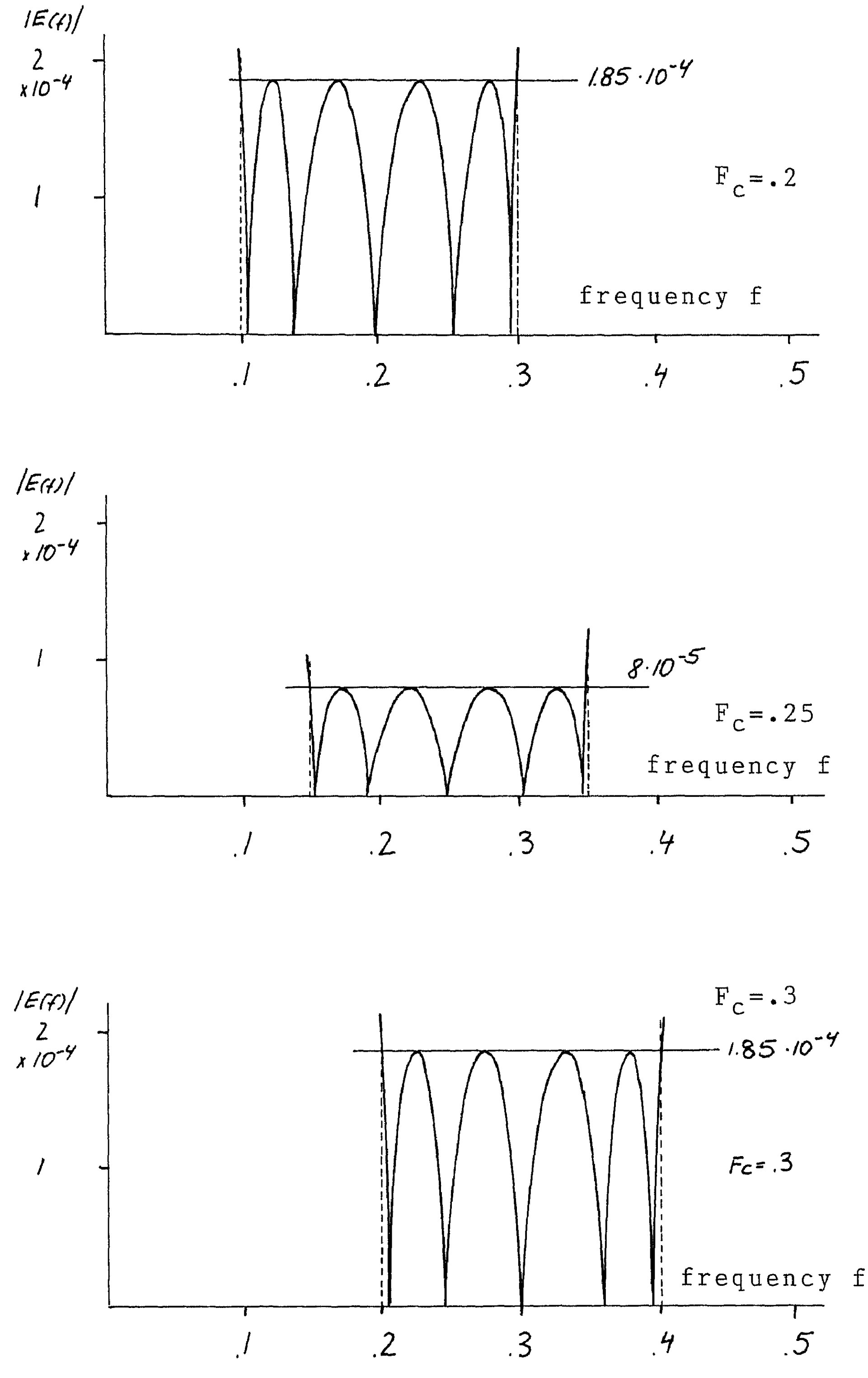

Fig 4-14 Magnitude of filter ripple of selected filters (case 1)

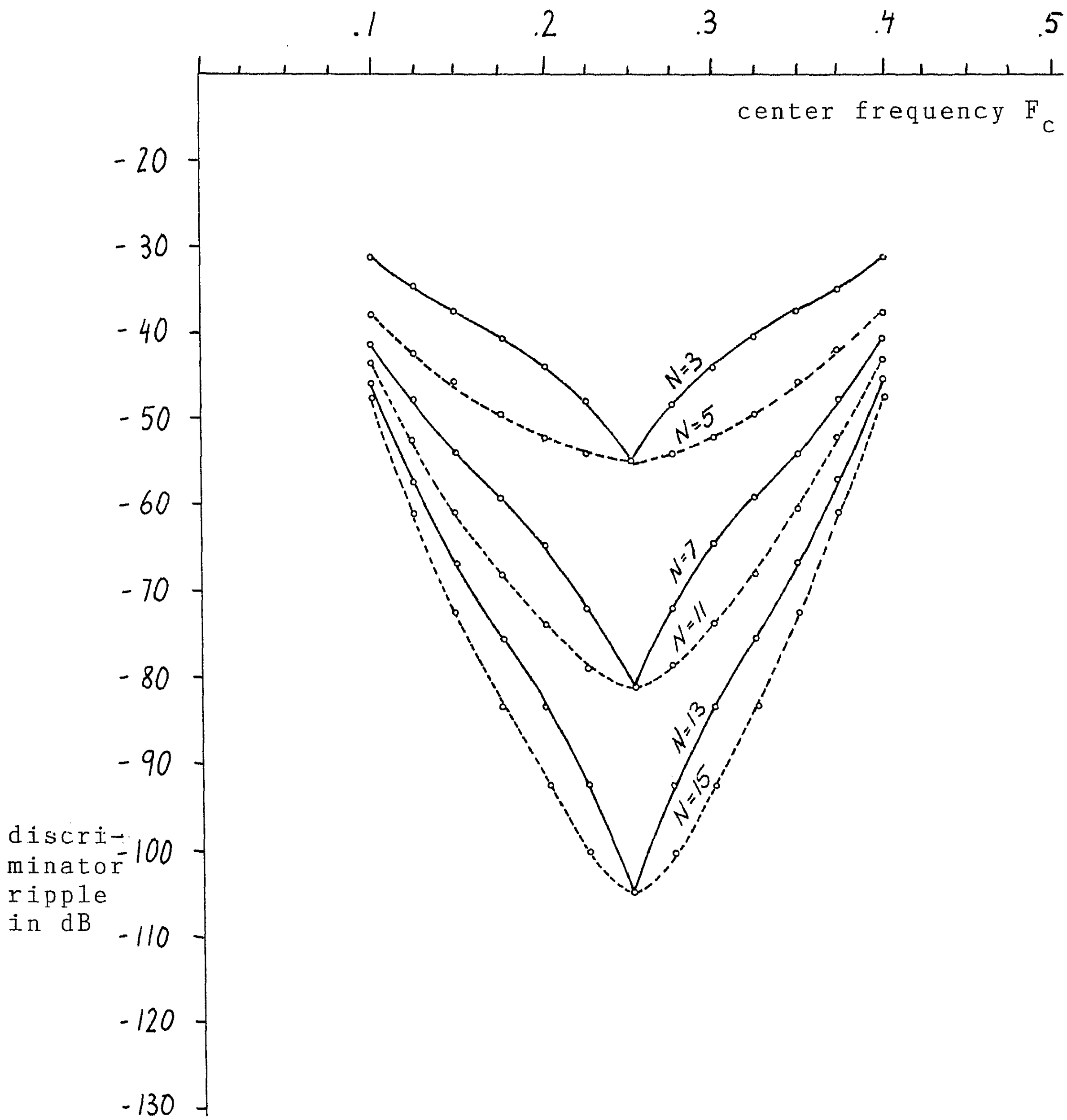

Fig. 4-15 Discriminator ripple vs. shifted center frequency F_c for case 1 filters ($s = 1$, $F_{bw} = 0.2$)

FIR DISCRIMINATOR, CASE 2 (EVEN N, POSITIVE SYMMETRY)

The chosen filter length N for this filter case is 8.

C	fc = 0.15	fc = 0.2	fc = 0.25	fc = 0.3
C4 = C5	0.0198361	-0.0146140	-0.0507390	-0.1688000
C3 = C6	-0.0887728	-0.0685610	-0.0691100	-0.1332000
C2 = C7	0.0232419	0.0115700	0.0081768	-0.0333900
C1 = C8	-0.0135686	-0.0071039	-0.0119500	-0.0313800

Cn = C(n)

Table 4-14 Coefficients of selected filters for shifted center frequency (case 2)

The frequency responses of these discriminators are presented in Fig. 4-16. Fig 4-17 shows the magnitude of the discriminator ripple for the selected filters.

The table shows the filter ripple for case 2 filters of different filter length N and shifted frequencies.

Fc	filter length N					
	4	6	8	10	12	14
0.1	-35.5	-38.3	-41.7	-44.0	-45.9	-47.4
0.125	-38.9	-43.2	-49.2	-54.0	-58.3	-62.2
0.15	-44.2	-46.7	-56.3	-62.2	-68.6	-74.4
0.175	-46.4	-48.6	-63.8	-68.9	-78.2	-85.0
0.2	-41.6	-48.5	-71.0	-72.7	-89.3	-93.4
0.225	-37.8	-46.9	-62.5	-71.1	-86.2	-94.2
0.25	-34.9	-44.5	-56.5	-66.3	-77.5	-87.3
0.275	-32.5	-41.5	-51.5	-60.5	-69.9	-78.7
0.3	-30.3	-38.3	-46.5	-54.5	-62.1	-69.7
0.325	-28.1	-34.9	-41.5	-47.8	-54.0	-60.2
0.35	-25.9	-30.9	-35.8	-40.6	-45.1	-49.7
0.375	-23.3	-26.3	-29.2	-32.8	-34.6	-37.3
0.4	-22.9	-23.2	-23.3	-23.4	-23.4	-23.5

Table 4-15 Discriminator ripple for center frequencies of the discriminator between 0.1 and 0.4 for case 2 filter, s = 1, F_{bw} = 0.2

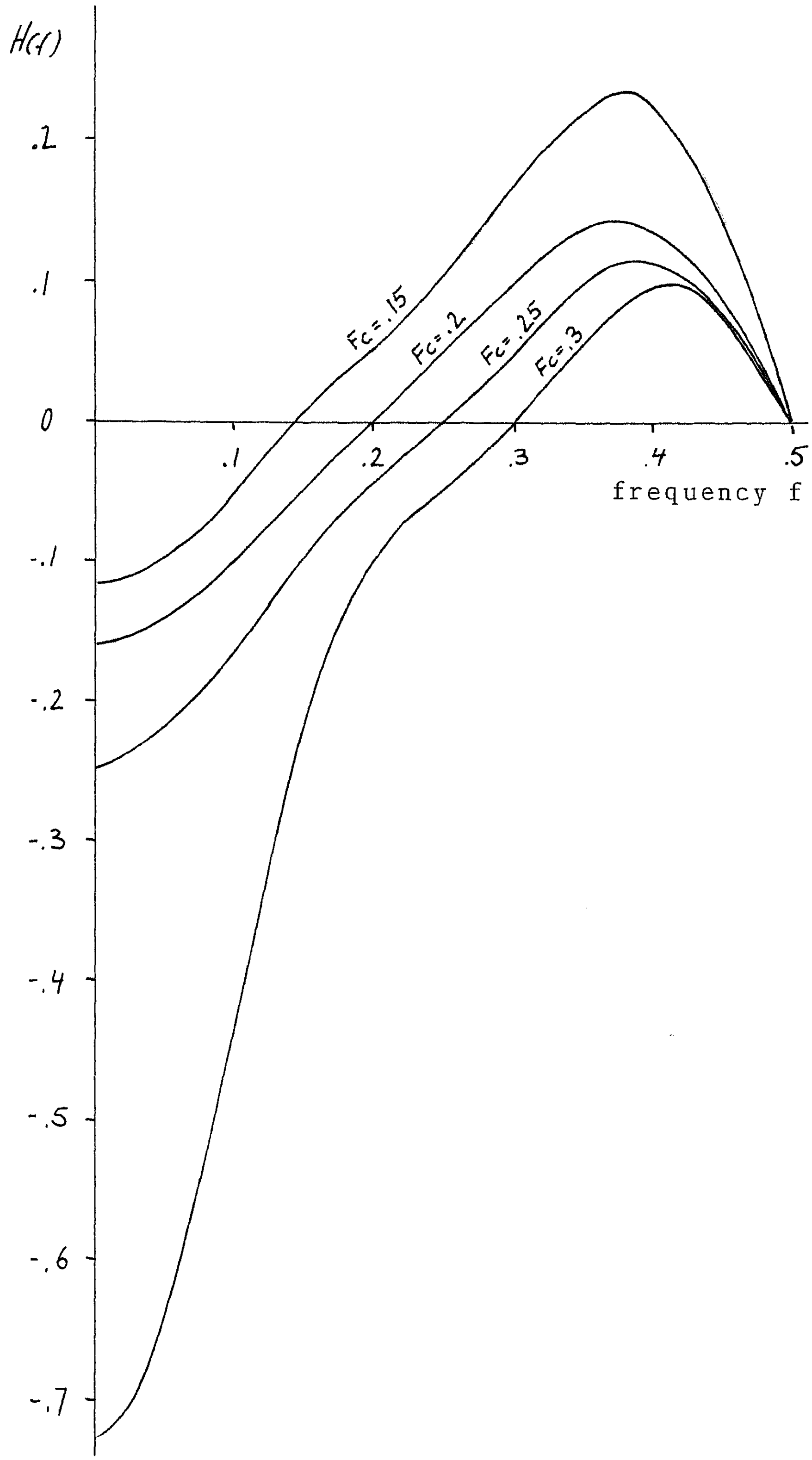

Fig 4-16 Frequency responses of selected filters (filter case 2)

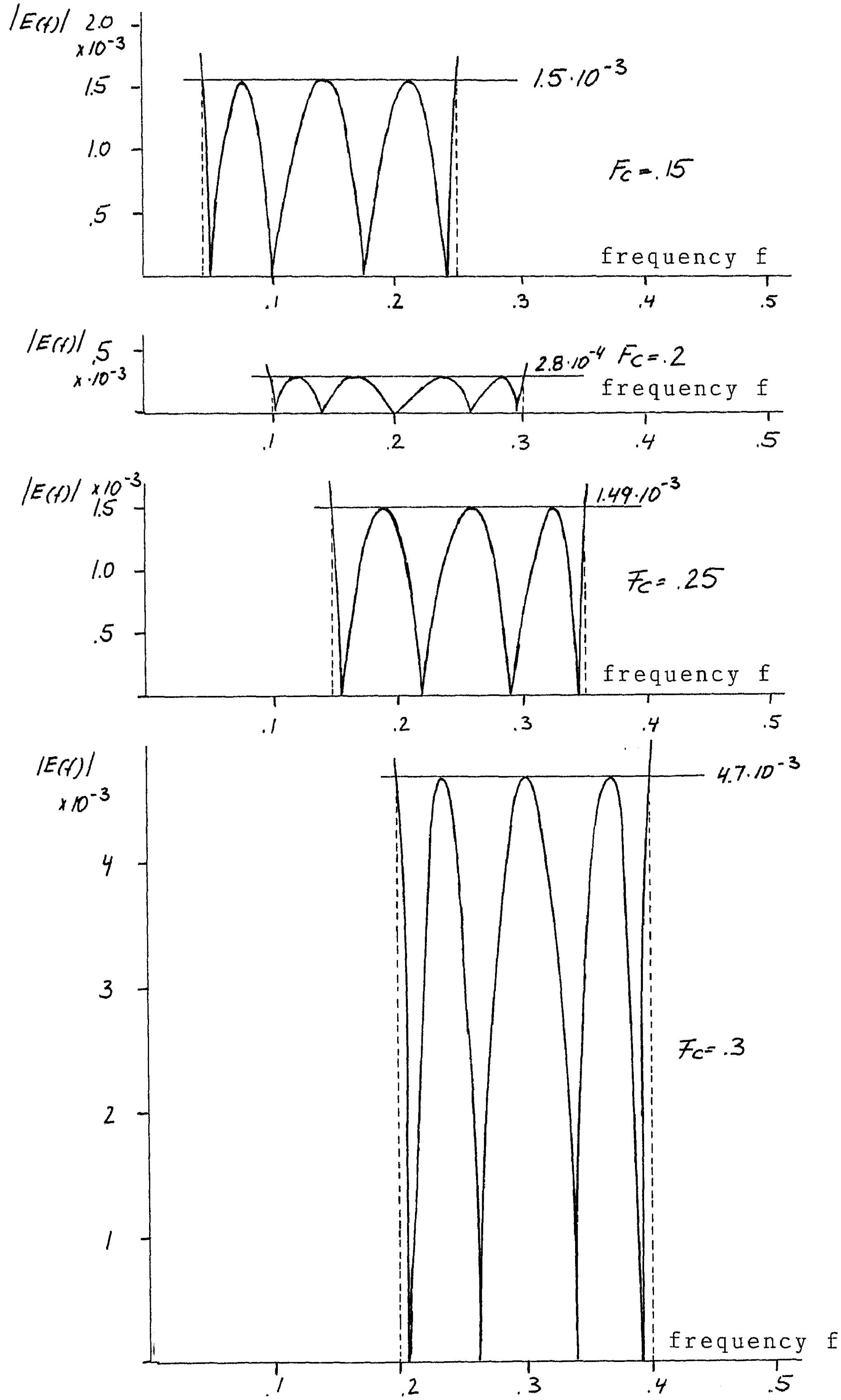

Fig 4-17 Magnitude of filter ripple of selected filters (case 2)

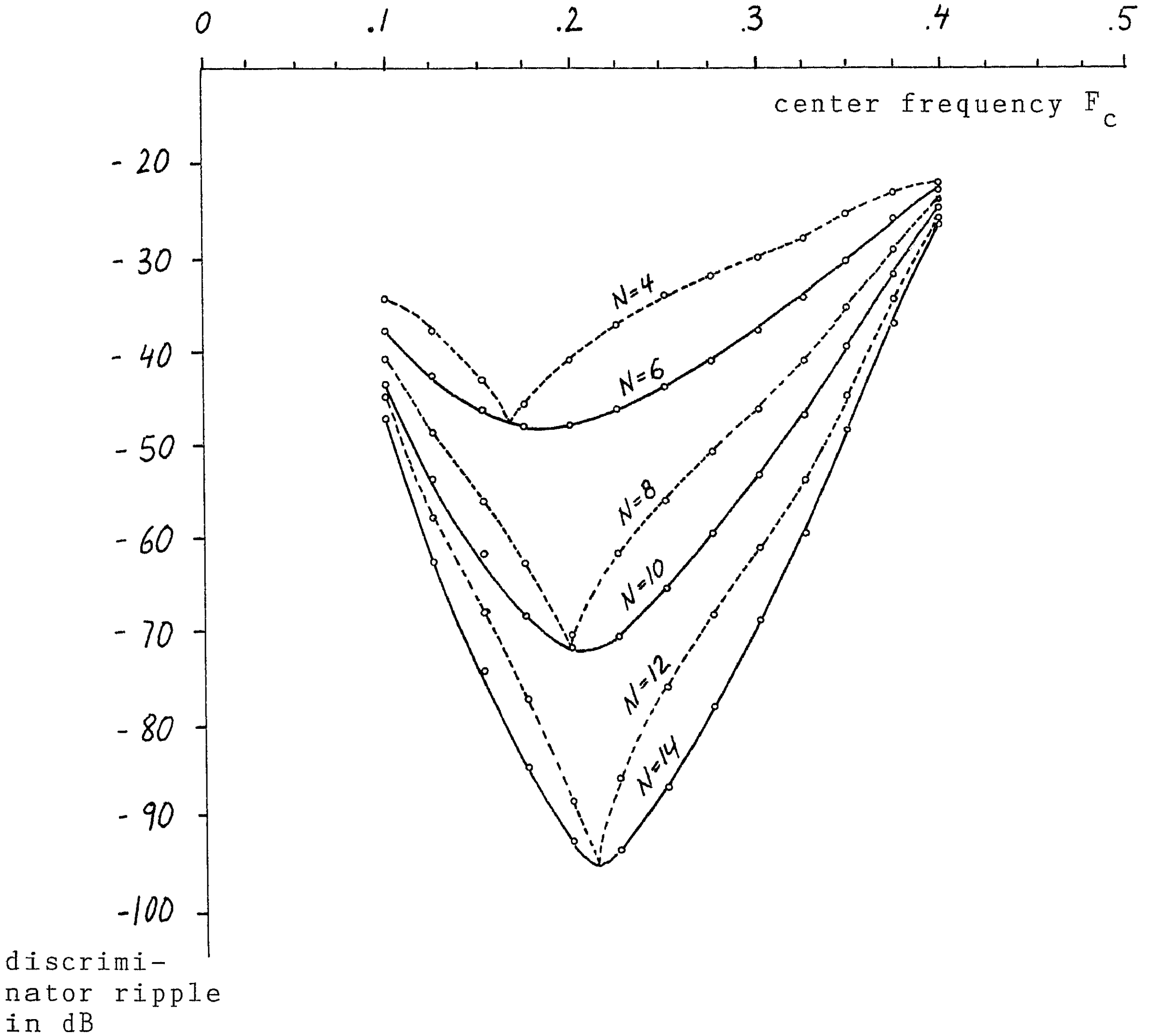

Fig. 4-18 Discriminator ripple vs. shifted center frequency F_c for case 2 filters ($s = 1$, $F_{bw} = 0.2$)

FIR DISCRIMINATOR, CASE 3 (ODD N, NEGATIVE SYMMETRY)

The chosen filter length N for this filter case is 9.

C	fc = 0.2	fc = 0.25	fc = 0.3
C5	0	0	0
C4 =-C6	0.03572285	0.00000005	-0.03572244
C3 =-C7	-0.07317143	-0.05839286	-0.07317072
C2 =-C8	0.01215681	-0.00000001	-0.01215624
C1 =-C9	-0.01890966	-0.00993923	-0.01890829

Cn = C(n)

Table 4-16 Coefficients of selected filters for shifted center frequency (case 3)

The frequency responses of these discriminators are presented in Fig. 4-19. Fig 4-20 shows the magnitude of the discriminator ripple for the selected filters.

The table shows the filter ripple for case 3 filters of different filter length N and shifted center frequencies.

Fc	filter length N					
	3	5	7	9	11	13
0.1	-23.1	-23.3	-23.4	-23.5	-23.5	-23.5
0.125	-23.7	-26.6	-29.5	-32.3	-35.0	-37.5
0.15	-26.9	-31.6	-36.6	-41.2	-45.8	-50.3
0.175	-30.1	-35.8	-42.7	-48.9	-55.1	-61.2
0.2	-33.6	-39.2	-48.6	-55.7	-63.8	-71.2
0.225	-37.8	-41.8	-55.2	-61.5	-72.8	-80.2
0.25	-42.7	-42.7	-64.2	-64.2	-85.2	-85.2
0.275	-37.8	-41.8	-55.2	-61.5	-72.8	-80.2
0.3	-33.6	-39.2	-48.6	-55.7	-63.8	-71.2
0.325	-30.1	-35.8	-42.7	-48.9	-55.1	-61.2
0.35	-26.9	-31.6	-36.6	-41.2	-45.8	-50.3
0.375	-23.7	-26.6	-29.5	-32.3	-35.0	-37.5
0.4	-23.1	-23.3	-23.4	-23.5	-23.5	-23.5

Table 4-17 Discriminator ripple for center frequencies of the discriminator between 0.1 and 0.4 for case 3 filter, s = 1, F_{bw} = 0.2

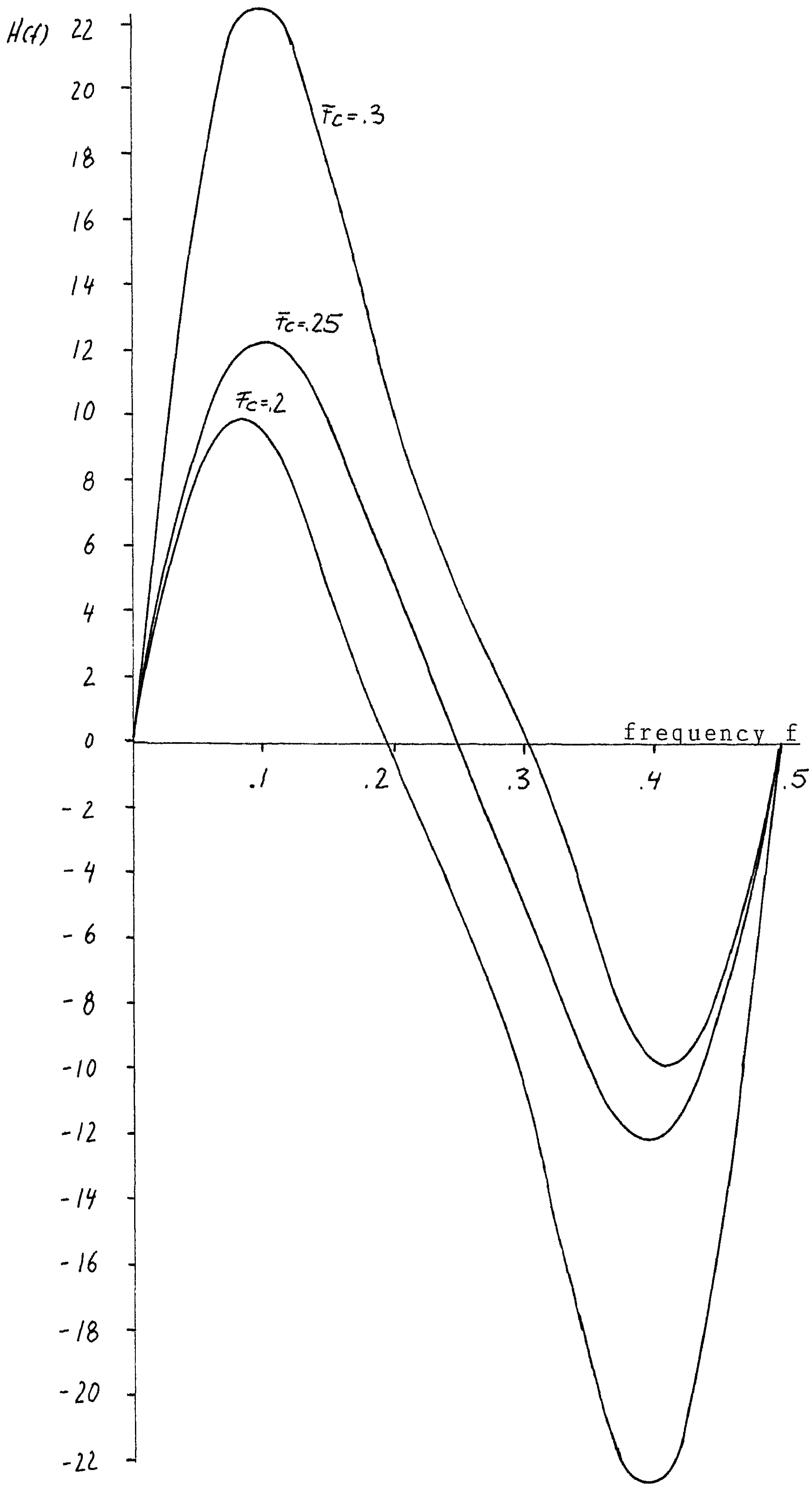

Fig 4-19 Frequency responses of selected filters (filter case 3)

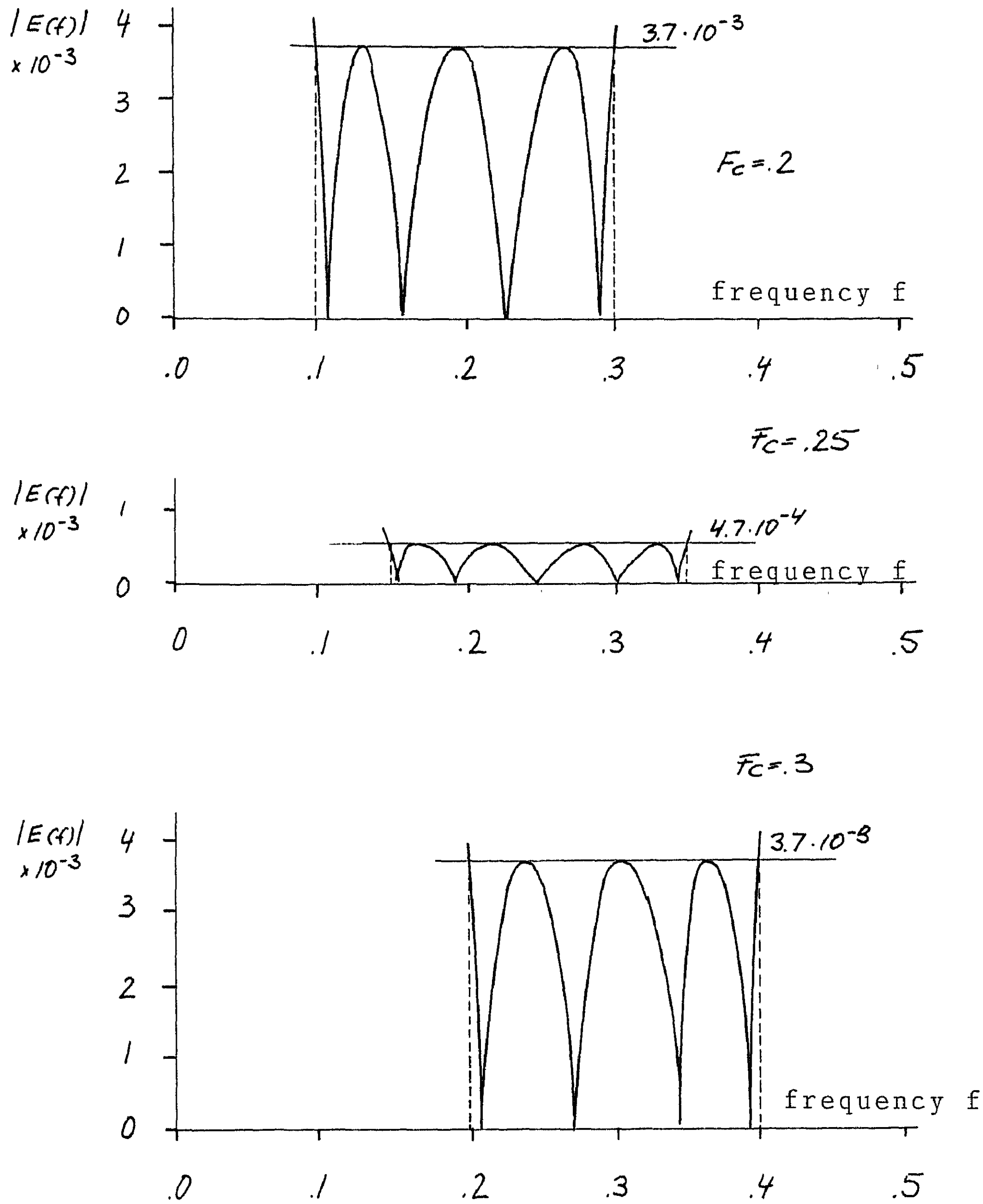

Fig 4-20 Magnitude of filter ripple of selected filters (case 3)

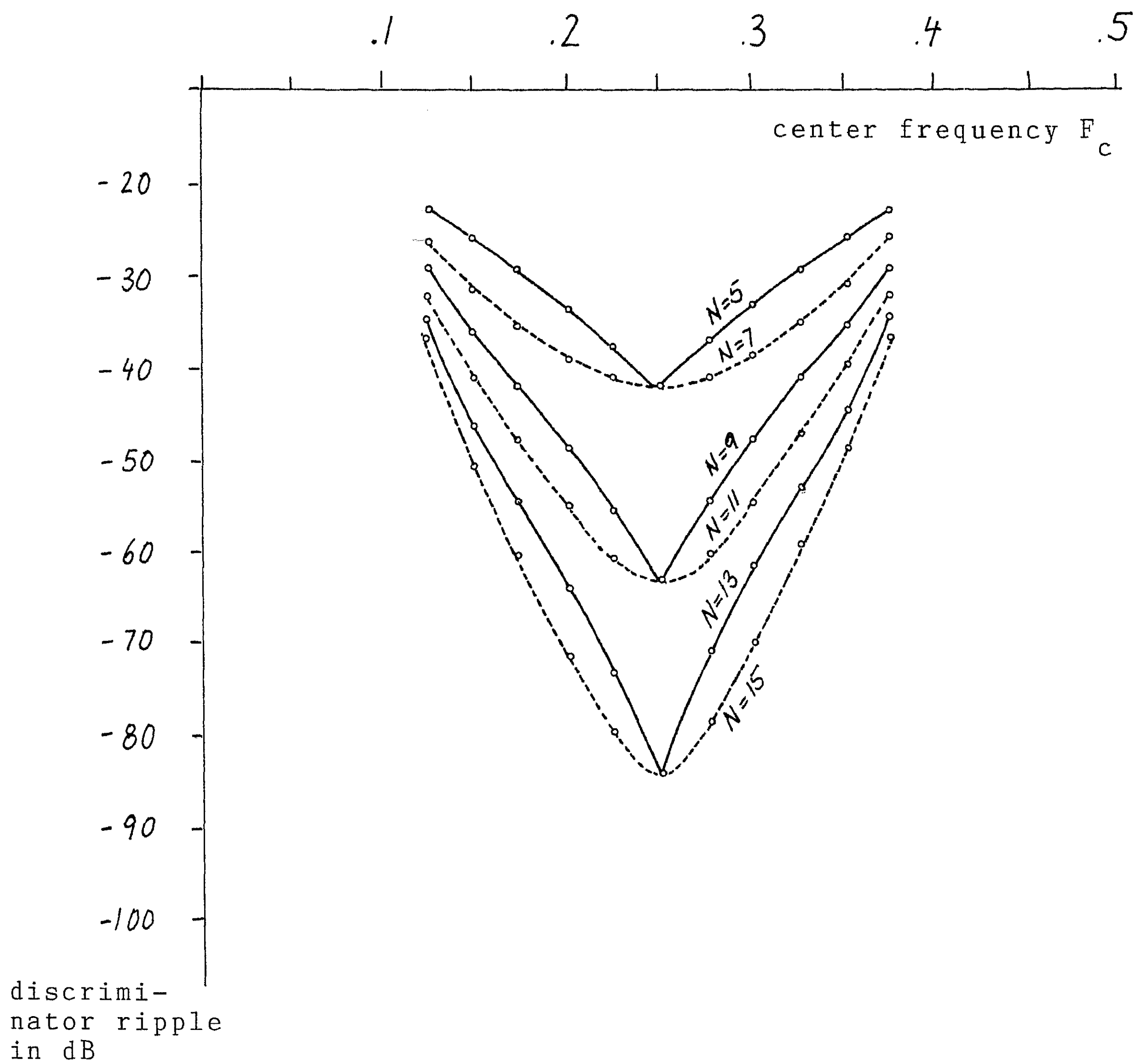

Fig. 4-21 Discriminator ripple vs. shifted center frequency F_c for case 3 filters ($s = 1$, $F_{bw} = 0.2$)

FIR DISCRIMINATOR, CASE 4 (EVEN N, NEGATIVE SYMMETRY)

The chosen filter length N for this filter case is 8.

C	fc = 0.2	fc = 0.25	fc = 0.3	fc = 0.35
C4 =-C5	0.16889440	0.05073904	-0.00710340	-0.09836000
C3 =-C6	-0.13329600	-0.06911039	-0.01157000	-0.08877300
C2 =-C7	0.03339469	-0.00817786	-0.06856000	-0.02324200
C1 =-C8	-0.03138814	-0.01119553	0.01461400	-0.01356800

Cn = C(n)

Table 4-18 Coefficients of selected filters for shifted center frequency (case 4)

The frequency responses of these discriminators are presented in Fig. 4-22. Fig 4-23 shows the magnitude of the discriminator ripple for the selected filters.

The table shows the filter ripple for case 4 filters of different filter length N and shifted center frequencies.

Fc	filter length N					
	4	6	8	10	12	14
0.1	-22.9	-23.2	-23.3	-23.4	-23.4	-23.5
0.125	-23.3	-26.3	-29.2	-32.8	-34.6	-37.3
0.15	-25.9	-30.9	-35.8	-40.6	-45.1	-49.7
0.175	-28.1	-34.9	-41.5	-47.8	-54.0	-60.2
0.2	-30.3	-38.3	-46.5	-54.5	-62.1	-69.7
0.225	-32.5	-41.5	-51.5	-60.5	-69.9	-78.7
0.25	-34.9	-44.5	-56.5	-66.3	-77.5	-87.3
0.275	-37.8	-46.9	-62.5	-71.1	-86.2	-94.2
0.3	-41.6	-48.5	-71.0	-72.7	-89.3	-93.4
0.325	-46.4	-48.6	-63.8	-68.9	-78.2	-85.0
0.35	-44.2	-46.7	-56.3	-62.2	-68.6	-74.4
0.375	-38.9	-43.2	-49.2	-54.0	-58.3	-62.2
0.4	-35.5	-38.3	-41.7	-44.0	-45.9	-47.4

Table 4-19 Discriminator ripple for center frequencies of the discriminator between 0.1 and 0.4 for case 4 filter, s = 1, F_{bw} = 0.2

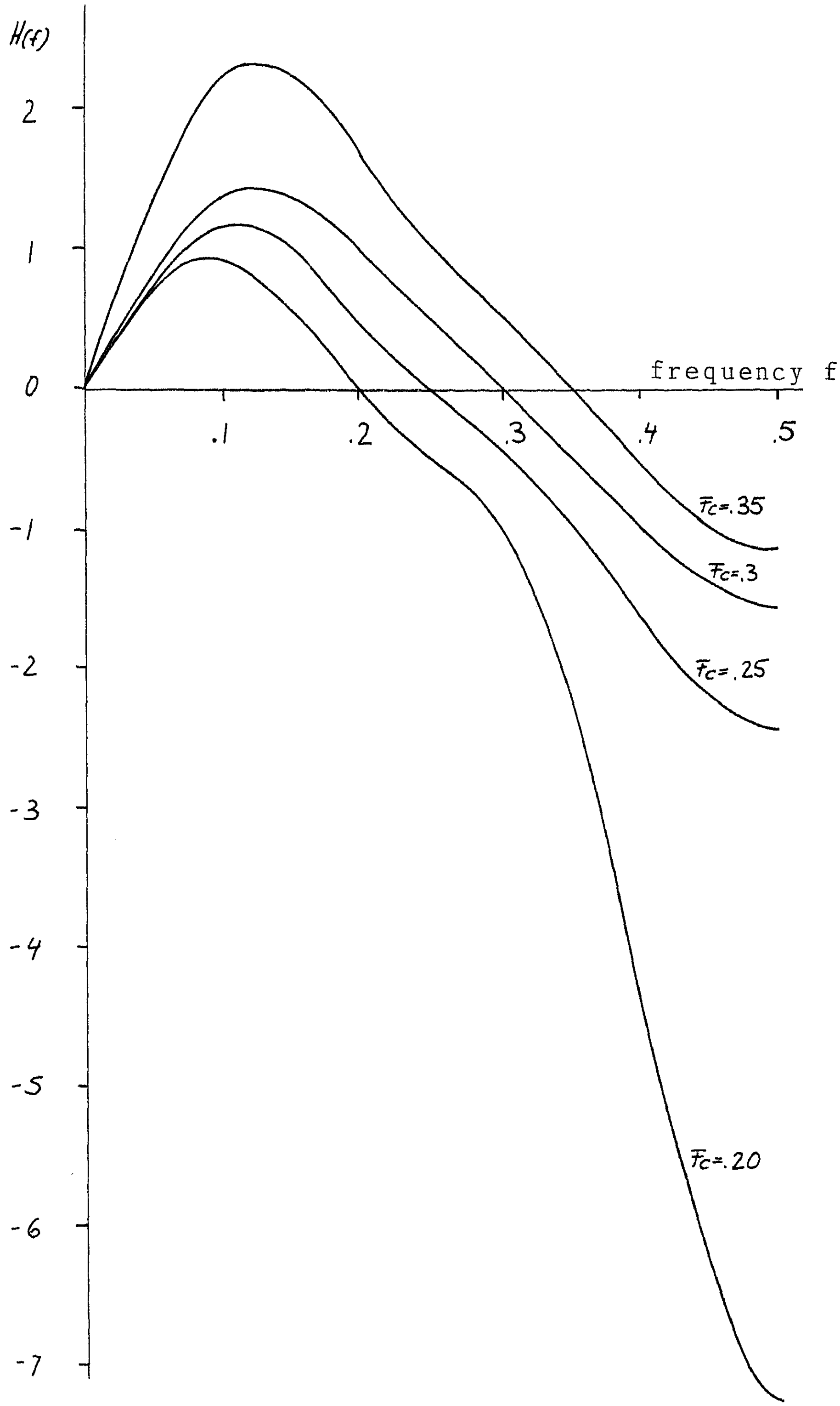

Fig 4-22 Frequency responses of selected filters (filter case 4)

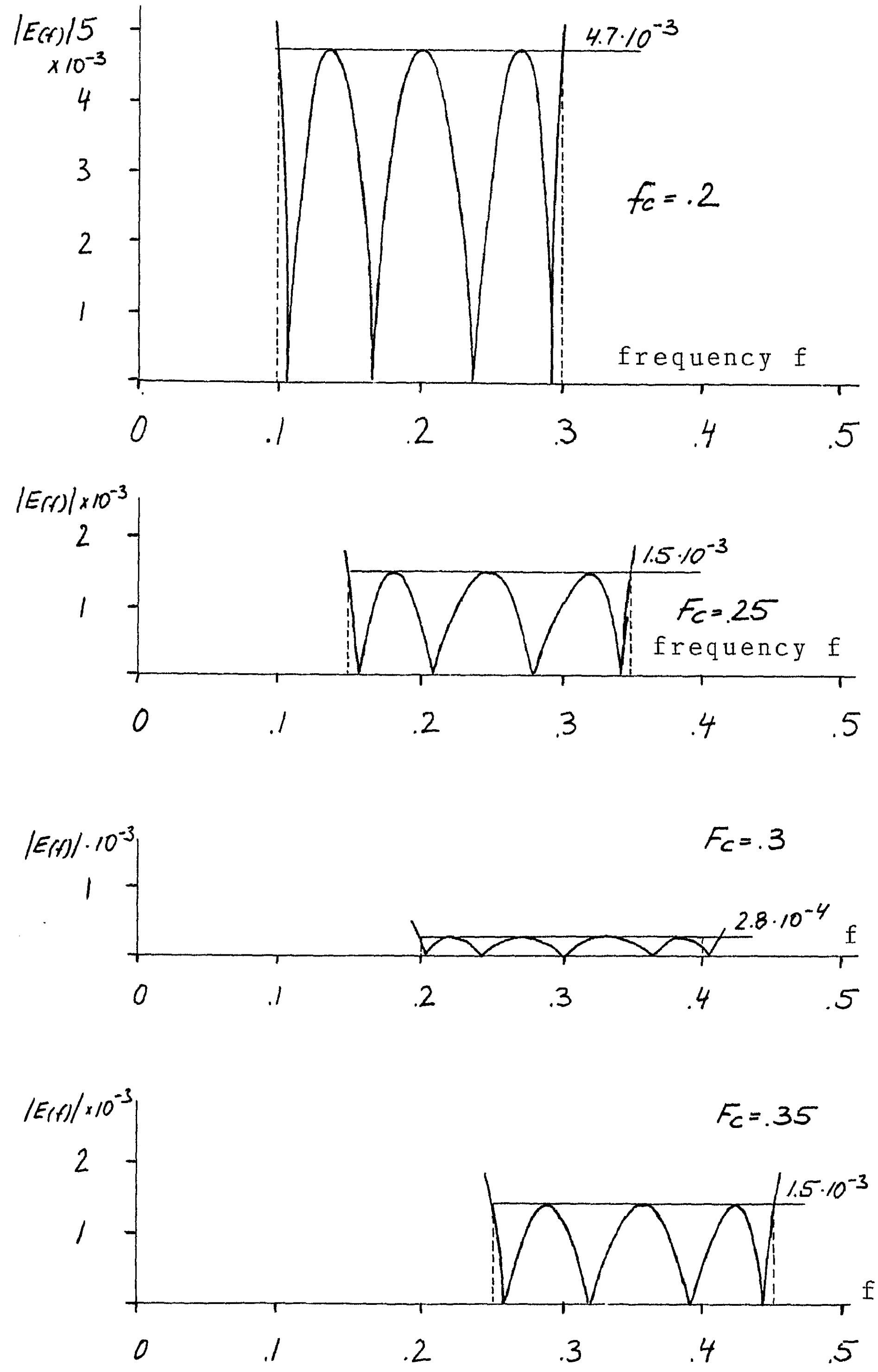

Fig 4-23 Magnitude of filter ripple of selected filters (case 4)

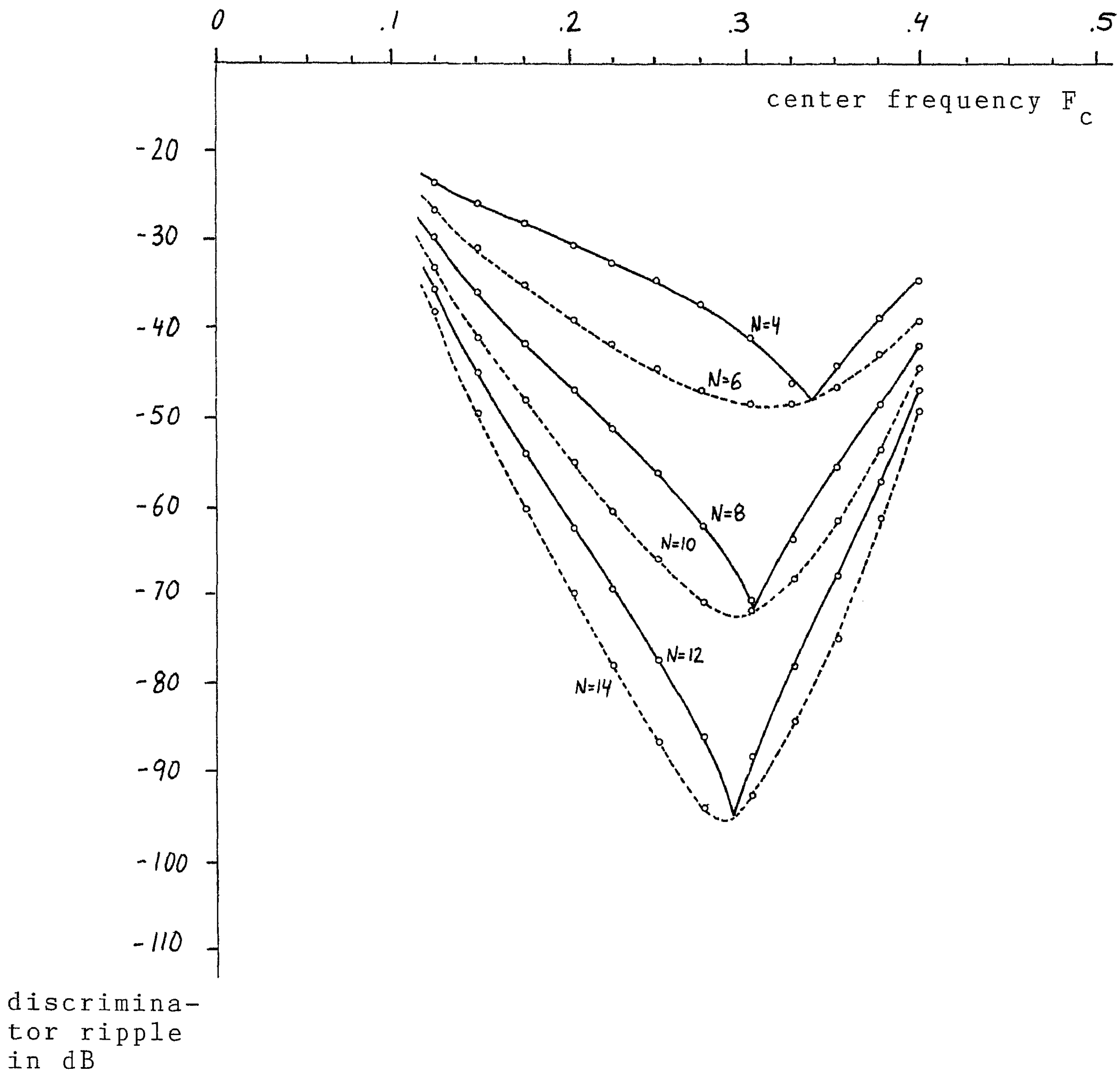

Fig. 4-24 Discriminator ripple vs. shifted center frequency F_c for case 4 filters ($s = 1$, $F_{bw} = 0.2$)

Previous figures and tables show the interdependence between center frequency and discriminator ripple for four different filter cases. The discriminator bandwidth and the slope of the discriminator remained constant for this set of calculations. The FIR discriminators have the following characteristics:

- discriminators with odd length (case 1 and case 3) have their minimum ripple at the frequency F_C=0.25.
- the minimum ripple for discriminators with even length (case 2) is below the frequency f=0.25 and approaches f=0.25 with increasing discriminator length.
- the minimum ripple for discriminators with even length (case 4) is above the frequency f=0.25 and approaches f=0.25 with increasing filter length.
- the ripple of discriminators of odd length (cases 1 and 3) are symmetrical to f=0.25. There is no difference in resulting discriminator ripple whether the new center frequency lies a certain amount below or above the frequency f=0.25.
- the ripple of FIR discriminators of even length with positive (case 2) and negative (case 4) coefficient symmetry are symmetrical to each other relative to f=0.25. This result can be explained by having a closer look at the coefficients of the selected fil-

ters in case 2 and case 4.

- coefficients of case 2 filters at a center frequency of 0.15 have the same coefficients as case 4 filters at the center frequency of f=0.35 except an inverted sign for every other coefficient.

- coefficients of case 2 filters at a center frequency of 0.20 have the same coefficients as case 4 filters at the center frequency for f=0.3 except an inverted sign of every other coefficient.

This result can be applied to any other frequency for this kind of FIR discriminators. The inverting of every other coefficient is equivalent to a flipping of the frequency axis at f=0.25. Figures 4-16 and 4-22 illustrate this. These figures are symmetrical to each other relative to f=0.25.

- filters with positive symmetry of coefficients show better discriminator ripple than filters with negative coefficient symmetry of the same length.

- for a certain filter length case 1, case 2 or case 4 filters show the minimum discriminator ripple depending on the chosen center frequency

4.3 Weighted Error Functions for FIR Discriminators

4.3.1 Reasons for Weighted Error Functions

All earlier presented FIR discriminators have a constant weighting function in common. This constant weighting function leads to an equiripple error of the calculated FIR discriminator. Sometimes it is desirable to influence the weighting function to change the tolerance scheme. This part of the thesis presents two newly developed approaches to shape the tolerance scheme of the error of an FIR discriminator in a selected discriminator bandwidth.

The first approach is the "split optimized bandwidth approach"; the second is the "closed solution approach". Both approaches utilize either features of the original computer program or require additional changes in the listing. Appendices A, B and C present the changes in the computer program.

Design requirements might require a nonconstant tolerance scheme within the FIR discriminator bandwidth. In other cases it is desirable to have an output of nearly zero at certain frequencies. Tougher specifications in a defined frequency band result in meeting this requirement.

4.3.2 Split Optimized Bandwidth Approach

The computer program with the Remez exchange algorithm gives the opportunity to specify weighting factors for selected frequency regions. This feature is usually used to give different weightings to the passband and stopband of lowpass, highpass or bandpass filters. This feature is used here to shape the weighting function in the discriminator bandwidth by splitting it into several frequency regions with different weighting factors.

The following example illustrates the procedure. The chosen discriminator bandwidth is $F_{bw}=0.2$ with a center frequency of $F_c=0.25$. Discriminator A has a constant weighting function. The bandwidth of discriminator B is split into three regions. Table 4-21 shows the weighting factors of the discriminators in the different bandwidths.

discriminator type	frequency band		
	0.15...0.2	0.2...0.3	0.3...0.35
A uniform	1	1	1
B split	1	10	1

Table 4-20 Weighting factors for uniform and split weighting function

Table 4-22 gives the coefficients of two discriminators calculated with uniform and split weighting function. The discriminators have the filter length N=13 and negative symmetry of the coefficients.

D(n)	weighting function constant	split
D(7)	0	0
D(6) = -D(8)	0.00000003	-0.00000083
D(5) = -D(9)	-0.06360787	-0.06294256
D(4) = -D(10)	0.00000011	-0.00000097
D(3) = -D(11)	-0.01547262	-0.01475467
D(2) = -D(12)	0.00000009	-0.00000007
D(1) = -D(12)	-0.00242904	-0.00213461

Table 4-21 Coefficients of FIR discriminators with constant and split weighting function

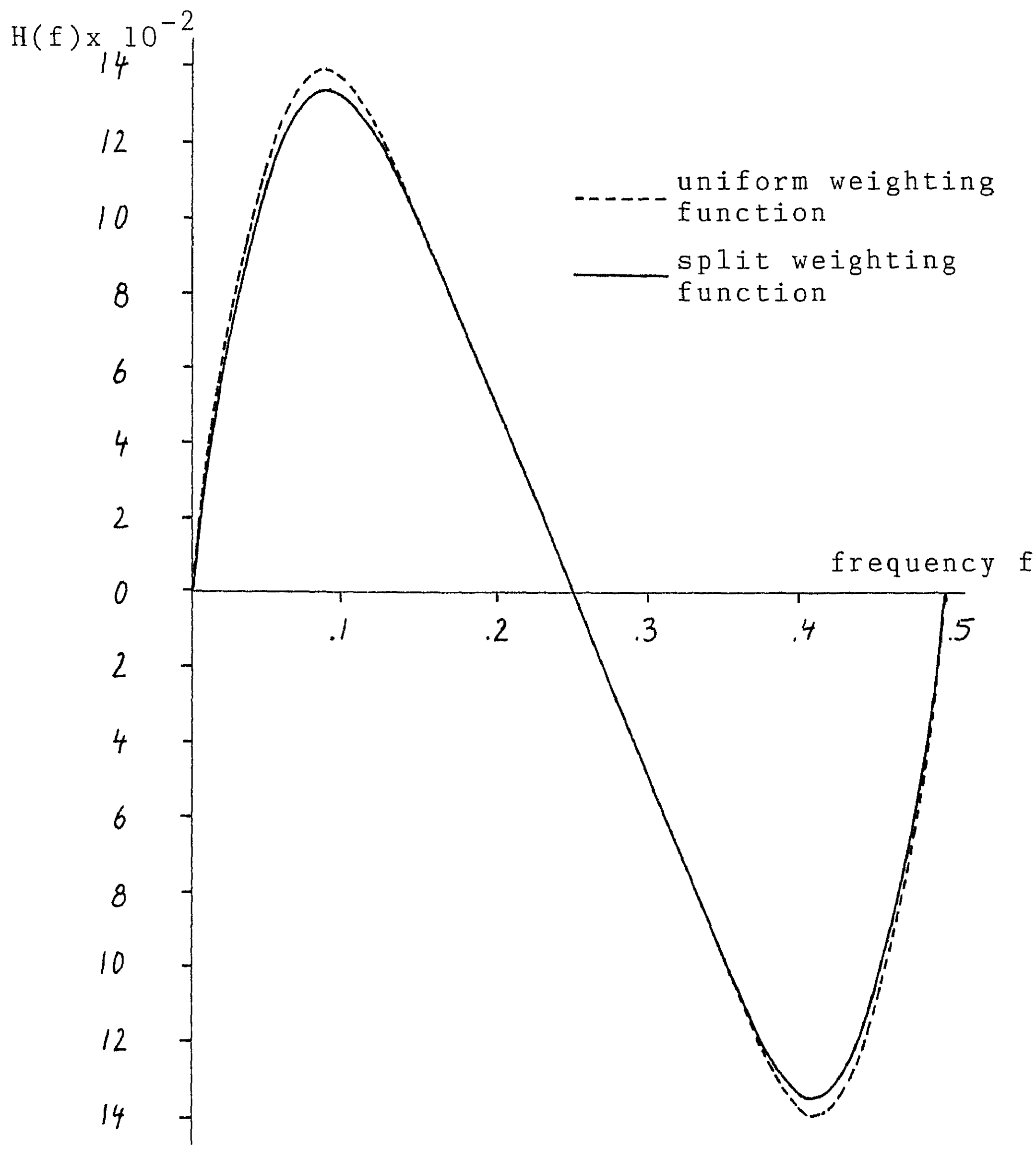

Fig. 4-25 Frequency response of FIR discriminator with constant and split weighting function

Figures 4-25 and 4-26 show the frequency response and the magnitude of the discriminator ripple of the FIR discriminators with constant and split weighting function.

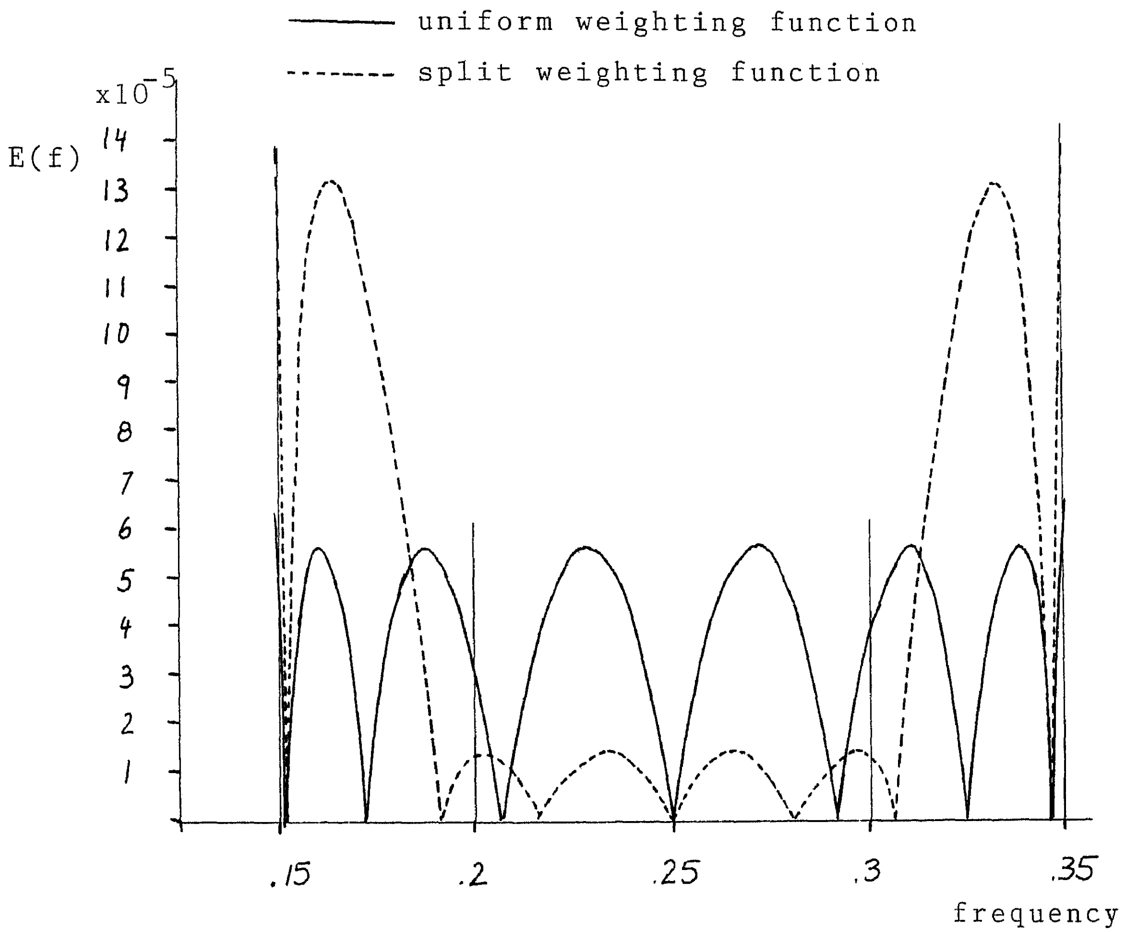

Fig. 4-26 Magnitude of error of FIR discriminator with constant and split weighting function

The minimum ripple of the FIR discriminator with a constant weighting factor has the value 5.5 10^{-5}. Between the frequencies 0.15 and 0.2 and between 0.3 and 0.35 the ripple has the value of 1.3 10^{-4}. In the range with weighting factor 10 between 0.2 and 0.3 the value of the ripple is 1.13 10^{-5}.

This example shows that the split weighting function is a powerful tool to shape the tolerance scheme. In a region of the discriminator bandwidth with a higher weighting factor, the resulting value of the equiripple error has a lower value compared to other regions. Several discriminators with up to 5 different frequency bands showed in experiments the good performance of the design program. This approach has the advantage of easy use and few changes in the original program.

4.3.3 Closed Solution Approach

McClellan, Parks, and Rabiner [1] describe with an example an arbitrary weighting function for a bandpass filter. The "closed solution approach" utilizes this technique for FIR discriminators to shape the tolerance scheme.

The following FIR discriminator has a filter length of N=13, a desired slope of s=1, a center frequency of F_C=0.25 and a error function with x^2-weighting, symmetrical to the center frequency F_C=0.25. W(f) is defined by

$$W(f) = f^2 \qquad \text{for} \qquad 0 \le f \le 0.25 \qquad (4\text{-}7)$$

$$W(f) = (f - 0.5)^2 \qquad \text{for} \qquad 0.25 < f \le 0.5 \qquad (4\text{-}8)$$

Figure 4-27 shows this weighting function. The frequency f=0.25 has the largest weighting. The weighting function decreasinges towards f=0 and f=0.5.

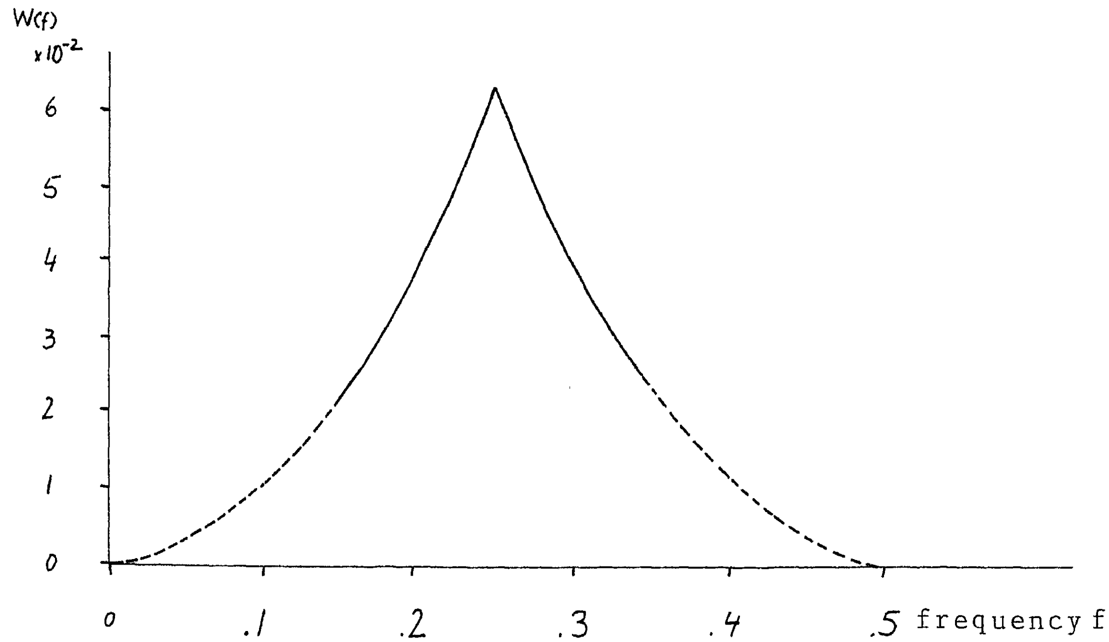

Fig. 4-27 x^2-weighting function

The optimized FIR discriminator with oddfilter length and negative filter symmetry has the coefficients D(n).

	coefficient
D(7)	0
D(6) = -D(8)	-0.00000005
D(5) = -D(9)	-0.06340289
D(4) = -D(10)	-0.00000004
D(3) = -D(11)	-0.01525602
D(2) = -D(12)	-0.00000001
D(1) = -D(13)	-0.00233613

Table 4-22 Coefficients of FIR discriminator with x^2-weighting function

The next two figures show the frequency response and the magnitude of the discriminator with an x^2-weighting function for the discriminator error.

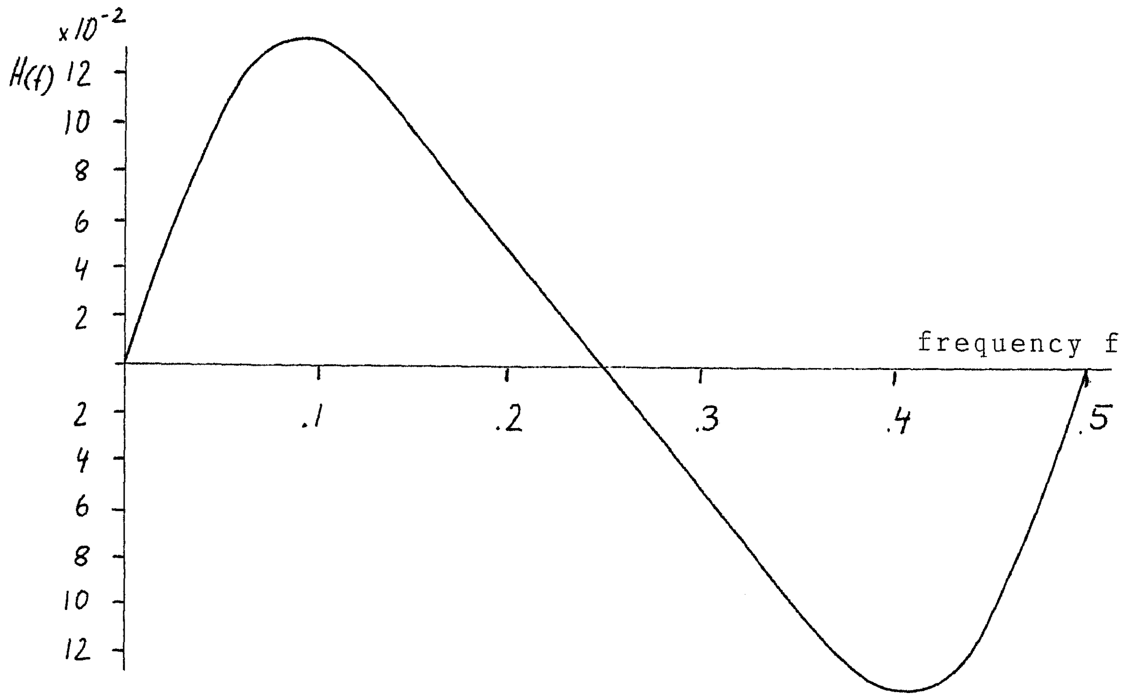

Fig. 4-28 Frequency response of a FIR discriminator with x^2-error weighting function

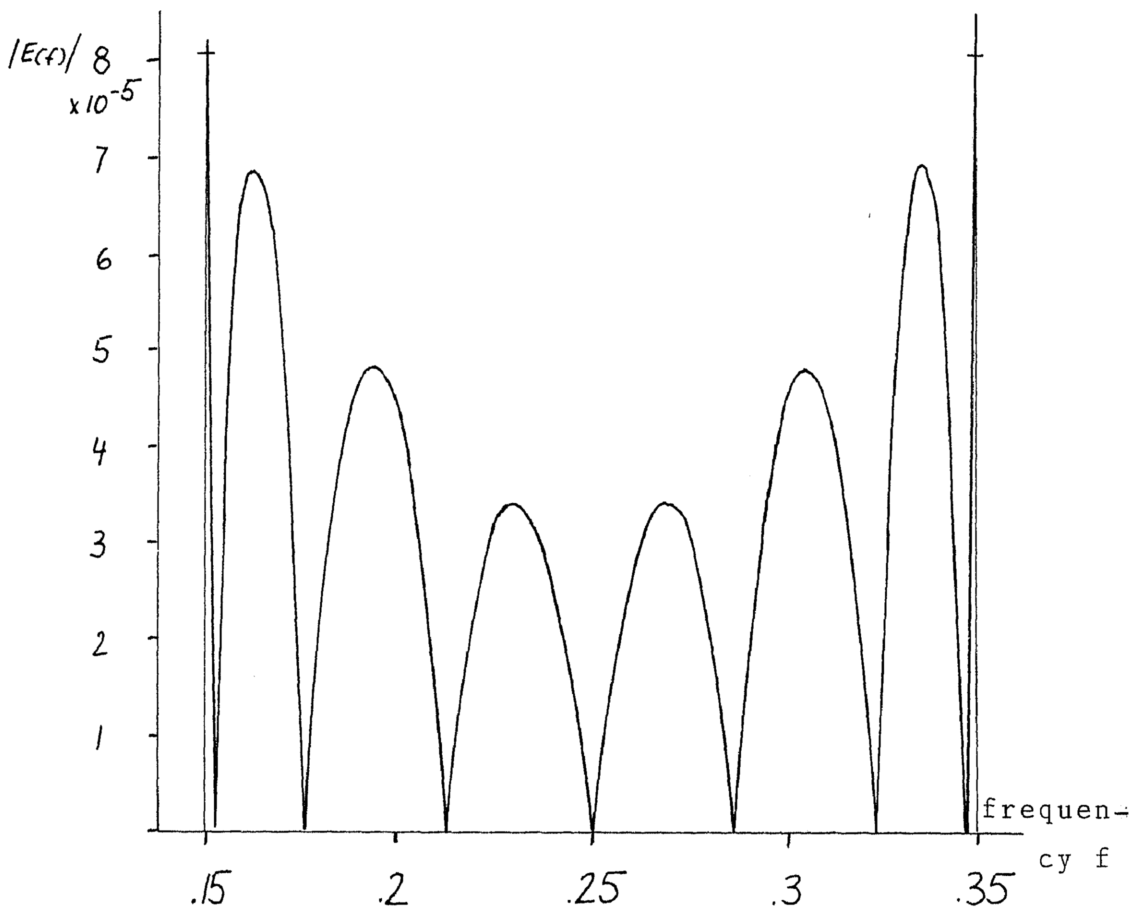

Fig. 4-29 Magnitude of FIR discriminator ripple with x^2-weighting function

4.3.4 Possible Weighting Functions

Previous example shows an FIR discriminator with an error function defined as a function of frequency in the discriminator bandwidth. There are no restrictions to the kind of weighting function. For example all kinds of X^n - functions are conceivable. A higher weighting factor close to f=0.25 leads to a smaller resulting ripple in this frequency range.

There are also almost no restrictions on the use of either split weighting functions or closed solution weighting functions. Even arbitrary weighting functions are thinkable. The computer program gives this possiblibity by replacing the functions in the WATE function by an array that carries the arbitrary weighting coefficients. This approach needs larger changes in the program and also an additional input of the coefficients.

Depending on the required tolerance scheme, one of the suggested approaches has to be chosen, implemented, and the resulting FIR discriminators evaluated. All calculated FIR discriminators should be checked by plotting the discriminator error. The reason is a possible drastic improvement of discriminator ripple by slight changes in the desired tolerance scheme.

4.4 Experiments on Sensitivity

The FIR discriminator gives the opportunity to influence the sensitivity and resulting error of the FIR discriminator by simple multiplication of the original coefficients D(n) by a factor k. This procedure results in the new coefficients $D^*(n)$.

$$D^*(n) = k \cdot D(n) \quad \text{for} \quad 1 \leq n \leq N \qquad (4\text{-}9)$$

The following table shows the coefficients D(n) of the original discriminator. This discriminator has a slope of s=1, the center frequency at F_c=0.25, the filter length N=7 and the discriminator bandwidth F_{bw}=0.2.

	coefficient
D(4)	0
D(3) = -D(5)	-0.01525602
D(2) = -D(6)	-0.00000001
D(1) = -D(7)	-0.00233613

Table 4-23 Filter coefficients of FIR discriminator with slope s=1

Calculated filters have the factors k of 1, 0.5, 2 and -0.5. The following figures show the frequency response and the resulting discrimiator ripple of the 4 filters.

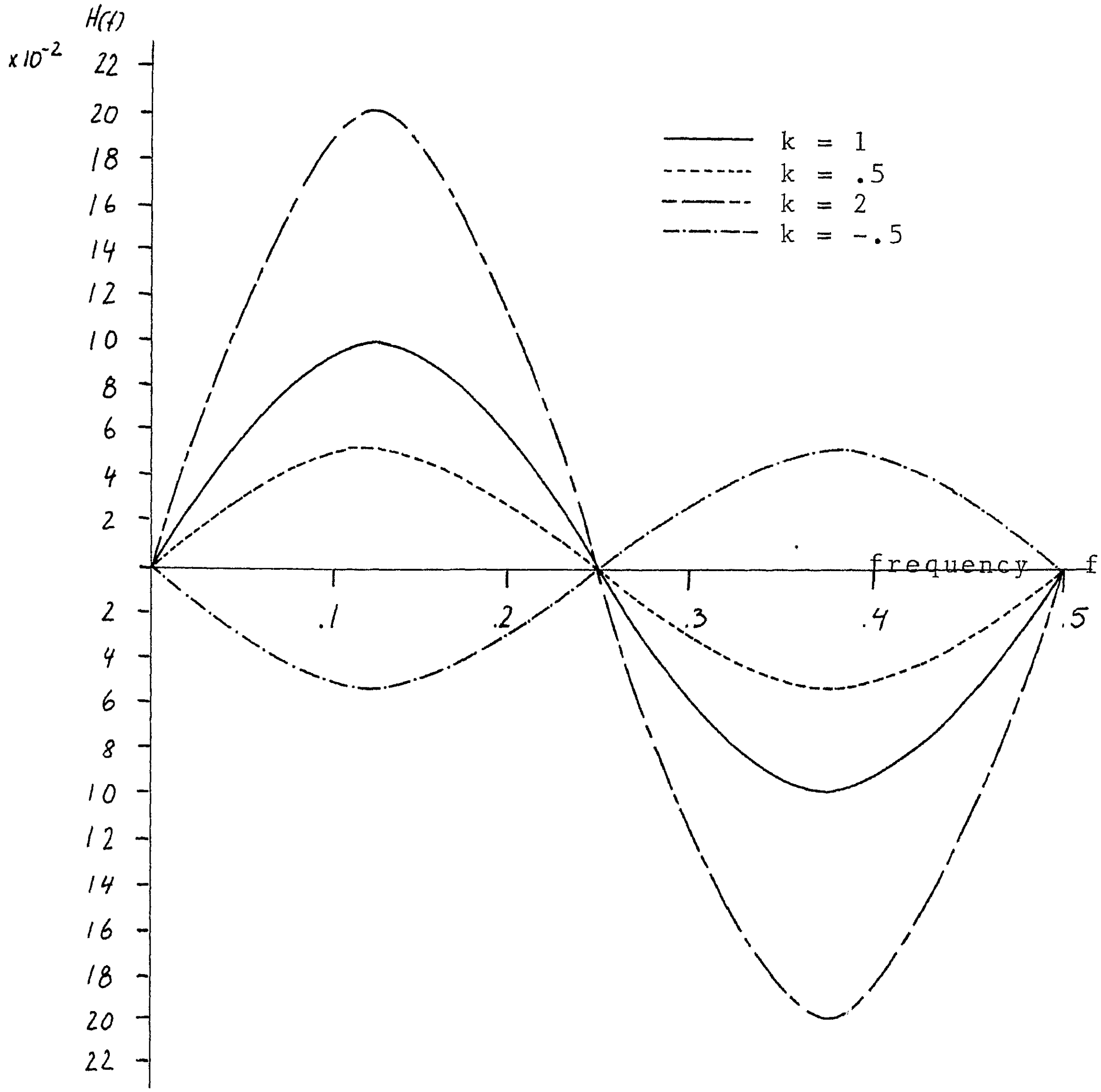

Fig. 4-30 Frequency response of original discriminator and discriminators with k-factors of 0.5, 2 and -0.5

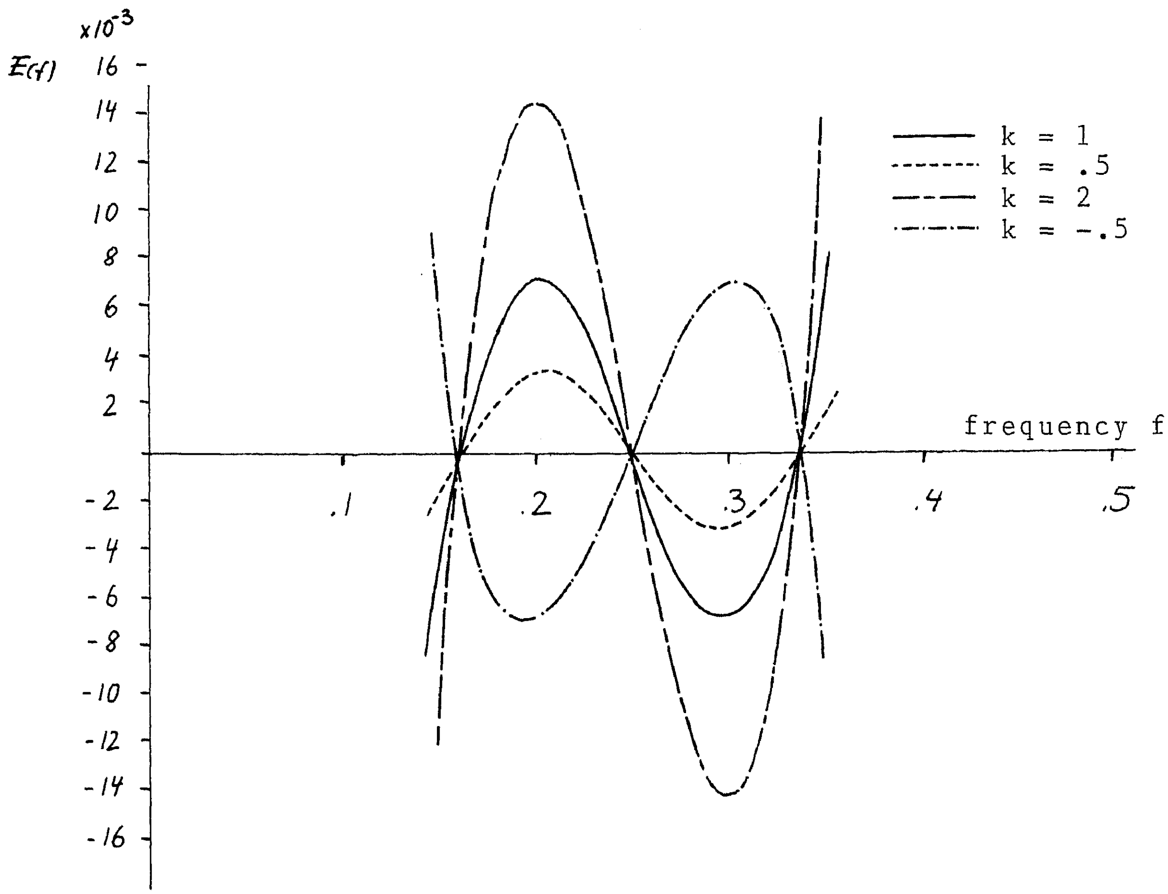

Fig. 4-28 Discrimiator ripple of original discriminator and discriminators with k-factors of 0.5, 2 and -0.5

Plotting the frequency response and the resulting error of these 4 different discriminators shows

- multiplication of the coefficients of the FIR discriminator by a factor k is equivalent to a multiplication of the frequency response by a factor k. Expanding equation 2-1 on both sides by a factor k proves this:

$$\sum_{n=0}^{N-1} k\ D(n)\ e^{-j\omega n} = k \sum_{n=0}^{N-1} D(n)\ e^{-j\omega n} = k\ H(e^{j\omega})$$

with $D^{*}(n) = k \cdot D(n)$ (4-9)

- the same is true for the resulting FIR discriminator ripple. A multiplication of the coefficients D(n) by k results in a multiplication of the ripple by the factor k.

- modifications to obtain a desired output voltage can be easily done by changing the set of coefficients of the FIR discriminator.

4.5 Discusion of the Results on the FIR Discriminator Design

4.5.1 Comparison of Different Filter Cases

As shown earlier, 4 different filter cases are possible with different characteristics in the resulting discriminators. A decision towards odd/even filter length or positive or negative symmetry of the coefficients puts a certain number of restrictions in the frequency domain on the optimization of the FIR discriminator.

One characteristic value for FIR discriminators is the resulting ripple. The suggested procedure minimizes this ripple. Research shows that the ripple is dependent on the chosen filter length and filter case. The fewer restrictions in the frequency domain, the better the results concerning FIR discriminator ripple.

The over all performance of FIR discriminators with odd filter length and positive symmetry of the coefficients has the fewest restrictions in the frequency domain and the best performance of calculated filters, because there are no restrictions at the frequencies f=0 or f=0.5. So this filter fits best for the design of an FIR FM demodulator.

But there are also limitations. The selection of the filter case 1 for FIR discriminators does turn out to be not always the best choice. In applications with a shifted center frequency, two other filter cases of the same discriminator length N show better results depending on the chosen center frequency. For this reason it is desirable to calculate all four filter cases for a specific application and make the decision, which case to choose, after evaluation of all 4 filters.

4.5.2 Discussion of Constant/Non-constant Weighting Functions

The choice between constant and nonconstant weighting functions gives the opportunity to design standard FIR discriminators with equiripple error or FIR discriminators with controllable error function. Shaping the error function by defining a tolerance scheme is an important design feature to influence the computated discriminator. Appendix C describes the necessary modifications of the original design program to create a computer program capable to design FIR discriminators with uniform and weighted error functions. It can be used as a powerful tool to adapt the design of an FIR discriminator to the design requirements. The two suggested approaches to shape the tolerance scheme can be used to control the ripple within the FIR discriminator bandwidth.

4.5.3 Evaluation of FIR Discrimator Design

The design of FIR discriminators with equiripple error in a chosen discriminator bandwidth is a new solution of a problem in the field of digital signal processing. The problem is to find a minimum digital system capable to perform FM to AM conversion and detect the baseband signal. The easy implementation and possibility to change the characteristic of the discriminator by simply changing the set of coefficients is one big advantage of this approach.

The FIR discriminator shows a very good performance at different center frequencies and different slopes of the discriminator. Minor changes in an available FIR filter design program lead to a program capable to design equiripple FIR discriminators.

The computed coefficients can be used for digital filters, but also for SAW filters or any other kind of "tapped delay line"-filter.

The presented figures can be used as design charts for FIR discriminators. A selection of desired ripple, weighting function, bandwidth and center frequency, for example will lead to the required filter length N.

4.5.4 Comparison of the Digital Klapper-Kratt Detector with the Digital FIR Discriminator

A direct comparison between the Klapper-Kratt detector and the suggested FIR FM demodulator is not fully possible. Both types of discriminators use linear filters for the demodulation of FM signals, but only with the Klapper-Kratt detector a cancellation of the carrier is possible. Fig. 2.1 shows the block diagram of the digital Klapper Kratt detector. The following figure shows the block diagram of the FIR FM discriminator with lowpass filter to recover the baseband signal.

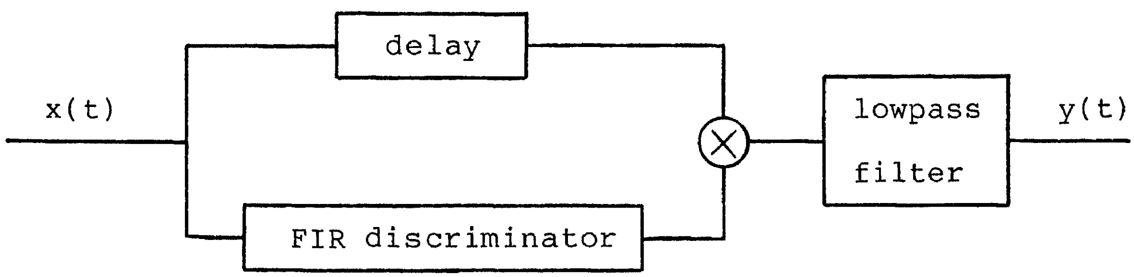

Fig. 4-29 FIR FM detector

Using this configuration, only FIR discriminators of the filter cases 1 and 3 can be realized, as only in these filter cases the calculated values lie exactly on the sampling points. A processing of signals generated by case 2 or 4 filters is in the suggested block diagram not possible.

The following two block diagrams show sections of the Klapper-Kratt detector, comparable to the FIR FM discriminator.

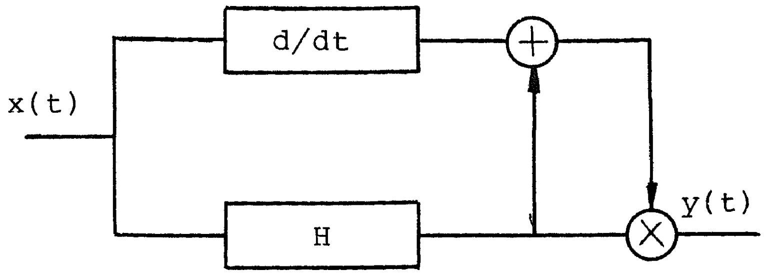

Fig 4-30 FM discriminator configurations of the Klapper-Kratt detector

The complexity of one of these kind of detectors is proportional to the required delays and number of coefficients. The FIR FM discriminator delay in Figure 4-32 does not have to be built seperately, because this delay line has to be implemented to build the FIR discriminator. This minimizes the complexity of the device.

The linearity of the detector is proportional to the number of impulses at the output. For the FIR FM discriminator in Fig. 4-32, the FIR discriminator has to have N coefficients, to get N impulses at the output. The other configurations require 2 N coefficients. This is equivalent to an improvement of 50 per cent.

The comparison shows that there are good results possible calculating FIR discriminators. The higher value of ripple for the over all optimized Klapper-Kratt discriminator can be explained, as it consists of two filters, each having two restrictions in the frequency domain. The best

FIR discriminator does not have any restriction in the frequency domain.

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CHAPTER V

PRACTICAL DESIGN CONSIDERATIONS

5.1 Introduction

The selection of an adequate technique is an important decision in the design procedure. Three solutions are possible:

- software solution
- combined hardware/software solution
- hardware solution

The desired sampling speed and the need for an instantaneous output have an impact on the decision. Low speed applications are in favor of the software solution. Increasing sampling speed works with a hardware solution.

Parallel processing and the utilization of the FIR discriminator symmetry make the design easier. One example in this chapter shows the simplification of a FIR filter by utilizing the filter symmetry.

5.2 Remarks on FIR Filter Design

Parallel processing splits complicated arithmetic structures into smaller pieces and implements these smaller pieces. This technique can be used to increase the processing speed, as the execution time is also split.

Another helpful approach to increase the speed and to simplify the implementation is to derive a modified structure from the original FIR filter structure. Figures 5-1 and 5-2 show the original and modified FIR filter structure.

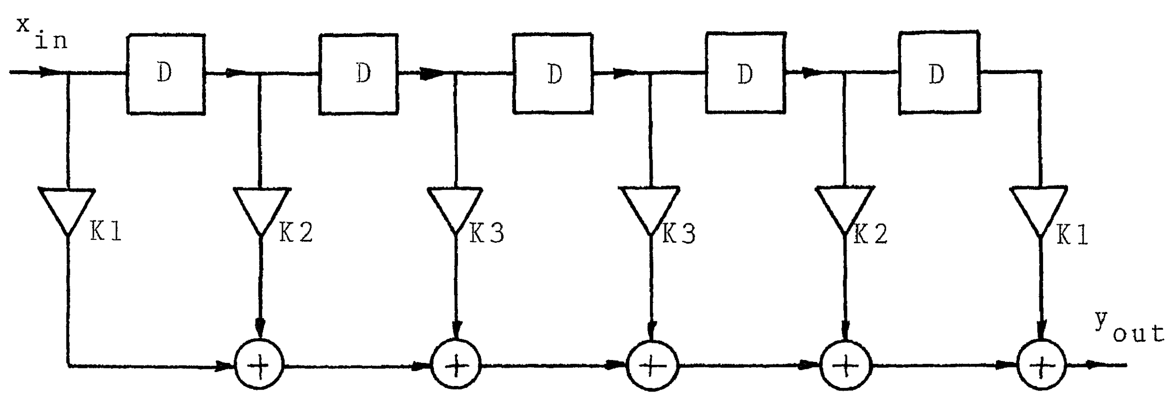

Fig 5-1 Original FIR filter structure

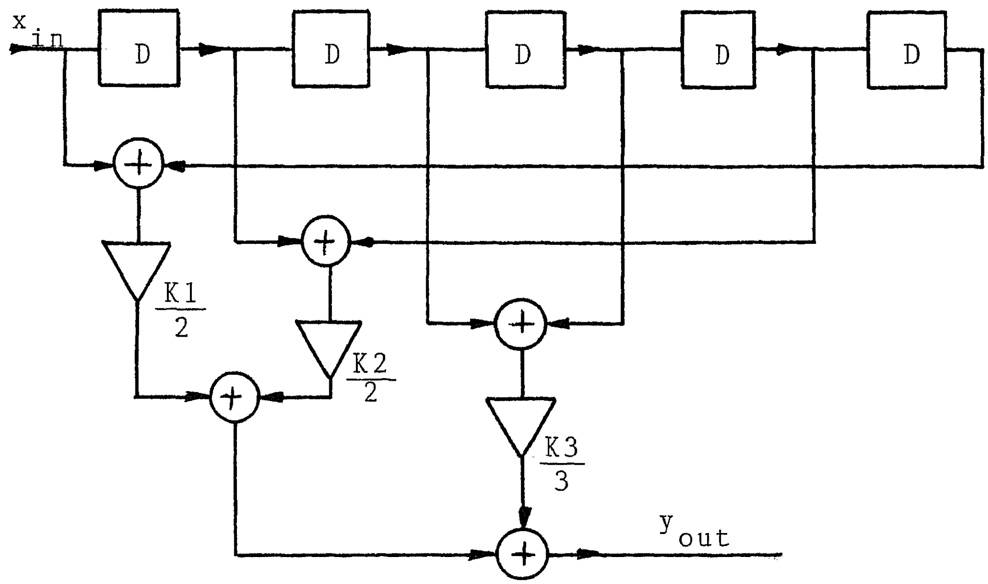

Fig 5-2 Modified FIR filter structure

The second figure shows only three coefficients, compared to six for the original structure. The minimization of the number of multiplications is desirable, as multiplication of digital signals require sophisticated subroutines or special purpose integrated hardware circuits. This is only one example how a simplification can be done.

Depending on the sampling rate, different kind of simplifications are possible. A combination of distributed arithmetic and simplification by utilization of filter symmetry shows good results.

CHAPTER VI

CONCLUSIONS

6.1 FIR discriminator - A New Approach

The trend in communication goes to the digital processing of analog signals. Fink, Hoelzel, and Kammeyer [1]-[3] describe the digital demodulation of FM signals, using interpolation techniques. Narasimha and Peterson [4] simplify interpolation filters , and Iwase, Kumata and Hashimoto [5] support the hardware solution by introducing digital signal processing integrated circuits for very high sampling frequencies. Deubert [6] and Lerner [7] apply digital signal processing to the digital demodulation of TV signals.

The objective of this thesis is to introduce the optimized FIR discriminator and to compare it with the digital Klapper-Kratt discriminator. The FIR discriminator is a direct approach towards the demodulation of FM signals with extreme linearity.

The FIR discriminator can be considered as one basic element of the digital signal processing. Applications include FM to AM conversion. Interpolation techniques may make this approach even more powerful. An AM detection technique is still needed to recover the baseband signal.

6.1 Future Efforts

There exist more areas that need further investigation. All of this thesis deals with non-quantitized input signals and coefficients. The performance of the filter with quantitized input signals and quantitized coefficients needs further investigation. Experiments with shifted center frequency should be extended with discriminator bandwidths that differ from $F_{bw}=0.2$.

The integration of the FIR discriminator into a system that uses interpolation to retrieve the original signals needs also further investigation.

The most important point to mention here is the implementation of all the suggested design procedures in one universal digital signal processing program package. This program should

- have all features of the original WEQFIR
- have all features of the modified WEQFIR
- offer the opportunity to plot frequency response and the error function of WEQFIR and modified WEQFIR
- offer the choice between different weighting functions.

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APPENDIX

APPENDIX A - CHANGES IN WEQFIR

WEQFIR is a design program, capable to design equiripple FIR bandpass filter, differentiators, and Hilbert-Transformers. This FIR design program is the basis for the program to design FIR discriminators. This FORTRAN program runs with minor modifications on any mainframe, mini- or personal computer. The designer of WEQFIR gave the users the opportunity to change the functions EFF and WATE. EFF calculates the desired transfer characteristic of ideal lowpass, highpass, bandpass filters, differentiators, or Hilbert-Transformers. WATE calculates the desired error weighting characteristic.

The main changes to modify the computer program into a computer program capable to design FIR discriminators are done by modifying the functions EFF and WATE. Changes in the main section of WEQFIR are not necessary. The programmers of WEQFIR give the user the choice to select filter length, filtertype, grid density, desired frequency bands for optimization and the weighting factor in a chosen frequency band. The program is designed to calculate lowpass, highpass and bandpass filter with positive symmetry of coefficients, discriminators and Hilbert-transformers with negative coefficient symmetry. I use this feature and select either bandpass filter mode or the Hilbert-transforme/differentiator mode to select the kind of filter symmetry. Changes in EFF and WATE are necessary to compute FIR discriminators.

The variable JTYPE in the program represents the selected filter type:

1 stands for multiple passband/stopband filter

2 stands for differentiators,

3 stands for Hilbert-transformer.

"1" as an input for the variable JTYPE is chosen to calculate FIR discriminators with positive symmetry of the coefficients, "2" for an FIR filter with negative symmetry of the coefficients. To get a neat output, in the output section "bandpass filter" can be replaced by "differentiator case 1/3" and "differentiator" by "differentiator case 2/4"

The result of this thesis is not a perfect new computer program, this being suggested as future efforts, but rather a procedure to change and use the modified program WEQFIR to design FIR discriminators. Chapter 3.3 gives an example for the modification of EFF and WATE, till reaching the desired frequency response and error ripple.

APPENDIX B - CHANGES IN EFF

The following listing shows a modified example of the function EFF:

```
DIMENSION FX(5), WTX(5)
EFF=FX(LBAND)*FREQ-X
RETURN
END
```

This program has always to be changed. The desired magnitude of the FIR discriminator does not match the ideal magnitudes of the program WEQFIR.

The variable "X" stands for any frequency between 0 and 0.5 and determines the desired center frequency. In this thesis, F_c can directly be replaced by "X". They are identical. "X" stands for the center frequency of the FIR discriminator with slope s=1.

APPENDIX C - CHANGES IN WATE

This program has to be changed to create the equiripple error function.

The following listing shows a modified example of the function EFF:

```
      DIMENSION FX(5), WTX(5)
      WATE=WTX(LBAND)
      RETURN
```

Changes up till now are already sufficient to calculate equiripple FIR discriminators. This modified function EFF is also capable to deal with the "split weighting function approach" to design FIR discriminators with split weighting funtion in the optimized bandwidth.

The closed solution approach to design to calculate FIR discriminators with non-uniform weighting function requires depending on the chosen weighting function further modifications.

The following example shows the FORTRAN listing of WATE with an x^2-error weighting function.

```
      DIMENSION FX(5), WTX(5)
      IF(JTYPE.EQ.2) GO TO 2
      WATE=WTX(LBAND)
      RETURN
    1 IF(FX(LBAND).LT.0.0001) GO TO 2
      WATE=WTX(LBAND)/FREQ
      RETURN
    2 IF(FREQ.LE.0.25) WATE=WTX(LBAND)*FREQ
      IF(FREQ.GT.0.25) WATE=WTX(LBAND)*(0.5-FREQ)*2
      RETURN
      END
```

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