Behavior of L-shaped reinforced concrete columns under combined biaxial bending and compression

Amar Shah

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BEHAVIOR OF L-SHAPED REINFORCED CONCRETE
COLUMNS UNDER COMBINED BIAXIAL BENDING
AND COMPRESSION

by

Amar Shah

Thesis submitted to the Faculty of the Graduate
School of the New Jersey Institute of Technology
in partial fulfillment of the requirement
for the degree of
Master of Science in Civil Engineering
1984
APPROVAL SHEET

Title of Thesis: Behavior of L-shaped Reinforced Concrete Columns under Combined Biaxial Bending and Compression

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5/11/94

Signatures of other members of the thesis committee:

5/11/94

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Degree and date to be conferred: MSCE, 1984.


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Major: Civil Engineering.
Title of Thesis: Behavior of L-shaped Reinforced Concrete Columns under Combined Biaxial Bending and Compression.

Amar Shah, Master of Science in Civil Engineering, 1984

Thesis directed by: Dr. C. T. Thomas Hsu, Associate Professor of Civil Engineering.

Combined biaxial and axial compression for L-shaped reinforced concrete short columns is a common design problem. Current code provisions and the available design aids do not offer an insight into the determination of strength and ductility of biaxially loaded reinforced concrete column. An experimental and analytical investigation of the moment-deformation behavior of biaxially loaded L-shaped short columns were undertaken. Four 1/2 scaled specimens were tested till failure. Moment-curvature and load-deflection curves were developed from the experimental and the analytical results. The analytical results were obtained using a computer program developed by Hsu(1). From the investigation it is deduced that the computer program developed by Hsu(1) can be used to find the ultimate strength, the moment-deformation
characteristics, the stress and the strain distributions across the section of L-shaped, biaxially loaded column with large and small eccentricities.
To
my Parents and Krupa
ACKNOWLEDGEMENTS

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\[ \theta_x \] - Curvature produced due to bending moment \( M_x \)
\[ \theta_y \] - Curvature produced due to bending moment \( M_y \)
\[ \delta_x \] - deflection in X-direction
\[ \delta_y \] - deflection in Y-direction
\[ N_u \] - Ultimate normal force
\[ \varepsilon_s \] - Strain in reinforcing steel
CHAPTER I
GENERAL INTRODUCTION AND SCOPE OF RESEARCH

A. GENERAL INTRODUCTION

Structural members subjected to axial load and biaxial bending are encountered in design practice from time to time; a typical example is the corner column in a framed structure. In recent years the idea of using irregularly shaped column (e.g., L-shaped column) at corner of the framed structure and at enclosure of elevator shaft has drawn the attention of investigators.

Unfortunately, little is known about the analytical and experimental behavior of irregularly shaped columns subjected to combined biaxial bending and axial compression; further, most investigations into the behavior of columns under combined biaxial bending and axial compression states have been primarily concerned with the determination of the ultimate strength of concrete and relatively few studies have been made of deformational characteristics of concrete columns.
subjected to biaxial bending and axial compression.

It is felt that current code provisions and available methods do not offer an insight into the determination of strength and ductility of biaxially loaded reinforced concrete columns. This study lays a special emphasis on L-shaped columns, as the use of such columns can be expected to increase in future. To design such structural members the following provisions are needed:

1. Design aids such as interaction diagrams or modified load contour design equations for cross section other than rectangular or circular, from which computer models can be developed.

2. Verification of mathematical modelling transcribed into computer programs by experimental testing.

3. The stress strain relationship of concrete and reinforcing steel must be reexamined in its application to columns other than of standard shapes.

4. Load-deflection characteristics must be studied and mathematical equations should be proposed.

5. In addition to these, the moment-curvature
characteristics at every stage of loading would also be helpful to understand the complete behavior of the structural member.

B. RESEARCH OBJECTIVE

The investigation described here was carried out to find the possible answers of some of the above problems. The primary objective of this project is to study the strength and deformational behavior of L-shaped column under combined biaxial bending and axial compression experimentally, and to assess the accuracy of a computer program developed by Hsu (1) on the basis of equilibrium of forces based on input material stress-strain curves and strain compatibility. A modified Newton-Raphson numerical method was used to achieve computation procedure for Hsu's computer program.

The experimental result will form a basis for a recommended analysis and design technique. For experimental purpose four reinforced concrete columns were tested. Moment-curvature relationships are derived from these experimental results and compared with those obtained by using a computer program.
C. DESIGN CRITERIA AND PRACTICE

The extensive research work done by many investigators has made it possible to develop different design criteria for eccentrically loaded columns such as working stress design, ultimate strength design, and limit design. Early recognition that compression limit at the extreme fibers of concrete cross sections produced unacceptably low estimates of allowable load preceded the adoption of a strength formulation of an allowable stress for the design of non-slender columns.

The present ACI Building Code (ACI 318-83)( 9 ) and design aids follow the strength criteria as a basis for designing concrete columns in which failure is defined in terms of a limiting strain or stress in the concrete and the reinforcing steel. In the above criteria, the stress distribution in the compression zone of a section is defined in terms of the stress block parameters $K_1$, $K_2$, $K_3$ where these parameters are determined experimentally. According to ACI $K_1=0.85$, $K_2=0.5$, $K_3=0.85$ for certain values of $f'_c$ and $f_y$. 
The methods available for the design of biaxially loaded columns are: (1) Trial and error procedure and (2) Determination of ultimate loads from failure surfaces in columns. The former method essentially involves a trial and error procedure for obtaining the position of an inclined neutral axis, hence this method is quite complex so that no formula can be easily developed for practical use. The concept of using failure surfaces has been presented by Bresler(12) and Pannel(13). Fannel(13) has shown that equivalent uniaxial moment $M_{uxo}$ of the radial moment $M_u$ corresponding to any ultimate load $P_u$ can be determined with the aid of the parameter $N$, the deviation factor and the ratio of $M_{ux}/M_{uy}$. The calculated uni-axial moment is then determined from the major axis interaction diagrams. This procedure, namely, determining the load from the moments, is likely to give rise to possible errors in the estimation of the ultimate load, especially when failure is controlled by tension. Bresler proposed two approaches. Of these, the Load-contour method gives the general nondimensional equation at constant $P$ as follows:

$$
\begin{bmatrix}
M_{nx} \\
M_{ox}
\end{bmatrix} + \begin{bmatrix}
M_{ny} \\
M_{oy}
\end{bmatrix} = 1
$$

\ldots\ldots\ldots\ldots\ldots(1.1)$
Bresler (12) suggested that it is acceptable to take $\alpha_1 = \alpha_2 = \infty$ and reported the calculated values of $\infty$ to vary from 1.15 to 1.55. Bresler (12) suggested another simple equation using the reciprocal method which is:

$$\frac{1}{P_i} = \frac{1}{P_{ux}} + \frac{1}{P_{uy}} + \frac{1}{P_o} \quad \ldots \ldots \ldots (1.2)$$

This equation gives surprisingly satisfactory results.

A modification of extended Newton-Raphson method or method of successive approximation has been used by investigators for determination of strain and curvature distributions at reinforced concrete section of column under biaxial bending and axial load. Under this method the typical definition of failure was suggested by Cranston (19) who considered that if the maximum strains in the concrete or steel reinforcement exceed certain predefined maximum values, the section is
considered to have failed. Hsu and Mirza (21) modified and extended Cranston’s (19) numerical approach and the stress-strain curves to include the descending branch of the concrete stress-strain curve and developed a computer program which is used in this study. This program uses the material property of the concrete and reinforcing steel and the section geometry as an input features. The idealization of the stress-strain curve of the steel was done by piece-wise linear approximation. The output features of the program include moment-curvature behavior of a structural member under biaxial bending and compression. This program was compared with rectangular column tests by Anderson and Lee (23), Bresler (12), Ramamurthy (14) and Hsu (1). Excellent agreement was obtained between experimental and analytical results according to Hsu (1).

Design aids for L-shaped columns have been developed by Marin (2). Marin (2) presented three sample design charts from group of 50 to be published.

Recently Ramamurthy and Khan (22) presented two methods to represent the load-contours in L-shaped columns and to use them to determine the ultimate load.
Method (I) is based on the failure surfaces in a column and the actual shapes of load-contours are developed using an inverse method of analysis. Method (II) proposes to be replaced by the simple analysis of an equivalent square or rectangular column.

There are very few test results of L-shaped column to study its behavior since most experimental work in column research for biaxial bending and compression was limited primarily to rectangular, circular and octagonal cross sections.

A state of the art in the inelastic behavior of irregularly shaped columns is gaining momentum, as it is foreseeable that in future there will be an increased use of irregularly shaped columns. There is greater need of design aids and computer programs for column under biaxial bending and compression.
Fig. - 1.1 COLUMN SECTION WITH BIAXIAL BENDING AT
THE ULTIMATE LOAD
CHAPTER II
TEST PROGRAM

A. DESCRIPTION OF TEST SPECIMENS

All together four specimens were tested. All columns were designed as short columns and were each six feet long. Physical characteristics of columns tested are shown in Table 5.1 and Fig. 2.1-2.2.

The brackets were heavily reinforced to prevent local failure. Three columns were reinforced longitudinally by 14 Grade 60 \# 3 bars and one by Grade 40 \# 3 bars as seen in Fig. 2.1. These longitudinal bars were held together by 1/8 in. ties at spacing of 3 inches center to center. The stirrups and longitudinal bars were tied together using 16 gauge binding wire. The reinforcement was assembled into a unit before it was placed in the mold.

B. MATERIALS AND FABRICATION:

1. Cement. High early strength type III Portland Cement was used for all concrete mixes.

2. Sand. Crushed quartz sand was used as
aggregates.

3. Concrete Mix. The concrete mix was of following proportions, specified by weight:
The water-cement ratio varied from 0.65 to 0.7.
The cement-sand ratio varied from 3 to 3.2.
Sand was used as aggregates. Coarse aggregates were not used.

4. Steel Reinforcement. Grade 40 and Grade 60 #3 bars (Diameter=0.375 in., Area=0.11 in.) were used in all columns for main reinforcing steel. Grade 40 #1 (Diameter=0.125 in., Area=0.1226 in.) were used for stirrups. The main reinforcement and stirrups were carefully bent to the required shapes. Gauge 16 binding wire was used to hold the main reinforcement and stirrups together.

C. FORMWORK, CASTING AND CURING

1. Formwork. The form was built using 5/8 in. thick plywood. The formwork was built in sections which were connected together by screws to ensure ease of removal of the cast specimens and to allow repeated use of the form. The plywood was braced horizontally and vertically using 2 x 4 in. wooden pieces to prevent bulging of the side walls. The form was
cleaned and oiled with a thin layer of motor oil to facilitate the easy removal of specimens.

2. Casting and curing. The test specimens were cast in horizontal position. This kind of casting is more practical as compared to the vertical casting. While a horizontal casting causes a strength differential across the column section, vertical casting will cause a differential in concrete quality along the column length. After the concrete was placed in the form, it was compacted by means of a high frequency vibrator. Standard 3x6 and 6x12 in. cylinders were cast.

The test specimens and the cylinders were cured in the molds two days before removing the molds. The test specimens and cylinders were then cured for six days.

D. INSTRUMENTATION

1. Strain and Curvature Measurements:

The measurements of strain and curvature were done by the Demec Gauze Method. Six inches-range dial gauze with a least count of 0.0001 in. was used to measure the strain between a pair of demec gauges.

2. Deflection Measurements:
Ames dial gauges (range=2 in., least count=0.001 in.) were used to measure the mid-span deflections.

3. Loading Method:

The columns were tested in the horizontal position. The columns were loaded using the Enerpac 100 ton capacity hydraulic cylinder ram (effective area = 20.63 in.²). Manual Enerpac pump model PEM 2042 with a maximum pressure of 10000 psi was used to drive the ram.
Fig. - 2.1 CROSS SECTION OF COLUMNS
Fig. - 2.2 TEST SPECIMEN DETAILS
CHAPTER III
TEST PROCEDURE

A. COLUMN TESTS

The points of application of load were marked on the faces of brackets. The specimens were placed in the area of the loading frame using a 1 ton crane and were supported on roller supports built up to the required height by the use of pieces of styrofoam and wooden blocks.

All the columns were carefully centered in the testing machine and steel plates were put against the faces of each bracket to transfer the load to the column. All the columns were pin ended. A small initial load was applied to hold the column and steel plates in place.

The Ames dial gauges were then placed. The demec gauges were slued to the specimen earlier. The initial readings were taken for all the instruments. The minimum and maximum increments in load were 100 psi and 300 psi respectively. The roller supports were taken out when the applied load reached a value of 1000 psi.
After each increment of load, the machine was operated so as to hold the load constant, until the deflections come to rest at a reading. Then all gauges were read. This continued until the failure of the specimen. The complete test duration excluding the time required for the experimental setup was about 1.5 hours.

In general all specimens were tested using "controlled load," rather than "controlled deformation." Stress and strain values for column 4 at each stage of loading are given in Table 5.3.

Standard 3 x 6 and 6 x 12 in. cylinders were cast for each batch of concrete. The cylinders were capped using a sulphur compound the day before the test. Then following the test the cylinders were tested. A soil Test 400,000 pound capacity hydraulic testing machine was used. Standard cylinder strengths ranged from 3900 to 4200 psi. The values of $f'_c$ for each column are given in Table 5.1.

**B. REINFORCEMENT TESTS:**

Random samples of the bars were taken from all batches and tested in a Universal Testing machine.
Twenty three in. test specimens were cut from the #3 bars and marked at two points equidistant from the center and 6 in. apart. Strain measurements were taken by using demec gage with a least count of 0.0001 in. The tests were "load-controlled". The resulting stress-strain curves for the main steel are shown in Figs. 3.1-3.2.
Fig. 3.1 STRESS-STRAIN CURVE FOR REINFORCEMENT (N3S #3 BARS)

\[ E = 35 \times 10^6 \]
Fig. 3.2 - STRESS-STRAIN CURVE FOR REINFORCEMENT (BV35 #3 BARS)

\[ E = 23 \times 10^6 \]
Fig. 3.3 ARRANGEMENT OF DEMEC GAUGES
Fig. 3.14 ARRANGEMENT OF DEFLECTION GAUGES
Fig. - 3.5 FAILURE ZONE OF COLUMN #2
Fig. - 3.6 FAILURE ZONE OF COLUMN #3
Fig. - 3.7  FAILURE ZONE OF COLUMN #1
Fig. - 3.8 TEST SPECIMENS
CHAPTER IV
THEORETICAL ANALYSIS AND COMPUTER PROGRAM

A. INTRODUCTION AND ASSUMPTIONS

Theoretical analysis of the moment-curvature and load-deformation characteristics was carried out using the computer program developed by Hsu (1). This computer program gives the information for the stress and strain distribution across the section, the ultimate load and interaction surface of biaxially loaded short columns. It can also calculate the load-deformation curves from zero to maximum moment capacity using a "load control" process in the case of biaxial bending and axial compression.

The following assumptions have been made in this theoretical analysis.

1. The bending moments are applied about the principal axes of the section.

2. Plane sections remain plane before and after bending.

3. The longitudinal stress at a point is a function of the longitudinal strain at that point. The effect
of creep and shrinkage are ignored.

4. The stress-strain curves for the materials used are known.

5. Stress reversal does not occur.

6. The effect of deformation due to shear and torsion and impact effects are negligible.

7. The section does not buckle before the ultimate load is attained.

8. Perfect bond exists between the concrete and reinforcing steel.

B. THEORETICAL DEVELOPMENT

The cross section of the structural member is divided into several small elements. Consider an element \( k \) with its centroid at point \((X, Y)\) referred to the axes of the symmetry (Fig. 4.2). The strain across the element \( k \) can be assumed to be uniform, since plane sections remain plane during bending,

\[
\varepsilon_k = \varepsilon_P + \varphi_x x_k + \varphi_y y_k \quad \ldots \ldots \ldots \ldots \ldots (4.1)
\]

Where,

\( \varepsilon_P \) = Uniform direct strain due to an axial load \( P \) (Fig. 4.2)

\( \varphi_x, \varphi_y \) = The curvature produced by the bending moment
component $M_x$ and is considered positive when it causes compressive strain in the positive $Y$-direction.

$\phi_y$ = The curvature produced by the bending moment component $M_y$ and is considered positive when it causes compression in the positive $X$-direction.

Hsu(1) has modified Cranston(19) and Chatterji's stress-strain curves for the concrete as shown in Fig. 4.3a. These curves account for the strain softening of concrete and the confined concrete elements maintained. The experimental stress-strain curve for steel has been idealized using piece-wise linear approximation to the curve in the strain-hardening region as shown in Fig. 4.3b.

Once the strain distribution across the section is established, the axial force $P$ and moment components $M_x$ and $M_y$ can be calculated using the following equations:

$$P(c) = \sum_{k=1}^{n} f_k a_k \quad \text{..................(4.2)}$$
$$M_x(c) = \sum_{k=1}^{n} f_k a_k Y_k \quad \text{..................(4.3)}$$
$$M_y(c) = \sum_{k=1}^{n} f_k a_k X_k \quad \text{..................(4.4)}$$
Subscript (c) indicates values of $P$, $M_x$ and $M_y$ calculated in an iteration cycle and $a_k$ is the area of element $K$. For a given section (known geometry and material properties) the stress resultants $P$, $M_x$ and $M_y$ can be expressed as functions of $\varphi_x$, $\varphi_y$ and $\varepsilon_p$ and given by the following equations:

\begin{align*}
    P &= P(\varphi_x, \varphi_y, \varepsilon_p) \\
    M_x &= M_x(\varphi_x, \varphi_y, \varepsilon_p) \\
    M_y &= M_y(\varphi_x, \varphi_y, \varepsilon_p) \\
    M_x(s) &= P(s) \varepsilon_y \\
    M_y(s) &= P(s) \varepsilon_x
\end{align*}

$P(s)$, $M_x(s)$ and $M_y(s)$ can be expressed in terms of $P(c)$, $M_x(c)$ and $M_y(c)$. Using Taylor's expansion retaining linear terms as follows:

\begin{equation}
    P(s) = P(c) + \frac{\partial P(c)}{\partial \varphi_x} \delta \varphi_x + \frac{\partial P(c)}{\partial \varphi_y} \delta \varphi_y + \frac{\partial P(c)}{\partial \varepsilon_p} \delta \varepsilon_p \quad \ldots(4.10)
\end{equation}
\[ M_x(s) = M_x(c) + \frac{\partial M_x(c)}{\partial \theta_x} \delta \theta_x + \frac{\partial M_x(c)}{\partial \theta_y} \delta \theta_y + \frac{\partial M_x(c)}{\partial \xi_p} \delta \xi_p \quad \text{...(4.11)} \]

\[ M_y(s) = M_y(c) + \frac{\partial M_y(c)}{\partial \theta_x} \delta \theta_x + \frac{\partial M_y(c)}{\partial \theta_y} \delta \theta_y + \frac{\partial M_y(c)}{\partial \xi_p} \delta \xi_p \quad \text{...(4.12)} \]

Let

\[ u' = P(c) - P(s) \quad \text{.................(4.13)} \]

\[ v' = M_x(c) - M_x(s) \quad \text{.................(4.14)} \]

\[ w' = M_y(c) - M_y(s) \quad \text{.................(4.15)} \]
Then equations (4.10-4.12) can be written as:

\[ u' = \frac{\partial P(c)}{\partial \varepsilon_p} \delta \varepsilon_p + \frac{\partial P(c)}{\partial \varphi_x} \delta \varphi_x + \frac{\partial P(c)}{\partial \varphi_y} \delta \varphi_y \quad \ldots \ldots (4.16) \]

\[ v' = \frac{\partial M_x(c)}{\partial \varepsilon_p} \delta \varepsilon_p + \frac{\partial M_x(c)}{\partial \varphi_x} \delta \varphi_x + \frac{\partial M_x(c)}{\partial \varphi_y} \delta \varphi_y \quad \ldots \ldots (4.17) \]

\[ w' = \frac{\partial M_y(c)}{\partial \varepsilon_p} \delta \varepsilon_p + \frac{\partial M_y(c)}{\partial \varphi_x} \delta \varphi_x + \frac{\partial M_y(c)}{\partial \varphi_y} \delta \varphi_y \quad \ldots \ldots (4.18) \]

An increment in axial force \( \delta P(c) \) produces an increment of strain, \( \delta \varepsilon_p \), at each element in the section. The corresponding stress change at element \( k \) is therefore \( \delta \varepsilon_p (E_t)_k \). The resulting change \( \delta P(c) \) in \( P(c) \) is given by:

\[ \delta P(c) = \sum_{k=1}^{n} \left( (E_t)_k \right) a_k \delta \varepsilon_p \quad \ldots \ldots (4.19) \]

Therefore,

\[ \frac{\partial P(c)}{\partial \varepsilon_p} = \sum_{k=1}^{n} \left( (E_t)_k \right) a_k \quad \ldots \ldots (4.20) \]

Similarly, the changes \( M_x(c) \) and \( M_y(c) \) can be
expressed in terms of $\delta \mathcal{E}_p$ and lead to the equations:

\[
\frac{\partial M_x(c)}{\partial \mathcal{E}_p} = \sum_{k=1}^{n} (E_t)_k a_k y_k \quad \ldots \ldots \quad (4.21)
\]

\[
\frac{\partial M_y(c)}{\partial \mathcal{E}_p} = \sum_{k=1}^{n} (E_t)_k a_k x_k \quad \ldots \ldots \quad (4.22)
\]

Similar expressions can be derived for $\delta P(c)$, $\delta M_x(c)$ and $\delta M_y(c)$ in terms of changes $\delta \varphi_x$ and $\delta \varphi_y$ and yield the following results:

\[
\frac{\partial P(c)}{\partial \varphi_x} = \sum_{k=1}^{n} (E_t)_k a_k y_k \quad \ldots \ldots \quad (4.23)
\]

\[
\frac{\partial M_x(c)}{\partial \varphi_x} = \sum_{k=1}^{n} (E_t)_k a_k x_k y_k^2 \quad \ldots \ldots \quad (4.24)
\]

\[
\frac{\partial M_y(c)}{\partial \varphi_x} = \sum_{k=1}^{n} (E_t)_k a_k x_k y_k \quad \ldots \ldots \quad (4.25)
\]

\[
\frac{\partial P(c)}{\partial \varphi_y} = \sum_{k=1}^{n} (E_t)_k a_k x_k \quad \ldots \ldots \quad (4.26)
\]

\[
\frac{\partial M_x(c)}{\partial \varphi_y} = \sum_{k=1}^{n} (E_t)_k a_k x_k y_k \quad \ldots \ldots \quad (4.27)
\]

\[
\frac{\partial M_y(c)}{\partial \varphi_y} = \sum_{k=1}^{n} (E_t)_k a_k x_k^2 \quad \ldots \ldots \quad (4.28)
\]
Equations (4.19-4.28) and (4.16-4.18) can be arranged in a matrix form as shown in equations (4.29-4.31) to give the rates of change of $P$, $M_x$, and $M_y$ due to changes in $\epsilon_p$, $\phi_x$, and $\phi_y$;

\[
\begin{bmatrix}
\sum_{k=1}^{n} (E_t)_{k}a_k x_k & \sum_{k=1}^{n} (E_t)_{k}a_k y_k & \sum_{k=1}^{n} (E_t)_{k}a_k z_k \\
\sum_{k=1}^{n} (E_t)_{k}a_k y_k & \sum_{k=1}^{n} (E_t)_{k}a_k y_k & \sum_{k=1}^{n} (E_t)_{k}a_k z_k \\
\sum_{k=1}^{n} (E_t)_{k}a_k z_k & \sum_{k=1}^{n} (E_t)_{k}a_k z_k & \sum_{k=1}^{n} (E_t)_{k}a_k z_k
\end{bmatrix}
\begin{bmatrix}
\delta \epsilon_p \\
\delta \phi_x \\
\delta \phi_y
\end{bmatrix}
= \begin{bmatrix}
u' \\
v' \\
w'
\end{bmatrix}
\]

or

\[
\begin{bmatrix}
\delta \epsilon_p \\
\delta \phi_x \\
\delta \phi_y
\end{bmatrix}
= \begin{bmatrix}
u' \\
v' \\
w'
\end{bmatrix}
\]

\[
[K]
\]

\[
\begin{bmatrix}
\delta \epsilon_p \\
\delta \phi_x \\
\delta \phi_y
\end{bmatrix}
= -[K]^{-1}
\begin{bmatrix}
u' \\
v' \\
w'
\end{bmatrix}
\]

or

\[
\begin{bmatrix}
\delta \epsilon_p \\
\delta \phi_x \\
\delta \phi_y
\end{bmatrix}
= \begin{bmatrix}
u' \\
v' \\
w'
\end{bmatrix}
\]
The values of \( u', v', w' \) can be selected to suit the accuracy required and their substitution in equation (4.31) at the end of \( m \) iteration cycle yields the values of \( 6\varepsilon_p, \varphi_x \) and \( \varphi_y \) which lead to values of \( \varepsilon_p, \varphi_x \) and \( \varphi_y \) for the \((m+1)\)th iteration cycle as follows:

\[
\varepsilon_{p(m+1)} = \varepsilon_{p(m)} + 6\varepsilon_p \quad \ldots \ldots \ldots (4.32)
\]

\[
\varphi_{x(m+1)} = \varphi_{x(m)} + 6\varphi_x \quad \ldots \ldots \ldots (4.33)
\]

\[
\varphi_{y(m+1)} = \varphi_{y(m)} + 6\varphi_y \quad \ldots \ldots \ldots (4.34)
\]

Once convergence is obtained within specified tolerances the computer program takes up the next load level and repeats the entire procedure.

For further detailed information about theory refer to (1).

C. THE COMPUTER PROGRAM

The computer program has to have the initial loads and curvature to start it. This is within the main program. Then the load, \( M_x, M_y \), and \( \varphi_x, \varphi_y \) are calculated. Then the load is incremented by an amount that can be adjusted within the main program. Again
the $M_x$, $\varphi_x$, $M_y$ and $\varphi_y$ are calculated. This occurs until either subroutine which calculates inverses fails or divergence occurs. In this fashion the complete behavior of the column can be obtained.

2. Analytical Investigations. Hsu and Mirza(21) proposed the approximate equations using the well-known modified moment-area theorem to evaluate the central deflection and rotations. The equations are as follows:

\[ \delta_{2x} = \frac{\varphi_y l^2}{8} \]  \hspace{1cm} (4.35)

\[ \delta_{2y} = \frac{\varphi_x l^2}{8} \]  \hspace{1cm} (4.36)

The axial load is incremented by $P$ with the factors which related to the effect of the mid-span deflection. The equations are as follows:

(a) in case of loading condition as shown in Fig. 4.4.

\[ P_3 = \frac{P_1 (e_x^2 + e_y^2)}{((e_x + \delta_{2y})^2 + (e_y + \delta_{2x})^2)} \]  \hspace{1cm} (4.37)
Fig. 4.1 TYPICAL RELATIONSHIP BETWEEN M-ψ AND P-δ
CURVES FOR SHORT COLUMN
Fig. 4.2 IDEALIZATION OF A CROSS-SECTION SUBJECTED TO BIAXIAL BENDING AND AXIAL LOAD
Fig. 4.3a IDEALIZED STRESS-STRAIN CURVES FOR CONCRETE

Fig. 4.3b IDEALIZED STRESS-STRAIN CURVE FOR STEEL
Fig. - 4.4  LOADING CONDITIONS FOR BIAXIALLY LOADED SHORT COLUMN
Fig. - 4.5 - CROSS-SECTION SHOWING ALL ELEMENTS
CHAPTER V
COMPUTER AND TEST RESULTS

A. COMPUTER RESULTS

1. Computer program developed by Hsu(1) was used to find the stress, strain distribution across the section, the ultimate strength, moments about the principal axes and curvature in the principal axes. These values of moments and curvatures about the principal axes are transferred to the centroidal axes X and Y. The transferred values of computer results are given in Table 5.6-5.8.

2. Transformation Matrix:

Since the principal axes are taken for analytical purpose co-ordinates transformation is an important procedure. From the strength of materials, the following steps can be used for transformation of co-ordinates, moments, and curvatures:

1. Find moment of inertia $I_x$, $I_y$ and product moment of inertia $I_{xy}$.

2. Use equation $\tan2\theta = 2I_{xy} / (I_y - I_x)$ to determine the angle between the centroidal and the
Principal axes.

3. Use equation

\[
\begin{bmatrix}
  u \\
  v
\end{bmatrix} = [R] \begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]  

\[\cdots\cdots\cdots\cdots(5.1)\]

Where,

\[
[R] = \begin{bmatrix}
  \cos \theta & \sin \theta \\
  -\sin \theta & \cos \theta
\end{bmatrix}
\]  

\[\cdots\cdots\cdots\cdots(5.2)\]

Following these steps the data for the specimens used in this study can be determined as follows:

\[I_x = 144.8 \text{ in}^4,\]
\[I_y = 81.5 \text{ in}^4,\]
\[I_{xy} = -43.4 \text{ in}^4,\]
\[\theta = 27^\circ\]

From the above investigation, the load, moment and curvature with respect to the principal axes U and V can be found easily. For practical purpose these results should be transferred to the centroidal axes X and Y.

Now consider the centroidal axes X and Y as global co-ordinate axes and the principal axes U and V as
structural co-ordinate axes as shown in Fig. 5.1. The angle of rotation is considered in anticlockwise direction. The transformation matrix $R'$ can be obtained as follows (see Ref. 15):

1. For the case (a) shown in Fig. 5.1a, the transformation is given by:

$$
\begin{bmatrix}
R' \\
\end{bmatrix} =
\begin{bmatrix}
\cos\theta & \sin\theta \\
-\sin\theta & \cos\theta \\
\end{bmatrix}
\quad \ldots \ldots (5.3)
$$

Moments and curvatures about the centroidal axes in terms of the moment and the curvature about the principal axes can be given as follows:

$$
\begin{bmatrix}
M_x \\
M_y \\
\end{bmatrix} =
\begin{bmatrix}
R' \\
\end{bmatrix}
\begin{bmatrix}
M_u \\
M_v \\
\end{bmatrix}
\quad \ldots \ldots (5.4)
$$

$$
M_x = M_u \cos\theta + M_v \sin\theta 
\quad \ldots \ldots (5.5)
$$

$$
M_y = M_v \cos\theta - M_u \sin\theta 
\quad \ldots \ldots (5.6)
$$

and

$$
\begin{bmatrix}
\phi_x & \phi_{xy} \\
\phi_{xy} & \phi_y \\
\end{bmatrix} =
\begin{bmatrix}
R' \\
\end{bmatrix}^T
\begin{bmatrix}
\phi_u & \phi_{uv} \\
\phi_{uv} & \phi_v \\
\end{bmatrix}
\begin{bmatrix}
R' \\
\end{bmatrix} \ldots (5.7)
$$
since $U$ and $V$ are the principal axes, in both the cases (a) and (b) $\phi_{uv} = 0$, then,

\[
\begin{bmatrix}
\phi_x & \phi_{xy} \\
\phi_{xy} & \phi_y 
\end{bmatrix} =
\begin{bmatrix}
\cos\theta & -\sin\theta \\
\sin\theta & \cos\theta
\end{bmatrix}
\begin{bmatrix}
\phi_u & 0 \\
0 & \phi_v
\end{bmatrix}
\begin{bmatrix}
\cos\theta & \cos\theta \\
-\sin\theta & \cos\theta
\end{bmatrix}
\]

\[\ldots (5.8)\]

\[
\begin{bmatrix}
\cos\theta & -\sin\theta \\
\sin\theta & \cos\theta
\end{bmatrix}
\begin{bmatrix}
\phi_u & \phi_v \\
-\phi_v & \phi_u
\end{bmatrix}
\]

\[\ldots (5.9)\]

\[
\begin{bmatrix}
(\phi_u \cos^2\theta + \phi_v \sin^2\theta) & (\phi_u - \phi_v) \sin\theta \cos\theta \\
(\phi_u - \phi_v) \sin\theta \cos\theta & (\phi_u \sin^2\theta + \phi_v \cos^2\theta)
\end{bmatrix}
\]

\[\ldots (5.10)\]

Therefore,

\[
\phi_x = \phi_u \cos^2\theta + \phi_v \sin^2\theta \quad \ldots \ldots (5.11)
\]

\[
\phi_y = \phi_u \sin^2\theta + \phi_v \cos^2\theta \quad \ldots \ldots (5.12)
\]

2. For the case (b) shown in Fig. 5.1b, the transformation matrix is as follows:
Moments and curvatures about the centroidal axes can be expressed as follows:

\[
\mathbf{R} = \begin{bmatrix}
\cos \theta & -\sin \theta \\
\sin \theta & \cos \theta
\end{bmatrix}
\]  

\[\begin{align*}
M_x &= \mathbf{R} \begin{bmatrix} M_u \\ M_v \end{bmatrix} \\
M_y &= \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} M_u \\ M_v \end{bmatrix}
\end{align*}\]

\[M_x = M_u \cos \theta - M_v \sin \theta\]  

\[M_y = M_u \sin \theta + M_v \cos \theta\]

and

\[\begin{bmatrix}
\phi_x & \phi_{xy} \\
\phi_{xy} & \phi_y
\end{bmatrix} = \mathbf{R}^T \begin{bmatrix}
\phi_u & 0 \\
0 & \phi_v
\end{bmatrix} \mathbf{R}\]

\[\phi_x = \phi_u \cos^2 \theta + \phi_v \sin^2 \theta\]  

\[\phi_y = \phi_u \sin^2 \theta + \phi_v \cos^2 \theta\]

In this study the equations for moments and curvatures those in case (a) are used.
Since,  \( \theta = 27^\circ \)

\[
M_x = 0.891 M_u + 0.454 M_v \\
M_y = 0.891 M_v - 0.454 M_u \\
\vartheta_x = 0.794 \vartheta_u + 0.2061 \vartheta_v \\
\vartheta_y = 0.794 \vartheta_v + 0.2061 \vartheta_u
\] (5.20) 
(5.21) 
(5.22) 
(5.23) 

**B. TEST BEHAVIOR**

The tests proceeded smoothly following the uncontrolled deformation loading procedure with the axial load maintained constant at each loading stage.

The development of cracks increased slowly as the load was increased. No signs of crushing or spalling of concrete were seen until ultimate load was reached. When the ultimate load was reached, concrete spalling occurred at the critical section. Typical failure zone of section is shown in Figs. 3.5-3.7. In most cases 1/2 to 1 in. thick concrete portion spalled off near the critical sections. Concrete spalling was followed by the post buckling of the compression steel. When
the ultimate load was reached, large rotations and
strains took place before the total collapse of the
columns. Strains and rotations at the collapse could
not be measured.

The average value of concrete strain over a 6 in.
gauge length was 0.003936 in. / in. at the loading
stage before the collapse. The maximum strain measured
in the 6 in. gauge length was 0.004766 in. / in.

C. ANALYSIS OF TEST RESULTS

1. MOMENT CURVATURE RELATIONSHIP

The writer, in the present experimental
investigation uses an approach in which the moments and
curvatures are established along X and Y axes ( X and Y
axes pass through the centroid of the section ). The
moment-curvature relationships along X and Y axes are
then compared with the results obtained using the
computer program.

To obtain the strain distributions along X and Y
axes demec gauge method was used. The typical
demec-gauge arrangement for the measurement of strain
values along X and Y axes are shown in Fig. 3.3. The
demec gauge method is used as follows. The strain distribution across the XZ and YZ planes is found at each loading stage and is plotted against the distance between corresponding pair of demec gauges as shown in Fig. 5.2-5.7. When the strain distribution is nonlinear the curvature at particular loading stage is given by the following equation suggested by Mattock (25).

\[ \psi = \frac{\varepsilon_c}{k_d d} \]  
\[ \psi = \text{curvature} \]
\[ \varepsilon_c = \text{Maximum compressive concrete strain and} \]
\[ k_d d = \text{Distance from this maximum compressive concrete strain to the point of zero strain (or neutral axis)} \]

\( k_d \) is obtained by drawing lines through the maximum concrete strain and the other strains until the neutral axis is bisected.

Moments, \( M_x \) and \( M_y \) are calculated as follows:

\[ M_x = P (e_y + \delta_y) \]  
\[ M_y = P (e_x + \delta_x) \]  

48
\[ \delta_y = \text{Deflection in Y direction at mid-height of column} \]

\[ \delta_x = \text{deflection in X direction at mid-height of column} \]

The curves of \( \varepsilon_c \) v/s distance between pairs of demec gauges for all columns are listed below.

Fig. 5.2-5.3 - column # 2
Fig. 5.4-5.5 - column # 3
Fig. 5.6-5.7 - column # 4

Tables listed below show experimental results for \( M_x, \phi_x, M_y, \phi_y \). Here computer results for \( M_x, \phi_x, M_y, \phi_y \) are also included.

Table 5.6 - column # 2
Table 5.7 - column # 3
Table 5.8 - column # 4

The curves of \( M_x - \phi_x \) and \( M_y - \phi_y \) plotted for all columns are listed below.

Fig. 5.8 - column #2 - \( M_x \) v/s \( \phi_x \)
Fig. 5.9 - column #2 - \( M_y \) v/s \( \phi_y \)
Fig. 5.10 - column #3 - \( M_x \) v/s \( \phi_x \)
Fig. 5.11 - column #3 - \( M_y \) v/s \( \phi_y \)
Fig. 5.12 - column # 4 - $M_x$ v/s $\delta_x$
Fig. 5.13 - column # 4 - $M_y$ v/s $\delta_y$

A comparative study is discussed in chapter VI and in the conclusions.

2. LOAD - DEFLECTION RELATIONSHIPS

In fact it was not possible to measure the mid-height deflections. The deflections in X and Y direction were taken at sections few inches away from (6 in.) the mid-height.

Dial gauges with least count of 0.0005 and 0.001 in. were used. From the dial gauge readings at each loading stage deflections in X and Y directions are calculated. The tables listed below show the experimental results for load and deflection for column #4.

Table 5.4 - column #4 $P - \delta_x$
Table 5.5 - column #4 $P - \delta_y$

The figures listed below show the load-deflection curves.

Fig. 5.14 - column #2 $P - \delta_x$
Fig. 5.15 - column #2 $P - \delta_y$
Figs. 5.16 - column #3 $P-\delta_x$
Figs. 5.17 - column #3 $P-\delta_y$
Figs. 5.18 - column #4 $P-\delta_x$
Figs. 5.19 - column #4 $P-\delta_y$
Table 5.1  
SPECIMEN DETAILS

<table>
<thead>
<tr>
<th>Col.</th>
<th>Bars</th>
<th>$f_y$ (Ksi)</th>
<th>$A$ (in²)</th>
<th>$s$ (in.)</th>
<th>$f'_c$ (Psi)</th>
<th>$e_x$ (in.)</th>
<th>$e_y$ (in.)</th>
<th>$l$ (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>14 $#3$</td>
<td>67.0</td>
<td>0.01227</td>
<td>3</td>
<td>4200</td>
<td>1.53</td>
<td>5.0</td>
<td>72</td>
</tr>
<tr>
<td>3</td>
<td>14 $#3$</td>
<td>67.0</td>
<td>0.01227</td>
<td>3</td>
<td>4200</td>
<td>1.68</td>
<td>5.5</td>
<td>72</td>
</tr>
<tr>
<td>4</td>
<td>14 $#3$</td>
<td>58.0</td>
<td>0.01227</td>
<td>3</td>
<td>4000</td>
<td>1.68</td>
<td>6.5</td>
<td>72</td>
</tr>
<tr>
<td>Load</td>
<td>1</td>
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<td>2467</td>
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</table>
Table 5.3

STRAINS OF CONCRETE SURFACE BETWEEN PAIRS
OF DEMEC GAUGES FOR COLUMN #4

<table>
<thead>
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<th>Load (Psi.)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
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<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
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<tr>
<td>334</td>
<td>166.6</td>
<td>250.0</td>
<td>416.6</td>
<td>816.6</td>
<td>233.3</td>
<td>233.3</td>
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<td>600</td>
<td>383.3</td>
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<td>616.6</td>
<td>1350.0</td>
<td>466.6</td>
<td>433.3</td>
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<td>800</td>
<td>700.0</td>
<td>983.3</td>
<td>1233.3</td>
<td>2083.3</td>
<td>916.6</td>
<td>1033.3</td>
</tr>
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<td>1000</td>
<td>1000.0</td>
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<td>2050.0</td>
<td>3000.0</td>
<td>1583.3</td>
<td>1133.3</td>
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<td>2266.6</td>
<td>3466.6</td>
<td>4366.6</td>
<td>4766.6</td>
<td>3333.3</td>
<td>2283.3</td>
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<tr>
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<td>FAILURE</td>
<td></td>
<td></td>
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</table>
Table 5.4
LOAD V/S HORIZONTAL DEFLECTION CALCULATION FOR COLUMN #4

<table>
<thead>
<tr>
<th>Load (Psi.)</th>
<th>Load (Kips)</th>
<th>Horizontal Gauge (in.)</th>
<th>Horizontal Deflection (in.)</th>
</tr>
</thead>
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<tr>
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<td>3.09</td>
<td>013</td>
<td>0.0</td>
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<tr>
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<td>6.89</td>
<td>994</td>
<td>0.019</td>
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<td>600</td>
<td>12.37</td>
<td>932</td>
<td>0.081</td>
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<tr>
<td>800</td>
<td>16.50</td>
<td>857</td>
<td>0.094</td>
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<tr>
<td>Reset</td>
<td>859</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1000</td>
<td>20.62</td>
<td>793</td>
<td>0.160</td>
</tr>
<tr>
<td>1250</td>
<td>25.78</td>
<td>540</td>
<td>0.413</td>
</tr>
<tr>
<td>1300</td>
<td>FAILURE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>------------</td>
<td>----------</td>
<td>-------------------</td>
<td>-------------------</td>
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<tr>
<td>150</td>
<td>3.09</td>
<td>860</td>
<td>677</td>
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<td>334</td>
<td>6.89</td>
<td>858</td>
<td>638</td>
</tr>
<tr>
<td>600</td>
<td>12.37</td>
<td>800</td>
<td>574</td>
</tr>
<tr>
<td>800</td>
<td>16.50</td>
<td>723</td>
<td>495</td>
</tr>
<tr>
<td>Reset</td>
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<td>1000</td>
<td>20.62</td>
<td>596</td>
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<tr>
<td>1250</td>
<td>25.78</td>
<td>350</td>
<td>142</td>
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</tbody>
</table>

1300 FAILURE
Table: 5.6

**CALCULATIONS OF EXPERIMENTAL AND COMPUTER**

\[ M_x, \varphi_x, M_y, \varphi_y \] FOR COLUMN #2

<table>
<thead>
<tr>
<th>Load (Kips)</th>
<th>( M_x ) (K-in)</th>
<th>( \varphi_x ) (1/in. \times 10^{-4})</th>
<th>( M_y ) (K-in)</th>
<th>( \varphi_y ) (1/in. \times 10^{-4})</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.91</td>
<td>34.7</td>
<td>0.95</td>
<td>10.83</td>
<td>1.5</td>
</tr>
<tr>
<td>10.31</td>
<td>51.9</td>
<td>1.07</td>
<td>16.27</td>
<td>1.9</td>
</tr>
<tr>
<td>14.44</td>
<td>72.7</td>
<td>2.00</td>
<td>23.04</td>
<td>3.0</td>
</tr>
<tr>
<td>19.60</td>
<td>112.3</td>
<td>2.79</td>
<td>32.39</td>
<td>4.2</td>
</tr>
<tr>
<td>24.75</td>
<td>126.3</td>
<td>3.80</td>
<td>42.14</td>
<td>5.0</td>
</tr>
<tr>
<td>28.90</td>
<td>148.3</td>
<td>5.10</td>
<td>49.21</td>
<td>6.2</td>
</tr>
<tr>
<td>33.00</td>
<td>171.1</td>
<td>6.56</td>
<td>57.78</td>
<td>9.1</td>
</tr>
</tbody>
</table>

\( 38.16 \text{(Kips)} \) FAILURE

\( e_x = 1.53 \text{ in.} \)

\( e_y = 5 \text{ in.} \)

\( 37.85 \text{(Kips)} \) FAILURE
Table: 5.7
CALCULATIONS OF EXPERIMENTAL AND COMPUTER
\( M_x, \phi_x, M_y, \phi_y \) FOR COLUMN #3

<table>
<thead>
<tr>
<th>Load (Kips)</th>
<th>( M_x )</th>
<th>( \phi_x )</th>
<th>( M_y )</th>
<th>( \phi_y )</th>
<th>Load (Kips)</th>
<th>( M_x )</th>
<th>( \phi_x )</th>
<th>( M_y )</th>
<th>( \phi_y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.89</td>
<td>38.1</td>
<td>0.65</td>
<td>11.68</td>
<td>1.0</td>
<td>10.0</td>
<td>54.9</td>
<td>0.98</td>
<td>16.8</td>
<td>1.88</td>
</tr>
<tr>
<td>10.31</td>
<td>57.24</td>
<td>1.15</td>
<td>17.75</td>
<td>1.6</td>
<td>15.0</td>
<td>82.4</td>
<td>1.57</td>
<td>25.2</td>
<td>2.98</td>
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<tr>
<td>14.43</td>
<td>80.4</td>
<td>1.84</td>
<td>25.35</td>
<td>2.0</td>
<td>20.0</td>
<td>109.8</td>
<td>2.19</td>
<td>33.6</td>
<td>4.15</td>
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<tr>
<td>20.62</td>
<td>116.3</td>
<td>2.90</td>
<td>38.38</td>
<td>3.0</td>
<td>25.0</td>
<td>137.3</td>
<td>2.90</td>
<td>42.0</td>
<td>5.47</td>
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<tr>
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<td>4.86</td>
<td>47.05</td>
<td>4.4</td>
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<td>4.07</td>
<td>50.4</td>
<td>7.65</td>
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<tr>
<td>28.87</td>
<td>167.3</td>
<td>5.90</td>
<td>58.58</td>
<td>7.0</td>
<td>35.0</td>
<td>192.2</td>
<td>9.31</td>
<td>58.8</td>
<td>17.2</td>
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</tbody>
</table>

\( e_x = 1.68 \) in.
\( e_y = 5.5 \) in.
### Table 5.8

**CALCULATIONS OF EXPERIMENTAL AND COMPUTER Mx, \( \phi_x \), My, \( \phi_y \) FOR COLUMN #4**

<table>
<thead>
<tr>
<th>Load Kip</th>
<th>Mx K-in</th>
<th>( \phi_x ) 1/in.</th>
<th>My K-in</th>
<th>( \phi_y ) 1/in.</th>
<th>Experiment</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.89</td>
<td>44.9</td>
<td>1.40</td>
<td>11.70</td>
<td>1.0</td>
<td>10.0 65.0</td>
</tr>
<tr>
<td>12.37</td>
<td>81.2</td>
<td>2.39</td>
<td>21.87</td>
<td>1.7</td>
<td>15.0 97.5</td>
</tr>
<tr>
<td>16.50</td>
<td>110.5</td>
<td>4.0</td>
<td>29.27</td>
<td>3.4</td>
<td>20.0 124.0</td>
</tr>
<tr>
<td>20.62</td>
<td>138.1</td>
<td>7.80</td>
<td>37.95</td>
<td>5.4</td>
<td>25.0 162.5</td>
</tr>
<tr>
<td>25.78</td>
<td>178.9</td>
<td>11.25</td>
<td>54.00</td>
<td>11.7</td>
<td>25.3 164.4</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>25.4 165.3</td>
</tr>
</tbody>
</table>

**Load Mx Kip**

- 26.81(Kips) FAILURE
- 26.50(Kips) FAILURE

\[ e_x = 1.68 \text{ in.} \]

\[ e_y = 6.5 \text{ in.} \]
Fig. - 5.1 TRANSFORMATION OF AXES
Fig. 5.2 STRAIN DISTRIBUTION LEADING TO $\phi_x$
COLUMN #2
Fig. 5.3 STRAIN DISTRIBUTION LEADING TO $\varphi_y$
COLUMN #2
Fig. 5.14 STRAIN DISTRIBUTION LEADING TO $\phi_x$
COLUMN #3
Fig. 5.5 STRAIN DISTRIBUTION LEADING TO $\phi_y$
COLUMN #3
Fig. 5.6 STRAIN DISTRIBUTION LEADING TO $\varphi_x$
COLUMN #1
Fig. 5.7 STRAIN DISTRIBUTION LEADING TO $\phi_y$
COLUMN #14
Ultimate Load - 38.16 K
$e_x = 1.53$ in.
$e_y = 5.0$ in.

Experimental -
Computer ---

$M_x$ vs. $\phi_x$ CURVE FOR COLUMN #2

---

Fig. 5.8  $M_x$-$\phi_x$ CURVE FOR COLUMN #2
Fig. 5.9 $M_y - \phi_y$ CURVE FOR COLUMN #2

Ultimate Load = 38.16 K
$e_x = 1.53$ in.
$e_y = 5.0$ in.

Experiment ---
Computer ---

$M_y$ K-IN.

$\phi_y$ 1 / IN. ( x 10^{-1} )

68
Fig. - 5.10 $M_x$ - $\phi_x$ CURVE FOR COLUMN #3

Ultimate Load = 35.06 k

$e_x = 1.68$ in.

$e_y = 5.5$ in.

Experiment - 

Computer -

$M_x$ K-IN.

$\phi_x / \text{IN. (X } 10^{-8})$

1 2 3 4 5 6 7 8 9

69
Fig. - 5.11 \( M_y - \phi_y \) CURVE FOR COLUMN #3

Ultimate Load - 35.06 K
\( \alpha_x = 1.68 \) in.
\( \alpha_y = 5.5 \) in.

\( \phi_y \) 1 / IN. \( \times 10^{-4} \)
Fig. - 5.12 $M_x - \frac{\phi_x}{\text{IN.}}$ CURVE FOR COLUMN #4

Ultimate Load = 26.81 K
$e_x = 1.68$ in.
$e_y = 6.5$ in.

Experiment - ---
Computer - ---
Ultimate Load - 26.81 K
\( e_x = 1.68 \text{ in.} \)
\( e_y = 6.5 \text{ in.} \)

\[ M_y \quad \text{K-in.} \]
\[ \phi_y \quad 1 / \text{IN.} \times 10^{-6} \]

Fig. - 5.13 \( M_y - \phi_y \) CURVE FOR COLUMN #1
Fig. - 5.14 P - $\delta_x$ COLUMN #2

Ultimate Load - 38.16K

$\delta_x = 1.53$ in.

$\delta_y = 5.0$ in.

Experiment - ----

Computer - ---
Fig. 5.15 $P - \delta_y$ CURVE FOR COLUMN #2

Ultimate Load -38.16 K
$e_x = 1.53$ in.
$e_y = 5.0$ in.

Experiment - 
Computer -
Fig. - 5.16  $P - \delta_x$ CURVE FOR COLUMN #3

Ultimate Load -35.06 K
$e_x = 1.68$ in.
$e_y = 5.5$ in.

Experiment - ----
Computer - ----
Fig. - 5.17 P - δy CURVE FOR COLUMN #3

Ultimate Load - 35.06 K

εx = 1.68 in.
εy = 5.5 in.

Experiment –
Computer –
Fig. - 5.18  P - $\delta_x$ CURVE FOR #1

Ultimate Load - 26.81 K
$e_x = 1.68$ in.
$e_y = 6.5$ in.

Experiment - ----
Computer - ______

P Kips

$\delta_x$ IN.

0.2 0.4 0.6 0.8 1.0 1.2 1.4

77
Fig. -5.19 $P - \delta_y$ CURVE FOR COLUMN #1

Ultimate Load - 26.81 K
$
\delta_x = 1.68$ in.
$
\delta_y = 6.5$ in.

Experiment - ----
Computer - ---

$P$ Kips

$\delta_y$ IN.
CHAPTER VI
COMPARATIVE STUDY AND DISCUSSION

Since the ultimate load, \( M_x - \phi_x \), \( M_y - \phi_y \) and \( p - \phi \) curves are of primary interest in this chapter the writer concentrates on the discussion and the comparative study of the experimental and the computer results for the same.

1. As seen in the experimental \( M - \phi \) curves, final rupture of the specimens were preceded by rapid, large curvature increase.

2. After the maximum moment was attained, the measured \( P - M - \phi \) relationships differed significantly from those calculated using commonly accepted concrete stress-strain curves with strain limits of 0.003 in. / in.

3. In comparing \( M_x - \phi_x \) curves there was a good agreement except the theoretical curves show more ductility of the specimens than indicated by the test results. This behavior might be attributed to the load controlled test procedure.

4. In comparing \( M_y - \phi_y \) curves there was extremely
good agreement. Experimental $M_y-\phi_y$ curves are well above the theoretical $M_y-\phi_y$ curves and show more ductility of the specimens than that of predicted by the computer results. Therefore it can be concluded that the computer program is on the conservative side.

5. For both the curves, $M_x-\phi_x$, $M_y-\phi_y$ a good agreement was found between the experimental and theoretical results for the first 70% of the load increments. As the load increased toward the failure, difference was larger.

6. The computer program accurately predicts the ultimate strength.

7. The theoretical and experimental curves do not coincide. This behavior might be attributed to the experimental errors and the fact that the measurements of the strain distribution were done over a 3 in. range as shown in Fig. 3.3. In the previous study (see Ref. 16), the measurements of the strains were taken over a 6 and 7.5 in. ranges for $X$ and $Y$ directions respectively. Even then the difference between the experimental and the theoretical results is not much.

8. A few experimental load-deflection curves do not agree with the analytical curves. This might again be attributed to unavoidable experimental errors and the
fact that the deflections in both the directions could not be measured exactly at mid-height of the columns.
## Table 6.1

**COMPARATIVE STUDY OF EXPERIMENTAL AND COMPUTER RESULTS**

<table>
<thead>
<tr>
<th>Col f'_c</th>
<th>e_x</th>
<th>e_y</th>
<th>P Expt.</th>
<th>P Comp.</th>
<th>M_x Expt.</th>
<th>M_x Comp.</th>
<th>M_y Expt.</th>
<th>M_y Comp.</th>
</tr>
</thead>
<tbody>
<tr>
<td>no.</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

2 4200 1.53 5.0 34.0 31.60 183.8 158.0 65.7 48.3

3 4200 1.68 5.5 35.06 35.02 210.8 192.3 78.3 58.8

4 4000 1.68 6.5 26.81 25.44 188.2 165.3 56.1 42.7

* -For calculation of the ultimate moments, M_x and M_y deflection at the loading stage before the collapse was considered.
CHAPTER VII
CONCLUSIONS

From the experimental and analytical results the following conclusions can be deduced.

1. Theoretical analysis (the computer program) accurately predicts the ultimate strength.

2. In general a good agreement between the experimental P-M-\( \phi \) and P-\( \delta \) relations and those of analytical was found. Consequently it can be concluded that the computer program developed by Hsu(1) can be used to find the ultimate strength, the moment-deformational characteristics, the stress-strain distribution across the section and the interaction surface of L-shaped, short column loaded biaxially with large and small eccentricities.

3. The results of this investigation could be used to develop the strength interaction diagrams and the failure surfaces that are needed in determining the value of an exponent \( \alpha \) that appears in the non-dimensional equation(1.1) suggested by Bresler(12).

4. Further research may be conducted to consider the
effects of length of the member, shape of the section, and the torsion for the analytical procedure of the computer program.
### APPENDIX -1

**X, Y-COORDINATES AND AREA OF ELEMENTS OF THE CROSS SECTION**

<table>
<thead>
<tr>
<th>Element No.</th>
<th>X Coordinate (in.)</th>
<th>Y Coordinate (in.)</th>
<th>Area (in.²)</th>
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</thead>
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</tr>
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<td>1.506</td>
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<td>0.110</td>
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<td>-2.052</td>
<td>0.110</td>
</tr>
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<td>-2.502</td>
<td>-1.371</td>
<td>0.110</td>
</tr>
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<td>-1.821</td>
<td>-0.003</td>
<td>0.110</td>
</tr>
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<td>7</td>
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<td>1.302</td>
<td>0.110</td>
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<tr>
<td>8</td>
<td>-0.459</td>
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<td>0.110</td>
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<td>9</td>
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<td>0.110</td>
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<td>1.559</td>
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<td>0.110</td>
</tr>
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<tr>
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<td>-1.396</td>
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9. ACI Committee 318, "Building Code requirements for Reinforced Concrete (ACI 318-77)".


