Fall 1971

The effect of variable properties on the laminar flow of gases

Norman Shilling
New Jersey Institute of Technology

Follow this and additional works at: https://digitalcommons.njit.edu/dissertations

Part of the Mechanical Engineering Commons

Recommended Citation
https://digitalcommons.njit.edu/dissertations/1356

This Dissertation is brought to you for free and open access by the Theses and Dissertations at Digital Commons @ NJIT. It has been accepted for inclusion in Dissertations by an authorized administrator of Digital Commons @ NJIT. For more information, please contact digitalcommons@njit.edu.
Copyright Warning & Restrictions

The copyright law of the United States (Title 17, United States Code) governs the making of photocopies or other reproductions of copyrighted material.

Under certain conditions specified in the law, libraries and archives are authorized to furnish a photocopy or other reproduction. One of these specified conditions is that the photocopy or reproduction is not to be “used for any purpose other than private study, scholarship, or research.” If a user makes a request for, or later uses, a photocopy or reproduction for purposes in excess of “fair use” that user may be liable for copyright infringement.

This institution reserves the right to refuse to accept a copying order if, in its judgment, fulfillment of the order would involve violation of copyright law.

Please Note: The author retains the copyright while the New Jersey Institute of Technology reserves the right to distribute this thesis or dissertation.

Printing note: If you do not wish to print this page, then select “Pages from: first page # to: last page #” on the print dialog screen.
The Van Houten library has removed some of the personal information and all signatures from the approval page and biographical sketches of theses and dissertations in order to protect the identity of NJIT graduates and faculty.
INFORMATION TO USERS

This dissertation was produced from a microfilm copy of the original document. While the most advanced technological means to photograph and reproduce this document have been used, the quality is heavily dependent upon the quality of the original submitted.

The following explanation of techniques is provided to help you understand markings or patterns which may appear on this reproduction.

1. The sign or “target” for pages apparently lacking from the document photographed is “Missing Page(s)”. If it was possible to obtain the missing page(s) or section, they are spliced into the film along with adjacent pages. This may have necessitated cutting thru an image and duplicating adjacent pages to insure you complete continuity.

2. When an image on the film is obliterated with a large round black mark, it is an indication that the photographer suspected that the copy may have moved during exposure and thus cause a blurred image. You will find a good image of the page in the adjacent frame.

3. When a map, drawing or chart, etc., was part of the material being photographed the photographer followed a definite method in “sectioning” the material. It is customary to begin phoetoing at the upper left hand corner of a large sheet and to continue phoetoing from left to right in equal sections with a small overlap. If necessary, sectioning is continued again — beginning below the first row and continuing on until complete.

4. The majority of users indicate that the textual content is of greatest value, however, a somewhat higher quality reproduction could be made from “photographs” if essential to the understanding of the dissertation. Silver prints of “photographs” may be ordered at additional charge by writing the Order Department, giving the catalog number, title, author and specific pages you wish reproduced.

University Microfilms
300 North Zeib Road
Ann Arbor, Michigan 48106
A Xerox Education Company
SHILLING, Norman Zethward, 1944-
THE EFFECT OF VARIABLE PROPERTIES ON THE
LAMINAR FLOW OF GASES IN CYLINDRICAL TUBES AT
LOW WALL TO BULK TEMPERATURE RATIOS.

Newark College of Engineering, D.Eng.Sc., 1972
Engineering, mechanical

University Microfilms, A XEROX Company, Ann Arbor, Michigan
PLEASE NOTE:

Some pages may have
indistinct print.
Filmed as received.

University Microfilms, A Xerox Education Company
THE EFFECT OF VARIABLE PROPERTIES ON THE LAMINAR FLOW OF GASES IN CYLINDRICAL TUBES AT LOW WALL TO BULK TEMPERATURE RATIOS

BY

NORMAN ZETHWARD SHILLING

A DISSERTATION

PRESENTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

DOCTOR OF ENGINEERING SCIENCE IN MECHANICAL ENGINEERING

AT

NEWARK COLLEGE OF ENGINEERING

This thesis is to be used only with due regard to the rights of the author(s). Bibliographical references may be noted, but passages must not be copied without permission of the College and without credit being given in subsequent written or published work.

Newark, New Jersey

1971
APPROVAL OF THESIS

THE EFFECT OF VARIABLE PROPERTIES ON THE LAMINAR FLOW OF
GASES IN CYLINDRICAL TUBES AT LOW WALL TO BULK TEMPERATURE RATIOS

BY

NORMAN ZETHWARD SHILLING

FOR

DEPARTMENT OF MECHANICAL ENGINEERING

NEWARK COLLEGE OF ENGINEERING

BY

FACULTY COMMITTEE

APPROVED: CHAIRMAN

_________________________________________

_________________________________________

_________________________________________

_________________________________________

NEWARK, NEW JERSEY

NOVEMBER, 1971
ABSTRACT

The problem of heat transfer in laminar flow of a gas through a constant diameter cylindrical tube is treated. The gas is cooled by the tube walls held at constant temperature. Two tube inlet conditions are considered: (1) fully developed velocity and uniform temperature profiles (Graetz boundary condition) and (2) uniform velocity and temperature (UTV) profiles. Results of the theoretical and experimental phases of the work are presented.

The theoretical solution is based on the compressible boundary layer equations with varying transport and thermodynamic property terms retained. For the Graetz condition, an existing finite difference solution scheme is modified for improved prediction of gradients at the wall. For the UTV condition, a combined analytical-numerical solution scheme is utilized. Similarity conditions are assumed at the tube entrance continuing to a short distance downstream. The results of this analytic solution are then patched to the numerical finite difference scheme. Improved convergence over the finite difference scheme is thus obtainable.

Numerical calculations of velocity and temperature profiles as well as of friction factors were carried out for air and helium at wall-to-bulk temperature ratios ranging from 0.1 to 0.95 with inlet Mach numbers varying from 0.01 to 0.05.

The results of the calculations are presented in terms of Nusselt number and product of friction factor and Reynolds
number vs. Graetz number. The local Nusselt number is shown to be relatively insensitive to variation in inlet wall-to-bulk temperature ratio, whereas the local friction factor Reynolds number parameter showed some sensitivity to the variation of this ratio.

Empirical equations are given for the Nusselt-Graetz number relationship and the friction factor-Reynolds number and a modified Graetz number relationship (which includes the temperature ratio effect).

To substantiate the theoretical results, a limited experimental investigation was conducted. Local heat fluxes and static pressure drops at several points along a 0.3 in. diameter tube were measured. Data was obtained for air for inlet Reynolds numbers ranging from 815 to 1950 and inlet wall to bulk temperature ratios ranging from 0.4 to 1.0.

Heat transfer data for the Graetz boundary condition and friction factor data for the UTV boundary condition are in substantial agreement with the theoretical results. Close agreement also exists for heat transfer results in the entrance for the UTV boundary condition, but in the downstream region the data falls approximately 30% below the theoretical. Friction factor data for the Graetz condition are substantially less than the theoretical prediction in the entrance. This may be due to a slight discontinuity in tube diameters (about 0.02 in.) between the flow development and cooling sections.
ACKNOWLEDGEMENTS

The author wishes to thank his principal advisor, Dr. Richard C. Progelhof, for his helpful criticisms and suggestions made during this investigation. He also appreciates the interest and encouragement of Dr. Rong Chen and Professor Robert Jacobs while the work was in progress. He is grateful for the constructive criticism offered by Dr. G. Peyser, Professor E.H. Stamper, Dr. P.J. Florio and Dr. William Haberman during the preparation of the manuscript and he is also grateful for the support provided by the Foundation for the Advancement of Graduate Study in Engineering.

Cahit Kitaplioglu aided in the correlation of theoretical results. Special thanks are extended to Anita LaSalle for her encouragement and help in the programming and debugging. Paul Baham and Richard Baseil also provided assistance.

These acknowledgements would not be complete if mention were not made of my dear wife, Eleanor, who in addition to her sacrifice and inexhaustable supply of encouragement also did the typing.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Abstract</td>
<td>iii</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>v</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>vi</td>
</tr>
<tr>
<td>List of Illustrations</td>
<td>viv</td>
</tr>
<tr>
<td>List of Tables</td>
<td>xiv</td>
</tr>
<tr>
<td>Nomenclature</td>
<td>xv</td>
</tr>
<tr>
<td>1. Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1. Objective</td>
<td>1</td>
</tr>
<tr>
<td>1.2. Method</td>
<td>1</td>
</tr>
<tr>
<td>1.3. Scope and Reason for Work</td>
<td>3</td>
</tr>
<tr>
<td>1.4. Previous Theoretical Work</td>
<td>6</td>
</tr>
<tr>
<td>1.5. Previous Experimental Investigations</td>
<td>15</td>
</tr>
<tr>
<td>2. Analytical Problem</td>
<td>23</td>
</tr>
<tr>
<td>2.1. Statement of the Problem</td>
<td>23</td>
</tr>
<tr>
<td>2.2. The Worsoe-Schmidt Analysis</td>
<td>39</td>
</tr>
<tr>
<td>3. Finite Difference Solution - The Graetz Boundary Condition</td>
<td>55</td>
</tr>
<tr>
<td>3.1. Basic Considerations</td>
<td>55</td>
</tr>
<tr>
<td>4. Uniform Temperature and Velocity Profile Boundary Conditions - Analytical Solution</td>
<td>82</td>
</tr>
<tr>
<td>4.1. Background</td>
<td>82</td>
</tr>
<tr>
<td>4.2. Choice of Method of Solution</td>
<td>84</td>
</tr>
<tr>
<td>4.3. Similarity Solution - Compressible Variable Property Boundary Layer Growth with Pressure Gradient for Tube Flow</td>
<td>91</td>
</tr>
</tbody>
</table>
4.4. Integration Procedure ......................... 99
4.5. Results ........................................ 110
4.6. Dissipation Function ............................ 128

5. Experimental Investigation ............................ 133
5.1. Introduction ..................................... 133
5.2. Experimental Apparatus ........................... 133
   A. Air Supply .................................. 135
   B. Preheater .................................. 135
   C. Development Section .......................... 136
   D. Exit Mixing Section .......................... 139
   E. Metering .................................. 139
   F. Flow Control ................................. 141
   G. Test Section ................................ 142
5.3. Calibration ..................................... 146
   A. Calibration of Heat Flux Calorimeters .... 146
   B. Calibration of Thermocouples ............. 150
   C. Adiabatic Development Section ............ 150
   D. Mixing Section and Bellmouth ............ 156
5.4. Leak Tests ..................................... 156
5.5. Adiabatic Pressure Drop and Friction Factors ........................................ 157
5.6. Repeatability Test .............................. 160
5.7. Wall Temperature Uniformity ....................... 160
5.8. Experimental Procedure .......................... 160
5.9. Data Reduction Program ........................... 168
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.</td>
<td>Experimental Results</td>
<td>180</td>
</tr>
<tr>
<td>6.1</td>
<td>Graetz Boundary Condition</td>
<td>180</td>
</tr>
<tr>
<td>6.2</td>
<td>UTV Boundary Condition</td>
<td>190</td>
</tr>
<tr>
<td>7.</td>
<td>Summary and Conclusions</td>
<td>200</td>
</tr>
<tr>
<td>7.1</td>
<td>Summary</td>
<td>200</td>
</tr>
<tr>
<td>7.2</td>
<td>Conclusions</td>
<td>201</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>205</td>
</tr>
<tr>
<td>Appendix A</td>
<td>Variable Property and Non-Boundary Layer Terms</td>
<td>215</td>
</tr>
<tr>
<td>Appendix B</td>
<td>Thermodynamic and Transport Properties</td>
<td>218</td>
</tr>
<tr>
<td>Appendix C</td>
<td>Calorimeter Conductance: Error in One Dimensional Heat Conduction Equation Due to Thermocouple Location</td>
<td>223</td>
</tr>
<tr>
<td>Appendix D</td>
<td>Calorimeter Radiation Calibration - End Effects and Conduction Losses</td>
<td>226</td>
</tr>
<tr>
<td>Appendix E</td>
<td>Uncertainty Analysis - Nusselt Number Friction Factor Data</td>
<td>231</td>
</tr>
<tr>
<td>Appendix F</td>
<td>Solution of Similarity Boundary Layer Equations</td>
<td>241</td>
</tr>
<tr>
<td>Appendix G</td>
<td>Data Reduction Program</td>
<td>259</td>
</tr>
<tr>
<td>Appendix H</td>
<td>Data</td>
<td>275</td>
</tr>
<tr>
<td>Appendix I</td>
<td>Derivation of the Similar Boundary Layer Equations</td>
<td>316</td>
</tr>
</tbody>
</table>
## LIST OF ILLUSTRATIONS

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Idealized boundary conditions</td>
<td>24</td>
</tr>
<tr>
<td>2.</td>
<td>Boundary layer regions in internal tube flow</td>
<td>28</td>
</tr>
<tr>
<td>3.</td>
<td>Relative magnitude of terms in axial momentum and energy equations $r/r_0 = 0.10$, $T_w/T_0 = 0.10$, UTV boundary condition</td>
<td>36</td>
</tr>
<tr>
<td>4.</td>
<td>Relative magnitude of terms in axial momentum and energy equations $r/r_0 = 0.95$, $T_w/T_0 = 0.10$, UTV boundary condition</td>
<td>37</td>
</tr>
<tr>
<td>5.</td>
<td>Designation of mesh points</td>
<td>43</td>
</tr>
<tr>
<td>6.</td>
<td>Effect of change of radial mesh size on $fRe_m$ when evaluated from cubic polynomial</td>
<td>58</td>
</tr>
<tr>
<td>7.</td>
<td>Effect of change of radial mesh size on $Nu_m$ when evaluated from 5 point spline</td>
<td>58</td>
</tr>
<tr>
<td>8.</td>
<td>Variation of local Nusselt number with radial mesh size at a fixed axial point. Integrated energy equation and 5 point spline</td>
<td>64</td>
</tr>
<tr>
<td>9.</td>
<td>Variation of $fRe_m$ with radial mesh size at a fixed axial point. Integrated momentum equation and cubic polynomial</td>
<td>64</td>
</tr>
<tr>
<td>10.</td>
<td>Axial variation of $fRe_m$ and $Nu_m$ for air Graetz boundary condition, $M_0 = 0.03$</td>
<td>67</td>
</tr>
<tr>
<td>11.</td>
<td>Axial variation of $fRe_m$ and $Nu_m$ for helium Graetz boundary condition, $M_0 = 0.05$</td>
<td>68</td>
</tr>
<tr>
<td>12.</td>
<td>Axial variation of $Nu_m$ for carbon dioxide Graetz boundary condition, $M_0 = 0.01$</td>
<td>69</td>
</tr>
<tr>
<td>13.</td>
<td>Axial variation of $fRe_m$ for carbon dioxide Graetz boundary condition, $M_0 = 0.01$</td>
<td>70</td>
</tr>
<tr>
<td>14.</td>
<td>Local Nusselt Number and $fRe_m$ versus $x_m^+$ for air. Graetz boundary condition, $M_0 = 0.03$</td>
<td>71</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>15.</td>
<td>$fRe_m$ versus $T_w/T_0$ for helium. Graetz boundary condition. $M_0 = 0.05$</td>
<td>73</td>
</tr>
<tr>
<td>16.</td>
<td>Variation of $fRe_m$ with $T_w/T_0$ for air. Graetz boundary condition. $M_0 = 0.01$</td>
<td>74</td>
</tr>
<tr>
<td>17.</td>
<td>Exponent in $fRe_m$ correlation for air, helium and carbon dioxide versus $T_w/T_0$. Graetz boundary condition.</td>
<td>76</td>
</tr>
<tr>
<td>18.</td>
<td>Axial development of reduced temperature and $u/u_m$ for air at two inlet temperature ratios</td>
<td>78</td>
</tr>
<tr>
<td>19.</td>
<td>Dimensionless radial velocity profiles for developing flow of air at two wall to inlet temperature ratios. Graetz boundary condition.</td>
<td>79</td>
</tr>
<tr>
<td>20.</td>
<td>Axial variation of $Nu_m$ and $fRe_m$ for UTV boundary condition. Helium, $T_w/T_0 = 0.95$, $M_0 = 0.03$</td>
<td>83</td>
</tr>
<tr>
<td>21.</td>
<td>One dimensional extrapolation of finite difference (Runge-Kutta) solution to zero step size</td>
<td>86</td>
</tr>
<tr>
<td>22.</td>
<td>Comparison of axial velocity development from simplified analysis with that of Hornbeck</td>
<td>90</td>
</tr>
<tr>
<td>23.</td>
<td>Flow diagram for solution similarity from boundary layer equations</td>
<td>101</td>
</tr>
<tr>
<td>24.</td>
<td>Flow diagram for coupling of boundary layer development to internal tube flow</td>
<td>103</td>
</tr>
<tr>
<td>25.</td>
<td>Comparison of velocity boundary layer before and after patch to finite difference solution</td>
<td>111</td>
</tr>
<tr>
<td>26.</td>
<td>Axial variation of dimensionless pressure defect for patched and completely finite difference solution. Helium, UTV boundary condition. $T_w/T_0 = 0.01$</td>
<td>112</td>
</tr>
<tr>
<td>27.</td>
<td>Comparison of downstream Nusselt Number behavior from patched and completely finite difference solutions. Air, UTV boundary condition. $T_w/T_0 = 0.5$, $M_0 = 0.01$</td>
<td>115</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>28.</td>
<td>Axial variation of ( fRe_m ) and ( Nu_m ) for helium at several inlet temperature ratios, UTV boundary condition, ( M_o = 0.03 )</td>
<td>116</td>
</tr>
<tr>
<td>29.</td>
<td>Axial variation of ( fRe_m ) and ( Nu_m ) for air at several inlet temperature ratios, UTV boundary condition, ( M_o = 0.01 )</td>
<td>117</td>
</tr>
<tr>
<td>30.</td>
<td>Comparison of isothermal Nusselt number development from other investigations</td>
<td>119</td>
</tr>
<tr>
<td>31.</td>
<td>Development of centerline axial velocity from patching solution compared with Hornbeck, UTV boundary condition</td>
<td>121</td>
</tr>
<tr>
<td>32.</td>
<td>Axial development of reduced temperature and ( U/U_m ) for air at two inlet temperature ratios, UTV boundary condition</td>
<td>122</td>
</tr>
<tr>
<td>33.</td>
<td>Dimensionless radial velocity profiles for developing flow of helium at two inlet temperature ratios, UTV boundary condition</td>
<td>124</td>
</tr>
<tr>
<td>34.</td>
<td>Reduced axial velocity development with ( x_m^+ )</td>
<td>126</td>
</tr>
<tr>
<td>35.</td>
<td>( fRe_m/(fRe)_I ) versus ( T_w/T_m ). Air, UTV boundary condition</td>
<td>129</td>
</tr>
<tr>
<td>36.</td>
<td>Effect of additional terms in dissipation function on local heat transfer</td>
<td>131</td>
</tr>
<tr>
<td>37.</td>
<td>Schematic diagram of experimental apparatus</td>
<td>134</td>
</tr>
<tr>
<td>38.</td>
<td>Photograph of preheater</td>
<td>137</td>
</tr>
<tr>
<td>39.</td>
<td>Photograph of adiabatic development section</td>
<td>137</td>
</tr>
<tr>
<td>40.</td>
<td>Schematic of inlet development section apparatus</td>
<td>138</td>
</tr>
<tr>
<td>41.</td>
<td>Schematic and photograph of exit mixing plenum</td>
<td>140</td>
</tr>
<tr>
<td>42.</td>
<td>Test section, Detail of pressure tap and heat flux calorimeter</td>
<td>144</td>
</tr>
<tr>
<td>43.</td>
<td>Photograph of heat flux calorimeter Pair</td>
<td>144</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>44</td>
<td>Photograph of test section and apparatus immediately before radiation calibration</td>
<td>148</td>
</tr>
<tr>
<td>45</td>
<td>Axial variation of calorimeter conductance</td>
<td>149</td>
</tr>
<tr>
<td>46</td>
<td>Velocity profile measuring apparatus Photograph of microscope stage and plug</td>
<td>152</td>
</tr>
<tr>
<td>47</td>
<td>Exit velocity profile from adiabatic development section</td>
<td>154</td>
</tr>
<tr>
<td>48</td>
<td>Calibration of adiabatic section thermocouple</td>
<td>155</td>
</tr>
<tr>
<td>49</td>
<td>Velocity profile from bellmouth used in UTV development section</td>
<td>155</td>
</tr>
<tr>
<td>50</td>
<td>Isothermal dimensionless pressure drop along test section with adiabatic development section</td>
<td>158</td>
</tr>
<tr>
<td>51</td>
<td>Friction factor in downstream region with UTV development section in place</td>
<td>159</td>
</tr>
<tr>
<td>52</td>
<td>Facsimile of original data sheet</td>
<td>164</td>
</tr>
<tr>
<td>53</td>
<td>Effect on dimensionless pressure drop of displacing point at which flow development commences, UTV boundary condition</td>
<td>176</td>
</tr>
<tr>
<td>54</td>
<td>Experimental dimensionless pressure defect Air, Graetz boundary condition, $T_wT_0=0.6$</td>
<td>181</td>
</tr>
<tr>
<td>55</td>
<td>Experimental dimensionless pressure defect Air, Graetz boundary condition, $T_w/T_0=0.5$</td>
<td>182</td>
</tr>
<tr>
<td>56</td>
<td>Experimental dimensionless pressure defect Air, Graetz boundary condition, $T_w/T_0=0.4$</td>
<td>183</td>
</tr>
<tr>
<td>57</td>
<td>Experimental friction factor results Air, $T_w/T_0=0.5$, Graetz boundary condition</td>
<td>185</td>
</tr>
<tr>
<td>58</td>
<td>Experimental friction factor results Air, $T_w/T_0=0.4$, Graetz boundary condition</td>
<td>186</td>
</tr>
</tbody>
</table>
59. Experimental local $N_u$, versus $x^+$, Graetz boundary condition.......................... 189

60. Experimental $q^+$ versus $x^+$ for air. Graetz boundary condition, $T_w/T_0 = 0.60, 0.40$........ 191

61. Experimental $q^+$ versus $x^+$ for air. Graetz boundary condition, $T_w/T_0 = 0.40$............. 192

62. Experimental dimensionless pressure defect versus $x^+$ for air. UTV boundary condition $T_w/T_0 = 0.60, 0.50, 0.40$................................. 193

63. Experimental $f Re_m$ versus $x^+$. Air, UTV boundary condition. $T_w/T_0 = 0.60, 0.50, 0.40$........ 195

64. Experimental $N_u$, versus $x^+$. Air, UTV boundary condition. $T_w/T_0 = 0.6, 0.5, 0.4$........... 197

65. Experimental $q^+$ versus $x^+$ for air. UTV boundary condition. $T_w/T_0 = 0.6, 0.5, 0.4$.......... 198

66. Thermodynamic and transport properties for helium .................................................. 220

67. Thermodynamic and transport properties for air......................................................... 221

68. Thermodynamic and transport properties for carbon dioxide........................................ 222

69. Coordinate system for radiation calibration ............................................................... 226


LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1 Review of previous experimental results</td>
<td>19</td>
</tr>
<tr>
<td>3.1 Transport and thermodynamic properties</td>
<td>57</td>
</tr>
<tr>
<td>3.2 High ordered profile derivatives- heating and cooling</td>
<td>61</td>
</tr>
<tr>
<td>5.1 Test section dimensions</td>
<td>142</td>
</tr>
<tr>
<td>E.1 Uncertainty intervals- experimentally measured quantities</td>
<td>233</td>
</tr>
<tr>
<td>E.2 Uncertainty intervals for test run #44- local heat transfer data</td>
<td>236</td>
</tr>
<tr>
<td>E.3 Uncertainty intervals for non-dimensionalized pressure drop</td>
<td>240</td>
</tr>
</tbody>
</table>
## Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_1$, $A_2$</td>
<td>area</td>
</tr>
<tr>
<td>$A_n^I$, $A_n^{II}$</td>
<td>coefficient in linearized form of momentum and energy difference equations pertaining to radial node $n$</td>
</tr>
<tr>
<td>$a$</td>
<td>exponent in power law for specific heat, or exponent in wall parameter correlation</td>
</tr>
<tr>
<td>$B_n^I$, $B_n^{II}$</td>
<td>coefficient in linearized form of momentum or energy difference equations pertaining to radial node $n$</td>
</tr>
<tr>
<td>$b$</td>
<td>exponent in power law for viscosity, or exponent in wall parameter correlation</td>
</tr>
<tr>
<td>$C'$</td>
<td>coefficient in expression for local friction factor, $2\int \rho u^2 r dr / \rho_m u_m^2$</td>
</tr>
<tr>
<td>$C$</td>
<td>coefficient in transformation of axial coordinate in similar boundary layer solution</td>
</tr>
<tr>
<td>$C_n^I$, $C_n^{II}$</td>
<td>coefficient in linearized form of momentum or energy difference equations pertaining to radial node $n$</td>
</tr>
<tr>
<td>$c$</td>
<td>exponent in power law for thermal conductivity</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>$c_p^+$</td>
<td>dimensionless specific heat</td>
</tr>
<tr>
<td>$c_v$</td>
<td>specific heat at constant volume</td>
</tr>
<tr>
<td>$D$</td>
<td>diameter</td>
</tr>
<tr>
<td>$D_n^{II}$, $D_n^{I}$</td>
<td>term in linearized form of momentum or energy difference equations pertaining to radial node $n$</td>
</tr>
<tr>
<td>$E$</td>
<td>voltage, or sum-squared error</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>---------</td>
<td>---------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$E_n^I, E_n^I$</td>
<td>coefficient in recursive relationship for axial velocity or enthalpy</td>
</tr>
<tr>
<td>$e$</td>
<td>thermocouple output</td>
</tr>
<tr>
<td>$F$</td>
<td>hypothetical closed form solution for velocity or enthalpy</td>
</tr>
<tr>
<td>$F_1(\eta), F_2(\eta)$</td>
<td>right hand sides of momentum and energy similar boundary layer equations</td>
</tr>
<tr>
<td>$F_{2-1}$</td>
<td>viewfactor from heating element to tube wall in radiation calibration</td>
</tr>
<tr>
<td>$f$</td>
<td>friction factor based on local wall shear stress</td>
</tr>
<tr>
<td>$f_{\Delta p}$</td>
<td>friction factor based on static pressure gradient</td>
</tr>
<tr>
<td>$f$</td>
<td>velocity function, $U/U_e$</td>
</tr>
<tr>
<td>$G$</td>
<td>enthalpy function, $H/H_e$</td>
</tr>
<tr>
<td>$G_n^I, G_n^I$</td>
<td>term in recursive relationship for velocity or enthalpy</td>
</tr>
<tr>
<td>$H$</td>
<td>enthalpy</td>
</tr>
<tr>
<td>$H_1^+$</td>
<td>dimensionless enthalpy, $(H - H_o)/c_pT_o$</td>
</tr>
<tr>
<td>$H_2^+$</td>
<td>dimensionless enthalpy, $(H - H_w)/c_pT_o$</td>
</tr>
<tr>
<td>$l$</td>
<td>current</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity</td>
</tr>
<tr>
<td>$k^+$</td>
<td>dimensionless thermal conductivity, $k/k_o$</td>
</tr>
<tr>
<td>$K$</td>
<td>calorimeter conductance, $q_w/(T_i - T_o)$</td>
</tr>
<tr>
<td>$L$</td>
<td>length</td>
</tr>
<tr>
<td>$M$</td>
<td>total number of axial steps in finite difference solution</td>
</tr>
</tbody>
</table>
\( M_0 \) inlet Mach number

\( \dot{M} \) mass flow rate

\( m \) axial node index

\( N \) radial node index at wall

\( \text{Nu}_m \) local Nusselt number, \(-2r_0q''_w/k_m(T_w-T_m)\)

\( n \) radial node index

\( P \) dimensionless pressure defect, \((p_o-p)/\rho_oU_0^2\)

\( \text{Pe} \) Peclet modulus, \( Re Pr \)

\( \text{Pr} \) Prandtl number, \( c_p\mu/k \)

\( \text{Pr}^+ \) modified Prandtl number, \( c_p\mu^+/k^+ \)

\( \rho \) static pressure

\( \rho^+ \) dimensionless static pressure, \( \rho/\rho_o \)

\( Q_r \) radiative heat flux

\( q''_w \) heat flux at tube inner wall

\( q^+ \) dimensionless wall heat flux, \( r_0q''_w/k_oT_o \)

\( R \) gas constant in perfect gas law

\( R_1 \) ratio of neglected terms to viscosity variation terms in axial momentum equation

\( R_2 \) ratio of neglected terms to specific heat and thermal conductivity variation terms in energy equation

\( R_3 \) ratio of axial molecular momentum transfer to axial convective momentum transfer

\( R_4 \) ratio of axial molecular heat transfer to axial convective heat transfer

\( \text{Re}_o \) inlet Reynolds number, \( 2r_0U_0\rho_o/\mu_o \)

\( R_o, r_o \) tube radius

\( \text{Re}_m \) Reynolds number based on local mean properties, \( 2r_0U_m\rho_m/\mu_m \)
Symbols

- \( r \) radial coordinate
- \( r^+ \) dimensionless radial coordinate, \( r/r_0 \)
- \( T \) temperature
- \( U \) axial velocity
- \( U^+ \) dimensionless axial velocity, \( U/U_0 \)
- \( \bar{U} \) representative magnitude of axial velocity variation
- \( \bar{V} \) radial velocity
- \( V^+ \) dimensionless radial velocity, \((V/U_0)Re_0Pr_0\)
- \( V \) representative magnitude of radial velocity variation
- \( x \) axial coordinate
- \( x^+ \) dimensionless axial coordinate (modified Graetz parameter), \( x/r_0Re_0Pr_0 \)
- \( x_m^+ \) dimensionless axial coordinate based on local mean properties
- \( y^+ \) dimensionless distance from tube wall, \( l-r^+ \)
- \( \gamma \) thermodynamic or transport property, or dependent variable

Greek Letter Symbols

- \( \beta \) modified Falkner Skan parameter, \( \frac{2\xi}{U_0^+} dU_0^+ \)
- \( \Gamma \) ratio of specific heats, \( c_p/c_v \)
- \( \Delta p_x, \Delta p_r \) representative magnitudes of radial and axial pressure variation
- \( \Delta r^+, \Delta x^+ \) dimensionless radial and axial mesh steps
- \( \delta, \delta^2 \) radial difference operators
- \( \epsilon \) small parameter in error expansion for wall derivative of axial velocity
\( \zeta \)  
term in Navier Stokes equations

\( \eta \)  
similarity parameter, \( \frac{U_e^+}{\sqrt{2\zeta_0}} \int_0^{y^+} \rho^+ dy^+ \)

\( \theta \)  
temperature ratio, \( T/T_0 \)

\( \Lambda_n \)  
Lagrangian polynomial of degree n-1

\( \lambda \)  
term in transformed momentum and energy similarity boundary layer equations, \( \mu^+\rho^+C_{p\lambda}^+\mu^+_c \) or second coefficient of viscosity

\( \mu \)  
absolute viscosity

\( \mu^+ \)  
dimensionless viscosity, \( \mu/\mu_0 \)

\( \xi \)  
transformed axial coordinate, \( \int_0^{x^+} \rho^+ \mu^+_c dx^+ \)

\( \rho \)  
density

\( \rho^+ \)  
dimensionless density, \( \rho/\rho_0 \)

\( \sigma \)  
weighting factor in divided difference derivative representations, or Stefan-Boltzmann constant

\( \tau_{w^+}^+ \)  
dimensionless wall shear stress, \( r_0 \tau_w/\mu_0 U_0 \)

\( \tau_w \)  
local wall shear stress, \( \mu_w \frac{\partial U}{\partial r^+} \bigg|_{r=r_0} \)

\( \tau_{w,\Delta p} \)  
wall shear stress due to static pressure gradient

\( \Phi \)  
dependent variable

\( \phi \)  
angle, or dependent variable

\( \Omega \)  
dummy independent variable

\( \omega \)  
absolute uncertainty interval

Subscripts

cp constant property

e evaluated at edge of boundary layer

I isothermal
i  inner, or running index
k  running index
m  mean or bulk
m,n referring to axial node m, radial node n
o  outer
w  evaluated at wall
x  local
∞  far field
CHAPTER 1. INTRODUCTION

1.1. Objective

It is intended that the present investigation shall give definitive answers to the effect of high rate cooling on the local heat transfer and wall friction parameters for the laminar flow of gases through cylindrical tubes. Two commonly encountered gases - helium and air, are examined in detail. Temperature differences considered may be large enough such that substantial variations in gas transport and thermodynamic properties occur. The tube wall temperature is constant. Initially, radial temperature profiles are uniform and velocity profiles may be either fully developed (parabolic) or uniform at the point where cooling of the gas commences. In particular, it is desired to,

1. Obtain a theoretical prediction of the axial behavior of the developing flow.

2. Develop satisfactory design correlations for the results of the analysis.

3. Test the theoretical analysis by obtaining experimental data under conditions treated in the theoretical analysis.

1.2. Method

For the theoretical analysis, the reduction of the governing equations of motion, continuity and energy to the boundary layer equations is examined. A finite difference algorithm for solution of the combined continuity,
energy and (axial) momentum boundary layer equations with property variation has been obtained from Worsoe-Schmidt (ref. 100) and modified for use in the present investigation. Because of the non-convergence of the finite difference approach when a uniform inlet velocity profile is specified, an analytical boundary layer solution is applied at the tube entrance. This solution, which is based on a similarity assumption and which includes property variations, is patched to the finite difference solution at a downstream point. An improved method of evaluating wall parameters is examined. Correlation of the wall parameter results in terms of significant operational parameters is attempted along with criteria as to when property variations can be ignored.

For the experimental portion of the investigation, a cylindrical test section is fabricated and calibrated for the measurement of axial variation of local heat transfer and static pressures for laminar gas flow. Two inlet configurations provide approximately the two general sets of boundary conditions examined in the theoretical portion of the investigation. Experimental data will be used to verify, when possible, the assumptions inherent in the theoretical solution or to point out areas where the analysis may be deficient.

A additional advantage of the combined experimental
and theoretical approach is in the practical type of data that is supplied. The idealized boundary conditions treated in the theoretical portion are unattainable in a physical situation. A special effort was made in the design of the apparatus to approach the idealized conditions. Comparison of results with those from the theoretical solution will help to determine if the theoretical results can be applied to physical situations which also do not attain either of the idealized conditions, but are closer to one set than the other. In a crude sense, the derivative or sensitivity of the wall parameters to small deviations in the boundary conditions has been defined in addition to its value at the limiting cases. This type of data is of much greater value to the designer.

1.3. Scope and Reason for the Work

Recently, interest in the effect of variable fluid properties on internal laminar fluid flow has greatly increased. This can be attested to by the large number of investigations, both experimental and analytical, which have been addressed to this problem in the past decade. Advances in technology have extended the range of temperature at which gas flow is utilized from temperatures near the cryogenic range to several thousand degrees. Extreme temperature differences occurring in a gas flow
situation can make the available constant property solutions inapplicable for prediction of heat transfer and flow characteristics. The majority of works devoted to prediction of these characteristics when gas properties vary appreciably deal with heating of the gas. Only a small portion is concerned with cooling of the gas. This is surprising since in many applications where extreme heating occurs, extreme cooling is also obtained.

An example of a possibly very important future use of gas in a heat transfer application with extreme temperature differences can be seen in the development of fast breeder reactors. Fast breeder reactors operate at far higher temperatures than conventional reactors but the result is a higher thermodynamic efficiency and reduced thermal pollution. Also, the breeder reactor produces more fissionable material than it consumes. Gas cooling has a distinct advantage (as opposed to liquid cooling) in fast reactors since bubbles or voids cannot form in a gas (ref. 78). A bubble or void might lead to overheating in a localized area which cannot be detected and may result in consequent failure of a fuel pin or rupture of a coolant passage. This type of failure was responsible for the accident at the Enrico Fermi Nuclear Generating Plant in 1967. While turbulent flow conditions would normally be used for the operating mode, laminar
flow may exist for periods during shutdown, low power operation or loss of flow accidents. Increasing public concern over the safety of nuclear facilities restricts the margin for design error and demands that the designer have data available for all possible modes of operation.

Modern electronic technology has triumphed in its ability to miniaturize, but the result has been the creation of extremely high power densities in electronic equipment and a subsequent need for cooling. In order to realize the decrease in size, heat exchangers must of necessity also be kept small. Laminar gas flow is important in this application since the maximum ratio of heat transferred to pumping power required is obtained with laminar flow in compact heat exchangers. While the gas would be undergoing heating in the equipment, applications outside the earth's atmosphere would require recirculation, and subsequently, cooling of the gas.

Several high energy rocket propellants have been developed which can be solidified at very low temperatures and made suitable for use in solid fuel rockets. Prior to burning it is necessary that the fuel be raised to melting temperature. It has been proposed that this could be accomplished by passing high temperature gas through passages in the supercooled propellant. Precise control of the supply of molten propellant generated would
require an accurate estimate of the heat transfer from the gas.

Recent application of the Brayton cycle in aerospace applications requires recirculation and cooling of the gas. Since flow and heat transfer losses may make up a substantial portion of the energy expended by the working fluid in the cycle, it is important that precise correlations be available for the cooling as well as the heating of the gas. Here again laminar flow becomes attractive because of its efficiency.

1.4. Previous Theoretical Work

Theoretical consideration of heat transfer for a fluid in laminar flow in a cylindrical tube dates back to the first (correct) derivation of the partial differential equation for conservation of energy derived by Poisson (68) in 1835. The first solution to this equation was published fifty years later by Graetz (31) in 1885. Graetz assumed radial symmetry of the velocity and temperature profiles, constant fluid properties and that second order derivatives of the temperature and velocity in the axial direction could be neglected with respect to other terms. In addition, Graetz assumed the following set of boundary conditions which has come to be known as the Graetz condition:

1. At $x = 0$, the tube wall undergoes a step change
from $T_0$ to $T_w$ and remains constant at $T_w$ for $x \geq 0$.

2. For $x \leq 0$, the fluid temperature is uniform at $T_0$.

3. The velocity profile is fully developed (parabolic) at $x = 0$.

In his analysis, Graetz assumed constant fluid properties so that the axial velocity profile is invariant with axial displacement. Also implicit in this last condition is a zero radial velocity component for all $x$. Upon substitution of the parabolic velocity profile in the energy equation, a linear second order partial differential equation with temperature as the dependent variable is obtained. Graetz obtained an infinite series solution along with the first three eigenfunctions and eigenvalues for the series. Higher order eigenfunctions, eigenvalues and additional solutions for these boundary conditions can be found in papers by Drew (24), Jakob (40), Larkin (53), Lipkis (55) and Sellers, Tribus and Klein (79). The latter authors (79) also obtained a solution for the laminar flow of a gas in a cylindrical tube for the case of uniform energy input by a superposition of constant wall temperature solutions. A more direct approach to the uniform heat addition problem has been presented by Siegel, Sparrow and Hallman (80).

When the axial conduction term in the energy equation, $k \left( \frac{\partial^2 T}{\partial x^2} \right)$ is non-negligible, the energy equation reduces to a form for which the eigenfunctions are no
longer orthogonal. To circumvent this problem, Singh (82) obtained expansions of the appropriate eigenfunctions for the case of constant wall temperature in terms of eigenfunctions for an auxiliary equation satisfying identical boundary conditions. For the case of constant heat addition, Hsu (36) showed that the solution for the case with axial conduction can be reduced to the solution for zero axial conduction as a special case. Hsu also derived the solution in the same eigenfunction form as for the case of zero axial conduction—the only difference being in the magnitude of the eigenfunctions, eigenvalues and coefficients of terms in the infinite series. The results for both of these analyses showed that the effect of axial conduction is negligible for Peclet numbers, Pe (i.e., product of Reynolds number defined in terms of axial displacement (ReX) and Prandtl number (Pr) greater than 100).

To date, there does not appear to be any closed form analytical solution for the laminar flow of a fluid in a circular tube with simultaneous development of velocity and temperature profiles. Theoretical results presented are based either partially or totally upon numerical techniques. The first of the solutions for these inlet conditions was given by Kays (42). Kays neglected the radial velocity component and assumed constant properties. In this case the axial momentum equation becomes uncoupled
from the energy equation and use could be made of a solution for the developing velocity field in a tube previously obtained by Langhaar (52). Langhaar solved the momentum equation by making several linearizing assumptions. Kays integrated the energy equation numerically for Pr = 0.7. He found that there was a significant increase in the Nusselt number over that obtained for a fully developed profile. Ulrichson and Schmidt (92) refined the work of Kays to include the radial component of velocity. Their results indicated a significant decrease in the calculated Nusselt number from Kays' results at points near the entrance. An implicit total finite difference solution to the momentum equation was presented by Hornbeck (34). Fairly large variation was found compared to the velocity profiles by Langhaar. However, good agreement was found to exist between the axial pressure variations.

One of the first analytical attempts to account for the effect of property variations on the flow of a gas was made by Deissler (20) for the case of uniform heat flux. Deissler assumed fully developed velocity and temperature profiles, so that his analysis would apply only in a region far from the entrance. He removed axial dependence from the governing equations by neglecting acceleration terms in the axial direction and assuming 1. zero radial velocity, 2. constant axial gradient of the bulk gas temperature (uniform heat addition) and
3. that axial variations in fluid properties were negligible with respect to radial variations. Further assuming that the fluid density varied inversely with temperature and that viscosity and thermal conductivity varied as absolute temperature raised to the 0.68 power, Deissler solved the coupled energy and momentum equations simultaneously by an iterative procedure. Although Deissler did not check his results experimentally, the data in several other references indicate that at high wall to bulk temperature ratios the friction factor is significantly underestimated. Sze (87) refined Deissler's analysis by use of actual experimental transport property variations. His results were in substantial agreement with those of Deissler.

A combined experimental and analytical investigation of the laminar flow of carbon dioxide near its critical point was presented by Koppel and Smith (48). The authors essentially linearized the momentum equation by assuming the radial velocity component was negligible and that the product of density and axial velocity at any radial point is independent of the axial coordinate. These results are rather restricted in their applicability to the flow of other gases due to the severe and unique variation of the density and transport properties of CO₂ at its critical point.

Davenport (18) extended Deissler's analysis by
including a radial velocity component. In essence, Davenport concluded that the temperature and velocity profiles are never fully developed. In order to test his hypothesis, he derived a set of axially independent energy and momentum equations in which the radial velocity component was left as an arbitrary function subject to the conditions that the radial velocity be zero at the tube wall and centerline, and that at any radial point the outward convective flux cannot exceed the inward conduction heat transfer. By assuming different forms of the radial velocity distributions, Davenport solved the coupled equations by an iterative procedure. His results indicated that the postulated radial velocity was sufficient to account for the experimentally determined variation of the friction factor. The predicted effect on the Nusselt number was less pronounced but depended more heavily on the postulated variation of the radial velocity component.

Worsoe-Schmidt (100) using a finite difference solution with a variable implicitness to the continuity and coupled momentum and energy equations included the effect of variable fluid properties. Specific heat, viscosity and thermal conductivity were assumed to obey power law variations with absolute temperature ratio and the fluid density was assumed to obey the perfect gas law. Although the solution was quite satisfactory for gas heating
and a fully developed inlet velocity profile, a single example computed with uniform velocity at the entrance did not converge to the proper constant property solution for the Nusselt number downstream. Worsoe-Schmidt postulated that this was primarily due to large errors in the solution of the momentum and energy equations at points near the tube entrance. He also postulated that either a restrictively small finite difference mesh size or an appropriate analytical boundary layer solution at the entrance would remove this problem. However, for the Graetz boundary condition, the effect of the variable properties on the Nusselt number when based on properties evaluated at the local bulk temperature was rather small and in good agreement with experimental data. The predicted friction factor increased with heating rate, but not as rapidly as the experimentally measured values. Only one example was calculated for gas cooling, and this was for a fully developed inlet velocity profile.

Following Worsoe-Schmidt's example, several finite difference and finite volume solutions for laminar internal flow with variable fluid properties have appeared. A slightly different algorithm for integration of the same set of equations was published a short time later by Deissler and Presler (20) for the case of constant heat addition and uniform velocity and temperature profiles at the tube entrance. Convergence of the wall
parameters was obtained in the far downstream region, but provision was not made for inclusion of other boundary conditions. The effect of variable fluid properties here also showed slight effect on the heat transfer results, but marked effect on the shear stress. Since the boundary conditions examined differed from those in Worsoe-Schmidt's analysis, direct comparison of numerical results is not possible.

A recent numerical solution allowing for inclusion of an eddy exchange coefficient for turbulent motion in addition to the molecular terms for transport properties has been published by Bankston and McEligot (6). Sample calculations for laminar flow included a uniform temperature profile at the entrance and varying hydrodynamic entry lengths with the extremes of fully developed and uniform velocity profiles included. Provision was made for specification of arbitrary inlet profiles. The only wall condition provided for was that of specified heat flux, although this may be variable with the axial coordinate. No cases with gas cooling were presented.

Swearingen (86) in his Ph.D thesis presented a finite difference solution for laminar variable property flow between parallel plates along with experimental results for laminar flow heating in a cylindrical tube which will be discussed later. Swearingen, allowing for a radial pressure distribution, included the radial momentum equation
in the set of finite difference equations to be solved, although the other usual boundary layer assumptions were invoked. Flow between parallel plates bears several resemblances to flow in cylindrical tubes since: 1. the flow is two-dimensional, 2. the flow is internal, 3. the boundary layer equations apply at some distance from the entrance and 4. thermal and/or velocity boundary layers are present at the wall in the entrance. In Swearingen's case, only results for the case of specified wall heat flux and fully developed inlet velocity profiles were generated. Specification of a constant wall temperature for all cases with compressible flow resulted in oscillations of the wall parameters near the entrance which were large enough to render the solution of little value in this region. Attempts to remove this oscillation by application of an analytical starting solution at the first two axial steps were not successful. Surprisingly, no radial pressure distributions for any cases were presented.

In a finite volume solution also for variable property flow between parallel plates, Schade and McE1igot (73) were able to obtain solutions for specified wall temperatures and uniform wall heat flux. The radial momentum equation was neglected. Both uniform and fully developed inlet velocity profiles for heating and cooling of the gas were treated. Step changes in the wall temperature were approximated by increasing the wall temperature over the
first twenty axial steps until the desired value was reached. For several cooling cases specified with a fully developed inlet velocity, the pressure was seen to rise with axial distance in the thermal entrance region. For severe cooling and uniform inlet velocity, the pressure drop in the entrance was found to be very small.

Similar finite difference and finite volume numerical methods have been applied to laminar plasma flow. While property variations associated with plasma cooling are indeed extreme, the present study is addressed to laminar flow of gases at subplasma temperatures. Characteristics unique to plasmas limit the relevance of these investigations to the topic under consideration. These characteristics, along with a review of notable literature in this field will be reserved for a later section.

To this date, no comprehensive numerical solutions were found for cooling and for simultaneous development of velocity and temperature profiles with uniform wall temperatures for flow in cylindrical tubes.

1.5. Previous Experimental Investigations

Experimental results for the laminar flow of gases in circular tubes are meager. This is due in part to the low heat transfer rates encountered in laminar flow. Heat losses from the test section are usually large in comparison with the heat transfer to the gas and can be
difficult to account for. Of those experiments performed in apparatus designed to minimize free convection effects, the investigations of Kroll (51), Weiland and Lowdermilk (97), Taylor and Kirchgessener (88), Kays and Nicoll (43), Davenport (18), Dalle Donne and Bowditch (17), Taylor (89), Bergman and Koppel (7) and Swearingen (86) are the most notable. The conditions under which the data was taken are presented in Table I along with correlations proposed. Only one of these reports data for gas cooling (43). With one exception, (7), the experimentally measured Nusselt number and friction factor under the conditions of low to moderate heat flux and negligible natural convection effects were in relatively close agreement with the predicted values from the Graetz (31) and the Sellars, Tribus and Klein (79) solutions. Bergman and Koppel report lower heat transfer coefficients for uniform heat flux at low axial velocities than those predicted by the Sellars, Tribus and Klein analysis, and also a Reynolds number dependence which is not predicted in any of the cited references. They postulate that this is due to an increase in the importance of the radial velocity component at low axial velocities. The Reynolds number dependence may be explained by the reduced validity of the usual boundary layer assumptions for low Reynolds numbers (100).

However, when the heat flux becomes relatively large, the experimental results of Davenport, Dalle Donne and
Bowditch, Kays and Nicoll, and Taylor show significant deviation of the friction factor from values predicted from constant property results, whereas the Nusselt numbers for both uniform energy input and uniform surface temperature were found to be in relatively close agreement with the constant property values. Swearingen, while not taking any pressure drop data, found that the difference between his heat transfer results and Worsoe-Schmidt's predictions were within his estimated experimental uncertainty. Swearingen considered this as being a confirmation of the assumptions made in the Worsoe-Schmidt analysis. However, Searingen maintained a flow development section of 100 diameters prior to the test section. It would seem that a more critical test of Worsoe-Schmidt's assumptions could be obtained for simultaneous velocity and temperature profile development where large axial second derivatives would occur in the momentum as well as in the energy equation at points near the tube entrance.

Of these experimental works, only that of Kays and Nicoll deals with gas cooling. Mean, rather than local, Nusselt numbers were measured for air. No friction factor or pressure drop data were obtained. Velocity profiles were essentially fully developed at the point where cooling commenced since Kays used a development section of about 60 diameters. The bulk of the data was found to lie
about five per cent below the constant property solution for ratios of the logarithmic mean fluid temperature to wall temperature ranging from approximately 1.0 to 1.8. This deviation was within the estimated experimental uncertainty and Kays postulates that it was due to a fixed error in the measurement of the inlet air temperature. In none of these investigations were detailed measurements made of velocity and temperature profiles.

When gases are heated to temperatures sufficiently high such that the ionization fraction becomes non-negligible, the gas is described as a plasma. Recently a great deal of attention has been devoted to this topic. Plasma heat transfer differentiates itself from that of a non-dissociated gas in several ways (3, 26, 27). Radiation heat transfer is added to that by conduction to the wall. In regions where a high cooling rate predominates, a condition of thermal non-equilibrium can exist. Electron temperatures can exceed heavy particle temperatures by several thousand degrees (3). Also, because of appreciable concentration gradients there is a diffusion of electrons to the cool wall where recombination and consequents release of ionization energy can enhance heat transfer-- for this reason, a plasma must be treated as a reacting two component gas.
Table I. Review of Experimental Results

<table>
<thead>
<tr>
<th>Author</th>
<th>Gas</th>
<th>Reynolds number</th>
<th>Boundary Conditions</th>
<th>Boundary Conditions</th>
<th>L/D</th>
<th>Correlations, Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Davenport (18)</td>
<td>Helium</td>
<td>200-2200</td>
<td>Q</td>
<td>1.1-2.2</td>
<td>128</td>
<td>Nu=Nu&lt;sub&gt;cp&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>f&lt;sub&gt;f&lt;/sub&gt; = (T&lt;sub&gt;w&lt;/sub&gt;)&lt;sup&gt;1.35&lt;/sup&gt;</td>
</tr>
<tr>
<td>Kays &amp; Nicoll (42)</td>
<td>Air</td>
<td>664-1300</td>
<td>Q</td>
<td>1.1-1.9</td>
<td>80</td>
<td>Nu=Nu&lt;sub&gt;cp&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td>955-1300</td>
<td>T</td>
<td>0.56-0.85</td>
<td>66.7</td>
<td>Nu=Nu&lt;sub&gt;cp&lt;/sub&gt;</td>
</tr>
<tr>
<td>Dalle Donne &amp; Bowditch (17)</td>
<td>Air, Helium</td>
<td>100-2000</td>
<td>Q</td>
<td>1.1-1.80</td>
<td>90</td>
<td>Nu=Nu&lt;sub&gt;cp&lt;/sub&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>180</td>
<td>f&lt;sub&gt;f&lt;/sub&gt; = (T&lt;sub&gt;w&lt;/sub&gt;)&lt;sup&gt;1.67&lt;/sup&gt; (1/2&quot; D)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>f&lt;sub&gt;f&lt;/sub&gt; = (T&lt;sub&gt;w&lt;/sub&gt;)&lt;sup&gt;3.33&lt;/sup&gt; (1/4&quot; D)</td>
</tr>
<tr>
<td>Kroll (51)</td>
<td>Air</td>
<td>80-2300</td>
<td>T</td>
<td>1.1-1.35</td>
<td>11-98</td>
<td>Nu&lt;sub&gt;b&lt;/sub&gt;=1.5(we&lt;sub&gt;p,m&lt;/sub&gt;)&lt;sup&gt;0.40&lt;/sup&gt;</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>f= 16&lt;sub&gt;Ref&lt;/sub&gt;</td>
</tr>
<tr>
<td>Taylor (88)</td>
<td>Helium</td>
<td>370-1300</td>
<td>Q</td>
<td>1.1-3.4</td>
<td></td>
<td>f= 16&lt;sub&gt;Ref&lt;/sub&gt;</td>
</tr>
<tr>
<td>Bergman &amp; Koppel (7)</td>
<td>Nitrogen</td>
<td>N.A.</td>
<td>Q</td>
<td>1.1-1.13</td>
<td>330</td>
<td>Reynolds number dependence</td>
</tr>
<tr>
<td>Swearingen (86)</td>
<td>Helium</td>
<td>1442-2065</td>
<td>Q</td>
<td>1.1-1.9</td>
<td>1.2-97</td>
<td>Confirmation of Results in Ref. 100</td>
</tr>
</tbody>
</table>
Early theoretical treatments of plasma flow suffered from restrictive assumptions made in the formulation of the problem. In 1967, Watson and Pegot (96) published a numerical solution for the combined energy (with Ohm's law), momentum and continuity equations in the arc region of a plasma generator. Of greater relevance here is the excellent finite difference treatment of plasma flow in the arc free region of a circular tube published by Incropera, et al (38,39). Radiation and recombination effects were included in the analysis, but it was not possible to include thermal non-equilibrium and its effect on the thermal conductivity. It was postulated that this was one of the reasons that poor comparison with existing experimental results was found. It was not possible to correlate the heat transfer results in terms of variables which are effective for moderate temperature gas flow. Also, wall parameters were found to be extremely sensitive to the assumed inlet profiles.

Unfortunately, experimental investigations for plasmas suffer from a lack of consistent inlet conditions. For example, in the experimental studies by Johnson, Choksi and Eubank (41) the flow underwent an abrupt expansion immediately after the plasma generator. Also, a spin was imparted to the gas by the plasma generator in this investigation and those of Skrivan and Jaskowski (83) and Wethern and Brodkey (98). In no case was the magnitude
of the spin accurately measured or its effect on the heat transfer and flow characteristics isolated. In the experimental study by Cann (10), the constricted arc region was extended so that a smooth transition into the cooling section was obtained. However, a small but non-negligible stabilizing axial magnetic field was applied in the cooled section with a probable effect on the wall parameters. Due to the rapid deterioration of any plasma condition, the cooling sections in all these studies were relatively short. For example, in the Johnson et al study, the maximum length to diameter ratio was 6.

Additional problems associated with plasma experimentation are the cost and difficulty of measuring temperature and velocity profiles, the difficulty in measuring ionization level, and the lack of experimental data for gas properties at plasma temperatures which make it necessary to resort to purely theoretical correlations (25). In addition, variations in plasma transport properties with temperature may differ substantially from those of the same gases at moderate temperature levels. For example, the variation of the viscosity of argon with temperatures above 20,000°K is opposite to that at moderate temperatures (77).

There are a considerable number of papers referenced in the Bibliography which have not been discussed. How-
ever, it is believed that the papers discussed present a good picture of the major contributions to the analysis of the laminar flow of a gas in a cylindrical tube.
CHAPTER 2. ANALYTICAL PROBLEM

2.1. Statement of the Problem

We are considering the laminar flow of a non-reacting, non-absorbing, non-dissociated, single component, monatomic thermally perfect gas inside a cylindrical tube. The tube is axially symmetric, there is zero swirl and no body forces (i.e. free convection effects are negligible) and the flow is steady. The thermal conductivity, absolute viscosity and specific heat are considered to be functions of temperature only. Two sets of boundary conditions are to be studied (Fig. 1). In the first set, we consider the gas flowing from a point in the tube at $x = -\infty$. For $x < 0$, the wall temperature is constant and equal to the fluid temperature, $T_0$. Also for $x \leq 0$ the fluid temperature is uniform and the velocity profile is parabolic. At $x = 0$ the wall temperature undergoes a step change from $T_0$ to $T_w$ and remains at $T_w$ for $x > 0$. This set of boundary condition is referred to as the Graetz boundary condition.

In the second set both the velocity and temperature profiles are uniform at $x = 0$. This condition would be approximated by a fluid flowing directly from a reservoir into a tube in the absence of any development section or more closely approached by providing the tube with a bellmouth entrance. With this latter inlet
GRAETZ BOUNDARY CONDITION

\[ x = 0 \quad U = 2U_0(1 - (r/r_0)^2) \quad T = T_0 \]

\[ x < 0 \quad r = 0 \quad \partial U / \partial r = 0 \quad \partial T / \partial r = 0 \]

\[ x > 0 \quad r = r_0 \quad U = 0 \quad V = 0 \quad T = T_0 \]

\[ x < 0 \quad r = r_0 \quad U = 0 \quad V = 0 \quad T = T_0 \]

UNIFORM TEMPERATURE AND VELOCITY (UTV)

\[ x = 0 \quad U = U_0 \quad T = T_0 \]

\[ x < 0 \quad r = 0 \quad \partial U / \partial r = 0 \quad \partial T / \partial r = 0 \]

\[ x > 0 \quad r = r_0 \quad U = 0 \quad V = 0 \quad T = T_0 \]

\[ x < 0 \quad U = 0 \quad V = 0 \quad T = T_0 \]

FIGURE 1. IDEALIZED BOUNDARY CONDITIONS
condition the set will be referred to as the UTV (Uniform Temperature and Velocity profile) condition.\footnote{This set of boundary conditions is sometimes referred to as the simultaneous development case, but this cannot be considered as being sufficiently definitive since strictly speaking, both profiles in the Graetz condition also undergo development.}

Symmetry of the temperature and velocity profiles and the no slip and impermeability condition at the wall allows us to complete the two sets of boundary conditions which are summarized in Figure 1. Only the case of gas cooling will be considered;

\[\frac{T_w}{T_0} < 1\]

The boundary condition of constant heat removal from the gas was not considered since, unlike the case of gas heating, this is not a physically realizable situation. The analytical problem may now be identified as the determination of the heat transfer and fluid friction at the tube wall for these boundary conditions along with satisfactory methods of correlation of these results.

The differential equations governing the situation are the Navier Stokes, energy and continuity equations. For the cylindrical coordinate system in Figure 1 and incorporating the aforementioned assumptions, we may write these as (37);

radial momentum:

\[
\rho \left( \frac{\partial \mathbf{V}}{\partial t} + \mathbf{V} \cdot \nabla \mathbf{V} \right) = -\frac{\partial \mathbf{p}}{\partial r} + \frac{\partial}{\partial r} \left[ 2\mu \frac{\partial \mathbf{V}}{\partial r} + \left( \frac{\zeta - 2\mu}{3} \right) \left[ \frac{\partial \mathbf{V}}{\partial r} + \frac{\mathbf{V}}{r} + \frac{\partial \mathbf{U}}{\partial \theta} \right] \right] \\
+ \frac{\partial}{\partial \theta} \left[ \mu \left( \frac{\partial \mathbf{V}}{\partial r} + \frac{\partial \mathbf{U}}{\partial \theta} \right) \right] + \frac{2\mu}{r} \left[ \frac{\partial \mathbf{V}}{\partial r} - \frac{\mathbf{V}}{r} \right]
\]  

(2.1)
axial momentum:
\[ \rho (v \frac{\partial U}{\partial r} + u \frac{\partial U}{\partial x}) = -\frac{\partial p}{\partial x} + \frac{2}{\alpha x} \left[ 2\mu \frac{\partial U}{\partial x} + (\zeta - \frac{2}{3} \mu) \left( \frac{\partial V}{\partial r} + \frac{v}{r} + \frac{\partial U}{\partial x} \right) \right] + \frac{1}{r} \frac{\partial r}{\partial x} \left[ \mu r (\frac{\partial V}{\partial x} + \frac{\partial U}{\partial x}) \right] \quad (2.2) \]

energy:
\[ U \frac{\partial p}{\partial x} + V \frac{\partial p}{\partial r} + \Phi + \frac{1}{r} \frac{\partial r}{\partial r} \left( r k \frac{\partial T}{\partial r} + \frac{2}{\alpha x} (k \frac{\partial T}{\partial x}) \right) = \rho (v \frac{\partial H}{\partial r} + u \frac{\partial H}{\partial x}) \quad (2.3) \]

continuity:
\[ \frac{1}{r} \frac{\partial r}{\partial r} (\rho V) + \frac{\partial}{\partial x} (\rho U) = 0 \quad (2.4) \]

where \( \Phi \) = viscous(mechanical) dissipation function
\[ \Phi = \mu \left[ 2 \left( \frac{\partial V}{\partial r} + (\frac{V}{r})^2 + \frac{\partial U}{\partial x} \right) + (\frac{\partial V}{\partial x} - \frac{\partial U}{\partial r})^2 \right] + \lambda \left[ \frac{\partial V}{\partial r} + \frac{V}{r} + \frac{\partial U}{\partial x} \right]^2 \quad (2.5) \]

\( \lambda \) = second coefficient of viscosity (\( \lambda = -\frac{2}{3} \mu \) for a monatomic gas)
\( \mu \) = primary coefficient of viscosity
\( \zeta = \lambda + \frac{2}{3} \mu \) (\( \zeta = 0 \) for a monatomic gas).

For a thermally perfect gas we may write
\[ dH = c_p dT \]
(i.e. the specific heat may be removed from the differential operator). This allows us to write the energy equation solely in terms of enthalpy as the dependent variable:
\[ U \frac{\partial p}{\partial x} + V \frac{\partial p}{\partial r} + \Phi + \frac{1}{r} \frac{\partial r}{\partial r} \left( r k \frac{\partial H}{\partial r} + \frac{a k H}{\alpha x} \right) = \rho (v \frac{\partial H}{\partial r} + u \frac{\partial H}{\partial x}) \quad (2.6) \]

The equations in this form and generality impose a (presently) nearly unsolvable problem-- both in terms of a
closed form or numerical type solution. They form a set of non-linear equations by virtue of the product terms in the dependent variables. The energy equation is coupled to both momentum equations by virtue of its velocity terms, and the momentum equations are likewise coupled to the energy equation by virtue of the density and viscosity terms. The equations are elliptic in character due to the presence of second order derivatives in two spatial directions. The solution for the flow and temperature fields must be made "in toto"—that is, values of the dependent variables must be specified at the exit of the tube as well as at the walls and inlet. As is generally done, the boundary layer assumptions will be invoked which both reduces the number of equations to be solved and changes the classification of the equations.

The rationale underlying the application of the boundary layer assumptions will be reviewed. In certain flow situations, the variation of the velocity and temperature can be much greater in one spatial direction than another. We can identify two such regions in internal tube flow (Fig. 2). In the entrance region where a thin viscous and/or thermal boundary layer are developing at the tube wall, variation of these quantities can be expected to be much greater in a direction normal to the
FIGURE 2. BOUNDARY LAYER REGIONS IN INTERNAL TUBE FLOW
flow than in an axial direction. Similar considerations apply downstream of the region where the boundary layers which have been growing along the tube wall have met at the tube centerline. In regions such as these we can apply an approximate order of magnitude analysis. We denote \( \sim \) as meaning "of the order of magnitude of" rather than its usual meaning. We then write,

\[
\frac{\partial}{\partial r} \sim \frac{1}{r_0} \quad \frac{\partial}{\partial x} \sim \frac{1}{x}
\]

where \( r_0 \) is the radius of the tube. In the inlet region of a tube we would use a representative boundary layer thickness as a characteristic dimension rather than \( r_0 \). Since for the gases which will be considered the Prandtl numbers are fairly close to unity (i.e. the thermal and velocity boundary layer thickness should be approximately equal), the same characteristic dimension will apply to the energy and momentum equations.

Let \( \Delta P_r \) = representative magnitude of radial pressure variation

\( \Delta P_x \) = representative magnitude of axial pressure variation

\( \bar{U} \) = representative magnitude of axial velocity

\( \bar{V} \) = representative magnitude of radial velocity

\( \bar{\rho} \) = representative magnitude of density.

We further denote the operators \( \oplus \) and \( \ominus \) as being order of magnitude addition and subtraction and to simply mean that the order of magnitude of a sum (or difference) of two
terms connected by the operator is that of the larger term. If both are of the same order of magnitude, then the order of the sum will be the same as for either term. Expanding continuity (2.4) we have

\[ \frac{\rho V}{r} + \frac{\partial (\rho V)}{\partial r} + \frac{\partial (\rho U)}{\partial x} = 0 \]  

(2.7)

Since the first term becomes indeterminate at \( r = 0 \), we can apply L'Hôpital's rule at the centerline:

\[ \left. \frac{V}{r} \right|_{r=0} = \frac{\partial V}{\partial r} \left|_{r=0} \right. = \frac{\partial V}{\partial r} \left|_{r=0} \right. \]

Due to the crudeness of the analysis, not much will be lost if we use \( \bar{V}/r_0 \) to represent \( V/r \) as well as \( \partial V/\partial r \). This will also be extended to the \( V/r \) terms which are present in the axial and radial momentum equations. Also, not much will be lost if we treat the density as being constant. Applying an order of magnitude analysis to the continuity equation (2.7) yields

\[ \bar{V}/r_0 + \bar{V}/r_0 + \bar{U}/x = 0 \]  

(2.8)

or

\[ V \sim r_0 U/x \]  

(2.9)

Expanding the axial momentum equation (2.2) by an order of magnitude analysis (for \( \zeta = 0 \)):

\[ \frac{\bar{V} \bar{U}}{r_0} \Theta \bar{U} \Theta x \sim -\frac{1}{P} \frac{\Delta P}{x} \Theta \frac{1}{P} \Theta x \left[ \frac{2}{x} \Theta 2 \left( \frac{\bar{V}}{r_0} \Theta \bar{U} \right) \Theta 2 \frac{\bar{V}}{r_0} \Theta \frac{\bar{U}}{x} \right] \Theta 2 \frac{\mu}{P} \Theta \frac{\bar{V}}{r_0} \Theta \frac{\bar{U}}{x} \]  

(2.10)
Substituting the magnitude of $\bar{V}$ in terms of $\bar{U}$ (1.9)

$$
\frac{\bar{U}^2}{x} \oplus \frac{\bar{U}^2}{x} \sim -\frac{1}{p} \frac{\Delta p}{x} \oplus 2\mu \left[ \frac{\bar{U}}{x^2} \oplus \frac{\bar{U}}{x^2} \oplus \frac{\bar{U}}{x^2} \right] + \frac{2\mu}{\rho} \left[ \frac{\bar{U}^2}{x^2} \oplus \frac{\bar{U}^2}{x^2} \right] + \frac{2\mu}{\rho} \left[ \frac{\bar{U}^2}{x^2} \oplus \frac{\bar{U}^2}{x^2} \right]
$$

The order of magnitude representation of the radial momentum equation (2.1) can be written:

$$
\frac{\bar{V}}{r_0} \oplus \frac{\bar{V}}{x^2} \sim -\frac{1}{p} \frac{\Delta p}{r_0} \oplus 2\mu \left[ \frac{\bar{V}}{r_0^2} \oplus \frac{1}{3} \left( \frac{\bar{V}}{r_0^2} \oplus \frac{\bar{U}}{x_0} \right) \right] \oplus \mu \left[ \frac{\bar{V}}{r_0^2} \oplus \frac{\bar{U}}{x_0} \right] + \frac{2\mu}{\rho} \left[ \frac{\bar{V}}{r_0} \oplus \frac{\bar{V}}{r_0} \right]
$$

and substituting the relationship $\bar{V}$,

$$
\frac{\bar{U}}{x} \oplus \frac{\bar{U}}{r_0} \sim -\frac{1}{p} \frac{\Delta p}{r_0} \oplus 2\mu \left[ \frac{\bar{U}}{r_0^2} \oplus \frac{1}{3} \left( \frac{\bar{U}}{r_0^2} \oplus \frac{\bar{U}}{x_0} \right) \right] \oplus \mu \left[ \frac{\bar{U}}{r_0^2} \oplus \frac{\bar{U}}{x_0} \right] + 2\mu \left[ \frac{\bar{U}}{r_0^2} \oplus \frac{\bar{U}}{x_0} \right]
$$

In the same case that $r_0 \ll l$, certain terms become small when compared with others. If these terms are neglected the following equations are obtained;

axial momentum:

$$
\frac{\bar{U}^2}{x} \sim -\frac{1}{p} \frac{\Delta p}{x} \oplus 2\mu \frac{\bar{U}}{x^2}
$$

radial momentum:

$$
\frac{\bar{U}^2}{x} \sim -\frac{1}{p} \frac{\Delta p}{r} \oplus \frac{2\mu}{\rho} \frac{\bar{U}}{x}
$$
It can be seen that both of the terms which could determine the order of magnitude of the term $\frac{-1}{\rho} \Delta P r/r_o$ differ by a factor $r_o/x$ from similar terms in the axial momentum equation. We can reasonably expect, therefore, that the term representing the radial pressure gradient will differ by a similar factor from the axial pressure gradient. For $r_o/x << 1$ we can also reasonably expect that the neglect of the radial pressure variation would not effect the solution greatly. This allows us to discard the radial momentum equation insofar as it provides information about this variation.\(^2\)

In the energy equation, we can use the same type of representation for the variation of the enthalpy. Assuming

$$\frac{\partial H}{\partial r} \sim \frac{\Delta H}{r_o} \quad \text{and} \quad \frac{\partial H}{\partial x} \sim \frac{\Delta H}{x}$$

\(^2\)From another standpoint, it would seem that more than simplification is gained from this assumption when finite difference or element techniques are used for solution. For compressible laminar flow between parallel plates, Swearingen (\textit{82}) included the transverse momentum equation by combining the radial and axial momentum equations through elimination of the pressure terms in each. This requires that cross derivatives of the pressure be taken which raises the order of the equation representing momentum transfer from second to third, and for the case of uniform inlet velocity and temperature profiles, introduces a higher order singularity in boundary conditions at $x = 0$ which must be accommodated by the solution. As noted earlier, in Swearingen's case, large scale oscillations were obtained near the entrance for compressible flow.
where $\Delta H$ is a representative magnitude of the enthalpy variation, we obtain after substitution of 2.9 into the energy equation,

$$
\bar{u} \frac{\Delta p}{\Delta x} + \bar{u} \left( \frac{\partial \Delta p}{\partial x} \right) + \bar{u} \frac{\partial \Delta H}{\partial x} + \bar{u} \frac{\partial \Delta H}{\partial r} + \bar{u} \frac{\partial \Delta H}{\partial \theta} + \bar{u} \frac{\partial \Delta H}{\partial \phi} (2.16)
$$

It can be seen that the second and fifth terms on the left hand side which represent $V \frac{\partial p}{\partial r}$ and $\frac{\partial}{\partial x}(r \frac{\partial H}{\partial r})$ respectively are negligible with respect to the terms derived from $U \frac{\partial p}{\partial x}$ and $\frac{\partial}{\partial r}(r \frac{\partial H}{\partial r})$. Concerning the dissipation function $\phi \ (2.5)$, the term $(\frac{\partial U}{\partial r})^2$ can be shown to be the controlling term when a boundary layer analysis can be applied. However, inclusion of the additional terms will not affect the results of the simplification that is being developed here--that is the problem will remain an initial value problem so that the additional terms in the dissipation can be included almost free of charge. Thse terms can become significant in the entrance region for the UTV boundary condition, so discussion will be withheld until Chapter 3 where this boundary condition will be reviewed.

When terms which have been shown to be small are neglected, the usual boundary layer equations are obtained axial momentum:

$$
\rho \left( \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial r} \right) = - \frac{dp}{dx} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \mu \frac{\partial U}{\partial r} \right) (2.17)
$$
continuity:

\[ \frac{\partial}{\partial x}(\rho U) + \frac{1}{r} \frac{\partial}{\partial r}(\rho V) = 0 \]  

(2.18)

energy:

\[ \rho (U \frac{\partial H}{\partial x} + V \frac{\partial H}{\partial r}) = U \frac{dp}{dx} + \frac{1}{r} \frac{\partial}{\partial r}(k \frac{\partial H}{\partial r} + \Phi) \]  

(2.19)

An additional consideration becomes important before we can fully justify the boundary layer assumptions. If those non-boundary layer terms which we eliminated in our simplification of the governing equations are larger or of the same order of magnitude as the terms which are due solely to property variation, then the solutions should be treated with caution. Those terms due to property variation whose magnitude relative to neglected terms can be calculated are 1.) the ratio of terms that were eliminated from the axial momentum equation to the term due the viscosity variation;

\[ R_1 = \left[ \frac{\partial}{\partial r} \left( \frac{\partial \mu V}{\partial x} \right) + \frac{4}{3} \frac{\partial}{\partial x} \left( \frac{\partial \mu \frac{\partial U}{\partial x}}{\partial x} \right) - \frac{2}{3} \frac{\partial}{\partial x} \left( \frac{\partial \mu \frac{\partial U}{\partial r}}{\partial r} \right) \right] \]  

\[ \int \frac{\partial U \frac{\partial \mu}{\partial r}}{\partial r \partial r} \]  

(2.20)

and the ratio of the terms that were eliminated from the energy equation to the term which is due to thermal conductivity and specific heat variation;

\[ R_2 = \left[ \frac{\partial}{\partial x} \left( k \frac{\partial H}{\partial x} \right) \right] \]  

\[ \int \frac{\partial H \frac{\partial k}{\partial r}}{\partial r \partial r} \]  

(2.21)
In order to estimate the magnitude of these ratios for verification of our assumptions, we must anticipate the solution of equations 2.17 through 2.19. In figure 3 and 4 these ratios are plotted as a function of distance from the tube entrance for the flow of helium and the UTB boundary condition at $T_w/T_0 = 0.1$. This case represents an extreme in terms of both numerators and denominators in 2.20 and 2.21. The derivatives were evaluated by means of radial and axial centered difference operators with dependent variables obtained from a combined analytical finite difference algorithm to be presented herein. It should be noted that we are estimating terms from a solution of equations that neglect them. Along with these ratios are plotted

$$R_3 = \frac{\frac{\partial}{\partial x} \mu \frac{\partial U}{\partial x}}{\rho \frac{\partial U}{\partial x}} \quad (2.22)$$

which represents the ratio of axial molecular momentum transfer to axial convective momentum transfer, and

$$R_4 = \frac{\frac{\partial}{\partial x} \frac{k}{\sigma_p} \frac{\partial H}{\partial x}}{\rho \frac{\partial H}{\partial x}} \quad (2.23)$$

which represents the ratio of the axial molecular heat transfer to the axial convective heat transfer. Both numerators again represent terms eliminated from the governing equations. $Re_0$ and $Pr_0$ represent the inlet Reynolds and Prandtl numbers respectively. A feel for the physical significance of these results can be obtained by choosing $Re_0 Pr_0 = 1000$. As a sample case, at $r/r_0 = 0.95$
HELIUM  $R/R_0=0.10$  $T_w/T_0=0.10$
$M_0=0.03$  UTV B.C.

$R_1$: non b.l. terms/viscosity variation
  (momentum equation)

$R_2$: non b.l. terms/conductivity, sp. heat variation
  (energy equation)

$R_3$: axial molec. conv./axial convection
  (momentum equation)

$R_4$: axial molec. transfer/axial convection
  (energy equation)

Figure 3. Relative magnitude of terms in axial momentum and energy equations. ($x^+ = x/Re_0 Pr_0 r_0$)
Figure 4. Relative magnitude of terms in axial momentum and energy equations. \( R/R_0 = 0.95 \), UTV boundary condition.
it can be seen that all of these ratios are reduced to magnitudes less than 0.10 at ten diameters, whereas for $r/r_0 = 0.95$, $R_1$ does not become less than 1 until 40 diameters downstream and $R_3$ does not become less than 1 until 50 diameters from the tube entrance. The behavior of $R_1$ and $R_2$ is due to the fact that the velocity and temperature profiles remain extremely flat in the central region of the tube for an axial distance which depends upon the severity of the cooling. Radial derivatives in this region will be quite small. It is not easy to determine what represents an unacceptable ratio. Basing our decision upon the ratios of terms in the core flow would lead us to discard the solution almost altogether. However, the absolute magnitude of the derivatives in this region are several orders of magnitude smaller than those occurring in the region near the tube wall and their effect will probably be small. An ultimate quantitative answer will have to wait for a solution to equations 2.1 through 2.4 or for experimental verification. The results for the case shown are not representative of other temperature ratios or boundary conditions.

A finite difference solution to the equations 2.17, 2.18 and 2.19 was published by P.M. Worsoe-Schmidt (100) in his Ph.D dissertation, and his algorithm will be made the basis of the numerical portion of the solution to be presented here. In the next section, the important points of the Worsoe-Schmidt analysis are reviewed.
2.2. The Worsoe-Schmidt Analysis

After Worsoe-Schmidt, we transform the boundary layer equations in terms of the following non-dimensional variables.

Independent variables:

\[ x^+ = \frac{x}{r_0 R_0 P_0} \]
\[ r^+ = \frac{r}{r_0} \]

Dependent variables:

\[ U^+ = \frac{U}{U_0} \]
\[ V^+ = \frac{V}{U_0} R_0 P_0 \]
\[ P = \frac{(p_0 - p)}{p_0 U_0^2} \]
\[ p^+ = \frac{p}{p_0} \]
\[ \theta = \frac{T}{T_0} \]

where \( r_0 \) = tube radius

\( R_0 = \) inlet Reynolds number = \( U_0^2 r_0 p_0 / \mu_0 \)

Subscript \( o \) will otherwise be taken to denote gas properties evaluated at the inlet temperature.

Non-dimensionalized gas properties:

\[ C_p^+ = \frac{C_p}{C_p_0} \]
\[ \mu^+ = \frac{\mu}{\mu_0} \]
\[ \rho^+ = \frac{\rho}{\rho_0} \]

The following expressions are assumed to adequately (and most generally) represent the relationships between the properties and thermodynamic quantities for the gases under consideration.
For air and helium, these assumed power laws are quite good (c.f. Appendix B). We define two non-dimensionalized enthalpies,

\[ H_1^+ = \frac{H - H_\infty}{c_p\theta_0} = \int_{c_p}^{\theta} d\theta = \frac{1}{1+\alpha} \left\{ \frac{1}{\theta^{1+\alpha}} \right\} \]

\[ H_2^+ = \frac{H - H_w}{c_p\theta_w} = \int_{c_p}^{\theta} d\theta = \frac{1}{1+\alpha} \left\{ \frac{1}{\theta^{1+\alpha}} - \theta_w^{1+\alpha} \right\} \]

Subscript \( w \) refers to conditions at the tube wall. The reasons for use of the two definitions of the enthalpy will become clear in Chapter 4. The form in 2.28 is that used throughout the Worsoe-Schmidt analysis. The form 2.29 will become necessary when we consider a similarity boundary layer solution. For the present, we will be using \( H_1^+ \), although the form of the energy equation will be unchanged since we have merely changed the zero
reference. We also define,

\[ \gamma_0 = \text{ratio of specific heats} = \frac{c_{p,0}}{c_{v,0}} \]

\[ M_0 = \text{inlet Mach no.} = \frac{U_0}{\sqrt{\gamma_0 R T_0}} \]

\[ q^+_w = \text{non-dimensionalized heat flux} = r_0 q^+_w / k_0 T_0 \]

With these new quantities, the governing equations become;

\[ \rho^+ (U^+ \frac{\partial U^+}{\partial x^+} + V^+ \frac{\partial U^+}{\partial r^+}) = \frac{\partial P}{\partial x^+} + 2 Pr_0 \left[ \frac{1}{r^+} \frac{\partial}{\partial r^+} (r^+ \mu^+ \frac{\partial U^+}{\partial r^+}) \right] \] (2.30)

\[ \frac{\partial}{\partial x^+} (\rho^+ U^+) + \frac{1}{r^+} \frac{\partial}{\partial r^+} (r^+ V^+ \rho^+ ) = 0 \] (2.31)

\[ \rho^+ (U^+ \frac{\partial H^+}{\partial x^+} + V^+ \frac{\partial H^+}{\partial r^+}) = (1 - \gamma_0) M_0^2 U^+ \frac{\partial P}{\partial x^+} + 2 \frac{\partial}{\partial r^+} (r^+ \frac{k^+}{c_p^+} \frac{\partial H^+}{\partial r^+}) + 2 (\gamma_0 - 1) M_0^2 \mu^+ Pr_0 \phi^+ \] (2.32)

where

\[ \phi^+ = \left( \frac{\partial U^+}{\partial r^+} \right)^2 + \frac{2}{Re_0 Pr_0} \left( \frac{\partial V^+}{\partial r^+} \right)^2 + \frac{\left( \frac{V^+}{r^+} \right)^2}{r^+} - 2 \left( \frac{\partial V^+}{\partial r^+} + \frac{V^+}{r^+} \right)^2 \] (2.33)
The boundary conditions in terms of the non-dimensionalized variables are

at \( x^+ = 0 \) \( u^+ = 2(1-r^+2) \) or \( u^+ = 1 \) \( \theta = 1 \)

\[ \begin{align*}
    &x^+ < 0 \quad r^+ = 1 \\
    &u^+ = 0 \quad v^+ = 0 \quad \theta = 1
\end{align*} \]

\[ \begin{align*}
    &x^+ \geq 0 \quad r^+ = 1 \\
    &u^+ = 0 \quad v^+ = 0 \quad \theta = \theta_w
\end{align*} \]

\[ \begin{align*}
    &x^+ \leq 0 \quad r^+ = 0 \\
    &\frac{\partial u^+}{\partial r^+} = 0 \quad v^+ = 0 \quad \frac{\partial \theta}{\partial r^+} = 0
\end{align*} \]

The unknowns in these equations are \( U^+, V^+ \) and \( H^+ \) which are function of two space variables \( x^+ \) and \( r^+ \), and the non-dimensionalized pressure \( P(x^+) \) which is a function of only the axial co-ordinate. At first sight, there would seem to be one equation less than that required for solution, since there are only three equations here. Before application of the boundary layer assumptions, the fourth equation was provided by the radial momentum equation. The fourth equation in this case comes from the integrated continuity equation,

\[ \int_0^1 \rho^+ u^+ r^+ dr^+ - 1 = 0 \quad (2.34) \]

which defines the flow as being confined. Integration over the space variable \( r^+ \) provides an equation in terms
of one variable, $x^+$. Worsoe Schmidt used a two level finite difference scheme with variable implicitness to generate a marching solution in the axial direction. If the velocity and temperature profiles are known at one axial point, then the profiles at the next step can be obtained with application of appropriate boundary conditions and so on down the tube until the profile development is sufficiently complete. Various discrete radial and axial points in the tube are specified as node points (Fig. 4) where we either know or are solving for values of the dependent variables. We define $\psi_{m,n}$ as being the value of a dependent variable corresponding to the node $m,n$ whose spatial point in the tube is given as $(m\Delta x^+, n\Delta r^+)$.  

![Designation of Mesh Points](image)

Figure 5. Designation of Mesh Points

---

3 This represents a simplification in that the radial and axial mesh steps changed at different points in the tube. If the axial step were $\Delta x_1$ for $m_1$ steps and $\Delta x_2$ for $m_2$ steps, the axial point would be $m_1 \Delta x_1 + m_2 \Delta x_2$.  

Although we are solving for the dependent variables at these points, this does not necessitate writing the finite difference representation of the differential equations to apply to these points. In the Worsoe-Schmidt solution, the equations are radially centered at $n \Delta r^+$, but if the dependent variables are known at the $m$'th axial point and we are solving for them at $m+1$, then the equations are written so as to apply to the point $m+\sigma$, where $\sigma$ is a constant less than or equal to 1. The reason for doing this, and the significance of $\sigma$ can be seen in the following analysis. The radial difference operators $\delta$ and $\delta^2$ are defined by

\[
\delta (\Phi_{m,n}) = \Phi_{m+1,n} - \Phi_{m,n} \tag{2.35}
\]
\[
\delta^2 (\Phi_{m,n}) = \Phi_{m+1,n} - 2\Phi_{m,n} + \Phi_{m,n-1} \tag{2.36}
\]

By means of Taylor series expansions, the analytical derivatives at the point $m+\sigma, n$ can be related to values of the dependent variables and their derivatives at the axial points $m$ and $m+1$. After some manipulation, the following relationships can be derived;

\[
\frac{\Phi_{m+1,n}-\Phi_{m,n}}{\Delta x^+} = \frac{\partial \Phi}{\partial x^+} + \left[ \frac{1}{2} (1-2\sigma) \frac{\partial^2 \Phi}{\partial x^2} \Delta x^+ + \frac{(1-3\sigma+3\sigma^2)}{6} \frac{\partial^3 \Phi}{\partial x^3} \Delta x^2 \right. \tag{2.37}
\]
\[
\left. + O(\Delta x^3) \right]
\]
\[
\frac{\sigma \delta (\Phi_{m+1,n}) + (1-\sigma) \delta (\Phi_{m,n})}{2 \Delta r^+} = \frac{\partial \Phi}{\partial r^+} + \left[ \frac{1}{2} \sigma (1-\rho) \frac{\partial^3 \Phi}{\partial x^2 \partial r^2} \Delta x^2 \right.
\]
\[
+ \frac{1}{6} \frac{\partial^3 \Phi}{\partial r^3} \Delta r^2 + O(\Delta x^4) + O(\Delta r^4) \right]
\]

(2.38)

\[
\frac{\sigma \delta^2 (\Phi_{m+1,n}) + (1-\sigma) \delta^2 (\Phi_{m,n})}{\Delta r^+} = \frac{\partial^2 \Phi}{\partial r^2} + \left[ \frac{1}{2} \sigma (1-\sigma) \frac{\partial^4 \Phi}{\partial x^2 \partial r^2} \Delta x^2 \right.
\]
\[
+ \frac{1}{12} \frac{\partial^4 \Phi}{\partial r^4} \Delta r^4 + O(\Delta x^4) + O(\Delta x^2 \Delta r^2) + O(\Delta r^4) \right]
\]

(2.39)

where all analytical derivatives apply to the point 
\((m+\sigma)\Delta x^+ n \Delta r^+. The bracketed terms on the right hand sides of these expressions can be considered as representing the error if the difference quotients on the right hand sides are substituted in place of the analytical derivatives in the differential equations. The value of \(\sigma\) can be seen to have a direct influence on the magnitude of these terms, and the value of \(\sigma\) should be chosen with this in mind. Choosing \(\sigma=1/2\) will minimize all the coefficients in which \(\sigma\) appears. This would appear to be an optimum value if it were not for the fact that the
solution was found to be unstable in this case. A compromise value of $\sigma = 3/4$ was chosen and was found to yield stable results in all cases. While it is possible that a stable solution could have been chosen closer to $1/2$, it is questionable whether it would have been worthwhile to have devoted the time to determining this $\sigma$. Similar results were found by Worsoe-Schmidt and are discussed by him (100). It must be remembered that an optimum determined for one set of boundary conditions may not be stable for another set.

The values of the dependent variables at $m+1$ are evaluated by assuming a linear variation between neighboring axial points;

$$\Phi_{m+1,n} = \sigma \Phi_{m+1,n+1} + (1-\sigma) \Phi_{m,n}$$

(2.40)

In regions where large second and higher order axial derivatives occur, this expression becomes less acceptable. This is in addition to the error incurred by dropping the second derivatives in the original equations. When these representations are substituted into the partial differential equations 2.29, 2.30 and 2.31, a relationship combining $\Phi_{m+1,n}$, $\Phi_{m+1,n+1}$ and $\Phi_{m+1,n-1}$ is obtained at each radial node for $\sigma > 0$. For this case there are $3N + 1$ simultaneous equations in order to solve for $3N + 1$ unknowns. For the particular case $\sigma = 0$, we
have an expression explicitly in $\Phi_{m+1,n}$ and known quantities at each radial node.

The difference equations obtained will still contain products of the unknown variables. This non-linearity can be neatly eliminated if we allow of linearization by means of iteration. Wherever a product of the same two dependent variables occur, we will substitute the best available value for one of them and then solve the linearized equation for the succeeding value. For example if the first solution at $(m+\sigma)\Delta x^+$ is being performed, the quantity $\Phi_{m,n}\Phi_{m+1,n}$ will be used in place of $\Phi_{m+1,n}^2$. On the next iteration at this point, $\Phi_{m+1,n}$ from the first solution of the linearized equation would be used, and so on until convergence is obtained.

A further linearization allows us to uncouple the equations at each iteration, insofar as products of different dependent variables occur. The details of this linearization depend upon the sequence in which the equations are solved. The energy equation (2.32) is the first equation to be solved at an axial point, so where products of enthalpies and velocities occur, velocities from the previous axial point are used. The integrated continuity equation 2.3/1 can then be arranged to bring out explicitly $p_{m+1}$ which is contained in $\rho_{m+1,n}$. Values of enthalpy from the present solution of the energy equation
are used in evaluating the temperature dependent quantities in the momentum equation. One exception to this is the term $\rho^+ U^+$. Generally, $\rho^+$ and $U^+$ vary in opposite directions along the tube. Using the new enthalpy and pressure for the evaluation of $\rho^+$ and the old value of $U^+$ would roughly give us $\rho^+_{m+1,n} U^+_{m,n}$ which overestimates $\rho^+_{m+1,n} U^+_{m+1,n}$. A better approximation can be obtained by using $\rho^+_{m+1,n} U^+_{m,n}$ until estimates of both $\rho^+_{m+1,n}$ and $U^+_{m+1,n}$ are available.\footnote{An extreme example of this type of linearization was used in the theoretical analysis of Koppel and Smith (48) where it was assumed that the product of velocity and density, $\rho u$, at any radius is independent of the axial coordinate.} After the energy and momentum equations, the differential form of the continuity equation (2.31) can be used for the evaluation of radial velocities for use on the next iteration or at the next step. Using this procedure reduces the problem to the solving of 3 sets of $N$ linear simultaneous equations plus the total continuity equation.

Considering that upwards of 320 radial mesh divisions were necessary for the solution of the most severe boundary conditions, the solution of this many simultaneous equations would still be prohibitive if it were not for the fact that the coefficient matrices for the dependent variables were of a particularly simple form. After the aforementioned linearizations are made, the general form
of the relationships that holds among the velocities and
enthalpies can be written as:

\[ D_n^{I} = -A_n^{I} U_{m+1,n-1}^{+} + B_n^{I} U_{m+1,n}^{+} - C_n^{I} U_{m+1,n+1}^{+} - P_{m+1} \] (2.41)

\[ D_n^{II} = -A_n^{II} H_{m+1,n-1}^{+} + B_n^{II} H_{m+1,n}^{+} - C_n^{II} H_{m+1,n+1}^{+} \] (2.42)

where the coefficients are functions of \( \sigma \) and known
values of enthalpies from the previous step and/or the
last iteration. For the present the solution for the
pressure defect and the radial velocities is skipped.
At the centerline, consideration of symmetry allows a
relationship to be written among two of the dependent
variables:

\[ B_{o}^{I} U_{m+1,0}^{+} - C_{o}^{I} U_{m+1,1}^{+} - P_{m+1} = D_{0}^{I} \] (2.43)

\[ B_{o}^{II} H_{m+1,0}^{+} - C_{o}^{II} H_{m+1,1}^{+} = D_{0}^{II} \] (2.44)

Since there is specified wall temperature (and enthalpy)
and no slip (zero axial velocity) at the wall, the equa­
tions are written at the wall are:
\[ P = D \]

\[ \text{where subscript } N \text{ refers to the node point at the wall.} \]

The coefficient matrices for the enthalpy and velocity are of the form:

\[
\begin{bmatrix}
XX000 \\
0XXX0 \\
00XXX
\end{bmatrix}
\]

which is a matrix of the tri-diagonalized type. Since many of the elements are zero, inversion could be accomplished by means of one of the many available computer inversions, particularly one which makes use of zero checks. However, further simplification can be obtained by assuming that relationships of the form:

\[
U_{m+1,n}^+ = E_{n}^I U_{m+1,n+1}^+ + F_{n} P_{m+1} + G_{n}^I \quad (2.47)
\]

\[
H_{m+1,n}^+ = E_{n}^{II} H_{m+1,n+1}^+ + G_{n}^{II} \quad (2.48)
\]
exist. If such a relationship for \( U_{m+1,n-1}^+ \) is substituted in terms of \( U_{m+1,n}^+ \) and \( H_{m+1,n-1}^+ \) is substituted in terms of \( H_{m+1,n}^+ \) in equations (2.45) and (2.46) respectively, solution may be made explicitly for \( U_{m+1,n}^+ \) and \( H_{m+1,n}^+ \).

\[
U_{m+1,n}^+ = \frac{C_n^I}{B_{n}^{-}A_{n}^{-}E_{n-1}^{-}} U_{m+1,n+1}^+ + \frac{(1+A_n^{P_n-1})}{B_{n}^{-}A_{n}^{-}E_{n-1}^{-}} P_{m+1} \quad (2.49)
\]

\[
+ \frac{D_n^{I}+A_n^{G_nI}}{B_{n}^{-}A_{n}^{-}E_{n-1}^{-}}
\]

\[
H_{m+1,n}^+ = \frac{C_n^{II}}{B_{n}^{-}A_{n}^{-}E_{n-1}^{-}} H_{m+1,n+1}^+ + \frac{D_{n}^{II}+A_n^{G_nII}}{B_{n}^{-}A_{n}^{-}E_{n-1}^{-}} \quad (2.50)
\]

where coefficients on the right hand sides can be directly associated with the coefficients in equations (2.47) and (2.48). The following coefficients can be identified:

\[
E_n^I = \frac{C_n^I}{B_{n}^{-}A_{n}^{-}E_{n-1}^{-}} \quad (2.51) \quad E_n^{II} = \frac{C_n^{II}}{B_{n}^{-}A_{n}^{-}E_{n-1}^{-}} \quad (2.54)
\]

\[
P_n^I = \frac{1+A_n^{P_n-1}}{B_{n}^{-}A_{n}^{-}E_{n-1}^{-}} \quad (2.52) \quad G_n^{II} = \frac{D_{n}^{II}+A_n^{G_nII}}{B_{n}^{-}A_{n}^{-}E_{n-1}^{-}} \quad (2.55)
\]
where all coefficients in the recursive relationship corresponding to radial node \( n \) can be written in terms of the coefficients for radial node \( n-1 \) and other known quantities. It can be seen that if the values of these coefficients are known at the tube centerline, then all coefficients can be evaluated successively out to the tube wall. These coefficients are directly available in the momentum and energy difference equations as written for the tube centerline in equations 2.43 and 2.44. The following identities can be made:

\[
G^n_I = \frac{D^n_I + A^n_I G^n_{n-1}}{B^n_I - A^n_IE^n_{n-1}} \quad (2.53)
\]

where all coefficients in the recursive relationship corresponding to radial node \( n \) can be written in terms of the coefficients for radial node \( n-1 \) and other known quantities. It can be seen that if the values of these coefficients are known at the tube centerline, then all coefficients can be evaluated successively out to the tube wall. These coefficients are directly available in the momentum and energy difference equations as written for the tube centerline in equations 2.43 and 2.44. The following identities can be made:

\[
E^n_0 = C^n_0 / B^n_0 \quad (2.56)
\]

\[
F^n_0 = 1 / B^n_0 \quad (2.57)
\]

\[
G^n_0 = D^n_0 / B^n_0 \quad (2.58)
\]

\[
E^{II}_0 = C^{II}_0 / B^{II}_0 \quad (2.59)
\]

\[
G^{II}_0 = C^{II}_0 / B^{II}_0 \quad (2.60)
\]

Once all the coefficients are known and after applying the boundary conditions at the wall (i.e. known \( U^+_{m+1, N} \) and \( H^+(\theta_w)_{m+1, N} \)), the enthalpies and axial velocities can be evaluated, this time from the wall successively out to the tube centerline.
The differential form of the continuity equation provides the radial velocities. Worsoe-Schmidt wrote the difference quotients for this equation so as to apply to the point $m+\sigma, n-1/2$. A new difference operator is defined as

$$\delta'(\Phi_{m,n+\frac{1}{2}}) = \Phi_{m,n} - \Phi_{m,n-1}$$  \hspace{1cm} (2.61)

In this case, the quotient representation of the partial differential equation (2.31) contains only two unknown quantities, $V_{m+1,n-1}^+$ and $V_{m+1,n}^+$ so that an explicit solution may be made for $V^+$. Returning to the solution for the pressure defect, an integration of the continuity equation consistent with the finite difference scheme was obtained by successive elimination of the radial velocities in the difference representation of the continuity equation. This results in:

$$\frac{1}{8}(\rho^+ U^+)_{m+1,0} + \sum_{n=1}^{N-1} n(\rho^+ U^+)_{m+1,n} = \frac{1}{8}(\rho^+ U^+)_{m,0} + \sum_{n=1}^{N-1} n(\rho^+ U^+)_{m,n}$$  \hspace{1cm} (2.62)

Extraction of a common term $P_{m+1}$ from the density terms on the left hand side of (2.62) and substitution of the recursive relationships for the axial velocities results in an equation with $P_{m+1}$ as the only unknown. Once this quantity is determined the solution of the momentum
A note should be mentioned concerning the way in which radial derivatives of the temperature and velocity profiles were obtained at the wall. These quantities are necessary for the calculation of the fluid friction and heat transfer at the wall. Worsoe-Schmidt evaluated these terms by taking the derivative of a third order polynomial in $r^+$ fitted to the velocities and enthalpies at the 4 radial node points closest to the wall. In terms of the quantities at these nodes:

$$\left.\frac{\partial U^+}{\partial r^+}\right|_w = \frac{1}{6\Delta r^+}(18U_{N-1}^+-9U_{N-2}^+ + 2U_{N-3}^+) \quad (2.63)$$

$$\left.\frac{\partial H^+}{\partial r^+}\right|_w = \frac{1}{6\Delta r^+}(11H_{N-1}^++18H_{N-2}^+-9H_{N-3}^++2H_{N-3}^+) \quad (2.64)$$

where subscript $N$ refers to the node point at the wall.

Subscript $m$ is absent since all variables pertain to the same axial point. The term $U_N^+$ is absent from 2.63 since $U_N^+=0$. The third order insures that inflection points can be accommodated.

The order of solution and basic features of the inversion for equations 2.45 and 2.46 and as described by Worsoe-Schmidt in reference 100 were used without major modification. Those requiring a more detailed review than that presented here should consult that reference.
CHAPTER 3. FINITE DIFFERENCE SOLUTION

THE GRAETZ BOUNDARY CONDITION

3.1. Basic Considerations

The variation with axial distance and inlet wall to bulk temperature ratio of the local Nusselt number $Nu_m$ and friction factor $f$ is sought. In terms of flow quantities and fluid properties, these are defined by:

\[ f \text{Re}_m = \left( \frac{-\tau_w}{\frac{1}{2}(\rho U)_m U_m} \right) \cdot \frac{2r_0 U_m \rho_m}{\mu_m} = \frac{-2 \tau_w^+}{\mu_m \int_0^1 U^+ r^+ dr^+} \]  \hspace{1cm} (3.1)

\[ Nu_m = \frac{2r_0 h}{k_m} = \frac{-2 r_0 q''_w}{k_m(T_w - T_m)} = \frac{2 q^+}{k_m^+ (\theta_w - \theta_m)} \]  \hspace{1cm} (3.2)

where subscript $m$ refers to quantities or properties evaluated at the bulk fluid temperature at an axial point. The non-dimensionalized heat transfer and wall shear stress are defined in terms of the temperature and velocity profiles by:

\[ \tau_w^+ = -\mu^+ \frac{\partial U^+}{\partial r^+} \bigg|_{r^+=1} \]  \hspace{1cm} (3.3)

\[ q_w^+ = -k^+ \frac{\partial \theta}{\partial r^+} \bigg|_{r^+=1} \]  \hspace{1cm} (3.4)

respectively.
Values of the power law exponents \( a, b, c \) (2.24, 2.25, 2.26) and \( \gamma_0 \) and \( \rho_0 \) were evaluated from published property data for three gases: air, helium and carbon dioxide. (See Appendix B) The transport properties of helium follow the power law almost exactly. While air and CO\(_2\) are not monatomic gases, the transport properties for air can still be fairly represented by the power law. These representations are not very good for CO\(_2\), but this type of variation was assumed to hold true anyway so as to provide a rough idea of the behavior of the gas. This gas is of some interest since its transport properties vary much more severely than the other two gases. Due to this approximation, correlation of the wall parameters for CO\(_2\) was not attempted. The properties and exponents were evaluated from a least squares fit to the tabulated data. The exponents were chosen so as to minimize the sum-squared error for all reference (subscript zero) points chosen in the range of tabulated data. Also, the properties are weak functions of pressure. The data was chosen for a pressure of 1 atmosphere which corresponds closely with the conditions run in the experimental apparatus, although this data should represent the properties quite accurately up to several atmospheres. The data for the three gases is summarized in Table 3.1.
Table 3.1. Transport and Thermodynamic Properties

<table>
<thead>
<tr>
<th>Gas</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>Pr₀</th>
<th>γ₀</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air</td>
<td>0.12</td>
<td>0.64</td>
<td>0.71</td>
<td>0.71</td>
<td>1.36</td>
</tr>
<tr>
<td>He</td>
<td>0.00</td>
<td>0.69</td>
<td>0.69</td>
<td>0.67</td>
<td>1.67</td>
</tr>
<tr>
<td>CO₂</td>
<td>0.29</td>
<td>0.74</td>
<td>1.38</td>
<td>0.71</td>
<td>1.21</td>
</tr>
</tbody>
</table>

Straightforward application of the finite difference program obtained from Worsoe-Schmidt and corresponding to the description given in reference 100 typically resulted in the behavior of fRe,m as shown in Figure 6. Similar behavior was obtained for Nu,m. The results are for He at an inlet wall to bulk temperature ratio of 0.10. The parametric curves in each plot correspond to different radial mesh divisions which are indicated on the graphs—also, the discontinuities in each curve correspond to the point where the number of mesh nodes was halved. At first it was suspected that this behavior might be due to a local instability or error in the profiles due to the mesh change, but examination of the profiles directly before and after the change revealed little or no noticeable difference over what could be considered as normal axial development. It should be noted from these plots that the effect of the step change is diminished as the mesh is refined. Also, the change to a coarser mesh in each
Figure 6. Effect of change of radial mesh size on $fRe_m$ when evaluated from a cubic polynomial.

Figure 7. Effect of change of radial mesh size on $Nu_m$ when evaluated from a 5 point spline.
case yields results that would be obtained if the coarser mesh had been used entirely. For example, the wall parameters after a change from 80 to 40 radial divisions have the same value as the results obtained for 40 radial divisions throughout. This would seem to indicate an invariance of the solution of the governing equations and points to a deficiency in the method of evaluating the wall derivatives.

In an effort to correct this, many types and orders of curvefits (for example— splines, Chebyshev polynomials, Lagrangian polynomials, ratios of rational polynomials, etc.) were tried in place of the cubic polynomial used by Worsoe-Schmidt and none were found to significantly improve the behavior. For example, values of $\text{Nu}_{\text{m}}$ obtained from a 5 point spline for varying radial meshes are shown in Figure 7. The reason for this failure in the cooling case and not in the heating case can be illustrated by examination of the expression for the error incurred when a first derivative is evaluated from taking the derivative of a Lagrangian polynomial of order $n-1$. The polynomial is fitted at $n$ tabular points of an analytical function $F$. The true derivative is $F'$. At a tabular point, we can write the error as (3.5):

$$\frac{d}{dr} F(r)_{r=r_i} - \frac{d}{dr} F(r)_{r=r_i} = \frac{F_{n+1}(r_i)}{n!}$$

\[(3.5)\]
where \( n \) = number of tabular points
\[ n-1 = \text{order of the polynomial fit} \]
\[ \Gamma_n(r) = \prod_{i=1}^{n-1}(r-r_i) \]

\[ \Lambda_n(r) = \text{Lagrangian polynomial of order } n-1 \]
\[ r_i = \text{tabular point} \]
\[ r' = \text{a value of the independent variable included} \]
\[ \text{in the range spanned by the tabular points} \]
\[ F = \text{hypothetical closed form solution for the velocity or enthalpy profile} \]

Although it is impossible to evaluate \( r' \) in this case, maximum values of the higher order derivatives of the temperature and velocity profiles near the tube wall as evaluated from difference quotients are summarized in Table 3.2. These maximum values occur in the region that would be included by polynomials of the degrees indicated for 80 radial mesh points and were evaluated from a solution using 320 radial mesh points for the severest cooling case (\( \theta_w = 0.10 \)) considered here. Magnitudes of the derivatives for the severest heating case considered in the Worsoe-Schmidt analysis (\( q^+_{w} = 20 \)) are included for comparison. Since the factor \( \frac{\ln n}{n!} \) will be the same for both the heating and cooling cases when the same order polynomial and mesh sizes are used, the errors will be proportioned to \( F(r') \). Polynomials of degree greater than 5 were not included because of generally poor suitability of high ordered polynomials for the calculation of derivatives.
Both results are for helium with $M_0 = 0.03$. It can be seen that the higher ordered derivatives in our case are greater by as much as three orders of magnitude, and are on the average, one degree of magnitude greater. In particular, for the third order polynomial, the applicable derivative of the velocity profile is 35 times greater and for the temperature profile more than 250 times greater for the cooling case.

Table 3.2. High Ordered Profile Derivatives

<table>
<thead>
<tr>
<th>n</th>
<th>Degree of Fit</th>
<th>Velocity Profile</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\left</td>
<td>\frac{\Delta^n}{\Delta r^n} \left( \frac{U}{U_0} \right)_{\text{max}} \right</td>
</tr>
<tr>
<td>3</td>
<td>2</td>
<td>$0.786 \times 10^4$</td>
<td>$0.332 \times 10^3$</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>$0.723 \times 10^7$</td>
<td>$0.211 \times 10^6$</td>
</tr>
<tr>
<td>5</td>
<td>4</td>
<td>$0.158 \times 10^{10}$</td>
<td>$0.133 \times 10^9$</td>
</tr>
<tr>
<td>6</td>
<td>5</td>
<td>$0.387 \times 10^{12}$</td>
<td>$0.731 \times 10^{11}$</td>
</tr>
</tbody>
</table>
An independent method of evaluating the shear stress and heat transfer can be obtained if a momentum and energy balance is performed between two axial mesh points. Radial integration of the momentum and energy equations 2.30 and 2.32 results in the following expressions for the average heat transfer $q_w^+$ and wall shear stress $\tau_w^+$ between two adjacent axial points:

$$ q_w^+ = \frac{1}{2\Delta x^+} \left[ \left( \int_0^{r^+} \rho^+ U_1^+ \frac{r^+}{x_1} dr^+ \right)_{m+1} - \left( \int_0^{r^+} \rho^+ U_1^+ \frac{r^+}{x_1} dr^+ \right)_m \right]$$

$$ + \frac{1}{2} (\gamma - 1) \frac{M_0^2}{P_{m+1} - P_m} \left[ \left( \int_0^{r^+} r^+ dr^+ \right)_{m+1} - \left( \int_0^{r^+} r^+ dr^+ \right)_m \right]$$

$$ - (\gamma - 1) \frac{M_0^2}{P_{m+1} - P_m} \left[ \left( \int_0^{r^+} \left( \frac{\partial U}{\partial r} \right)^2 r^+ dr^+ \right)_{m+1} \right.$$  
$$ + \left. \left( \int_0^{r^+} \left( \frac{\partial U}{\partial r} \right) r^+ dr^+ \right)_m \right] \Delta x^+ $$

$$ \tau_w^+ = \frac{1}{2Pr_o \Delta x^+} \left[ \left( \int_0^{r^+} \rho^+ U^+ r^+ dr^+ \right)_{m+1} - \left( \int_0^{r^+} \rho^+ U^+ r^+ dr^+ \right)_m \right]$$

$$ + \frac{1}{4Pr_o \Delta x^+} (P_m - P_{m+1}) $$

The radial integrations were evaluated by means of Simpson's rule with a resultant error of the order $(\Delta r^+)^5$ which is smaller by a factor $(\Delta r^+)^3$ than that of the finite-difference
scheme. Values of the Nusselt number and \( f_{Re,m} \) as evaluated from these expressions at one axial point are shown in Figures 8 and 9 respectively as functions of the maximum number of radial mesh points. Wall parameters from the curvefit method are included for comparison. In both cases, variation of the integrated values are quite small, while the curvefit results are asymptotically approaching these values. The error between the curvefit and the true first derivative will decrease with decreasing \( \Delta r^+ \) by virtue of the function \( \Gamma_n(r) \) in equation 3.7. However, for the plotted values, a small difference between the two methods would still be present even for infinitesimal \( \Delta r^+ \) since the integrated parameters apply to the point \( (m - \frac{1}{2}) \Delta x^+ \) rather than \( m \Delta x^+ \). At most points though, the value of \( \Delta x^+ \) was sufficiently small with respect to \( x^+ \) such that these values could be considered as point values. The results presented herein were plotted with this correction at small \( x^+ \).

Two additional considerations arising from this analysis should be noted here. The first deals with convergence checks that Worsoe-Schmidt was able to use. An independent check of how well the solution is satisfying conservation of total momentum and energy in the tube can be obtained by comparison of two sides of the equalities obtained from the double integration of the momentum and
Figure 8. Variation of local Nusselt number with radial mesh size at a fixed axial point. Integrated energy equation and 5 point spline.

Figure 9. Variation of $fRe_m$ with radial mesh size at a fixed axial point. Integrated energy equation and cubic polynomial.
energy equations;

Total Momentum:

\[
2 \left[ (\int_0^{x^+} \rho^+ U^+ r^+ dr^+) x^+ \right] - (\int_0^{x^+} (\rho^+ U^+ r^+ dr^+) x^+ \right]_{x^+=0} = p - 4 Pr \int_0^{x^+} \tau^w dx^+ 
\]

Total Energy:

\[
2 \left[ (\int_0^{x^+} (\rho^+ H^+ r^+ dr^+) x^+ \right] = \frac{4}{5} \int_0^{x^+} \rho^+ U^+ r^+ dr^+ dx^+ = \frac{2}{5} (\chi - 1) M_o^2 \left\{ \int_0^{x^+} \delta^+ \left( \frac{2 U^+}{3 dr^+} \right)^2 r^+ dr^+ dx^+ \right\} (3.9)
\]

where Worsoe-Schmidt evaluated the \( \tau^w \) and \( q^w \) terms from the curvefit method. Since this method has been shown to be unacceptable in our case, the only independent check remaining is that of the conservation of mass equation;

\[
2 \int_0^1 (\rho^+ U^+ r^+ dr^+) - 1 = 0 (3.10)
\]

The conservation of mass is incorporated into the solution on a local basis (2.62) so that equation 3.10 represents a measure of the drift of the solution. It is also likely that large errors in 3.10 would reflect large errors in the overall conservation of axial momentum. In addition, a good indication of how well the solution is progressing
can be provided by observing whether the wall parameters converge to their correct asymptotic values in the downstream region and how the solution in the developing region behaves with varying mesh size and \( \sigma \). These latter methods were used quite liberally throughout the generation of results.

Wall parameters obtained from equations 3.6 and 3.7 are shown plotted as a function of \( x^* \) in Figures 10 and 11 for air and helium respectively and Figures 12 and 13 for carbon dioxide. The parametric curves correspond to different inlet wall to bulk temperature ratios which range from sum 0.90 to 0.1.\(^5\) The same results for air are plotted in Figure 14 as a function of the non-dimensionalized axial co-ordinate \( x_m^* \) based on local rather than inlet conditions.

No clear advantage of one representation over the other exists. While the parameters converge to their asymptotic values more rapidly when plotted against \( x_m^* \), the effect of wall to bulk temperature ratio is augmented on \( \text{Nu}_m \). No improvement or degradation of the product \( fRe \) occurs in the entrance since these curves are very

\(^5\)It was not possible to evaluate the fully isothermal Nusselt number since the term \( \theta_w - \theta_m \) becomes zero in the denominator of equation 3.2. Also, small absolute errors in the solution for the temperature profile would result in large errors in \( \theta_w - \theta_m \) if \( \theta_w \) were specified as, say 0.99.
Figure 10. Axial variation of $f\text{Re}_m$ and $Nu_m$ for air. Graetz boundary condition, $M_0 = 0.03$
Figure 11. Axial variation of $fRe_m$ and $Nu_m$ for Helium. Graetz boundary condition, $M_0 = 0.05$.
Figure 12. Axial variation of $N_{u,m}$ for carbon dioxide. Graetz boundary condition, $M_o = 0.01$.
Figure 13. Axial variation of $fRe_m$ for carbon dioxide. Graetz boundary condition, $M_o = 0.01$
Figure 14. Local Nusselt number and $f\text{Re}_m$ versus $X_m^+$ for air. Graetz boundary condition $M_0 = 0.03$.
nearly horizontal—relative axial displacement does not affect the vertical spacing. When plotted as a function of $x^+$, $\text{Nu}_m$ shows surprisingly little variation with respect to the inlet wall to bulk temperature ratio. For example, the maximum decrease in this quantity resulting from an almost ten fold decrease in the temperature ratio is on the order of 14% for air and helium and 33% for CO$_2$. The Nusselt number reaches the fully developed value more rapidly with reduced $\theta_w$. A simple linear variation with inlet wall to bulk temperature ratio will describe the theoretical Nusselt number behavior to within 5%.

$$\text{Nu}_m = (3.67 + Ax^+ B e^{-\beta x^+})(1-C\left(\frac{T_0}{T_w} - 1\right)) \quad (3.11)$$

0.001 $\leq x^+ \leq 0.35$

$$\text{Nu}_m = 3.67 \quad x^+ > 0.35 \quad (3.12)$$

where for air, $A = 0.198 \quad B = -0.584 \quad C = 0.13 \quad \beta = -20.8$

He $\quad A = 0.201 \quad B = -0.584 \quad C = 0.15$

The variation of the friction factor is more pronounced and required a different type of correlation. If the product $fRe_m$ is plotted as a function of local wall to mean temperature ratio, Figures 15 for helium and 16 for air result. The different curves correspond to different
Figure 15. $f_{Re,m}$ versus $T_w/T_m$. Helium, Graetz boundary condition, $M_0 = 0.05$
Figure 16. $fRe_m$ versus $Tw/Tm$. Air, Graetz boundary condition, $M_o = 0.01$.
inlet wall to bulk temperature ratios and were approximated by straight line segments passing through \( fRe_m = 16.0 \) at \( T_w/T_m = 1.0 \). The slopes of these lines were determined by a least squares criterion for nearly equally spaced \( T_w/T_m \). These straight line approximations would seem to be fairly good with the exception of \( T_w/T_o = 0.10 \). The significance of the slope \( a \) of these log-log plots is defined in the equation:

\[
fRe_m = 16 \left( \frac{T_w}{T_m} \right)^a \quad (3.13)
\]

In Figure 17, \( a \) for each gas is plotted against \( T_w/T_o \) on log-log paper and a correlation of the form

\[
a = b \left( \frac{T_w}{T_o} \right)^c \quad (3.14)
\]

is excellent. The values of \( b \) and \( c \) were also chosen by a least squares criterion. The final form of the friction factor correlations are, for air,

\[
fRe_m = 16 \left( \frac{T_w}{T_m} \right)^a \quad a = 0.904 \left( \frac{T_w}{T_o} \right)^{0.257} \quad (3.15)
\]

and for helium

\[
a = 0.957 \left( \frac{T_w}{T_o} \right)^{0.251} \quad (3.16)
\]

In the range \( 0.001 \leq x^+ \leq 0.5 \). These correlations will
Figure 17. Exponent in $f_{Re_m}$ correlation versus $T_w/T_0$ for air, helium and carbon dioxide. Graetz boundary condition.
describe the theoretical results within 9% for $T_w/T_0 = 0.10$ and within 5% for $T_w/T_0 \geq 0.20$. The initial curvature at the beginning of each of the curves in Figure 15 is probably due to starting errors in the finite difference solution.

The axial development of the axial velocity non-dimensionalized with respect to the local mean velocity and the reduced temperature, $T_{\text{red}}$, where

$$T_{\text{red}} = \frac{T - T_w}{T_m - T_w}$$  \hspace{1cm} (3.17)

is shown in Figure 18 for air at inlet wall to bulk temperature ratios of 0.10 and 0.90. Radial velocity $V^+$ development for the same cases are shown in Figure 19. The physical reason for the observed behavior of the wall parameters can be seen from these figures. For example, the radial derivative of this reduced temperature,

$$\frac{\partial T_{\text{red}}}{\partial r^+} = \frac{\partial}{\partial r^+} \left( \frac{T_w - T}{T_w - T_m} \right) = \frac{\partial T_{\text{red}}}{\partial r^+}$$  \hspace{1cm} (3.18)

is a term in the expression for $Nu_{,m}$

$$Nu_{,m} = \left( \frac{2k_w}{k_m} \right) \frac{\partial T_{\text{red}}}{\partial r^+}$$  \hspace{1cm} (3.19)
Figure 18. Axial development of reduced temperature and velocity profiles for two wall to inlet temperature ratios. Air, Graetz boundary condition.
Figure 19. Dimensionless radial velocity profiles for developing flow of air at two wall to inlet temperature ratios. Graetz boundary condition.
As the wall to bulk inlet temperature ratio is reduced, the magnitude of the radial velocity increases in an outward direction along with a convected energy. The increasing density of the gas at the wall tends to augment the heat transfer, while the reduced thermal conductivity at the wall tends to decrease it. The temperature profiles are seen to remain flatter for a further axial distance with decreasing $\theta_w$ along with a corresponding increase in the magnitude of $\frac{\partial T_{red}}{\partial r^+}$ which tends to offset the decrease in thermal conductivity. On the other hand, the slope of the non-dimensionalized velocity $U/U_m$ is relatively insensitive to changes in $\theta_w$.

The product $fRe_m$ may be written as,

$$fRe_m = \frac{4\mu_w \partial(U/U_m)}{\mu_m \partial r^+}$$  \hspace{1cm} (3.20)

where since $\partial(U/U_m)/\partial r^+$ shows little change, the controlling factor will be the term $\mu_w/\mu_m$. For example, at small $x^+$ we have $T_w/T_m \approx T_w/T_0$. On this basis, for $\theta_w = 0.10$, $fRe_m$ should differ by a factor approximately equal to;

$$\frac{\mu_w}{\mu_m} = \left(\frac{m_w}{T_m}ight)^b \approx \left(\frac{T_w}{T_0}\right)^b = (0.10)^{0.69} = 0.21$$
from the isothermal product \( f \text{Re}_m \) for helium. In the actual case, this factor is approximately equal to 0.28 at \( x^+ = 0.001 \) and becomes less than this and closer to the above value for axial displacements less than this. However, wall parameters obtained in the region before this occur only after a few axial steps and should be treated with a great deal of caution. For all inlet wall to bulk temperature ratios, a region of increasing static pressure occurred in the entrance—the magnitude of the rise and the extent of the region depended on the magnitude of the temperature ratio. Where a step change in the wall temperature occurs, the axial derivative of the bulk fluid temperature and the fluid bulk density will be infinite. The deceleration of the flow in the entrance will be of sufficient magnitude to overcome the static pressure drop due to wall friction. This will be discussed further in Chapter 5. The Mach numbers that were specified for the results presented (0.05 for helium and 0.03 for air) may be considered as being on the high side for laminar flow, although the effect of halving these values was found to have a negligible influence on the results.
CHAPTER 4

UNIFORM TEMPERATURE AND VELOCITY PROFILE BOUNDARY CONDITIONS

ANALYTICAL SOLUTION

4.1. Background

Typical results obtained from the finite difference solution when the boundary condition of uniform inlet velocity and temperature profile is specified are shown by the dotted line in Figure 20 for helium at an inlet wall to bulk temperature ratio of 0.95. Mesh sizes and points where changes occur are indicated. The solution has not converged to yield the correct asymptotic value of $N_u_m$ while not enough is known to determine if the friction factor is correct. Due to the coupling of the momentum to the energy equation, it is quite likely that error exists. For the case $\theta_w = 0.95$, small absolute errors in the temperature profile due both to computational truncation in the computer and truncation of the terms in the derivative representations 2.37, 2.38 and 2.39 would result in large errors in the evaluation of $\theta_w - \theta_m$ and $N_u_m$. However, divergent results in the downstream region for wall to bulk temperature ratios down to 0.10 were obtained. Absolute errors in the temperature profile would have to be an order of magnitude greater to affect the results for $\theta_w = 0.10$. This would seem to rule out computational truncation as being responsible in this
Figure 20. Axial variation of $\text{Nu}_m$ and $\text{fRe}_m$ for UTV boundary condition. Helium, $T_w/T_0 = 0.95$, $M_0 = 0.03$
case, since this error will probably remain of the same
order of magnitude for both temperature ratios since
for the same mesh size, the number of computations will
stay roughly the same. The increased severity of the
cooling will result in larger magnitudes of the higher
order derivatives and increased magnitudes for the error
terms in equations 2.37, 2.38 and 2.39. This could be
the cause of the error. For a case Worsoe-Schmidt ran
with uniform temperature and velocity and constant heat
addition, Nu,m converged to the wrong asymptotic value.
He conjectured that this was due to large errors incurred
in the solution of the energy and momentum equations at
the tube entrance. Several cases run here with different
mesh sizes showed that in general, refinement of the mesh
in the entrance region improved the downstream results,
whereas refinement of the downstream mesh had little or
no effect. Little change was noted from varying σ or
use of double precision arithmetic. These would seem to
support Worsoe-Schmidt’s conjecture. While improvement
was noted, results were still unacceptable even using 320
radial mesh points and Δx* as small as 1.0x10⁻⁵ near the
entrance (x* ≤ 0.001).

4.2. Choice of Method of Solution

Two methods of resolving this difficulty were con-
sidered; 1.) by improvement and continued use of a
completely finite difference solution or 2.) by use of an analytical boundary layer solution in the tube entrance. There are many methods by which the numerical solution could be improved. For example, the program could have been rewritten so that it would check its own convergence and choose its own mesh size to achieve convergence, and/or a variable radial mesh which would allow a much finer radial step to be used near the wall where the variables are undergoing far more variation than in the center region of the tube. In order to test the suitability of the completely finite difference scheme, a technique was used which is often applied in the numerical solution of ordinary differential equations (71). As an illustration, consider an ordinary differential equation of the form:

$$\frac{dY}{dx} = F(x, y) \quad Y(x_o) = Y_o \quad (4.1)$$

which we are integrating from $x_o$ to $x_e$ using a numerical scheme--for example, a Runge-Kutta method. If we obtain values of $Y(x_e)$ from use of three different step sizes for the variable $x$, we can plot the value of $Y(x_e)$ versus step size $\Delta x$ (Figure 21a). If our numerical scheme is stable and consistent with the differential equation, we would expect that a better estimate of $Y(x_e)$ (i.e., closer to the exact solution of the differential equation) could be obtained by fitting a curve through these values at $x_e$ and extrapolating to $Y_e$ for zero step size. For a
Figure 21. a. One dimensional extrapolation of finite difference solution to zero mesh size.

Figure 21. b. Two dimensional extrapolation of finite difference solution to zero mesh size.
function of two variables, say $U(x,y)$, the value of $U(x_e,y_e)$ for different step sizes in $x$ and $y$ would define a surface (Figure 21b) and an estimate of the value $U(x,r)$ corresponding to $\Delta x=0$, $\Delta r=0$ would be obtained by extrapolating the surface to a line in the $\Delta y=0$ plane, then extrapolating the line to $\Delta x=0$. For the case considered here, a relation of the form

$$\phi_{m,n} = \Phi_{m,n}(1+A_{m,n}^{x} \Delta x^{2}+B_{m,n}^{x} \Delta x^{\mu}+\cdots)(1+A_{m,n}^{r} \Delta r^{2}+B_{m,n}^{r} \Delta r^{\mu}+\cdots)$$

(4.2)

was assumed to exist. $\phi_{m,n}$ is the value of the dependent variable (temperature or velocity) obtained at the node $m,n$ by use of step sizes $\Delta x^{+}$ and $\Delta r^{+}$ in the numerical solution, $\Phi_{m,n}$ represents the value that would be obtained for zero step sizes and $A_{m,n}^{x}$, $B_{m,n}^{x}$, $A_{m,n}^{r}$ and $B_{m,n}^{r}$ are constants which must be determined for each node. In the actual implementation of this equation to solve for $\phi_{m,n}$, the series in $\Delta x^{+}$ and $\Delta r^{+}$ were truncated after three terms. The constants were determined for all the radial nodes at the axial point $x^{+}=0.001$ by running 5 separate solutions to this point for the same boundary conditions, but with varying $\Delta x^{+}$ and $\Delta r^{+}$, (the finest mesh used was $\Delta x^{+}=10^{-5}$ with 320 radial nodes). This allowed solution for the values of the radial and axial velocity, the enthalpy profiles and the pressure defect that
correspond to a mesh much finer than the smallest actually used. These refined profiles were reinserted into the finite difference program and the solution continued in a normal manner. Double precision arithmetic and ten iterations at each axial step for the complete set of equations 2.30, 2.31, 2.32 up to $x^+ = 0.20$ were used. The results are shown in Figure 20 for $\theta_w = 0.95$. Similar improvement was found for $\theta_w = 0.10$, but in both cases, complete convergence was not obtained. Differences between the extrapolated profiles and those from the finest mesh size used were surprisingly small and occurred in the second decimal place.

The method of using an analytical boundary solution was evaluated by applying the Blasius solution for the growth of a thermal and velocity boundary layer with zero pressure gradient and constant properties. The velocity and temperature fields are assumed to undergo a normal boundary layer growth at the tube wall. Outside of this boundary layer lies a potential flow field with uniform temperature and velocity. After determination of a similarity parameter for use in the Blasius solution consistent with the non-dimensionalized form of the boundary layer equations, the axial velocity in the core, $U_e^+$, and the pressure defect, $(p_0 - p)/\rho_0 U_0^2$ were evaluated at any axial point by solving the total momentum and
continuity equations.

The axial velocity profile development as obtained from this solution is compared with that of Hornbeck (35) for the isothermal case in Figure 22. For small $x^+$, the comparison is very good. The profiles obtained from this method were patched to the numerical solution at several axial points. A wide range of axial points was found for which convergence in the downstream region was significantly improved. Results for this method applied at $x^+ = 0.00025$ and $\theta_w = 0.95$ are shown plotted in Figure 20 along with those from the rational extrapolation method. On the basis of these results, it was decided to proceed with an improved analytical boundary layer solution at the entrance.

The application of the constant property boundary layer growth for $\theta_w = 0.10$ seemed to result in a significant overestimate of $\text{Nu}_m$ at the entrance (with respect to the completely numerical solution) for an extended distance after a patch to the finite difference solution. Properties in the boundary layer solution were evaluated at a film temperature midway between the wall and inlet bulk temperatures. It should be noted that the wall parameter results at the entrance obtained from the finite difference solution were found to be insensitive to mesh size. This lends confidence to the finite difference results as being correct there and indicates that Blasius
Figure 22. Comparison of axial velocity development from simplified analysis with that from Hornbeck.
profiles are not suitable for the non-isothermal case.

An analytical boundary layer solution which includes property variation and pressure gradient is described in the next section.

4.3. Similarity Solution—Compressible Variable Property Boundary Layer Growth with Pressure Gradient for Tube Flow

A solution for the thermal and velocity boundary layer growth at the wall in the entrance of a cylindrical tube is sought along with a method of coupling this solution to the internal flow. Similarity methods have been shown to yield satisfactory results for many situations even where the requirements for similarity are not satisfied exactly (23, 85). More will be said about these requirements after the transformation of the boundary layer equations in terms of the similarity parameter.

At points where the boundary layer thickness is small with respect to the tube radius, \( \delta/\delta_0 \ll 1 \), the boundary layer behaves as though it were developing on a flat plate. When an order of magnitude analysis is applied to the boundary layer equations in cylindrical co-ordinates, certain terms can be shown to be negligible. When they are neglected the following non-dimensional equations obtain:
Momentum;

\[ \rho^+(U^+ \frac{\partial U^+}{\partial x^+} + V^+ \frac{\partial U^+}{\partial y^+}) = \frac{dP}{dx^+} + 2Pr_0 \frac{\partial}{\partial y^+}(\mu^+ \frac{\partial U^+}{\partial y^+}) \]  

(4.3)

Continuity

\[ \frac{\partial}{\partial x^+}(\rho^+ U^+) + \frac{\partial}{\partial y^+}(\rho^+ V^+) = 0 \]  

(4.4)

Energy;

\[ \rho^+(U^+ \frac{\partial H_2^+}{\partial x^+} + V^+ \frac{\partial H_2^+}{\partial y^+}) = \frac{\partial}{\partial y^+}(\frac{k^+}{\rho c_p^+} \frac{\partial H_2^+}{\partial y^+}) - (\gamma - 1)M_o^2 \left[ U^+ \frac{dP}{dx^+} - 2Pr_0 \mu^+ \left( \frac{\partial U^+}{\partial y^+} \right)^2 \right] \]

where the non-dimensionalized variables are the same as in equations 2.30, 2.31 and 2.32 with the exception of

\[ H_2^+ = (H - H_w)/c_p o T_o \]  

(4.6)

and

\[ y^+ = 1 - r^+ \]  

(4.7)

where \( y^+ \) represents the distance measured in a positive sense away from the wall. The transverse velocity maintains the usual boundary layer convention of being positive in a direction away from the wall. The transverse velocity is related to the radial velocity by \( V^+ = -V^+ \).

We transform to the following non-dimensionalized
independent variables after Dewey and Forbes (23):

\[ \xi = \int_0^{x^+} \frac{\rho^+ u_e^+ U_e^+}{\sqrt{2\xi e}} \, dx^+ \]  
(4.8)

\[ \eta = \frac{U_e^+}{\sqrt{2\xi}} \int_0^{y^+} \rho^+ dy^+ \]  
(4.9)

where subscript \( e \) refers to conditions at the edge of the boundary layer or in the central core flow. \( \eta \) is the similarity parameter. We assume that at any point \( x^+; \)

\[ \frac{U^+}{U_e^+} = \frac{U}{U_e} = f(\eta) \]

\[ H_2^+/H_2, e = G(\eta) \]

where \( f(\eta) \) and \( G(\eta) \) are functions of \( \eta \) only. The differential continuity equation 4.4 can be eliminated by solving for \( V^- \) in terms of \( f \) and \( G \). After the required transformations are made (c.f. Appendix I), the following differential equations are obtained:

Momentum:

\[ 2Pr_0 (\lambda f''(\eta))' + f(\eta) f''(\eta) = \beta (f(\eta) - \frac{\rho_0}{\rho}) \]  
(4.10)
Energy:

\[ 2 \left( \frac{\Lambda \gamma'(\eta)}{Pr^+} \right)' + C(\eta) f(\eta) = (\gamma - 1) \frac{M_o^2}{H_e^+} \frac{U^2_e}{\rho} \beta f(\eta) - 2 Pr_o \Lambda f^2 \]  

(4.11)

where \[ \lambda = \rho_e / \rho \]

\[ \beta = \text{modified Falkner-Skan parameter} = \frac{2 \xi dU^+_e}{U^+_e d \xi} \]  

(4.12)

(4.13)

the boundary conditions to be satisfied are

at \( \eta = 0 \)  
\[ f'(\eta) = 0 \quad \text{and} \quad G(\eta) = 0 \]

and at \( \eta = \infty \)  
\[ f'(\eta) = 1 \quad \text{and} \quad G(\eta) = 1 \]

The equations in this form represent a pair of coupled, non-linear ordinary differential equations. In addition they are of the two point boundary value type rather than an initial value problem. The terms in these equations which provide coupling to the internal flow are \( \beta \) in the momentum equation and \( U^+_e \) and \( \beta \) in the energy equation.

It should be noted that an attempt was made to use the Probstein-Elliott-Levy-Lees Transformation where it is assumed that

\[ U/U_e^+ = f(\eta, \xi) \]

and

\[ H^+/H_e^+ = G(\eta, \xi) \]

The prime still represents differentiation with respect to \( \eta \). The differential equations which result will contain derivatives in both the \( \eta \) and \( \xi \) directions,
but are treated as differential equations in $\eta$ only. The $\xi$ derivatives are written as axial centered difference operators in terms of the unknown dependent variables and their values on the previous step. This allows a stronger coupling to conditions outside the boundary layer, or for example, to an axial variation of the wall temperature. The problem came with representation of the profiles at the previous step for evaluation of the axial derivatives. The values of the dependent variables there are known at specified and equal $\eta$ intervals, while the integration procedure at the new step solves the equations and requires evaluation of the non-$\eta$-derivative terms at intermediate steps which are determined by the convergence of the equations and are not known beforehand. At the edge of the boundary layer, all terms in the differential equations become extremely small. Evidently, small error or inflections in the interpolation schemes used were sufficient to give the integration routine a great deal of difficulty in this region and the routine often would report non-convergence at large values of $\eta$. When convergence could be obtained, little difference was seen in the profile from a test solution where the $\xi$ dependence in $\hat{f}$ and $G$ was removed so the variation terms were removed altogether.

The assumption that the profiles are similar with
respect to \( \eta \) is satisfied exactly if these differential equations are functions of \( \eta \) only. This requirement places the following restriction on some of the terms:

\[
\lambda = \lambda(\eta) \quad (4.14)
\]

\[
\beta \left( f^2 - \frac{\rho}{\rho} \right) = F_1(\eta) \quad (4.15)
\]

\[
(\gamma - 1) M_o^2 \left( \frac{U_e}{H_e} \right)^2 \left( \frac{\rho}{\rho} \beta f' \right)^2 - 2 Pr_0 \lambda f'' = F_2(\eta) \quad (4.16)
\]

Since the static pressure is assumed constant across the boundary layer,

\[
\lambda = \left( \frac{T_c}{T} \right) \left( \frac{T}{T_e} \right)^b \quad (4.17)
\]

In the central core, we will have \( T_e = T_o \) and using equation 2.29 we have

\[
\frac{T}{T_e} = \frac{T}{T_o} = \theta = (G(1 - \theta_w^{a+1}) + \theta_w^{a+1})^{1/\alpha+1} \quad (4.18)
\]

and

\[
\lambda = (\theta_w^{a+1} + (1 - \theta_w^{a+1}) G)^{\alpha-1/\alpha+1} \quad (4.19)
\]

where \( G = G(\eta) \) is the only dependent variable present
so that equation 4.14 is satisfied exactly. Clearly, equations 4.15 and 4.16 cannot be satisfied exactly, but we note that in the entrance region, it can be shown by using a constant property boundary layer solution in conjunction with the total continuity equation, that $\beta$ behaves approximately as $\sqrt{\xi}$ and is equal to zero at $x^+ = 0$. Since we are applying this solution only for small $x^+$, $\beta$ will always be small. The addition of variable properties is not expected to change these results significantly. In fact for the cooling case, the increased density at the wall results in a reduced velocity boundary layer displacement thickness and correspondingly, a reduced $dU_e^+/d\xi$ and $\beta$ since the flow to the core is reduced. Our requirements are then met more closely in the cooling than in the heating case. At small $x^+$, expression 4.15 should be approximately equal to zero. For laminar flow, the Mach numbers considered are in the range 0.01 to 0.05 so that the multiplication factor $M_o^2$ is very small (0.0001 to 0.0025). Also, the magnitude of $U_e^+/H_e^+$ does not differ significantly from 1 in the region under consideration. It is important to note that for some types of equations, the inclusion of a small term can change the solution entirely, for example, if the terms govern the order of the equation. This is not the case here. It would be extremely difficult to give a definitive answer as to the error that is
incurred by the assumption that at least local similarity is satisfied. However, in an analysis of the incompressible momentum equation with an external pressure gradient, Dewey and Gross (23) by using an approach based on the work of Mecksyn were able to show that the derivative of the velocity at the wall \( f'(0) \) from a solution assuming local similarity can be written in terms of the exact solution of the equation \( f''(0) \) by,

\[
f''(0) = f''(0) \left[ 1 - 0.053\epsilon + O(\epsilon^2) + \ldots \right]
\]

where prime again denotes differentiation with respect to \( \eta \). The small parameter \( \epsilon \) is defined by

\[
\epsilon(\beta) = 2\xi \frac{d\beta}{d\xi} = 2\xi \frac{d}{d\xi} \left( \frac{2U_e^+}{\sqrt{\xi}} \right)
\]

which measures the departure of the solutions from complete similarity. Expanding \( \epsilon \) and noting that \( dU_e^+/d\xi \sim -\frac{1}{\sqrt{\xi}} \) in the tube entrance, we obtain

\[
\epsilon(\beta) \sim 2(\beta - \beta^2 + \frac{\xi^2}{U_e^+} \frac{d^2U_e^+}{d\xi^2}) = 0 \quad \text{at} \quad \xi = 0
\]

This means that relaxation effects due to increasing or decreasing \( \beta \) will be zero at the entrance and can be expected to remain small for small \( x^+ \). This adds further confidence to the use of this method for the present
problem.

4.4. Integration Procedure

The program for the integration of equations 4.10 and 4.11 was written so that an available algorithm for the integration of ordinary differential equations of the initial value type could be used (15). Reduction of these equations to initial value problems and the coupling to the internal flow is described in this section. It is assumed initially that the correct $U_e^+$ and $\beta$ are known at the axial point where the boundary layer profiles are being determined. For the solution at the first point, $x^+ = 0$ ($\xi = 0$), the exact values are known to be;

$$\beta = 0 \quad U_e^+ = 1 \quad (4.23)$$

The integration is started by specifying the known values $f'(0) = 0$, $G(0) = 0$, guessing initial values of $f''(0)$ and $G'(0)$ and integrating the equations to a relatively large value of the independent variable $\eta_e$ where the boundary layer growth is considered to be essentially complete. For the Blasius solution and for the variable property cases, the solutions were within 0.01% of fully developed for $\eta = 7.0$. At $\eta = 7$, a check was made on the quantities $f'(\eta_e)-1$ and $G(\eta_e)-1$. If these were both less than 0.0001 in absolute magnitude, then the values of $f''(0)$ and $G'(0)$ were considered as the correct values for the specified $U_e^+$ and $\beta$ and solution could be transferred to
determination of new values of $\beta$ and $U_e^+$ (Figure 23). If not, each of the initial guesses $f''(0)$ and $G'(0)$ were perturbed—the magnitude of the perturbation dependent on how many previous times solution had been attempted for $\beta$ and $U_e^+$ at this axial point. For example, on the first solution, initial guesses for $f''(0)$ and $G'(0)$ are made from the Blasius solution. On the next guess, $f'(0)$ is perturbed by an absolute amount 0.01, and on the third guess, $G'(0)$ and $f'(0)$ are both perturbed by 0.01 from their initial values. We can solve for the terms in the matrix

$$
\begin{bmatrix}
\frac{\partial G(\eta_e)}{\partial f''(0)} & \frac{\partial G(\eta_e)}{\partial G'(0)} \\
\frac{\partial f(\eta_e)}{\partial f''(0)} & \frac{\partial f(\eta_e)}{\partial G'(0)} \\
\frac{\partial G(\eta_e)}{\partial f'(0)} & \frac{\partial G(\eta_e)}{\partial G'(0)} \\
\frac{\partial f(\eta_e)}{\partial f'(0)} & \frac{\partial f(\eta_e)}{\partial G'(0)} \\
\end{bmatrix}
\approx
\begin{bmatrix}
\Delta G(\eta_e) \\
\Delta f''(0) \\
\Delta G'(0) \\
\Delta f'(0) \\
\end{bmatrix}
\begin{bmatrix}
\Delta f''(0) & \Delta G'(0) \\
\Delta f'(0) & \Delta G(\eta_e) \\
\end{bmatrix}
$$

and, on a linear basis, can solve for the $\Delta f''(0)$ and $\Delta G'(0)$ which will make the dependent variables assume their correct free stream magnitudes. In matrix form,

$$
\begin{bmatrix}
\frac{\partial G(\eta_e)}{\partial f'(0)} & \frac{\partial G(\eta_e)}{\partial G'(0)} \\
\frac{\partial f(\eta_e)}{\partial f'(0)} & \frac{\partial f(\eta_e)}{\partial G'(0)} \\
\frac{\partial G(\eta_e)}{\partial f''(0)} & \frac{\partial G(\eta_e)}{\partial G'(0)} \\
\frac{\partial f(\eta_e)}{\partial f''(0)} & \frac{\partial f(\eta_e)}{\partial G'(0)} \\
\end{bmatrix}
\begin{bmatrix}
\Delta f'(0) \\
\Delta G'(0) \\
\end{bmatrix}
= 
\begin{bmatrix}
1.0-G(\eta_e) \\
1.0-f'(\eta_e) \\
\end{bmatrix}
$$
Figure 23. Flow diagram for solution of similarity boundary layer equations.
The pair of initial values $G'(0)$ and $f''(0)$ from the last trio of initial values which provided the best free stream values for $G$ and are designated the base solutions onto which $f''(0)$ and $G'(0)$ will be added in order to form the next coefficient matrix. A flow chart of this procedure is shown in Figure 24.

After the correct boundary values have been chosen and the profiles obtained for a given $\beta$ and $U_e^{+}$, control is returned to the part of the program which will calculate a new $\beta$ and $U_e^{+}$ corresponding to these profiles (Figure 24). The new value of $U_e^{+}$ and static pressure must be determined by applying total conservation of momentum;

$$2 \left[ \int_0^1 \rho U^+ r^+ dr^+ - \left( \int_0^1 \rho U^+ r^+ dr^+ \right)_{x^+=0} \right] - P$$

$$+ 4 Pr \int_0^{x^+} \bar{w}^+ dx^+ = 0 \tag{4.26}$$

and conservation of mass (2.34). We denote $y_e^{+}$ as that value of displacement corresponding to $U_e^{+}$, or:

$$y_e^{+} = \frac{\sqrt{2 \xi}}{U_e^{+}} \int_0^{\eta_e} \frac{\eta d\eta}{\rho^+} = \frac{\sqrt{2 \xi}}{U_e^{+}} \frac{\nu}{\rho} \int_0^{\eta_e} \frac{\eta d\eta}{\rho}$$

$$= \frac{\nu \sqrt{2 \xi}}{\rho U_e^{+}} \int_0^{\eta_e} \left[ (1 - \theta_w^{a+1}) G + \theta_w^{a+1} \right]^{1/\alpha+1} d\eta \tag{4.27}$$
Figure 24. Flow diagram for coupling of boundary layer development to internal tube flow.

- Evaluate $G(\eta), f(\eta)$ for first step ($\beta = 0$)
- Evaluate $\beta, U_\infty^+,$ $P/P_0$ from integrated momentum and continuity equations
- Is this the last step?
- Yes: Stop
- No: Evaluate $\beta, U_\infty^+,$ $P/P_0$
- Solve differential energy and momentum equations for $f(\eta), G(\eta)$
- Do these values of $\beta, U_\infty^+,$ $P/P_0$ differ by less than specified amount from values at last iteration?
- Yes: Print (or punch) $\beta, U_\infty^+, P/P_0, G(\eta), f'(\eta), f(\eta), \eta, y^+ x^+$
- No: Repeat
In the radial integrations, the following values of the dependent variables were used:

\[ r^+ = 0 \quad \text{to} \quad r^+ = 1 - \gamma^+_e \quad U^+ = U^+_e \]

\[ r^+ = 1 - \gamma^+_e \quad \text{to} \quad r^+ = 1 \quad U^+ = U^+_e f' \quad \theta = \left[ (1 - \theta^+_w)^{\alpha^+} + \theta^+_w \right]^{1/\alpha^+} \]

The wall shear stress is given by

\[
\tau^+_w = \frac{\mu^+_w \nu \gamma^+_e U^+_e f''(0)}{\sqrt{2\xi}} \tag{4.28}
\]

The following equations are obtained from 2.34 and 4.26:

**Total continuity;**

\[
\left( \frac{p}{p_o} U^+_e \right)^2 + \left( \frac{p}{p_o} U^+_e \right) \left( 2\sqrt{2\xi} \left( f'(\gamma^+_e) - \int_0^{\gamma^+_e} \theta d\eta \right) \right) + 2\xi \left( \int_0^{\gamma^+_e} \theta d\eta \right)^2 - 2 \int_0^{\gamma^+_e} \int_0^\gamma \theta(\xi) d\Omega d\eta = 0 \tag{4.29}
\]

**Total momentum;**

\[
\left( \frac{p}{p_o} \right)^2 - \left( \frac{p}{p_o} \right)(1 + \chi^2) + \left( \frac{p}{p_o} U^+_e \right) \gamma^2 \left[ 2\sqrt{2\xi} \left( \int_0^{\gamma^+_e} f''(\gamma^+_e) - \int_0^{\gamma^+_e} \theta(\xi) d\eta \right) \right] + 4\xi \frac{\mu^+_w p_o}{\mu^+_w} \int_0^{\xi^+_e} \frac{U^+_e}{\sqrt{2\xi}} f''(0) d\xi = 0 \tag{4.30}
\]
Once the correct wall parameters were determined at a given axial step, an additional integration of the equations 4.10 and 4.11 was performed with evaluation of the dependent variables at equal $\eta$ intervals. This was so that all radial integrations could be carried out by Simpson's rule. The analytical solution was applied in a stepwise manner at equally spaced intervals $\Delta \xi$. Once a solution was complete at an axial point, the independent axial variable was incremented by $\Delta \xi$ and the values of the profile dependent terms in equations 4.29 and 4.30 were approximated on this first solution at $\xi + \Delta \xi$ by their values from the previous axial point. This allowed for an initial estimate of $\beta$ and $U_e^+$ and control would be returned to the integration procedure. On proceeding iterations, new values for these terms would be used. The last integral in the equation was linearized by applying a modified midpoint rule.

$$
\int_{0}^{\xi} \frac{U_e^+}{\sqrt{2\xi}} f''(\xi) d\xi = \int_{0}^{\xi - \Delta \xi} \frac{U_e^+}{\sqrt{2\xi}} f''(\xi) d\xi \\
+ \left( f'(\xi) \left. + \right|_{\xi - \Delta \xi} \frac{f'(\xi)}{2} \right) \left( \frac{U_e^+}{\sqrt{2\xi - \Delta \xi}} \right) \left( \sqrt{2\xi} - \sqrt{2(\xi - \Delta \xi)} \right)
$$

(4.31)
We note that the continuity equation is a quadratic in the product of unknowns \( \frac{p}{p_0} U_e^+ \) for which solution may be made directly. The momentum equation is a quadratic both in \( \frac{p}{p_0} \) and \( \frac{p}{p_0} U_e^+ \), and solution for the former unknown may be made directly after determination of \( \frac{p}{p_0} U_e^+ \). The unknown \( U_e^+ \) can then be determined.

The parameter \( \beta \) (4.13) was evaluated at each axial point by use of local \( \xi \) and \( U_e^+ \) and using

\[
\frac{dU_e^+}{d\xi} = \frac{U_e^+|_{\xi+\Delta\xi} - U_e^+|_{\xi-\Delta\xi}}{2\Delta\xi}
\]  

(4.32)

for the derivative term. Solution could have been made for this quantity directly at \( \xi \) by taking the \( \xi \)-derivatives of equations 4.29 and 4.30 and solving the non-linear simultaneous equations in \( dU_e^+/d\xi \) and \( d(p/p_0)/d\xi \) which result, but it is questionable whether this approach would be worth the effort. In the entrance region, the higher order derivatives will be rapidly decreasing in magnitude with axial distance. Consider the evaluation of the first axial derivative by use of this difference quotient (2.37). The coefficients of the higher order derivatives in the error term are monotonically increasing with \( \sigma \) for \( \sigma > 1/2 \). At the same time, the decrease in the magnitude of the derivatives with
increasing $\sigma$ (increasing $x^+$) will partially offset this so that the minimum error will occur at a point in-between $(m + \sigma)\Delta x^+$ and $(m + 1)\Delta x^+$. After several axial steps, the relative difference in $(m + \sigma)\Delta x^+$ and $(m + 1)\Delta x^+$ will be negligible. Also, 4.32 is consistent with the way axial derivatives were evaluated in the finite difference solution.

Convergence on the two iteration levels was considered complete at an axial point when successive values of the parameter $\beta$ differed by less than 0.1% and successive values of $U_e^+$ and $p/p_0$ differed by less than 0.00005 in absolute magnitude. The program was coded in Fortran IV and run on the RCA Spectra 70 computer at the college. Integration of equations 4.10 and 4.11 took about 8 seconds and, on the average, 4 such solutions were needed for convergence of $G'(0)$ and $f''(0)$. Approximately 3 or 4 of these converged solutions were needed to complete iteration for $\beta$, $U_e^+$ and $p/p_0$ so that a total of about 1.5 minutes was needed for each axial step. The axial increment used for all patching solutions was $\Delta \xi = 5 \times 10^{-5}$. No change in the free stream value of $U_e^+$ was found for the axial step. Also, changing the value of $\eta_e$ from 7 to 14 resulted in a change in absolute value of $U_e^+$ of less than $10^{-5}$ at the same axial displacement.

Comparison of the boundary layer profiles generated
from this solution was made, when possible, with published boundary layer data. For the constant property case with zero pressure gradient, agreement was found to be perfect within the 5 decimal place accuracy for the enthalpy and velocity profiles of the Blasius solution presented in Schlichting(75). Also, no difference was found between present results for Pr = 1, T_w/T_o = 0.20 and the results of Reshotko and Cohen (72) for \( \beta = 0 \).

The joining of this analytical solution to the finite difference solution was made in a two step patch at \( \xi = 0.00025 \) and \( \xi = 0.00030 \). These particular points were chosen because previously the best downstream behavior was obtained when the Blasius solution was patched to the finite difference solution in this region. At \( \xi = 0.00025 \), complete radial and axial velocity and enthalpy profiles along with \( p/p_o \) generated by the similarity solution were inserted as initial values into the Worsoe-Schmidt program.

It can be shown that the function \( f \) is related to the Cartesian stream function \( \psi \) by

\[
\psi = 2\xi f \quad (4.33)
\]

and the radial velocity in terms of the stream function is,

\[
V_r = -\frac{1}{\rho \frac{\partial \psi}{\partial x^+}} = \rho \mu^+ \frac{U^+}{U_e^+} \sqrt{\frac{2\xi}{\eta}} \left[ \frac{f}{2\xi} - \frac{\eta}{2\xi} \frac{dU_e^+}{d\xi} - \frac{U_e^+}{2\xi} \right] \quad (4.34)
\]
This was the expression used to evaluate the transverse velocity in the boundary layer. Outside of the boundary layer, the radial velocity is obtained from integration of the continuity equation from the centerline out to a radius $r^+$:

$$v^+ = -\frac{1}{\rho^+} \int_0^{r^+} (\rho^+ U^+) dr^+$$

(4.35)

Since in the core flow, $\rho_e^+ U_e^+ = F(x^+)$ only, $\frac{\partial}{\partial x^+}(\rho_e^+ U_e^+)$ is a function of $x^+$ only in the region from $r^+=0$ to $1-y_e^+$ where $y_e^+$ denotes the edge of the velocity boundary layer:

$$v^+ = \frac{-r^+ \frac{\partial}{\partial x^+}(\rho_e U_e^+)}{2\rho_e \frac{\partial}{\partial x^+}(\rho U_e^+)} = -\frac{r^+ \rho_0 \frac{\partial}{\partial x^+}(\frac{p}{p_0} U_e^+)}{2 \rho \frac{\partial}{\partial x^+}(\frac{p}{p_0} U_e^+)}$$

(4.36)

The radial velocity is seen to be a linear function of the radius. The axial derivative was evaluated by the difference quotient 4.32. Once the profiles were patched, the finite difference program was allowed to generate all profiles for the next axial step. However, at this step the axial velocities and enthalpies were re-entered from the analytical entrance solution to begin the solution at the following step. Radial velocities from the finite difference solution were retained. This was done in order to help smooth the patch. The solution for all proceeding steps continued
in a normal manner. The effect of the patch on the velocity boundary layer development can be seen in Figure 22 where the velocity boundary layer as developed by the finite difference solution after a typical patch is shown compared with the solution from further independent development of the similarity solution. The difference is quite small and it would seem to indicate that the two solutions are at least compatible. A listing of the computer program used for the entrance region solution is given in Appendix F.

4.5. Results

The largest discrepancy between the present entrance solution and the finite difference solution can be seen in the variation of the static pressure with axial distance. In Figure 26, the non-dimensionalized pressure defect is shown for He at $\theta_w = 0.1$ from the similar boundary layer growth and for two finite difference solutions— one being the results from the rational extrapolation procedure noted earlier. The most obvious difference is in the difference in signs of the static pressure drop. The present analytical solution predicts a pressure rise in the entrance due to deceleration from the severe cooling. However, note that if the finite difference solutions are visually extrapolated to $x^+ = 0$, a non zero pressure defect is the result. This is not physically possible.
Figure 25. Comparison of velocity boundary layers before and after patch to finite difference solution.
Figure 26. Axial variation of dimensionless pressure defect for patched and complete finite difference solution. Air, UTV boundary condition, $M_0 = 0.03$.

1. finite difference - no modification. 80 radial divisions, $\Delta x^+ = 0.0005$
2. rational extrapolation to zero mesh size.
3. analytical boundary layer plus finite difference.
It should be noted that refinement of the mesh was found to reduce this pressure defect and displace it closer to the results from the similarity solution. The finite difference program normally begins the iterations at the second step by using a pressure defect calculated on the basis of a constant property boundary layer growth. This implies a static pressure drop for all cases. Changing the magnitude and sign of this initial guess was found to make no difference in the final pressure defect obtained after several iterations at the first step. It should be noted that the shape of the curves from both solutions are the same. Very little difference was found in the wall parameters $\text{Nu}_m$ and $\text{fRe}_m$ near the entrance for all solutions. The closest agreement was obtained from the rational extrapolation procedure. This would also lend confidence to the present results. Perhaps it is not without merit to reiterate that all we are actually doing is providing a better solution to the set of equations 2.30 - 2.32. The question is still open as to the applicability of these equations in the entrance region.

Downstream convergence was improved considerably by this method, although absolute convergence (i.e. stability of the wall parameters to infinite $x^+$) was not obtained. In all cases, the asymptotic Nusselt number was attained first and stayed at this value until the friction factor
differed by only a few percent from its fully developed value. Downstream results from this method are compared with those from the finite difference method in Figure 27 for He at $T_W/T_0 = 0.50$. Roughly the same mesh sizes were used in the patching solution and in the completely finite difference solution. The same magnitude of deviation indicated by the horizontal line is developed in the solution with the analytical boundary layer growth at an axial point which is more than twice as far downstream than the completely numerical method. Comparison of the axial variation of $\theta_w - \theta_m$ and $q_w^+$ for the two showed that the error in $\text{Nu}_m$ is about equally divided between these two quantities and is not due solely to the error in either quantity. For example, if a large error in the local heat flux is responsible, then the problem could be identified as a local one. Since we can reasonably expect that $\text{Nu}_m$ will remain constant once it has reached the fully developed value, continuation of the solution might be made by specifying as a boundary condition that $\text{Nu}_m = 3.67$ and evaluating a heat flux on this basis. An error in the wall to bulk temperature ratio represents an accumulation of small errors whose presence and origin are hard to detect and correct.

Wall parameters for the flow of helium and air are shown plotted versus $x^+$ in Figures 28 and 29 respectively.
Figure 27. Comparison of downstream Nusselt number behavior from patched and complete finite difference solutions. Air, UTV boundary condition, $T_W/T_o = 0.5$. 
Figure 28. Axial variation of $f_{Re,m}$ and $Nu_m$ for Helium at several wall to inlet temperature ratios. UTV boundary condition, $M_o = 0.03$
Figure 29. Axial variation of $f\Re_m$ and $\text{Nu}_m$ for air at several wall to inlet temperature ratios. UTV boundary condition, $M_o = 0.01$
The Nusselt number is almost completely insensitive to the temperature ratio. This is not completely surprising since the flow in the entrance of the tube is of a boundary layer character and results from variable property external boundary layer solutions for gases (72,23) have shown little change with severe cooling. Here, for helium $Nu_m$ actually exhibits a slight increase with increased cooling. The variation of the product $fRe_m$ with $\theta_w$ is large in comparison with that of $Nu_m$, but still rather small in an absolute sense. For example, a tenfold decrease in the inlet wall to bulk temperature ratio results in less than a 50% decrease in $fRe_m$. The friction factor variation is nearly identical for the two gases. The extremely slow convergence of this parameter for the case of $\theta_w = 0.1$ should be noted. For example, for both gases, $fRe_m$ still differs from its fully developed value by 25% at $x^+ = 0.85$. To translate this to physical terms, consider the flow of He at $Re_0 = 1000$. Then $x^+ = 0.85$ corresponds to an axial displacement of more than 300 diameters. For the Graetz condition and this same temperature ratio, this magnitude of deviation from the isothermal $fRe_m$ corresponds to approximately half this displacement. Data for the isothermal case for $Nu_m$ and $Pr_0 = 0.70$ is also plotted from references 43, 57 and 93 in Figure 30. Present results fall midway between the results of Manohar and Ulrichson and Schmitz.
Figure 30. Comparison of Isothermal Nusselt number development from other investigations.
In Figure 31, the centerline axial velocity development from this solution is compared with that from Hornbeck (35). Since, for reasons given previously, it was not possible to run the fully isothermal case here, the case of $\theta_w = 0.95$ was used for comparison. Agreement here is excellent.

Representative axial velocity and temperature profiles are shown in Figure 32 for $\theta_w = 0.10$ and 0.50 and radial velocity profiles in Figure 33 for $\theta_w = 0.10$ and 0.95 and the flow of helium. With the exception of helium, where for the case shown of $\theta_w = 0.10$ an outward radial velocity existed for a short distance from the entrance, the displacement of gas in the velocity boundary layer is responsible for an inward radial velocity. For gas cooling, an outward radial velocity would bring gas at a higher temperature from the core towards the wall. The result is a flatter temperature profile and an increased magnitude of temperature gradient and heat transfer at the wall. In an incompressible UTV case, the inward radial velocity profile reverses this effect, and in a sense, effectively 'insulates' the wall. When compressibility and cooling are introduced, the magnitude of this inward velocity is reduced. Qualitatively, the heat transfer is augmented for the reasons previously stated and the net effect is to partially offset the decrease in the thermal conductivity ratio at the wall,
Figure 31. Development of centerline axial velocity from patching solution compared with results of Hornbeck. UTV boundary condition
Figure 32. Axial development of reduced temperature and axial velocity profiles for air at two inlet temperature ratios. UTV boundary condition
Figure 33.
Dimensionless radial velocity profiles for developing flow of helium at two inlet temperature ratios. UTV boundary condition, $M_0 = 0.03$
$T_w / T_o = 0.95$

$T_w / T_o = 0.10$

HELUM $M_o = 0.03$ UTV B.C.
\( \frac{k_w}{k_m} \) (3.21). This is probably the primary reason for the insensitivity of \( \mathrm{Nu}_m \) to temperature ratio.

Near the center of the tube, the axial velocity profiles show the most variation with wall to bulk temperature ratio (Figure 31). As is the case with the temperature profile, cooling flattens the profile. It is difficult to argue through the reasons why this behavior is present since there are many possibly cancelling effects. For example, as cooling is increased the static pressure drop along the tube is increased. The density near the wall is increased which in the absence of a radial velocity tends to decrease the axial velocity and its gradient at the wall. The magnitude of the outward radial velocity component and the viscosity ratio at the wall \( \frac{\mu_w}{\mu_m} \) are also all decreased.

For the examples shown, at \( x^+ = 0.490 \) the velocity profiles have essentially reached the fully developed state, while the product \( f\mathrm{Re}_m \) still differs from its fully developed value by more than 50%—this difference must be attributable to the factor \( \frac{\mu_w}{\mu_m} \) again. If the velocity profile development is plotted as a function of \( x_m^+ \) instead of \( x^+ \) (Figure 34), the fully developed state is reached more quickly. Also the distortion of the profiles with \( x_m^+ \) is reduced when presented on this basis. Using \( x_m^+ \) for the representation of \( f\mathrm{Re}_m \) and \( \mathrm{Nu}_m \) is questionable. Even though convergence of these quantities
Figure 34. Reduced axial velocity development with $X_m^+$. 

$X_m^+ = 0.196$

$X_m^+ = 0.051$

$X_m^+ = 0.012$

AIR • UTV B.C.

$T_w/T_0 = 0.50$

$T_w/T_0 = 0.10$
is also quicker. The displacement of the curves with respect to each other at points intermediate between the entrance and fully developed regime would be increased. Also, the excellent correlation of $Nu_m$ with $x^+$ for all $T_w/T_o$ should not be sacrificed.

For $Nu_m$, a single correlation for all inlet wall to bulk temperature ratios is recommended (maximum error 3%),

$$Nu_m = 3.67 + 0.246x^+ - 0.592e^{-20.6x^+}$$ \quad (4.37)

$$0.001 \leq x^+ \leq 0.50$$

$$Nu_m = 3.67$$ \quad (4.38)

$$x^+ > 0.50$$

for both helium and air. For the local friction factor, the following correlation is proposed;

$$1 - \left(\frac{f_{Re,m}}{f_{Re}}\right)_I = \left[1.067(1-\theta_w)x^+ - 0.576\right]e^{-\beta x^+}$$ \quad (4.39)

for $x^+ > 0.002$

where

$$\beta = 7.70\theta_w^{0.675}$$ \quad (4.40)

The coefficients were determined from a least squares multiple regression analysis. The quantity $f_{Re}$ represents the isothermal quantity whose variation with axial distance is well represented by,

$$f_{Re} = 16.0 + 0.694x^+ - 0.576e^{-22.9x^+}$$ \quad (4.41)
for both gases in the range \( x^+ > 0.001 \). An attempt was made to isolate the effect temperature ratio has on the friction factor by plotting the ratio \((fRe_m)/(fRe)_I\) at the same axial \( (x^+) \) points as a function of local wall to mean temperature ratio. This is shown in Figure 35 for air and shows that for developing flow, correlation is not possible on this basis.

4.6. Dissipation Function

The form of the dissipation function used in the Worsoe-Schmidt analysis was,

\[
\Phi^+ = \mu^+ \left( \frac{\partial u^+}{\partial r^+} \right)^2
\]

(4.42)

However, in the assumed core flow for the UTV boundary condition \( \partial u^+/\partial r^+ \) was assumed to be zero for the similarity inlet solution and in the finite difference solution, this term was found to be extremely small in the core. Since for an acceleration or deceleration of the mass flow in the core, the continuity equation predicts a non-zero radial velocity in the core even when \( \partial u^+/\partial r^+ = 0 \), a re-examination of the complete dissipation function showed that for the UTV boundary condition the form

\[
\Phi^+ = \mu^+ \left[ \frac{(\partial u^+)^2}{(\partial r^+)} + \frac{2}{(Re_o Pr_0)^2} \left( \frac{v^+}{r^+} \right)^2 + \frac{2}{3} \left( \frac{\partial v^+}{\partial r^+} \right)^2 \right]
\]

(4.43)
should be used. For the isothermal UTV condition, the
term $\frac{\partial v^+}{\partial r^+}$ can be shown to be larger by a factor $r_o/\delta$
than the term $\frac{\partial u^+/\partial x^+}$ at the entrance. Examination
of these terms from the numerical solution showed that
the axial derivative could still be neglected when
property variations were present. At the tube centerline
($r^+ = 0$) application of L'Hospital's rule and symmetry
yields:

$$\frac{4\mu^+}{3(Re_0 Pr_0)^2} \left( \frac{\partial v^+}{\partial r^+} \right)^2 = \Phi^+$$  (4.44)

This dissipation function is operating over a fairly
long axial distance in the core and the integrated
effect on the temperature profile and the wall parameters
may be non-negligible. An interesting point in the
inclusion of the factor $Re_0 Pr_0$ which requires the speci-
fication of the Reynolds number when the additional terms
are included. Generally, Reynolds number dependence is
a characteristic of non-boundary layer flow. For example,
inclusion of axial second derivatives also requires speci-
fication of $Re_o$. The initial value nature of the problem
is not changed by the inclusion of these terms. The
variation of $Nu_{m}$ for helium at $\theta_w = 0.90$ is shown in
Figure 36 for several inlet Reynolds numbers. For $Re_o >$
100, the change in $Nu_{m}$ is negligible. While Reynolds
Figure 36. Effect of additional terms in dissipation function $\Phi$ on heat transfer.
numbers lower than this are not of any practical im-
portance, solutions are presented for Reynolds numbers
less than this for the sake of completeness. For Reynolds
numbers of this magnitude, the second order axial terms
would probably be of such magnitude as to make these
results of academic interest only. The effect will be
reduced for lower wall to bulk temperature ratios due to
1.) the decrease in the magnitude of the radial velocity
component and 2.) the increasing magnitude of boundary
layer terms relative to these terms, so that it was not
necessary to test further cases. It is interesting to
note that the effect of the new dissipation function is
felt immediately in the entrance. This indicates that
the increased magnitude of Nu,m is probably due to the
dissipation in the boundary layer at or near the wall
rather than in the core. Local viscous energy generation
at the wall would raise the gas temperature near the wall.
Perhaps it would be more applicable to define a convective
heat transfer coefficient using a wall to local film
temperature difference. Such a film temperature could
be defined, for example, by using the bulk temperature
in the thermal boundary layer rather than across the
whole tube. The effective "film" to wall temperature
difference is increased by a greater factor than the
ordinary wall to bulk temperature difference.
CHAPTER 5. EXPERIMENTAL INVESTIGATION

5.1. Introduction

In this chapter the procedure and apparatus used to obtain experimental data for gas cooling with the sets of boundary conditions examined in the theoretical portion of the investigation is described.

5.2. Experimental Apparatus

The apparatus was designed to measure the local heat transfer and static pressure at several axial points along a constant temperature cylindrical tube for cooling of a gas with severe transverse temperature gradients. The flow diagram is shown in Figure 37. Air supplied from a reciprocating air compressor flows into supply plenum, through a filter, scrubber and regulator and into a settling tank. The flow than passes through a resistively heated inconel tube into a mixing plenum where its temperature and pressure are measured before passing into a development section mounted directly before the test section. The gas temperature is measured in a mixing plenum mounted directly after the cooling section. It then passes through a constant temperature bath after which its temperature is measured. Finally the flow is metered by a laminar flow meter and vented to atmosphere.
Figure 37. Schematic diagram of experimental apparatus.
A. Air Supply

A Worthington two stage air compressor was connected into the supply line. The gas was initially dried in a water jacketed condenser after the high pressure cylinder in the compressor. The compressor ran continuously during each test. Primary regulation of the supply plenum pressure was accomplished by varying the bleed flow from the supply plenum. This method provided an extremely steady flow. The pressure was maintained in the plenum at approximately 90 psig.

A King Model 2260-1 filter fitted with a King Model 9326 polisher cartridge was mounted in line directly before a Denver-Harris model 1503-C two stage pressure regulator.

B. Preheater

The preheater consisted of a 1/8"D x 0.020" wall x 5' inconel tube mounted in a steel cylinder loosely packed with MgO powder and externally insulated with magnesite sheath (Figure 38). Power is supplied from a Transtat catalog no. 29145 single phase voltage regulator through specially fabricated taps mounted at opposite ends of the inconel tube. The preheater was electrically insulated from the test section by a special flange fabricated from 316 S.S. and a Cermacast pottable ceramic. The power input to the tube was measured with a Weston voltmeter-
ammeter combination. Maximum exit gas temperatures obtained were on the order of 1800 F.

C. Development Section

Two flow development sections were used. The first provided a fully developed velocity and uniform temperature profile to the test section and is shown in a photograph in Figure 39 and schematic in Figure 40. The entering gas temperature was measured by a chromel-alumel thermocouple mounted downstream of a pair of mixing baffles. The thermocouple was fitted with a cylindrical stainless steel radiation shield so that it effectively "saw" only the center portion of these baffles and the development tube centered in the downstream region. The flow divided into a portion which flowed through an isolated central tube leading into the cooling section and a portion which flowed in an annulus surrounding this tube. This flow was vented to the atmosphere through a needle valve. The annular flow served as insulation to assure that the flow development was adiabatic. The length to diameter ratio of the section was well over 100. For the second development section, the annular section was removed from the plenum. A bellmouth entrance was used to provide nearly uniform velocity and temperature profiles to the test section. The inlet bulk temperature of the gas was measured by bleeding air from the supply
Figure 38. Photograph of preheater

Figure 39. Photograph of adiabatic development section
Figure 40. Schematic of inlet development section apparatus
plenum around a long stem 1/16 inch diameter chromel alumel thermocouple (Figure 40). Pressure in the plenum was measured by a Meriam 40 inch air-over-mercury manometer. Magnesite sheath of approximately 1-1/2 inch thickness was bonded to both development sections with a refractory cement. Several inches of exterior fiberglass insulation was added. All tube and thermocouple fittings used were Gyrolok 316 stainless.

D. Exit Mixing Section

A schematic of this apparatus and a photograph showing the section mounted in its insulating case are shown in Figure 41. After passing through the test section, the gas flows through a short length of tube in which several mixing baffles are mounted and over a long stem, small diameter Cu-Con thermocouple probe. This air then flows back in an annulus around the tube to serve as an insulator and then passes over the rear of the thermocouple stem so the conduction losses are reduced. This mixing portion was mounted in a box filled with several inches of MgO powder and vermiculite insulation. The entire apparatus was then covered with fiberglass insulation.

E. Metering

Prior to metering, the gas passes through several turns of 1/4 inch D copper tubing immersed in a room
Figure 41. Schematic and photograph of exit mixing plenum
temperature water bath. This insures that the temperature correction term for the flowmeter will be small. The flow rate is measured with a Meriam model 50 MW 20-1 factory calibrated laminar flow element which provides a linear differential pressure output with air flow rates up to 8 S.C.F.M. The output pressure is measured by a Teltrue type A micromanometer with a resolution as low as 0.001 inch H$_2$O. Gas temperature at the flowmeter was measured by a long stem Cu-Con thermocouple mounted so that the junction was inside the flowmeter and the gas flow was along the stem.

F. Flow Control

The flow through the test section is controlled by three needle valves. One is mounted in line between the test section and flowmeter, one is upstream of the preheater and one is mounted on the bleed line from the development section. The bleed flow rate was maintained at a much larger value than the flow through the test section. This was done so that changes in this latter flow rate through the test section made by adjusting the downstream valve would make only small relative changes in the total flow rate through the preheater. Since the power input to the preheater was not normally changed during a test series for a fixed inlet temperature, this procedure assured a fairly steady output temperature.
G. Test Section

The test section is a 1" O.D. x 0.294" I.D. 304 stainless steel tube. Pressure taps made from 1/16 D x 0.006 inch wall stainless tubes are brazed into the tube at 7 axial points. Actual entry into the inner tube is made by 0.040 inch diameter holes in the tube wall. Six heat flux calorimeters are clamped to the tube at 6 axial points where grooves are turned into the section. Axial locations of the pressure taps and calorimeters are given in Table 5.1.

Table 5.1. Test Section Dimensions

<table>
<thead>
<tr>
<th>A. Axial location of pressure taps (in.)</th>
<th>1.250</th>
<th>6.550</th>
<th>16.150</th>
<th>25.749</th>
<th>34.349</th>
<th>44.949</th>
<th>50.248</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.981 in. additional with bellmouth)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| B. Axial location of calorimeters (to center of each) | 1.730 | 11.330 | 20.930 | 30.529 | 40.129 | 49.728 |

The calorimeters consist of 304 S.S. semi-circle sections 0.380 inch I.D. x 1.000 inch O.D. x 0.500 inch width fabricated from the same tube stock as the test section. Thermocouple holes 0.030 inch D x 0.250 inch deep are drilled into each section at radii of 0.250 inch and 0.437 inch. Teflon insulated 36 gage Cu-Con thermocouples
made from thermocouple wire supplied by Thermo Electric Company are mounted in the holes which are packed with a high conductivity GE silicone grease. The thermocouples are mounted in matched pairs formed by cutting the wire and welding leads directly on either side of the cut. This insures that each pair of thermocouples will have leads of essentially the same composition since thermocouple wire may vary even from the same spool. The thermocouples are soldered with a 60-30 resin solder.

A 1/16 inch thick balsa facing is bonded with epoxy to the face of each calorimeter (see photograph, Figure 43). The thermocouple leads are epoxied into grooves cut into this facing such that thermocouple conduction error is reduced and the leads are protected from abrasion. The calorimeters are pressed against the test section by means of simple clamps fabricated from 1/8 inch t 304 S.S. sheet stock. Contact resistance between the calorimeters and the test section was reduced by liberal application of silicone grease to all contact surfaces prior to mounting.

The test section is immersed in a bath of H₂O which is maintained at a pool boiling condition by approximately 12 immersion heaters. The test section plus constant

---

6 Initial attempts at using liquid N₂ as the boiling medium were unsuccessful due to the difficulty of maintaining a good thermal bond of the calorimeters to the test section at extremely low temperature.
pressure tap: 1/16" o.d.
s.s. tube brazed into
test section wall

0.040" hole thru

balsa facing

0.500

0.540

cross section
through calorimeter

thermocouple hole 0.030" D

clamp

high conductivity silicone
grease filler

Figure 42. Test Section Pressure Tap and Calorimeter - Detail

Balsa Facing

Radial Grooves

Figure 43. Heat flux calorimeter pair - photograph
temperature bath is mounted in a large rectangular vermiculite filled box. Maximum values of the ratio $Gr/Re^2$ obtained in the test section were on the order of $2.5 \times 10^{-3}$ so that free convection effects are expected to be negligible (50,63). $Gr$ is the Grashof number calculated on the basis of maximum wall to gas temperature difference, tube diameter and using gas properties evaluated at temperatures midway between wall and maximum gas temperature.

Provision was made for measurement of the static pressure drop between any pair of pressure taps by means of a pair of pressure switching banks. Pressure drops were measured by a Teltrue type A micromanometer.

The thermocouple outputs are measured on a recently calibrated Leeds and Northrup type K3 potentiometer and type 9834 null detector. External reference junctions for the thermocouples were placed in an ice bath. Absolute rather than differential EMFs from the calorimeter thermocouples are measured because most of the measuring junctions were grounded to the test section.

Where possible, electrostatic shielding is applied to thermocouple leads. External thermocouple leads are glass on teflon insulated. Leakage currents are minimized through extension of the internal guard circuit of the potentiometer to its power supply and standard
cell. These are mounted on a capacitor formed from sheets of polymethyl methacrylate and aluminum. This was necessary due to the high humidity in the laboratory from the boiling off of the H₂O in the test section bath. Ground loops are eliminated by use of a 0.01 microfarad mica capacitor inserted between the potentiometer and earth ground.

5.3. Calibration

A series of calibrations performed on portions of the apparatus are described in this section.

A. Calibration of the Heat Flux Calorimeters

An analysis of the possible error in using a one dimensional heat conduction equation to evaluate calorimeter conductances is presented in Appendix C. This result necessitates a calibration of the calorimeters. The calibration was performed on the calorimeters after mounting on the test section by applying a known heat flux to the inside wall of the test section and measuring the corresponding ΔT across the calorimeters. The inside wall of the test section was coated with several layers of a flat black refractory enamel in order that the absorptivity of the wall would be uniform. A 1/8 in. D thin walled stainless steel tube whose surface was uniformly roughened on a lathe with a #500 grit emery cloth was mounted along the centerline of the test
section. Figure 44 is a photograph of the calibration setup. Thin ceramic spacers were mounted at points midway between successive calorimeters in order to insure centering of the wire. An analysis of the error introduced by these spacers is presented in Appendix D and is shown to be negligible. In order to eliminate sag at high temperatures, a tension was applied to the wire by a spring mounted in a vacuum chamber at the end of the test section. Power leads and voltage taps were introduced into the chamber containing the tensioning spring by means of Conax sealing glands. Power was provided by a Variac model W20MT3 autotransformer and was measured with a Weston 0-25 volt range voltmeter and 0-30 amp range precision ammeter. The system was evacuated by a mechanical vacuum pump and pressure was measured with a Scientific Glass no. 1-759 tilting type McLeod gage. Pressure in the test section was maintained at a maximum of approximately 0.02 mm Hg in order to minimize conduction and convection heat transfer.

The test was performed with boiling H2O in the test section bath. Tare thermocouple readings with zero power to the heating tube were subtracted from readings in the power-on test to correct for possible spurious heat losses. The conductances as determined from this procedure are shown plotted as a function of axial position in Figure 45. Error limits with respect to the
Figure 44. Photograph of test section and apparatus for radiation calibration
Figure 45. Axial variation of calorimeter conductance.
mean value of all the conductances (Appendix C) are shown plotted as horizontal lines. All conductances are seen to lie within these limits.

B. Calibration of Thermocouples

The copper-constantan reference junction was obtained from Conax Corporation with a factory calibration in accordance with ASTM procedure E220-64 against a National Bureau of Standards calibrated Platinum versus Platinum-Rhodium thermocouple. The deviation at 32 °F was 0.00°F. This thermocouple was used for calibration of the flowmeter and exit bulk temperature measuring thermocouples. The deviations were too small (<0.25°F) to make a noticeable difference in the results. No calibration was necessary for the test section thermocouples.

C. Adiabatic Development Section

Velocity profiles were measured at the exit of the adiabatic development section which was used for generation of fully developed velocity profiles. This measurement was taken for two reasons. 1.) Since the velocity profile development in the section will depend on L/(DRe_o) where L/D is the length to diameter ratio of the development section, it was necessary to find the maximum Reynolds number for which the flow could be treated as fully developed and 2.) it could be used for a check on the flowmeter. During the test all bleed valves on the
development section were closed and the flowmeter was mounted upstream of the section.

A total pressure probe was fabricated from 0.020 inch O.D. x 0.010 inch I.D. 316 stainless steel hypodermic needle stock. The probe was mounted on a microscope vernier control stage and output was measured on a type A micromanometer (see photograph, Figure 46). The control stage was set for a traverse across a diameter of the tube by means of a cylindrical brass plug which fit into the end of the test section. A 0.022 inch wide rectangular groove was cut into the plug. The probe was moved into this groove and the microscope stage adjusted until a traverse could be made by moving one of the verniers without touching the sides of the channel. An electrical circuit was set up with the probe connected to one side of a battery and the test section and plug connected to the other polarity. An ammeter was placed in the circuit so that when contact of the probe and plug occurred, the ammeter would give a non-zero reading. Static pressure at the exit was assumed equal to atmospheric. Corrections for measurements near the tube wall as describe in reference 95 were applied. The data showed a flattening of the profile at the centerline occurring at approximately $Re_o = 1800$. A third order polynomial identically satisfying the zero slip condition at the wall was fitted to the data.
Figure 46. Velocity profile measuring apparatus
Microscope stage and plug
at $Re_0 = 1585$ by a least squares criterion (Figure 47) and inserted as an initial velocity profile along with a uniform temperature profile in the finite difference solution. The effect on $Nu_m$ and $fRe_m$ was negligible. No difference was seen in $fRe_m$ at $x^+ = 0.001$ while $Nu_m$ was about 1% higher than that in the idealized case. Also, an integration of this profile yielded a mass flow rate within 3% of that indicated by the Meriam flowmeter. This agreement is excellent considering that velocity measurements near the tube wall have the least accuracy and a greater weight in a flow rate calculation than points near the center of the tube.

A second calibration was performed after the plenum and development section were run for several hours at 1800 °F. This was done so that any incipient change in the calibration of the chromel alumel thermocouple in the mixing section of the development section would be triggered. The bulk temperature measuring chamber described in section 5.2. was fitted with a chromel-alumel thermocouple and fitted onto the end of the development section. The flowmeter was mounted downstream of the chamber. For a Reynolds number of 1500 with the bleed open, the exit temperature was correlated with the output from the upstream thermocouple. The final exit temperature correlation with upstream thermocouple output is shown in Figure 48 which includes a correlation after a two point
Figure 47. Exit velocity profile from adiabatic development section.
Figure 48. Calibration of adiabatic development section.

Figure 49. Velocity profile from bellmouth used in UTV development section.
calibration at 32 °F and 212 °F of the chromel-alumel thermocouple used in the bulk temperature measuring device. This curve was used in the data reduction program for correction of the upstream thermocouple output.

D. Mixing Section and Bellmouth

The velocity profiles from several bellmouths with slightly varying geometry were measured with the total pressure probe apparatus described in the previous section. The bellmouth giving the most uniform velocity profile (Figure 49) was mounted in the mixing supply plenum for the portion of the experiment dealing with the UTV boundary condition. Output of the chromel-alumel thermocouple mounted in the bleed flow was monitored as a function of its depth of immersion in the bleed flow to determine the depth at which conduction error becomes negligible.

5.4. Leak Tests

Prior to the beginning of the two test series, the test section was pressurized to 35 psig and all connections were covered with a soap solution. This pressure was well in excess of the maximum pressure (20 psig,) used during the actual testing. Tygon plastic tubing was used throughout and G.E. RTV silicone sealant was used at all plastic-metal tube connections.
5.5 Adiabatic Pressure Drop and Friction Factor

With the adiabatic development section in place, two sets of pressure drop data were taken at $\theta_w = 1.0$ for the entering gas and test section both at room temperature and $212^\circ\text{F}$. The non-dimensionalized pressure defect is plotted as a function of $x^+ - x_0^+$ along with the theoretical constant property pressure defect which, due to the low experimental Mach numbers, should correspond to the experimental data. $x_0^+$ is the position of the first pressure tap measured from the point where cooling was assumed to commence (Figure 50). Pressure drops are taken with respect to the first tap. The second tap is reading low and yields an average friction factor between the first two taps which is almost 10% low. No burrs were evident at the second tap. Other friction factors agreed to within 5% of fully developed.

With the UTV development section in place, low Reynolds number friction factors calculated on the basis of a least squares fit to the pressure drop from pressure taps 5 and 6 are shown in Figure 51. Since these taps are so far downstream (~100 diameters), the fully developed friction factor should be present. This is borne out by the excellent agreement in Figure 51. In addition to serving as a check on individual taps, the adiabatic friction factor is important in another respect.
Figure 50. Isothermal dimensionless pressure drop along test section with adiabatic development section in place.
Figure 51. Friction factor in downstream region with UTV development section in place.
Since in a plot of $f$ versus $Re$, the tube diameter effectively enters the reduction calculation in the fifth power, errors in the measurement of this diameter will be evident here.

5.6. Repeatability Test

In order to determine what effect the arrangement of the immersion heaters in the constant temperature bath have, two additional tests were performed with a re-arranged immersion heater configuration for the UTV b.c. for $\theta_w = 0.50$. (tests #50 and 51). No apparent effect was found on either the heat transfer or the friction factor results.

5.7. Wall Temperature Uniformity

The axial wall temperature drop between the first and last calorimeter was, in the worst case, approximately 15°F. The local axial wall temperature gradients are estimated to be approximately two orders of magnitude less than the radial gradients.

5.8. Experimental Procedure

Prior to the beginning of each set of test runs at a given inlet wall to bulk temperature ratio, the bleed flow from the development section was opened, the valve downstream of the test section was closed and the supply pressure adjusted to 60 psig. The power level to the
preheater was adjusted in accordance with a bulk temperature power level curve that had been obtained from a previous test series. About 7 hours of operation in this state was necessary for elimination of all thermal transients in the development section. Zero points on the flowmeter and pressure drop micromanometers were set. The integrity of the lines from the pressure taps and the pressure switching banks were checked by seeing that the micromanometer zero was maintained for pressure measurements between several pairs of taps in the pressurized zero flow condition. Also at the no flow condition, power was supplied to the immersion heaters and the constant temperature bath was brought up to and held at the pool boiling condition for approximately a half an hour. This allowed a complete set of tare thermocouple readings to be taken. This data was used as a correction for the readings with gas heat transfer for that day.

The valve downstream of the test section was set for the maximum inlet Reynolds number to be run and about 45 minutes was allowed before data was taken. Boiloff from the pool was replaced by boiling water from a separate heater and tank in the laboratory. The following data was taken;

1. Static pressure drops. With the adiabatic
development section in place, the pressure drop between the first and 6 succeeding pressure taps were taken. With the bellmouth inlet, pressure drops were recorded between the inlet plenum and 2nd taps, the first and second tap, and then between the second and 5 succeeding taps. This series was made necessary by fluctuation in the plenum pressure that were not present in the downstream regions of the tube. These fluctuations were dampened by insertion of a laminarizing element in the plenum and the insertion of a large volume in the line from the inlet pressure tap so than an integrated pressure difference was measured; rather an instantaneous value.

2. Thermocouple outputs from both thermocouples in each of the twelve calorimeters, the upstream and exit mixing chamber and the flowmeter.

3. Pressure in the inlet plenum, atmospheric and supply pressure.

A typical data sheet is shown in Figure 52. At the end of a test series which usually included 5 or 6 mass flow rates, the downstream valve was closed and an additional set of zero gas flow thermocouple readings was taken. The purpose of this second set of tare readings was to detect any abnormal thermocouple output rather than to
Figure 52. Facsimile of original data sheet.
Variable Property Gasflow Test # 11

Date 4/1/71

V(in) volts 22.0
I(in) amps 2.0

Supply Pressure (lbf/in²) 65,0
Atmospheric Pressure (in, hg) 30.20

Tare Thermocouple Readings - Boiling, zero flow

<table>
<thead>
<tr>
<th>Cal.</th>
<th>Station</th>
<th>Output(mv)</th>
<th>Cal.</th>
<th>Station</th>
<th>Output(mv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td><strong>SEE NOTE #4-4/12/71</strong></td>
<td>7</td>
<td>13</td>
<td><strong>SEE NOTE #4-4/12/71</strong></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td>8</td>
<td>14</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td></td>
<td>9</td>
<td>15</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td></td>
<td>10</td>
<td>16</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td></td>
<td>11</td>
<td>17</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td></td>
<td>12</td>
<td>18</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td></td>
<td>13</td>
<td>19</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td></td>
<td>20</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td></td>
<td>21</td>
<td>21</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td></td>
<td>22</td>
<td>22</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td></td>
<td>23</td>
<td>23</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td></td>
<td>24</td>
<td>24</td>
<td></td>
</tr>
</tbody>
</table>

Flow Test

Differential Pressure - Flowmeter (in, H₂O) 0.406
Flowmeter Temperature Cu-Con (mv) 0.376
Bulk Temperature Cu-Con (mv) 4.871
P₁ (in, hg.) Left 3.60 Right 3.70
Upstream Temperature Cr-Al (mv) 13.412

Static Pressure Drop (in, H₂O) - BUCK MONOMETER FLUID

\[
\begin{align*}
P₁ - P₂ &= 0.015 \\
P₁ - P₃ &= 0.071 \\
P₁ - P₄ &= 0.035
\end{align*}
\]

Thermocouple Output

<table>
<thead>
<tr>
<th>Cal.</th>
<th>Station</th>
<th>Output(mv)</th>
<th>Cal.</th>
<th>Station</th>
<th>Output(mv)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>4.2930</td>
<td>7</td>
<td>13</td>
<td>7.3034</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>4.2957</td>
<td>8</td>
<td>14</td>
<td>7.2924</td>
</tr>
<tr>
<td>3</td>
<td>3</td>
<td>4.2941</td>
<td>9</td>
<td>15</td>
<td>4.3074</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4.2929</td>
<td>10</td>
<td>16</td>
<td>7.3273</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>4.2928</td>
<td>11</td>
<td>17</td>
<td>7.3423</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>4.2915</td>
<td>12</td>
<td>18</td>
<td>7.3513</td>
</tr>
<tr>
<td>7</td>
<td>7</td>
<td>4.2914</td>
<td></td>
<td>19</td>
<td>7.3646</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>4.2917</td>
<td></td>
<td>20</td>
<td>7.3721</td>
</tr>
<tr>
<td>9</td>
<td>9</td>
<td>4.2917</td>
<td></td>
<td>21</td>
<td>4.3056</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
<td>4.2917</td>
<td></td>
<td>22</td>
<td>4.3073</td>
</tr>
<tr>
<td>11</td>
<td>11</td>
<td>4.2904</td>
<td></td>
<td>23</td>
<td>4.3073</td>
</tr>
<tr>
<td>12</td>
<td>12</td>
<td>4.2916</td>
<td></td>
<td>24</td>
<td>4.3016</td>
</tr>
</tbody>
</table>

Taken by N. Sheehan 4/1/71
serve as a tare reading.

5.9. Data Reduction Program

A computer program written in Fortran IV was used in the reduction of all the data. It contained provisions so that data from both inlet sections could be treated. A listing of the program is included in Appendix G along with a list of significant I/O and intermediate variables. Many descriptive comment cards are distributed throughout the listing. The program initially prints out all input data for an echo check. Complete input and reduced data for all tests are included in Appendix H. Third degree polynomial least squares fits were used to represent various calibrations and property variations. Coefficients for these fits are initially read in as punched data. Gas properties were taken from reference 32. A two section fit to the Cu-Con thermocouple tables in reference 62 was used for better accuracy. Also, the x/D ratios at which the pressure taps and calorimeters are located are read in as initial data. Since the test section is maintained at a uniform temperature, no thermal expansion corrections to the non-dimensionalized displacements are necessary. Initially, the program converts and corrects the inlet and flowmeter gas temperatures, calculates a corrected mass flow rate, and uses the expression

$$Re_0 = \frac{\dot{m}}{\pi \mu_0}$$  \hspace{1cm} (5.1)
for the inlet Reynolds number where $\dot{M}$ is the mass flow rate (lbm/min). The inlet air density (assuming zero radial pressure variation) is calculated from the perfect gas law using the corrected inlet bulk temperature. For the tests using the adiabatic development section, the first pressure tap on the cooling section was used as the reference for the pressure drops. Friction factor data is measured from the point where cooling was assumed to commence. Since the pressure drop from the beginning of the adiabatic development section to the cooling section will be on the order of 0.5 in. of H$_2$O, the error in the density will be small from assuming the absolute pressure is equal to the inlet plenum pressure (about 20 psig). The inlet velocity,

$$U_0 = \frac{4\dot{M}}{\pi D^2 \rho_0}$$

(5.2)

and Prandtl number $Pr_0$ are calculated. An additional word should be mentioned concerning the precise points where the cooling and flow development were assumed to start. The initial point of temperature profile development was essentially the same for the two inlet sections. Modified 316 S.S. Gyrolok 3/8 inch to 1/4 inch tube fittings were used to connect the test and development sections. For the adiabatic development section, cooling was assumed to begin at the end of a small lip on the cooling section on which the ferrules
for the connecting tube fitting clamped. Little was known about the type of thermal contact present, so for the UTV development section, a high conductivity epoxy was used to seal all fitting components. For this case, cooling was assumed to begin midway between the downstream face of the mixing plenum and the outside face of the test section bath. For both development sections the connection was insulated with MgO powder. Actually, the tube wall temperature will decrease along the connection from nearly inlet bulk temperature to $212^\circ F$ near the face of the cooling section. The difference in starting the cooling midway or at the lip is quite small, amounting to only a few percent difference in the location of the first calorimeter and pressure tap and almost indistinguishable in the graphs presented herein. However, the measured velocity profile for the bellmouth (Figure 45) shows that a finite velocity boundary layer thickness has developed at the end of the bellmouth. The point where the velocity field development begins was therefore taken upstream of this. In order to make this point correspond to a physical point on the test section, displacements of the pressure taps are measured from the upstream tip of the bellmouth. The effect of

---

*A detailed measurement of the profile with a boundary layer probe might have allowed solving for an equivalent point from which a constant property layer would have reached the same displacement thickness. However, the difference between this approach and the present is not expected to be large.*
this displacement for the pressure drop data for a UTV test run is shown in Figure 53.

A. Heat Transfer Data Reduction

The emf difference across each calorimeter is calculated and the tare emf difference is subtracted to leave the differential emf due only to heat transfer from the gas. The output from the thermocouple at the inner radius of each calorimeter is converted to a temperature (deg. F) and the tube wall temperature used in the Nusselt number is calculated from the one dimensional heat conduction equation in cylindrical co-ordinates;

\[ T_w = T_i + \frac{(T_i - T_0)}{26.0} \frac{\ln(r_i/0.147)}{\ln(r_o/r_i)} \quad \text{(deg. F)} \tag{5.3} \]

Subscript \( o \) refers to the mean radius at which the outer thermocouple is located and \( i \) refers to the inner thermocouple. The factor 26.0 represents the thermoelectric power of a Cu-Con thermocouple with reference junction at 32°F and measuring junction at 212°F obtained by a visual fit to plotted data in reference (62). TT refers to thermocouple output (mv) and 0.147 represents the inside radius of the test section (in.).

Provision is included in the program for the elimination of calorimeters whose thermocouples were giving
spurious output. In such a case, this is noted on the data sheet. The position of the bad calorimeters were read into the data reduction program. If the response of only one of the pair of calorimeters at each axial position is poor, then the unit heat flux and wall temperature for that point are calculated solely from the good half. When both calorimeters are inoperative, the axial point is skipped altogether. This is usually indicated by a negative or obviously incorrect heat flux in the reduced data. During the testing for the Graetz b.c., the response of thermocouples in both calorimeters at the third axial location from the entrance were consistently spurious. Testing of these thermocouples showed that those giving poor response were not grounded to the test section. Although great care was taken in the composition of the electrical measuring system, this is undoubtably the cause of the trouble. An ungrounded thermocouple at the second axial position intermittently gave spurious output. A filter improved response somewhat for the UTV tests.

An important factor in the reduction of the heat transfer and friction data is the method by which the bulk gas temperature is evaluated at any axial point. For the case of gas heating, the usual experimental facility consists of a resistively heated tube for which the local rate of heat transfer to the gas at every
point along the tube can be calculated fairly well once allowance is made for losses. In the present case, we are provided with the local heat transfer rates at discrete axial points rather than as a continuous function. The method of fitting a function $q^+_w(x^+)$ to the heat transfer data for each run was used. The curve may then be integrated to any axial point to obtain the net heat lost up to that point. This was applied for many assumed forms of $q^+_w(x^+)$. Candidate functions examined were those that could attain large magnitudes at the entrance with a rapid decay. Downstream the function had to approach zero asymptotically. Typical functions tested were combinations of exponentials and powers of $x^+$ with exponents less than 1. No function was found to be satisfactory for the data at all axial points. There are several problems associated with this method. With the exception of the first calorimeter location, the local heat transfer coefficients were found to be extremely sensitive to the form of the assumed function with the sensitivity increasing at the downstream calorimeter locations. Also, since the bulk temperature and hence the heat transfer coefficients at downstream positions depend upon the results from the upstream calorimeters, there will be an integration of errors. This can lead to a great relative error in the wall to bulk temperature difference when this latter quantity becomes
small. This problem is magnified in some cases due to the absence of readings from the third calorimeter and the low differential outputs in the downstream region. An uncertainty analysis is presented in Appendix F which shows that the uncertainty in $\text{Nu}_{m}$ at the second calorimeter is already of the order of 13%.

The test section was designed with a length to diameter ratio far in excess of that required for the laminar heat transfer tests. This was done so that tests could be performed at a later time over an adequate range of axial displacements for flow in the transition and turbulent regime\(^7\). It is possible, however, by varying the inlet Reynolds number to obtain a range of the modified Graetz parameter $x^+$ sufficient for comparison with the theoretical results. Values of $x^+$ obtained at the second calorimeter can be made to extend well into the theoretical fully developed region. Emphasis was therefore placed on the reduction of data from the first two calorimeters.

\(^7\)For turbulent heat transfer, the problem of experimental uncertainty is somewhat reduced due to the high heat transfer rates which can be maintained further downstream. The high mass flow rates insure a much larger wall to bulk temperature difference and a lower drop in gas bulk temperature at a given displacement. In the experimental study by Brim (9) in an apparatus similar to that used here, but for turbulent flow, these problems were not as acute.
The bulk temperatures were finally evaluated by the integration of an analytical function fitted to the heat flux only at the first two calorimeters. A further restriction placed on the function was that this integrated flux should yield the bulk exit temperature as measured in the exit mixing chamber. It was required that the variation of the heat flux should closely approach the shape of the theoretical variation. Most important, it was necessary that when the function was fitted to theoretical values of the heat flux and bulk temperature for both boundary conditions, the theoretical $\text{Nu}_m$ could be retrieved. This is important past the second calorimeter for evaluation of the local friction factor based on total wall shear stress. The fitting of the bulk temperature insures that the tail of the heat flux curve will not shoot off unbounded. Between the second and fifth calorimeters, the wall to bulk temperature difference is only on the order of 100°F. Large relative errors in this difference make up only small errors in the absolute temperature level. The function which best satisfied these criteria out of nearly one hundred forms tested was:

$$q_w'' = P(x) = \frac{A}{(\frac{x}{D})^{.45}} + \frac{B}{(\frac{x}{D})^{.39}} + \frac{C}{(\frac{x}{D})^{.25}}$$  \hspace{1cm} (5.4)
where the three constant $A$, $B$, and $C$ are determined from

\[
q_w^{''} \bigg|_{x=x_1} = F(x_1)
\]

\[
q_w^{''} \bigg|_{x=x_2} = F(x_2)
\]

and

\[
\int_{T_0}^{T_{exit}} M c_p dT = \int_{0}^{1} q_w^{''} d(x/D)
\]

where $x_1$, $x_2$ are the axial displacements of the first and second calorimeters respectively.

$T_{exit} = \text{exit gas temperature from downstream mixing plenum}$

$L = \text{total length of the test section}$

The specific heat was represented by a cubic polynomial in temperature.

\[
c_p(T) = A(T) + B(T)T + C(T)T^2 + D(T)T^3
\]

The bulk temperature $T_{m,x}$ at any $x$ is evaluated from,

\[
\int_{T_0}^{T_{mx}} M c_p(T) dT = \left. A(T)T + \frac{B(T)T^2}{2} + \frac{C(T)T^3}{3} + \frac{D(T)T^4}{4} \right|_{T_0}^{T_{mx}}
\]

\[
= \int_{0}^{x} F(x) d(x/D)
\]

\[8\text{For laminar flow the kinetic energy term } \rho u_0^2/2g_c J \text{ can be shown to be negligible.} \]
This equation was solved for $T_{m,x} (°K)$ by an iteration process in the computer program which stopped when successive values of $T_{m,x}$ differed by less than $1/4$ deg. K. Next, the local heat transfer coefficient is calculated from

$$h = q_w^\prime/(T_{m,x} - T_{w,x})$$

and the Nusselt number

$$Nu = hD/k_m$$

where $k_m$ = thermal conductivity evaluated at bulk temperature = 0.01395 $A(\theta) + B(\theta)T_{m,x}^2 + C(\theta)T_{m,x}^3 + D(\theta)T_{m,x}^4$

$T_{w,x} = \text{wall temperature evaluated from equation 5.3}$

and the non-dimensionalized heat flux from

$$q_w^+ = q_w^\prime r_0/k_o T_0$$

where $T_0 = \text{inlet temperature (deg. R)}$

$k_o = \text{thermal conductivity (BTU/hr ft F)}$

Also, the local modified Graetz parameter at each calori­meter was evaluated from

$$x_m^+ = x^+ \left( \frac{c_{p,o}}{c_{p,m}} \right)^\prime \left( \frac{k_m}{k_o} \right)$$

(5.6)

The heat flux fit (5.4) was also used in the reduction of the friction factor and pressure drop data.

With the adiabatic development section in place, the dimensionless pressure drop is;

$$P = (p_0 - p)/\rho_o U_o^2$$

(5.7)

where $p_0$ refers to the static pressure at the first tap
location. Dimensionless displacements $x^+$ are measured with respect to this point. For the UTV boundary condition, the dimensionless pressure defect calculated is

$$P = \frac{P_0 - p - \frac{1}{2} \rho_0 U_0^2}{\rho_0 U_0^2}$$

(5.8)

and, as mentioned previously, $x^+$ is measured from the forward tip of the bellmouth. The reference pressure $p_0$ is the supply plenum static pressure. The term $\frac{1}{2} \rho_0 U_0^2$ in the numerator of (5.8) is included to account for the acceleration of the gas from zero velocity in the supply plenum to $U_0$ at the end of the bellmouth where the transition into the test section is completed.\footnote{An attempt was made to determine the initial pressure drop experimentally since, due to velocity profile distortion in the bellmouth, the pressure drop may differ from $V_2(\kappa U_0^2)$. It is assumed that this initial pressure drop can be written as $K \rho_0 U_0^2$ where $K$ is a constant which is a function of the bellmouth geometry. The pressure drop actually measured is $p_0 - p - K \rho_0 U_0^2$ where $p_0 - p$ is the viscous parasitic pressure loss from the point where the bellmouth joins the section to the first pressure tap. It can be expected that the term $(p_0 - p) \rho_0 U_0^2$ will decrease with increasing Reynolds number. For large Reynolds numbers, the governing term will be $K$ and if the total non-dimensionalized pressure drop is plotted as a function of $x^+$, it should approach $K$ for small $x^+$ (high $Re_0$). For several isothermal high Reynolds number runs, the dimensionless pressure did seem to be approaching $1/2$, but at a very slow rate. Large pressure fluctuations in the inlet plenum for high flow rates limited the maximum Reynolds number for which this test could be run.} Two types
Figure 53. Effect on dimensionless pressure drop of displacing point at which flow development commences. UTV boundary condition.
of manometer fluids (specific gravities 0.826 and 0.797) were used during the experiment and an index JJ is included in the data reduction program to indicate which is used. The micromanometers were scaled to read directly in inches of H\textsubscript{2}O when blue manometer fluid (s.g. 0.797) was used.

In addition to the dimensionless pressure drop, two types of local friction factor are calculated. The first is based on the portion of the total wall shear stress due to the static pressure drop only:

$$\tau_{w,\Delta p} = -\frac{r_0}{2} \frac{dp}{dx}$$  \hspace{1cm} (5.9)

A corresponding friction factor $f_{\Delta p}$ is defined in terms of the dimensionless pressure gradient by

$$f_{\Delta p} = \frac{\tau_{w,\Delta p}}{\frac{1}{2}\rho_m U_m^2} = \frac{r}{\frac{1}{2}\rho_m U_m^2} \frac{\rho_m d}{\rho_0 dx} \left(\frac{p_0 - p}{\rho_0 U_0^2}\right)$$  \hspace{1cm} (5.10)

and in terms of our non-dimensionalized variables,

$$f_{\Delta p \text{Re}_m} = \frac{1}{Pr_0} \frac{dP}{dx^+} \left(\frac{\rho_m^+}{\mu_m^+}\right)$$  \hspace{1cm} (5.11)

The total wall shear stress is given by

$$\tau_w = \frac{X_{ad}}{2} \frac{dp}{dx^+} \left(1 + \frac{C \sigma^2}{\rho_m g_c}\right)$$  \hspace{1cm} (5.12)
and a local friction factor based on total wall shear stress by

\[ f = \frac{\tau_w}{2 \rho_m U_m^2} = r \frac{\rho_m}{\rho_0} \frac{d \tau}{dx} + \left( \rho - C' \frac{\rho_0}{\rho_m} \right) \]  \hspace{1cm} (5.13)

where \( C \) is the mean mass velocity which is a constant along the tube. The coefficient \( C' \) is defined by,

\[ C' = \sqrt{2 \int_0^1 \rho U^2 r^+ dx^+ / \rho_m U_m^2} \] \hspace{1cm} (5.14)

and is a measure of the non-uniformity of the velocity profile. For a uniform profile, \( C' = 1 \) and for a parabolic profile \( C' = 4/3 \). A value of \( C' \) less than \( 4/3 \) indicates a flattened profile. Since the experimental profiles will be undergoing development, \( C' \) is a function of \( x^+ \) and should be kept within the differential operator in equations 5.12 and 5.13. The actual value of \( C \) is known only at the entrance and approximately in the downstream regions— it is not known at intermediate points. In the entrance for the fully developed inlet velocity, this momentum change due to profile development comprises a substantial portion of the total friction factor. For the UTV bondition, it is less important. For present purposes, \( C' \) was assumed constant at \( 4/3 \) for the parabolic velocity profile since it will begin at this value and reapproach it in the downstream region.
For the UTV case, the actual $C'$ will begin at 1 and also asymptotically approach $4/3$. A value of $C' = 7/6$ which is midway between these limits was used in the data reduction.

In order to evaluate the derivative terms in 5.11 and 5.13, a third order least squares polynomial was fitted to the terms in the parenthesis at the pressure tap locations and differentiated. The bulk properties were evaluated at bulk temperatures obtained by using the same curvefit used in the heat transfer calculations. All densities correspond to inlet static pressure and local bulk temperature. Even though this approach would allow plotting of the friction factors as continuous functions of $x^+$, they are calculated and printed out only at the pressure tap locations. It is felt that this better reflects the experimental nature of the data.

Experimental inlet Mach numbers ranged from 0.009 to 0.023 which are somewhat less than that treated in the theoretical analysis (i.e. $M_0 = 0.03$). For cooling, the Mach number based on the mean axial velocity will decrease along the tube. For example, for $T_w/T_0 = 0.5$, the Mach number downstream of the thermal development region will be reduced by approximately 30% from its initial value. Compressibility effects were shown to be small in the theoretical analysis and are expected to have little effect on the experimental results.
6.1. Graetz Boundary Condition

For both boundary conditions, heat transfer and pressure drop data for air was obtained for inlet wall to bulk temperature ratios of 0.6, 0.5 and 0.4 and pressure drop data only for the additional isothermal cases. For the Graetz boundary condition, the non-dimensionalized pressure defect $P$ is shown plotted against $x^+$ in Figures 54, 55 and 56 along with the same quantity from the finite difference solution. This defect was considered as the best quantity for comparison for three reasons. 1.) The experimental defect requires the least amount of computational reduction. 2.) There is minimal dependence on additional experimental or inferred quantities such as the heat transfer and bulk temperature. 3.) The defect, rather than the wall shear stress, would be the most significant parameter to a designer. Similar to the friction factor, for laminar flow it is a function solely of $x^+$ for a given gas and $\theta_o$, and hence, maintains the same generality.

In these plots one immediately notes that there is a pressure rise in the entrance for both the experimental and theoretical results. The reason is that when the gas undergoes cooling, the resulting increase in bulk density causes a net deceleration of the flow. If the
Figure 5. Experimental dimensionless pressure defect versus $X^+$. Air, Graetz boundary condition, $\frac{T_w}{T_0} = 0.60$. 

- Run 10
- All other runs
- Finite difference, $\frac{T_w}{T_0} = 0.60$
Figure 55. Experimental dimensionless pressure defect versus $X^+$. Air, Graetz boundary condition, $T_w/T_o = 0.5$
Figure 56. Experimental dimensionless pressure drop, Graetz boundary condition,

\( \frac{T_w}{T_0} = 0.40 \)
cooling is severe enough, the deceleration pressure rise can be great enough to offset the frictional pressure drop. The experimental results show a pressure rise greater in magnitude and extending over a further displacement than the theory predicts. The pressure rise at the entrance has profound effect on the character of the flow. Separate curves diverging from the bulk of the data are shown in Figure 54 and 55 for some of the higher Reynolds number tests. Similar divergent data corresponding to high Reynolds number tests for the other wall to bulk temperature ratios was obtained. It should be noted that for gas cooling, the Reynolds number increases with axial distance. Although the data shown is for an inlet Reynolds number less than 2000, this magnitude will be exceeded at some point. All the results for several Re₀ are seen to plot on single curves and no Reynolds number dependence is present. Evidently, the divergence of the pressure defect for the higher Reynolds number is the result of a transition to the turbulent regime. It is not possible to determine the precise point at which transition was triggered. It is interesting to note that the comparison between the theoretical and the experimental results improve as wall to inlet bulk temperature ratio decreases. The friction factor for these same tests for Tₘ/T₀ = 0.50 and 0.40 are shown in Figure 57 and 58. The experimental fReₘ is
Figure 57. Experimental $fRe_m$ versus $x^+$. Air, Graetz boundary condition, $T_w/T_o = 0.50$
Figure 58. Experimental $f_{Re_m}$ versus $x^+$. Air, Graetz boundary condition,
$T_w/T_o = 0.40$
much smaller in the entrance and increases at a much
greater rate than the theoretical results for all
temperature ratios. The fully developed \( f\Re_m \) is
reached more quickly than the theoretical in each case.
Of necessity, the adiabatic development section was
designed so as to be 0.009" smaller in diameter than the
test section at room temperature. For the higher inlet
temperatures, the adiabatic section will be at a
higher temperature and will grow due to thermal expan­sion, so the transition between the two sections will
become smoother. The increasing static pressure may
have a profound effect in the presence of such a dis­continuity. Another possible reason for this behavior
may be the factor \( C' \) in equation 5.13 which may be
underestimated by choosing \( C' = \frac{4}{3} \). This is not how­ever, considered a probable reason since the theoretical
velocity profiles for the Graetz condition show a dis­tinct flattening with decreasing \( \theta_w \). This results in a
value of \( C' \) closer to 1. Also, the pressure tap at
the entrance is reading a static pressure along the wall.
In a region of severe cooling and possible non-negligible
radial pressure gradient, this pressure may not be
representative of the mean pressure existing across the
radius. Since the radial velocity will be in a radially
outward direction, we can reasonably expect that the
static pressure will decrease from the centerline to the
wall. If this is the case, then the pressure measured at the wall is underestimated so that pressure drops along the tube would also be underestimated and the magnitude of the pressure rise overestimated. Use of the mean pressure would tend to move the experimental and theoretical pressure drop and friction factor closer to each other.

The local Nusselt number data from the first two calorimeters are shown in Figure 59 for $\theta_w = 0.40$, 0.50 and 0.60 along with theoretical results for $\theta_w = 0.40$ and 0.60. Agreement is good, although the experimental results show more sensitivity to inlet temperature ratio. Also, the variation with $x^+$ is greater than the theoretical. This can be explained in terms of the configuration of the test section. There was a short, insulated section between the annular section of the development section and the cooling section. In the absence of heat transfer from the gas in this section, a linear temperature gradient could be expected. With gas flowing in the section, the average temperature of the section would probably increase. If the point at which cooling is assumed to commence is taken as the centroid of the temperature-displacement curve, this point would move downstream. This displacement would increase with higher flow rates and Reynolds numbers. In terms of the experimental results shown in Figure 59, the data
Figure 59. Experimental local Nu, m versus x+. Graetz boundary condition.
points at the left side of each of the two data clusters correspond to higher Reynolds numbers than the other points. The displacement of these points from the point of cooling is reduced, so that these points should be displaced to the left. Since the abscissa is logarithmic, points in the left cluster will be affected more than those in the right hand cluster. The 'floating' point at which cooling starts will affect the results since the integration of the heat flux curve used to determine the bulk temperature begins at a fixed point.

The indeterminateness of the bulk temperature is not present in the comparison of theoretical and experimental dimensionless heat flux in Figure 60 for $\theta_w = 0.5$ and 0.6 and in Figure 61 for $\theta_w = 0.40$. Agreement is excellent with the exception of some low experimental points from the second calorimeter in Figure 62. Output was rather spurious from this calorimeter during this test series, but the data is included for completeness. These results would seem to indicate that the spread of the data for $\text{Nu}_m$ may be due to the bulk temperature calculation.

6.2. UTV Boundary Condition

Non-dimensionalized pressure defect data for the UTV boundary condition is plotted in Figure 63 for $\theta_w = 0.60$, 0.50 and 0.40 along with theoretical results for $\theta_w = 0.50$. 
Figure 60. Experimental $q^+_w$ versus $x^+$ for air. Graetz boundary condition. $T_w/T_0 = 0.6, 0.5$
DIMENSIONLESS HEAT FLUX $q_w^+$ VERSUS $X^+$ GRAETZ B.C.

AIR: $T_w/T_0 = 0.40$

Figure 61. Experimental $q_w^+$ versus $X^+$ for air. Graetz boundary condition, $T_w/T_0 = 0.40$
Figure 62. Experimental dimensionless pressure defect versus $X^+$ for air.

UTV boundary condition. $T_w/T_0 = 0.6, 0.5, 0.4$. 

FINITE DIFFERENCE

$T_w/T_0 = 0.5$
The experimental results for $\theta_w = 0.50$ lie approximately 8% below the theoretical. It should be noted that for the UTV inlet geometry, the pressure drop is measured from the plenum. At the first pressure tap, it is possible that a radial pressure gradient exists. In this case, the deceleration of the flow at the wall and the inward radial velocity at the wall is indicative of a static pressure which decreases from wall to centerline. This means that the first pressure tap would be reading relatively high. Using the mean pressure across the section would result in a higher pressure drop from the plenum to the first tap. Although the situation at the first tap is reversed from what it was in the Graetz boundary condition, use of a mean pressure would again move the experimental results closer to the theoretical.

Friction factor results are shown in Figure 63. Theoretical results are for $\theta_w = 0.50$ are included for comparison. Agreement is very good. It should be noted that the use of a mean or lower pressure at the first tap would tend to raise the friction factor slightly at the first pressure tap ($x^+ < 0.022$) and lower the data at $x^+ \sim 0.05$. This would make the comparison even better. In the downstream region ($x^+ > 0.2$) the effect of temperature ratio is small, but somewhat greater than the theory predicts. This may be partially due to
Figure 63. Experimental $f_{Re, m}$ versus $x^+$ for air. UTV boundary condition.

$T_w/T_0 = 0.6, 0.5, 0.4$.
use of a value of $C'(1.1/6)$ in equation 5.13 which is lower than the actual values in this region. Further downstream, the axial variation of density will be so small that the correct asymptotic value of $fRe_m$ would still be reached. It should be noted that friction factor data at the last pressure tap (#7) was not included in the results. Values of the friction factor at this tap for the isothermal flow tests were found to be about 20% high. Similar results were obtained at this tap during the cold flow tests. Since this tap is so far downstream, it is highly unlikely that the actual $fRe_m$ at this point differs by more than a few percent from 16.0.

Local $Nu_m$ is shown in Figure 64. The exaggerated variation with $x^+$ of the data in each cluster is probably due to the variation of wall temperature in the transition from the plenum to the test section. The darkened data points are for run #44 and the uncertainty interval for this run is indicated. This was discussed in section 5.1. A more direct comparison of theoretical and experimental data is included in the axial variation of plotted in Figure 65 for all the experimental UTV data. Again, for run #44, data points are darkened and the uncertainty interval is shown. The data falls a maximum of approximately 35% below the theoretical. Data from the third calorimeter from the entrance (which was
Figure 64. Experimental $\text{Nu}_m$ versus $x^+$ for air. UTV boundary condition,
$T_w/T_\infty = 0.6, 0.5, 0.4$. 
Figure 65. Experimental $q_w^+$ versus $X^+$ for air. UTV boundary condition. $T_w/T_0 = 0.6, 0.5, 0.4$
not operative during most of the testing for the Graetz b.c.) was relatively low for those tests in which it was operative. It is hard to argue that this difference is due to an error in one calorimeter (for example— a low calibration of the conductance of the third calorimeter) since the transition of data between neighboring calorimeters is continuous and the data is consistently lower. Also, the good results obtained from the calorimeters for the Graetz b.c. would add confidence to the calibration.
CHAPTER 7. SUMMARY AND CONCLUSIONS

7.1. Summary

The results of a combined experimental and analytical investigation of heat transfer and flow characteristics for the laminar flow of gases in cylindrical tubes at low wall to bulk temperature ratios has been presented. For the theoretical analysis, gas transport and thermodynamic properties were treated as variable. It was possible to modify an existing finite difference solution to the boundary layer equations with property variation terms for use in cooling cases. For the case when inlet temperature and velocity profiles were uniform, the conditions for similar thermal and velocity variable property boundary layers to exist at the tube wall were found to be closely obtained for small distances from the tube entrance. An entrance region solution based on the similarity assumption was patched to the finite difference solution and downstream convergence of the wall parameters from the finite difference solution was seen to be significantly improved. This improvement is believed due to reduced error in maintaining net conservation of energy in the presence of initial singularities in both the thermal and velocity boundary conditions. Wall friction and heat transfer results obtained for air and helium can be best discussed in four categories.
7.2. Conclusions

7.2.1. Heat Transfer (Graetz Boundary Condition)

Theoretical heat transfer results as expressed in the axial variation of the Nusselt number for fully developed inlet velocity profiles were found to be relatively insensitive to temperature ratio. Experimental Nusselt number and dimensionless heat flux, \( q_w^+ \) data for air supports this conclusion. Maximum variation of the theoretical \( \text{Nu}_m \) from the isothermal \( \text{Nu}_m \) occurring at an inlet wall to bulk temperature ratio of 0.10 was found to be a decrease of approximately 13% for air and 15% for helium. The theoretical axial variation of \( \text{Nu}_m \) can be correlated within \( \pm 5\% \) by the following equations:

For air: \( 0.001 \leq x^+ < 0.35 \)
\[
\text{Nu}_m = (3.67 + 0.198x^{+\cdot 584}e^{-20.8x^+}) (1-0.13(\theta_w-1))
\]

For helium: \( 0.001 \leq x^+ < 0.35 \)
\[
\text{Nu}_m = (3.67 + 0.201x^{+\cdot 584}e^{-20.8x^+})(1-0.15(\theta_w-1))
\]

For both gases: \( x^+ \geq 0.35 \)
\[
\text{Nu}_m = 3.67
\]

7.2.2. Friction Factor (Graetz Boundary Condition)

Friction factors for fully developed inlet velocity profiles are affected more severely by temperature ratio, but the experimental and theoretical results are not in agreement as to the degree of this variation. Experimental friction factors and pressure drops were significantly
lower in the entrance than the theoretical. This may have been due to several factors. A pressure rise in the entrance in the presence of a discontinuity in the tube diameter may have changed the character of the flow in this region. The theoretical variation of the total friction factor is well represented by the following equation:

\[ fRe_m = 16 \left( \frac{T_w}{T_m} \right)^n \]

where

- for air: \( n = 0.904 \left( \frac{T_w}{T_0} \right)^2 \)
- for helium: \( n = 0.957 \left( \frac{T_w}{T_0} \right)^{2.51} \)

This correlation is offered with the reservation that it does not represent the experimental variation.

7.2.3. Heat Transfer (UTV Boundary Condition)

For the UTV boundary condition, the theoretical Nusselt number, \( Nu_m \), was found to be almost totally insensitive to cooling throughout the flow development region. Constant property correlations for friction factor are recommended. In the entrance region this behavior could have been predicted, at least on a qualitative basis, by variable property external boundary layer results for cooling (23). Entrance effects predominate throughout the thermal development region and correlation in terms of local significant temperature ratios was not possible. The experimental heat transfer data confirmed these results in the entrance region where experimental uncertainties were smallest, but in the mid and downstream region, experimental
values of the local heat flux were found to be about 30% below the predicted. The difference is not within the estimated uncertainty and the consistency of the data is difficult to explain. A single correlation applies to the theoretical variation of $\text{Nu}_m$ for helium and air at all wall to bulk temperature ratios:

$$\text{Nu}_m = 3.67 + 0.246x^+ \cdot 592 \cdot e^{-20.6x^+} \quad (0.001 \leq x^+ < 0.5)$$

$$\text{Nu}_m = 3.67 \quad (x^+ \geq 0.5)$$

This is offered with the reservation that it does not represent the downstream experimental data well.

7.2.4. Friction Factor (UTV Boundary Condition)

Again, the small variation of the friction factor with temperature ratio at the entrance could have been predicted from variable property external boundary layer results. However, the axial pressure gradient has a greater weight in the momentum equation than on the energy equation. This gradient was found to be strongly effected by temperature ratio along with a greater sensitivity of the flow characteristics to temperature ratio. The flow development region was found to be substantially lengthened with extreme cooling. Maximum decrease of the friction factor-Reynolds product for air and helium was approximately 45%. Excellent agreement between theoretical and experimental pressure drop and friction factor variation was
obtained for the UTV boundary condition. The theoretical friction factor results for air and helium are well correlated by:

\[ 1 - \frac{(f_{Re_m})}{(f_{Re})_I} = 1.067(1 - \theta_w) x^{0.675} e^{-0.576x^+} \]  

where \( \beta = 7.70\theta_w^{0.675} \)

and \((f_{Re})_I = \text{isothermal friction factor-Reynolds number product},\)  
\[ (f_{Re})_I = 16.0 + 0.694x^+ - 22.9x^+ \]

In general, for both boundary conditions, friction factor and flow characteristics for both gases were found to be much more sensitive to temperature ratio than heat transfer results. In an absolute sense, variation of all theoretical wall parameters were found to be relatively insensitive to temperature ratio when the severity of the cooling is considered. The modified Graetz parameter based on inlet properties was deemed a better independent variable for representation of the results than \( x_m^+ \) which is based on local properties evaluated at the mean temperature.
REFERENCES


12. Cholette, S., See Ref. 30


85. Smith, A.M.O., "Rapid Laminar Boundary Layer Calculations by Piecewise Application of Similar Solutions," Jour. of Aerospace Science, 23, 1956

86. Swearingen, T.B., Internal Laminar Heat Transfer to a Gas with Temperature Dependent Properties, Ph.D. Thesis, Engineering Experiment Station, Arizona University, Tucson, 1969


89. Taylor, M.F., Experimental Local Heat Transfer Data for Precooled Hydrogen and Helium at Surface Temperatures up to 5300 R, NASA TN-D-2595, 1965

91. Taylor, M.F., *A Method of Correlating Local and Average Friction Coefficients for both Laminar and Turbulent Flow of Gases through a Smooth Tube with Surface to Fluid Bulk Temperature Ratios from 0.35 to 7.35*, NASA, TR-R267, 1967


100. Worsoe-Schmidt, P.M., *Finite Difference Solution for Laminar Flow of Gas in a Tube at High Heating Rate*, Ph.D. Thesis, Department of Mechanical Engineering, Stanford University, 1965

102. Worsoe-Schmidt, P.M., Personal Communication, March 7, 1969


Appendix A

Variable Property and Non-Boundary Layer Terms

When the full momentum and energy equations 2.2 and 2.3 are non-dimensionalized with the same variables that were used to non-dimensionalize the boundary layer equations in Chapter 2, the following forms result:

Axial momentum:

\[
\rho^+ \left( \frac{u^+ \partial u^+}{\partial x^+} + v^+ \partial u^+ \right) = - \frac{\rho_0}{\rho_0 U_0} \frac{\partial p}{\partial x^+} + 2 \Pr_0 \mu^+ \partial^2 u^+ \partial r^+ + 2 \Pr_0 \left[ \frac{\partial u^+}{\partial r^+} + \frac{1}{(\Re_0 \Pr_0)^2} \left( \frac{\partial}{\partial r^+} \mu^+ \frac{\partial V}{\partial x^+} \right) \right] + 4 \frac{\partial}{\partial x^+} \mu^+ \frac{\partial u^+}{\partial x^+} - \frac{2}{3} \frac{\partial}{\partial x^+} \mu^+ \frac{\partial (r^+ U^+)}{\partial r^+}
\]

Energy equation:

\[
\rho^+ \left( \frac{u^+ \partial H_1^+}{\partial x^+} + v^+ \partial H_1^+ \right) = (1 - \gamma_0) M_0^2 \mu^+ \frac{\partial p}{\partial x^+} + \frac{2}{\Re_0 \Pr_0} \frac{\partial}{\partial x^+} \left( c_p \partial \mu^+ \right) + \frac{2}{\Re_0 \Pr_0} \frac{\partial}{\partial r^+} \left( c_p \partial H_1^+ \right) + 2 \frac{\partial}{\partial r^+} \left( \frac{k^+}{\Re_0 \Pr_0} \frac{\partial H_1^+}{\partial r^+} \right) + 2(\gamma_0 - 1) M_0^2 \Pr_0 \mu^+ \Phi^+
\]

where \( \Phi^+ = \) mechanical dissipation function

\[
\Phi^+ = \left[ \left( \frac{\partial u^+}{\partial r^+} \right)^2 + \frac{2}{(\Re_0 \Pr_0)^2} \left( \frac{\partial v^+}{\partial r^+} \right)^2 + \left( \frac{v^+}{r^+} \right)^2 - \frac{2}{3} \left( \frac{\partial v^+}{\partial r^+} \right)^2 \right]
\]

We note that as the Reynolds number decreases, the importance of the dissipation function as usually defined \( \left( \frac{\partial u^+}{\partial r^+} \right)^2 \) will decrease in relation to the other terms in the function, however, if the decrease in Reynolds number is due to a
decrease in the mass flow rate, the decrease in $M_0^2$ will offset this rise. The term which is retained in the momentum boundary layer equation and can be identified as being due to property variation (excepting the density) is

$$aP_0 \frac{\partial u^*}{\partial r} \frac{\partial T^*}{\partial r}$$

The ratio of the non-boundary layer terms to this is,

$$R_1 = \frac{\frac{2}{3} \frac{\partial \mu^*}{\partial x^*} \frac{\partial \mu^*}{\partial x^*} \frac{\partial \mu^*}{\partial x^*} \frac{\partial \mu^*}{\partial x^*} - \frac{\mu^*}{3} \frac{\partial \mu^*}{\partial x^*}}{(\text{Re}_P \rho_0)^2 \frac{\partial T^*}{\partial r} \frac{\partial u^*}{\partial r}}$$

Using the power law representation for viscosity,

$$\mu^* = \theta^b$$

This becomes

$$R_1 = \frac{b \theta^{-1} \frac{\partial \theta}{\partial x^*} \frac{\partial \theta}{\partial x^*} \frac{\partial \theta}{\partial x^*} \frac{\partial \theta}{\partial x^*} - \frac{\theta}{3} \frac{\partial \theta}{\partial x^*}}{(\text{Re}_P \rho_0)^2 b \theta^{-1} \frac{\partial \theta}{\partial r} \frac{\partial u^*}{\partial r}}$$

Similarly, expanding the ratio of the non-boundary layer terms in the energy equation to the property variation terms (neglecting the dissipation function),

$$R_2 = \frac{2}{\text{Re}_P \rho_0} \frac{\partial \rho^*}{\partial x^*} \frac{\partial \rho^*}{\partial x^*} - \frac{1}{\text{Re}_P \rho_0^2} \frac{(c-a)^{-1} \frac{\partial \rho^*}{\partial x^*} \frac{\partial \rho^*}{\partial x^*} + \frac{\partial \rho^*}{\partial x^*}}{(c-a)^{-1} \frac{\partial \rho^*}{\partial r} \frac{\partial u^*}{\partial r}}$$

The ratio of molecular to convective axial momentum transfer is

$$R_3 = \frac{2 \frac{\partial \rho^*}{\partial x^*} \frac{\partial \rho^*}{\partial x^*}}{(\text{Re}_P \rho_0)^2} \frac{\rho^* U^* \frac{\partial u^*}{\partial x^*}}{\rho^* U^* \frac{\partial u^*}{\partial x^*}} = \frac{a(b \theta^{-1} \frac{\partial \theta}{\partial x^*} \frac{\partial \theta}{\partial x^*} \frac{\partial \theta}{\partial x^*} - \frac{\theta}{3} \frac{\partial \theta}{\partial x^*})}{(\text{Re}_P \rho_0)^2 \rho^* U^* \frac{\partial u^*}{\partial x^*}}$$

and the ratio of molecular conduction to axial convective heat transfer is:

$$R_4 = \frac{2}{(\text{Re}_P \rho_0)^2} \frac{\frac{\partial \rho^*}{\partial x^*} \frac{\partial \rho^*}{\partial x^*}}{\rho^* U^* \frac{\partial u^*}{\partial x^*}}$$
The radial and axial derivatives were evaluated by using central difference operators and values of dependent variables from the finite difference solution with $\Delta x^+ = 10^{-4}$ and $\Delta r^+ = 1/320$. 
Appendix B

Gas Thermodynamic and Transport Properties

Data was drawn from several sources in the evaluation of transport and thermodynamic properties for air, helium and CO₂. The exponents in the power law representation with temperature were chosen so as to minimize the least square error for all reference points (subscript zero) in the desired ranges. In a usual least squares fit to N tabulated values of a property \( Y \), the quantity

\[
E = \sum_{i=1}^{n} \left( \frac{y_i}{y_0} - \left( \frac{T_i}{T_0} \right)^{\exp} \right)^2
\]

would be minimized by appropriate choice of \( \exp \). Subscript 0 quantities are reference values. However, in the present investigation the quantity which was minimized in most instances was

\[
E = \sum_{k=1}^{n} \sum_{i=1}^{n} \left( \frac{y_i}{y_k} - \left( \frac{T_i}{T_k} \right)^{\exp} \right)^2
\]

which means that the exponent is also an optimum with respect to all reference points in the tabulated range. An exponent chosen by such a criterion will differ somewhat from that chosen by a ordinary least squares fit or a visual fit to plotted data. Properties which did not require the use of a reference quantity, namely \( \rho_0 \) and \( \gamma_0 \) were chosen by an ordinary least squares criterion. In several cases data from more than one source is plotted in order to extend the temp-
temperature range or to serve as a confirmation of data from the prime source. Graphical plots of the data are given in Figures 66, 67, and 68 for helium, air, and CO₂ respectively. The correlations used in the theoretical portion of the investigation are represented by solid lines.
Figure 66. Thermodynamic and transport properties of helium.
Figure 67. Thermodynamic and transport properties for air.
Figure 68. Thermodynamic and transport properties for carbon dioxide.

DATA OF △ Hilsenrath, et al

▼ Vines

\[ \mu/\mu_0 \]
\[ \mu_0 = 9.2067 \times 10^{-6} \text{lb/ft \cdot sec.} \]

\[ k/k_0 \]
\[ k_0 = 8.40 \times 10^{-3} \text{BTU/hr \cdot ft \cdot °F} \]

\[ Cp/Cv \]

\[ R = 0.04512 \text{ BTU/lb \cdot °R} \]
Appendix C

Calorimeter Conductance:

Error in One Dimensional Heat Conduction Equation

Due to Thermocouple Location

A schematic of the calorimeter used to measure local $q_w''$ is shown in Figure 42. We assume that all thermocouples are homogeneous and thermocouple beads are infinitesimally small. We are trying to determine a maximum range of deviation for the calorimeter conductance defined as

$$K_{cal} = \frac{q_w''}{(T_i - T_o)}$$

due solely to thermocouple bead location. $T_i$ and $T_o$ are the temperatures measured by thermocouples in the inner and outer holes respectively. When a thermocouple with a bead diameter less than the hole diameter is inserted into a calorimeter, the exact location of the thermocouple is unknown. The uncertainty in the thermocouple location is the sum of two uncertainties—1.) the location of the thermocouple hole and 2.) the location of the thermocouple in the holes. The holes drilled were 0.030"D. A realistic maximum error in the location of the holes with respect to the tube centerline is 0.005". On the basis of the one dimensional heat conduction equation in cylindrical coordinates:

$$K_{cal} = \frac{k}{r t \ln(r_i/r_o)}$$
where \( r_i \) = radius where the center of the inner thermocouple hole is located

\( r_o \) = radius where the center of the outer thermocouple hole is located

\( r_t \) = inside radius of tube

\( k \) = thermal conductivity of calorimeter material

We assume that the temperature field in the calorimeter is unaffected by the presence of holes, that the conductivity of the calorimeter material is not a function of radius and that the thermocouple bead will be at the temperature of the point of the wall where it is touching. This is not a bad approximation since although the holes were packed with a high conductivity grease, the conductivity of the thermocouples are greater by almost two orders of magnitude than the filler. Also, there was a possibility of voids existing in the packing. The minimum value of conductance occurs with the tolerances and free play of the thermocouples in the holes acting so as to provide a maximum distance between the thermocouple beads. The ratio of minimum calorimeter conductance to that calculated using nominal dimensions can be shown to be,

\[
\frac{K_{\text{cal.min}}}{K_{\text{cal.nom}}} = \frac{\ln\left(\frac{0.4375}{0.2500}\right)}{\ln\left(\frac{0.4375 + 0.015 + 0.005}{0.2500 - 0.015 - 0.005}\right)} = 0.80
\]

and the maximum ratio obtainable

\[
\frac{K_{\text{cal.max}}}{K_{\text{cal.nom}}} = \frac{\ln\left(\frac{0.4375}{0.2500}\right)}{\ln\left(\frac{0.4375 - 0.015 - 0.005}{0.2500 + 0.015 + 0.005}\right)} = 1.28
\]
So that the total range of variation can be as large as 48%. In Figure 44 these error limits are shown drawn with respect to the average value of all the calorimeter conductances. While additional uncertainties (i.e. power level during radiation test, uncertainty of thermocouple output) could have been added to increase the limits, there seems to be no reason for doing this since the above uncertainty is sufficient to include all the conductance scatter.
A. Radiation

For the calibration, a 1/8 inch diameter stainless steel tube was extended down the center of the test section. The section was evacuated with a mechanical vacuum pump and a voltage was applied to the heating element. Knowledge of the power input to the element, assumption of uniform irradiation to the tube inner wall and measurement of the temperature difference corresponding to this known \( q_w \) allows calculation of the calorimeter conductance \( K \) where

\[
K = \frac{q_w}{\Delta \varepsilon}
\]

where \( \Delta \varepsilon \) is the corrected difference in thermocouple emf across each calorimeter half. In order to insure proper centering of the heating element, several ceramic spacers were mounted at points midway between the calorimeter locations. These will reduce the radiative heat transfer in two ways. 1.) The viewfactor from the wire to the wall is reduced and 2.) the local temperature of the wire at the spacer is reduced due to thermal conduction through the spacer. Concerning the viewfactor, we consider the geometry and co-ordinate system illustrated in Figure 69. The elemental cylindrical area \( dA \), is at a point directly
Figure 69. Coordinate system for radiation calibration.
under the calorimeter. The ceramic spacers are mounted at \( x = \frac{-L}{2} \) and it is desired to calculate the viewfactor from the finite length of wire between these limits to \( dA_1 \). The angle \( \phi_1 \) is the included angle between an inward facing normal from the tube inner wall and a line segment of length \( r \) connecting \( dA_2 \) and \( dA_1 \) where \( dA_1 \) is an element of area on the heater. Angle \( \phi_2 \) lies between this same line segment and \( dA_2 \). The inside radius of the test section is \( R_0 \) and \( r_0 \) is the radius of the heating element. The geometric shape factor from the heater to \( dA_1 \) is:

\[
A_{2F2-1} = \int_{-L}^{L} \frac{\cos \phi_1 \cos \phi_2 dA_2}{\pi r^2} dA_1
\]

writing \( \cos \phi_1 \), \( \cos \phi_2 \) in terms of geometric quantities:

\[
A_{2F2-1} = \int_{-L}^{L} \frac{1}{(R_0^2 + x^2)^2} \frac{R_0^2 r_0 dx}{dA_2}
\]

\[
= \pi r_0^2 \left[ \frac{x}{R_0^2 + x^2} + \frac{1}{R_0} \tan^{-1}\left( \frac{x}{R_0} \right) \right]_{-L}^{+L}
\]

The percentage difference between the shapefactor for \( L = \) and the shapefactor for the particular test section dimensions is:

\[
\left| \frac{(A_{2F2-1})_L = 4.8" - (A_{2F2-1})_L = \infty}{(A_{2F2-1})_L = \infty} \right| < 0.01
\]
It must be remembered that this does not include reflection or reradiation from the spacer.

B. Conduction Losses

A total of 4 spacers, 1/16 inch thick x 1/8 inch I.D. x 0.294 inch O.D. were used to center the heating wire at the centerline of the test section. Here we attempt to calculate the thermal conduction loss through these spacers from the wire to the wall. First, a temperature of the heating wire must be determined. For radiation between two grey bodies which see only each other, the total heat transfer may be written

\[ q = \sigma \left( T_1^4 - T_2^4 \right) \left( \frac{\rho_1 A_1 + \frac{1}{A_1} + \frac{\rho_2}{A_2}}{\epsilon_1 A_1 + \frac{1}{A_1} + \frac{\epsilon_2 A_2}{A_2}} \right) \]

where \( \rho_1 \) = reflectivity of heating tube (Ref. 49. Table 2.5 316 s.s. 'as received') = 0.39

\( \rho_2 \) = absorptivity of inside tube wall (taken as equal to that of black enamel) = 0.95 (Ref. 50)

\( \epsilon_1 \) = emissivity of heating tube = 1-0.61 = 0.39

\( \epsilon_2 \) = emissivity of inside tube wall = 1-0.95 = 0.05

\( A_1 \) = surface area of heating wire = \( \pi \times (58"L) \times (1/8"D) \) in\(^2\)

\( A_2 \) = surface area of inside tube wall = \( \pi \times (58"L) \times (0.294"D) \) in\(^2\)

\( T_1 \) = temperature of inside tube wall = 212 \(^\circ\)F = 671 \(^\circ\)R

The power input to the heater during the radiation test was 1167 BTU/hr. Using these quantities in the above expression yields \( T_1 = 1190 \ ^\circ\)F. For the ceramic spacer, taking the thermal conductivity as being approximately
approximately equal to that of glass \((0.40 \text{ BTU/hr ft F})\), the conducted heat transfer through the disks is

\[
q_{\text{conduction}} = 4 \times \frac{1}{16} \times 2\pi \times 0.40 \frac{(1190^\circ F - 212^\circ F)}{\ln \left(\frac{0.294}{0.125}\right)} \text{ BTU/hr}
\]

\[= 58 \text{ BTU/hr}\]

This amounts to about \(5\%\) of the total heat transfer. However, a correction for this loss was not included in the calibration for two reasons. First, in the vicinity of the spacers, the temperature of the wire is reduced so the wire-to-wall temperature difference is reduced. Second, this analysis assumes perfect thermal contact of wire with spacer and spacer with tube wall. It is probable that the conduction losses are a fraction of the above, but there is no way to calculate this quantity precisely.
Appendix E

Uncertainty Analysis - Nusselt Number, Friction Factor Data

The relationship between the uncertainty interval or precision index $w_i$ of a calculated quantity or dependent variable $R$ and the uncertainty intervals $w_i$ of the independent variables or measured quantities, $x_i$, is given by (46),

$$w_R^2 = \sum_{i=1}^{n} \left( \frac{\partial R}{\partial x_i} \right)^2 w_i^2 \quad (E.1)$$

Since the uncertainty intervals of most of the instruments whose outputs are combined to produce $R$ are not known, the recommendation of Kline and McClintock (46) will be used. An interval is estimated for each instrument or measurement for which it is felt the probability is 1 to 20 that the true value of the measured quantity lies outside of this interval centered at the measured value. For gages such as for pressure or voltage, the uncertainty is taken as $1/2$ of the least division on the dial. For thermocouples, the uncertainty interval is taken as the ISA calibration. Since the uncertainty interval for the wall parameters will vary with each run, the time and effort needed to treat nearly 40 tests and several hundred data points would be prohibitive. Test run #44 was chosen as an example for uncertainty calculations. Since this particular run was one of the highest in terms of
pressure drop and local heat fluxes, the uncertainty intervals will be low in relation to those for other runs. If the uncertainty interval for this test can be shown to span most of the data for the lower flux tests, then there is no need to calculate these additional intervals.

The uncertainty in the calorimeter conductances must be known. The expression for the conductance of a calorimeter in terms of experimentally measured quantities in the calibration is,

\[ K_i = \frac{EI/2\pi r_0 L}{(\Delta e_i,\text{test} - \Delta e_i,\text{tare})} \times 3.413 \text{ BTU/hr}^2\text{mv} \]

where \( \Delta e_i,\text{test} \) is the difference in thermocouple output across calorimeter \( i \) corresponding to an electrical input \( E \times I \) to the heating wire and \( \Delta e_i,\text{tare} \) is the difference in readings when there is no input to the heating wire. The factor 3.413 is a conversion between watts and BTU/hr.

Application of equation E.1 yields

\[ \left( \frac{w_{K_i}}{K_i} \right)^2 = \left( \frac{w_E}{E} \right)^2 + \left( \frac{w_I}{I} \right)^2 + \left( \frac{w_{R_0}}{R_0} \right)^2 + 4 \left( \frac{w_{\Delta e_i}}{\Delta e_i} \right)^2 \]

The coefficient 4 is present in front of the thermocouple emf term since the differential emf will actually be a combination of 4 thermocouple readings—2 tare or zero heat flux readings and 2 readings with heat flux. Absolute uncertainty intervals which were deemed appropriate for all significant measured quantities are given in Table E.1. Some word should be mentioned concerning some
Table E.1.

Uncertainty Intervals. Experimentally Measured Quantities

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Experimental Level</th>
<th>Uncertainty (Absolute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E</td>
<td>15.1 volts</td>
<td>0.1 V</td>
</tr>
<tr>
<td>I</td>
<td>22.6 amps</td>
<td>0.1 amp</td>
</tr>
<tr>
<td>T₀</td>
<td>0.294 inches</td>
<td>0.001 inches</td>
</tr>
<tr>
<td>L</td>
<td>58.625 inches</td>
<td>0.0625 inches</td>
</tr>
<tr>
<td>x₁, x₂</td>
<td>1.75 inches, 11.33 inches</td>
<td>0.005 inches</td>
</tr>
<tr>
<td>(Δe₁)<em>{test} - (Δe₁)</em>{tare}</td>
<td>0.055mv (avg.)</td>
<td>0.0010 mv.</td>
</tr>
<tr>
<td>M</td>
<td>0.4 - S.C.P.M.</td>
<td>1%</td>
</tr>
<tr>
<td>T₀</td>
<td>1670 °R (avg.)</td>
<td>5°R</td>
</tr>
<tr>
<td>P₀</td>
<td>order of 8 inches Hg</td>
<td>0.05 inches Hg</td>
</tr>
<tr>
<td>P₀ - p</td>
<td>order of 0.10 inches H₂O</td>
<td>0.002 inches H₂O</td>
</tr>
</tbody>
</table>
of these quantities. The interval for the thermocouple output $w_e$ may seem to be extremely small, but it should be remembered that we are dealing with a precision rather than an accuracy error in this case. The ISA calibration is essentially an accuracy term. The accuracy error is effectively eliminated through subtraction of the tare readings. The quantity $w_e$ was determined by a test on the instrumentation during an actual run. It was found that ten consecutive readings from the same thermocouple could be included within an interval of 0.0010 mv. However, there were sporadic intervals when electrical interference or power fluctuations would cause a much greater variation. For most calorimeters, these periods were the exception rather than the rule. The Meriam flowmeter was calibrated to within a 0.5% of a Meriam standard flow device. The error in the flow measurement will increase due to errors in measurement of the output pressure and quantities necessary for calculation of correction factors. 1% should be quite representative when these additional uncertainties are considered.

Substitution of the quantities in Table E.1 into the above equation yields for an average conductance of approximately 55.0 BTU/hrft^2-mv

$$\left(\frac{w_{K_i}}{K_i}\right)_{\text{avg.}} = 0.037$$
or a deviation of approximately 3.7%. The expression for the wall heat flux at any point is given by

\[ q_w^n \bigg|_{x_1} = K_i \left( \Delta e_{i, \text{test}} - \Delta e_{i, \text{tare}} \right) \]

The uncertainty interval for the local heat transfer is

\[ \left( \frac{w_{q^n}}{q^n} \right)^2 = 4 \left( \frac{w_e}{(\Delta e_{i, \text{test}} - \Delta e_{i, \text{tare}})} \right)^2 + \left( \frac{w_{K_i}}{K_i} \right)^2 \]

The uncertainty, as can be expected, is a function of the output during the test. For UTV test #44, the uncertainty intervals in the local heat flux are shown in Table E.2. Since the local results presented in the text are averaged from two calorimeters, the uncertainty should be somewhat less than these values. Since the output from the fourth and further calorimeters from the entrance are much smaller, the uncertainty interval is rather large for these fluxes.

This problem is compounded in the calculation for the gas bulk temperature, which besides the local heat transfer rate, is the most important quantity in the evaluation of \( Nu_m \). Since the local heat fluxes are combined in a rather complicated fashion in the integration for the bulk temperature, a simplified analysis is used. Since Nusselt number data is presented only for the first two calorimeters, the uncertainty will be calculated

---

1This should not be confused with the uncertainty calculated in Appendix C which applies to the calorimeter before calibration.
### Table E.2.
Uncertainty Intervals for Test Run # 44

**Local Heat Transfer Rates**

<table>
<thead>
<tr>
<th>Calorimeter</th>
<th>$(\Delta e_i)<em>{test} - (\Delta e_i)</em>{tare}$ ($\mu$V)</th>
<th>$K_i^*$</th>
<th>$q_w^{\prime \prime}$ (BTU/hr ft$^2$)</th>
<th>$\frac{\sigma q^{\prime \prime}}{q}$ %</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 entrance</td>
<td>144.0</td>
<td>45.2</td>
<td>6520</td>
<td>3.85</td>
</tr>
<tr>
<td>11</td>
<td>123.1</td>
<td>60.2</td>
<td>7380</td>
<td>4.05</td>
</tr>
<tr>
<td>10</td>
<td>37.3</td>
<td>58.5</td>
<td>2180</td>
<td>6.61</td>
</tr>
<tr>
<td>9</td>
<td>40.7</td>
<td>59.0</td>
<td>2400</td>
<td>6.10</td>
</tr>
<tr>
<td>8</td>
<td>20.3</td>
<td>48.6</td>
<td>988</td>
<td>10.1</td>
</tr>
<tr>
<td>7</td>
<td>15.2</td>
<td>63.9</td>
<td>970</td>
<td>13.3</td>
</tr>
<tr>
<td>6</td>
<td>11.9</td>
<td>52.2</td>
<td>620</td>
<td>17.0</td>
</tr>
<tr>
<td>5</td>
<td>11.5</td>
<td>60.8</td>
<td>700</td>
<td>17.6</td>
</tr>
<tr>
<td>4</td>
<td>6.9</td>
<td>57.9</td>
<td>400</td>
<td>29.0</td>
</tr>
<tr>
<td>3</td>
<td>8.8</td>
<td>55.7</td>
<td>485</td>
<td>22.9</td>
</tr>
<tr>
<td>2 exit</td>
<td>5.9</td>
<td>57.6</td>
<td>340</td>
<td>34.0</td>
</tr>
<tr>
<td>1</td>
<td>-0.5</td>
<td>50.0</td>
<td>-25</td>
<td>40.0</td>
</tr>
</tbody>
</table>

*$^{\text{BTU/hr ft}^2 \text{mv}}$
only for these two points. We assume that the heat transfer from the gas between the test section entrance and first calorimeter is given by,

\[ Q_1 = 2\pi r_0 q_w^{*} x_1 \]

and up to the second calorimeter by

\[ Q_2 = 2\pi r_0 (x_2 - x_1) q_w^{*} x_2 + Q_1 \]

which represents an integration by means of Simpson's rule. We also make the simplification that the specific heat \( c_p \) of the gas is constant so that the bulk temperature may be written as,

\[ T_{m1} = T_o - 2\pi r_0 x_1 q_w^{*} / M c_p \]

and

\[ T_{m2} = T_o - \left( 2\pi r_0 x_1 q_w^{*} x_1 + (x_2 - x_1) q_w^{*} x_2 \right) / M c_p \]

where \( M \) is the mass flow rate, \( x_1 \) and \( x_2 \) are the locations of calorimeters 1 and 2 respectively. Use of the uncertainty intervals in Table E.2 results in

\[ \frac{w_{T_{m1}}}{T_{m1}} = 0.0402 \quad \frac{w_{T_{m2}}}{T_{m2}} = 0.0545 \]

Proceeding in this manner, the uncertainty in the Nusselt numbers at these points are:

\[ \frac{w_{Nu_1}}{Nu_1} = 0.078 \quad \frac{w_{Nu_2}}{Nu_2} = 0.132 \]

The bulk temperature enters the computation for the friction factor (eqn. 5.13) by way of the density term. Friction factor data is presented for axial points past the second calorimeter and the uncertainty analysis for the bulk temperature is not extended to this region.
However, the second term in the brackets in equation 5.13, $\rho_o/\rho_m$ rapidly decreases in importance along the tube. For example by the fourth calorimeter in test #40, the axial gradient of this term accounts for less than 25% of the total friction factor. The bulk temperature is asymptotically approaching the limiting value of the exit bulk temperature. This limiting value would tend to bound the error on the negative side of the bulk temperature so that continuation of the preceding analysis downstream would overestimate the error. The evaluation of derivatives for discrete data can be a risky business such as is performed here for the local friction factor. The limiting error for the derivative is extremely difficult to determine and primarily for this reason an uncertainty analysis is not performed on the local friction factor. The non-dimensionalized pressure drop $P$ easily admits to such an analysis, however. The expression for this quantity in terms of experimentally measured quantities is,

$$P = (2\pi p_o (p_o-p)r_o^2) / RT_o \bar{M}^2$$

where $R$ is the gas constant for air. The uncertainty interval is given by

$$\left( \frac{w_p}{P} \right)^2 = 2 \left( \frac{w_{r_o}}{r_o} \right)^2 + \left( \frac{w_T}{T_o} \right)^2 + \left( \frac{w(p_o-p)}{p_o-p} \right)^2 + \left( \frac{w_{p_o}}{p_o} \right)^2 + 2 \left( \frac{w_{\bar{M}}}{\bar{M}} \right)^2$$

where we have made a distinction between the uncertainty interval for $p_o$ and $p_o-p$ since the former quantity was
measured by an air over mercury manometer and the latter quantity was read with a micromanometer. Using the uncertainty interval used previously for $p_0$ and a 0.001" absolute uncertainty for the micromanometer reading, we obtain the results in Table E.3 for runs #44 and #25. Run #25 is at about the middle of operational parameters run in the Graetz boundary condition test series. We note that the uncertainty level decreases with axial displacement since the pressure defect is an integrated quantity.
Table E.3.

Uncertainty Intervals for Non-Dimensionalized Pressure Drop

<table>
<thead>
<tr>
<th>Test Run # 44</th>
<th>Test Run # 25</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^+ )</td>
<td>( x^+ )</td>
</tr>
<tr>
<td>( p )</td>
<td>( p )</td>
</tr>
<tr>
<td>( \frac{w_p}{p} ) (%)</td>
<td>( \frac{w_p}{p} )</td>
</tr>
<tr>
<td>-----------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>0.0120</td>
<td>0.0362</td>
</tr>
<tr>
<td>0.136</td>
<td>-0.085</td>
</tr>
<tr>
<td>1.69</td>
<td>4.5</td>
</tr>
<tr>
<td>0.0404</td>
<td>0.1018</td>
</tr>
<tr>
<td>0.266</td>
<td>0.039</td>
</tr>
<tr>
<td>1.66</td>
<td>9.2</td>
</tr>
<tr>
<td>0.0919</td>
<td>0.1674</td>
</tr>
<tr>
<td>0.471</td>
<td>0.185</td>
</tr>
<tr>
<td>1.65</td>
<td>2.5</td>
</tr>
<tr>
<td>0.1434</td>
<td>0.2330</td>
</tr>
<tr>
<td>0.648</td>
<td>0.398</td>
</tr>
<tr>
<td>1.65&lt;</td>
<td>1.88</td>
</tr>
<tr>
<td>0.1949</td>
<td>0.2986</td>
</tr>
<tr>
<td>0.807</td>
<td>0.647</td>
</tr>
<tr>
<td>1.65&lt;</td>
<td>1.73</td>
</tr>
<tr>
<td>0.2465</td>
<td>0.3349</td>
</tr>
<tr>
<td>0.969</td>
<td>0.796</td>
</tr>
<tr>
<td>1.65&lt;</td>
<td>1.70</td>
</tr>
<tr>
<td>0.2749</td>
<td></td>
</tr>
<tr>
<td>1.083</td>
<td></td>
</tr>
<tr>
<td>1.65&lt;</td>
<td></td>
</tr>
</tbody>
</table>
## Appendix F

### Computer Program

#### Solution of Similarity Boundary Layer Equations

The basic initial value integration program (DESP) requires that the simultaneous fourth order equations 4.10 and 4.11 be written as a set of simultaneous first order equations. To this end let,

\[
\begin{align*}
    f &= f_1 \\
    f' &= \frac{df_1}{d\eta} = f_2 \\
    f'' &= \frac{df_2}{d\eta} = f_3 \\
    f''' &= \left\{ \beta_0 \left[ \rho_0 \left( \frac{\rho}{\rho_0} \right)^{\frac{2}{3}} - f_3 f_5 - \lambda f_2 P_0 \right] \right\} \lambda / P_0 \\
    G &= f_4 \\
    G' &= \frac{df_4}{d\eta} = f_5 \\
    G'' &= \frac{df_5}{d\eta} \left\{ (x-1) M_0^2 \frac{u_s^2}{H_2 e} \left[ \beta_0 \left( \frac{\rho}{\rho_0} \right)^{\frac{2}{3}} - 2 f_5 f_3 - 2 f_5 \left( \frac{\lambda}{P_0} \right)^{\frac{1}{2}} \right] \right\} / 2 \left( \frac{\lambda}{P_0} \right)
\end{align*}
\]

The following is a list of variables used in the computer program.

<table>
<thead>
<tr>
<th>Program name</th>
<th>Meaning</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>exponent in power law for specific heat = a</td>
</tr>
<tr>
<td>B</td>
<td>exponent in power law for viscosity = b or</td>
</tr>
<tr>
<td></td>
<td>= coefficient of $u_s \rho / \rho_0$ - total continuity equation (4.29)</td>
</tr>
<tr>
<td></td>
<td>= coefficient of $p / p_0$ - total momentum equation (4.30)</td>
</tr>
<tr>
<td>BETA</td>
<td>$\frac{\partial u_s}{\partial x}$</td>
</tr>
<tr>
<td>BUZZ</td>
<td>$(2g)^{\frac{1}{2}} - (2g - \Delta g)^{\frac{1}{2}}$</td>
</tr>
</tbody>
</table>
C = exponent in power law for conductivity \( c \) or

\[ = (2\frac{\kappa}{\rho_0} U_e^2 M_0^2) \left\{ \int_0^{\eta_e} \eta_e^{-\frac{\kappa}{\rho_0} U_e^2 M_0^2} \left( \int_0^{\eta_e} \eta_e^{-\frac{\kappa}{\rho_0} U_e^2 M_0^2} \right) \right\} \]

or

\[ C = \text{term in total momentum equation (4.30)} \]

\[ = p \frac{\rho_0}{\rho_e} U_e x_o M_0^2 \left\{ (2\frac{\kappa}{\rho_0} U_e^2 M_0^2) \int_0^{\eta_e} \eta_e^{-\frac{\kappa}{\rho_0} U_e^2 M_0^2} \left( \int_0^{\eta_e} \eta_e^{-\frac{\kappa}{\rho_0} U_e^2 M_0^2} \right) \right\} \]

where subscript \( o \) refers to values at the last axial step.

COEP1 = \( \gamma_0 \) M_0^2

COEP2 = \( \gamma_0 M_0^2 \)

COEP3 = \( \gamma_0 M_0^2 \)

COEP4 = \( \mu_e \rho_0^2 \mu_e \rho_0^2 \)

DELP3 = \( \Delta f''(o) \), perturbation in derivative of velocity at wall

DELP5 = \( \Delta g''(o) \), perturbation in derivative of enthalpy at wall

DETA = \( \Delta \eta \), stepsize in similarity parameter

DZETA = \( \Delta \xi \), stepsize in transformed axial coordinate

ETA = \( \eta_e \), point where boundary layer growth is assumed to be complete

EXP1 = \( 1/(A + 1) \)

EXP2 = \( (B-A-2)/(A+1) \)

EXP3 = \( (B-1)/(A+1) \)
\[ \text{EXP4} = \frac{(C-A-1)}{(A+1)} \]

\[ \text{ERRUE} = \text{acceptable absolute error} \quad | \frac{U^r_{e-1}}{U^*_e} | \quad \text{at} \quad \eta = \eta_e \]

\[ \text{ERRHE} = \text{acceptable absolute error} \quad | \frac{H^r_{e-1}}{H^*_e} | \quad \text{at} \quad \eta = \eta_e \]

\[ \text{ERPPO} = \text{acceptable absolute difference in two successive iterated values of } \frac{p}{p_0} \quad \text{at } \eta \]

\[ \text{ERRUUE} = \text{acceptable absolute difference in two successive values of } | \frac{U^r_{e-1}}{U^*_e} | \quad \text{at} \quad \eta = \eta_e \]

\[ \text{FUNC}(1,1) = f((r-0)^6) \]

\[ \text{FUNC}(2,1) = f((r-1)^6) \]

\[ \text{FUNC}(3,1) = G((r-1)^6) \]

\[ \text{IGRAL1} = \int_0^{\eta_e} \phi d\eta \]

\[ \text{IGRAL2} = \int_0^{\eta_e} \phi \left( \int_0^\eta \phi \, d\eta \right) \, d\eta \]

\[ \text{IGRAL3} = \int_0^{\eta_e} \phi \, d\eta \]

\[ \text{IGRAL4} = \int_0^{\eta_e} \phi (\eta)^2 \left( \int_0^\eta \phi \, d\eta \right) \, d\eta \]

\[ \text{GAMMA} = \gamma_0 \]

\[ \text{MACH} = \text{M}_e \]

\[ \text{PDIF}(1,1) = \Delta f'(\eta_e) / \Delta f''(0) \]

\[ \text{PDIF}(1,2) = \Delta f'(\eta_e) / \Delta G'(0) \]

\[ \text{PDIF}(2,1) = \Delta G(\eta_e) / \Delta f''(0) \]

\[ \text{PDIF}(2,2) = \Delta G(\eta_e) / \Delta G'(0) \]

\[ \text{PPO} = \frac{p}{p_0} \]

\[ \text{PPOO} = \left( \frac{p}{p_0} \right)_0 = \text{value of pressure ratio from last iteration} \]

\[ \text{PP000} = \frac{p}{p_0} \text{ at last axial step} \]
STAR(1) = \( V^+ \) (obtained by interpolation from equal \( \eta \) intervals to equal \( y^+ \) intervals)

STAR(2) = \( U^+ \) (obtained by interpolation from equal \( \eta \) intervals to equal \( y^+ \) intervals)

STAR(3) = \( T/T_0 \) (obtained by interpolation from equal \( \eta \) intervals to equal \( y^+ \) intervals)

TAIW = \( \theta_\omega^+ \)

TAW1 = \( 1 - TA1W \)

UEO = \( U_e^+ \) value of free stream dimensionless velocity from last iteration

UEOO = \( U_e^+ \) value of free stream dimensionless velocity from last axial step

UE = \( U_e^+ \) value of free stream dimensionless velocity at present step

\( V = V^+ \) radial velocity

\( X = X^+ \)

\( Y(1) = f \)

\( Y(2) = f' \)

\( Y(3) = f'' \)

\( Y(4) = G \)

\( Y(5) = G' \)

YEND1 = \( f(\eta_e) \)

YEND2 = \( f'(\eta_e) \)

YEND3 = \( G(\eta_e) \)

YSRT3 = \( f''(\eta_e) \)

YSRT5 = \( G'(0) \) from guess or correction routine

YST3T0 = \( f''(0)_o \) = value of velocity derivative at wall at last axial step
SUBROUTINE FEOUTI(Y, DY, N, X, STYPE, *)

**********************************************************************

THIS SUBROUTINE PULLS OUT VALUES OF U/UE, F (STREAM FUNCTION)
AND P/HE AT ETA (END OF THE B.L.) FOR USE IN CONVERGENCE CHECK
AND CORRECTIVE PROCEDURE

**********************************************************************

DIMENSION Y(5), DY(5), F(101), FUNC(7, 101)
COMMON F, FUNC, NOENS, ETA, TA1W, DZETA, PR, COEF1, COEF3, EXP1, EXP2, EXP3,
  LEXP4, YEND1, YFND2, YFND4, XEND, UE, ZETA, TWTO, ETA, DFETA, UECO, COEF2, COEF4
  L, TA1W, IGRAL1, IGRAL2, IGRAL3, IGRAL4, IGRAL5, II
REAL IGRAL1, IGRAL2, IGRAL3, IGRAL4, IGRAL5
LOGICAL STYPE
IF (.NOT. STYPE) RETURN
IF (X.EQ.0) RETURN
YEND1 = Y(1)
YEND2 = Y(2)
YEND4 = Y(4)
WRITE (6, 1) YEND1, YEND2, YEND4
1 FORMAT (20H AT ETA=END, Y(1) = , F8.4, 8H Y(2) = , F8.4, 7H Y(5) = , F8.4
 1)
RETURN
END
<

a.

2^6.

o
in
N
O
H

li­
ft
li­
ft
li­
ft

a
k

li­
ft

OO
00

li­
ft
li­
ft
ft
ft

C\J
m

ii

LU
k
<

ft
•»

<X

X

k
—%

z

c
LU
LL

•*

>
—
rv

C
LU
X

C
CM

Cj
Cr:
rc
X)
00

-J
<

X
k

2
<
CL
<

a
-1
<
cc

c
LL
k
2
I—

C*
UJ
k
k

ft
•li­
ft
•li­
ft
•li­
ft
■li­
ft

LU
k
<
c
—J X
<

<

u
LL

k
— i

c
a

2
3
k
2
UJ
2
u
21

cc
a
LL

X
O
CC
G.
>
C.

i/i

<
►— 1
Cl
X
1L— 1
QC
CJ

<r
X
hZ
LU
Cl

O
Z
Uj
O

>

<1
a
Uj
k
>
<
tCL •— i
X
C
t o LL •_.
>— 4 k
_l
LU
X
2
to—
k
>
O

c
cj

>
Lu
i/)
►— i

QC
X
lX
or
»*■»

t—
<1
OO

li­
ft

•*
<
t—
k
U j to—
to* •*
in
-j
<

c..

CL

k

». o
< r to— 1
•*
k
ro O
k
UJ L U
W.
to* N J M
u
•j
to* <
2
m
C
t
o
*
L
U
C
L
3
3
U- <
3
3
C'
•> c: pH
•* *— « p 4
«> CL
<
C
c

z

ft
ft

o

UUJ
o
o
c
to* k
►
3
CL

pH
o
-j
f"l <
p. C L

li­
ft
li­
ft
li­
ft

ft
li­
ft
li­

<
k
UJ

►

-j
<
a
c
to-

li­
ft
ft
li­
ft

ft
li­
ft
li­
ft

z z

CL

o
u
in

ft
li­
ft
li­
ft

li­

QC
LU

X
X

X
UJ

ft
li­
ft
li­

cc
o
x

<M

a
X

a
c
X
a
UJ LU
•> 3
to*
m

ft
li­

X
I—

nj

UJ o
to* U J
to*

•li­
ft
■li­

LU

a:
3
LO

li­
ft

loo

a.

3
LO
z
1—4 LLI
k
G
*— »
2

li­

G
_J

CL

a
LL)

X
UJ

o
UJ o
to* to*

ft
•li­
ft
ft
ft
•li­
ft
•li­
ft
•Si­
ft
■li­
ft

LO
2” k
3
a
M
C
k
k
<
2
*— 4
3

Q.
X

■li­
ft
•li­
ft

LL
• ►—

L0

ft
•li­
ft
ft
ft
ft

h-

o
LL

UJ
k
LU

ft
•li­
ft

LU
i:
_

00
to-H

ft
li­
ft
•ft
•ft

li­
ft
•li­
ft
■li­

t—
3

O
00

c l

li­
ft
li­
ft
li­
ft
ft
ft

>•
c
••

CL

LO

ft
ft

Uj
CL
>
hCL
1/0

k
00

ft
li­

ft
•ft
ft

Q

UJ
00

CL

ro

ft
ft

3

li­
ft

r-

UJ
a

li­
ft
li­
ft

CO

z
to—

2
UJ
a
Q<
X

p»

ft
ft
ft
ft
•li­
ft
ft
•li­

UJ
CL
>
1Q.
OO

UJ
a
k
k
a
<s>

p— i *—
2
k
to* •* U J
c
p j C\J <
X
w
to*
-J k
U J -t
LL <
*» a
CC c
»■ 2
c
IP ► - 0 0 L U
w
to- 2
>top-H LL
>
►

CL

O

>

•> 3
U•*
cc

2

9C

k

►— *

<■
LU
*—

a

LO,

(S

— 1

z
O

z

m
_j
<

i.L

z

2
c
• U J _l 5
2L <
5.
LL. K-l L U C
*— < C
CC L J
2

CL

«
>

««■

to*
(XI

w

-J
<

>
*

rr.

c.
2

G
G
—*
O
toH
•
P**4 < L
CL
<
o
o
•
z" C
k
U j to— *— G
to* II LL
>
to* * H 2
•
Cl X
<}■
<
G *—*
Q
X
k
2 LL
to- »— to—
u.
—M

p

2
a: a
5.
o

2

CL
G
to—
+
a
2
IU

k
II
Cl
?
LU
k

wk

k
■»>

•K

-p*
(XI
>-

(X
—»
k

c

o

UJ —

(X »— k
C'j *—* •Tf #
Cj k > C o
•
•
k
nT
CJ O
• + •f
a o
H
(X
o
— pH + -J -J
<
1 ^L. <a <'
• U a cc.
k
LU o IL o c
« LU X to­ to­
• u
ll ll
G
LU OL II pH CvJ
• to— 2 — J
X x : U) < <
*w — * UJ r ' cr:
LL LL X C O
— to—
to— to-* G ►
(XI

pH k

3
<
k
+
— I 3

nj

OJ fV

#
—
— *

2
LU
k
•ft
—

#— 4—

X
UJ

CL
O
to*—

C\j C L
Q
G
to—
► 2
to*
UJ
G
— *J
2
>

-J C
<
2

LO
w

_J

c

•>

<

&

#

to

•

•f

+
m
-j

<
<
CL
CL
c
o
to­
to­
ll »— ll m
cn Oil.
_j 1 — L .
<
II <r k
cr

X

c

C to— C- C '
►
— c
►-

o

to­
ll

•
o
II
OL
G

y

UJ

to— X
x: G 1

1

IGOAL 1 = 0 . 0
■ TFMP=IM0PN/2
IGRAL2=0.0
IGP AL 3 = 0 . 0
I G p A L4 = 0 .0

-O
O
S

LU

>
—J
O
UJ

U

O

U

G

G

G

G

G

U

G

LU

—J

o
o

o

(Xm
oo
o CJ
oo

*
f
rIT.
oO
oO
oo

*c
o
o
c

t-OOff O'-'tVifn'tlTivOh-Or. C T O
i CO

i*~4

«— 1 p—

r— 1 r— 4 r—

4 — •* *” 4 (\j

O O O C O C C ' C C O O O C J O
O C . O C O O O C C O < ~ I O O O

002 1
0022
0023
0024
0025

>

t
o
—
2
<1
CC
k
CL
c:


M=1
2 IGRL1=IGRL1+INGRN
   CHECK=CHECK+Y(2)
   IGRL2=Y(2)*TEMP+IGRL2
   IGRL3=IGRL3+Y(2)*Y(2)
   IGRL4=IGRL4+Y(2)*Y(2)*TEMP
   GO TO 5
3 IGRL1=IGRL1+2.0*INGRN
   CHECK=CHECK+2.0*Y(2)
   IGRL2=IGRL2+2.0*TEMP+Y(2)
   IGRL4=IGRL4+2.0*Y(2)*Y(2)*TEMP
   IGRL3=IGRL3+2.0*Y(2)*Y(2)
   KIK=-KIK
5 M=M+1
0040 FUNC(1,M)=Y(1)
0041 FUNC(2,M)=Y(2)
0042 FUNC(3,M)=Y(4)
0043 IF(X.NE.ETA)GO TO 6
0044 CHECK=CHECK*DETA/3.0
0045 WRITE(6,100)CHECK,YEND1
0046 100 FORMAT(9H CHECK =,F8.6,PH CHECK =,F8.6)
0047 IGRL1=IGRL1*DETA/3.0
0048 IGRL2=IGRL2*DETA*DETA/3.0
0049 IGRL3=IGRL3*DETA/3.0
0050 IGRL4=IGRL4*DETA*DETA/3.0
0051 6 RETURN
0052 END
SUBROUTINE FEVAL(Y,X,DY)

******************************************************************************

THIS SUBROUTINE EVALUATES THE RIGHT HAND SIDE OF THE BOUNDARY LAYER EQUATIONS AFTER CONVERSION TO A SET OF SIMULTANEOUS FIRST ORDER DIFFERENTIAL EQUATIONS AND IS CALLED BY DESP

******************************************************************************

DIMENSION Y(5),DY(5),F(101),FUNC(3,101),FPRM(3)
COMMON F,FUNC,NOFNS,RETA,TA1W,TA2T,ETA,PR,COE1,COE2,COE3,EX5,EXP1,EXP2,EXP3, EXP4,YEND1,YEND2,YEND3,YEND4,XEND,UE,ZFTA,TWTD,TETA,DETA,UECO,COE4,COE5,COE6, 1,TA1W,IGR1L,IGR2L,IGR3L,IGR4L,IGR5L,II
REAL IGR1L,IGR2L,IGR3L,IGR4L,IGR5L

DY(1)=Y(2)
DY(2)=Y(3)
HARCH=TA1W+TA2T*Y(4)
DY(3)=ETA*(Y(2)+Y(3)-HARCH**EXP1)-Y(3)*Y(1)-2.0*PR*EXP3*HARCH**EXP1*TA1W*Y(3)*Y(5))/((2.0)*PR*HARCH**EXP3)

DY(4)=Y(5)

DY(5)=(COE1*UE*UE*(BETA*HARCH**EXP1*Y(2)-2.0*PR*HARCH**EXP3*Y(3)*Y(3))-2.0*EXP4*HARCH**EXP4*EXP4-1.0)*TA1W*Y(5)*Y(5)-Y(5)*Y(1))/(2.0*HARCH)**EXP4)

RETURN
END
**MAIN PROGRAM**

THIS PROGRAM INTEGRATES THE SIMILARITY DIFFERENTIAL EQUATIONS FOR LAMINAR FLOW OF A GAS IN A TUBE WITH TEMPERATURE DEPENDENT PROPERTIES.

INPUT VARIABLES ARE NAMED AS follows:

- $\gamma$: RATIO OF SPECIFIC HEATS $CP/\text{CV}$
- $M$: INLET MACH NO.
- $T_{W1}$: INLET WALL TO BULK TEMPERATURE RATIO
- $P_{R}$: PRANDTL NO.
- $a$: EXPONENT IN POWER LAW FOR SPECIFIC HEAT
- $b$: EXPONENT IN POWER LAW FOR VISCOSITY
- $c$: EXPONENT IN POWER LAW FOR CONDUCTIVITY
- $\Omega$: AXIAL MESH STEP IN TRANSFORMED COORDINATE SYSTEM
- $\eta$: VALUE OF TRANSFORMED $Y$ VARIABLE AT WHICH THE VELOCITY AND ENTHALPY BOUNDARY LAYERS ARE ASSUMED TO REACH THEIR FREE STREAM VALUES
- $\mu$: NO. OF AXIAL MESH POINTS AT WHICH SOLUTIONS WILL BE COMPLETED
- $F_{R}$: ACCEPTABLE ABSOLUTE DIFFERENCE BETWEEN NON DIMENSIONALIZED AXIAL VELOCITY $U/U_{1}$ AND 1 AT ETA
- $F_{R}$: ACCEPTABLE ABSOLUTE DIFFERENCE BETWEEN NON DIMENSIONALIZED ENTHALPY AND 1 AT ETA
- $F_{R}$: ACCEPTABLE ABSOLUTE DIFFERENCE IN TWO SUCCESSIVE VALUES OF THE PRESSURE RATIO AT ZETA
REAL MISS1, MISS2, NETA, IGRAL1, IGRAL2, IGRAL3, IGRAL4, IGRAL5, MACH
REAL SILT
DIMENSION F(101), FLAP(3), GLAP(3), PDIF(2, 2), ERR(3), FUNC(3, 101), Y(101)
1, DN(100), STAR(101), Y(101)
COMMON F, FUNC, NOFNS, RETA, TAW, DZETA, PR, COEF1, COEF2, EXP1, EXP2, EXP3,
1 EXP4, YFNC1, YEND2, YEND4, XEND, UE, ZETA, TW0, ETA, DETA, UE2, COEF2, COEF4
1, TAW1, IGRAL1, IGRAL2, IGRAL3, IGRAL4, IGRAL5, II
EXTERNAL FEVAL, AF, FFUT1, FFUT2
DOUBLE PRECISION B, C, PPOUE, PPD
LOGICAL FIRST, SPP
READ(5, 1) GAMMA, A, B, C, PR
1 FORMAT(5F6.4)
 READ(5, 2) MACH, TW0
2 FORMAT(2F4.2)
 READ(5, 3) DZETA, M, N, XEND
3 FORMAT(6.2, 213, F5.2)
 READ(5, 8) FRRUE, ERRHE, ERPP0, ERUUE
8 FORMAT(4F8.5)
C
WRITE(6, 37) GAMMA, A, B, C
37 FORMAT(9H GAMMA = ,F8.4, 5H A = ,F8.4, 5H B = ,F8.4, 5H C = ,F8.4)
 WRITE(6, 38) MACH, TW0, PR
38 FORMAT(12H MACH NO. = ,F8.4, 8H T/TO = ,F8.4, 6H PR = ,F8.4)
 WRITE(6, 39) DZETA, M, ETA
39 FORMAT(9H DZETA = ,F8.4, 17H # AXIAL STEPS = ,14, 21H ETA (FREE STRE
1AM) = ,F8.4)
40 FORMAT(9H ERRUE = ,E9.3, 9H ERRHE = ,E9.3, 9H ERPP0 = ,E9.3, 9H ERUUE
 WRITE(6, 40) FRRUE, ERRHE, ERPP0, ERUUE
 TA1W = TW0** (A + 1.0)
 DETA = 0.10
 DWS = 1.0 / 320.0
 ETA = XEND
 II = ETA / DETA + 2

***************************************************************
CALCULATE COEFFICIENTS AND EXPONENTS WHICH ARE FREQUENTLY USED
IN CALCULATIONS
***************************************************************
0030    TAW1=(1.0-TAW1)
0031    PPON=1.00
0032    COEF1=MACH*MACH
0033    COEF2=ГАММА*COEF1
0034    COEF3=1.0+ГАММА*COEF1
0035    IF(TAW1.EQ.0.0)GO TO 700
0036    COEF1=(ГАММА-1.0)*COEF1*(A+1.0)/TAW1
0037    GO TO 701
0038
0039
0040    700 COEF1=0.0
0041    701 CONTINUE
0042    COEFF4=TWTC**(B-1.0)
0043    FXP1=1.0/(1.0+A)
0044    FXP2=(B-A-2.0)/(A+1.0)
0045    FXP3=(B-1.0)/(A+1.0)
0046    FXP4=(C-A-1.0)/(A+1.0)
0047    NN=5
0048    H=0.10
0049    MM=0
0050    ZETA=0.0
0051    UFN0=1.0
0052    XX=0.0
0053    XRAL5=0.0
0054    ETA=0.0
0055    R1=0.332
0056    33 XSTART=0.0
0057    ETA=XEND
0058    YSPT3=0.332
0059    IF(TAW1.EQ.0.0)GO TO 702
0060    702 YSPT5=C.0
0061    703 CONTINUE
0062    5 DELF3=0.001
0063    DELF5=0.001
0064    KK=0
0065    IF(MM.GE.4)DELF3=0.0001
0066    IF(MM.GE.4)DELF5=0.0001
6 Y(3)=YSRT3
0068 Y(5)=YSRT5
0069 ETA=XEND
0070 XSTART=0.00
0071    Y(1)=0.0
0072    Y(2)=0.0
0073    Y(4)=0.0
0074 CALL DESP(ETA,FEVAL,NN,Y,XSTART,ETA,H,ERRUE,AE,FEOU1,800)

******************************************************************************

C CALCULATE PARTIAL DERIVATIVES OF U/UE AND H/HE AT THE OUTER EDGE
C OF BOUNDARY LAYER WITH RESPECT TO D(U/UE)/D(ETA) AND D(H/HE)/D(ETA)
C
******************************************************************************

C FRR1=ABS(1.0-YEND2)
0075 FRR2=ABS(1.0-YEND4)
0076 IF(ERR1.LE.ERRUE.AND.ERR2.LE.ERRHE)GO TO 9
0077 IF(KK.EQ.0)GO TO 10
0078 IF(KK.FQ.1)GO TO 11
0079 FLAP(3)=YEND2
0080 GLAP(3)=YEND4
0081 GO TO 12
0082 10 YSRT3=YSRT3+DELF3
0083 FLAP(1)=YEND2
0084 KK=KK+1
0085 GLAP(1)=YEND4
0086 GO TO 6
0087 11 YSRT5=YSRT5+DELF5
0088 FLAP(2)=YEND2
0089 GIAP(2)=YEND4
0090 KGAP(2)=YEND2
0091 GO TO 6
0092

******************************************************************************

C SET UP CORRECTION MATRIX TO CALCULATE NEW VALUES OF WALL DERIVATIVES
C
******************************************************************************
0095 \texttt{PDIF}(2,1)= (GLAP(2) - GLAP(1))/DELF3
0096 \texttt{PDIF}(2,2)= (GLAP(3) - GLAP(2))/DELF5
0097 DO 13 I=1,3
0098 \texttt{MISS1}=ABS(1.0-FLAP(I))
0099 \texttt{MISS2}=ABS(1.0-GLAP(I))
0100 \texttt{ERR}(I)=SQRT(\texttt{MISS1}**2.0*\texttt{MISS2}**2.0)
0101 I=1
0102 IF(\texttt{ERR}(I).GE.\texttt{ERR}(2)) I=2
0103 IF(\texttt{ERR}(I).GE.\texttt{ERR}(3)) I=3
0104 IF(I.EQ.1.OR.I.EQ.2) YSRT5=YSRT5-DELF5
0105 IF(I.EQ.1) YSRT3=YSRT3-DELF3
0106 FLAP(1)=FLAP(I)
0107 GLAP(1)=GLAP(I)
0108 KK=1
0109 DET=PDIF(1,1)*PDIF(2,2)-PDIF(1,2)*PDIF(2,1)
0110 DELF3=(PDIF(2,2)*(1.0-FLAP(I))-PDIF(1,2)*(1.0-GLAP(I)))/DET
0111 DELF5=(PDIF(1,1)*(1.0-GLAP(I))-PDIF(2,1)*(1.0-FLAP(I)))/DET
0112 YSRT3=YSRT3+DELF3
0113 GO TO 6
0114 Y(1)=0.0
0115 Y(2)=0.0
0116 Y(3)=YSRT3
0117 Y(4)=0.0
0118 Y(5)=YSRT5
0119 KK=0

C**********************************************************************
C
C GET INTEGRATED BOUNDARY LAYER PROFILES
C**********************************************************************
C
0120 CALL DESP(DETA,FFVAL,NN,Y,XSTART,ETA,H,ERRUF,AE,FEDOUT2,6800)
0121 IF(MM,NE,0)GO TO 24
0122 ZETA=DZETA
C**********************************************************************
C
SOLVE INTEGRATED CONTINUITY AND MOMENTUM EQUATIONS
C FOR U/UF AND P/PD
24  B=2.0*(2.0*ZETA)**0.5*(YEND1-IGRAL1)-1.0
0123     C=2.0*ZETA*(IGRAL1**2.0-2.0*IGRAL2)
0124     PPOUF=(-B+DSRT(P**2.0-4.0*C))/2.0
0125     IF(MM.EQ.0)YST3TO=YSRT3
0126     BUZZ=(2.0*ZETA)**0.5-(2.0*(ZETA-DZETA))**0.5
0127     B=-COEF3+4.0*P*COFF2*COEF4*(IGRAL5+BUZZ*(YST3TO+YSRT3)*UE00/4.0)
0128     C=PPOUF-COEF2*(2.0*(2.0*ZETA)**0.5*(IGRAL3-IGRAL1)+PPOUF+PR*COEF4*
0129     1BUZZ*(YST3TO+YSRT3))=COFF2*(2.0*ZETA)*(IGRAL1**2.0-2.0*IGRAL4)
0130     PPO=(-B+DSRT(B**2.0-4.0*C))/2.0
0131     U=U=PPPOF/PPPO
0132     WRITE(6,501)U,PPPO
0133     50 FORMAT(8H U/UE = ,F9.5, 8H P/PO = ,F9.5)

0134     IF(FIRST.OR.MM.EQ.0) GO TO 18
0135     ERR5=ABS(U-E=U)
0136     ERR6=ABS(PPPO-PPPO)
0137     IF(ERR5.EQ.ERR6.AND.ERR6.LE.ERRPO) GO TO 19
0138     U=U=U
0139     PPPO=PPPO
0140     RETA=2.0*ZETA/UE*(UE-UE00)/DZETA
0141     GO TO 5
0142     18 UEC=U
0143     PPPO=PPPO
0144     FIRST=.FALSE.
0145     RETA=2.0*ZETA/UE*(UE-UE00)/DZETA
0146     IF(MM.EQ.0) GO TO 20
**** IF THIS IS X+=0, THEN BOUNDARY LAYERS CAN ONLY BE PRINTED AS ****
FUNCTIONS OF ETA SINCE IN THE Y+-X+ SYSTEM, THE BOUNDARY LAYER ****
THICKNESS IS THEORETICALLY ZERO ****
****
0147 GO TO 5
0148 20 MM=1
0149 WRITF(6,21)
0150 21 FORMAT(3EH ZETA = 0.0 UF/U0=1.0 P/PO=1.0 X=0 //)
0151 WRITF(6,22)
0152 22 FORMAT(35H ETA Y(1) U/UE H/HE)
0153 ETA=0.0
0154 DO 23 I=1,II
0155 WRITF(6,37)ETA, FUNC(1,I),FUNC(2,I),FUNC(3,I)
0156 32 FORMAT(3X,F5.2,4X,F8.5,2(2X,F8.5))
0157 ETA=ETA+DETA
0158 F(J)=FUNC(1,J)
0159 CONTINUE
0160 YST3TO=YRT3
0161 GO TO 33
0162 19 XX=XX+DZETA/(UF+U0)*((PPCN+PPD)
0163 WRITE(6,25)ZETA,UE,PPD,XX
0164 25 FORMAT(1H ZETA = ,F8.5,8H U/UE = ,F8.5,8H P/PO = ,F8.5,6H X = ,F8
1.5)
0165 ETA=2.0*ZETA*(UF-U00)/(DZETA*JE)
0166 NETA=0.0
0167 ON(1)=0.00
0168 WRITF(6,28)
0169 DO 26 I=1,II
0170 !=FUNC(2,I)*UF
0171 TEMP=1TA1W+TAW1*FUNC(3,I)*#EXP1
0172 X=NETA
DO 67 J=1,II
DEX=X/DETA-0.00001
IND=DEX
IND=IND+1
LKJ=II-3
IF(IND.GF.LKJ)IND=LKJ
SILT=IND*DETA-DETA
IF(IND.LF.1)GO TO 60
IND=IND+1
FY=-(X-SILT-DETA)*(X-SILT-2.0*DETA)*(X-SILT-3.0*DETA)*F(IND-1)/6.0
1*(X-SILT)*(X-SILT-2.0*DETA)*(X-SILT-3.0*DETA)*F(IND)/2.0-(X-SILT)*
1*(X-SILT-DETA)*(X-SILT-3.0*DETA)*F(IND+1)/2.0+(X-SILT)*(X-SILT-DETA)
1)*(Y-SILT-2.0*DETA)*F(IND+2)/6.0
FY=FY/DETA**3.0
GO TO 71
60 FY=-X*(X-2.0*DETA)*F(2)/DETA**2.0
FY=FY+Y*(X-DETA)*F(3)/(2.0*DETA**2.0)
71 CONTINUE
V(I)=TEMP*UE*(2.0*ZETA)**0.5*((NFTA*(1.0-BFTA)*FUNC(2,I)-FUNC(1,I)
1)/(2.0*ZETA))
WRITE(6,31)DN(I),NETA,FUNC(2,I),U,TEMP,FUNC(3,I),V(I)
DN(I+1)=DN(I)+(2.0*ZETA)**0.5*DETA/(UE*PPO)*TEMP
NETA=NFTA+DETA
26 CONTINUE
31 FORMAT(1X,F6.4,F8.4,1X,F9.6,4(3X,F9.6))
28 FORMAT(3X,3H Y,4X,5H NETA,3X,5H U/UE,7X,5H U/UO,7X,5H T/T0,7X,5H
1/H/HE,7X,3H V+/+J)
LAP=K-1
IF(MN.LT,LAP)GO TO 400
YY=0.00
KLAPO=II-2
NDMON'S=1
406 DO 401 K=1,KLAP
400 KULP=K+1
IF(YY.GF.DN(K).AND.YY.LT.DN(KULP))GO TO 402
GO TO 401
I = K
GO TO 407
CONTINUE

************************************************************

SINCE OUTPUT FROM INTEGRATION ROUTINE IS AT EQUAL INTERVALS
IT IS NECESSARY TO INTERPOLATE FOR VALUES OF THE DEPENDENT
VARIABLES AT EQUALLY SPACED Y = 1-R/R0 VALUES. USE PARABOLA FIT
THRU LAST AND TWO FORWARD TABULATED VALUES OF DEPENDENT VARIABLE

************************************************************

DO 403 J = 2, 3
STAR(J) = (YY - DN(I+1)) * (YY - DN(I+2)) * FUNC(I, I) / ((DN(I) - DN(I+1)) * (DN(I+1) - DN(I+2)))
 + (YY - DN(I+2)) * (YY - DN(I)) * FUNC(I+1) / ((DN(I+1) - DN(I)) * (DN(I+2) - DN(I+1)))

403 CONTINUE

STAR(1) = (YY - DN(I+1)) * (YY - DN(I+2)) * V(I) / ((DN(I) - DN(I+1)) * (DN(I+1) - DN(I+2)))
 + (YY - DN(I+2)) * (YY - DN(I)) * V(I+1) / ((DN(I+1) - DN(I)) * (DN(I+2) - DN(I+1)))

STAR(2) = UE * STAR(2)
STAR(3) = (TA1W + TAW1 * STAR(3)) * EXP1

PUNCH PROFILES FOR USE IN FINITE DIFFERENCE PROGRAM

WRITE(9#, 404) STAR(2), STAR(1), STAR(3)
IF (YY .GT. DN(KLAP)) GO TO 405
NOMCNS = NOMCNS + 1

YY = YY + DNS
404 FORMAT(F11.7, F11.4, F11.7)

GO TO 406

CONTINUE

WRITE(9#, 408) X, PPO

FISH = (PPO * UE - PPO * UE00) / (2.0 * PPO * DZETA)
WRITE(9#, 410) NOMCNS, UE, FISH

CONTINUE

410 FORMAT(I4, 2F8.5)
258.

```
0226  409 CONTINUE
0227  IF(MM.EQ.M) GO TO 30
0228  IG=AL5=IGAL5+0.25*(UE+UENO)*(YST3TO+YSRT3)*(ZETA**0.5-(ZETA-DZETA)
0229   I)**0.5)
0230  DO 248 J=1,II
0231     F(J)=FUNC(1,J)
0232     248 CONTINUE

C
c
C     ************************************************************************************************************************************
C     INCENT AXIAL DISPLACEMENT
C     AND INITIALIZE PROFILE DERIVATIVES
C
C     ************************************************************************************************************************************

0232   ZETA=ZETA+DZETA
0233   YST3TO=YST3T3
0234   FIRST=.TRUE.
0235   MM=MM+1
0236   PP000=PP0
0237   UENO=UE
0238   GO TO 24
0239   800 CALL ERROR
0240   30 STOP
0241   END
```
Appendix G

This appendix is a listing of the data reduction program described in Chapter 5. Major I/O and intermediate variables and different stages of the calculations are identified through use of comment cards.
VARIABLE PROPERTY LAMINAR FLOW DATA REDUCTION
FINALIZED 6/18/71
ALL CUPVEFFTS ARE OF POLYNOMIAL THIRD DEGREE TYPE A+BX+CX2+DX3
ALL THERMOCOUPLE CORRELATIONS ARE FOR 32.2 DEG. F.
REFERENCE JUNCTION TEMPERATURE

THE FOLLOWING SUBSCRIPTS APPLY AS FOLLOWS,
1 REFERS TO CU-CON THERMOCOUPLE OUTPUT, RANGE 200 TO 600 DEG F
(DEG. F. VS. MV)
2 REFERS TO CR-AL THERMOCOUPLE OUTPUT, RANGE 200 TO 1880 DEG. F.
(DEG. F. - MV)
3 REFERS TO CALIBRATION OF UPSTREAM CR-AL THERMOCOUPLE
(DEG. F. (TRUE) VS. DEG. F. (MEASURED))
4 REFERS TO CU-CON THERMOCOUPLE OUTPUT, RANGE 200 TO 600 DEG F.
(DEG. F. - MV)
5 REFERS TO TEMPERATURE CORRECTION FACTOR FOR LAMINAR FLOW ELEMENT
(CORRECTION FACTOR VS. DEG. F.) RANGE 50-120 DEG. F.
ALL TRANSPORT PROPERTY CORRELATIONS OF DATA IN NBS 564
6 REFERS TO ABSOLUTE VISCOSITY RATIO VERSUS TEMPERATURE
MU/MU0 VS. DEG. K
RANGE 200-1800 DEG. F.
7 REFERS TO THERMAL CONDUCTIVITY RATIO K/K0 VS. DEG. K.
RANGE 200-1800 DEG. F. K0=0.01395 BTU/HR FT DEG. F.
RANGE 200-1800 DEG. F.
8 REFERS TO SPECIFIC HEAT RATIO CP/R VS. DEG. K.
RANGE 200-1800 DEG. F. R=GAS CONSTANT=0.0685 BTU/LBM DEG. K
K=NO. OF RUN FROM WHICH TAPE TEST WAS TAKEN
II=NO. OF RUN
JJ=0 MEANS BLUE MAN. FLUID AT MICROMANOMETER
JJ=1 MEANS RED MAN. FLUID AT MICROMANOMETER
KKK=0 MEANS PVUT BOUNDARY CONDITION
KKK=1 MEANS UVT BOUNDARY CONDITION

**************************************************************************************************
DIMENSION A(9), B(9), C(9), D(9), COND(12), XDC(6), XDP(7), DELP(8), TA(12 1,2), TT(12,2), IREAD(6), TW(12), FLUX(12), DELT(12), TB(7), PHI(4,4), XDP( 18), XXDC(6), RHS(4), PDIV(7), X(7), AAA(100), ANS(4), EFF(2,8)

EQUIVALENCE(PHI, AAA)

DO 1 I=1,8

1 CONTINUE

100 FORMAT(4F12,7)

************************************************************************************

READ IN CALORIMETER CONDUCTANCES BEGINNING WITH EXIT BOTTOM
CALORIMETER TO INLET TOP CALORIMETER (BTU/HRFT2 MV(CU-CON))

************************************************************************************

READ(5,101)(COND(I),I=1,12)

101 FORMAT(4F8.2)

************************************************************************************

READ IN X/D RATIOS WHERE CALORIMETERS ARE LOCATED BEGINNING WITH CALORIMETER CLOSEST TO EXIT

************************************************************************************

READ(5,102)(XXDC(I),I=1,6)

102 FORMAT(3F7.3)

************************************************************************************

READ IN X/D RATIOS WHERE PRESSURE TAPS ARE LOCATED BEGINNING WITH TAP CLOSEST ENTRANCE

************************************************************************************

READ(5,157)(XDP(I),I=1,7)

157 FORMAT(F7.3)

507 READ(5,160)II, JJ, KKK, K

WRITE(6,301)

301 FORMAT(1H1)

160 FORMAT(4I4)

IF(II.GT.500)GO TO 508
READ IN TEST DATA AS FOLLOWS
PATM=ATMOSPHERIC PRESSURE (IN. HG)
PDM=PRESSURE DIFFERENTIAL ACROSS LAMINAR FLOW ELEMENT (IN. H2O)
TFM=OUTPUT FROM CU-CON THERMOCOUPLE LOCATED AT FLOW ELEMENT
P1=PRESSURE AT STATIC TAP #1 (IN. HG) LEFT COLUMN
P2=PRESSURE AT STATIC TAP #1 (IN. HG) RIGHT COLUMN
TUP=OUTPUT FROM UPSTREAM CR-AL THERMOCOUPLE (MV)
TBULK = OUTPUT FROM THERMOCOUPLE AT EXIT

READ(5,103)PATM,PDM,TFM,TBULK,P1,P2,TUP
103 FORMAT(F6.2,F6.3,2F7.4,2F5.2,F7.3)

READ IN STATIC PRESSURE DROP DATA

IF(KKK)500,500,501
500 WRITE(6,505)II
501 WRITE(6,506)II
504 FORMAT(4F6.3)
505 FORMAT(/10X,35H PARABOLIC INLET VELOCITY TEST NO. ,I3)
506 FORMAT(/10X,35H SIMULTANEOUS DEVELOPMENT TEST NO. ,I3)
502 CONTINUE
WRITE(6,6)K
6 FORMAT(10X,30H TARE TEST TAKEN FROM RUN NO. ,I3)

READ IN TARE TEST THERMOCOUPLE READINGS
C READ(5,104)(TA(I,1),TA(I,2),I=1,12)
C ***********************************************
C READ IN THERMOCOUPLE READING FOR TEST WITH GASFLOW (MV)
C ***********************************************
C READ(5,104)(TT(I,1),TT(I,2),I=1,12)
C ECHO CHECK
C 104 FORMAT(2F7.4)
C WRITE(6,805)
C 805 FORMAT(10X,30H TARE THERMOCOUPLE OUTPUT (MV))
C WRITE(6,808)
C DO 806 I=1,6
C 806 CONTINUE
C K=I+6
C WRITE(6,807)I,TA(I,1),TA(I,2),K,TA(K,1),TA(K,2)
C 807 FORMAT(10X,2(2X,I3,2(3X,F8.4)))
C 808 FORMAT(10X,5H 1,3X,8H TA(I,1),4X,8H TA(I,2),5H I ,2X,8H TA(I, 11),4X,8H TA(I,2) /
C 809 FORMAT(10X,30H TEST THERMOCOUPLE OUTPUT (MV))
C WRITE(6,550)
C 550 FORMAT(10X,5H 1,3X,8H TT(I,1),4X,8H TT(I,2),5H I ,2X,8H TT(I, 11),4X,8H TT(I,2) /
C DO 810 I=1,6
C 810 CONTINUE
C K=I+6
C WRITE(6,807)I,TT(I,1),TT(I,2),K,TT(K,1),TT(K,2)
C 811 FORMAT(10X,42H DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = F6.3)
C WRITE(6,912)TFM,TRULK
C 912 FORMAT(10X,23H FLOWMETER TEMP (MV) = F7.4,23H RULK EXIT TEMP (MV) 1 = F7.4)
C WRITE(6,814)P1,P2
C 814 FORMAT(10X,27H INLET PRESSURE MAN , LEFT ,F5.2,7H RIGHT ,F5.2,7H IN. HG)
C WRITE(6,615)TUP
C 815 FORMAT(10X,25H INLET RULK TEMP CR-AL = F7.3)
0063 IF(KK.EQ.0)GO TO 817
0064  WRITE(6,819)DELP(1),DELP(2),DELP(3)
0066  WRITE(6,820)DELP(4),DELP(5),DELP(6)
0068  WRITE(6,821)DELP(7)
0069  821 FORMAT(10X,9H P2-P7 = ,F6.3)
0070  GO TO 822
0071  817 WRITE(6,823)DELP(1),DELP(2),DELP(3)
0072  WRITE(6,824)DFL(4),DELP(5),DELP(6)
0075  822 CONTINUE
0076  IF(JJ.EQ.0)GO TO 825
0077  WRITE(6,826)
0078  GO TO 827
0079  825 WRITE(6,828)
0080  827 CONTINUE
0081  826 FORMAT(10X,32H RED MANOMETER FLUID SP GR 0.826)
0082  828 FORMAT(10X,33H BLUE MANOMETER FLUID SP GR 0.797)

******************************************************************************

C
C CALCULATE MASS FLOW RATE , MACH AND REYNOLDS NUMBERS
C QMAS=CORRECTED MASS FLOW RATE (LB/MI)
C TFM=GAS BULK TEMPERATURE AT FLOWMETER

******************************************************************************

C
C TFM=A(4)+B(4)*TFM+C(4)*TFM**2.+D(4)*TFM**3.0
QMAS=0.0751*PFM*4.37/4.00*(A(5)+B(5)*TFM+C(5)*TFM**2.+D(5)*TFM**3)

******************************************************************************

C
C CALCULATE INLET REYNOLDS NUMBER
C TIN=INLET GAS TEMPERATURE=CORRECTED UPSTREAM ADIABATIC DEVELOPMENT
C SECTION TEMPERATURE
C TUP=UPSTREAM MIXING CHAMBER GAS TEMPERATURE (DEG. F.)

******************************************************************************
TUP=A(2)+B(2)*TUP+C(2)*TUP**2.0+D(2)*TUP**3.0
TIN=A(3)+B(3)*TUP+C(3)*TUP**2.0+D(3)*TUP**3.0
WRITE(6,829)TIN
TIN=TIN+459.7
SPEC=671.0/TIN
829 FORMAT(10X,30H INLET TEMPERATURE (DEG. F) = ,F9.2)
TAP=TIN/1.8
VISC=0.00001153*(A(6)+B(6)*TAP+C(6)*TAP**2.0)
TBULK=A(1)+B(1)*TBULK+C(1)*TRULK**2.0+D(1)*TRULK**3.0

******************************************************************************

CALCULATE MACH NO. AT INLET
DENSGAS DENSITY AT INLET (LBM/FT3)
UIN=GAS VELOCITY AT INLET (FT/SEC)

******************************************************************************

DFNS=0.071*(PATM+P1+P2)/29.92*559.7/TIN
UIN=576.0*OMAS/(10.294*2.0*3.1415*60.0*DENSG)
V*MACH=35.4*OMAS/(DFNS*SORT(2405.0*TIN))
WRITE(6,106)SPEC,V*MACH
106 FORMAT(10X,G9.3,12H MACH NO. = ,F6.3)
CSURP=0.0685*(A(8)+B(8)*TAP+C(8)*TAP**2.0+D(8)*TAP**3.0)
COND=0.01395*(A(7)+B(7)*TAP+C(7)*TAP**2.0+D(7)*TAP**3.0)
PR=VISC*CSURP/COND*3600.0
REYN=4.0*OMAS/(60.0*3.1414*VISC*0.294)*12.0
WRITE(6,300)PR,REYN
300 FORMAT(10X,G8.6,8H PR = ,F8.8H REYN = ,F8.1/

******************************************************************************

READ IN CALORIMETERS WHICH ARE BELIEVED TO BE PROVIDING SPURIOUS
OUTPUT, N=NUMBER OF SUCH CALORIMETERS

******************************************************************************

READ(5,108)N
108 FORMAT(14)
TF(N)200,201,200
200 CONTINUE
DO 11 I=1,N
READ(5,108)IRAD(I)
11 CONTINUE
201 CONTINUE

******************************************************************************
C C C
C R1=POSITION(RADIUS) OF INNER THERMOCOUPLE
C R2=POSITION (RADIUS) OF OUTER THERMOCOUPLE (IN.)
******************************************************************************

0113 R1=0.2500
0114 R2=0.4375
0115 DO 12 I=1,12

******************************************************************************
C C C
C AMP=TEMPERATURE REGISTERED BY INNER THERMOCOUPLE (DEG. F.)
******************************************************************************

0116 AMP=A(1)+B(1)*TT(I,2)+C(1)*TT(I,2)**2.0+D(1)**TT(I,2)**3.0
0117 TW(I)=AMP+(TT(I,2)-TT(I,1))/26.0*LOG(R1/0.147)/LOG(R2/R1)
0118 DELT(I)=TT(I,2)-TT(I,1)-TA(I,2)+TA(I,1)
0119 FLUX(I)=COND(I)*DELT(I)+1000.0
0120 12 CONTINUE

******************************************************************************
C C C
C NEXT REMOVE SPURIOUS CALORIMETER READINGS, REPLACE WITH READING
C OF OPPOSITE CALORIMETER. AT LEAST ONE CALOTERI MUST BE
C OPERATIVE AT EACH AXIAL POSITION. INTEGRATE HEAT FLUX CURVE TO
C OBTAIN GAS BULK TEMPERATURE AT EACH AXIAL POSITION
******************************************************************************
0121    IF(N)401,401,400
0122    400 CONTINUE
0123         K=N
0124    DO 13 N=1,K
0125     KCHEK=(TBA(N)+1)/2-IBAD(N)/2
0126    IF(KCHEK)15,15,14
0127    14      KK=IBAD(N)
0128     FLUX(KK)=FLUX(KK+1)
0129     TW(KK)=TW(KK+1)
0130    GO TO 13
0131    15      KK=IBAD(N)
0132     FLUX(KK)=FLUX(KK-1)
0133     TW(KK)=TW(KK-1)
0134    13 CONTINUE
0135    401 CONTINUE
0136    DO 18 I=1,11,2
0137         K=(I+1)/2
0138     FLUX(K)=(FLUX(I)+FLUX(I+1))/2.0
0139     TW(K)=(TW(I)+TW(I+1))/2.0
0140    18 CONTINUE
0141    DO 193 I=1,6
0142    IF(KKK)180,180,181
0143    181 XXD(I)=XXDC(I)+3.337-3.450
0144    GO TO 183
0145    180 XXD(I)=XXDC(I)
0146    183 CONTINUE
TRULK=(TRULK+459.7)/1.8

******************************************************************************

ASSUME COOLING OF GAS CEASES JUST INSIDE FORWARD FACE OF BULK
TEMPERATURE MEASURING PLENUM 1.25 IN. FROM LAST CALORIMETER
EVALUATE THE TOTAL HEAT TRANSFER FROM THE GAS FROM ENTRANCE
TO EXIT

******************************************************************************

XDCEND=XDC(1)+1.250/0.294
QTOT=(60.0*0.0695*1.8*QMAS)*(A(8)*(TAP-TRULK)+B(8)*(TAP**2.0-TBULK
1**2.6)/?C+C(3)*(TAP**3.0-TBULK**3.0)/3.0)

******************************************************************************

HEAT FLUX VARIATION GIVEN BY DA/X**0.425+BB/X**0.39+CC/X**0.25

******************************************************************************

EXP1=0.450
EXP2=0.390
EXP3=0.290
P1H1(1,1)=1.0/XDC(6)**EXP1
P1H1(2,1)=1.0/XDC(5)**EXP1
P1H1(3,1)=XDCEND**(1.0-EXP1)/(1.0-EXP1)
P1H1(1,2)=1.0/XDC(6)**EXP2
P1H1(2,2)=1.0/XDC(5)**EXP2
P1H1(3,2)=XDCEND**(1.0-EXP2)/(1.0-EXP2)
P1H1(1,3)=1.0/XDC(6)**EXP3
P1H1(2,3)=1.0/XDC(5)**EXP3
P1H1(3,3)=XDCEND**(1.0-EXP3)/(1.0-EXP3)
Q1=ABS(FLUX(6))
Q2=ABS(FLUX(5))
Q3=527.9*QTOT
0180 WR ITE(6,839)
0181 839 FORMAT(12X,3H X+,4X,6H (X+)M,3X,3H 0+,5X,4H NUM,4X,6H TBULK,2X,6H
    1TW/TR,2X,6H (REM)
0182 WR ITE(6,840)
0183 840 FORMAT(43X,8H DEG. F.)
0184 DO 23 I=1,6
0185 1W(I)=Tw(I)+459.7
0186 TAPw=Tw(I)/1.8
0187 CONDP=0.1395*(A(7)+B(7)*TB(I)+C(7)*TB(I)**2.0+D(7)*TB(I)**3.0)
0188 H=FLUX(I)/(1.8*TB(I)-Tw(I))
0189 RNUSS=H*0.0245/CONDP

C***********************************************************************
C CALCULATE INVERSE GRAFTZ PARAMETERS BASED ON INLET AND
C LOCAL BULK PROPERTIES
C***********************************************************************
0190 XINC=XOC(I)*2.0/(REYD*PR)
0191 CSUBP=0.0685*(A(8)+B(8)*TB(I)+C(8)*TB(I)**2.0+D(8)*TB(I)**3.0)
0192 VISCR=0.00000153*(A(6)+B(6)*TB(I)+C(6)*TB(I)**2.0)
0193 REYR=4.0*12.0*CMAS/(3.1414*0.294*VISCR*60.0)
0194 XLOC=CONDP*XOC(I)/(REYR*VISCR*CSUBP*1800.0)
0195 RATIO=Tw(I)/(TB(I)**1.8)
0196 QNON=0.147/12.0*FLUX(I)/(TIN*CONDP)
0197 TRB=1.8*TB(I)-459.7
0198 WR ITE(6,150)XINC,XLOC,QNON,RNUSS,TRB,RATIO,REYR
0199 150 FORMAT(10X,3F8.4,F9.4,F8.2,F7.3,F8.1)
0200 23 CONTINUE
0201 WR ITE(6,830)
0202 830 FORMAT(/10X,34H NON DIMENSIONALIZED PRESSURE DROP)
0203 WR ITE(6,841)
0204 841 FORMAT(10X,6H X+,4X,6H P+)
0205 IF(KK)172,172,173
0206 172 DO 8 I=1,6
0207 XDP(I+1)=(XDP(I+1)-XDP(I))/(REYD*PR)*2.0
0208 IF(I(JJ)*0.1)DFLP(I)=DFLP(I)*0.826/0.797
0209 DFLP(I)=32.2*62.4*DFLP(I)/(12.0*DENS*UIN**2.0)
0210 WR ITE(6,107)XDP(I+1),DFLP(I)
0211 8 CONTINUE
GO TO 174

ASSUME PRESSURE DROP DATA FOR UVT GIVEN AS P0-P2,P1-P2,
P2-P3,P2-P4,P2-P5,P2-P6,P2-P7

ASSUME PRESSURE DROP IN BELLMOUTH GIVEN BY DYNAMIC VELOCITY HEAD
ASSUME VELOCITY FIELD DEVELOPMENT BEGINS AT END OF BELLMOUTH
NOW DETERMINE LOCAL GATEZ PARAMETERS CORRESPONDING TO EACH
OF THE PRESSURE TAP LOCATIONS

DELPL1)=DELPL1)-.50
XXOP(1)=(XXOP(1)+3.337)/(REYM*PR)*2.0
WRITE(5,107)XXOP(1),DELPL1)
107 FORMAT(10X,F3.4,F9.3)
CONTINUE
174 CONTINUE
IF(K*XK.K,E.1)GO TO 113
XXOP(1)=XXOP(1)/(REYM*PR)*2.0
WRITE(6,150)XXOP(1)
150 FORMAT(10X,34E99) POSITION OF FIRST PRESSURE TAP = ,F8.6)
CONTINUE
113 CONTINUE
IF(KKK,KK,E.1)GO TO 411
30 TO 412
117 CONTINUE
C237 411 CASP=DELPL1)
C238 412 CASP=DELPL1)
C239 413 I=1.7
C240 414 X(1)=XXP(1)+3.337-3.450
C241 415 DELPL1)=DELPL1)-CASPL1)
**CALCULATE MEAN REYNOLDS NUMBER AND PRANDTL NUMBER AT PRESSURE TAP #1**
C
C ******************************************************************************
C 0262   422 CONTINUE
C 0263   IT=0
C 0264   802 CONTINUE
C 0265   DUSP=DELP(1)
C 0266   IT=IT+1
C
C ******************************************************************************
C NOW FIT THIRD ORDER LEAST SQUARES POLYNOMIAL THROUGH NON-DIMENSIONALIZED PRESSURE AS A FUNCTION OF LOCAL GRAFTZ PARAMETER
C ASSIGN DOUBLE WEIGHT TO DELP(2) IN LEAST SQUARES FIT
C
C ******************************************************************************
C 0267   DISP=XXDP(2)-XXDP(1)
C 0268   DO 700 I=1,4
C 0269   DO 701 J=1,4
C 0270   PHI(I,J)=DISP**(I+J)
C 0271   701 CONTINUE
C 0272   700 CONTINUE
C 0273   RHS(I)=(DELP(2)-DUSP)*DISP**I
C 0274   DO 702 I=1,7
C 0275   DISP=XXDP(I)-XXDP(1)
C 0276   DO 703 J=1,4
C 0277   DO 704 K=1,4
C 0278   PHI(J,K)=PHI(J,K)+DISP**(J+K)
C 0279   704 CONTINUE
C 0280   RHS(J)=RHS(J)+(DELP(I)-DUSP)*DISP**J
C 0281   703 CONTINUE
C 0282   702 CONTINUE
C 0283   IM=4
C 0284   L=4
C 0285   CALL SPINV(AAA,L,IM,IS)
C
C ******************************************************************************
C NOW CALCULATE DERIVATIVE OF NON-DIMENSIONALIZED PRESSURE
C DROP AT TAP POINTS WITH RESPECT TO GRAFTZ PARAMETER
0323       GO TO 507
0324       510 FORMAT (34H BULK TEMPERATURE DID NOT CONVERGE)
0325       509 WRITE (6, 510)
0326       GO TO 507
0327       508 STOP
0328       END
Appendix H. Data

Raw and reduced experimental data is included in this appendix. Tests numbered consecutively are in chronological order. Reynolds numbers well above 2000. Some results in the transition region are included.
### PARABOLIC INLET VELOCITY TEST NO. 1

**TAKE TEST TAKEN FROM RUN NO. 1**

**TAKE THERMOCOUPLE OUTPUT (MV)**

<table>
<thead>
<tr>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2515</td>
<td>4.2548</td>
<td>7</td>
<td>4.2745</td>
<td>4.2770</td>
</tr>
<tr>
<td>2</td>
<td>4.2412</td>
<td>4.2367</td>
<td>8</td>
<td>4.2771</td>
<td>4.2775</td>
</tr>
<tr>
<td>3</td>
<td>4.2765</td>
<td>4.2745</td>
<td>9</td>
<td>4.2730</td>
<td>4.2680</td>
</tr>
<tr>
<td>4</td>
<td>4.2619</td>
<td>4.2816</td>
<td>10</td>
<td>4.2776</td>
<td>4.2781</td>
</tr>
<tr>
<td>5</td>
<td>4.2813</td>
<td>4.2801</td>
<td>11</td>
<td>4.2926</td>
<td>4.2919</td>
</tr>
<tr>
<td>6</td>
<td>4.2720</td>
<td>4.2654</td>
<td>12</td>
<td>4.2919</td>
<td>4.2889</td>
</tr>
</tbody>
</table>

**TEST THERMOCOUPLE OUTPUT (MV)**

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2520</td>
<td>4.2546</td>
<td>7</td>
<td>4.2838</td>
<td>4.2919</td>
</tr>
<tr>
<td>2</td>
<td>4.2669</td>
<td>4.2367</td>
<td>8</td>
<td>4.2820</td>
<td>4.2936</td>
</tr>
<tr>
<td>3</td>
<td>4.2784</td>
<td>4.2796</td>
<td>9</td>
<td>4.2919</td>
<td>4.3026</td>
</tr>
<tr>
<td>4</td>
<td>4.2817</td>
<td>4.2824</td>
<td>10</td>
<td>4.2979</td>
<td>4.3076</td>
</tr>
<tr>
<td>5</td>
<td>4.2769</td>
<td>4.2815</td>
<td>11</td>
<td>4.3549</td>
<td>4.3800</td>
</tr>
<tr>
<td>6</td>
<td>4.2691</td>
<td>4.2695</td>
<td>12</td>
<td>4.3515</td>
<td>4.3790</td>
</tr>
</tbody>
</table>

**DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.300**

**FLOWMETER TMP. (MV) = 1.0240**

**BULK EXIT TEMP (IN.) = 4.5796**

**INLET PRESSURE MAN. LEFT = 3.45, RIGHT = 3.50 IN. Hg**

**INLET BULK TEMP CR-AL = 12.300**

**STATIC PRESSURE DROP (IN.)**

| PL-P2 = -0.026 | PL-P3 = 0.029 | PL-P4 = 0.053 |
| PL-P5 = 0.092   | PL-P6 = 0.152  |               |

**BLUE PANNEUMETER FLUID SP GR 0.797**

**INLET TEMP. PRESSURE (DEG. F.) = 578.18**

**TW/Tb = 0.647 MACH #N. = 0.011**

**PR = 0.707995 REY. = 1108.4**

### DIMENSIONALIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>A+M</th>
<th>Q+</th>
<th>HUM</th>
<th>TBULK</th>
<th>TW/Tb</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4301</td>
<td>0.3360</td>
<td>0.2508</td>
<td>0.1256</td>
<td>25.66</td>
<td>0.977</td>
</tr>
<tr>
<td>0.3470</td>
<td>0.2763</td>
<td>0.2563</td>
<td>0.1252</td>
<td>243.75</td>
<td>0.954</td>
</tr>
<tr>
<td>0.2640</td>
<td>0.2161</td>
<td>0.1744</td>
<td>0.0997</td>
<td>272.42</td>
<td>0.916</td>
</tr>
<tr>
<td>0.1810</td>
<td>0.1591</td>
<td>0.2152</td>
<td>0.1043</td>
<td>316.66</td>
<td>0.865</td>
</tr>
<tr>
<td>0.0980</td>
<td>0.0981</td>
<td>0.2560</td>
<td>0.3904</td>
<td>387.46</td>
<td>0.793</td>
</tr>
<tr>
<td>0.0150</td>
<td>0.0146</td>
<td>0.7027</td>
<td>4.8038</td>
<td>528.66</td>
<td>0.683</td>
</tr>
</tbody>
</table>

### FIRST PRESSURE TAP = 0.010811

<table>
<thead>
<tr>
<th>X+</th>
<th>A+M</th>
<th>F</th>
<th>F(REM)</th>
<th>FP</th>
<th>FP(REM)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0193</td>
<td>0.0190</td>
<td>0.0006</td>
<td>0.1256</td>
<td>-0.00356</td>
<td>-4.02</td>
</tr>
<tr>
<td>0.0651</td>
<td>0.0503</td>
<td>0.0053</td>
<td>6.03</td>
<td>0.00173</td>
<td>2.13</td>
</tr>
<tr>
<td>0.0981</td>
<td>0.0859</td>
<td>0.0087</td>
<td>12.52</td>
<td>0.00422</td>
<td>11.06</td>
</tr>
<tr>
<td>0.2312</td>
<td>0.2093</td>
<td>0.0093</td>
<td>11.53</td>
<td>0.00422</td>
<td>11.06</td>
</tr>
<tr>
<td>0.3142</td>
<td>0.2593</td>
<td>0.0087</td>
<td>12.19</td>
<td>0.00071</td>
<td>10.85</td>
</tr>
<tr>
<td>0.3972</td>
<td>0.3134</td>
<td>0.0103</td>
<td>15.56</td>
<td>0.01035</td>
<td>14.70</td>
</tr>
<tr>
<td>0.4930</td>
<td>0.3464</td>
<td>0.0104</td>
<td>14.40</td>
<td>0.01097</td>
<td>45.74</td>
</tr>
<tr>
<td>0.4430</td>
<td>0.3464</td>
<td>0.0104</td>
<td>14.40</td>
<td>0.01097</td>
<td>45.74</td>
</tr>
</tbody>
</table>

**DEG. F.**
### PARABOLIC INLET VELOCITY TEST NO. 2

<table>
<thead>
<tr>
<th>TARE TEST TAKEN FROM RUN NO. 2</th>
<th>TARE THERMOCOUPLE OUTPUT (MV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TA(I, 1)</td>
</tr>
<tr>
<td>1</td>
<td>4.1606</td>
</tr>
<tr>
<td>2</td>
<td>4.1715</td>
</tr>
<tr>
<td>3</td>
<td>4.2064</td>
</tr>
<tr>
<td>4</td>
<td>4.2190</td>
</tr>
<tr>
<td>5</td>
<td>4.2091</td>
</tr>
<tr>
<td>6</td>
<td>4.2084</td>
</tr>
</tbody>
</table>

### TEST THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th></th>
<th>TT(I, 1)</th>
<th>TT(I, 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.1946</td>
<td>4.1959</td>
</tr>
<tr>
<td>2</td>
<td>4.1604</td>
<td>4.1807</td>
</tr>
<tr>
<td>3</td>
<td>4.2161</td>
<td>4.2206</td>
</tr>
<tr>
<td>4</td>
<td>4.2212</td>
<td>4.2221</td>
</tr>
<tr>
<td>5</td>
<td>4.2175</td>
<td>4.2244</td>
</tr>
<tr>
<td>6</td>
<td>4.2084</td>
<td>4.2091</td>
</tr>
</tbody>
</table>

**DIFFERENTIAL PRESSURE-FLOWMETER (IN) = 0.385**

**FLOWMETER TEMP (MV) = 0.8439**

**BULK EXIT TEMP (MV) = 4.9611**

**INLET PRESSURE MAN. LEFT 2.95 RIGHT 3.05 IN. HG**

**INLET BULK TEMP CR-AL = 13.249**

**STATIC PRESSURE DROP (IN.)**

<table>
<thead>
<tr>
<th>PI-P2</th>
<th>PI-P3</th>
<th>PI-P4</th>
<th>PI-P5</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.010</td>
<td>0.033</td>
<td>0.064</td>
<td>0.118</td>
</tr>
</tbody>
</table>

**BLUE MANOMETER FLUID SP GP = 0.797**

**INLET TEMPERATURE (DEG. F) = 618.91**

**TW/TU = 0.622 MACH IN. = 0.016**

**PR = 0.712679**

**KLYD = 1435.1**

### NON DIMENSIONALIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+1)R</th>
<th>W+</th>
<th>NUM</th>
<th>T BULK</th>
<th>TW/TU</th>
<th>(P/EM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3357</td>
<td>0.2294</td>
<td>0.0564</td>
<td>1.4972</td>
<td>240.81</td>
<td>0.953</td>
<td>1466.9</td>
</tr>
<tr>
<td>0.2694</td>
<td>0.2135</td>
<td>0.3863</td>
<td>4.1117</td>
<td>264.99</td>
<td>0.923</td>
<td>1425.6</td>
</tr>
<tr>
<td>0.2030</td>
<td>0.1676</td>
<td>0.1263</td>
<td>3.9456</td>
<td>300.56</td>
<td>0.879</td>
<td>1659.2</td>
</tr>
<tr>
<td>0.1392</td>
<td>0.1231</td>
<td>0.2892</td>
<td>3.1211</td>
<td>352.68</td>
<td>0.824</td>
<td>1691.1</td>
</tr>
<tr>
<td>0.0753</td>
<td>0.0687</td>
<td>0.3534</td>
<td>3.9160</td>
<td>432.42</td>
<td>0.751</td>
<td>1597.2</td>
</tr>
<tr>
<td>0.0115</td>
<td>0.0113</td>
<td>0.8745</td>
<td>5.2496</td>
<td>581.44</td>
<td>0.647</td>
<td>1462.0</td>
</tr>
</tbody>
</table>

### POSITION N1 FIRST PRESSURE TAP = 0.008313

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+1)R</th>
<th>F</th>
<th>F(P/EM)</th>
<th>FP</th>
<th>(P/EM)M</th>
<th>T BULK</th>
<th>TB/TU</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0148</td>
<td>0.0147</td>
<td>0.00131</td>
<td>1.91</td>
<td>-0.00296</td>
<td>4.29</td>
<td>1450.6</td>
<td>594.2</td>
</tr>
<tr>
<td>0.0521</td>
<td>0.0574</td>
<td>0.00412</td>
<td>6.33</td>
<td>0.00089</td>
<td>1.37</td>
<td>1535.7</td>
<td>849.3</td>
</tr>
<tr>
<td>0.0319</td>
<td>0.0110</td>
<td>0.0063</td>
<td>10.92</td>
<td>0.00679</td>
<td>7.28</td>
<td>1490.2</td>
<td>884.7</td>
</tr>
<tr>
<td>0.0178</td>
<td>0.0151</td>
<td>0.0068</td>
<td>11.56</td>
<td>0.00541</td>
<td>9.36</td>
<td>1630.4</td>
<td>829.0</td>
</tr>
<tr>
<td>0.0216</td>
<td>0.0165</td>
<td>0.00553</td>
<td>11.37</td>
<td>0.00520</td>
<td>9.33</td>
<td>1597.5</td>
<td>810.0</td>
</tr>
<tr>
<td>0.0385</td>
<td>0.0216</td>
<td>0.00812</td>
<td>15.35</td>
<td>0.00751</td>
<td>13.86</td>
<td>1345.0</td>
<td>611.6</td>
</tr>
<tr>
<td>0.3467</td>
<td>0.2762</td>
<td>0.0843</td>
<td>46.08</td>
<td>0.00859</td>
<td>14.94</td>
<td>1865.2</td>
<td>839.8</td>
</tr>
</tbody>
</table>
### PARABOLIC INLET VELOCITY TEST

<table>
<thead>
<tr>
<th>TEST NO.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>THERM.</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td>OUTPUT</td>
<td>4.2819</td>
<td>4.2866</td>
<td>4.2811</td>
<td>4.2853</td>
<td>4.2999</td>
<td>4.2994</td>
</tr>
</tbody>
</table>

### DIFFERENTIAL PRESSURE - FLOWMETER (IN.)

<table>
<thead>
<tr>
<th>PRESSURE DROP (IN.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.442</td>
</tr>
</tbody>
</table>

### FLOWMETER TEMP (MV) = 0.9812 BULK EXIT TEMP (MV) = 4.9150

### INLET BULK TEMP - AL | 13.920

### STATIC PRACTICE DROP (IN.)

<table>
<thead>
<tr>
<th>P1-P2 =</th>
<th>P1-P3 =</th>
<th>P1-P4 =</th>
<th>P1-P5 =</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.086</td>
<td>0.068</td>
<td>0.220</td>
<td></td>
</tr>
</tbody>
</table>

### BLUE MANOMETER FLUID SP GR = 0.797

### INLET TEMPERATURE (DEG. F) = 647.82

| MACH NO. = | KEYD = | 1592.9
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.018</td>
<td></td>
</tr>
</tbody>
</table>

### X+ (X+M) | W+ | NUM | T'BULK | T/W'TB (REL) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2960</td>
<td>0.2960</td>
<td>0.0432</td>
<td>4.0359</td>
<td>740.22</td>
</tr>
<tr>
<td>0.2390</td>
<td>0.1909</td>
<td>0.0827</td>
<td>4.1344</td>
<td>771.11</td>
</tr>
<tr>
<td>0.1823</td>
<td>0.1504</td>
<td>0.1769</td>
<td>4.2866</td>
<td>310.11</td>
</tr>
<tr>
<td>0.1250</td>
<td>0.1175</td>
<td>0.2943</td>
<td>4.3976</td>
<td>361.29</td>
</tr>
<tr>
<td>0.0677</td>
<td>0.0712</td>
<td>0.3414</td>
<td>3.9608</td>
<td>434.14</td>
</tr>
<tr>
<td>0.013</td>
<td>0.0160</td>
<td>0.0652</td>
<td>0.3098</td>
<td>571.63</td>
</tr>
</tbody>
</table>

### MIN UNLIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>0.0216</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1</td>
<td></td>
</tr>
<tr>
<td>0.0310</td>
<td></td>
</tr>
<tr>
<td>0.0896</td>
<td></td>
</tr>
<tr>
<td>0.1683</td>
<td></td>
</tr>
<tr>
<td>0.2386</td>
<td></td>
</tr>
<tr>
<td>0.2659</td>
<td></td>
</tr>
<tr>
<td>0.2926</td>
<td></td>
</tr>
</tbody>
</table>

### POSITION OF FIRST PRESSURE TAP = 0.007464

<table>
<thead>
<tr>
<th>X+</th>
<th>0.0133</th>
<th>0.0130</th>
<th>0.00119</th>
<th>1.95</th>
<th>-0.02979</th>
<th>-0.86</th>
<th>1425.5</th>
<th>0.643</th>
</tr>
</thead>
<tbody>
<tr>
<td>P</td>
<td>0.0453</td>
<td>0.0530</td>
<td>0.04030</td>
<td>0.419</td>
<td>-0.00004</td>
<td>-0.07</td>
<td>1732.4</td>
<td>0.086</td>
</tr>
<tr>
<td>F (REL)</td>
<td>0.01038</td>
<td>0.00972</td>
<td>0.00553</td>
<td>0.973</td>
<td>0.00766</td>
<td>6.99</td>
<td>1543.9</td>
<td>0.803</td>
</tr>
<tr>
<td>F (REL)</td>
<td>0.01465</td>
<td>0.01345</td>
<td>0.00669</td>
<td>1.292</td>
<td>0.00559</td>
<td>10.72</td>
<td>1931.2</td>
<td>0.333</td>
</tr>
<tr>
<td>0.2169</td>
<td>0.1760</td>
<td>0.06672</td>
<td>14.58</td>
<td>0.00617</td>
<td>12.37</td>
<td>2155.2</td>
<td>0.813</td>
<td>0.697</td>
</tr>
<tr>
<td>0.2473</td>
<td>0.2154</td>
<td>0.00767</td>
<td>15.87</td>
<td>0.00690</td>
<td>14.24</td>
<td>2369.5</td>
<td>0.854</td>
<td>0.940</td>
</tr>
<tr>
<td>0.3573</td>
<td>0.2564</td>
<td>0.00372</td>
<td>18.12</td>
<td>0.00777</td>
<td>15.74</td>
<td>2101.5</td>
<td>0.897</td>
<td>0.962</td>
</tr>
</tbody>
</table>
### PARABOLIC INLET VELOCITY TEST NO. 9

TAKEN TEST TAKEN FROM RUN NO. 4

TAKEN THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th></th>
<th>TAU(1,1)</th>
<th>TAU(1,2)</th>
<th>TAU(1,1)</th>
<th>TAU(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2389</td>
<td>4.2403</td>
<td>7</td>
<td>4.2861</td>
</tr>
<tr>
<td>2</td>
<td>4.2724</td>
<td>4.2212</td>
<td>8</td>
<td>4.2854</td>
</tr>
<tr>
<td>3</td>
<td>4.2684</td>
<td>4.2695</td>
<td>9</td>
<td>4.2837</td>
</tr>
<tr>
<td>4</td>
<td>4.2674</td>
<td>4.2729</td>
<td>10</td>
<td>4.2885</td>
</tr>
<tr>
<td>5</td>
<td>4.2898</td>
<td>4.2893</td>
<td>11</td>
<td>4.3042</td>
</tr>
<tr>
<td>6</td>
<td>4.2612</td>
<td>4.2754</td>
<td>12</td>
<td>4.3039</td>
</tr>
</tbody>
</table>

### TEST THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th></th>
<th>TTT(1,1)</th>
<th>TTT(1,2)</th>
<th>TTT(1,1)</th>
<th>TTT(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2400</td>
<td>4.2393</td>
<td>7</td>
<td>4.3055</td>
</tr>
<tr>
<td>2</td>
<td>4.2849</td>
<td>4.2403</td>
<td>8</td>
<td>4.3100</td>
</tr>
<tr>
<td>3</td>
<td>4.2851</td>
<td>4.2903</td>
<td>9</td>
<td>4.3101</td>
</tr>
<tr>
<td>4</td>
<td>4.2900</td>
<td>4.2915</td>
<td>10</td>
<td>4.3180</td>
</tr>
<tr>
<td>5</td>
<td>4.2998</td>
<td>4.3076</td>
<td>11</td>
<td>4.5058</td>
</tr>
<tr>
<td>6</td>
<td>4.2904</td>
<td>4.2942</td>
<td>12</td>
<td>4.5301</td>
</tr>
</tbody>
</table>

#### DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.566

FLOWMETER TEMP (MV) = 0.8602 BULK EXIT TEMP (MV) = 4.3450

INLET BULK TEMP MAN, LEFT 5.75 RIGHT 5.90 IN. HG

INLET TEMPERATURE (DEG. F) = 652.18

INLET TEMPERATURE (DEG. F) = 14.021

STATIC PRESSURE DROP (IN.)

P1-P2 = 0.001 P1-P3 = 0.002 P1-P4 = 0.005

P1-P5 = 0.004 P1-P6 = 0.005

BLUE MANOMETER FLUID SP CR 0.797

VCR = 0.715622 REYD = 2067.5

### NON DIMENSIONALIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)*M</th>
<th>Q+</th>
<th>NUM</th>
<th>TBULK</th>
<th>TW/TB</th>
<th>(RE)m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2205</td>
<td>0.1712</td>
<td>-0.5541</td>
<td>-976.164</td>
<td>211.66</td>
<td>0.997</td>
<td>2911.9</td>
</tr>
<tr>
<td>0.1045</td>
<td>0.1751</td>
<td>0.0918</td>
<td>-14.8064</td>
<td>191.29</td>
<td>1.031</td>
<td>2472.5</td>
</tr>
<tr>
<td>0.1204</td>
<td>0.1200</td>
<td>0.2137</td>
<td>-26.0078</td>
<td>184.58</td>
<td>1.042</td>
<td>2427.3</td>
</tr>
<tr>
<td>0.0962</td>
<td>0.0711</td>
<td>0.2279</td>
<td>-56.1272</td>
<td>199.75</td>
<td>1.023</td>
<td>2396.8</td>
</tr>
<tr>
<td>0.0521</td>
<td>0.0409</td>
<td>0.4440</td>
<td>30.8898</td>
<td>257.00</td>
<td>0.939</td>
<td>2689.6</td>
</tr>
<tr>
<td>0.0000</td>
<td>0.0073</td>
<td>2.3427</td>
<td>25.2938</td>
<td>458.10</td>
<td>0.745</td>
<td>2297.5</td>
</tr>
</tbody>
</table>

### MANOMETER LINE BLEW:

Dummy Values.

POSITION 1: FIRST PRESSURE TAP = 0.005748

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)*M</th>
<th>F</th>
<th>F(RE)m</th>
<th>FP</th>
<th>FP(RE)m</th>
<th>(RE)m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0143</td>
<td>0.0036</td>
<td>0.0082</td>
<td>8.67</td>
<td>0.0000</td>
<td>0.28</td>
<td>2756.2</td>
</tr>
<tr>
<td>0.0346</td>
<td>0.0288</td>
<td>0.0056</td>
<td>11.18</td>
<td>0.0000</td>
<td>0.37</td>
<td>2549.0</td>
</tr>
<tr>
<td>0.0788</td>
<td>0.0596</td>
<td>0.0344</td>
<td>4.00</td>
<td>0.0000</td>
<td>0.46</td>
<td>2182.5</td>
</tr>
<tr>
<td>0.1229</td>
<td>0.0698</td>
<td>0.0327</td>
<td>0.27</td>
<td>0.0000</td>
<td>0.49</td>
<td>2793.4</td>
</tr>
<tr>
<td>0.1670</td>
<td>0.1217</td>
<td>0.0327</td>
<td>0.27</td>
<td>0.0000</td>
<td>0.49</td>
<td>2489.7</td>
</tr>
<tr>
<td>0.2112</td>
<td>0.1564</td>
<td>0.0000</td>
<td>-2.91</td>
<td>0.0000</td>
<td>0.50</td>
<td>2541.5</td>
</tr>
<tr>
<td>0.2359</td>
<td>0.1769</td>
<td>0.0000</td>
<td>-2.65</td>
<td>0.0000</td>
<td>0.51</td>
<td>2504.0</td>
</tr>
</tbody>
</table>

### NON DIMENSIONALIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>P</th>
<th>Manometer Line BLEW:</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0244</td>
<td>0.002</td>
<td></td>
</tr>
<tr>
<td>0.0683</td>
<td>0.005</td>
<td></td>
</tr>
<tr>
<td>0.1126</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>0.1568</td>
<td>0.013</td>
<td></td>
</tr>
<tr>
<td>0.2009</td>
<td>0.012</td>
<td></td>
</tr>
<tr>
<td>0.2253</td>
<td>0.015</td>
<td></td>
</tr>
</tbody>
</table>

### FIRST PRESSURE TAP = 0.005748

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)*M</th>
<th>F</th>
<th>F(RE)m</th>
<th>FP</th>
<th>FP(RE)m</th>
<th>(RE)m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0143</td>
<td>0.0036</td>
<td>0.0082</td>
<td>8.67</td>
<td>0.0000</td>
<td>0.28</td>
<td>2756.2</td>
</tr>
<tr>
<td>0.0346</td>
<td>0.0288</td>
<td>0.0056</td>
<td>11.18</td>
<td>0.0000</td>
<td>0.37</td>
<td>2549.0</td>
</tr>
<tr>
<td>0.0788</td>
<td>0.0596</td>
<td>0.0344</td>
<td>4.00</td>
<td>0.0000</td>
<td>0.46</td>
<td>2182.5</td>
</tr>
<tr>
<td>0.1229</td>
<td>0.0698</td>
<td>0.0327</td>
<td>0.27</td>
<td>0.0000</td>
<td>0.49</td>
<td>2793.4</td>
</tr>
<tr>
<td>0.1670</td>
<td>0.1217</td>
<td>0.0327</td>
<td>0.27</td>
<td>0.0000</td>
<td>0.49</td>
<td>2489.7</td>
</tr>
<tr>
<td>0.2112</td>
<td>0.1564</td>
<td>0.0000</td>
<td>-2.91</td>
<td>0.0000</td>
<td>0.50</td>
<td>2541.5</td>
</tr>
<tr>
<td>0.2359</td>
<td>0.1769</td>
<td>0.0000</td>
<td>-2.65</td>
<td>0.0000</td>
<td>0.51</td>
<td>2504.0</td>
</tr>
</tbody>
</table>
### Parabolic Inlet Velocity Test

**Test No. 10**

**Test Taken From Run No. 4**

**Thermocouple Output (MV)**

<table>
<thead>
<tr>
<th>Test</th>
<th>TA(i,1)</th>
<th>TA(i,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2189</td>
<td>4.2011</td>
</tr>
<tr>
<td>2</td>
<td>4.2224</td>
<td>4.2156</td>
</tr>
<tr>
<td>3</td>
<td>4.2864</td>
<td>4.2865</td>
</tr>
<tr>
<td>4</td>
<td>4.2749</td>
<td>4.2749</td>
</tr>
<tr>
<td>5</td>
<td>4.2908</td>
<td>4.2893</td>
</tr>
<tr>
<td>6</td>
<td>4.2812</td>
<td>4.2754</td>
</tr>
</tbody>
</table>

**Thermocouple Output (MV)**

<table>
<thead>
<tr>
<th>Test</th>
<th>TT(i,1)</th>
<th>TT(i,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2519</td>
<td>4.2546</td>
</tr>
<tr>
<td>2</td>
<td>4.2407</td>
<td>4.2390</td>
</tr>
<tr>
<td>3</td>
<td>4.2818</td>
<td>4.2859</td>
</tr>
<tr>
<td>4</td>
<td>4.2660</td>
<td>4.2686</td>
</tr>
<tr>
<td>5</td>
<td>4.2941</td>
<td>4.2995</td>
</tr>
<tr>
<td>6</td>
<td>4.2839</td>
<td>4.2858</td>
</tr>
</tbody>
</table>

**Differential Pressure - Flowmeter (In.) = 0.492**

**Flowmeter Temp. (MV) = 0.076**

**Bulk Exit Temp. (MV) = 4.6130**

**Inlet Pressure Man, Left = 4.50 Right = 4.65 In. HG**

**Inlet Bulk Temp. CR-AL = 14.023**

**Static Pressure Drop (In.)**

- P1-P2 = -0.019
- P1-P3 = 0.012
- P1-P4 = 0.081
- P1-P5 = 0.167
- P1-P6 = 0.264

**Blue Manometer Fluid Sp Gr = 0.797**

**Inlet Temperature (Deg. F) = 652.35**

**TW/Th = 0.603**

**Mach No. = 0.019**

**PR = 0.715**

**Reynolds = 1794.8**

### Non-Dimensionalized Pressure Drop

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)(M)</th>
<th>Q+</th>
<th>NUMER</th>
<th>T-Bulk</th>
<th>TW/Th</th>
<th>REYN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2634</td>
<td>0.211</td>
<td>0.074</td>
<td>1.002</td>
<td>229.64</td>
<td>0.972</td>
<td>297.4</td>
</tr>
<tr>
<td>0.2125</td>
<td>0.1675</td>
<td>0.0701</td>
<td>4.2423</td>
<td>261.47</td>
<td>0.931</td>
<td>292.2</td>
</tr>
<tr>
<td>0.1617</td>
<td>0.1224</td>
<td>0.1713</td>
<td>9.4422</td>
<td>302.64</td>
<td>0.880</td>
<td>292.0</td>
</tr>
<tr>
<td>0.1169</td>
<td>0.0749</td>
<td>0.4492</td>
<td>2.6432</td>
<td>357.65</td>
<td>0.822</td>
<td>275.2</td>
</tr>
<tr>
<td>0.0690</td>
<td>0.0343</td>
<td>0.4164</td>
<td>4.7837</td>
<td>436.75</td>
<td>0.750</td>
<td>209.2</td>
</tr>
<tr>
<td>0.0292</td>
<td>0.0189</td>
<td>1.1219</td>
<td>7.0979</td>
<td>583.92</td>
<td>0.649</td>
<td>145.4</td>
</tr>
</tbody>
</table>

**Position of First Pressure Tap = 0.006621**

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)(M)</th>
<th>F</th>
<th>FP</th>
<th>FP(RE)(M)</th>
<th>T-Bulk</th>
<th>TB/Th</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0118</td>
<td>0.0116</td>
<td>0.0009</td>
<td>0.17</td>
<td>-0.00423</td>
<td>-7.78</td>
<td>183.2</td>
</tr>
<tr>
<td>0.0399</td>
<td>0.0373</td>
<td>0.00291</td>
<td>5.66</td>
<td>-0.00031</td>
<td>-0.59</td>
<td>194.7</td>
</tr>
<tr>
<td>0.0907</td>
<td>0.0795</td>
<td>0.00614</td>
<td>12.79</td>
<td>0.00435</td>
<td>9.06</td>
<td>209.3</td>
</tr>
<tr>
<td>0.1416</td>
<td>0.1186</td>
<td>0.00721</td>
<td>15.80</td>
<td>0.00593</td>
<td>12.79</td>
<td>219.0</td>
</tr>
<tr>
<td>0.1924</td>
<td>0.1546</td>
<td>0.00722</td>
<td>16.48</td>
<td>0.00598</td>
<td>13.65</td>
<td>221.0</td>
</tr>
<tr>
<td>0.2433</td>
<td>0.1888</td>
<td>0.0078</td>
<td>19.37</td>
<td>0.00687</td>
<td>16.20</td>
<td>235.9</td>
</tr>
<tr>
<td>0.2913</td>
<td>0.2271</td>
<td>0.00416</td>
<td>20.56</td>
<td>0.00729</td>
<td>17.45</td>
<td>239.7</td>
</tr>
<tr>
<td>0.3413</td>
<td>0.2671</td>
<td>0.00416</td>
<td>20.56</td>
<td>0.00729</td>
<td>17.45</td>
<td>239.7</td>
</tr>
</tbody>
</table>
**PARABOLIC INLET VELOCITY TEST NO. 11**

TARE TEST TAKEN FROM RUN NO. 4

<table>
<thead>
<tr>
<th>TARE THERMOCOUPLE OUTPUT (MV)</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2389</td>
<td>4.2403</td>
<td>4.2861</td>
<td>4.2892</td>
</tr>
<tr>
<td>2</td>
<td>4.2224</td>
<td>4.2212</td>
<td>4.2854</td>
<td>4.2893</td>
</tr>
<tr>
<td>3</td>
<td>4.2084</td>
<td>4.2095</td>
<td>4.2837</td>
<td>4.2804</td>
</tr>
<tr>
<td>4</td>
<td>4.2749</td>
<td>4.2729</td>
<td>4.2885</td>
<td>4.2865</td>
</tr>
<tr>
<td>5</td>
<td>4.2098</td>
<td>4.2893</td>
<td>4.3042</td>
<td>4.3048</td>
</tr>
</tbody>
</table>

**TEST THERMOCOUPLE OUTPUT (MV)**

<table>
<thead>
<tr>
<th>TEST THERMOCOUPLE OUTPUT (MV)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2530</td>
<td>4.2567</td>
<td>4.3034</td>
<td>4.3034</td>
</tr>
<tr>
<td>2</td>
<td>4.2421</td>
<td>4.2416</td>
<td>4.3039</td>
<td>4.3149</td>
</tr>
<tr>
<td>3</td>
<td>4.2009</td>
<td>4.2867</td>
<td>4.3199</td>
<td>4.3423</td>
</tr>
<tr>
<td>4</td>
<td>4.2832</td>
<td>4.2887</td>
<td>4.3274</td>
<td>4.3421</td>
</tr>
<tr>
<td>5</td>
<td>4.2891</td>
<td>4.3048</td>
<td>4.3866</td>
<td>4.4257</td>
</tr>
<tr>
<td>6</td>
<td>4.2899</td>
<td>4.2913</td>
<td>4.3818</td>
<td>4.4267</td>
</tr>
</tbody>
</table>

**DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.400**

**FLOWMETER TEMP (MV) = 0.8700 BULK EXIT TEMP (MV) = 4.8080**

**INLET PRESSURE MAN., LEFT 3.40 RIGHT 3.70 IN. HG**

**INLET BULK TEMP CR-AL = 13.942**

**STATIC PRESSURE DROP (IN.)**

- $P_1-P_2 = -0.015$
- $P_1-P_3 = 0.011$
- $P_1-P_6 = 0.058$
- $P_1-P_5 = 0.118$
- $P_1-P_6 = 0.208$

**BLUE MANOMETER FLUID SPGR 0.797**

**INLET TEMPERATURE (DEG. F.) = 648.77**

**$W/TU = 0.7868$ MACH NO. = 0.016**

**PR = 6.71527 REYN = 1461.5**

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+M)</th>
<th>Q+</th>
<th>NUM</th>
<th>TBULK</th>
<th>W/TB (REL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3236</td>
<td>0.2496</td>
<td>0.1327</td>
<td>3.3447</td>
<td>238.50</td>
<td>0.959</td>
</tr>
<tr>
<td>0.3211</td>
<td>0.2467</td>
<td>0.1370</td>
<td>4.9302</td>
<td>264.54</td>
<td>0.927</td>
</tr>
<tr>
<td>0.1687</td>
<td>0.1632</td>
<td>0.1610</td>
<td>4.7507</td>
<td>305.41</td>
<td>0.870</td>
</tr>
<tr>
<td>0.0362</td>
<td>0.1175</td>
<td>0.2311</td>
<td>4.0768</td>
<td>366.85</td>
<td>0.804</td>
</tr>
<tr>
<td>0.0737</td>
<td>0.0678</td>
<td>0.4186</td>
<td>4.2293</td>
<td>461.70</td>
<td>0.701</td>
</tr>
<tr>
<td>0.0113</td>
<td>0.0112</td>
<td>0.9355</td>
<td>5.0145</td>
<td>633.84</td>
<td>0.619</td>
</tr>
</tbody>
</table>

**NON DIMENSIONALIZED PRESSURE DROP**

<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
<th>0.084</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0970</td>
<td>0.062</td>
<td></td>
</tr>
<tr>
<td>0.1594</td>
<td>0.323</td>
<td></td>
</tr>
<tr>
<td>0.2219</td>
<td>0.655</td>
<td></td>
</tr>
<tr>
<td>0.2843</td>
<td>0.970</td>
<td></td>
</tr>
<tr>
<td>0.3183</td>
<td>1.169</td>
<td></td>
</tr>
</tbody>
</table>

**POSITION OF FIRST PRESSURE TAP = 0.00134**

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+M)</th>
<th>F</th>
<th>F(REL)</th>
<th>FP</th>
<th>FP(REL)</th>
<th>TP</th>
<th>TP(REL)</th>
<th>TBULK</th>
<th>TB/TUB</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0145</td>
<td>0.0145</td>
<td>0.33</td>
<td>0.0043</td>
<td>6.30</td>
<td>1649.5</td>
<td>647.2</td>
<td>0.087</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0493</td>
<td>0.0493</td>
<td>4.92</td>
<td>0.0038</td>
<td>5.25</td>
<td>1548.0</td>
<td>731.4</td>
<td>0.678</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1115</td>
<td>0.0992</td>
<td>11.39</td>
<td>0.0062</td>
<td>11.74</td>
<td>1574.9</td>
<td>40.4</td>
<td>0.174</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1739</td>
<td>0.1646</td>
<td>14.55</td>
<td>0.0066</td>
<td>11.85</td>
<td>1764.3</td>
<td>23.6</td>
<td>0.840</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2364</td>
<td>0.1906</td>
<td>15.46</td>
<td>0.0070</td>
<td>13.03</td>
<td>1815.7</td>
<td>87.5</td>
<td>0.950</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2999</td>
<td>0.2353</td>
<td>16.84</td>
<td>0.0078</td>
<td>15.60</td>
<td>1909.5</td>
<td>249.0</td>
<td>0.947</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3334</td>
<td>0.2572</td>
<td>16.72</td>
<td>0.0064</td>
<td>16.94</td>
<td>1931.5</td>
<td>237.4</td>
<td>0.964</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
PARABOLIC INLET VELOCITY TEST NO. 12
TARE TEST TAKEN FROM RUN NO. 4
TARE THERMOCOUPLE OUTPUT (IV)

<table>
<thead>
<tr>
<th>I</th>
<th>T(I,1)</th>
<th>T(I,2)</th>
<th>T(I,1)</th>
<th>T(I,1)%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.289</td>
<td>4.2403 / 7</td>
<td>4.2861</td>
<td>4.2892</td>
</tr>
<tr>
<td>2</td>
<td>4.2724</td>
<td>4.2212 / 8</td>
<td>4.2854</td>
<td>4.2894</td>
</tr>
<tr>
<td>3</td>
<td>4.2684</td>
<td>4.2695 / 9</td>
<td>4.2837</td>
<td>4.2894</td>
</tr>
<tr>
<td>4</td>
<td>4.2749</td>
<td>4.2729 / 10</td>
<td>4.2885</td>
<td>4.2896</td>
</tr>
<tr>
<td>5</td>
<td>4.2698</td>
<td>4.2893 / 11</td>
<td>4.3042</td>
<td>4.3078</td>
</tr>
<tr>
<td>6</td>
<td>4.2612</td>
<td>4.2754 / 12</td>
<td>4.3039</td>
<td>4.3078</td>
</tr>
</tbody>
</table>

TEST THERMOCOUPLE OUTPUT (IV)

<table>
<thead>
<tr>
<th>I</th>
<th>T(I,1)</th>
<th>T(I,2)</th>
<th>T(I,1)</th>
<th>T(I,1)%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2514</td>
<td>4.2560 / 7</td>
<td>4.3007</td>
<td>4.3071</td>
</tr>
<tr>
<td>2</td>
<td>4.2521</td>
<td>4.2420 / 8</td>
<td>4.2986</td>
<td>4.3030</td>
</tr>
<tr>
<td>3</td>
<td>4.2839</td>
<td>4.2892 / 9</td>
<td>4.3105</td>
<td>4.3041</td>
</tr>
<tr>
<td>4</td>
<td>4.2895</td>
<td>4.2898 / 10</td>
<td>4.3163</td>
<td>4.3275</td>
</tr>
<tr>
<td>5</td>
<td>4.2992</td>
<td>4.3041 / 11</td>
<td>4.3813</td>
<td>4.4180</td>
</tr>
<tr>
<td>6</td>
<td>4.2884</td>
<td>4.2905 / 12</td>
<td>4.3809</td>
<td>4.4226</td>
</tr>
</tbody>
</table>

DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.350
FLOWMETER TEMP (HV) = 0.8065 BULK EXIT TEMP (HV) = 4.957;
INLET PRESSURE MAN. LEFT 3.40 RIGHT 3.50 IN. HG
INLET BULK TEMP CR-AL = 13.810
STATIC PRESSURE DROP (IN.)
P1-P2 = 0.010 P1-P3 = 0.021 P1-P4 = 0.055
P1-P5 = 0.143 P1-P6 = 0.174
BLUE MANOMETER FLUID SP GR 0.797
INLET TEMPERATURE (DEG. F.) = 643.96
PR = 0.714032; KEYO = 1282.4

X+ (A+M) Q+ NUM TBULK TW/TA (RE):

<table>
<thead>
<tr>
<th>X+</th>
<th>(A+M)</th>
<th>Q+ NUM</th>
<th>TBULK</th>
<th>TW/TA (RE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3690</td>
<td>0.2657</td>
<td>0.0475</td>
<td>4.7922</td>
<td>240.35</td>
</tr>
<tr>
<td>0.2973</td>
<td>0.2351</td>
<td>0.0793</td>
<td>4.7922</td>
<td>240.35</td>
</tr>
<tr>
<td>0.2266</td>
<td>0.1843</td>
<td>0.1728</td>
<td>4.7922</td>
<td>240.35</td>
</tr>
<tr>
<td>0.1553</td>
<td>0.1320</td>
<td>0.1728</td>
<td>3.6322</td>
<td>244.18</td>
</tr>
<tr>
<td>0.0841</td>
<td>0.0759</td>
<td>0.3337</td>
<td>3.9530</td>
<td>248.43</td>
</tr>
<tr>
<td>0.0128</td>
<td>0.0126</td>
<td>0.8874</td>
<td>5.3149</td>
<td>259.31</td>
</tr>
</tbody>
</table>

NON DIMENSIONALIZED PRESSURE DROP

X+ P+
<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0393</td>
<td>-0.075</td>
</tr>
<tr>
<td>0.1106</td>
<td>-0.149</td>
</tr>
<tr>
<td>0.1810</td>
<td>-0.237</td>
</tr>
<tr>
<td>0.2531</td>
<td>-0.371</td>
</tr>
<tr>
<td>0.3243</td>
<td>-1.035</td>
</tr>
<tr>
<td>0.3636</td>
<td>-1.253</td>
</tr>
</tbody>
</table>

POSITION OF FIRST PRESSURE TAP = 0.009277
X+ (A+M) F F(RE)M FP IP(RE)M (RE)M TBULK TB/TL

<table>
<thead>
<tr>
<th>X+</th>
<th>(A+M)</th>
<th>F</th>
<th>F(RE)M</th>
<th>FP</th>
<th>IP(RE)M (RE)M</th>
<th>TBULK</th>
<th>TB/TL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0166</td>
<td>0.0164</td>
<td>-0.00066</td>
<td>0.89</td>
<td>-0.000415</td>
<td>-5.39</td>
<td>138.57</td>
<td>-1.10</td>
</tr>
<tr>
<td>0.0556</td>
<td>0.0526</td>
<td>-0.00046</td>
<td>5.63</td>
<td>0.000045</td>
<td>0.62</td>
<td>138.57</td>
<td>-0.705</td>
</tr>
<tr>
<td>0.1271</td>
<td>0.1112</td>
<td>-0.000742</td>
<td>11.40</td>
<td>0.000547</td>
<td>8.18</td>
<td>145.45</td>
<td>-0.7097</td>
</tr>
<tr>
<td>0.1944</td>
<td>0.1649</td>
<td>-0.000768</td>
<td>12.38</td>
<td>0.000661</td>
<td>10.62</td>
<td>157.59</td>
<td>-1.52</td>
</tr>
<tr>
<td>0.2690</td>
<td>0.260</td>
<td>-0.000753</td>
<td>12.29</td>
<td>0.000644</td>
<td>10.51</td>
<td>163.30</td>
<td>-1.74</td>
</tr>
<tr>
<td>0.3408</td>
<td>0.2665</td>
<td>-0.000934</td>
<td>15.61</td>
<td>0.000683</td>
<td>14.43</td>
<td>167.23</td>
<td>-24.16</td>
</tr>
<tr>
<td>0.3802</td>
<td>0.2945</td>
<td>-0.001142</td>
<td>16.72</td>
<td>0.000854</td>
<td>16.17</td>
<td>168.69</td>
<td>-23.96</td>
</tr>
<tr>
<td>TEST THERMOCOUPLE OUTPUT (MV)</td>
<td>( TA(1,1) )</td>
<td>( TA(1,2) )</td>
<td>( TA(1,1) )</td>
<td>( TA(1,2) )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td>-------------</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 4.2389 4.2403 7 4.2861 4.2882</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 4.224 4.212 8 4.254 4.253</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 4.284 4.2695 9 4.283 4.2834</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 4.2749 4.2729 10 4.2845 4.2866</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 4.2898 4.2893 11 4.3042 4.3048</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 4.2612 4.2754 12 4.3039 4.3078</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.300
FLOWMETER TEMP (MV) = 0.6968 BULK EXIT TEMP (MV) = 4.5213
INLET PRESSURE MAN, LEFT 3.50 RIGHT 3.70 IN. HG
INLET BULK TEMP CR-AL = 14.020
STATIC PRESSURE DROP (IN.)
\( P1-P2 = -0.097 \)
\( P1-P3 = 0.023 \)
\( P1-P4 = 0.052 \)
\( P1-P5 = 0.038 \)
\( P1-P6 = 0.129 \)
BLUE HANDHELD FLUID SP GR = 0.797
INLET TEMP/REACTURE (DEG. F) = 652.14
\( \text{In}/\text{Out} = 0.604 \)
\( \text{MACH NO.} = 0.012 \)
\( \text{PR} = 0.715623 \)

<table>
<thead>
<tr>
<th>( x^+ )</th>
<th>( (x^+)^2 )</th>
<th>( q^+ )</th>
<th>( \text{NUM} )</th>
<th>( \text{TBULK} )</th>
<th>( \text{In}/\text{Out} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4229</td>
<td>0.312</td>
<td>0.0093</td>
<td>0.1914</td>
<td>223.67</td>
<td>0.980</td>
</tr>
<tr>
<td>0.3435</td>
<td>0.253</td>
<td>0.0458</td>
<td>0.2723</td>
<td>244.37</td>
<td>0.952</td>
</tr>
<tr>
<td>0.2591</td>
<td>0.224</td>
<td>0.1904</td>
<td>0.3734</td>
<td>279.41</td>
<td>0.908</td>
</tr>
<tr>
<td>0.1776</td>
<td>0.1495</td>
<td>0.3184</td>
<td>0.5036</td>
<td>335.45</td>
<td>0.845</td>
</tr>
<tr>
<td>0.0961</td>
<td>0.0864</td>
<td>0.3113</td>
<td>0.7767</td>
<td>426.56</td>
<td>0.789</td>
</tr>
<tr>
<td>0.0147</td>
<td>0.0144</td>
<td>0.3157</td>
<td>0.8157</td>
<td>4783.6</td>
<td>0.694</td>
</tr>
</tbody>
</table>

DIMENSIONALIZED PRESSURE DROP
<table>
<thead>
<tr>
<th>( x^+ )</th>
<th>( P^+ )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0450</td>
<td>-0.067</td>
</tr>
<tr>
<td>0.1264</td>
<td>0.217</td>
</tr>
<tr>
<td>0.2979</td>
<td>0.485</td>
</tr>
<tr>
<td>0.2894</td>
<td>0.822</td>
</tr>
<tr>
<td>0.3718</td>
<td>1.219</td>
</tr>
<tr>
<td>0.4154</td>
<td>1.419</td>
</tr>
</tbody>
</table>

POSITION OF FIRST PRESSURE TAP = 0.010608
<table>
<thead>
<tr>
<th>( x^+ )</th>
<th>( (x^+)^2 )</th>
<th>( F )</th>
<th>( F(\text{RE}) )</th>
<th>( \text{FP} )</th>
<th>( \text{FP}(\text{RE}) )</th>
<th>( \text{FP}(\text{RE}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0149</td>
<td>0.0188</td>
<td>0.00135</td>
<td>1.53</td>
<td>-0.00374</td>
<td>-4.25</td>
<td>1134.4</td>
</tr>
<tr>
<td>0.0639</td>
<td>0.0099</td>
<td>0.00487</td>
<td>5.91</td>
<td>0.00100</td>
<td>1.22</td>
<td>1037.8</td>
</tr>
<tr>
<td>0.1454</td>
<td>0.1693</td>
<td>0.03849</td>
<td>11.18</td>
<td>0.00635</td>
<td>6.37</td>
<td>1537.6</td>
</tr>
<tr>
<td>0.2268</td>
<td>0.1662</td>
<td>0.00910</td>
<td>12.70</td>
<td>0.00770</td>
<td>10.74</td>
<td>1739.4</td>
</tr>
<tr>
<td>0.3038</td>
<td>0.2429</td>
<td>0.09078</td>
<td>17.74</td>
<td>0.00753</td>
<td>10.94</td>
<td>1951.6</td>
</tr>
<tr>
<td>0.3908</td>
<td>0.2988</td>
<td>0.13245</td>
<td>15.57</td>
<td>0.00960</td>
<td>14.31</td>
<td>1976.1</td>
</tr>
<tr>
<td>0.4347</td>
<td>0.3300</td>
<td>0.00997</td>
<td>14.99</td>
<td>0.00945</td>
<td>14.21</td>
<td>1804.8</td>
</tr>
</tbody>
</table>

DEG F
### Parabolic Inlet Velocity Test No. 20

**Tare Test Taken From Run No. 16**

**Tare Thermocouple Output (MV)**

<table>
<thead>
<tr>
<th>I</th>
<th>TA(1,1)</th>
<th>TA(1,2)</th>
<th>TA(1,3)</th>
<th>TA(1,4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2323</td>
<td>4.2321</td>
<td>4.2768</td>
<td>4.2771</td>
</tr>
<tr>
<td>2</td>
<td>4.2180</td>
<td>4.2156</td>
<td>4.2749</td>
<td>4.2785</td>
</tr>
<tr>
<td>3</td>
<td>4.2685</td>
<td>4.2686</td>
<td>4.2793</td>
<td>4.2739</td>
</tr>
<tr>
<td>4</td>
<td>4.2739</td>
<td>4.2719</td>
<td>4.2837</td>
<td>4.2524</td>
</tr>
<tr>
<td>5</td>
<td>4.2864</td>
<td>4.2846</td>
<td>4.2966</td>
<td>4.2969</td>
</tr>
<tr>
<td>6</td>
<td>4.2741</td>
<td>4.2688</td>
<td>4.2908</td>
<td>4.2937</td>
</tr>
</tbody>
</table>

**Test Thermocouple Output (MV)**

<table>
<thead>
<tr>
<th>I</th>
<th>TT(1,1)</th>
<th>TT(1,2)</th>
<th>TT(1,3)</th>
<th>TT(1,4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>2</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>3</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>4</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>5</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
<tr>
<td>6</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
<td>4.0000</td>
</tr>
</tbody>
</table>

**Differential Pressure = Flowmeter (In.)** = 0.394

**Flowmeter Temp (MV) = 0.8760**

**Bulk Exit Temp (MV) = 4.2753**

**Inlet Pressure Man., Left = 3.70** Right = 3.85 In. Hg

**Inlet Bulk Temp CR-AL = 7.985**

**Static Pressure Drop (In.)**

- P1-P2 = 0.017
- P1-P3 = 0.045
- P1-P4 = 0.103
- P1-P5 = 0.151
- P1-P6 = 0.206

**Blue Manometer Fluid Sp Gr = 0.797**

**Inlet Temperature (Deg. F.)** = 397.35

**TW/TH = 0.783** Mach No. = 0.014

**PR = 0.70695**

**Revd. = 1063.8**

### Pressure Drop

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)M</th>
<th>3+</th>
<th>NIM</th>
<th>Tbulk</th>
<th>TW/TH (REL.)</th>
<th>DEG. F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.293</td>
<td>0.247</td>
<td>0.043</td>
<td>373</td>
<td>214.23</td>
<td>0.983</td>
<td>794.3</td>
</tr>
<tr>
<td>0.235</td>
<td>0.242</td>
<td>0.036</td>
<td>345</td>
<td>226.61</td>
<td>0.987</td>
<td>125.1</td>
</tr>
<tr>
<td>0.178</td>
<td>0.156</td>
<td>0.120</td>
<td>477.6</td>
<td>274.34</td>
<td>0.952</td>
<td>173.0</td>
</tr>
<tr>
<td>0.122</td>
<td>0.121</td>
<td>0.121</td>
<td>749</td>
<td>69.87</td>
<td>0.920</td>
<td>184.9</td>
</tr>
<tr>
<td>0.066</td>
<td>0.251</td>
<td>0.251</td>
<td>4.183</td>
<td>328.51</td>
<td>0.874</td>
<td>174.2</td>
</tr>
<tr>
<td>0.011</td>
<td>0.000</td>
<td>0.610</td>
<td>6.310</td>
<td>380.25</td>
<td>0.480</td>
<td>108.7</td>
</tr>
</tbody>
</table>

**Non-dimensionalized Pressure Drop**

<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
<th>0.0309</th>
<th>0.089</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0069</td>
<td>0.338</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1429</td>
<td>0.766</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1989</td>
<td>1.122</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2549</td>
<td>1.932</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2958</td>
<td>1.775</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Position Dr. First Pressure Tap = 0.007292**

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)M</th>
<th>F</th>
<th>F(RE)M</th>
<th>FP</th>
<th>F(RE)</th>
<th>FP</th>
<th>TBUFLK</th>
<th>TBFLK</th>
<th>1B/TF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0139</td>
<td>0.0129</td>
<td>0.00358</td>
<td>8.00</td>
<td>0.00102</td>
<td>3.71</td>
<td>1074.9</td>
<td>1046.3</td>
<td>6.794</td>
<td></td>
</tr>
<tr>
<td>0.0439</td>
<td>0.0471</td>
<td>0.00529</td>
<td>10.20</td>
<td>0.00342</td>
<td>6.76</td>
<td>1740.2</td>
<td>1740.2</td>
<td>9.843</td>
<td></td>
</tr>
<tr>
<td>0.0999</td>
<td>0.0971</td>
<td>0.00711</td>
<td>17.91</td>
<td>0.00609</td>
<td>13.21</td>
<td>1787.8</td>
<td>1787.8</td>
<td>55.9</td>
<td></td>
</tr>
<tr>
<td>0.1559</td>
<td>0.1489</td>
<td>0.00755</td>
<td>14.48</td>
<td>0.00707</td>
<td>13.21</td>
<td>1787.8</td>
<td>1787.8</td>
<td>55.9</td>
<td></td>
</tr>
<tr>
<td>0.2119</td>
<td>0.1649</td>
<td>0.00790</td>
<td>15.06</td>
<td>0.00732</td>
<td>13.95</td>
<td>1787.8</td>
<td>1787.8</td>
<td>55.9</td>
<td></td>
</tr>
<tr>
<td>0.2679</td>
<td>0.2300</td>
<td>0.00837</td>
<td>14.20</td>
<td>0.00798</td>
<td>15.44</td>
<td>1924.6</td>
<td>1924.6</td>
<td>211.4</td>
<td></td>
</tr>
<tr>
<td>0.2988</td>
<td>0.2549</td>
<td>0.00893</td>
<td>17.73</td>
<td>0.00879</td>
<td>15.73</td>
<td>1945.7</td>
<td>1945.7</td>
<td>713.7</td>
<td></td>
</tr>
</tbody>
</table>

**Inlet Pressure = 3.70 In. Hg**

**Static Pressure Drop = 0.017 In.**

**Flowmeter Temp = 0.8760**

**Bulk Exit Temp = 4.2753**

**Inlet Temperature = 397.35 Deg. F.**

**TW/TH = 0.783**

**Mach No. = 0.014**

**PR = 0.70695**

**Revd. = 1063.8**
PARABOLIC INLET VELOCITY TEST NO. 14
TARE TEST TAKEN FROM RUN NO. 4
TARE THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2389</td>
<td>4.2403</td>
<td>7</td>
<td>4.2861</td>
<td>4.2882</td>
</tr>
<tr>
<td>2</td>
<td>4.2224</td>
<td>4.2212</td>
<td>8</td>
<td>4.2854</td>
<td>4.2893</td>
</tr>
<tr>
<td>3</td>
<td>4.2684</td>
<td>4.2695</td>
<td>9</td>
<td>4.2837</td>
<td>4.2804</td>
</tr>
<tr>
<td>4</td>
<td>4.2749</td>
<td>4.2729</td>
<td>10</td>
<td>4.2845</td>
<td>4.2665</td>
</tr>
<tr>
<td>5</td>
<td>4.2698</td>
<td>4.2893</td>
<td>11</td>
<td>4.3042</td>
<td>4.3048</td>
</tr>
<tr>
<td>6</td>
<td>4.2512</td>
<td>4.2754</td>
<td>12</td>
<td>4.3039</td>
<td>4.3028</td>
</tr>
</tbody>
</table>

TEST THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2327</td>
<td>4.2317</td>
<td>7</td>
<td>4.2800</td>
<td>4.2600</td>
</tr>
<tr>
<td>2</td>
<td>4.2189</td>
<td>4.2176</td>
<td>8</td>
<td>4.2910</td>
<td>4.2910</td>
</tr>
<tr>
<td>3</td>
<td>4.2059</td>
<td>4.2093</td>
<td>9</td>
<td>4.2923</td>
<td>4.3009</td>
</tr>
<tr>
<td>4</td>
<td>4.2211</td>
<td>4.2178</td>
<td>10</td>
<td>4.2999</td>
<td>4.3084</td>
</tr>
<tr>
<td>5</td>
<td>4.2330</td>
<td>4.2858</td>
<td>11</td>
<td>4.3655</td>
<td>4.3570</td>
</tr>
<tr>
<td>6</td>
<td>4.2737</td>
<td>4.2732</td>
<td>12</td>
<td>4.3601</td>
<td>4.3039</td>
</tr>
</tbody>
</table>

DIFFERENTIAL PRESSURE - FLOWMETER (IN. ) = 0.251
FLOWMETER RATE (MV) = 0.8295 BULK EXIT TEMP (MV) = 4.5960
INLET PRESSURE MAN. LEFT 3.79 RIGHT 3.85 IN. HG
INLET BULK TEMP CR-AL = 13.971

STATIC PRESSURE DROP (IN. )
P1-P2 = -0.003 P1-P3 = 0.022 P1-P4 = 0.049
P1-P5 = 0.079 P1-P6 = 0.101 P1-P6 = 0.130

BLUE MANOMETER FLUID SP GR 0.797
INLET TEMPERATURE (DEG. F. ) = 65.02

X/TU = 0.609 MACH NO. = 0.010
PR = 1.715437 REYD = 918.0

NON DIMENSIONALIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+M)</th>
<th>M+</th>
<th>R+</th>
<th>NUM</th>
<th>T BULK</th>
<th>X/TU</th>
<th>AREN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5151</td>
<td>0.3917</td>
<td>0.2254</td>
<td>0.1080</td>
<td>225.16</td>
<td>0.977</td>
<td>153.4</td>
<td></td>
</tr>
<tr>
<td>0.4151</td>
<td>0.3199</td>
<td>0.2484</td>
<td>0.0926</td>
<td>230.79</td>
<td>0.963</td>
<td>158.3</td>
<td></td>
</tr>
<tr>
<td>0.3962</td>
<td>0.2602</td>
<td>0.1883</td>
<td>0.0853</td>
<td>264.55</td>
<td>0.926</td>
<td>164.6</td>
<td></td>
</tr>
<tr>
<td>0.2168</td>
<td>0.1796</td>
<td>0.1627</td>
<td>0.0785</td>
<td>315.65</td>
<td>0.865</td>
<td>132.2</td>
<td></td>
</tr>
<tr>
<td>0.1176</td>
<td>0.1242</td>
<td>0.2627</td>
<td>0.3545</td>
<td>401.80</td>
<td>0.776</td>
<td>155.2</td>
<td></td>
</tr>
<tr>
<td>0.0179</td>
<td>0.0176</td>
<td>0.7351</td>
<td>4.3993</td>
<td>597.89</td>
<td>0.639</td>
<td>94.0</td>
<td></td>
</tr>
</tbody>
</table>

POSITION ON FIRST PRESSURE TAP = 0.012949

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+M)</th>
<th>F</th>
<th>F(RED)</th>
<th>F培</th>
<th>F培(RE)</th>
<th>F培(RE)</th>
<th>T BULK</th>
<th>X/TU</th>
<th>AREN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0231</td>
<td>0.0229</td>
<td>0.00174</td>
<td>1.62</td>
<td>-0.00387</td>
<td>-3.61</td>
<td>0.947</td>
<td>515.2</td>
<td>0.625</td>
<td></td>
</tr>
<tr>
<td>0.0780</td>
<td>0.0725</td>
<td>0.00624</td>
<td>6.27</td>
<td>0.00203</td>
<td>2.09</td>
<td>1405.8</td>
<td>0.797</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>0.1774</td>
<td>0.1519</td>
<td>0.01077</td>
<td>11.80</td>
<td>0.00363</td>
<td>9.45</td>
<td>1451.7</td>
<td>154.5</td>
<td>0.25</td>
<td></td>
</tr>
<tr>
<td>0.2769</td>
<td>0.2239</td>
<td>0.01120</td>
<td>12.99</td>
<td>0.01902</td>
<td>11.02</td>
<td>1597.7</td>
<td>168.6</td>
<td>0.99</td>
<td></td>
</tr>
<tr>
<td>0.3764</td>
<td>0.2934</td>
<td>0.01031</td>
<td>12.78</td>
<td>0.00947</td>
<td>11.37</td>
<td>1601.3</td>
<td>240.2</td>
<td>0.749</td>
<td></td>
</tr>
<tr>
<td>0.4758</td>
<td>0.3638</td>
<td>0.01199</td>
<td>14.67</td>
<td>0.01133</td>
<td>14.48</td>
<td>1223.7</td>
<td>227.2</td>
<td>0.975</td>
<td></td>
</tr>
<tr>
<td>0.5307</td>
<td>0.4440</td>
<td>0.01289</td>
<td>15.85</td>
<td>0.01266</td>
<td>15.56</td>
<td>1290.0</td>
<td>194.9</td>
<td>0.981</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>TA(I,1)</td>
<td>TA(I,2)</td>
<td>TA(I,1)</td>
<td>TA(I,2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>----</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td>---------</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4.241</td>
<td>4.245</td>
<td>4.258</td>
<td>4.259</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4.233</td>
<td>4.229</td>
<td>4.262</td>
<td>4.264</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.252</td>
<td>4.251</td>
<td>4.256</td>
<td>4.256</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.232</td>
<td>4.259</td>
<td>4.250</td>
<td>4.252</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4.270</td>
<td>4.270</td>
<td>4.285</td>
<td>4.287</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.250</td>
<td>4.268</td>
<td>4.310</td>
<td>4.310</td>
</tr>
<tr>
<td>2</td>
<td>4.251</td>
<td>4.250</td>
<td>4.325</td>
<td>4.325</td>
</tr>
<tr>
<td>3</td>
<td>4.274</td>
<td>4.281</td>
<td>4.336</td>
<td>4.352</td>
</tr>
<tr>
<td>4</td>
<td>4.238</td>
<td>4.292</td>
<td>4.338</td>
<td>4.359</td>
</tr>
<tr>
<td>5</td>
<td>4.300</td>
<td>4.307</td>
<td>4.562</td>
<td>4.673</td>
</tr>
<tr>
<td>6</td>
<td>4.256</td>
<td>4.294</td>
<td>4.607</td>
<td>4.761</td>
</tr>
</tbody>
</table>

### Differential Pressure - Flowmeter (IN.) = 0.576

Flowmeter infl (IN) = 0.8599 Bulk Exit Temp (IN.) = 6.3303

Inlet Pressure Max Left 5.80 Right 6.00 IN. HG

Inlet Bulk Temp CRAL = 19.192

Static Pressure Drop (IN.)

\[ p_{1-P2} = -0.045 \]
\[ p_{1-P3} = 0.012 \]
\[ p_{1-P4} = 0.190 \]
\[ p_{1-P5} = 0.317 \]

Blue Manometer Fluid Sp Gr = 0.797

Inlet Temperature (Deg. F.) = 1875.64

\[ T_{WTU} = 0.502 \]

Mach No. = 0.023

\[ PR = 0.740267 \]

Reyu = 1943.0

### Non-Dimensionalized Pressure Drop

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)* M</th>
<th>Q+</th>
<th>Num</th>
<th>TBulk</th>
<th>TWTU</th>
<th>(RELM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.252</td>
<td>-0.111</td>
<td>0.959</td>
<td>7.473</td>
<td>286.50</td>
<td>0.693</td>
<td>257.45</td>
</tr>
<tr>
<td>0.189</td>
<td>-0.148</td>
<td>0.115</td>
<td>7.215</td>
<td>275.21</td>
<td>0.913</td>
<td>267.96</td>
</tr>
<tr>
<td>0.144</td>
<td>-0.110</td>
<td>0.178</td>
<td>9.825</td>
<td>283.31</td>
<td>0.904</td>
<td>257.90</td>
</tr>
<tr>
<td>0.099</td>
<td>-0.078</td>
<td>0.227</td>
<td>11.95</td>
<td>321.32</td>
<td>0.661</td>
<td>259.54</td>
</tr>
<tr>
<td>0.033</td>
<td>-0.047</td>
<td>0.517</td>
<td>8.970</td>
<td>414.69</td>
<td>0.771</td>
<td>249.42</td>
</tr>
<tr>
<td>0.042</td>
<td>-0.079</td>
<td>2.225</td>
<td>14.373</td>
<td>679.30</td>
<td>0.604</td>
<td>277.94</td>
</tr>
</tbody>
</table>

### Position of Test Pressure Tap = 0.005912

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)* M</th>
<th>F</th>
<th>F(RELM)</th>
<th>FP</th>
<th>FP(RELM)</th>
<th>CREM</th>
<th>TBulk</th>
<th>TWTU</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0126</td>
<td>0.3103</td>
<td>0.00238</td>
<td>4.87</td>
<td>0.006756</td>
<td>-11.80</td>
<td>2058.2</td>
<td>21.3</td>
<td>0.574</td>
</tr>
<tr>
<td>0.0356</td>
<td>0.0520</td>
<td>0.00477</td>
<td>10.01</td>
<td>-0.001102</td>
<td>2.32</td>
<td>2269.7</td>
<td>504.2</td>
<td>0.697</td>
</tr>
<tr>
<td>0.0810</td>
<td>0.0064</td>
<td>0.00727</td>
<td>18.22</td>
<td>0.00526</td>
<td>13.19</td>
<td>2506.2</td>
<td>350.3</td>
<td>0.821</td>
</tr>
<tr>
<td>0.1264</td>
<td>0.0236</td>
<td>0.00763</td>
<td>20.10</td>
<td>0.00706</td>
<td>18.61</td>
<td>2738.1</td>
<td>297.6</td>
<td>0.867</td>
</tr>
<tr>
<td>0.1718</td>
<td>0.0124</td>
<td>0.00711</td>
<td>19.08</td>
<td>0.00669</td>
<td>17.97</td>
<td>2454.7</td>
<td>276.3</td>
<td>0.913</td>
</tr>
<tr>
<td>0.2172</td>
<td>0.0115</td>
<td>0.00738</td>
<td>17.77</td>
<td>0.00788</td>
<td>21.11</td>
<td>2741.7</td>
<td>276.5</td>
<td>0.909</td>
</tr>
<tr>
<td>0.2423</td>
<td>0.0142</td>
<td>0.00754</td>
<td>20.05</td>
<td>0.00797</td>
<td>21.19</td>
<td>2504.3</td>
<td>287.5</td>
<td>0.999</td>
</tr>
<tr>
<td>I</td>
<td>TAI1</td>
<td>TAI2</td>
<td>TAI1</td>
<td>TAI2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td>------</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4.2441</td>
<td>4.2452</td>
<td>7</td>
<td>4.2585</td>
<td>4.2594</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4.2323</td>
<td>4.2290</td>
<td>8</td>
<td>4.2620</td>
<td>4.2645</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.2526</td>
<td>4.2521</td>
<td>9</td>
<td>4.2562</td>
<td>4.2500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.2032</td>
<td>4.2598</td>
<td>10</td>
<td>4.2550</td>
<td>4.2525</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4.2705</td>
<td>4.2700</td>
<td>11</td>
<td>4.2865</td>
<td>4.2879</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4.2592</td>
<td>4.2533</td>
<td>12</td>
<td>4.2854</td>
<td>4.2843</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>I</th>
<th>TAI1</th>
<th>TAI2</th>
<th>TAI1</th>
<th>TAI2</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2562</td>
<td>4.2609</td>
<td>7</td>
<td>4.2868</td>
</tr>
<tr>
<td>2</td>
<td>4.2456</td>
<td>4.2451</td>
<td>8</td>
<td>4.3054</td>
</tr>
<tr>
<td>3</td>
<td>4.2659</td>
<td>4.2699</td>
<td>9</td>
<td>4.3314</td>
</tr>
<tr>
<td>4</td>
<td>4.2764</td>
<td>4.2792</td>
<td>10</td>
<td>4.3407</td>
</tr>
<tr>
<td>5</td>
<td>4.2054</td>
<td>4.2819</td>
<td>11</td>
<td>4.4286</td>
</tr>
<tr>
<td>6</td>
<td>4.2716</td>
<td>4.2726</td>
<td>12</td>
<td>4.4384</td>
</tr>
</tbody>
</table>

**DIFFERENTIAL PRESSURE - FLOWMETER (IN.)** = 0.505

**FLOWMETER TEMP (MV)** = 0.875

**BULK EXIT TEMP (MV)** = 5.8791

**INLET PRESSURE (MN.)**

<table>
<thead>
<tr>
<th>R</th>
<th>LEFT</th>
<th>RIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1-P2</td>
<td>-0.057</td>
<td>P1-P3</td>
</tr>
<tr>
<td>P1-P4</td>
<td>-0.057</td>
<td>P1-P5</td>
</tr>
</tbody>
</table>

**INLET TEMPERATURE DEG. F.** = 874.13

**X/W** = ...0.503 NACH NN. = ...0.021

**PR** = ...7.409...71 REYD = 1700.8

<table>
<thead>
<tr>
<th>X+</th>
<th>(A+M)</th>
<th>Q+</th>
<th>NUM</th>
<th>TBULK</th>
<th>TH/TR</th>
<th>(RE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0286</td>
<td>0.2254</td>
<td>0.0941</td>
<td>4.4070</td>
<td>276.94</td>
<td>0.909</td>
<td>2315.5</td>
</tr>
<tr>
<td>0.2169</td>
<td>0.1730</td>
<td>0.0777</td>
<td>4.4720</td>
<td>326.17</td>
<td>0.954</td>
<td>2293.5</td>
</tr>
<tr>
<td>0.1665</td>
<td>0.1289</td>
<td>0.1122</td>
<td>4.4720</td>
<td>397.37</td>
<td>0.783</td>
<td>2132.0</td>
</tr>
<tr>
<td>0.1131</td>
<td>0.1913</td>
<td>0.1267</td>
<td>4.3256</td>
<td>499.05</td>
<td>0.709</td>
<td>1994.3</td>
</tr>
<tr>
<td>0.0612</td>
<td>0.0084</td>
<td>0.6232</td>
<td>4.2617</td>
<td>647.85</td>
<td>0.603</td>
<td>1874.7</td>
</tr>
<tr>
<td>0.0093</td>
<td>0.0094</td>
<td>1.1856</td>
<td>4.6779</td>
<td>891.49</td>
<td>0.503</td>
<td>1975.0</td>
</tr>
</tbody>
</table>

**NON DIMENSIONALIZED PRESSURE DROP**

<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0286</td>
<td>-0.175</td>
</tr>
<tr>
<td>0.0875</td>
<td>-0.039</td>
</tr>
<tr>
<td>0.1374</td>
<td>0.173</td>
</tr>
<tr>
<td>0.1843</td>
<td>0.384</td>
</tr>
<tr>
<td>0.2362</td>
<td>0.624</td>
</tr>
<tr>
<td>0.2648</td>
<td>0.761</td>
</tr>
</tbody>
</table>

**LEAK III PRESSURE TAP # 1.**

**POSITION OF FIRST PRESSURE TAP = 0.006756**

<table>
<thead>
<tr>
<th>X+</th>
<th>(A+M)</th>
<th>F</th>
<th>F(RE)M</th>
<th>FP</th>
<th>I(P(RE))</th>
<th>(RE)M</th>
<th>TBULK</th>
<th>IB/IT</th>
<th>(RE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0121</td>
<td>0.6121</td>
<td>0.00222</td>
<td>7.93</td>
<td>0.00186</td>
<td>-3.29</td>
<td>1706.2</td>
<td>2318.6</td>
<td>271.9</td>
<td>0.491</td>
</tr>
<tr>
<td>0.0467</td>
<td>0.3400</td>
<td>0.00322</td>
<td>8.39</td>
<td>0.00326</td>
<td>3.96</td>
<td>1906.1</td>
<td>2318.6</td>
<td>271.9</td>
<td>0.491</td>
</tr>
<tr>
<td>0.0920</td>
<td>0.2057</td>
<td>0.0727</td>
<td>13.95</td>
<td>0.00439</td>
<td>8.42</td>
<td>1906.1</td>
<td>2318.6</td>
<td>271.9</td>
<td>0.491</td>
</tr>
<tr>
<td>0.1449</td>
<td>0.1255</td>
<td>0.00782</td>
<td>16.14</td>
<td>0.00568</td>
<td>11.72</td>
<td>2002.8</td>
<td>2318.6</td>
<td>271.9</td>
<td>0.491</td>
</tr>
<tr>
<td>0.1963</td>
<td>0.1097</td>
<td>0.0642</td>
<td>14.06</td>
<td>0.00472</td>
<td>10.35</td>
<td>2139.0</td>
<td>2318.6</td>
<td>271.9</td>
<td>0.491</td>
</tr>
<tr>
<td>0.2482</td>
<td>0.1538</td>
<td>0.00758</td>
<td>17.46</td>
<td>0.00640</td>
<td>14.92</td>
<td>2307.0</td>
<td>2318.6</td>
<td>271.9</td>
<td>0.491</td>
</tr>
<tr>
<td>0.2769</td>
<td>0.2115</td>
<td>0.00865</td>
<td>20.12</td>
<td>0.00685</td>
<td>17.32</td>
<td>2352.9</td>
<td>2748.9</td>
<td>0.914</td>
<td></td>
</tr>
</tbody>
</table>
# PARABOLIC INLET VELOCITY TEST NO. 24

<table>
<thead>
<tr>
<th>I</th>
<th>TAI(I,1)</th>
<th>TAI(I,2)</th>
<th>I</th>
<th>TAI(I,1)</th>
<th>TAI(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2441</td>
<td>4.2452</td>
<td>7</td>
<td>4.2585</td>
<td>4.2594</td>
</tr>
<tr>
<td>2</td>
<td>4.2323</td>
<td>4.2290</td>
<td>8</td>
<td>4.2620</td>
<td>4.2645</td>
</tr>
<tr>
<td>3</td>
<td>4.2526</td>
<td>4.2521</td>
<td>9</td>
<td>4.2562</td>
<td>4.2500</td>
</tr>
<tr>
<td>4</td>
<td>4.2632</td>
<td>4.2598</td>
<td>10</td>
<td>4.2550</td>
<td>4.2524</td>
</tr>
<tr>
<td>5</td>
<td>4.2705</td>
<td>4.2700</td>
<td>11</td>
<td>4.2865</td>
<td>4.2879</td>
</tr>
<tr>
<td>6</td>
<td>4.2592</td>
<td>4.2533</td>
<td>12</td>
<td>4.2854</td>
<td>4.2843</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.268</td>
<td>4.2730</td>
<td>7</td>
<td>4.3050</td>
<td>4.3050</td>
</tr>
<tr>
<td>2</td>
<td>4.2525</td>
<td>4.2566</td>
<td>8</td>
<td>4.3164</td>
<td>4.3164</td>
</tr>
<tr>
<td>3</td>
<td>4.2698</td>
<td>4.2700</td>
<td>9</td>
<td>4.3300</td>
<td>4.3300</td>
</tr>
<tr>
<td>4</td>
<td>4.2632</td>
<td>4.2636</td>
<td>10</td>
<td>4.3467</td>
<td>4.3474</td>
</tr>
<tr>
<td>5</td>
<td>4.2942</td>
<td>4.3056</td>
<td>11</td>
<td>4.4222</td>
<td>4.4850</td>
</tr>
<tr>
<td>6</td>
<td>4.2750</td>
<td>4.2959</td>
<td>12</td>
<td>4.4330</td>
<td>4.5048</td>
</tr>
</tbody>
</table>

**Differential Pressure -Flowmeter (IN.) = 0.451**

**Flowmeter Temp (MV) = 0.8501**

**Bulk Exit Temp (MV) = 5.6150**

**Inlet Pressure Man, Left 3.90 Right 4.10 IN. HG**

**Inlet Bulk Temp CR-AL = 19.545**

**Static Pressure Drop (IN.)**

\[ P_1 - P_2 = -0.020 \]

\[ P_1 - P_3 = -0.008 \]

\[ P_1 - P_4 = 0.033 \]

\[ P_1 - P_5 = 0.087 \]

\[ P_1 - P_6 = 0.151 \]

**Blue Manometer Fluid SP CR 0.797**

**Inlet Temperature (Deg. F) = 895.78**

**Tw/To = 0.497. Mach No. = 0.020**

**Pr = 0.742642**

**Reyn = 1509.3**

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)*M</th>
<th>Q+</th>
<th>NUM</th>
<th>TBulk</th>
<th>Tw/To (REL)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3020</td>
<td>0.2287</td>
<td>0.1063</td>
<td>7.4717</td>
<td>264.90</td>
<td>0.920</td>
</tr>
<tr>
<td>0.2437</td>
<td>0.1933</td>
<td>0.1399</td>
<td>4.9133</td>
<td>321.00</td>
<td>0.855</td>
</tr>
<tr>
<td>0.1854</td>
<td>0.1546</td>
<td>0.1710</td>
<td>3.5238</td>
<td>387.11</td>
<td>0.792</td>
</tr>
<tr>
<td>0.1271</td>
<td>0.1121</td>
<td>0.3244</td>
<td>0.3244</td>
<td>473.54</td>
<td>0.720</td>
</tr>
<tr>
<td>0.0688</td>
<td>0.0643</td>
<td>0.4367</td>
<td>3.5695</td>
<td>596.65</td>
<td>0.639</td>
</tr>
<tr>
<td>0.0105</td>
<td>0.0104</td>
<td>1.1010</td>
<td>5.2187</td>
<td>807.97</td>
<td>0.536</td>
</tr>
</tbody>
</table>

**Non-dimensionalized Pressure Drop**

<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
<th>-0.073</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0322</td>
<td>-0.029</td>
<td></td>
</tr>
<tr>
<td>0.1488</td>
<td>0.122</td>
<td></td>
</tr>
<tr>
<td>0.2071</td>
<td>0.317</td>
<td></td>
</tr>
<tr>
<td>0.2654</td>
<td>0.549</td>
<td></td>
</tr>
<tr>
<td>0.2975</td>
<td>0.686</td>
<td></td>
</tr>
</tbody>
</table>

**Position of First Pressure Tap = 0.007591**

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)*M</th>
<th>F</th>
<th>F(RE)</th>
<th>FP</th>
<th>FP(RE)</th>
<th>M</th>
<th>REL</th>
<th>TBulk</th>
<th>1B/1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0135</td>
<td>0.0135</td>
<td>0.00163</td>
<td>7.51</td>
<td>0.00335</td>
<td>-5.15</td>
<td>1534.8</td>
<td>320.7</td>
<td>0.527</td>
<td></td>
</tr>
<tr>
<td>0.0457</td>
<td>0.0041</td>
<td>0.00286</td>
<td>4.64</td>
<td>0.00102</td>
<td>-1.65</td>
<td>1619.4</td>
<td>370.5</td>
<td>0.595</td>
<td></td>
</tr>
<tr>
<td>0.1040</td>
<td>0.0045</td>
<td>0.0072</td>
<td>8.23</td>
<td>0.00234</td>
<td>4.08</td>
<td>1745.2</td>
<td>328.0</td>
<td>0.680</td>
<td></td>
</tr>
<tr>
<td>0.1623</td>
<td>0.1194</td>
<td>0.00610</td>
<td>11.33</td>
<td>0.00427</td>
<td>7.94</td>
<td>1859.1</td>
<td>427.1</td>
<td>0.757</td>
<td></td>
</tr>
<tr>
<td>0.2206</td>
<td>0.1797</td>
<td>0.00708</td>
<td>13.92</td>
<td>0.00531</td>
<td>10.43</td>
<td>1964.7</td>
<td>351.9</td>
<td>0.828</td>
<td></td>
</tr>
<tr>
<td>0.2789</td>
<td>0.2163</td>
<td>0.00793</td>
<td>16.36</td>
<td>0.00655</td>
<td>13.51</td>
<td>2062.6</td>
<td>293.4</td>
<td>0.892</td>
<td></td>
</tr>
<tr>
<td>0.3111</td>
<td>0.2354</td>
<td>0.00842</td>
<td>17.76</td>
<td>0.00704</td>
<td>14.88</td>
<td>2113.2</td>
<td>260.4</td>
<td>0.925</td>
<td></td>
</tr>
</tbody>
</table>
### Parabolic Inlet Velocity Test No. 25

**Tare Test Taken From Run No. 22**

**Tare Thermocouple Output (HV)**

<table>
<thead>
<tr>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2441</td>
<td>4.2452</td>
<td>4.2585</td>
<td>4.2594</td>
</tr>
<tr>
<td>2</td>
<td>4.2423</td>
<td>4.2470</td>
<td>4.2620</td>
<td>4.2645</td>
</tr>
<tr>
<td>3</td>
<td>4.2526</td>
<td>4.2521</td>
<td>4.2502</td>
<td>4.2506</td>
</tr>
<tr>
<td>4</td>
<td>4.2632</td>
<td>4.2598</td>
<td>4.2550</td>
<td>4.2525</td>
</tr>
<tr>
<td>5</td>
<td>4.2705</td>
<td>4.2700</td>
<td>4.2865</td>
<td>4.2879</td>
</tr>
<tr>
<td>6</td>
<td>4.2592</td>
<td>4.2533</td>
<td>4.2854</td>
<td>4.2443</td>
</tr>
</tbody>
</table>

**Test Thermocouple Output (HV)**

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2600</td>
<td>4.2664</td>
<td>4.2954</td>
<td>4.2954</td>
</tr>
<tr>
<td>2</td>
<td>4.2466</td>
<td>4.2494</td>
<td>4.3071</td>
<td>4.3071</td>
</tr>
<tr>
<td>3</td>
<td>4.2642</td>
<td>4.2712</td>
<td>4.3194</td>
<td>4.3484</td>
</tr>
<tr>
<td>4</td>
<td>4.2639</td>
<td>4.2768</td>
<td>4.3189</td>
<td>4.3192</td>
</tr>
<tr>
<td>5</td>
<td>4.2556</td>
<td>4.2955</td>
<td>4.4126</td>
<td>4.4184</td>
</tr>
<tr>
<td>6</td>
<td>4.2698</td>
<td>4.2754</td>
<td>4.4270</td>
<td>4.4948</td>
</tr>
</tbody>
</table>

**Differential Pressure - Flowmeter (IN.) = 0.40**

**Flowmeter Temp (MV) = 0.9475 Bulk Exit Temp (MV) = 5.4448**

**Inlet Pressure Man., Left 3.25 Right 3.35 IN. HG**

**Inlet Bulk Temp CR-AL = 19.485**

**Static Pressure Drop (IN.)**

- P1-P2 = -0.0152 P1-P3 = -0.009 P1-P4 = -0.041
- P1-P5 = -0.0319 P1-P6 = -0.179

**Blue Hand Temperature Fluid Sp Gr = 0.797**

**Inlet Temperature (Deg. F) = 888.21**

**In/TU = 0.498 Mach No. = 0.018**

**PR = 0.741292 Reynolds = 1341.7**

### nondimensionalized pressure drop

**x+**

<table>
<thead>
<tr>
<th>X+ (x+M)</th>
<th>Q+</th>
<th>NUM</th>
<th>TRLNK</th>
<th>TWH/TH (PLM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3399</td>
<td>0.2557</td>
<td>0.3064</td>
<td>7.0674</td>
<td>261.78 0.922 1188.0</td>
</tr>
<tr>
<td>0.2742</td>
<td>0.2156</td>
<td>0.1239</td>
<td>4.4415</td>
<td>300.79 0.871 1184.7</td>
</tr>
<tr>
<td>0.2086</td>
<td>0.1129</td>
<td>0.1896</td>
<td>4.8146</td>
<td>376.51 0.802 1715.0</td>
</tr>
<tr>
<td>0.1438</td>
<td>0.1538</td>
<td>0.2815</td>
<td>4.7149</td>
<td>469.12 0.723 1689.6</td>
</tr>
<tr>
<td>0.0774</td>
<td>0.0726</td>
<td>0.4393</td>
<td>3.4752</td>
<td>604.12 0.633 1491.2</td>
</tr>
<tr>
<td>0.0018</td>
<td>0.0118</td>
<td>1.0158</td>
<td>4.5037</td>
<td>840.01 0.522 1459.9</td>
</tr>
</tbody>
</table>

**Position 0: First Pressure Tap = 0.008543**

<table>
<thead>
<tr>
<th>X+ (X+M)</th>
<th>F</th>
<th>F(RE)M</th>
<th>FP</th>
<th>FP(RE)M (PE)M</th>
<th>TRLNK</th>
<th>TB/TH (PLM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0152</td>
<td>0.0152</td>
<td>0.36149</td>
<td>1.95</td>
<td>-0.00378</td>
<td>-5.11</td>
<td>1350.6 858.0 0.509</td>
</tr>
<tr>
<td>0.0515</td>
<td>0.0499</td>
<td>0.00359</td>
<td>5.12</td>
<td>-0.00059</td>
<td>-0.84</td>
<td>1428.5 699.8 0.579</td>
</tr>
<tr>
<td>0.1171</td>
<td>0.1635</td>
<td>0.06631</td>
<td>9.31</td>
<td>0.00334</td>
<td>5.18</td>
<td>1569.3 529.3 0.679</td>
</tr>
<tr>
<td>0.1827</td>
<td>0.1561</td>
<td>0.06616</td>
<td>11.24</td>
<td>0.00281</td>
<td>8.00</td>
<td>1661.2 418.7 0.765</td>
</tr>
<tr>
<td>0.2483</td>
<td>0.2034</td>
<td>0.0697</td>
<td>12.23</td>
<td>0.00527</td>
<td>9.29</td>
<td>1761.3 340.4 0.839</td>
</tr>
<tr>
<td>0.3139</td>
<td>0.2144</td>
<td>0.20845</td>
<td>15.62</td>
<td>0.00371</td>
<td>13.50</td>
<td>1847.7 282.8 0.903</td>
</tr>
<tr>
<td>0.3591</td>
<td>0.2032</td>
<td>0.26326</td>
<td>17.70</td>
<td>0.00776</td>
<td>16.64</td>
<td>1892.9 254.9 0.934</td>
</tr>
<tr>
<td>I</td>
<td>TA(I,1)</td>
<td>TA(I,2)</td>
<td>I</td>
<td>TA(I,1)</td>
<td>TA(I,2)</td>
<td></td>
</tr>
<tr>
<td>---</td>
<td>---------</td>
<td>---------</td>
<td>---</td>
<td>---------</td>
<td>---------</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>4.2491</td>
<td>4.2492</td>
<td>7</td>
<td>4.2585</td>
<td>4.2494</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>4.2323</td>
<td>4.2290</td>
<td>8</td>
<td>4.2608</td>
<td>4.2608</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>4.2625</td>
<td>4.2521</td>
<td>9</td>
<td>4.2562</td>
<td>4.2500</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4.2632</td>
<td>4.2598</td>
<td>10</td>
<td>4.2550</td>
<td>4.2525</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>4.2705</td>
<td>4.2700</td>
<td>11</td>
<td>4.2865</td>
<td>4.2879</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>4.2922</td>
<td>4.2533</td>
<td>12</td>
<td>4.2854</td>
<td>4.2843</td>
<td></td>
</tr>
</tbody>
</table>

**TEST THERMOCOUPLE OUTPUT (MV)**

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2566</td>
<td>4.2615</td>
<td>7</td>
<td>4.2884</td>
<td>4.2884</td>
</tr>
<tr>
<td>2</td>
<td>4.2460</td>
<td>4.2434</td>
<td>8</td>
<td>4.3015</td>
<td>4.3015</td>
</tr>
<tr>
<td>3</td>
<td>4.2677</td>
<td>4.2734</td>
<td>9</td>
<td>4.3134</td>
<td>4.3376</td>
</tr>
<tr>
<td>4</td>
<td>4.2774</td>
<td>4.2805</td>
<td>10</td>
<td>4.3178</td>
<td>4.3331</td>
</tr>
<tr>
<td>5</td>
<td>4.2869</td>
<td>4.2925</td>
<td>11</td>
<td>4.4057</td>
<td>4.4581</td>
</tr>
<tr>
<td>6</td>
<td>4.2716</td>
<td>4.2748</td>
<td>12</td>
<td>4.4124</td>
<td>4.4724</td>
</tr>
</tbody>
</table>

**DIFFERENTIAL PRESSURE - FLOWMETER (MV) = 0.353**

**FLOWMETER TEMP (MV) = 0.8676 BULK EXIT TEMP (MV) = 5.177**

**INLET PRESSURE MAN, LEFT 3.15 RIGHT 3.30 IN., HG**

**INLET BULK TEMP CR-AL = 19.124**

**STATIC PRESSURE DROP (IN.)**

<table>
<thead>
<tr>
<th>P1-P2</th>
<th>P1-P3</th>
<th>P1-P4</th>
<th>P1-P5</th>
<th>P1-P6</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.011</td>
<td>0.007</td>
<td>0.046</td>
<td>0.138</td>
<td>0.167</td>
</tr>
</tbody>
</table>

**BLUE MANOMETER FLUID SP GR 0.797**

**INLET TEMPERATURE (DEG., F) = 872.71**

**T/W/TU = 0.504 MACH NO. = 0.016**

**PR = 0.739086 REYN = 1190.8**

### NON DIMENSIONALIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+M)</th>
<th>Q+</th>
<th>NUM</th>
<th>T/BULK</th>
<th>T/W/TB</th>
<th>REYN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3840</td>
<td>0.2656</td>
<td>0.0675</td>
<td>-0.1067</td>
<td>751.82</td>
<td>0.942</td>
<td>1684.5</td>
</tr>
<tr>
<td>0.3082</td>
<td>0.2912</td>
<td>0.1159</td>
<td>-0.2044</td>
<td>297.24</td>
<td>0.886</td>
<td>1616.3</td>
</tr>
<tr>
<td>0.2357</td>
<td>0.1927</td>
<td>0.1527</td>
<td>3.1357</td>
<td>356.38</td>
<td>0.822</td>
<td>1639.4</td>
</tr>
<tr>
<td>0.1616</td>
<td>0.1396</td>
<td>-0.0094</td>
<td>4.0094</td>
<td>435.83</td>
<td>0.750</td>
<td>1533.1</td>
</tr>
<tr>
<td>0.0875</td>
<td>0.0604</td>
<td>0.3347</td>
<td>3.1135</td>
<td>550.53</td>
<td>0.666</td>
<td>1355.2</td>
</tr>
<tr>
<td>0.0134</td>
<td>0.0131</td>
<td>0.9371</td>
<td>4.7688</td>
<td>785.67</td>
<td>0.553</td>
<td>1231.4</td>
</tr>
</tbody>
</table>

**INLET TEMPERATURE (DEG., F) = 872.71**

**T/W/TU = 0.504 MACH NO. = 0.016**

**PR = 0.739086 REYN = 1190.8**

### NON DIMENSIONALIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
<th>(X+M)</th>
<th>Q+</th>
<th>NUM</th>
<th>T/BULK</th>
<th>T/W/TB</th>
<th>REYN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0419</td>
<td>-0.064</td>
<td>0.1150</td>
<td>0.041</td>
<td>0.1882</td>
<td>0.266</td>
<td>0.309</td>
<td></td>
</tr>
<tr>
<td>0.3374</td>
<td>0.797</td>
<td>0.3783</td>
<td>0.962</td>
<td>0.00658</td>
<td>1.36</td>
<td>0.0136</td>
<td></td>
</tr>
</tbody>
</table>

**INLET TEMPERATURE (DEG., F) = 872.71**

**T/W/TU = 0.504 MACH NO. = 0.016**

**PR = 0.739086 REYN = 1190.8**
**PARABOLIC INLET VELOCITY TEST NO. 27**

**I ARE TEST I AKFN FROM RUN NO. 22**

**I ARE THERMOCUPLUE OUTPUT (MV)**

<table>
<thead>
<tr>
<th></th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2441</td>
<td>4.2452</td>
<td>4.2585</td>
<td>4.2594</td>
</tr>
<tr>
<td>2</td>
<td>4.2323</td>
<td>4.2290</td>
<td>4.2608</td>
<td>4.2608</td>
</tr>
<tr>
<td>3</td>
<td>4.2526</td>
<td>4.2521</td>
<td>4.2562</td>
<td>4.2500</td>
</tr>
<tr>
<td>4</td>
<td>4.2632</td>
<td>4.2598</td>
<td>4.2550</td>
<td>4.2525</td>
</tr>
<tr>
<td>5</td>
<td>4.2705</td>
<td>4.2700</td>
<td>4.2865</td>
<td>4.2879</td>
</tr>
<tr>
<td>6</td>
<td>4.2592</td>
<td>4.2533</td>
<td>4.2854</td>
<td>4.2843</td>
</tr>
</tbody>
</table>

**I TEST THERMOCUPLUE OUTPUT (MV)**

<table>
<thead>
<tr>
<th></th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2580</td>
<td>4.2616</td>
<td>4.2848</td>
<td>4.2848</td>
</tr>
<tr>
<td>2</td>
<td>4.2633</td>
<td>4.2498</td>
<td>4.2855</td>
<td>4.2978</td>
</tr>
<tr>
<td>3</td>
<td>4.2655</td>
<td>4.2700</td>
<td>4.3061</td>
<td>4.3274</td>
</tr>
<tr>
<td>4</td>
<td>4.2734</td>
<td>4.2754</td>
<td>4.3121</td>
<td>4.3275</td>
</tr>
<tr>
<td>5</td>
<td>4.2840</td>
<td>4.2875</td>
<td>4.4010</td>
<td>4.4500</td>
</tr>
<tr>
<td>6</td>
<td>4.2702</td>
<td>4.2728</td>
<td>4.4075</td>
<td>4.4643</td>
</tr>
</tbody>
</table>

**DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.300**

**FLOWMETER TEMP. (MV) = 0.8576**

**BULK EXIT TEMP. (MV) = 4.7630**

**INLET PRESSURE: MAN. LEFT 3.20 RIGHT 3.35 IN. HG**

**STATIC PRESSURE DROP (IN.)**

<table>
<thead>
<tr>
<th>P1-P2</th>
<th>-0.008</th>
<th>P1-P3</th>
<th>0.015</th>
<th>P1-P4</th>
<th>0.040</th>
</tr>
</thead>
<tbody>
<tr>
<td>P1-P5</td>
<td>-0.032</td>
<td>P1-P5</td>
<td>0.123</td>
<td>P1-P6</td>
<td>0.147</td>
</tr>
</tbody>
</table>

**BLUE MANOMETER FLUID SP GR = 0.797**

**INLET TEMPERATURE (DEG. F.) = 87.740**

**TW/T0 = 0.502**

**MACH NO. = 0.013**

**PR = 0.740497**

**X+**

<table>
<thead>
<tr>
<th>NUM</th>
<th>T(BULK)</th>
<th>TW/TB</th>
<th>REYM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4515</td>
<td>235.17</td>
<td>0.964</td>
<td>1456.2</td>
</tr>
<tr>
<td>0.3644</td>
<td>273.33</td>
<td>0.715</td>
<td>1104.7</td>
</tr>
<tr>
<td>0.2772</td>
<td>330.98</td>
<td>0.848</td>
<td>1393.6</td>
</tr>
<tr>
<td>0.1900</td>
<td>416.86</td>
<td>0.766</td>
<td>1272.4</td>
</tr>
<tr>
<td>0.1079</td>
<td>549.69</td>
<td>0.660</td>
<td>1153.5</td>
</tr>
<tr>
<td>0.0157</td>
<td>800.84</td>
<td>0.533</td>
<td>1035.1</td>
</tr>
</tbody>
</table>

**X+**

<table>
<thead>
<tr>
<th>NUM</th>
<th>T(BULK)</th>
<th>TW/TB</th>
<th>REYM</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4515</td>
<td>235.17</td>
<td>0.964</td>
<td>1456.2</td>
</tr>
<tr>
<td>0.3644</td>
<td>273.33</td>
<td>0.715</td>
<td>1104.7</td>
</tr>
<tr>
<td>0.2772</td>
<td>330.98</td>
<td>0.848</td>
<td>1393.6</td>
</tr>
<tr>
<td>0.1900</td>
<td>416.86</td>
<td>0.766</td>
<td>1272.4</td>
</tr>
<tr>
<td>0.1079</td>
<td>549.69</td>
<td>0.660</td>
<td>1153.5</td>
</tr>
<tr>
<td>0.0157</td>
<td>800.84</td>
<td>0.533</td>
<td>1035.1</td>
</tr>
</tbody>
</table>

**DIMENSIONALIZED PRESSURE DROP**

<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
<th>0.0481</th>
<th>-0.063</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1353</td>
<td>0.120</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2224</td>
<td>0.316</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3090</td>
<td>0.652</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3968</td>
<td>0.974</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4449</td>
<td>1.164</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**POSITION OF FIRST PRESSURE TAP = 0.011350**

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+M)</th>
<th>F</th>
<th>F(RE)</th>
<th>FP</th>
<th>FP(RE)</th>
<th>(RE)</th>
<th>TP</th>
<th>CLG</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0203</td>
<td>0.00202</td>
<td>0.00287</td>
<td>2.95</td>
<td>-0.000309</td>
<td>-3.17</td>
<td>1926.5</td>
<td>0.228</td>
<td>0.524</td>
</tr>
<tr>
<td>0.0684</td>
<td>0.0053</td>
<td>0.00494</td>
<td>5.31</td>
<td>0.00016</td>
<td>0.17</td>
<td>1996.6</td>
<td>648.3</td>
<td>0.066</td>
</tr>
<tr>
<td>0.1555</td>
<td>0.0176</td>
<td>0.00756</td>
<td>9.99</td>
<td>0.00476</td>
<td>5.73</td>
<td>1263.3</td>
<td>475.1</td>
<td>0.719</td>
</tr>
<tr>
<td>0.2427</td>
<td>0.0205</td>
<td>0.00917</td>
<td>11.87</td>
<td>0.000716</td>
<td>9.26</td>
<td>1294.7</td>
<td>369.4</td>
<td>0.010</td>
</tr>
<tr>
<td>0.3299</td>
<td>0.0257</td>
<td>0.01007</td>
<td>13.81</td>
<td>0.00815</td>
<td>11.17</td>
<td>1317.9</td>
<td>297.2</td>
<td>0.485</td>
</tr>
<tr>
<td>0.4170</td>
<td>0.0315</td>
<td>0.01103</td>
<td>15.78</td>
<td>0.00953</td>
<td>13.64</td>
<td>1430.7</td>
<td>252.1</td>
<td>0.494</td>
</tr>
<tr>
<td>0.4652</td>
<td>0.03610</td>
<td>0.01114</td>
<td>14.77</td>
<td>0.01015</td>
<td>14.79</td>
<td>1465.6</td>
<td>233.6</td>
<td>0.969</td>
</tr>
</tbody>
</table>
PARABOLIC INLET VELOCITY TEST NO. 28
TARE TEST TAKEN FROM RUN NO. 22
TAKE THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2441</td>
<td>4.2452</td>
<td>7</td>
<td>4.2585</td>
<td>4.2594</td>
</tr>
<tr>
<td>2</td>
<td>4.2323</td>
<td>4.2290</td>
<td>8</td>
<td>4.2608</td>
<td>4.2562</td>
</tr>
<tr>
<td>3</td>
<td>4.2526</td>
<td>4.2521</td>
<td>9</td>
<td>4.2562</td>
<td>4.2500</td>
</tr>
<tr>
<td>4</td>
<td>4.2632</td>
<td>4.2598</td>
<td>10</td>
<td>4.2550</td>
<td>4.2525</td>
</tr>
<tr>
<td>5</td>
<td>4.2705</td>
<td>4.2700</td>
<td>11</td>
<td>4.2865</td>
<td>4.2879</td>
</tr>
<tr>
<td>6</td>
<td>4.2592</td>
<td>4.2533</td>
<td>12</td>
<td>4.2854</td>
<td>4.2843</td>
</tr>
</tbody>
</table>

TEST THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2522</td>
<td>4.2556</td>
<td>7</td>
<td>4.2769</td>
<td>4.2769</td>
</tr>
<tr>
<td>2</td>
<td>4.2372</td>
<td>4.2366</td>
<td>8</td>
<td>4.2804</td>
<td>4.2926</td>
</tr>
<tr>
<td>3</td>
<td>4.2533</td>
<td>4.2568</td>
<td>9</td>
<td>4.2990</td>
<td>4.3140</td>
</tr>
<tr>
<td>4</td>
<td>4.2650</td>
<td>4.2654</td>
<td>10</td>
<td>4.3062</td>
<td>4.3198</td>
</tr>
<tr>
<td>5</td>
<td>4.2776</td>
<td>4.2803</td>
<td>11</td>
<td>4.3921</td>
<td>4.4362</td>
</tr>
<tr>
<td>6</td>
<td>4.2654</td>
<td>4.2662</td>
<td>12</td>
<td>4.4000</td>
<td>4.4484</td>
</tr>
</tbody>
</table>

DIFFERENTIAL PRESSURE - FLOW METER (IN.) = 0.281
FLOWMETER TEP (MV) = 0.867
BULK EXIT TEMP (MV) = 4.868

INLET PRESSURE - HAN, LEFT 3.45 RIGHT 3.35 IN. HG
INLET BULK TEMP CR-AL = 19.160

STATIC PRESSURE DROP (IN.)
P1-P2 = -0.007
P1-P3 = 0.019
P1-P4 = 0.042
P1-P5 = 0.016
P1-P6 = 0.090

BLUE MANOMETER FLUID SP GR 0.797
INLET TEMPERATURE (DEG. F) = 874.26
TW/TA = 0.503
MACH NO. = 0.011
PR = 0.740

\[
\begin{align*}
X &+ (X+)^M & 0+ & NUM & TRULK TW/TA (PR)M \\
\hline
0.5396 & 0.3570 & 0.0430 & & 237.66 & 0.960 & 124.7 \\
0.4356 & 0.3300 & 0.0714 & & 267.82 & 0.921 & 118.5 \\
0.3314 & 0.2633 & 0.1122 & & 321.05 & 0.859 & 112.7 \\
0.2272 & 0.1926 & 0.2612 & & 407.48 & 0.774 & 105.4 \\
0.1230 & 0.1126 & 0.3022 & & 548.20 & 0.667 & 96.5 \\
0.0108 & 0.0187 & 0.7712 & & 813.24 & 0.536 & 86.8 \\
\end{align*}
\]

NON DIMENSIONALIZED PRESSURE DROP

\[
\begin{align*}
X &+ P+ & 0.0575 & -0.083 \\
0.1617 & 0.220 & 0.2660 & 0.486 & 0.3762 & 0.871 & 0.4744 & 1.256 \\
0.5319 & 1.036 & 0.5396 & 0.3570 & 0.0430 & & 237.66 & 0.960 & 124.7 \\
0.4356 & 0.3300 & 0.0714 & & 267.82 & 0.921 & 118.5 \\
0.3314 & 0.2633 & 0.1122 & & 321.05 & 0.859 & 112.7 \\
0.2272 & 0.1926 & 0.2612 & & 407.48 & 0.774 & 105.4 \\
0.1230 & 0.1126 & 0.3022 & & 548.20 & 0.667 & 96.5 \\
0.0108 & 0.0187 & 0.7712 & & 813.24 & 0.536 & 86.8 \\
\end{align*}
\]

LEAK IN PRESSURE

TAP #7

POSITION OF FIRST PRESSURE TAP = 0.013570

\[
\hline
0.0242 & 0.0242 & 0.00413 & 3.53 & -0.00214 & -1.83 & 853.6 & 140.9 & 0.516 \\
0.0080 & 0.0080 & 0.00531 & 4.86 & 0.00033 & 0.30 & 914.3 & 654.4 & 0.603 \\
0.1865 & 0.1860 & 0.00924 & 7.24 & 0.00628 & 6.34 & 1010.4 & 419.0 & 0.724 \\
0.2922 & 0.2380 & 0.1309 & 14.28 & 0.01109 & 12.10 & 1091.3 & 459.0 & 0.820 \\
0.3944 & 0.2357 & 0.1225 & 14.14 & 0.01052 & 11.54 & 1154.6 & 290.9 & 0.895 \\
0.4980 & 0.3719 & 0.0121 & 14.04 & 0.00004 & 0.14 & 1198.4 & 256.2 & 0.946 \\
0.5561 & 0.4091 & 0.147 & 14.37 & 0.01160 & 14.31 & 1214.3 & 236.6 & 0.965 \\
\]
<table>
<thead>
<tr>
<th>I</th>
<th>T(A,1)</th>
<th>T(A,2)</th>
<th>T(A,1)</th>
<th>T(A,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2514</td>
<td>4.2527</td>
<td>7</td>
<td>4.2568</td>
</tr>
<tr>
<td>2</td>
<td>4.2570</td>
<td>4.2350</td>
<td>8</td>
<td>4.2588</td>
</tr>
<tr>
<td>3</td>
<td>4.2604</td>
<td>4.2590</td>
<td>9</td>
<td>4.2714</td>
</tr>
<tr>
<td>4</td>
<td>4.2713</td>
<td>4.2708</td>
<td>10</td>
<td>4.2768</td>
</tr>
<tr>
<td>5</td>
<td>4.2822</td>
<td>4.2818</td>
<td>11</td>
<td>4.3079</td>
</tr>
<tr>
<td>6</td>
<td>4.2731</td>
<td>4.2671</td>
<td>12</td>
<td>4.3064</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>I</th>
<th>T(B,1)</th>
<th>T(B,2)</th>
<th>T(B,1)</th>
<th>T(B,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2739</td>
<td>4.2824</td>
<td>7</td>
<td>4.3231</td>
</tr>
<tr>
<td>2</td>
<td>4.2621</td>
<td>4.2677</td>
<td>8</td>
<td>4.3221</td>
</tr>
<tr>
<td>3</td>
<td>4.2789</td>
<td>4.2890</td>
<td>9</td>
<td>4.3504</td>
</tr>
<tr>
<td>4</td>
<td>4.2909</td>
<td>4.2972</td>
<td>10</td>
<td>4.3573</td>
</tr>
<tr>
<td>5</td>
<td>4.3105</td>
<td>4.3270</td>
<td>11</td>
<td>4.5043</td>
</tr>
<tr>
<td>6</td>
<td>4.2892</td>
<td>4.2948</td>
<td>12</td>
<td>4.5019</td>
</tr>
</tbody>
</table>

**Differential Pressure - Flowmeter (in.)** = 0.461
Flowmeter Temp. (MV) = 0.8049
Bulk Exit Temp. (MV) = 5.9850
Inlet Pressure Man. Left = 4.10
Right = 4.30

**Inlet Pressure Drop (IN.)**:
PI-P2 = -0.038
PI-P3 = -0.002
PI-P4 = 0.006
PI-P5 = 0.097
PI-P6 = 0.163

**Blue Manometer Fluid Sp Gr.** = 0.797

**Inlet Temperature (Deg. F.)** = 118.18

**IN/TU = 0.409**
**Mach No.** = 0.022

**PR = 0.756**

**Reynolds = 1487.5**

**Non Dimensionalized Pressure Drop**

**Position of First Pressure Tap = 0.007265**

**PLG F**
PARABOLIC INLET VELOCITY TEST NO. 30
TARE TEST TAKEN FROM RUN NO. 29
TARE THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2514</td>
<td>4.2527</td>
<td>7</td>
<td>4.2568</td>
</tr>
<tr>
<td>2</td>
<td>4.2370</td>
<td>4.2350</td>
<td>8</td>
<td>4.2588</td>
</tr>
<tr>
<td>3</td>
<td>4.2604</td>
<td>4.2609</td>
<td>9</td>
<td>4.2714</td>
</tr>
<tr>
<td>4</td>
<td>4.2713</td>
<td>4.2708</td>
<td>10</td>
<td>4.2768</td>
</tr>
<tr>
<td>5</td>
<td>4.2622</td>
<td>4.2818</td>
<td>11</td>
<td>4.3079</td>
</tr>
<tr>
<td>6</td>
<td>4.2731</td>
<td>4.2671</td>
<td>12</td>
<td>4.3064</td>
</tr>
</tbody>
</table>

TEST THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2696</td>
<td>4.2761</td>
<td>7</td>
<td>4.3120</td>
</tr>
<tr>
<td>2</td>
<td>4.2568</td>
<td>4.2610</td>
<td>8</td>
<td>4.3157</td>
</tr>
<tr>
<td>3</td>
<td>4.2740</td>
<td>4.2828</td>
<td>9</td>
<td>4.3378</td>
</tr>
<tr>
<td>4</td>
<td>4.2660</td>
<td>4.2919</td>
<td>10</td>
<td>4.3459</td>
</tr>
<tr>
<td>5</td>
<td>4.2913</td>
<td>4.3036</td>
<td>11</td>
<td>4.4939</td>
</tr>
<tr>
<td>6</td>
<td>4.2728</td>
<td>4.2799</td>
<td>12</td>
<td>4.4784</td>
</tr>
</tbody>
</table>

DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.402
FLOWMETER TEMP (MV) = 0.8140
BULK EXIT TEMP (MV) = 5.5973
INLET PRESSURE MAN/LEFT 3.40
RIGHT 3.60
INLET BULK TEMP CR-AL = 26.325
STATIC PRESSURE DROP (IN.)
P1-P2 = -0.024
P1-P3 = -0.019
P1-P6 = 0.020
P1-P5 = 0.070
P1-P6 = 0.162
BLUE MANOMETER FLUID SP GR 0.797
INLET TEMPERATURE (DEG. F) = 1172.28
Tw/TD = 0.411
MACH NO. = 0.020
PR = 0.755439
KEYD = 1296.4

NON DIMENSIONALIZED PRESSURE DROP

X+ (X+M) Q+ NUM TRBULK TW/TB (RE)M

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+M)</th>
<th>Q+</th>
<th>NUM</th>
<th>TRBULK</th>
<th>TW/TB (RE)M</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3322</td>
<td>0.2547</td>
<td>0.0738</td>
<td>6.4213</td>
<td>271.57</td>
<td>0.917</td>
</tr>
<tr>
<td>0.2681</td>
<td>0.2195</td>
<td>0.1055</td>
<td>3.5906</td>
<td>348.51</td>
<td>0.831</td>
</tr>
<tr>
<td>0.2040</td>
<td>0.1784</td>
<td>0.1651</td>
<td>3.2029</td>
<td>443.76</td>
<td>0.743</td>
</tr>
<tr>
<td>0.1398</td>
<td>0.1309</td>
<td>0.1499</td>
<td>2.2162</td>
<td>565.70</td>
<td>0.656</td>
</tr>
<tr>
<td>0.0757</td>
<td>0.0746</td>
<td>0.4280</td>
<td>3.1092</td>
<td>733.12</td>
<td>0.565</td>
</tr>
<tr>
<td>0.0116</td>
<td>0.0117</td>
<td>1.1569</td>
<td>4.3422</td>
<td>1028.42</td>
<td>0.458</td>
</tr>
</tbody>
</table>

POSITION OF FIRST PRESSURE TAP = 0.008352
X+ (X+M) F F(RE)M FP IP(RE)M (RE)M TRBULK TB/I

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+M)</th>
<th>F</th>
<th>F(RE)M</th>
<th>FP</th>
<th>IP(RE)M</th>
<th>(RE)M</th>
<th>TRBULK</th>
<th>TB/I</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0149</td>
<td>0.0151</td>
<td>0.00225</td>
<td>2.95</td>
<td>-0.00368</td>
<td>-4.83</td>
<td>1311.9</td>
<td>1.055.5</td>
<td>0.443</td>
</tr>
<tr>
<td>0.0533</td>
<td>0.0507</td>
<td>0.00337</td>
<td>4.62</td>
<td>-0.00133</td>
<td>-1.02</td>
<td>1323.4</td>
<td>649.6</td>
<td>0.513</td>
</tr>
<tr>
<td>0.1444</td>
<td>0.1100</td>
<td>0.00513</td>
<td>7.61</td>
<td>0.00220</td>
<td>3.26</td>
<td>1482.7</td>
<td>641.3</td>
<td>0.610</td>
</tr>
<tr>
<td>0.1786</td>
<td>0.1616</td>
<td>0.00659</td>
<td>10.52</td>
<td>0.00427</td>
<td>3.94</td>
<td>1596.9</td>
<td>604.4</td>
<td>0.700</td>
</tr>
<tr>
<td>0.2427</td>
<td>0.2557</td>
<td>0.00779</td>
<td>13.35</td>
<td>0.00545</td>
<td>9.34</td>
<td>1714.2</td>
<td>593.2</td>
<td>0.788</td>
</tr>
<tr>
<td>0.3069</td>
<td>0.2434</td>
<td>0.0885</td>
<td>16.22</td>
<td>0.00704</td>
<td>12.92</td>
<td>1833.7</td>
<td>507.9</td>
<td>0.875</td>
</tr>
<tr>
<td>0.3423</td>
<td>0.2618</td>
<td>0.0979</td>
<td>17.87</td>
<td>0.00730</td>
<td>14.44</td>
<td>1900.1</td>
<td>267.8</td>
<td>0.923</td>
</tr>
</tbody>
</table>
PARABOLIC INLET VELOCITY TEST NO. 21
TARE TEST TAKEN FROM RUN NO. 29
TARE THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2514</td>
<td>4.2527</td>
<td>7</td>
<td>4.2568</td>
<td>4.2559</td>
</tr>
<tr>
<td>2</td>
<td>4.2370</td>
<td>4.2350</td>
<td>8</td>
<td>4.2588</td>
<td>4.2613</td>
</tr>
<tr>
<td>3</td>
<td>4.2604</td>
<td>4.2609</td>
<td>9</td>
<td>4.2714</td>
<td>4.2654</td>
</tr>
<tr>
<td>4</td>
<td>4.2713</td>
<td>4.2708</td>
<td>10</td>
<td>4.2768</td>
<td>4.2744</td>
</tr>
<tr>
<td>5</td>
<td>4.2822</td>
<td>4.2818</td>
<td>11</td>
<td>4.3079</td>
<td>4.3085</td>
</tr>
<tr>
<td>6</td>
<td>4.2731</td>
<td>4.2671</td>
<td>12</td>
<td>4.3064</td>
<td>4.3096</td>
</tr>
</tbody>
</table>

TEST THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2089</td>
<td>4.2731</td>
<td>7</td>
<td>4.3056</td>
<td>4.3056</td>
</tr>
<tr>
<td>2</td>
<td>4.2562</td>
<td>4.2582</td>
<td>8</td>
<td>4.3310</td>
<td>4.3310</td>
</tr>
<tr>
<td>3</td>
<td>4.2783</td>
<td>4.2839</td>
<td>9</td>
<td>4.3355</td>
<td>4.3710</td>
</tr>
<tr>
<td>4</td>
<td>4.2873</td>
<td>4.2908</td>
<td>10</td>
<td>4.3397</td>
<td>4.3665</td>
</tr>
<tr>
<td>5</td>
<td>4.2699</td>
<td>4.2966</td>
<td>11</td>
<td>4.4098</td>
<td>4.5752</td>
</tr>
<tr>
<td>6</td>
<td>4.2727</td>
<td>4.2782</td>
<td>12</td>
<td>4.4751</td>
<td>4.5669</td>
</tr>
</tbody>
</table>

DIFFERENTIAL PRESSURE - FLUXMETER (IN.) = 0.305
FLOWMETER TEMP. (MV) = 0.8158 BULK EXIT TEMP. (MV) = 5.409
INLET PRESSURE MAN. LEFT 3.50 RIGHT 3.65 IN. HG
INLET BULK TEMP CR-AL = 26.710
STATIC PRESSURE DROP (IN.)

\[ P_{1-2} = -0.017 \quad P_{1-P_3} = -0.008 \quad P_{1-P_4} = 0.032 \]
\[ P_{1-P_5} = 0.073 \quad P_{1-P_6} = 0.191 \]

BLUE MANOMETER FLUID SP GR = 0.797
INLET TEMPERATURE (DEG. F) = 1187.49
\[ \frac{TW}{TU} = 0.407 \quad MACH NO. = 0.015 \]
\[ PR = 0.7488 \quad KEYNO. = 982.3 \]

\[ X^+ \quad (X^+)_M \quad Q^+ \quad NUM. \quad TBULK \quad TW/TB \quad (RE)_M \]

\[ 0.4370 \quad 0.3217 \quad 0.0444 \quad \text{deg. f.} \]
\[ 0.3527 \quad 0.2777 \quad 0.0615 \quad 3.6631 \quad 298.79 \quad 0.085 \quad 1403.9 \]
\[ 0.2083 \quad 0.2243 \quad 0.1432 \quad 4.2488 \quad 372.86 \quad 0.806 \quad 1372.0 \]
\[ 0.1839 \quad 0.1561 \quad 0.1977 \quad 0.3184 \quad 495.72 \quad 0.704 \quad 1214.3 \]
\[ 0.0990 \quad 0.0574 \quad 0.0407 \quad 3.2735 \quad 697.47 \quad 0.583 \quad 1099.1 \]
\[ 0.0152 \quad 0.0154 \quad 1.1026 \quad 4.2987 \quad 185.32 \quad 0.441 \quad 792.3 \]

NON DIMENSIONALIZED PRESSURE DROP

\[ X^+ \quad P^+ \]
\[ 0.0466 \quad -0.105 \]
\[ 0.139 \quad -0.050 \]
\[ 0.2153 \quad 0.200 \]
\[ 0.2997 \quad 0.460 \]
\[ 0.3840 \quad 0.760 \]
\[ 0.4306 \quad 0.950 \]

POSITION DI FIRST PRESSURE TAP = 0.01086

\[ X^+ \quad (X^+)_M \quad F \quad F(RE)_M \quad FP \quad FP(RE)_M \quad (RE)_M \quad TBULK \quad TB/TB \]

\[ 0.0196 \quad 0.0198 \quad 0.00219 \quad 2.16 \quad -0.00526 \quad -5.19 \quad 987.0 \quad 111.5 \quad 0.426 \]
\[ 0.0062 \quad 0.0063 \quad 0.00458 \quad 4.76 \quad -0.00152 \quad 1.58 \quad 1040.8 \quad 850.3 \quad 0.513 \]
\[ 0.1566 \quad 0.1419 \quad 0.00864 \quad 9.30 \quad 0.00429 \quad 4.96 \quad 1156.3 \quad 583.2 \quad 0.644 \]
\[ 0.2349 \quad 0.2041 \quad 0.01003 \quad 12.74 \quad 0.00750 \quad 9.53 \quad 1269.9 \quad 426.6 \quad 0.758 \]
\[ 0.3193 \quad 0.2587 \quad 0.01031 \quad 14.89 \quad 0.00884 \quad 12.07 \quad 1346.7 \quad 320.7 \quad 0.850 \]
\[ 0.4036 \quad 0.3115 \quad 0.01185 \quad 16.97 \quad 0.01082 \quad 15.49 \quad 1491.7 \quad 275.1 \quad 0.914 \]
\[ 0.4587 \quad 0.3417 \quad 0.01321 \quad 18.24 \quad 0.01440 \quad 17.61 \quad 1585.8 \quad 257.5 \quad 0.937 \]
### PARABOLIC INLET VELOCITY TEST NO. 32

**TARE TEST TAKEN FROM RUN NO. 29**

**TARE THERMOCouple OUTPUT (MV)**

<table>
<thead>
<tr>
<th></th>
<th>DA(I,1)</th>
<th>DA(I,2)</th>
<th></th>
<th>DA(I,1)</th>
<th>DA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2514</td>
<td>4.2527</td>
<td>7</td>
<td>4.2568</td>
<td>4.2559</td>
</tr>
<tr>
<td>2</td>
<td>4.2370</td>
<td>4.2350</td>
<td>8</td>
<td>4.2588</td>
<td>4.2613</td>
</tr>
<tr>
<td>3</td>
<td>4.2604</td>
<td>4.2609</td>
<td>9</td>
<td>4.2714</td>
<td>4.2654</td>
</tr>
<tr>
<td>4</td>
<td>4.2713</td>
<td>4.2708</td>
<td>10</td>
<td>4.2768</td>
<td>4.2744</td>
</tr>
<tr>
<td>5</td>
<td>4.2822</td>
<td>4.2818</td>
<td>11</td>
<td>4.3079</td>
<td>4.3085</td>
</tr>
<tr>
<td>6</td>
<td>4.2731</td>
<td>4.2671</td>
<td>12</td>
<td>4.3064</td>
<td>4.3066</td>
</tr>
</tbody>
</table>

**TEST THERMOCouple OUTPUT (MV)**

<table>
<thead>
<tr>
<th></th>
<th>DA(I,1)</th>
<th>DA(I,2)</th>
<th></th>
<th>DA(I,1)</th>
<th>DA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2572</td>
<td>4.2608</td>
<td>7</td>
<td>4.2924</td>
<td>4.2924</td>
</tr>
<tr>
<td>2</td>
<td>4.2464</td>
<td>4.2464</td>
<td>8</td>
<td>4.3164</td>
<td>4.3164</td>
</tr>
<tr>
<td>3</td>
<td>4.2725</td>
<td>4.2776</td>
<td>9</td>
<td>4.3186</td>
<td>4.3527</td>
</tr>
<tr>
<td>4</td>
<td>4.2824</td>
<td>4.2836</td>
<td>10</td>
<td>4.3310</td>
<td>4.3546</td>
</tr>
<tr>
<td>5</td>
<td>4.2765</td>
<td>4.2821</td>
<td>11</td>
<td>4.4771</td>
<td>4.5564</td>
</tr>
<tr>
<td>6</td>
<td>4.2631</td>
<td>4.2705</td>
<td>12</td>
<td>4.4572</td>
<td>4.5475</td>
</tr>
</tbody>
</table>

### DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.304

**FLOWMETER ENP (MV) = 0.8088**

**BULK EXIT TEMP (MV) = 4.6971**

**INLET PRESSURE MAN. LEFT 3.65 RIGHT 3.80 IN. HG**

**INLET BULK TEMP CR-AL = 27.051**

**STATIC PRESSURE DROP (IN.)**

\[ P_1 - P_2 = -0.0016, P_1 - P_3 = -0.0003, P_1 - P_4 = 0.037 \]

\[ P_1 - P_5 = 0.073, P_1 - P_6 = 0.137 \]

**BLUE MANOMETER FLUID SP GR = 0.7979**

**INLET TEMPERATURE (DEG. F) = 1200.87**

\[ \frac{T}{T_0} = 0.404 \]

**MACH NO. = 0.015**

**PR = 0.750376**

**KEYD = 979.1**

### NON DIMENSIONALIZED PRESSURE DROP

**\( x^+ \)**

\[ \frac{\pi}{\theta^+} \]

\[ 0.3472, 0.3426, 0.0269, 0.00152, 235.41, 0.964, 1484.9 \]

\[ 0.3528, 0.2776, 0.0418, 0.00222, 295.71, 0.884, 1490.3 \]

\[ 0.2684, 0.2626, 0.1651, 0.00333, 381.35, 0.797, 1310.5 \]

\[ 0.1844, 0.1672, 0.0224, 0.00444, 503.90, 0.697, 1207.5 \]

\[ 0.0996, 0.0574, 0.3590, 2.9800, 688.61, 0.587, 1180.6 \]

\[ 0.0152, 0.0155, 1.3398, 4.3687, 1040.08, 0.454, 979.1 \]

### POSITION OF FIRST PRESSURE TAP = 0.010989

**\( x^+ \)**

\[ \frac{\pi}{\theta^+} \]

\[ 0.0199, 0.0199, 0.00213, 2.11, 0.00499, -0.95, 900.3, 107.8, 0.438 \]

\[ 0.0062, 0.0067, 0.00401, 4.82, 0.00108, -1.13, 1040.3, 92.7, 0.523 \]

\[ 0.1570, 0.1423, 0.0975, 9.16, 0.00454, 5.23, 1158.9, 88.4, 0.643 \]

\[ 0.2353, 0.2558, 0.00990, 12.07, 0.00710, 8.93, 1259.3, 437.6, 0.749 \]

\[ 0.3104, 0.2601, 0.01308, 13.68, 0.00763, 10.36, 1457.1, 336.6, 0.844 \]

\[ 0.4333, 0.3686, 0.01998, 15.42, 0.00892, 12.83, 1446.6, 262.7, 0.930 \]

\[ 0.4573, 0.3341, 0.00110, 1.47, 0.00321, 4.77, 1486.7, 232.7, 0.970 \]
<table>
<thead>
<tr>
<th>X+</th>
<th>(1.1)</th>
<th>Q+</th>
<th>NUM.</th>
<th>TBUKLK TW/TB (RE)M</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5244</td>
<td>0.3350</td>
<td>0.0202</td>
<td>0.0727</td>
<td>223.51</td>
</tr>
<tr>
<td>0.4232</td>
<td>0.3243</td>
<td>0.0339</td>
<td>0.3545</td>
<td>266.28</td>
</tr>
<tr>
<td>0.3219</td>
<td>0.2635</td>
<td>0.0993</td>
<td>0.6973</td>
<td>341.60</td>
</tr>
<tr>
<td>0.2257</td>
<td>0.1664</td>
<td>0.1833</td>
<td>3.2626</td>
<td>464.14</td>
</tr>
<tr>
<td>0.1190</td>
<td>0.1161</td>
<td>0.3314</td>
<td>2.9339</td>
<td>664.56</td>
</tr>
<tr>
<td>0.0182</td>
<td>0.0185</td>
<td>0.9573</td>
<td>3.9162</td>
<td>1059.97</td>
</tr>
</tbody>
</table>

**PR = 0.7518**, **REV = 815.5**

**X+ (1.1) M Q+ NUM. TBUKLK TW/TB (RE)M**

**NON DIMENSIONALIZED PRESSURE DROP.**

<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0557</td>
<td>0.073</td>
</tr>
<tr>
<td>0.1571</td>
<td>0.117</td>
</tr>
<tr>
<td>0.2583</td>
<td>0.383</td>
</tr>
<tr>
<td>0.3590</td>
<td>0.633</td>
</tr>
<tr>
<td>0.4608</td>
<td>0.996</td>
</tr>
<tr>
<td>0.5167</td>
<td>1.138</td>
</tr>
</tbody>
</table>

**POSITION OF FIRST PRESSURE TAP = 0.013182**

<table>
<thead>
<tr>
<th>X+</th>
<th>(1.1)</th>
<th>F</th>
<th>F(1.1)M</th>
<th>FP</th>
<th>FP(1.1)M</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0235</td>
<td>0.039</td>
<td>0.00351</td>
<td>2.89</td>
<td>-0.00437</td>
<td>-3.60</td>
</tr>
<tr>
<td>0.0794</td>
<td>0.040</td>
<td>0.00674</td>
<td>5.89</td>
<td>0.00033</td>
<td>0.29</td>
</tr>
<tr>
<td>0.1876</td>
<td>0.038</td>
<td>0.01038</td>
<td>10.15</td>
<td>0.00654</td>
<td>6.39</td>
</tr>
<tr>
<td>0.2819</td>
<td>0.029</td>
<td>0.01120</td>
<td>12.03</td>
<td>0.00853</td>
<td>9.71</td>
</tr>
<tr>
<td>0.3391</td>
<td>0.030</td>
<td>0.01101</td>
<td>12.83</td>
<td>0.00857</td>
<td>9.99</td>
</tr>
<tr>
<td>0.4343</td>
<td>0.022</td>
<td>0.01331</td>
<td>14.34</td>
<td>0.01173</td>
<td>14.41</td>
</tr>
<tr>
<td>0.542</td>
<td>0.0365</td>
<td>0.01440</td>
<td>14.74</td>
<td>0.01328</td>
<td>15.46</td>
</tr>
</tbody>
</table>

**DEG F**

| 0.0235 | 0.0039 | 0.00351 | 2.89 | -0.00437 | -3.60 |
| 0.0794 | 0.0040 | 0.00674 | 5.89 | 0.00033 | 0.29 |
| 0.1876 | 0.0038 | 0.01038 | 10.15 | 0.00654 | 6.39 |
| 0.2819 | 0.0029 | 0.01120 | 12.03 | 0.00853 | 9.71 |
| 0.3391 | 0.0030 | 0.01101 | 12.83 | 0.00857 | 9.99 |
| 0.4343 | 0.0022 | 0.01331 | 14.34 | 0.01173 | 14.41 |
| 0.542 | 0.00365 | 0.01440 | 14.74 | 0.01328 | 15.46 |

**DEG F**
### Differential Pressure - Flowmeter (IN.) = 0.408

<table>
<thead>
<tr>
<th>Test Thermocouple Output (MV)</th>
<th>Test Thermocouple Output (MV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>TA(I,1)</td>
</tr>
<tr>
<td>1</td>
<td>4.2480</td>
</tr>
<tr>
<td>2</td>
<td>4.2269</td>
</tr>
<tr>
<td>3</td>
<td>4.2369</td>
</tr>
<tr>
<td>4</td>
<td>4.2596</td>
</tr>
<tr>
<td>5</td>
<td>4.2628</td>
</tr>
<tr>
<td>6</td>
<td>4.2498</td>
</tr>
</tbody>
</table>

Differential Pressure - Flowmeter (IN.) = 0.408

Inlet Pressure Man., Left 3.60 Right 3.70 in. HG

### Static Pressure Drop (IN.)

\[ P_0 - P_2 = 0.102 \]
\[ P_1 - P_2 = 0.049 \]
\[ P_2 - P_3 = 0.074 \]
\[ P_2 - P_4 = 0.131 \]
\[ P_2 - P_5 = 0.189 \]
\[ P_2 - P_6 = 0.241 \]

### Fluid Speed

- Reynolds Number = 1455.7

### Mach Number

- Mach No. = 0.017

### Inlet Temperature (Deg. F.)

- Inlet Temp. = 688.22

### Fluid Properties

- Bulk Temp = 1445.7

### Dimensionalized Pressure Drop

\[ X^+ (\Delta P)^M \]
\[ Q^+ \]

### Inlet Temperature (Deg. F.)

- Inlet Temp. = 688.22

### Reynolds Number

- Reynolds Number = 1455.7

### Mach Number

- Mach No. = 0.017
### SIMULTANEOUS DEVELOPMENT TEST NO. 35

**TARE TEST TAKEN FROM RUN NO. 34**

**TARE THERMOCouple OUTPUT (MV)**

<table>
<thead>
<tr>
<th></th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2480</td>
<td>4.2506</td>
<td>4.2506</td>
<td>4.2534</td>
</tr>
<tr>
<td>2</td>
<td>4.2369</td>
<td>4.2332</td>
<td>4.2502</td>
<td>4.2555</td>
</tr>
<tr>
<td>3</td>
<td>4.2569</td>
<td>4.2571</td>
<td>4.2510</td>
<td>4.2406</td>
</tr>
<tr>
<td>4</td>
<td>4.2596</td>
<td>4.2591</td>
<td>4.2543</td>
<td>4.2539</td>
</tr>
<tr>
<td>5</td>
<td>4.2628</td>
<td>4.2613</td>
<td>4.2746</td>
<td>4.2777</td>
</tr>
<tr>
<td>6</td>
<td>4.2498</td>
<td>4.2452</td>
<td>4.2705</td>
<td>4.2780</td>
</tr>
</tbody>
</table>

### TEST THERMOCouple OUTPUT (MV)

<table>
<thead>
<tr>
<th></th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2488</td>
<td>4.2553</td>
<td>4.2506</td>
<td>4.2700</td>
</tr>
<tr>
<td>2</td>
<td>4.2401</td>
<td>4.2375</td>
<td>4.2639</td>
<td>4.2724</td>
</tr>
<tr>
<td>3</td>
<td>4.2569</td>
<td>4.2606</td>
<td>4.2664</td>
<td>4.2785</td>
</tr>
<tr>
<td>4</td>
<td>4.2599</td>
<td>4.2594</td>
<td>4.2711</td>
<td>4.2815</td>
</tr>
<tr>
<td>5</td>
<td>4.2662</td>
<td>4.2690</td>
<td>4.3580</td>
<td>4.3972</td>
</tr>
<tr>
<td>6</td>
<td>4.2519</td>
<td>4.2509</td>
<td>4.3613</td>
<td>4.4098</td>
</tr>
</tbody>
</table>

**DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.350**

**FLOWMETER TEMP (MV) = 0.9268 BULK EXIT TEMP (MV) = 4.3294**

**INLET PRESSURE MAN. LEFT 3.800 RIGHT 4.000 IN. HG**

**INLET BULK TEMP...CR-AL = 14.785**

**STATIC PRESSURE DROP (IN.)**

\[ p_0 - p_2 = 0.100 \]
\[ p_1 - p_2 = 0.031 \]
\[ p_2 - p_3 = 0.065 \]
\[ p_2 - p_4 = 0.125 \]
\[ p_2 - p_5 = 0.194 \]

**RED MANOMETER FLUID SP GR 0.626**

**INLET TEMPERATURE (DEG. F) = 605.19**

\[ T/W/T = 0.566 MACH. NO. = 0.014 \]

\[ PR = 0.718650 REYD = 1269.5 \]

### NON DIMENSIONALIZED PRESSURE DROP

\[ X+ (X+M) = 0.169 \]
\[ Q+ = 0.272 \]
\[ NUM = 0.946 \]
\[ TAULK = 0.942 \]
\[ T/W/T = 1657.6 \]

\[ T/W/T (REL) = 1712.0 \]

### PRESSURE DROP

\[ X+ (X+M) = 0.0169 \]
\[ F = 0.01107 \]
\[ F_0 = 0.00487 \]
\[ F_0 = 9.02 \]
\[ T/W/T = 1311.6 \]

\[ T/W/T = 578.5 \]

### FLOW RATE

\[ F_0 = 0.00487 \]
\[ F_0 = 11.60 \]
\[ F_0 = 1397.8 \]
\[ F_0 = 1471.0 \]
\[ F_0 = 10.93 \]

\[ F_0 = 10.90 \]
\[ F_0 = 1.08 \]
\[ F_0 = 0.920 \]
\[ F_0 = 0.988 \]
\[ F_0 = 0.947 \]

\[ F_0 = 0.993 \]
### SIMULTANEOUS DEVELOPMENT TEST NO. 36

**TAKE TEST TAKEN FROM RUN NO. 34**

**TAKE THERMOCUPLUE OUTPUT (MV)**

<table>
<thead>
<tr>
<th>Test</th>
<th>TA(1,1)</th>
<th>TA(1,2)</th>
<th>TA(1,1)</th>
<th>TA(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2480</td>
<td>4.2506</td>
<td>7</td>
<td>4.2506</td>
</tr>
<tr>
<td>2</td>
<td>4.2369</td>
<td>4.2332</td>
<td>8</td>
<td>4.2502</td>
</tr>
<tr>
<td>3</td>
<td>4.2569</td>
<td>4.2571</td>
<td>9</td>
<td>4.2510</td>
</tr>
<tr>
<td>4</td>
<td>4.2596</td>
<td>4.2591</td>
<td>10</td>
<td>4.2543</td>
</tr>
<tr>
<td>5</td>
<td>4.2628</td>
<td>4.2613</td>
<td>11</td>
<td>4.2746</td>
</tr>
<tr>
<td>6</td>
<td>4.2498</td>
<td>4.2452</td>
<td>12</td>
<td>4.2785</td>
</tr>
</tbody>
</table>

**TEST THERMOCUPLUE OUTPUT (MV)**

<table>
<thead>
<tr>
<th>Test</th>
<th>TT(1,1)</th>
<th>TT(1,2)</th>
<th>TT(1,1)</th>
<th>TT(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2542</td>
<td>4.2562</td>
<td>7</td>
<td>4.2613</td>
</tr>
<tr>
<td>2</td>
<td>4.2417</td>
<td>4.2399</td>
<td>8</td>
<td>4.2607</td>
</tr>
<tr>
<td>3</td>
<td>4.2538</td>
<td>4.2605</td>
<td>9</td>
<td>4.2681</td>
</tr>
<tr>
<td>4</td>
<td>4.2622</td>
<td>4.2614</td>
<td>10</td>
<td>4.2706</td>
</tr>
<tr>
<td>5</td>
<td>4.2538</td>
<td>4.2560</td>
<td>11</td>
<td>4.3518</td>
</tr>
<tr>
<td>6</td>
<td>4.2363</td>
<td>4.2359</td>
<td>12</td>
<td>4.3570</td>
</tr>
</tbody>
</table>

**DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.300**

**FLOWMETER TEMP (MV) = 0.9209**

**BULK EXIT TEMP (MV) = 4.2962**

**INLET/EXIT TEMP**<br>**LEFT 3.90 RIGHT 4.10 IN. HG**

**INLET BULK TEMP (CR-AL) = 14.788**

**STATIC PRESSURE DROP (IN.)**

- $p_0 - p_2 = 0.113$
- $p_1 - p_2 = 0.033$
- $p_2 - p_3 = 0.045$
- $p_2 - p_4 = 0.083$
- $p_2 - p_5 = 0.119$
- $p_2 - p_6 = 0.164$
- $p_2 - p_7 = 0.192$

**RED MANOMETER FLUID SP G = 0.826**

**INLET TEMPERATURE (DEG. F) = 685.32**

**TW/TO = 0.586**

**MACH NO. = 0.012**

**PR = 0.718069**

**RE**

### NON DIMENSIONALIZED PRESSURE DROP

**X+ ($x^+$) M Q+ NUM TBULK TW/TO (REL/M**

<table>
<thead>
<tr>
<th>$x^+$</th>
<th>M</th>
<th>Q+</th>
<th>NUM</th>
<th>TBULK</th>
<th>TW/TO (REL/M**</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.4389</td>
<td>0.3271</td>
<td>0.0163</td>
<td>216.45</td>
<td>0.991</td>
<td>141.69</td>
</tr>
<tr>
<td>0.3541</td>
<td>0.2716</td>
<td>0.0134</td>
<td>243.98</td>
<td>0.952</td>
<td>143.20</td>
</tr>
<tr>
<td>0.2693</td>
<td>0.2137</td>
<td>0.0093</td>
<td>279.32</td>
<td>0.906</td>
<td>138.67</td>
</tr>
<tr>
<td>0.1846</td>
<td>0.1527</td>
<td>0.0055</td>
<td>327.04</td>
<td>0.852</td>
<td>147.89</td>
</tr>
<tr>
<td>0.0998</td>
<td>0.0741</td>
<td>0.0032</td>
<td>398.50</td>
<td>0.782</td>
<td>125.85</td>
</tr>
<tr>
<td>0.0150</td>
<td>0.0143</td>
<td>0.8135</td>
<td>595.10</td>
<td>0.566</td>
<td>114.15</td>
</tr>
</tbody>
</table>

**X+ ($x^+$) M F F(REL/M** FP $F(P(REL/M** (REL/M** TBULK TW/TO**

<table>
<thead>
<tr>
<th>$x^+$</th>
<th>M</th>
<th>F</th>
<th>F(REL/M**</th>
<th>FP</th>
<th>F(P(REL/M**</th>
<th>(REL/M**</th>
<th>TBULK</th>
<th>TW/TO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0197</td>
<td>0.0190</td>
<td>0.01551</td>
<td>17.51</td>
<td>0.01097</td>
<td>12.39</td>
<td>1129.1</td>
<td>171.7</td>
<td>0.051</td>
</tr>
<tr>
<td>0.0665</td>
<td>0.0603</td>
<td>0.1325</td>
<td>16.03</td>
<td>0.01008</td>
<td>12.20</td>
<td>1209.9</td>
<td>153.3</td>
<td>0.735</td>
</tr>
<tr>
<td>0.1513</td>
<td>0.1285</td>
<td>0.00900</td>
<td>17.76</td>
<td>0.00834</td>
<td>10.81</td>
<td>1295.2</td>
<td>135.2</td>
<td>0.621</td>
</tr>
<tr>
<td>0.2361</td>
<td>0.1913</td>
<td>0.00900</td>
<td>17.21</td>
<td>0.00799</td>
<td>10.85</td>
<td>1357.4</td>
<td>101.1</td>
<td>0.883</td>
</tr>
<tr>
<td>0.3209</td>
<td>0.2505</td>
<td>0.01464</td>
<td>14.73</td>
<td>0.00933</td>
<td>13.13</td>
<td>1408.0</td>
<td>260.4</td>
<td>0.933</td>
</tr>
<tr>
<td>0.4057</td>
<td>0.3069</td>
<td>0.01310</td>
<td>19.70</td>
<td>0.01231</td>
<td>17.85</td>
<td>1450.6</td>
<td>229.4</td>
<td>0.975</td>
</tr>
<tr>
<td>0.4525</td>
<td>0.3372</td>
<td>0.14247</td>
<td>20.54</td>
<td>0.14207</td>
<td>14.71</td>
<td>215.1</td>
<td>0.995</td>
<td></td>
</tr>
</tbody>
</table>
THERE EXISTED A SIMULTANEOUS DELIVERY TEST NO. 37.

Differential \( DLV \) and \( HP \) Test are taken from RUN NO. 34.

**Thermocouple Output (MV)**

<table>
<thead>
<tr>
<th>I</th>
<th>( TA(1,1) )</th>
<th>( TA(1,2) )</th>
<th>( TA(1,1) )</th>
<th>( TA(1,2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2480</td>
<td>4.2506</td>
<td>7</td>
<td>4.2506</td>
</tr>
<tr>
<td>2</td>
<td>4.2369</td>
<td>4.2332</td>
<td>8</td>
<td>4.2502</td>
</tr>
<tr>
<td>3</td>
<td>4.2569</td>
<td>4.2571</td>
<td>9</td>
<td>4.2510</td>
</tr>
<tr>
<td>4</td>
<td>4.2596</td>
<td>4.2591</td>
<td>10</td>
<td>4.2543</td>
</tr>
<tr>
<td>5</td>
<td>4.2528</td>
<td>4.2613</td>
<td>11</td>
<td>4.2746</td>
</tr>
<tr>
<td>6</td>
<td>4.2498</td>
<td>4.2452</td>
<td>12</td>
<td>4.2785</td>
</tr>
</tbody>
</table>

**Test Thermocouple Output (MV)**

<table>
<thead>
<tr>
<th>I</th>
<th>( TT(1,1) )</th>
<th>( TT(1,2) )</th>
<th>( TT(1,1) )</th>
<th>( TT(1,2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2491</td>
<td>4.2508</td>
<td>7</td>
<td>4.2644</td>
</tr>
<tr>
<td>2</td>
<td>4.2551</td>
<td>4.2327</td>
<td>8</td>
<td>4.2644</td>
</tr>
<tr>
<td>3</td>
<td>4.2576</td>
<td>4.2579</td>
<td>9</td>
<td>4.2695</td>
</tr>
<tr>
<td>4</td>
<td>4.2607</td>
<td>4.2580</td>
<td>10</td>
<td>4.2724</td>
</tr>
<tr>
<td>5</td>
<td>4.2537</td>
<td>4.2545</td>
<td>11</td>
<td>4.3012</td>
</tr>
<tr>
<td>6</td>
<td>4.2387</td>
<td>4.2376</td>
<td>12</td>
<td>4.3550</td>
</tr>
</tbody>
</table>

**Differential Pressure Flowmeter (IN.)** = 0.250

Flowmeter \( F_p \) (MV) = 0.9384. Bulk Exit Temp (MV) = 0.260d

**Inlet Pressure Man. Left \( 4.10 \) Right \( 4.30 \) IN. HG**

**Inlet Bulk Temp CR-AL = 14.959**

**Static Pressure Drop (IN.)**

\( P_0 - P_2 = 0.076 \), \( P_1 - P_2 = 0.023 \), \( P_2 - P_3 = 0.035 \), \( P_2 - P_4 = 0.068 \), \( P_2 - P_5 = 0.127 \), \( P_2 - P_7 = 0.148 \).

**Red Manometer Fluid Sp Gr = 0.926**

**Inlet Temperature (Deg. F.) = 692.72**

**TW/TS = 0.582. Mach No. = 0.010**

**PR = 0.71937. Keyd. = 888.4.**

**X** | \( (X+M) \) | \( Q_+ \) | \( M_+ \) | \( TBULK \) | \( TW/TA \) (REL) | DEG. F. |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0237</td>
<td>0.0327</td>
<td>0.0206</td>
<td>4.1567</td>
<td>214.85</td>
<td>0.993</td>
<td>125.5</td>
</tr>
<tr>
<td>0.0766</td>
<td>0.3256</td>
<td>0.0295</td>
<td>2.6570</td>
<td>240.67</td>
<td>0.957</td>
<td>119.5</td>
</tr>
<tr>
<td>0.3246</td>
<td>0.2599</td>
<td>0.0349</td>
<td>3.1885</td>
<td>274.34</td>
<td>0.912</td>
<td>115.7</td>
</tr>
<tr>
<td>0.2227</td>
<td>0.1127</td>
<td>0.0344</td>
<td>3.9245</td>
<td>320.37</td>
<td>0.859</td>
<td>111.7</td>
</tr>
<tr>
<td>0.1252</td>
<td>0.1422</td>
<td>0.1837</td>
<td>2.8848</td>
<td>391.20</td>
<td>0.789</td>
<td>105.9</td>
</tr>
<tr>
<td>0.0181</td>
<td>0.0711</td>
<td>0.7012</td>
<td>5.2505</td>
<td>550.02</td>
<td>0.669</td>
<td>95.1</td>
</tr>
</tbody>
</table>

**Non Dimensionalized Pressure Drop**

<table>
<thead>
<tr>
<th>X</th>
<th>( P_+ )</th>
<th>( P_2 )</th>
<th>( P_4 )</th>
<th>( P_5 )</th>
<th>( P_7 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.023</td>
<td>0.215</td>
<td>0.543</td>
<td>1.035</td>
<td>1.504</td>
<td>1.903</td>
</tr>
<tr>
<td>0.048</td>
<td>2.348</td>
<td>0.543</td>
<td>1.035</td>
<td>1.504</td>
<td>1.903</td>
</tr>
<tr>
<td>0.545</td>
<td>2.641</td>
<td>0.823</td>
<td>1.504</td>
<td>1.903</td>
<td>1.903</td>
</tr>
</tbody>
</table>

**Non Dimensionalized Pressure Drop**

<table>
<thead>
<tr>
<th>X</th>
<th>( (X+M) )</th>
<th>( F )</th>
<th>( F(RE) )</th>
<th>( FP )</th>
<th>( FP(RE) )</th>
<th>( TPULK )</th>
<th>( TB/T )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0237</td>
<td>0.028</td>
<td>0.01519</td>
<td>14.28</td>
<td>0.01046</td>
<td>9.84</td>
<td>940.2</td>
<td></td>
</tr>
<tr>
<td>0.0482</td>
<td>0.0722</td>
<td>0.01413</td>
<td>14.28</td>
<td>0.01085</td>
<td>10.77</td>
<td>1049.6</td>
<td></td>
</tr>
<tr>
<td>0.1823</td>
<td>0.1537</td>
<td>0.01171</td>
<td>12.60</td>
<td>0.01028</td>
<td>11.13</td>
<td>1032.8</td>
<td></td>
</tr>
<tr>
<td>0.2845</td>
<td>0.2889</td>
<td>0.01688</td>
<td>12.12</td>
<td>0.00976</td>
<td>11.07</td>
<td>1313.3</td>
<td></td>
</tr>
<tr>
<td>0.3867</td>
<td>0.3000</td>
<td>0.01123</td>
<td>13.20</td>
<td>0.01019</td>
<td>11.98</td>
<td>1753.5</td>
<td></td>
</tr>
<tr>
<td>0.4489</td>
<td>0.3682</td>
<td>0.01303</td>
<td>15.75</td>
<td>0.01237</td>
<td>14.96</td>
<td>1209.1</td>
<td></td>
</tr>
<tr>
<td>0.5453</td>
<td>0.449</td>
<td>0.01046</td>
<td>17.41</td>
<td>0.01344</td>
<td>17.04</td>
<td>1275.4</td>
<td></td>
</tr>
</tbody>
</table>

**Static Pressure Drop (IN.)**

\( X + (X+M) = 0.027 \), \( Q + = 0.031 \), \( M + = 0.035 \).
### Test 30

#### Tare Test

<table>
<thead>
<tr>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>J</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2515</td>
<td>4.2548</td>
<td>7</td>
<td>4.2601</td>
<td>4.2629</td>
</tr>
<tr>
<td>2</td>
<td>4.2418</td>
<td>4.2361</td>
<td>8</td>
<td>4.2594</td>
<td>4.2622</td>
</tr>
<tr>
<td>3</td>
<td>4.2601</td>
<td>4.2605</td>
<td>9</td>
<td>4.2577</td>
<td>4.2544</td>
</tr>
<tr>
<td>4</td>
<td>4.2628</td>
<td>4.2606</td>
<td>10</td>
<td>4.2592</td>
<td>4.2584</td>
</tr>
<tr>
<td>5</td>
<td>4.2585</td>
<td>4.2561</td>
<td>11</td>
<td>4.2890</td>
<td>4.2923</td>
</tr>
<tr>
<td>6</td>
<td>4.2387</td>
<td>4.2384</td>
<td>12</td>
<td>4.2936</td>
<td>4.2954</td>
</tr>
</tbody>
</table>

#### Test Thermocouple Output (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>J</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2712</td>
<td>4.2760</td>
<td>7</td>
<td>4.2928</td>
<td>4.3063</td>
</tr>
<tr>
<td>2</td>
<td>4.2577</td>
<td>4.2591</td>
<td>8</td>
<td>4.2957</td>
<td>4.3105</td>
</tr>
<tr>
<td>3</td>
<td>4.2753</td>
<td>4.2806</td>
<td>9</td>
<td>4.3047</td>
<td>4.3283</td>
</tr>
<tr>
<td>4</td>
<td>4.2781</td>
<td>4.2812</td>
<td>10</td>
<td>4.3106</td>
<td>4.3339</td>
</tr>
<tr>
<td>5</td>
<td>4.2632</td>
<td>4.2905</td>
<td>11</td>
<td>4.4675</td>
<td>4.5481</td>
</tr>
<tr>
<td>6</td>
<td>4.2479</td>
<td>4.2749</td>
<td>12</td>
<td>4.4770</td>
<td>4.5744</td>
</tr>
</tbody>
</table>

**Differential Pressure & Flowmeter (IN.)** = 0.500

**Flowmeter Temp (MV) =** 0.9405

**Tare Global Temp (MV) =** 4.0551

**Inlet Pressure (IN.)** Left: 6.80 Right: 7.00

**Inlet Bulk Temp (C) =** 19.485

**Static Pressure Drop (IN.)**

- $P_0 - P_2 = 0.026$  
- $P_1 - P_2 = 0.054$  
- $P_2 - P_3 = 0.097$

**Red Manometer Fluid SP Gr:** 0.826

**Inlet Temperature (Deg. F)** = 888.21

**Tw/Tb =** 0.498  
**Mach NO. =** 0.019  
**Pr =** 1.741

**X+ (X+)M**  
**Q+ Num**  
**Tbulk Tw/Tb**  

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)M</th>
<th>Q+</th>
<th>Num</th>
<th>TbULK</th>
<th>Tw/Tb</th>
<th>(Re)M</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2739</td>
<td>0.2023</td>
<td>0.0579</td>
<td>243.33</td>
<td>0.954</td>
<td>238.23</td>
<td></td>
</tr>
<tr>
<td>0.221</td>
<td>0.1701</td>
<td>0.2915</td>
<td>286.22</td>
<td>0.900</td>
<td>229.73</td>
<td></td>
</tr>
<tr>
<td>0.1691</td>
<td>0.1357</td>
<td>0.1293</td>
<td>341.80</td>
<td>0.837</td>
<td>218.65</td>
<td></td>
</tr>
<tr>
<td>0.1152</td>
<td>0.0981</td>
<td>0.2051</td>
<td>416.65</td>
<td>0.767</td>
<td>265.35</td>
<td></td>
</tr>
<tr>
<td>0.0623</td>
<td>0.0365</td>
<td>0.4454</td>
<td>526.37</td>
<td>0.697</td>
<td>192.67</td>
<td></td>
</tr>
<tr>
<td>0.0094</td>
<td>0.092</td>
<td>1.4193</td>
<td>7.7514</td>
<td>0.565</td>
<td>173.99</td>
<td></td>
</tr>
</tbody>
</table>

**Non Dimensionalized Pressure Drop**

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)M</th>
<th>F</th>
<th>F(Re)M</th>
<th>FP</th>
<th>FP(Re)M</th>
<th>(Re)M</th>
<th>Tbulk</th>
<th>Tw/Tb</th>
<th>(Re)M</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0123</td>
<td>0.123</td>
<td>0.1237</td>
<td>21.32</td>
<td>0.00729</td>
<td>12.56</td>
<td>172.33</td>
<td>160.5</td>
<td>0.547</td>
<td></td>
</tr>
<tr>
<td>0.0656</td>
<td>0.0691</td>
<td>0.00938</td>
<td>18.22</td>
<td>0.00612</td>
<td>11.27</td>
<td>135.58</td>
<td>50.7</td>
<td>0.629</td>
<td></td>
</tr>
<tr>
<td>0.0944</td>
<td>0.0630</td>
<td>0.0598</td>
<td>15.94</td>
<td>0.06501</td>
<td>10.90</td>
<td>199.68</td>
<td>469.2</td>
<td>0.726</td>
<td></td>
</tr>
<tr>
<td>0.1473</td>
<td>0.1222</td>
<td>0.0718</td>
<td>15.25</td>
<td>0.00576</td>
<td>12.23</td>
<td>212.43</td>
<td>376.8</td>
<td>0.604</td>
<td></td>
</tr>
<tr>
<td>0.2622</td>
<td>0.1079</td>
<td>0.0974</td>
<td>16.64</td>
<td>2.13</td>
<td>59.5</td>
<td>317.0</td>
<td>0.670</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2824</td>
<td>0.1410</td>
<td>0.1147</td>
<td>12.54</td>
<td>0.00719</td>
<td>17.14</td>
<td>238.31</td>
<td>741.3</td>
<td>0.518</td>
<td></td>
</tr>
</tbody>
</table>
### Simultaneous Development Test No. 39

**Tare Test Taken from Run No. 38**

**Tare Thermocouple Output (mV)**

<table>
<thead>
<tr>
<th></th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2515</td>
<td>4.2548</td>
<td>4.2601</td>
<td>4.2629</td>
</tr>
<tr>
<td>2</td>
<td>4.2418</td>
<td>4.2381</td>
<td>4.2594</td>
<td>4.2622</td>
</tr>
<tr>
<td>3</td>
<td>4.2601</td>
<td>4.2605</td>
<td>4.2577</td>
<td>4.2544</td>
</tr>
<tr>
<td>4</td>
<td>4.2628</td>
<td>4.2606</td>
<td>4.2592</td>
<td>4.2584</td>
</tr>
<tr>
<td>5</td>
<td>4.2585</td>
<td>4.2561</td>
<td>4.2890</td>
<td>4.2923</td>
</tr>
<tr>
<td>6</td>
<td>4.2397</td>
<td>4.2304</td>
<td>4.2936</td>
<td>4.2956</td>
</tr>
</tbody>
</table>

**Test Thermocouple Output (mV)**

<table>
<thead>
<tr>
<th></th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2690</td>
<td>4.2735</td>
<td>4.2893</td>
<td>4.3015</td>
</tr>
<tr>
<td>2</td>
<td>4.2559</td>
<td>4.2569</td>
<td>4.2919</td>
<td>4.3062</td>
</tr>
<tr>
<td>3</td>
<td>4.2725</td>
<td>4.2775</td>
<td>4.3000</td>
<td>4.3210</td>
</tr>
<tr>
<td>4</td>
<td>4.2765</td>
<td>4.2799</td>
<td>4.3066</td>
<td>4.3273</td>
</tr>
<tr>
<td>5</td>
<td>4.2811</td>
<td>4.2873</td>
<td>4.4347</td>
<td>4.5069</td>
</tr>
<tr>
<td>6</td>
<td>4.2708</td>
<td>4.2735</td>
<td>4.4471</td>
<td>4.5371</td>
</tr>
</tbody>
</table>

**Differential Pressure - Flowmeter (IN.) = 0.452**

**Flowmeter LPM (mV) = 0.943**

**Bulk Exit Temp (mV) = 5.765**

**Inlet Pressure Man Left = 7.10**

**Right = 7.30 IN. HG**

**Inlet Bulk Temp CR-AL = 19.738**

**Static Pressure Drop (IN.)**

- P0-P2 = 0.215
- P2-P3 = 0.043
- P2-P4 = 0.114
- P2-P5 = 0.169
- P2-P6 = 0.219

**Red Manometer Fluid SP Gr = 0.826**

**Inlet Temperature (Deg. F) = 899.05**

**TW/TO = 0.494**

**Mach No. = 0.017**

**Pr = 1.743**

**Reynolds (RE) = 1499.1**

**Non Dimensionalized Pressure Drop**

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+M)</th>
<th>Q+</th>
<th>NUM</th>
<th>Tbulk</th>
<th>TW/TO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0136</td>
<td>0.0235</td>
<td>0.0460</td>
<td>0.740</td>
<td>0.1632</td>
<td>0.906</td>
</tr>
<tr>
<td>0.2218</td>
<td>1.143</td>
<td>0.2844</td>
<td>1.354</td>
<td>0.3127</td>
<td>1.517</td>
</tr>
<tr>
<td>0.00136</td>
<td>0.01344</td>
<td>0.00142</td>
<td>17.74</td>
<td>0.00638</td>
<td>9.91</td>
</tr>
<tr>
<td>0.0460</td>
<td>0.00433</td>
<td>0.00998</td>
<td>16.56</td>
<td>0.00627</td>
<td>10.41</td>
</tr>
<tr>
<td>0.1046</td>
<td>0.00924</td>
<td>0.00736</td>
<td>13.20</td>
<td>0.00545</td>
<td>9.78</td>
</tr>
<tr>
<td>0.1632</td>
<td>0.00263</td>
<td>0.00626</td>
<td>11.91</td>
<td>0.00490</td>
<td>9.34</td>
</tr>
<tr>
<td>0.2218</td>
<td>0.01674</td>
<td>0.01674</td>
<td>13.49</td>
<td>0.00530</td>
<td>10.61</td>
</tr>
<tr>
<td>0.2844</td>
<td>0.02137</td>
<td>0.00843</td>
<td>17.17</td>
<td>0.00726</td>
<td>14.17</td>
</tr>
<tr>
<td>0.3127</td>
<td>0.02333</td>
<td>0.00000</td>
<td>10.44</td>
<td>0.00000</td>
<td>18.31</td>
</tr>
</tbody>
</table>

**X + (X+M) F (F(REM) FP FP(REM) REYN T BULK TB/T DEG. F)**

<p>| 0.00136 | 0.01344 | 0.01142 | 17.74  | 0.00638 | 9.91  | 1553.6 | 775.3 | 0.944 |
| 0.0460 | 0.00433 | 0.00998 | 16.56  | 0.00627 | 10.41 | 1659.1 | 615.8 | 0.625 |
| 0.1046 | 0.00924 | 0.00736 | 13.20  | 0.00545 | 9.78  | 1793.3 | 674.9 | 0.719 |
| 0.1632 | 0.00263 | 0.00626 | 11.91  | 0.00490 | 9.34  | 1924.1 | 387.0 | 0.793 |
| 0.2218 | 0.01674 | 0.01674 | 13.49  | 0.00530 | 10.61 | 2060.4 | 323.3 | 0.558 |
| 0.2844 | 0.02137 | 0.00843 | 17.17  | 0.00726 | 14.17 | 2187.1 | 275.0 | 0.914 |
| 0.3127 | 0.02333 | 0.00000 | 10.44  | 0.00000 | 18.31 | 2319.4 | 252.0 | 0.943 |</p>
<table>
<thead>
<tr>
<th></th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2515</td>
<td>4.2548</td>
<td>4.2601</td>
<td>4.2629</td>
</tr>
<tr>
<td>2</td>
<td>4.2118</td>
<td>4.2381</td>
<td>4.2594</td>
<td>4.2622</td>
</tr>
<tr>
<td>3</td>
<td>4.2601</td>
<td>4.2605</td>
<td>4.2577</td>
<td>4.2544</td>
</tr>
<tr>
<td>4</td>
<td>4.2620</td>
<td>4.2606</td>
<td>4.2592</td>
<td>4.2584</td>
</tr>
<tr>
<td>5</td>
<td>4.2585</td>
<td>4.2561</td>
<td>4.2890</td>
<td>4.2923</td>
</tr>
<tr>
<td>6</td>
<td>4.2387</td>
<td>4.2384</td>
<td>4.2936</td>
<td>4.2954</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2700</td>
<td>4.2745</td>
<td>4.2859</td>
<td>4.2971</td>
</tr>
<tr>
<td>2</td>
<td>4.2579</td>
<td>4.2579</td>
<td>4.2915</td>
<td>4.3046</td>
</tr>
<tr>
<td>3</td>
<td>4.2724</td>
<td>4.2754</td>
<td>4.2980</td>
<td>4.3158</td>
</tr>
<tr>
<td>4</td>
<td>4.2761</td>
<td>4.2804</td>
<td>4.3049</td>
<td>4.3231</td>
</tr>
<tr>
<td>5</td>
<td>4.2813</td>
<td>4.2852</td>
<td>4.4312</td>
<td>4.4980</td>
</tr>
<tr>
<td>6</td>
<td>4.2695</td>
<td>4.2729</td>
<td>4.4430</td>
<td>4.5298</td>
</tr>
</tbody>
</table>

DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.402
FLOWMETER TEMP (MV) = 0.9492
INLET PRESSURE MAN., LEFT 7.50, RIGHT 7.60 IN. HG
INLET BULK TEMP CR-AL = 20.00
STATIC PRESSURE DROP (IN.)
P0-P2 = 0.169
P1-P2 = 0.027
P2-P3 = 0.070
P2-P4 = 0.100
P2-P5 = 0.160
P2-P6 = 0.216
P2-P7 = 0.238
REYNOLDS NUMBER FLUID SP GR 0.826
INLET TEMPERATURE (DEG. F.) = 910.65
H/W = 0.490
MACH NO. = 0.015
PR = 0.7449
REYN. = 1328.4

<table>
<thead>
<tr>
<th>X+</th>
<th>(A+)+M</th>
<th>0+</th>
<th>NUM</th>
<th>TBULK</th>
<th>TW/TB</th>
<th>(P)+M</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3416</td>
<td>0.2520</td>
<td>0.0449</td>
<td>0.2539</td>
<td>243.61</td>
<td>0.953</td>
<td>1912.7</td>
</tr>
<tr>
<td>0.2756</td>
<td>0.2118</td>
<td>0.0803</td>
<td>0.2135</td>
<td>285.87</td>
<td>0.900</td>
<td>1839.3</td>
</tr>
<tr>
<td>0.2096</td>
<td>0.1687</td>
<td>0.0888</td>
<td>0.1710</td>
<td>340.05</td>
<td>0.839</td>
<td>1756.9</td>
</tr>
<tr>
<td>0.1436</td>
<td>0.1119</td>
<td>0.1618</td>
<td>0.1642</td>
<td>412.70</td>
<td>0.770</td>
<td>1663.4</td>
</tr>
<tr>
<td>0.0777</td>
<td>0.0701</td>
<td>0.3426</td>
<td>3.7291</td>
<td>519.76</td>
<td>0.687</td>
<td>1553.4</td>
</tr>
<tr>
<td>0.0117</td>
<td>0.0914</td>
<td>1.1798</td>
<td>6.6980</td>
<td>742.03</td>
<td>0.566</td>
<td>1399.2</td>
</tr>
</tbody>
</table>

NON DIMENSIONALIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0153</td>
<td>0.0275</td>
<td></td>
</tr>
<tr>
<td>0.0518</td>
<td>0.0423</td>
<td></td>
</tr>
<tr>
<td>0.1178</td>
<td>0.0866</td>
<td></td>
</tr>
<tr>
<td>0.1838</td>
<td>0.0693</td>
<td></td>
</tr>
<tr>
<td>0.2497</td>
<td>0.0297</td>
<td></td>
</tr>
<tr>
<td>0.3157</td>
<td>0.0604</td>
<td></td>
</tr>
<tr>
<td>0.3522</td>
<td>0.1721</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X+</th>
<th>(A+)+M</th>
<th>F</th>
<th>F(RE)+M</th>
<th>FP</th>
<th>FP(RE)+M</th>
<th>T(BULK)</th>
<th>T/B/T</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0153</td>
<td>0.0151</td>
<td>0.01117</td>
<td>15.48</td>
<td>0.00587</td>
<td>8.14</td>
<td>1385.0</td>
<td>1706.5</td>
</tr>
<tr>
<td>0.0518</td>
<td>0.0485</td>
<td>0.00946</td>
<td>14.64</td>
<td>0.00599</td>
<td>8.90</td>
<td>1485.3</td>
<td>500.8</td>
</tr>
<tr>
<td>0.1178</td>
<td>0.1030</td>
<td>0.00417</td>
<td>13.14</td>
<td>0.00625</td>
<td>10.06</td>
<td>1607.1</td>
<td>59.2</td>
</tr>
<tr>
<td>0.1838</td>
<td>0.1519</td>
<td>0.00836</td>
<td>14.30</td>
<td>0.00702</td>
<td>12.91</td>
<td>1709.6</td>
<td>373.4</td>
</tr>
<tr>
<td>0.2497</td>
<td>0.1965</td>
<td>0.00935</td>
<td>16.80</td>
<td>0.00787</td>
<td>14.15</td>
<td>1797.0</td>
<td>311.1</td>
</tr>
<tr>
<td>0.3157</td>
<td>0.2379</td>
<td>0.00913</td>
<td>17.11</td>
<td>0.00804</td>
<td>15.08</td>
<td>1874.6</td>
<td>263.4</td>
</tr>
<tr>
<td>0.3522</td>
<td>0.2596</td>
<td>0.00720</td>
<td>17.46</td>
<td>0.00635</td>
<td>12.15</td>
<td>1913.6</td>
<td>241.6</td>
</tr>
</tbody>
</table>
THERE IS AN ERROR IN THE DOCUMENT. THE TEXT SEEMS TO BE A MIXTURE OF MEASUREMENTS AND UNREADABLE CHARACTERS. PLEASE PROVIDE THE CORRECT TEXT OR A CLEARER VERSION OF THE DOCUMENT.
SIMULTANEOUS DEVELOPMENT TEST NO. 42
TAKE TEST TAKEN FROM RUN NO. 43
TAKE THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>I</th>
<th>TA(I',1)</th>
<th>TA(I',2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2620</td>
<td>4.2628</td>
<td>7</td>
<td>4.2620</td>
<td>4.2642</td>
</tr>
<tr>
<td>2</td>
<td>4.2484</td>
<td>4.2461</td>
<td>8</td>
<td>4.2638</td>
<td>4.2660</td>
</tr>
<tr>
<td>3</td>
<td>4.2640</td>
<td>4.2648</td>
<td>9</td>
<td>4.2650</td>
<td>4.2612</td>
</tr>
<tr>
<td>4</td>
<td>4.2679</td>
<td>4.2664</td>
<td>10</td>
<td>4.2677</td>
<td>4.2656</td>
</tr>
<tr>
<td>5</td>
<td>4.2572</td>
<td>4.2555</td>
<td>11</td>
<td>4.2964</td>
<td>4.3000</td>
</tr>
<tr>
<td>6</td>
<td>4.2390</td>
<td>4.2391</td>
<td>12</td>
<td>4.2989</td>
<td>4.3015</td>
</tr>
</tbody>
</table>

TEST THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>I</th>
<th>TT(I',1)</th>
<th>TT(I',2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2661</td>
<td>4.2685</td>
<td>7</td>
<td>4.2779</td>
<td>4.2506</td>
</tr>
<tr>
<td>2</td>
<td>4.2532</td>
<td>4.2520</td>
<td>8</td>
<td>4.2930</td>
<td>4.2925</td>
</tr>
<tr>
<td>3</td>
<td>4.2706</td>
<td>4.2727</td>
<td>9</td>
<td>4.2880</td>
<td>4.2688</td>
</tr>
<tr>
<td>4</td>
<td>4.2745</td>
<td>4.2752</td>
<td>10</td>
<td>4.2949</td>
<td>4.3076</td>
</tr>
<tr>
<td>5</td>
<td>4.2652</td>
<td>4.2664</td>
<td>11</td>
<td>4.4018</td>
<td>4.4491</td>
</tr>
<tr>
<td>6</td>
<td>4.2484</td>
<td>4.2468</td>
<td>12</td>
<td>4.4158</td>
<td>4.4878</td>
</tr>
</tbody>
</table>

DIFFERENTIAL PRESSURE FLOWMETER (IN.) = 0.300
FLOWMETER TEMP. (MV) = 0.9510 BULK EXIT TEMP. (MV) = 4.2465
INLET PRESSURE MAN., LEFT 8.10 RIGHT 8.30 IN. HG
INLET BULK TEMP. (F) = 19.269

STATIC PRESSURE DROP (IN.)
P1-P2 = 0.024  P1-P3 = 0.032
P2-P4 = 0.068  P2-P5 = 0.091
P2-P7 = 0.144

HUD MANOMETER FLUID SP GR 0.826
INLET TEMPERATURE (DEG. F) = 378.94
IW/TL = 0.501 MACH. NO. = 0.010
PR = 0.740099 KEYD = 999.3

X+  (X+1)M  O+  NUM  TBULK  IW/TA  (F+M)  DEG. F.
0.0458  0.3282  0.0226  37.3399  216.07  0.292  1466.5  0.0205
0.3685  0.2757  0.0319  2.7027  255.34  0.938  1411.2  0.2225
0.2803  0.2198  0.0540  2.1605  306.39  0.675  1357.9  0.0156
0.1921  0.1592  0.1127  2.4778  375.83  0.803  1374.8  0.0156
0.1038  0.0920  0.2577  3.1961  479.90  0.715  1187.0  0.0156
0.0156  0.0156  0.9203  5.5040  702.05  0.584  1160.3  0.0156

NON DIMENSIONALIZED PRESSURE DROP

X+  P+
0.0205  0.225
0.0692  0.472
0.1574  0.805
0.2457  1.171
0.3339  1.410
0.4221  1.853
0.4709  1.956

X+  (X+1)M  O+  NUM  TBULK  IW/TA  (F+M)  DEG. F.
0.0205  0.2200  0.01450  15.20  0.00894  9.37  1048.4  0.0205
0.0692  0.0639  0.01203  13.62  0.00800  9.05  1131.6  0.0692
0.1574  0.1349  0.00913  11.25  0.00720  8.37  1241.4  0.1574
0.2457  0.1981  0.00924  12.51  0.00823  10.79  1311.1  0.2457
0.3339  0.2559  0.01120  15.44  0.00973  13.42  1378.9  0.3339
0.4221  0.3097  0.01054  15.16  0.00951  13.68  1437.9  0.4221
0.4709  0.3381  0.00461  14.71  0.00564  13.71  1497.8  0.4709
<table>
<thead>
<tr>
<th>THERMOCOUPLE OUTPUT (MV)</th>
<th>TA(1,1)</th>
<th>TA(1,2)</th>
<th>TA(1,1)</th>
<th>TA(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2620</td>
<td>4.2628</td>
<td>7</td>
<td>4.2620</td>
</tr>
<tr>
<td>2</td>
<td>4.2668</td>
<td>4.2666</td>
<td>8</td>
<td>4.2638</td>
</tr>
<tr>
<td>3</td>
<td>4.2640</td>
<td>4.2648</td>
<td>9</td>
<td>4.2650</td>
</tr>
<tr>
<td>4</td>
<td>4.2679</td>
<td>4.2664</td>
<td>10</td>
<td>4.2667</td>
</tr>
<tr>
<td>5</td>
<td>4.2722</td>
<td>4.2656</td>
<td>11</td>
<td>4.2664</td>
</tr>
<tr>
<td>6</td>
<td>4.2390</td>
<td>4.2391</td>
<td>12</td>
<td>4.2989</td>
</tr>
<tr>
<td>THERMOCOUPLE OUTPUT (MV)</td>
<td>TT(1,1)</td>
<td>TT(1,2)</td>
<td>TT(1,1)</td>
<td>TT(1,2)</td>
</tr>
<tr>
<td>-------------------------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
<td>----------</td>
</tr>
<tr>
<td>1</td>
<td>4.2627</td>
<td>4.2644</td>
<td>7</td>
<td>4.2724</td>
</tr>
<tr>
<td>2</td>
<td>4.2686</td>
<td>4.2767</td>
<td>8</td>
<td>4.2768</td>
</tr>
<tr>
<td>3</td>
<td>4.2677</td>
<td>4.2767</td>
<td>9</td>
<td>4.2609</td>
</tr>
<tr>
<td>4</td>
<td>4.2711</td>
<td>4.2714</td>
<td>10</td>
<td>4.2907</td>
</tr>
<tr>
<td>5</td>
<td>4.2621</td>
<td>4.2611</td>
<td>11</td>
<td>4.3822</td>
</tr>
<tr>
<td>6</td>
<td>4.2668</td>
<td>4.2470</td>
<td>12</td>
<td>4.4065</td>
</tr>
</tbody>
</table>

DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.252
FLOWMETER TEMP. (MV) = 0.9521 BULK EXIT TEMP (MV) = 4.3865
INLET PRESSURE MAN, LEFT 8.50 RIGHT 8.70 IN. HG
INLET BULK TEMP CR-AL = 19484
STATIC PRESSURE DROP (IN.)
P0-P2 = 0.070 P1-P2 = -0.011 P2-P3 = 0.031
P2-P4 = 0.041 P2-P5 = 0.072 P2-P6 = 0.108
P2-P7 = 0.122
RED MANOMETER FLUID SP GR 0.826
INLET TEMPERATURE (DEG. F.) = 884.17
TW/TH = 0.498 MACH NO. = 0.009
PR = 0.741915. REYD = 337.2

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+M)</th>
<th>Q+</th>
<th>NUM</th>
<th>T BULK</th>
<th>TW/TH</th>
<th>(RE)m</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5443</td>
<td>0.3926</td>
<td>0.0188</td>
<td>690335</td>
<td>220.39</td>
<td>0.985</td>
<td>1262.3</td>
</tr>
<tr>
<td>0.4391</td>
<td>0.3717</td>
<td>0.0064</td>
<td>595324</td>
<td>251.73</td>
<td>0.943</td>
<td>11803</td>
</tr>
<tr>
<td>0.3340</td>
<td>0.2593</td>
<td>0.0183</td>
<td>567672</td>
<td>295.73</td>
<td>0.887</td>
<td>1142.6</td>
</tr>
<tr>
<td>0.2289</td>
<td>0.1730</td>
<td>0.0189</td>
<td>522334</td>
<td>359.66</td>
<td>0.819</td>
<td>1084.0</td>
</tr>
<tr>
<td>0.1237</td>
<td>0.1689</td>
<td>0.0144</td>
<td>292374</td>
<td>461.41</td>
<td>0.729</td>
<td>1008.7</td>
</tr>
<tr>
<td>0.0186</td>
<td>0.1800</td>
<td>0.0829</td>
<td>507866</td>
<td>693.42</td>
<td>0.587</td>
<td>893.7</td>
</tr>
</tbody>
</table>

DIMENSIONALIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0044</td>
<td>0.097</td>
</tr>
<tr>
<td>0.0082</td>
<td>0.038</td>
</tr>
<tr>
<td>0.0187</td>
<td>0.090</td>
</tr>
<tr>
<td>0.2928</td>
<td>1.149</td>
</tr>
<tr>
<td>0.3979</td>
<td>1.650</td>
</tr>
<tr>
<td>0.5030</td>
<td>2.138</td>
</tr>
<tr>
<td>0.5611</td>
<td>2.346</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+M)</th>
<th>F</th>
<th>F(RE)m</th>
<th>FP</th>
<th>FP(RE)m</th>
<th>(RE)m</th>
<th>T BULK</th>
<th>IB/T</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0024</td>
<td>0.0028</td>
<td>-0.0005</td>
<td>-0.44</td>
<td>-0.0064</td>
<td>-5.71</td>
<td>219.8</td>
<td>0.569</td>
<td></td>
</tr>
<tr>
<td>0.0082</td>
<td>0.0075</td>
<td>0.0009</td>
<td>1.40</td>
<td>0.00029</td>
<td>0.28</td>
<td>959.3</td>
<td>0.543</td>
<td>0.0078</td>
</tr>
<tr>
<td>0.0187</td>
<td>0.0187</td>
<td>0.0036</td>
<td>11.16</td>
<td>0.00083</td>
<td>9.15</td>
<td>1067.1</td>
<td>0.403</td>
<td>0.0078</td>
</tr>
<tr>
<td>0.0292</td>
<td>0.0293</td>
<td>0.0126</td>
<td>14.06</td>
<td>0.00137</td>
<td>12.66</td>
<td>1131.6</td>
<td>0.324</td>
<td>0.0057</td>
</tr>
<tr>
<td>0.0397</td>
<td>0.0326</td>
<td>0.0126</td>
<td>14.97</td>
<td>0.00134</td>
<td>13.22</td>
<td>1165.7</td>
<td>0.271</td>
<td>0.0119</td>
</tr>
<tr>
<td>0.0503</td>
<td>0.0389</td>
<td>0.0142</td>
<td>15.02</td>
<td>0.00177</td>
<td>13.61</td>
<td>1267.0</td>
<td>0.234</td>
<td>0.0067</td>
</tr>
<tr>
<td>0.0561</td>
<td>0.0447</td>
<td>0.0120</td>
<td>14.93</td>
<td>0.00144</td>
<td>14.01</td>
<td>1267.0</td>
<td>0.219</td>
<td>0.0090</td>
</tr>
</tbody>
</table>
SIMULTANEOUS DEVELOPMENT TEST NO. 44
TARE TEST TAKEN FROM RUN NO. 44
TARE THERMOCUPE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2534</td>
<td>4.2600</td>
<td>7</td>
<td>4.2434</td>
<td>4.2474</td>
</tr>
<tr>
<td>2</td>
<td>4.2462</td>
<td>4.2426</td>
<td>8</td>
<td>4.2468</td>
<td>4.2485</td>
</tr>
<tr>
<td>3</td>
<td>4.2621</td>
<td>4.2614</td>
<td>9</td>
<td>4.2591</td>
<td>4.2546</td>
</tr>
<tr>
<td>4</td>
<td>4.2661</td>
<td>4.2650</td>
<td>10</td>
<td>4.2603</td>
<td>4.2588</td>
</tr>
<tr>
<td>5</td>
<td>4.2647</td>
<td>4.2651</td>
<td>11</td>
<td>4.3038</td>
<td>4.3068</td>
</tr>
<tr>
<td>6</td>
<td>4.2573</td>
<td>4.2524</td>
<td>12</td>
<td>4.3030</td>
<td>4.3176</td>
</tr>
</tbody>
</table>

TEST THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2700</td>
<td>4.2761</td>
<td>7</td>
<td>4.3007</td>
<td>4.3199</td>
</tr>
<tr>
<td>2</td>
<td>4.2555</td>
<td>4.2578</td>
<td>8</td>
<td>4.2981</td>
<td>4.3261</td>
</tr>
<tr>
<td>3</td>
<td>4.2776</td>
<td>4.2837</td>
<td>9</td>
<td>4.3268</td>
<td>4.3630</td>
</tr>
<tr>
<td>4</td>
<td>4.2796</td>
<td>4.2854</td>
<td>10</td>
<td>4.3222</td>
<td>4.3570</td>
</tr>
<tr>
<td>5</td>
<td>4.2467</td>
<td>4.2982</td>
<td>11</td>
<td>4.3335</td>
<td>4.3996</td>
</tr>
<tr>
<td>6</td>
<td>4.2725</td>
<td>4.2795</td>
<td>12</td>
<td>4.5178</td>
<td>4.6644</td>
</tr>
</tbody>
</table>

DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.508
FLOWMETER TEMP (MV) = 0.9780 BULK EXIT TEMP (MV) = 5.4841
INLET PRESSURE MAN. LEFT 4.70 RIGHT 4.85 IN. HG
INLET BULK TEMP CR-AL = 27.375.
STATIC PRESSURE DROP (IN.)
\[ p_{0} - p_{2} = 0.310 \]
\[ p_{1} - p_{2} = 0.053 \]
\[ p_{2} - p_{3} = 0.083 \]
\[ p_{2} - p_{4} = 0.155 \]
\[ p_{2} - p_{5} = 0.219 \]
\[ p_{2} - p_{6} = 0.285 \]
\[ p_{2} - p_{7} = 0.331 \]
BLUE MANOMETER FLUID SP GR 0.797
INLET TEMPERATURE (DEG. F) = 1213.49
TW/TO = 0.140 MACH NO. = 0.023
PR = 0.752284 REYN = 1599.6

X+ \( (X+)_M \) Q+ NUM TBULK TW/TH (RI)_M

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>DEG. F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.2667</td>
<td>0.2047</td>
<td>0.3944</td>
<td>3.4242</td>
<td>266.27</td>
<td>0.924</td>
</tr>
<tr>
<td>0.2151</td>
<td>0.1751</td>
<td>0.4362</td>
<td>4.2669</td>
<td>332.97</td>
<td>0.847</td>
</tr>
<tr>
<td>0.1633</td>
<td>0.1417</td>
<td>0.4924</td>
<td>4.4924</td>
<td>417.28</td>
<td>0.765</td>
</tr>
<tr>
<td>0.1171</td>
<td>0.1034</td>
<td>0.228d</td>
<td>3.1752</td>
<td>523.31</td>
<td>0.681</td>
</tr>
<tr>
<td>0.0666</td>
<td>0.0594</td>
<td>0.4964</td>
<td>4.1730</td>
<td>687.56</td>
<td>0.587</td>
</tr>
<tr>
<td>0.0091</td>
<td>0.0093</td>
<td>1.6269</td>
<td>7.3769</td>
<td>1,000.08</td>
<td>0.469</td>
</tr>
</tbody>
</table>

NON DIMENSIONALIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>( (X+)_M )</th>
<th>F</th>
<th>F(RELM)</th>
<th>FP</th>
<th>FP(RL)_M</th>
<th>(RELM)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0120</td>
<td>0.0122</td>
<td>0.01210</td>
<td>16.45</td>
<td>0.00414</td>
<td>6.75</td>
<td>1632.7</td>
</tr>
<tr>
<td>0.0464</td>
<td>0.0407</td>
<td>0.04010</td>
<td>15.67</td>
<td>0.00455</td>
<td>7.84</td>
<td>1727.5</td>
</tr>
<tr>
<td>0.0919</td>
<td>0.0875</td>
<td>0.08709</td>
<td>13.27</td>
<td>0.00400</td>
<td>8.60</td>
<td>1816.7</td>
</tr>
<tr>
<td>0.1474</td>
<td>0.1283</td>
<td>0.12809</td>
<td>11.74</td>
<td>0.00444</td>
<td>9.24</td>
<td>1902.2</td>
</tr>
<tr>
<td>0.1943</td>
<td>0.1638</td>
<td>0.16370</td>
<td>10.44</td>
<td>0.00443</td>
<td>9.84</td>
<td>1983.4</td>
</tr>
<tr>
<td>0.2467</td>
<td>0.2150</td>
<td>0.21501</td>
<td>10.24</td>
<td>0.00443</td>
<td>10.44</td>
<td>2064.1</td>
</tr>
<tr>
<td>0.2749</td>
<td>0.2160</td>
<td>0.21601</td>
<td>9.74</td>
<td>0.00443</td>
<td>10.24</td>
<td>2144.8</td>
</tr>
</tbody>
</table>

INLET THERMOMETER READ |

<table>
<thead>
<tr>
<th>X+</th>
<th>( (X+)_M )</th>
<th>DEG F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0120</td>
<td>0.0122</td>
<td>0.01210</td>
</tr>
<tr>
<td>0.0464</td>
<td>0.0407</td>
<td>0.04010</td>
</tr>
<tr>
<td>0.0919</td>
<td>0.0875</td>
<td>0.08709</td>
</tr>
<tr>
<td>0.1474</td>
<td>0.1283</td>
<td>0.12809</td>
</tr>
<tr>
<td>0.1943</td>
<td>0.1638</td>
<td>0.16370</td>
</tr>
<tr>
<td>0.2467</td>
<td>0.2150</td>
<td>0.21501</td>
</tr>
<tr>
<td>0.2749</td>
<td>0.2160</td>
<td>0.21601</td>
</tr>
</tbody>
</table>
### SIMULTANEOUS DEVELOPMENT TEST NO. 45

**TAKEN TEST TAKEN FROM RUN NO.** 44

<table>
<thead>
<tr>
<th>THERMOCOUPLE OUTPUT (MV)</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2534</td>
<td>4.2600</td>
<td>7</td>
<td>4.2434</td>
</tr>
<tr>
<td>2</td>
<td>4.2462</td>
<td>4.2426</td>
<td>8</td>
<td>4.2468</td>
</tr>
<tr>
<td>3</td>
<td>4.2621</td>
<td>4.2614</td>
<td>9</td>
<td>4.2591</td>
</tr>
<tr>
<td>4</td>
<td>4.2661</td>
<td>4.2650</td>
<td>10</td>
<td>4.2503</td>
</tr>
<tr>
<td>5</td>
<td>4.2647</td>
<td>4.2651</td>
<td>11</td>
<td>4.3038</td>
</tr>
<tr>
<td>6</td>
<td>4.2573</td>
<td>4.2524</td>
<td>12</td>
<td>4.3030</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>THERMOCOUPLE OUTPUT (MV)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2692</td>
<td>4.2600</td>
<td>7</td>
<td>4.2899</td>
</tr>
<tr>
<td>2</td>
<td>4.2485</td>
<td>4.2530</td>
<td>8</td>
<td>4.3098</td>
</tr>
<tr>
<td>3</td>
<td>4.2706</td>
<td>4.2776</td>
<td>9</td>
<td>4.3161</td>
</tr>
<tr>
<td>4</td>
<td>4.2741</td>
<td>4.2794</td>
<td>10</td>
<td>4.3454</td>
</tr>
<tr>
<td>5</td>
<td>4.2891</td>
<td>4.2887</td>
<td>11</td>
<td>4.6416</td>
</tr>
<tr>
<td>6</td>
<td>4.2663</td>
<td>4.2724</td>
<td>12</td>
<td>4.6499</td>
</tr>
</tbody>
</table>

**DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.452**

**FLOWMETER TEMP. (MV) = 0.9829 BULK EXIT TEMP (MV) = 5.372**

**INLET PRESSURE: MAN. LEFT 5000 RIGHT 525 IN. HG**

**INLET BULK TEMP. CRAL = 27220**

**STATIC PRESSURE DROP (IN.)**

<table>
<thead>
<tr>
<th>P0-P2 = 0.245</th>
<th>P1-P2 = 0.036</th>
<th>P2-P3 = 0.068</th>
</tr>
</thead>
<tbody>
<tr>
<td>P2-P4 = 0.175</td>
<td>P2-P5 = 0.237</td>
<td></td>
</tr>
<tr>
<td>P2-P7 = 0.272</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**BLUE MANOMETER FLUID SP. GR. A.797**

**INLET TEMPERATURE (DEG. F) = 1207.46**

**TW/TO = 0.402 MACH NO. = 0.020**

**PR = 0.75152 KEYD = 1022.8**

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+1)M</th>
<th>Q+</th>
<th>NUM</th>
<th>TBULK</th>
<th>TW/TR</th>
<th>(X+1)M</th>
<th>DEG F.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3002</td>
<td>0.2292</td>
<td>6.5133</td>
<td>+.1427</td>
<td>261.78</td>
<td>0.929</td>
<td>3.1049</td>
<td></td>
</tr>
<tr>
<td>0.2422</td>
<td>0.1939</td>
<td>2.9746</td>
<td>+.1653</td>
<td>326.38</td>
<td>0.853</td>
<td>1.0891</td>
<td></td>
</tr>
<tr>
<td>0.1842</td>
<td>0.1592</td>
<td>1.247</td>
<td>+.2092</td>
<td>409.95</td>
<td>0.772</td>
<td>1.6660</td>
<td></td>
</tr>
<tr>
<td>0.1262</td>
<td>0.1159</td>
<td>0.1872</td>
<td>+.2450</td>
<td>522.05</td>
<td>0.685</td>
<td>1.7349</td>
<td></td>
</tr>
<tr>
<td>0.0682</td>
<td>0.0667</td>
<td>0.4548</td>
<td>3.8285</td>
<td>684.90</td>
<td>0.589</td>
<td>1.5024</td>
<td></td>
</tr>
<tr>
<td>0.0102</td>
<td>0.0104</td>
<td>1.4619</td>
<td>6.5465</td>
<td>1003.81</td>
<td>0.467</td>
<td>1.4578</td>
<td></td>
</tr>
</tbody>
</table>

**NON DIMENSIONALIZED PRESSURE DROP**

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+1)M</th>
<th>P+</th>
<th>0.0135</th>
<th>0.169</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0455</td>
<td>0.283</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1035</td>
<td>0.499</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1615</td>
<td>0.687</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2195</td>
<td>0.842</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2774</td>
<td>1.038</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3395</td>
<td>1.159</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+1)M</th>
<th>F</th>
<th>F(REA)M</th>
<th>FP</th>
<th>FP(REA)M</th>
<th>(REA)M</th>
<th>T(BULK)</th>
<th>TBULK</th>
<th>18/TO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0135</td>
<td>0.0137</td>
<td>0.00935</td>
<td>13.82</td>
<td>0.00352</td>
<td>5.10</td>
<td>1447.6</td>
<td>1.3605</td>
<td>0.449</td>
<td></td>
</tr>
<tr>
<td>0.0455</td>
<td>0.0458</td>
<td>0.00896</td>
<td>13.71</td>
<td>0.00433</td>
<td>6.63</td>
<td>1530.6</td>
<td>0.0493</td>
<td>0.531</td>
<td></td>
</tr>
<tr>
<td>0.1035</td>
<td>0.0983</td>
<td>0.00742</td>
<td>12.39</td>
<td>0.00485</td>
<td>8.16</td>
<td>1661.3</td>
<td>0.9432</td>
<td>0.673</td>
<td></td>
</tr>
<tr>
<td>0.1615</td>
<td>0.1437</td>
<td>0.00674</td>
<td>12.13</td>
<td>0.00481</td>
<td>8.66</td>
<td>1799.9</td>
<td>4.613</td>
<td>0.729</td>
<td></td>
</tr>
<tr>
<td>0.2195</td>
<td>0.1832</td>
<td>0.00713</td>
<td>13.73</td>
<td>0.00505</td>
<td>9.72</td>
<td>1925.9</td>
<td>369.2</td>
<td>0.814</td>
<td></td>
</tr>
<tr>
<td>0.2774</td>
<td>0.2182</td>
<td>0.00644</td>
<td>17.06</td>
<td>0.00641</td>
<td>12.73</td>
<td>2155.4</td>
<td>292.0</td>
<td>0.894</td>
<td></td>
</tr>
<tr>
<td>0.3395</td>
<td>0.2359</td>
<td>0.00670</td>
<td>14.33</td>
<td>0.00770</td>
<td>15.10</td>
<td>2155.0</td>
<td>256.7</td>
<td>0.935</td>
<td></td>
</tr>
</tbody>
</table>
### TARE TEST TAKEN FROM RUN NO. 44

#### TARE THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2534</td>
<td>4.2600</td>
<td>7</td>
<td>4.2434</td>
</tr>
<tr>
<td>2</td>
<td>4.2462</td>
<td>4.2426</td>
<td>8</td>
<td>4.2468</td>
</tr>
<tr>
<td>3</td>
<td>4.2621</td>
<td>4.2614</td>
<td>9</td>
<td>4.2591</td>
</tr>
<tr>
<td>4</td>
<td>4.2661</td>
<td>4.2650</td>
<td>10</td>
<td>4.2603</td>
</tr>
<tr>
<td>5</td>
<td>4.2647</td>
<td>4.2651</td>
<td>11</td>
<td>4.3043</td>
</tr>
<tr>
<td>6</td>
<td>4.2573</td>
<td>4.2524</td>
<td>12</td>
<td>4.3030</td>
</tr>
</tbody>
</table>

#### TEST THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2602</td>
<td>4.2651</td>
<td>7</td>
<td>4.2830</td>
</tr>
<tr>
<td>2</td>
<td>4.2450</td>
<td>4.2482</td>
<td>8</td>
<td>4.2850</td>
</tr>
<tr>
<td>3</td>
<td>4.2656</td>
<td>4.2714</td>
<td>9</td>
<td>4.3085</td>
</tr>
<tr>
<td>4</td>
<td>4.2705</td>
<td>4.2736</td>
<td>10</td>
<td>4.3083</td>
</tr>
<tr>
<td>5</td>
<td>4.2754</td>
<td>4.2823</td>
<td>11</td>
<td>4.4998</td>
</tr>
<tr>
<td>6</td>
<td>4.2631</td>
<td>4.2676</td>
<td>12</td>
<td>4.4814</td>
</tr>
</tbody>
</table>

#### DIFFERENTIAL PRESSURE -FLOWMETER (IN.) = 0.402

- FLOWMETER TEMP (MV) = 0.9822
- BULK EXIT TEMP (MV) = 5.0653

#### INLET PRESSURE MAN. LEFT 5.35 RIGHT 5.50 IN. HG

#### INLET BULK TEMP CR-AL = 27.290

#### STATIC PRESSURE DROP (IN.)

- P0-P2 = 0.20
- P1-P2 = 0.01
- P2-P3 = 0.01

#### BULK MANOMETER FLUID SP CR 0.797

#### INLET TEMPERATURE (DEG. F.) = 1210.19

#### TW/TH = 0.402 MACH NO. = 0.018

#### PR = 0.752065 REYD = 1265.4

### Non Dimensionalized Pressure Drop

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)^2</th>
<th>Q+</th>
<th>NUM</th>
<th>TBUK</th>
<th>TW/TH</th>
<th>(REL)M</th>
<th>DEG F</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0373</td>
<td>0.0573</td>
<td>0.15</td>
<td>0.16</td>
<td>0.577</td>
<td>0.944</td>
<td>1392.1</td>
<td></td>
</tr>
<tr>
<td>0.0272</td>
<td>0.0572</td>
<td>0.15</td>
<td>0.16</td>
<td>0.577</td>
<td>0.944</td>
<td>1392.1</td>
<td></td>
</tr>
<tr>
<td>0.0207</td>
<td>0.0507</td>
<td>0.15</td>
<td>0.16</td>
<td>0.577</td>
<td>0.944</td>
<td>1392.1</td>
<td></td>
</tr>
<tr>
<td>0.0141</td>
<td>0.0561</td>
<td>0.15</td>
<td>0.16</td>
<td>0.577</td>
<td>0.944</td>
<td>1392.1</td>
<td></td>
</tr>
<tr>
<td>0.0077</td>
<td>0.0557</td>
<td>0.15</td>
<td>0.16</td>
<td>0.577</td>
<td>0.944</td>
<td>1392.1</td>
<td></td>
</tr>
<tr>
<td>0.0015</td>
<td>0.0593</td>
<td>0.15</td>
<td>0.16</td>
<td>0.577</td>
<td>0.944</td>
<td>1392.1</td>
<td></td>
</tr>
</tbody>
</table>

### X+  | (X+)^2 | F  | F(REL)M | FP | FP(REL)M | (REL)M | TBUK | IB/TH |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0051</td>
<td>0.0102</td>
<td>0.0010</td>
<td>13.87</td>
<td>0.0043</td>
<td>13.66</td>
<td>1295.1</td>
<td>1002.8</td>
<td>0.459</td>
</tr>
<tr>
<td>0.0051</td>
<td>0.0102</td>
<td>0.0010</td>
<td>13.87</td>
<td>0.0043</td>
<td>13.66</td>
<td>1295.1</td>
<td>1002.8</td>
<td>0.459</td>
</tr>
<tr>
<td>0.0141</td>
<td>0.0194</td>
<td>0.0072</td>
<td>11.94</td>
<td>0.0054</td>
<td>11.72</td>
<td>1295.1</td>
<td>1002.8</td>
<td>0.459</td>
</tr>
<tr>
<td>0.0141</td>
<td>0.0194</td>
<td>0.0072</td>
<td>11.94</td>
<td>0.0054</td>
<td>11.72</td>
<td>1295.1</td>
<td>1002.8</td>
<td>0.459</td>
</tr>
<tr>
<td>0.0246</td>
<td>0.0347</td>
<td>0.0077</td>
<td>13.49</td>
<td>0.0056</td>
<td>13.34</td>
<td>1295.1</td>
<td>1002.8</td>
<td>0.459</td>
</tr>
<tr>
<td>0.0246</td>
<td>0.0347</td>
<td>0.0077</td>
<td>13.49</td>
<td>0.0056</td>
<td>13.34</td>
<td>1295.1</td>
<td>1002.8</td>
<td>0.459</td>
</tr>
<tr>
<td>0.0318</td>
<td>0.0491</td>
<td>0.0081</td>
<td>16.40</td>
<td>0.0071</td>
<td>16.15</td>
<td>1295.1</td>
<td>1002.8</td>
<td>0.459</td>
</tr>
<tr>
<td>0.0318</td>
<td>0.0491</td>
<td>0.0081</td>
<td>16.40</td>
<td>0.0071</td>
<td>16.15</td>
<td>1295.1</td>
<td>1002.8</td>
<td>0.459</td>
</tr>
<tr>
<td>0.0478</td>
<td>0.1256</td>
<td>0.0100</td>
<td>18.53</td>
<td>0.0072</td>
<td>18.28</td>
<td>1295.1</td>
<td>1002.8</td>
<td>0.459</td>
</tr>
</tbody>
</table>
**SIMULTANEOUS DEVELOPMENT TEST NO. 47**

TARE TEST TAKEN FROM RUN NO. 44

TARE THERMOCouple OUTPUT (MV)

<table>
<thead>
<tr>
<th></th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>I</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2522</td>
<td>4.2530</td>
<td>7</td>
<td>4.2532</td>
<td>4.2563</td>
</tr>
<tr>
<td>2</td>
<td>4.2373</td>
<td>4.2355</td>
<td>8</td>
<td>4.2550</td>
<td>4.2582</td>
</tr>
<tr>
<td>3</td>
<td>4.2550</td>
<td>4.2550</td>
<td>9</td>
<td>4.2562</td>
<td>4.2523</td>
</tr>
<tr>
<td>4</td>
<td>4.2588</td>
<td>4.2529</td>
<td>10</td>
<td>4.2592</td>
<td>4.2569</td>
</tr>
<tr>
<td>5</td>
<td>4.2603</td>
<td>4.2585</td>
<td>11</td>
<td>4.2923</td>
<td>4.2976</td>
</tr>
<tr>
<td>6</td>
<td>4.2518</td>
<td>4.2485</td>
<td>12</td>
<td>4.2937</td>
<td>4.3006</td>
</tr>
</tbody>
</table>

TARE THERMOCouple OUTPUT (MV)

<table>
<thead>
<tr>
<th></th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>I</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2593</td>
<td>4.2616</td>
<td>7</td>
<td>4.2764</td>
<td>4.2978</td>
</tr>
<tr>
<td>2</td>
<td>4.2451</td>
<td>4.2467</td>
<td>8</td>
<td>4.2790</td>
<td>4.2991</td>
</tr>
<tr>
<td>3</td>
<td>4.2885</td>
<td>4.2616</td>
<td>9</td>
<td>4.2982</td>
<td>4.3066</td>
</tr>
<tr>
<td>4</td>
<td>4.2613</td>
<td>4.2654</td>
<td>10</td>
<td>4.3026</td>
<td>4.3296</td>
</tr>
<tr>
<td>5</td>
<td>4.2709</td>
<td>4.2761</td>
<td>11</td>
<td>4.4837</td>
<td>4.5782</td>
</tr>
<tr>
<td>6</td>
<td>4.2604</td>
<td>4.2641</td>
<td>12</td>
<td>4.4645</td>
<td>4.5805</td>
</tr>
</tbody>
</table>

**DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.350**

FLOWMETER TEMPERATURE (MV) = 0.9795
BULK EXIT TEMP (MV) = 4.7076

INLET PRESSURE MAN. LEFT = 5.70
RIGHT = 5.90
IN. HG

INLET BULK TEMP. CR-CAL = 27.513

STATIC PRESSURE DROP (IN.)

<table>
<thead>
<tr>
<th></th>
<th>P0-P2</th>
<th>P1-P2</th>
<th>P2-P3</th>
<th>P3-P4</th>
<th>P4-P5</th>
<th>P5-P6</th>
<th>P6-P7</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.156</td>
<td>0.031</td>
<td>0.048</td>
<td>0.028</td>
<td>0.124</td>
<td>0.167</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.190</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**BLUE MANOMETER FLUID SP GR. 0.797**

INLET TEMPERATURE (DEG. F) = 121.85

TW/TH = 0.400
NACH = 0.015

PR = 0.7532

### Dimensionless Pressure Drop

<table>
<thead>
<tr>
<th></th>
<th>x+</th>
<th>(x+1)</th>
<th>M</th>
<th>Q+</th>
<th>NUM</th>
<th>TBULK</th>
<th>TW/TH (F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3868</td>
<td>0.2195</td>
<td>0.0313</td>
<td>2.7559</td>
<td>23.68</td>
<td>0.962</td>
<td>1570.1</td>
<td></td>
</tr>
<tr>
<td>0.3120</td>
<td>0.2471</td>
<td>0.0874</td>
<td>2.7544</td>
<td>299.22</td>
<td>0.883</td>
<td>1576.7</td>
<td></td>
</tr>
<tr>
<td>0.2373</td>
<td>0.2205</td>
<td>0.0921</td>
<td>2.4853</td>
<td>380.04</td>
<td>0.799</td>
<td>1677.3</td>
<td></td>
</tr>
<tr>
<td>0.1626</td>
<td>0.1471</td>
<td>0.1291</td>
<td>3.1043</td>
<td>488.93</td>
<td>0.704</td>
<td>1712.2</td>
<td></td>
</tr>
<tr>
<td>0.0879</td>
<td>0.0852</td>
<td>0.3460</td>
<td>3.2219</td>
<td>649.25</td>
<td>0.607</td>
<td>1660.8</td>
<td></td>
</tr>
<tr>
<td>0.0132</td>
<td>0.0135</td>
<td>1.1991</td>
<td>5.6405</td>
<td>977.22</td>
<td>0.475</td>
<td>1135.1</td>
<td></td>
</tr>
</tbody>
</table>

**NUMERICALIZED THERMOCouple OUTPUT**

<table>
<thead>
<tr>
<th></th>
<th>x+</th>
<th>(x+1)</th>
<th>M</th>
<th>F</th>
<th>F(RE)</th>
<th>FP</th>
<th>IP(RE)</th>
<th>IP(RE)</th>
<th>TP(RE)</th>
<th>TP(RE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0174</td>
<td>0.0177</td>
<td>0.01230</td>
<td>13.85</td>
<td>0.00584</td>
<td>6.58</td>
<td>1125.9</td>
<td>1947.7</td>
<td>0.456</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0586</td>
<td>0.0587</td>
<td>0.01949</td>
<td>13.12</td>
<td>0.00602</td>
<td>7.22</td>
<td>1179.8</td>
<td>769.8</td>
<td>0.546</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1333</td>
<td>0.1250</td>
<td>0.00844</td>
<td>11.10</td>
<td>0.00584</td>
<td>7.69</td>
<td>1315.8</td>
<td>559.7</td>
<td>0.650</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2047</td>
<td>0.1280</td>
<td>0.00799</td>
<td>11.74</td>
<td>0.00596</td>
<td>8.48</td>
<td>1412.7</td>
<td>525.8</td>
<td>0.755</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2827</td>
<td>0.2314</td>
<td>0.00828</td>
<td>13.50</td>
<td>0.00661</td>
<td>10.09</td>
<td>1525.8</td>
<td>536.7</td>
<td>0.843</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3575</td>
<td>0.2149</td>
<td>0.00910</td>
<td>15.74</td>
<td>0.00783</td>
<td>12.70</td>
<td>1622.1</td>
<td>265.2</td>
<td>0.926</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3917</td>
<td>0.2469</td>
<td>0.00444</td>
<td>14.19</td>
<td>0.00647</td>
<td>13.64</td>
<td>1672.7</td>
<td>233.7</td>
<td>0.969</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### Tare Test Tare from Run No. 44

#### Tare Thermocouple Output (MV)

<table>
<thead>
<tr>
<th></th>
<th>TA(1,1)</th>
<th>TA(1,2)</th>
<th>TA(1,1)</th>
<th>TA(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2522</td>
<td>4.2530</td>
<td>7.2532</td>
<td>4.2523</td>
</tr>
<tr>
<td>2</td>
<td>4.2373</td>
<td>4.2355</td>
<td>8.2562</td>
<td>4.2582</td>
</tr>
<tr>
<td>3</td>
<td>4.2550</td>
<td>4.2550</td>
<td>9.2562</td>
<td>4.2573</td>
</tr>
<tr>
<td>4</td>
<td>4.2588</td>
<td>4.2529</td>
<td>10.2592</td>
<td>4.2569</td>
</tr>
<tr>
<td>5</td>
<td>4.2503</td>
<td>4.2585</td>
<td>11.2923</td>
<td>4.2976</td>
</tr>
<tr>
<td>6</td>
<td>4.2518</td>
<td>4.2485</td>
<td>12.2937</td>
<td>4.3006</td>
</tr>
</tbody>
</table>

#### Test Thermocouple (Output (MV))

<table>
<thead>
<tr>
<th></th>
<th>TT(1,1)</th>
<th>TT(1,2)</th>
<th>TT(1,1)</th>
<th>TT(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2535</td>
<td>4.2554</td>
<td>7.2711</td>
<td>4.2836</td>
</tr>
<tr>
<td>2</td>
<td>4.2381</td>
<td>4.2389</td>
<td>8.2742</td>
<td>4.2628</td>
</tr>
<tr>
<td>3</td>
<td>4.2574</td>
<td>4.2516</td>
<td>9.2911</td>
<td>4.3102</td>
</tr>
<tr>
<td>4</td>
<td>4.2635</td>
<td>4.2646</td>
<td>10.2972</td>
<td>4.3176</td>
</tr>
<tr>
<td>5</td>
<td>4.2660</td>
<td>4.2698</td>
<td>11.4494</td>
<td>4.5307</td>
</tr>
<tr>
<td>6</td>
<td>4.2669</td>
<td>4.2597</td>
<td>12.4440</td>
<td>4.7463</td>
</tr>
</tbody>
</table>

### Differential Pressure - Flowmeter (IN.) = 0.305

**Flowmeter Temp (MV) = 0.9816**

**Bulk Exit Temp (MV) = 4.4940**

**Inlet Pressure Man, Left 6.00 Right 6.25 IN. HG**

**Inlet Bulk Temp CR-AL = 27.000**

### Static Pressure Drop (IN.)

\[ P_0-P_2 = 0.110 \]
\[ P_1-P_2 = 0.025 \]
\[ P_2-P_3 = 0.038 \]
\[ P_2-P_4 = 0.068 \]
\[ P_2-P_5 = 0.096 \]
\[ P_2-P_6 = 0.132 \]
\[ P_2-P_7 = 0.152 \]

### Blue Manometer Fluid SP CR 0.797

**Inlet Temperature (Deg. F) = 1198.87**

**TW/TB = 0.405 Mach No. = 0.013**

**PR = 0.750029 Reyd = 960.7**

### X+ (X+) M 0+ Num Tbulk Tw/Tb (PF) M Deg. F.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>0.4454</th>
<th>0.3283</th>
<th>0.0240</th>
<th>7.2532</th>
<th>7.5382</th>
<th>228.10</th>
<th>0.974</th>
<th>1.674</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.3934</td>
<td>0.2808</td>
<td>0.0755</td>
<td>8.2515</td>
<td>5.1244</td>
<td>288.04</td>
<td>0.896</td>
<td>1.671</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2733</td>
<td>0.2281</td>
<td>0.0779</td>
<td>6.2422</td>
<td>4.2242</td>
<td>368.19</td>
<td>0.810</td>
<td>1.295</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1873</td>
<td>0.1679</td>
<td>0.1029</td>
<td>4.1233</td>
<td>3.1233</td>
<td>478.59</td>
<td>0.745</td>
<td>1.233</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1012</td>
<td>0.0976</td>
<td>0.3140</td>
<td>2.9363</td>
<td>2.9363</td>
<td>643.56</td>
<td>0.609</td>
<td>1.101</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.0152</td>
<td>0.1555</td>
<td>1.0505</td>
<td>4.8555</td>
<td>4.8555</td>
<td>977.34</td>
<td>0.474</td>
<td>988.9</td>
</tr>
</tbody>
</table>

### Non Dimensionalized Pressure Drop

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>0.0200</th>
<th>0.130</th>
<th>0.313</th>
<th>0.595</th>
<th>0.2340</th>
<th>0.810</th>
<th>0.2550</th>
<th>1.025</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.6375</td>
<td>0.595</td>
<td>0.813</td>
<td>1.284</td>
<td>1.437</td>
<td>0.0128</td>
<td>12.57</td>
<td>0.00602</td>
</tr>
</tbody>
</table>

### X+ (X+) M F F(PE) M FP FP(PE) M Tbulk Tbl/Tb (PF) M Deg. F.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>0.0200</th>
<th>0.0204</th>
<th>0.0128</th>
<th>12.57</th>
<th>0.00602</th>
<th>6.20</th>
<th>981.0</th>
<th>1512.3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.0675</td>
<td>0.0673</td>
<td>0.0114</td>
<td>11.94</td>
<td>0.00462</td>
<td>6.71</td>
<td>1515.1</td>
<td>767.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1535</td>
<td>0.1430</td>
<td>0.0085</td>
<td>10.08</td>
<td>0.00005</td>
<td>6.87</td>
<td>1515.6</td>
<td>551.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.2390</td>
<td>0.2747</td>
<td>0.1080</td>
<td>10.11</td>
<td>0.00013</td>
<td>7.07</td>
<td>1542.2</td>
<td>414.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.3256</td>
<td>0.2630</td>
<td>0.0092</td>
<td>17.42</td>
<td>0.00716</td>
<td>9.61</td>
<td>1342.2</td>
<td>324.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.4117</td>
<td>0.3125</td>
<td>0.0109</td>
<td>15.64</td>
<td>0.00945</td>
<td>13.48</td>
<td>1426.5</td>
<td>255.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.4592</td>
<td>0.3379</td>
<td>0.1124</td>
<td>17.73</td>
<td>0.01228</td>
<td>15.44</td>
<td>1469.4</td>
<td>225.3</td>
</tr>
</tbody>
</table>
SIMULTANEOUS DEVELOPMENT TEST NO. 49
TAKE TEST TAKEN FROM RUN NO. 44
TAKE THERMOCOUPLP OUTPUT (MV)

<table>
<thead>
<tr>
<th></th>
<th>TA(1,1)</th>
<th>TA(1,2)</th>
<th>TA(1,1)</th>
<th>TA(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2522</td>
<td>4.2530</td>
<td>7</td>
<td>4.2532</td>
</tr>
<tr>
<td>2</td>
<td>4.2373</td>
<td>4.2550</td>
<td>8</td>
<td>4.2550</td>
</tr>
<tr>
<td>3</td>
<td>4.2550</td>
<td>4.2550</td>
<td>9</td>
<td>4.2562</td>
</tr>
<tr>
<td>4</td>
<td>4.2588</td>
<td>4.2529</td>
<td>10</td>
<td>4.2592</td>
</tr>
<tr>
<td>5</td>
<td>4.2603</td>
<td>4.2585</td>
<td>11</td>
<td>4.2923</td>
</tr>
<tr>
<td>6</td>
<td>4.2518</td>
<td>4.2485</td>
<td>12</td>
<td>4.2937</td>
</tr>
</tbody>
</table>

TEST THERMOCOUPLE OUTPUT (MV)

<table>
<thead>
<tr>
<th></th>
<th>TT(1,1)</th>
<th>TT(1,2)</th>
<th>TT(1,1)</th>
<th>TT(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2541</td>
<td>4.2554</td>
<td>7</td>
<td>4.2657</td>
</tr>
<tr>
<td>2</td>
<td>4.2388</td>
<td>4.2360</td>
<td>8</td>
<td>4.2689</td>
</tr>
<tr>
<td>3</td>
<td>4.2562</td>
<td>4.2592</td>
<td>9</td>
<td>4.2857</td>
</tr>
<tr>
<td>4</td>
<td>4.2604</td>
<td>4.2613</td>
<td>10</td>
<td>4.2932</td>
</tr>
<tr>
<td>5</td>
<td>4.2647</td>
<td>4.2645</td>
<td>11</td>
<td>4.4329</td>
</tr>
<tr>
<td>6</td>
<td>4.2557</td>
<td>4.2567</td>
<td>12</td>
<td>4.4315</td>
</tr>
</tbody>
</table>

DIFFERENTIAL PRESSURE - FLOWMETER (IN.) = 0.252
FLOWMETER TEMP (MV) = 0.9918
BULK EXIT TEMP (MV) = 4.2700
INLET PRESSURE MAN. LEFT 6.10 RIGHT 6.20 IN. HG
INLET BULK TEMP. CR-AL = 26.650

STATIC PRESSURE DROP (IN.)
P0-P2 = 0.076 P1-P2 = 0.076 P2-P3 = 0.078 P3-P4 = 0.108
P5-P7 = 0.138

BLUE HAND METER. FLUID SP GR. 0.797.
INLET TEMPERATURE (DEG. F.) = 1185.13
TW/TU = 0.408 MACH NO. = 0.011
PR = 0.247 M4/ REYD = 793.5

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+M)</th>
<th>Q+</th>
<th>NUM</th>
<th>TBULK</th>
<th>TW/TU (RE)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0439</td>
<td>0.3934</td>
<td>0.3998</td>
<td>7.2664</td>
<td>21.766</td>
<td>0.989</td>
</tr>
<tr>
<td>0.4364</td>
<td>0.3320</td>
<td>0.3670</td>
<td>6.7112</td>
<td>26.422</td>
<td>0.926</td>
</tr>
<tr>
<td>0.3319</td>
<td>0.2692</td>
<td>0.3537</td>
<td>2.1838</td>
<td>33.384</td>
<td>0.845</td>
</tr>
<tr>
<td>0.2274</td>
<td>0.1987</td>
<td>0.3915</td>
<td>1.9442</td>
<td>43.288</td>
<td>0.747</td>
</tr>
<tr>
<td>0.1230</td>
<td>0.1167</td>
<td>0.2684</td>
<td>2.7903</td>
<td>604.698</td>
<td>0.631</td>
</tr>
<tr>
<td>0.0185</td>
<td>0.0187</td>
<td>0.2689</td>
<td>4.3011</td>
<td>957.589</td>
<td>0.480</td>
</tr>
</tbody>
</table>

NON DIMENSIONALIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0243</td>
<td>1.500</td>
</tr>
<tr>
<td>0.0420</td>
<td>1.320</td>
</tr>
<tr>
<td>0.1065</td>
<td>1.199</td>
</tr>
<tr>
<td>0.2910</td>
<td>0.797</td>
</tr>
<tr>
<td>0.3954</td>
<td>1.003</td>
</tr>
<tr>
<td>0.4999</td>
<td>1.508</td>
</tr>
<tr>
<td>0.5570</td>
<td>1.847</td>
</tr>
</tbody>
</table>

FRAGMENTS OF TABLES AND TEXT.
<table>
<thead>
<tr>
<th></th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>TA(1,1)</th>
<th>TA(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2401</td>
<td>4.2410</td>
<td>4.2516</td>
<td>4.2531</td>
</tr>
<tr>
<td>2</td>
<td>4.2275</td>
<td>4.2261</td>
<td>4.2534</td>
<td>4.2571</td>
</tr>
<tr>
<td>3</td>
<td>4.2532</td>
<td>4.2537</td>
<td>4.2551</td>
<td>4.2513</td>
</tr>
<tr>
<td>4</td>
<td>4.2566</td>
<td>4.2561</td>
<td>4.2566</td>
<td>4.2560</td>
</tr>
<tr>
<td>5</td>
<td>4.2602</td>
<td>4.2597</td>
<td>4.2872</td>
<td>4.2896</td>
</tr>
<tr>
<td>6</td>
<td>4.2523</td>
<td>4.2489</td>
<td>4.2875</td>
<td>4.2904</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>TT(1,1)</th>
<th>TT(1,2)</th>
<th>TT(1,1)</th>
<th>TT(1,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2564</td>
<td>4.2594</td>
<td>4.2730</td>
<td>4.2835</td>
</tr>
<tr>
<td>2</td>
<td>4.2541</td>
<td>4.2427</td>
<td>4.2751</td>
<td>4.2874</td>
</tr>
<tr>
<td>3</td>
<td>4.2499</td>
<td>4.2620</td>
<td>4.2911</td>
<td>4.3067</td>
</tr>
<tr>
<td>4</td>
<td>4.2622</td>
<td>4.2649</td>
<td>4.2930</td>
<td>4.3112</td>
</tr>
<tr>
<td>5</td>
<td>4.2719</td>
<td>4.2769</td>
<td>4.4266</td>
<td>4.4910</td>
</tr>
<tr>
<td>6</td>
<td>4.2609</td>
<td>4.2626</td>
<td>4.4202</td>
<td>4.5001</td>
</tr>
</tbody>
</table>

**DIFFERENTIAL PRESSURE -FLOWMETER (IN.) = 0.402**

**FLUWMETER TEMP. (MV) = 0.9756**

**INLET PRESSURE MAN, LEFT 7.50 RIGHT 7.70 IN. HG**

**INLET BULK TEMP CR-AL = 19.589**

**STATIC PRESSURE DROP (IN.)**

<table>
<thead>
<tr>
<th></th>
<th>P0-P2</th>
<th>P1-P2</th>
<th>P2-P3</th>
<th>P2-P4</th>
<th>P2-P5</th>
<th>P2-P6</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.155</td>
<td>0.031</td>
<td>0.048</td>
<td>0.093</td>
<td>0.133</td>
<td>0.176</td>
</tr>
</tbody>
</table>

**BLUE MANOMETER FLUID SP GR = 0.797**

**INLET TEMPERATURE (DEG. F) = 892.67**

**TW/TT = 0.496, MACH NO. = 0.015**

**PR = 0.742313, REYD = 1330.0**

<table>
<thead>
<tr>
<th>X+ (X+)-</th>
<th>(X+)-M</th>
<th>Q+</th>
<th>NUM</th>
<th>TBULK</th>
<th>TW/TT (RF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.3423</td>
<td>0.2504</td>
<td>0.0342</td>
<td>234.11</td>
<td>0.965</td>
<td>1924.0</td>
</tr>
<tr>
<td>0.2762</td>
<td>0.2118</td>
<td>0.0431</td>
<td>282.44</td>
<td>0.903</td>
<td>1838.9</td>
</tr>
<tr>
<td>0.2101</td>
<td>0.1695</td>
<td>0.0944</td>
<td>341.92</td>
<td>0.837</td>
<td>1748.4</td>
</tr>
<tr>
<td>0.1439</td>
<td>0.1227</td>
<td>0.1651</td>
<td>418.55</td>
<td>0.764</td>
<td>1651.1</td>
</tr>
<tr>
<td>0.0778</td>
<td>0.1706</td>
<td>0.3457</td>
<td>526.69</td>
<td>0.681</td>
<td>1541.9</td>
</tr>
<tr>
<td>0.0117</td>
<td>0.1114</td>
<td>1.1146</td>
<td>740.03</td>
<td>0.566</td>
<td>1395.6</td>
</tr>
</tbody>
</table>

**NON DIMENSIONALIZED PRESSURE DROP**

<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0154</td>
<td>0.189</td>
</tr>
<tr>
<td>0.0519</td>
<td>0.361</td>
</tr>
<tr>
<td>0.1180</td>
<td>0.626</td>
</tr>
<tr>
<td>0.1841</td>
<td>0.878</td>
</tr>
<tr>
<td>0.2503</td>
<td>1.098</td>
</tr>
<tr>
<td>0.3169</td>
<td>1.337</td>
</tr>
<tr>
<td>0.3929</td>
<td>1.500</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X+ (X+)-</th>
<th>F</th>
<th>F(RE)-M</th>
<th>FP</th>
<th>FP(RE)-M</th>
<th>RE</th>
<th>TBULK</th>
<th>TB/TT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0154</td>
<td>0.0151</td>
<td>0.01336</td>
<td>14.32</td>
<td>0.00536</td>
<td>7.41</td>
<td>1382.2</td>
<td>762.7</td>
</tr>
<tr>
<td>0.0519</td>
<td>0.0488</td>
<td>0.00942</td>
<td>13.91</td>
<td>0.00574</td>
<td>8.47</td>
<td>1706.4</td>
<td>606.2</td>
</tr>
<tr>
<td>0.1180</td>
<td>0.1383</td>
<td>0.00765</td>
<td>12.22</td>
<td>0.00707</td>
<td>9.10</td>
<td>1596.5</td>
<td>466.9</td>
</tr>
<tr>
<td>0.1841</td>
<td>0.1520</td>
<td>0.00707</td>
<td>12.01</td>
<td>0.00559</td>
<td>9.49</td>
<td>1693.6</td>
<td>377.4</td>
</tr>
<tr>
<td>0.2503</td>
<td>0.1970</td>
<td>0.00769</td>
<td>13.77</td>
<td>0.00600</td>
<td>10.75</td>
<td>1792.0</td>
<td>510.4</td>
</tr>
<tr>
<td>0.3169</td>
<td>0.2372</td>
<td>0.00305</td>
<td>17.00</td>
<td>0.00759</td>
<td>14.26</td>
<td>1879.6</td>
<td>257.0</td>
</tr>
<tr>
<td>0.3929</td>
<td>0.2579</td>
<td>0.00344</td>
<td>10.71</td>
<td>0.00466</td>
<td>17.07</td>
<td>1925.8</td>
<td>231.7</td>
</tr>
</tbody>
</table>

**DEG. F**

| 0.0154 | 0.0151 | 0.01336 | 14.32 | 0.00536 | 7.41 | 1382.2 | 762.7 |
| 0.0519 | 0.0488 | 0.00942 | 13.91 | 0.00574 | 8.47 | 1706.4 | 606.2 |
| 0.1180 | 0.1383 | 0.00765 | 12.22 | 0.00707 | 9.10 | 1596.5 | 466.9 |
| 0.1841 | 0.1520 | 0.00707 | 12.01 | 0.00559 | 9.49 | 1693.6 | 377.4 |
| 0.2503 | 0.1970 | 0.00769 | 13.77 | 0.00600 | 10.75 | 1792.0 | 510.4 |
| 0.3169 | 0.2372 | 0.00305 | 17.00 | 0.00759 | 14.26 | 1879.6 | 257.0 |
| 0.3929 | 0.2579 | 0.00344 | 10.71 | 0.00466 | 17.07 | 1925.8 | 231.7 |
SIMULTANEOUS DEVELOPMENT TEST NO. 51
TARE TEST TAKEN FROM RUN NO. 50
TARE THERMOCOUPLE OUTPUT (MV).

<table>
<thead>
<tr>
<th></th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
<th>TA(I,1)</th>
<th>TA(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2401</td>
<td>4.2410</td>
<td>7</td>
<td>4.2516</td>
</tr>
<tr>
<td>2</td>
<td>4.2275</td>
<td>4.2261</td>
<td>8</td>
<td>4.2534</td>
</tr>
<tr>
<td>3</td>
<td>4.2532</td>
<td>4.2537</td>
<td>9</td>
<td>4.2551</td>
</tr>
<tr>
<td>4</td>
<td>4.2566</td>
<td>4.2561</td>
<td>10</td>
<td>4.2566</td>
</tr>
<tr>
<td>5</td>
<td>4.2602</td>
<td>4.2597</td>
<td>11</td>
<td>4.2872</td>
</tr>
<tr>
<td>6</td>
<td>4.2523</td>
<td>4.2489</td>
<td>12</td>
<td>4.2875</td>
</tr>
</tbody>
</table>

TEST THERMOCOUPLE OUTPUT (MV).

<table>
<thead>
<tr>
<th></th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
<th>TT(I,1)</th>
<th>TT(I,2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.2543</td>
<td>4.2564</td>
<td>7</td>
<td>4.2646</td>
</tr>
<tr>
<td>2</td>
<td>4.2394</td>
<td>4.2391</td>
<td>8</td>
<td>4.2689</td>
</tr>
<tr>
<td>3</td>
<td>4.2573</td>
<td>4.2609</td>
<td>9</td>
<td>4.2834</td>
</tr>
<tr>
<td>4</td>
<td>4.2615</td>
<td>4.2662</td>
<td>10</td>
<td>4.2886</td>
</tr>
<tr>
<td>5</td>
<td>4.2673</td>
<td>4.2699</td>
<td>11</td>
<td>4.4134</td>
</tr>
<tr>
<td>6</td>
<td>4.2572</td>
<td>4.2588</td>
<td>12</td>
<td>4.4123</td>
</tr>
</tbody>
</table>

DIFFERENTIAL PRESSURE -FLOWMETER (IN.) = 0.352
FLOWMETER TEMP (MV) = 0.9962 BULK EXIT TEMP (MV) = 4.4663
INLET PRESSURE MAN, LEFT 7.80, RIGHT 7.90 IN. HG
INLET BULK TEMP CR-AL = 19.650
STATIC PRESSURE DROP (IN.)

<table>
<thead>
<tr>
<th></th>
<th>P0-P2 =</th>
<th>0.126</th>
<th>P1-P2 =</th>
<th>0.000</th>
<th>P2-P3 =</th>
<th>0.042</th>
</tr>
</thead>
<tbody>
<tr>
<td>02-P4 =</td>
<td>0.081</td>
<td>P2-P5 =</td>
<td>0.152</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>02-P7 =</td>
<td>0.127</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

BLUE MANOMETER FLUID SP GR = 0.797
INLET TEMPERATURE (DEG. F.) = 895.28
TW/To = 0.495, NACH MN. = 0.013
PR = 0.742, DURATION = 1160.8

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)*M</th>
<th>Q+</th>
<th>NUM</th>
<th>TRULK</th>
<th>TW/Tr (RE)</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.392</td>
<td>0.2540</td>
<td>0.0191</td>
<td>5.4477</td>
<td>225.58</td>
<td>0.978</td>
<td>1695.5</td>
</tr>
<tr>
<td>0.3163</td>
<td>0.2392</td>
<td>0.0742</td>
<td>5.3326</td>
<td>267.83</td>
<td>0.921</td>
<td>1627.4</td>
</tr>
<tr>
<td>0.2426</td>
<td>0.1907</td>
<td>0.0647</td>
<td>2.2400</td>
<td>320.32</td>
<td>0.859</td>
<td>1554.2</td>
</tr>
<tr>
<td>0.1408</td>
<td>0.1377</td>
<td>0.1470</td>
<td>3.0165</td>
<td>388.46</td>
<td>0.791</td>
<td>1473.4</td>
</tr>
<tr>
<td>0.0891</td>
<td>0.0792</td>
<td>0.2787</td>
<td>3.4042</td>
<td>448.36</td>
<td>0.709</td>
<td>1378.5</td>
</tr>
<tr>
<td>0.0134</td>
<td>0.0129</td>
<td>1.0341</td>
<td>6.4253</td>
<td>700.23</td>
<td>0.589</td>
<td>1271.0</td>
</tr>
</tbody>
</table>

NON DIMENSIONALIZED PRESSURE DROP

<table>
<thead>
<tr>
<th>X+</th>
<th>P+</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0176</td>
<td>0.423</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.0394</td>
<td>0.423</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.1351</td>
<td>0.733</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2109</td>
<td>1.019</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.2802</td>
<td>1.253</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.3861</td>
<td>1.539</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.4922</td>
<td>1.720</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>X+</th>
<th>(X+)*M</th>
<th>F (RE)</th>
<th>Fp (RE)</th>
<th>Fp</th>
<th>(RE)</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0176</td>
<td>0.0172</td>
<td>0.00333</td>
<td>4.08</td>
<td>-0.00210</td>
<td>-2.51</td>
<td>1274.0</td>
</tr>
<tr>
<td>0.0594</td>
<td>0.0548</td>
<td>0.00667</td>
<td>9.06</td>
<td>0.00300</td>
<td>3.96</td>
<td>1334.4</td>
</tr>
<tr>
<td>0.1351</td>
<td>0.1164</td>
<td>0.00948</td>
<td>13.53</td>
<td>0.00769</td>
<td>10.97</td>
<td>1726.6</td>
</tr>
<tr>
<td>0.2129</td>
<td>0.1116</td>
<td>0.00866</td>
<td>13.10</td>
<td>0.00738</td>
<td>11.17</td>
<td>1913.1</td>
</tr>
<tr>
<td>0.2860</td>
<td>0.2220</td>
<td>0.00750</td>
<td>11.92</td>
<td>0.00547</td>
<td>9.49</td>
<td>1581.5</td>
</tr>
<tr>
<td>0.3629</td>
<td>0.2844</td>
<td>0.0125</td>
<td>17.02</td>
<td>0.00925</td>
<td>15.03</td>
<td>1659.0</td>
</tr>
<tr>
<td>0.4041</td>
<td>0.3296</td>
<td>0.1137</td>
<td>18.74</td>
<td>0.00977</td>
<td>16.92</td>
<td>1696.7</td>
</tr>
</tbody>
</table>

PR = 0.742, DURATION = 1160.8, DEG. F.
Appendix I
Derivation of the Similar Boundary Layer Equations

When the thermal or velocity boundary layer thickness on a body of revolution is small when compared to the local radius of the body, then the energy, continuity and momentum equations (2.30, 2.31, 2.32) may be written with the radius terms removed;

\[
\rho u^+ \frac{\partial u^+}{\partial x^+} + \rho v^+ \frac{\partial u^+}{\partial r^+} = \frac{dp}{dx^+} + 2Pr_0 \frac{\partial}{\partial r^+} \mu^+ \frac{\partial u^+}{\partial r^+} \tag{I.1}
\]

\[
\frac{\partial}{\partial x^+} \left( \rho u^+ \right) + \frac{\partial}{\partial r^+} \left( \rho v^+ \right) = 0 \tag{I.2}
\]

\[
\rho u^+ \frac{\partial H_2^+}{\partial x^+} + \rho v^+ \frac{\partial H_2^+}{\partial r^+} = 2 \frac{\nu^+}{\mu^+} \frac{\partial}{\partial r^+} \left( \rho H_2^+ \right) - (\nu^+ - 1) \frac{\partial}{\partial r^+} \left( u^+ \frac{dp}{dx^+} \right) + 2Pr_0 \mu^+ \left( \frac{\partial u^+}{\partial r^+} \right)^2 \tag{I.3}
\]

where the usual form of the dissipation term in the energy equation has been retained. The stream function \( \psi \) is introduced to eliminate the continuity equation;

\[
\frac{\partial \psi}{\partial x^+} = -\rho v^+ \tag{I.4}
\]

\[
\frac{\partial \psi}{\partial y^+} = \rho u^+ \tag{I.5}
\]

We introduce the transformation to new independent variables;

\[
\xi = \int_0^{x^+} \rho e^+ \mu e^+ u_e^+ \, dx^+ \tag{I.6}
\]

\[
\eta = \frac{u_e^+}{\sqrt{2\xi}} \int_0^{y^+} \rho e^+ \, dy^+ \tag{I.7}
\]

where \( y^+ = 1 - r^+ \) represents the dimensionless displacement from the tube wall and \( C \) is a constant. Subscript \( e \) refers to quantities at the edge of the boundary layer or in the central core. We also introduce two new dependent variables;
Expanding the axial derivative in terms of the new independent variables,

\[ \frac{\partial}{\partial x^+} = \frac{\partial \xi}{\partial x^+} \frac{\partial}{\partial \xi} + \frac{\partial \eta}{\partial x^+} \frac{\partial}{\partial \eta} \]  

and the radial derivative,

\[ \frac{\partial}{\partial r} = -\frac{\partial r}{\partial r} \frac{\partial}{\partial \eta} + \frac{\partial \xi}{\partial r} \frac{\partial}{\partial \xi} \]  

It can be shown that the stream function \( \psi \) is is related to the velocity function \( f \) by,

\[ \psi = \sqrt{2 \xi} f \]  

From the stream function we may solve for the radial velocity,

\[ v^+ = -\frac{1}{\rho^+} \frac{\partial \psi}{\partial x^+} = -\frac{1}{\rho^+} \left( \frac{\partial \xi}{\partial x^+} \frac{f}{\sqrt{2 \xi}} + \frac{\partial \eta}{\partial x^+} \frac{\partial}{\partial \eta} (\sqrt{2 \xi} f) \right) \]  

\[ = \frac{C \rho^+ \mu_e}{\rho^+} \left( \frac{f'}{\sqrt{2 \xi}} - \frac{f}{\sqrt{2 \xi}} \frac{\partial u_e^+}{\partial \xi} \right) \]  

where primes denote differentiation with respect to \( \eta \). We consider each term in the momentum equation separately. Starting with the first term on the left hand side of equation I.1.1:

\[ \rho^+ u_e^+ \frac{\partial u^+}{\partial x^+} = \rho^+ u_e^+ \left( \frac{\partial \xi}{\partial x^+} \frac{\partial u^+}{\partial \xi} + \frac{\partial \eta}{\partial x^+} \frac{\partial u^+}{\partial \eta} \right) \]  

\[ = \rho^+ u_e^+ \left( \frac{\partial \xi}{\partial x^+} \frac{\partial (u_e^+ f')}{\partial \xi} + \frac{\partial (u_e^+ f)}{\partial \eta} \frac{\partial \xi}{\partial \eta} \right) \]  

We note that \( \xi \) is a function only of the axial coordinate \( x^+ \) since all quantities of which it is composed are functions only of the axial coordinate. Continuing the expansion:

\[ \rho^+ u_e^+ \frac{\partial u^+}{\partial x^+} = C \rho^+ \mu_e u_e^+ \left( u_e^+ f^2 \frac{\partial u^+}{\partial \xi} + \eta f f' u_e^+ \frac{\partial u^+}{\partial \xi} - \eta f f'' u_e^+ \frac{\partial u^+}{\partial \xi} \right) \]  

\[ = C \rho^+ \mu_e u_e^+ \left( u_e^+ f^2 \frac{\partial u^+}{\partial \xi} + \eta f f' u_e^+ \frac{\partial u^+}{\partial \xi} - \eta f f'' u_e^+ \frac{\partial u^+}{\partial \xi} \right) \]  

\[ = C \rho^+ \mu_e u_e^+ \left( u_e^+ f^2 \frac{\partial u^+}{\partial \xi} + \eta f f' u_e^+ \frac{\partial u^+}{\partial \xi} - \eta f f'' u_e^+ \frac{\partial u^+}{\partial \xi} \right) \]
In the solution of the boundary layer equations at the tube wall, the convention for the sign of the transverse velocity \( v^+ \) was reversed. A positive value is away from the tube wall. This is in accordance with usual boundary layer convention.

\[
\rho v^+ \frac{\partial u^+}{\partial y^+} = C_{\rho e} \mu_{e} e \rho \left( \frac{v'}{\sqrt{2 \xi}} - \frac{1}{2} \frac{\partial}{\partial \xi} \frac{\partial u^+}{\partial \xi} \right) \frac{u^+}{\sqrt{2 \xi}} \quad (I.17)
\]

Next consider,

\[
\frac{\partial \mu \frac{\partial u^+}{\partial r^+}}{\partial r} = \frac{\partial \rho \frac{\partial u^+}{\partial y^+}}{\partial y} \quad (I.18)
\]

\[
= \frac{\partial u^+}{\partial y} \frac{\partial \mu}{\partial y} + \mu \frac{\partial^2 u^+}{\partial y^2} + \frac{\partial (\partial u^+ \partial y^+)}{\partial y^+} \frac{\partial^2 u^+}{\partial y^2} \quad (I.19)
\]

\[
= \frac{\rho U_e^4}{2 \xi} \left( \frac{\partial \rho}{\partial y} \mu f'' + \mu f' \frac{\partial \rho}{\partial y} + \mu f' \right) \quad (I.20)
\]

The pressure gradient term becomes;

\[
\frac{dp}{dx^+} = \frac{d}{dx^+} \left( \frac{P_{ref}}{\rho U_0^2} \right) = - \frac{dp}{dx^+} \frac{\rho_e}{\rho U_0^2} \quad (I.21)
\]

Since we are assuming the flow in the core is inviscid (i.e. potential flow),

\[
\frac{dp}{dx^+} = -\rho_e U_e \frac{dU_e}{dx^+} \quad (I.22)
\]

\[
\frac{dp}{dx^+} = \rho_e U_e \frac{dU_e}{dx^+} \frac{\rho_e}{\rho U_0^2} = \rho_e U_e \frac{dU_e}{dx^+} \quad (I.23)
\]
\[
\begin{align*}
\frac{dP}{d\xi} &= \frac{dP}{dx^*} \frac{dx^*}{d\xi} = \rho_e u_e^* \frac{du_e^*}{C_p e u_e^* dx^*} = \rho_e u_e^* \frac{du_e^*}{d\xi} \\
(1.24)
\end{align*}
\]

After combining all terms in the momentum equation and after division by a common factor \( C_p e \rho_e u_e^* / 2 \xi \), many terms are found to cancel. The final result is:

\[
2 \xi \frac{du_e^*}{u_e^* \frac{dx^*}{d\xi}} \left( f' - \frac{\rho_e^*}{\rho^*} \right) = 2 \text{Pr}_o \left( \frac{u_e^*}{C_p e u_e^*} \right) \left( \lambda f'' \right) + ff''
\]

(1.25)

Drawing the constant term \( C_p e \rho_e^* \) within the differential in the first term on the right hand side of this equation;

\[
2 \xi \frac{du_e^*}{u_e^*} \left( f' - \frac{\rho_e^*}{\rho^*} \right) = 2 \text{Pr}_o \left( \lambda f'' \right) + ff''
\]

(1.26)

where

\[
\lambda = \frac{\mu^* \rho^*}{C_p e \rho_e^*}
\]

(1.27)

For the energy equation, the free stream or "core" enthalpy is considered as being invariant with axial distance;

\[
\frac{\partial H_e^*}{\partial \xi} = \frac{\partial H_e^*}{\partial x^*} = 0
\]

(1.28)

Again considering separate terms in the energy equation;

\[
\rho u^* \frac{\partial H_e^*}{\partial x^*} = \rho u_e^* \frac{H_e^*}{u_e^*} \frac{\partial u_e^*}{\partial \eta} \frac{\partial \eta}{\partial \xi} \frac{\partial \xi}{\partial x^*} = C_p e \rho_e^* u_e^* \frac{2 H_e^*}{u_e^*} G \left( \frac{\eta}{u_e^* \frac{\partial u_e^*}{\partial \xi}} - \frac{\eta f}{2 \xi} \right)
\]

(1.29)

\[
\rho v^* \frac{\partial H_e^*}{\partial y^*} = \rho v_e^* \frac{H_e^*}{u_e^*} \frac{\partial u_e^*}{\partial \eta} \frac{\partial \eta}{\partial y^*}
\]

(1.30)

\[
= C_p e \rho_e^* u_e^* \left( \frac{\eta f}{\sqrt{2 \xi}} - \frac{f}{\sqrt{2 \xi}} \right) \frac{\partial u_e^*}{\partial \xi} \left( \frac{\partial u_e^*}{\partial \xi} \right) \frac{\partial u_e^*}{\partial \xi} G'
\]

(1.31)
\[ \frac{\partial}{\partial y} \left( k^+ \frac{\partial H^+}{\partial y} \right) = \frac{\partial}{\partial y} \left( \frac{\partial}{\partial \eta} \left( k^+ \frac{\partial}{\partial \eta} \frac{\partial H^+}{\partial \eta} \right) \right) \]  
\hfill (I.33)

\[ = \left( \frac{\rho e U_e}{\sqrt{2} \xi} \right)^2 H_{2,e}^+ G^+ \frac{\partial}{\partial \eta} \left( k^+ \frac{\partial}{\partial \eta} \right) + \left( \frac{\rho e U_e}{\sqrt{2} \xi} \right) k^+ \frac{\partial}{\partial \eta} \frac{\partial \rho^+}{\partial \eta} + \left( \frac{\rho e U_e}{\sqrt{2} \xi} \right) k^+ \frac{\partial}{\partial \eta} \frac{\partial H^+}{\partial \eta} \]  
\hfill (I.34)

\[ \frac{\rho e U_e}{2 \xi} H_{2,e}^+ \left[ \rho e G^+ \frac{\partial}{\partial \eta} \left( k^+ \frac{\partial}{\partial \eta} \right) + \left( \rho e \right) G^+ \frac{\partial}{\partial \eta} \left( k^+ \frac{\partial}{\partial \eta} \right) + \rho e \right] \]  
\hfill (I.35)

\[ = \frac{\rho e U_e^2}{2 \xi} \left( \rho e \right) \left( k^+ \frac{\partial}{\partial \eta} \right) \]  
\hfill (I.36)

\[ \frac{U^dP}{dX} = \frac{C R e \mu^+ U_e^3 f \frac{dU_e}{d\xi}}{d\xi} \]  
\hfill (I.37)

After combining these terms into a single expression, many terms are seen to cancel. The final result is;

\[ 2 \left( \frac{\rho e U_e^2 H_{2,e}^+ \left( k^+ \frac{\partial}{\partial \eta} G^+ \right)}{2 \xi} + \frac{\rho e H_{2,e}^+ c p U_e c p G^+ f}{2 \xi} \right) = (\gamma_0 - 1) M_0^2 \left[ \frac{\rho e^2 \mu^+ f \frac{dU_e}{d\xi}}{U_e^2} - 2 P_0 \frac{\rho e^2}{\mu^+} \frac{U_e^2}{H_{2,e}^+} \frac{\rho^2}{2 \xi} \right] \]  
\hfill (I.39)

Dividing by a common factor \( C R e \mu^+ U_e / 2 \xi \) yields;

\[ 2 \left( \frac{\rho G^+ k^+}{C R e \mu^+} \right) + G^+ f = (\gamma_0 - 1) M_0^2 \left[ \frac{\rho e^2 \mu^+ f \frac{dU_e}{d\xi}}{U_e^2} - 2 P_0 \frac{\rho e^2}{\mu^+} \frac{U_e^2}{H_{2,e}^+} \frac{\rho^2}{2 \xi} \right] \]  
\hfill (I.40)

Let

\[ \beta = \frac{2 \xi dU_e}{U_e d\xi} \]  
\hfill (I.41)

\[ P_r^+ = \chi_p \mu^+ k^+ \]  
\hfill (I.42)
The final form of the equation becomes

\[ 2 \left( \frac{\lambda G^\prime}{Pr^*} \right) + C f = (\gamma_0 - i) \mathcal{M}_0 \left[ \frac{\rho e}{p H_2 e} \beta f' - 2 \Pr_0 \frac{U_e^2}{H_2 e} f'' \right] \]  

(I.43)
NORMAN ZETHWARD SHILLING, son of Edward Shilling and Bessie Arlene Jenkins was born in New Jersey on September 1. He graduated from Lyndhurst High School, Lyndhurst, New Jersey in 1962 and attended Stevens Institute of Technology and Newark College of Engineering. He received his B.S. in M.E. from the latter institution with the designation Summa Cum Laude in June, 1966. He was elected to membership in Pi Tau Sigma and Tau Beta Pi honor societies.

In September of 1966 he entered Massachusetts Institute of Technology holding a National Science Foundation Traineeship. He was nominated and elected to the MIT chapter of Sigma Xi in May of 1967. His M.S. thesis topic was "Dynamic Compensation Techniques for Plenum Fluid Suspensions". He completed requirements for his M.S. in M.E. in August 1967.

On September 2, 1967 he married Mary Eleanor Powell of Lyndhurst, New Jersey and subsequently returned to Newark College of Engineering as a doctoral candidate in the Department of Mechanical Engineering. While completing requirements for the D.Sc. degree he was the recipient of NASA research and NDEA teaching fellowships. He began work on the present dissertation in June of 1969. All experimental work was performed in the Mechanical Engineering Department laboratories during the period from September 1969 to August 1971. Support was provided by the Foundation for the Advancement of Graduate Study in Engineering.