Spring 1977

Thermal stratification and circulation of water bodies subjected to thermal discharge

Mohammad A. Borhani
New Jersey Institute of Technology

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BY

MOHAMMAD A. BORHANI

A DISSERTATION
PRESENTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF ENGINEERING SCIENCE AT NEW JERSEY INSTITUTE OF TECHNOLOGY

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Newark, New Jersey
1977
ABSTRACT

A three-dimensional analytical model for large water bodies is presented. Time histories and spatial distribution of pressure, velocity and temperature in water bodies, subjected to thermal discharge, are determined employing a digital computer. The dynamic response is obtained for a rectangular water body by applying a finite difference method to the mass, momentum and energy balance equations. These partial differential equations are algebraically manipulated to obtain; 1) three parabolic differential equations integrated temporally to find the horizontal velocity components and temperature; 2) one algebraic integral equation to get the vertical velocity component; 3) one elliptic differential equation integrated spatially to find the pressure; and 4) one differential equation to get the water level. Numerical stability criteria are developed which facilitate the selection of space and time increments for stable numerical integration.

The distinctive feature of this analysis, as compared to previous studies, is the calculation of pressure and water level from equations of motion without simplifying assumptions such as hydrostatic pressure approximation and rigid-lid concept.

The mathematical formulation is verified by applying this analysis to cases where the final steady state flow patterns have been determined analytically or experimentally by others. In particular, the final steady state solution obtained from this
dynamic analysis is verified with existing flow measurements of laminar flow development in a square duct. Furthermore, the natural circulation flows developed by this analysis are verified with known flow patterns in partially heated ponds.

The problem of thermal discharge entering a river with known initial velocity and temperature distribution is then analyzed. The time histories of the velocity and temperature distribution as well as the velocity and temperature profiles are obtained. These results provide the values of temperature rise and the rate of temperature rise needed for the assessment of the extent of thermal pollution in water bodies.
APPROVAL OF DISSERTATION

THERMAL STRATIFICATION AND CIRCULATION OF
WATER BODIES SUBJECTED TO THERMAL DISCHARGE

BY

MOHAMMAD A. BORHANI

FOR

DEPARTMENT OF MECHANICAL ENGINEERING
NEW JERSEY INSTITUTE OF TECHNOLOGY

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March, 1977
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>DESCRIPTION</th>
<th>PAGE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td>i</td>
</tr>
<tr>
<td>APPROVAL OF DISSERTATION</td>
<td>iii</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td>x</td>
</tr>
</tbody>
</table>

## PART ONE

1. INTRODUCTION
   1.1 Review of Previous Work
   1.2 Scope and Objectives of the Present Study

2. MATHEMATICAL FORMULATION
   2.1 Simplifying Assumption
   2.2 Governing Equations
   2.3 Boundary Conditions

3. NUMERICAL SOLUTION
   3.1 Calculation of the Vertical Velocity Components
   3.2 Calculation of Pressure Distribution
   3.3 Calculation of Time Derivatives for the Horizontal Velocity Components and Temperature
   3.4 Calculation of the Time Derivatives for the Water Level, Horizontal Velocity Components and Temperature in Water Level Elements
TABLE OF CONTENTS (Cont'd)

<table>
<thead>
<tr>
<th>Section</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5 Calculation of the Horizontal Velocity Components and Temperature</td>
<td>52</td>
</tr>
<tr>
<td>3.6 Calculation of the Water Level, Velocity Components and Temperature in the Water Level Elements</td>
<td>52</td>
</tr>
<tr>
<td>3.7 Calculation of Density Distribution</td>
<td>53</td>
</tr>
<tr>
<td>4. NUMERICAL STABILITY AND CONVERGENCE</td>
<td>55</td>
</tr>
<tr>
<td>4.1 Stability Analysis of the Horizontal Momentum and Energy Equations</td>
<td>57</td>
</tr>
<tr>
<td>4.2 Convergence of Pressure Distribution</td>
<td>60</td>
</tr>
<tr>
<td>4.3 Stability of the Water Level Equation</td>
<td>61</td>
</tr>
<tr>
<td>5. PRESENTATION OF RESULTS</td>
<td>63</td>
</tr>
<tr>
<td>5.1 Verification Studies</td>
<td>63</td>
</tr>
<tr>
<td>5.2 Circulation and Stratification in Water Bodies</td>
<td>75</td>
</tr>
<tr>
<td>5.3 Three Dimensional Non-Buoyant Jet in a Cross Current</td>
<td>75</td>
</tr>
<tr>
<td>5.4 Three Dimensional Buoyant Jet in a Cross Current</td>
<td>90</td>
</tr>
<tr>
<td>6. CONCLUSIONS AND RECOMMENDATIONS</td>
<td>102</td>
</tr>
<tr>
<td>7. NOMENCLATURE</td>
<td>104</td>
</tr>
<tr>
<td>8. REFERENCES</td>
<td>107</td>
</tr>
<tr>
<td>APPENDICES</td>
<td></td>
</tr>
<tr>
<td>APPENDIX 1 Derivation of the Governing Equations (1) through (5)</td>
<td>111</td>
</tr>
<tr>
<td>APPENDIX 2 Verification of Stability Analysis</td>
<td>117</td>
</tr>
<tr>
<td>APPENDIX 3 Calculation of the First and Second Derivatives at the Boundary and in the Flow Field</td>
<td>122</td>
</tr>
</tbody>
</table>
TABLE OF CONTENTS (Cont'd)

PART TWO

<table>
<thead>
<tr>
<th></th>
<th>DESCRIPTION OF THERMA DIGITAL COMPUTER PROGRAM</th>
<th>PAGE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>DESCRIPTION OF THERMA DIGITAL COMPUTER PROGRAM</td>
<td>132</td>
</tr>
<tr>
<td>1.1</td>
<td>MAIN Program</td>
<td>204</td>
</tr>
<tr>
<td>1.2</td>
<td>DERIV Subroutine</td>
<td>205</td>
</tr>
<tr>
<td>1.3</td>
<td>PRESS Subroutine</td>
<td>205</td>
</tr>
<tr>
<td>1.4</td>
<td>SUM Subroutine</td>
<td>206</td>
</tr>
<tr>
<td>1.5</td>
<td>SI Function</td>
<td>206</td>
</tr>
<tr>
<td>1.6</td>
<td>OUTP Subroutine</td>
<td>207</td>
</tr>
<tr>
<td>2</td>
<td>DESCRIPTION OF THE INPUT DATA</td>
<td>208</td>
</tr>
<tr>
<td>2.1</td>
<td>Non-Subscripted Variables</td>
<td>208</td>
</tr>
<tr>
<td>2.2</td>
<td>Subscripted Variables</td>
<td>215</td>
</tr>
<tr>
<td>3</td>
<td>DESCRIPTION OF THE OUTPUT DATA</td>
<td>216</td>
</tr>
<tr>
<td>4</td>
<td>OPERATING PROCEDURE</td>
<td>218</td>
</tr>
<tr>
<td>5</td>
<td>PROGRAM NOMENCLATURE</td>
<td>220</td>
</tr>
<tr>
<td>6</td>
<td>PROGRAM LISTING AND SAMPLE RUN</td>
<td>226</td>
</tr>
<tr>
<td></td>
<td>VITA</td>
<td>276</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 1</td>
<td>Schematic Diagram of the Water Body</td>
<td>13</td>
</tr>
<tr>
<td>Figure 2</td>
<td>Velocity Development in a Square Duct</td>
<td>66</td>
</tr>
<tr>
<td>Figure 3</td>
<td>Natural Circulation at $y=100$ ft. and $t=40$ sec. in a Pond Partially Heated from Side</td>
<td>71</td>
</tr>
<tr>
<td>Figure 4</td>
<td>Natural Circulation at $x=240$ ft. and $t=40$ sec. in a Pond Partially Heated from Side</td>
<td>72</td>
</tr>
<tr>
<td>Figure 5</td>
<td>Natural Circulation at $y=100$ ft. and $t=25$ sec. in a Pond Partially Heated from Bottom</td>
<td>73</td>
</tr>
<tr>
<td>Figure 6</td>
<td>Natural Circulation at $t=25$ sec. in Vertical Diagonal Plane in a Pond Partially Heated from Bottom</td>
<td>74</td>
</tr>
<tr>
<td>Figure 7</td>
<td>Grid Work with Variable Mesh Size Superimposed on the Water Body</td>
<td>79</td>
</tr>
<tr>
<td>Figure 8</td>
<td>Time Histories of Variables at Point A in the Water Body Subjected to Non-Buoyant and Buoyant Jets</td>
<td>80</td>
</tr>
<tr>
<td>Figure 9</td>
<td>Time Histories of Variables at Point B in the Water Body Subjected to Non-Buoyant and Buoyant Jets</td>
<td>81</td>
</tr>
<tr>
<td>Figure 10</td>
<td>Time Histories of Variables at Point C in the Water Body Subjected to Non-Buoyant and Buoyant Jets</td>
<td>82</td>
</tr>
<tr>
<td>Figure 11</td>
<td>Time Histories of Variables at Point D in the Water Body Subjected to Non-Buoyant and Buoyant Jets</td>
<td>83</td>
</tr>
<tr>
<td>Figure 12</td>
<td>Time Histories of Variables at Point E in the Water Body Subjected to Non-Buoyant and Buoyant Jets</td>
<td>84</td>
</tr>
<tr>
<td>Figure 13</td>
<td>Time Histories of Variables at Point F in the Water Body Subjected to Non-Buoyant and Buoyant Jets</td>
<td>85</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page No.</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------</td>
<td>----------</td>
</tr>
<tr>
<td>Figure 14</td>
<td>Time histories of Variables at Point G in the Ware Body Subjected to Non-Buoyant and Buoyant Jets</td>
<td>86</td>
</tr>
<tr>
<td>Figure 15</td>
<td>Time Histories of Variables at Point H in the Water Body Subjected to Non-Buoyant and Buoyant Jets</td>
<td>87</td>
</tr>
<tr>
<td>Figure 16</td>
<td>Surface Velocity Field at t=500 sec. in the Water Body Subjected to a Buoyant Jet</td>
<td>93</td>
</tr>
<tr>
<td>Figure 17</td>
<td>Surface Isotherms at t=60 sec. in the Water Body Subjected to a Buoyant Jet</td>
<td>94</td>
</tr>
<tr>
<td>Figure 18</td>
<td>Vertical Isotherms at y=100 ft. and t=60 sec. in the Water Body Subjected to a Buoyant Jet</td>
<td>95</td>
</tr>
<tr>
<td>Figure 19</td>
<td>Vertical Isotherms at x=222.5 ft. and t=60 sec. in the Water Body Subjected to a Buoyant Jet</td>
<td>96</td>
</tr>
<tr>
<td>Figure 20</td>
<td>Surface Isotherms at t=500 sec. in the Water Body Subjected to a Buoyant Jet</td>
<td>97</td>
</tr>
<tr>
<td>Figure 21</td>
<td>Vertical Isotherms at y=100 ft. and t=500 sec. in the Water Body Subjected to a Buoyant Jet</td>
<td>98</td>
</tr>
<tr>
<td>Figure 22</td>
<td>Vertical Isotherms at x=222.5 ft. and t=500 sec. in the Water Body Subjected to a Buoyant Jet</td>
<td>99</td>
</tr>
<tr>
<td>Figure 23</td>
<td>Calculation of the First and Second Derivatives</td>
<td>123</td>
</tr>
<tr>
<td>Figure 24a-ll</td>
<td>Flow Chart of MAIN Program</td>
<td>133</td>
</tr>
<tr>
<td>Figure 25a-r</td>
<td>Flow Chart of Subroutine DERIV</td>
<td>171</td>
</tr>
<tr>
<td>Figure 26a-k</td>
<td>Flow Chart of Subroutine PRESS</td>
<td>188</td>
</tr>
<tr>
<td>Figure 27a-b</td>
<td>Flow Chart of Subroutine SUM</td>
<td>199</td>
</tr>
<tr>
<td>Figure 28a-c</td>
<td>Flow Chart of Subroutine OUTP</td>
<td>201</td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 1</td>
<td>Density versus Temperature</td>
<td>54</td>
</tr>
<tr>
<td>Table 2</td>
<td>The Geometric and Hydraulic Input Data for Laminar Flow in a Square Duct</td>
<td>65</td>
</tr>
<tr>
<td>Table 3</td>
<td>Grid Dimensions for the Laminar Flow in a Square Duct</td>
<td>67</td>
</tr>
<tr>
<td>Table 4</td>
<td>Geometric and Hydraulic Input Data for Partially Heated Pond</td>
<td>69</td>
</tr>
<tr>
<td>Table 5</td>
<td>Grid Dimensions for Partially Heated Pond</td>
<td>70</td>
</tr>
<tr>
<td>Table 6</td>
<td>Geometric and Hydraulic Input Data for Three-Dimensional Non-Buoyant and Buoyant Jets in a Cross Current</td>
<td>76</td>
</tr>
<tr>
<td>Table 7</td>
<td>Grid Dimensions for Three-Dimensional Non-Buoyant and Buoyant Jets in a Cross Current</td>
<td>78</td>
</tr>
<tr>
<td>Table 8</td>
<td>Mass and Energy Balance</td>
<td>101</td>
</tr>
</tbody>
</table>
1. INTRODUCTION

Thermal power and industrial processing plants use large quantities of cooling water from natural or artificial water bodies and return the water to the source at higher temperatures. Changes that occur, either in the temperature of the water or in temperature-related water quality constituents, may adversely affect the aquatic environment of the water source. When the thermal discharge from a plant, into a water body, raises the water temperature by such an extent as to damage aquatic life or other legitimate uses of the water source, some degree of thermal pollution exists.

To assess the extent of a thermal pollution, utilities, ecologists, federal and state agencies as well as the public are interested in determining the space and time distribution of temperature within a water body, (such as lakes, reservoirs and rivers) when it is subjected to a prescribed thermal load.

The temperature distribution within a water impoundment is intimately associated with the hydrodynamic behavior of the water mass. Fluid flow in the impoundment is, in turn, affected by the hydrodynamic boundary conditions, three-dimensionality of the water body and the degree of stratification imposed on the water mass. The problem is further complicated by the heat transfer process through the water surface, internal mixing, inflows and outflows. For these reasons, a quantitative prediction of the temperature field for a particular water source must consider the mass-transport,
momentum-transport, and heat-transport processes which produce the temperature changes.

The theoretical analysis of the dynamic behavior of a large water body, subjected to thermal discharge, is based on the solution of space- and time-dependent conservation and state equations. Conservation of mass, momentum, and energy for the water flow in the impoundment together with the equation of state, provide a sufficient number of partial differential equations and algebraic relations in space and time for the solution of the problem. Although, these equations have been known for over a century, due to their highly nonlinear nature, a direct closed form analytical solution is considered practically impossible for most general cases involving realistic geometries and boundary conditions. The available analytical solutions of problems involving thermal discharge is severely limited by the many simplifying assumptions made in order to achieve a solution. In fact, analytical solutions to the problems of thermal discharge are obtained only for problems involving simple geometry and boundary conditions. Solutions to more realistic geometries and boundary conditions can be sought only by the development of computer modeling in three-dimensions under transient conditions.

Current three-dimensional computer simulation of thermal discharge into water bodies are based on a number of simplifying assumptions discussed hereunder:

1. **Compliance with the Conservation of Mass and Momentum**
Generally, the three components of water velocity are found from the longitudinal, lateral, and vertical momentum equations. These three components of velocity should satisfy the conservation of mass equation. A major difficulty arises when simultaneous compliance with the principles of conservation of mass and momentum cannot be ensured. Under the Boussinesq approximation, the conservation of mass reduces to zero divergence for the water velocity (with no mass storage term) i.e., the sum of all inflows to and outflows from any fluid control volume in the flow field must vanish. Since the new velocity components, calculated by the integrations of the momentum equations do not necessarily satisfy the continuity equation, the mass balance would be disturbed, thereby yielding a small surplus or deficit inflow at certain control volumes. To resolve this dilemma, Brady and Geyer (2)\textsuperscript{1} hypothesize that the surplus or deficit inflows redistribute in all directions by equal magnitude. Obviously, the disadvantage of this velocity adjustment technique is that the adjusted velocities no longer satisfy the momentum equations. Considering a water body, such as a river, flowing longitudinally with a laterally uniform velocity distribution, and applying the approximate boundary conditions on the lateral velocities at the thermal discharge location, they found that the out-of-balance surplus inflow to each adjacent cell is exceptionally large. When their model attempts to distribute this surplus inflow among the surrounding

\textsuperscript{1}Underlined numerals in parenthesis designate references listed in Section 8.
fluid cells, the resulting vertical velocity adjustment becomes excessively large which aborts the run. Brady and Geyer referred to this problem as the "vertical over-responsiveness of the model" and devised schemes to suppress the associated undesirable vertical fluctuations by either: 1) temporarily suppressing vertical redistribution such that each fluid layer achieves its own mass balance independently; or 2) temporarily enforcing a rigid lid at the fluid free surface such that a zero vertical velocity is maintained at the most upper layer of the model at every fluid cell. The rigid lid assumption is used by many authors to simplify the solution to the stratification and circulation in water bodies \((18, 20, 32)\). Obviously, this situation calls for an improved approach for a simultaneous compliance with the conservation equations of mass and momentum.

2. Surface Flow Distribution

The treatment of the slope variation at the water free surface constitutes another major difficulty. The upwelling (or downwelling) flow at the surface creates positive (or negative) surge waves \((36, 10)\). The mathematical modeling of these phenomena under unsteady flow conditions is complicated. To resolve this difficulty, Waldrop and Farmer \((39)\) first assumed that the vertical component of velocity near the surface will be redirected in the horizontal direction (in \(x\) and \(y\) directions) and the surface will move. Later, Waldrop and Farmer \((40)\), in a major effort to resolve the surface flow distribution problem, introduced a mass balance at the free surface and extended their horizontal momentum equations to elements near the water
surface. This situation calls for further studies of the behavior of the free surface under unsteady flow conditions.

3. Pressure Distribution

The velocity components in the flow field are extremely sensitive to values of nodal pressures. Slight changes in the pressure distribution would affect the circulation in the water body considerably. The calculation of the correct pressure distribution is, therefore, of paramount importance. Most authors (41, 42, 19, 4, 16, 23, 8) assume that the total dynamic pressure at each point in the flow is equal to hydrostatic pressure obtained under static conditions. This assumption, referred to as hydrostatic approximation, leads to inaccuracies in regions of severe upwelling and downwelling which will adversely affect the correctness of circulation velocities. Thus, a better technique for the computation of the pressure field is called for.

4. Numerical Stability

Calculations of space and time increments for obtaining a stable numerical solution of partial differential equations of mass, momentum and energy conservation is a difficult task. Most authors resort to overly simplified criteria for the establishment of the upper bound of the integration time step for an assumed set of space increments. Brady and Geyer (2) state that the maximum integration step appears to be limited by relationships between the velocities and the smallest cell dimensions in each direction. Waldrop and Farmer (39)
and Harlow and Welch (9) resort to a stability limit calculated on the basis of one-dimensional, unsteady, incompressible flow equations with a free surface. This limit imposes an upper bound on their integration time step equal to the ratio of the smallest cell size and the surface wave speed. More accurate means for the prediction of the space and time increments are needed.

Most authors believe that much work is still required to refine various features of the present unsteady three-dimensional computer models before the present models can be applied to situations involving significant stratification effects, or flow fluctuations, in water bodies.

1.1 History and Background

Wind-induced circulation in shallow waters has been studied by Liu and Perez (22), Liggett and Hadjitheodorou (18, 19, 20), among others. These studies neglect thermal stratification and employ unsteady momentum and continuity equations to determine the pressure and velocity distribution as functions of space and time in large water bodies. The problem of circulation in stratified lakes idealized as a two layer lake, in which epilimnion and hypolimnion are considered homogeneous has been analyzed by Liggett and Lee (15, 21, 10). These studies neglect the existence of the thermocline and are unrealistic.

The aforementioned studies do not take into account the coupled hydrometeorological phenomena controlling the disposition of energy at
the surface of water bodies and neglect the mechanisms that are responsible for distributing energy internally throughout the fluid mass.

Orlob and Selna (27) developed a digital computer program for the calculation of temperature variations in deep reservoirs. Their analysis is limited by the assumption that all transfer of heat energy is accomplished along the vertical axis. Stefan (35) presented a two-dimensional mathematical formulation consisting of continuity, momentum and energy equations for the modeling of heated water over lakes. However, he made no attempt to solve the governing equations.

Brady and Geyer (2) provided a comprehensive review of analytical modeling techniques for the solution of problems involving thermal discharges and presented a general computer model for simulating thermal discharge in three dimensions. They established the feasibility of their technique by a sample application to a river site where density effects are relatively insignificant. The compliance with the conservation of mass and momentum is obtained by redistributing the surplus or deficit inflows in a control volume in all directions by equal magnitudes. This feature was responsible for their observed "vertical over-responsiveness of the model" leading to excessive vertical velocities, as discussed earlier. They recommended further development work for situations involving significant stratification effects and fluctuating flows.

Waldrop and Farmer (39, 40), in a three dimensional study of
buoyant plumes, examined the flow field resulting from the interaction of a stream with the much larger body of flowing water into which it debouches. They expressed the local density of the water as function of salinity (39) and temperature (40). Three dimensional unsteady conservation equations, used in their study to describe the interaction, included the effects of buoyancy, inertia, and the difference in the hydrostatic heads of the two currents. They solved the conservation equations with a time-dependent finite difference technique. The treatment of the slope variation at the water free surface is accomplished as discussed earlier in surface flow distribution. The pressure distribution is calculated from the vertical momentum equations after neglecting the fluid vertical inertia.

Leendertse et al (16), in a study similar to that of Waldrop and Farmer (40), developed a three-dimensional model for estuaries, bays, and coastal seas in which non-isotropic density conditions exist. They performed a numerical integration of the finite difference equations of motion, transport, and continuity and applied their computational method to the analysis of a number of basins with boundaries of increasing complexity. They employ the hydrostatic approximation discussed earlier and neglect the vertical acceleration altogether.

Mercier (24, 25) developed a predictor-corrector method for the solution of incompressible viscous fluid and applied this method to the transient flow in a density stratified reservoir. His solution
is restricted to two-dimensional and adiabatic flow. Rather than considering the density as a function of temperature, Mercier obtains the water density by setting the material derivative of the density equal to zero.

Spradley and Churchill (34) examined the problem of pressure- and buoyancy-driven thermal convection in a rectangular enclosure for unsteady laminar compressible flow at various reduced levels of gravity. The enclosure was heated on one side and cooled on the opposite side. The conservation equations of mass, momentum, and energy were solved numerically for a compressible, heat-conducting ideal gas employing an explicit finite-difference technique. Their solution show that the thermally induced motion is acoustic in nature owing to the sonic character of the induced pressure waves.

Further reviews of the state of art for the solution of the conservation equations of mass, momentum, and energy as well as the study of the behavior of water bodies subjected to thermal discharge are available (28, 33, 30, 29) and interested readers are referred to these references for additional information.

A careful examination of the above references demonstrates that a completely adequate mathematical technique for the combined analysis of thermal stratification and circulation of water bodies is still lacking. A rigorous mathematical simulation for the prediction of the thermal and hydraulic behavior of impoundments is needed.
1.2 Scope and Objectives of the Present Study

The main objectives of the present study are as follows:

1) To develop a three-dimensional model for the mathematical description of a large rectangular water body free from the major shortcomings described earlier.

2) To develop a numerical integration technique for solving the above mathematical model on a digital computer.

3) To develop a digital computer program for predicting the thermal stratification and circulation phenomenon in a given rectangular water body subjected to thermal discharge and to determine the time histories and spatial distribution of pressure, velocity and temperature fields within the water body.

The major contributions of the present study, as compared with the previous works in this general area, can be summarized as follows:

1) Compliance with the conservation of mass and momentum is obtained by the introduction of two flow regions in the entire flow field: a) the water-level region containing a portion of water near the free surface in which the water-level rises or falls, as the case may be, during the dynamic solution; and b) the sub-water-level region located under the water-level region which remains totally filled with fluid at all times during the transient. As will be seen later, the superimposition of a three-dimensional grid on these two regions facilitates the simultaneous compliance of the principles of conservation of mass and momentum for the entire flow field without any mass imbalance.
2) A different set of differential equations for the conservation of mass, momentum and energy is applied to the cells located in the water-level and sub-water-level flow regions. This is necessary because the cells in the water-level region have a variable height while the cells in the sub-water level region have a constant height. This feature facilitates the calculation of surface flow distribution.

3) The pressure distribution is obtained by combining the momentum and continuity equations. This feature eliminates the need for the hydrostatic pressure approximation. It also eliminates the need for neglecting the acceleration terms in the momentum equations in the vertical direction.

4) A detailed numerical stability analysis is performed which provides accurate criteria for the selection of the space and time increments.
2. MATHEMATICAL FORMULATION

2.1 Simplifying Assumptions

The mathematical formulation in this study is based on the following assumptions:

1) The equations of motion, energy and continuity are applied in their time-smoothed form to turbulent incompressible three-dimensional flow with Cartesian coordinates, as shown in Figure 1, with \( z \) positive upward.

2) It is assumed that Boussinesq's approximation applies. In other words, densities are treated as constants except in terms involving gravity. This allows natural circulation to take place. Furthermore, density in the body force term is considered to be a function of temperature only.

3) The effects of turbulence are modeled by using eddy transport coefficients. Horizontal and vertical momentum eddy viscosities and thermal eddy diffusivities are considered as constants throughout the body of water though different magnitudes for horizontal and vertical directions.

4) It is assumed that there are no internal heat sources and the heat exchange between the water body and the atmosphere takes place near the water free surface.

5) Loss of mass due to evaporation at the surface and conductive heat transfer through the impoundment solid boundaries are generally small and are neglected.

6) The Coriolis forces acting on the water body is considered to
FIG 1. SCHEMATIC DIAGRAM OF THE WATER BODY
be negligible.

The boundary conditions of the problem involve: 1) the geometry of the impoundment including the thermal discharge inlet, as well as the river inflow and outflow configurations; 2) the mass flow rates of the inflows across the boundaries; and 3) the level, pressure and temperature along the inlet boundaries. It is assumed that all of the above parameters are known.

The initial conditions of the problem includes the pressure, velocity, level and temperature distributions of the impoundment at time \( t = 0 \). These variables are computed by the present program by first performing a dynamic analysis under no thermal discharge conditions and then using the calculated pressure, velocity, level and temperature distributions, as initial conditions, in a dynamic analysis involving a thermal discharge.

### 2.2 Governing Equations

The mathematical formulation of the sub-water-level flow region consists of the following fundamental equations derived in Appendix 1.

**Momentum equations:**

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \nu_h \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \nu_v \frac{\partial^2 u}{\partial z^2} \tag{1}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = - \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \nu_h \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \nu_v \frac{\partial^2 v}{\partial z^2} \tag{2}
\]
Continuity equation:

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho_0} \frac{\partial p}{\partial z} + \nu_h \left( \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 w}{\partial y^2} \right) + \nu_v \frac{\partial^2 w}{\partial z^2} - g \frac{\rho}{\rho_0}$$  \hspace{1cm} (3)

Energy equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$ \hspace{1cm} (4)

Equation of state:

$$\rho = \rho (T)$$ \hspace{1cm} (6)

In the above formulation, \( u, v \) and \( w \) are the local velocity components in the \( x, y \) and \( z \) directions respectively; \( p \) is the pressure; \( T \) is the local water temperature; \( \rho \) is the local density of water; \( \rho_0 \) is a fixed reference density to be defined later; and \( \nu_h, \nu_v, D_h \) and \( D_v \) are the horizontal and vertical components of eddy viscosity and eddy diffusivity of heat, respectively. The above differential equations are solved simultaneously, subject to the boundary conditions described in the following section. To solve the above mathematical formulation, the following steps will be taken:

1) Non-dimensionalization of the governing equations and derivation of time-derivative equations for velocity components \( u, v, w \) and temperature \( T \).

2) Derivation of the final equations including the pressure equation.
In this study, the thermal discharge parameters are used as reference values for the non-dimensionalization of the governing equations. Let \( U_0, \rho_0, T_0 \) and \( d_0 \) be the velocity, density, temperature and half width of thermal discharge flow respectively. The dimensionless quantities are defined by

\[
\begin{align*}
    x^* &= \frac{x}{d_0} \quad y^* = \frac{y}{d_0} \quad z^* = \frac{z}{d_0} \\
    u^* &= \frac{u}{U_0} \quad v^* = \frac{v}{U_0} \quad w^* = \frac{w}{U_0} \\
    t^* &= \frac{tU_0}{d_0} \quad T^* = \frac{T}{T_0} \quad p^* = \frac{p}{\rho_0 g d_0} \\
    \rho^* &= \frac{\rho}{\rho_0}
\end{align*}
\]

Employing the above dimensionless variables in the continuity, momentum and energy equations, one obtains the time derivatives for velocity components \( u, v, w \) and temperature \( T \) in parabolic differential form as follows:

\[
\begin{align*}
    \frac{\partial u^*}{\partial t} &= -u^* \frac{\partial u^*}{\partial x} - v^* \frac{\partial u^*}{\partial y} - w^* \frac{\partial u^*}{\partial z} - \frac{1}{F_0} \frac{\partial p^*}{\partial x} \\
    &+ \frac{v}{v_0} \frac{1}{R_0} \left( \frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} \right) + \frac{v}{v_0} \frac{1}{R_0} \frac{\partial^2 u^*}{\partial z^2} \\
    \frac{\partial v^*}{\partial t} &= -u^* \frac{\partial v^*}{\partial x} - v^* \frac{\partial v^*}{\partial y} - w^* \frac{\partial v^*}{\partial z} - \frac{1}{F_0} \frac{\partial p^*}{\partial y}
\end{align*}
\]
\[ + \frac{v_h}{v_0} \frac{1}{R_0} \left( \frac{a_{v}^2}{ax^2} + \frac{a_{v}^2}{ay^2} \right) + \frac{v_v}{v_0} \frac{1}{R_0} \frac{a_{v}^2}{az^2} \]  \hspace{1cm} (12)

\[ \frac{aw^*}{at} = -u^* \frac{aw^*}{ax} - v^* \frac{aw^*}{ay} - w^* \frac{aw^*}{az} - \frac{1}{F_0} \frac{ap^*}{az^2} - \frac{\rho^*}{F_0} \]  \hspace{1cm} (13)

\[ + \frac{v_h}{v_0} \frac{1}{R_0} \left( \frac{a_{w}^2}{ax^2} + \frac{a_{w}^2}{ay^2} \right) + \frac{v_v}{v_0} \frac{1}{R_0} \frac{a_{w}^2}{az^2} \]

\[ \frac{aT^*}{at} = -u^* \frac{aT^*}{ax} - v^* \frac{aT^*}{ay} - w^* \frac{aT^*}{az} \]  \hspace{1cm} (14)

with continuity and state equations given by

\[ \frac{au^*}{ax} + \frac{av^*}{ay} + \frac{aw^*}{az} = 0 \]  \hspace{1cm} (15)

\[ \rho^* = \rho^* (T^*) \]  \hspace{1cm} (16)

In the above equations, the following dimensionless numbers are used:

1) Froude number \( F_0 = U_0/(gd_0)^{1/2} \)  \hspace{1cm} (17)

2) Reynolds number \( R_0 = U_0d_0/\nu_0 \)  \hspace{1cm} (18)

Equations (11) to (16) are a set of six equations that will be used for the determination of six unknowns, \( u^* \), \( v^* \), \( w^* \), \( p^* \), \( \rho^* \), and \( T^* \), as functions of time and space.
The final equations, including the pressure equation, are obtained from the above equations as follows. First, the continuity equation is differentiated with respect to time to give

$$\frac{\partial}{\partial t} \left( \frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} + \frac{\partial w^*}{\partial z} \right) = 0 \quad (19)$$

The order of differentiation in equation (19) is then interchanged

$$\frac{\partial}{\partial x} \left( \frac{\partial u^*}{\partial t} + \frac{\partial v^*}{\partial y} + \frac{\partial w^*}{\partial z} \right) + \frac{\partial}{\partial y} \left( \frac{\partial v^*}{\partial t} + \frac{\partial u^*}{\partial x} \right) + \frac{\partial}{\partial z} \left( \frac{\partial w^*}{\partial t} + \frac{\partial u^*}{\partial x} \right) = 0 \quad (20)$$

To facilitate spatial differentiations indicated by equation (20), equations (11) through (13) are first rewritten in the following form

$$\frac{\partial u^*}{\partial t} = Q_x^* - \frac{1}{F_0} \frac{\partial p^*}{\partial x} \quad (21)$$

$$\frac{\partial v^*}{\partial t} = Q_y^* - \frac{1}{F_0} \frac{\partial p^*}{\partial y} \quad (22)$$

$$\frac{\partial w^*}{\partial t} = Q_z^* - \frac{1}{F_0} \frac{\partial p^*}{\partial z} \quad (23)$$

where

$$Q_x^* = - u^* \frac{\partial u^*}{\partial x} - v^* \frac{\partial u^*}{\partial y} - w^* \frac{\partial u^*}{\partial z}$$
\[ + \frac{v_h}{v_0} \frac{1}{R_0} \left( \frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2} \right) + \frac{v_v}{v_0} \frac{1}{R_0} \frac{\partial^2 u^*}{\partial z^2} \]  
\hspace{1cm} (24) 

\[ Q_y^* = -u^* \frac{\partial v^*}{\partial x} - v^* \frac{\partial v^*}{\partial y} - w^* \frac{\partial v^*}{\partial z} \]

\[ + \frac{v_h}{v_0} \frac{1}{R_0} \left( \frac{\partial^2 v^*}{\partial x^2} + \frac{\partial^2 v^*}{\partial y^2} \right) + \frac{v_v}{v_0} \frac{1}{R_0} \frac{\partial^2 v^*}{\partial z^2} \]  
\hspace{1cm} (25) 

\[ Q_z^* = -u^* \frac{\partial w^*}{\partial x} - v^* \frac{\partial w^*}{\partial y} - w^* \frac{\partial w^*}{\partial z} \]

\[ + \frac{v_h}{v_0} \frac{1}{R_0} \left( \frac{\partial^2 w^*}{\partial x^2} + \frac{\partial^2 w^*}{\partial y^2} \right) + \frac{v_v}{v_0} \frac{1}{R_0} \frac{\partial^2 w^*}{\partial z^2} - \frac{\rho^*}{F_0} \]  
\hspace{1cm} (26) 

with

\[ \frac{\partial w^*}{\partial x} = - \left( \frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial z} \right) \]  
\hspace{1cm} (27)

substituting equations (21) through (23) into (20), yields

\[ \nabla^2 p^* = F_0^2 \left( \frac{\partial Q_x^*}{\partial x} + \frac{\partial Q_y^*}{\partial y} + \frac{\partial Q_z^*}{\partial z} \right) \]  
\hspace{1cm} (28)

This elliptic differential equation will provide a means for the calculation of the pressure distribution in the water body. Equations (27), (24), (25), (26), (28), (21), (22), (14), and (16) constitute our final equations which will be solved for updated values of \( w^* \), \( Q_x^* \),
\(Q^*_y, Q^*_z, p^*, u^*, v^*, T^*,\) and \(\rho^*,\) respectively. It should be noted that the vertical momentum equation (23) is used indirectly in this computation through its usage in the derivation of the pressure equation (28). Since the water-level variation provides a vertical freedom for the water body, it is imperative to calculate the vertical velocity component \(w^*\) from the continuity equation and not by the application of the momentum equation in the vertical direction equation (23). However, the momentum balance in the vertical direction is fully satisfied since equation (23) is explicitly used in the derivation of the pressure equation (28). In fact, equation (23) may be easily derived from the final equations stated above.

The mathematical formulation for the water-level flow region is derived by the application of the fundamental equations of mass, momentum, and energy as follows. The water-level in cells located at the air-water interface varies with time and space. For this reason, the continuity equation, as given by equation (27), is not valid for these cells. Application of a mass balance to such a cell with all sides fixed in space except the top which moves with the water-level, yields

\[
\frac{\partial}{\partial t} (\rho \Delta x \Delta y H) = (\rho u \Delta y H)_x - (\rho u \Delta y H)_x + \Delta x \\
+ (\rho v \Delta x H)_y - (\rho v \Delta x H)_y + \Delta y \\
+ (\rho w \Delta x \Delta y)_z - (\rho w \Delta x \Delta y)_z + H
\] (29)
The last term represents the mass flux leaving the water-level surface. Since the evaporation losses are considered insignificant, this term is equal to zero. Dividing both sides of the above equation by \( \rho \Delta x \Delta y \), considering \( \rho \) to be constant and equal to \( \rho_0 \) consistent with Boussinesq approximation used earlier, and letting \( \Delta x, \Delta y \) approach zero, yields

\[
\frac{\partial H}{\partial t} = - \frac{\partial}{\partial x} (uH) - \frac{\partial}{\partial y} (vH) + w
\]  

(30)

The above equation will be integrated in time to obtain the water-level. In nondimensional form, the above equation becomes

\[
\frac{\partial H^*}{\partial t} = - \frac{\partial}{\partial x} (u^* H^*) - \frac{\partial}{\partial y} (v^* H^*) + w^*
\]  

(31)

where

\[
H^* = H / d_0
\]  

(32)

Similarly, the energy equation, as given by equation (14), is not valid for the water-level elements. Application of the energy balance to these cells yields

\[
\frac{\partial}{\partial t} (\rho C_p \Delta x \Delta y HT) = (\rho C_p \Delta x \Delta y HT)_x - (\rho C_p \Delta x \Delta y HT)_x + \Delta x
\]

\[
+ (\rho C_p \Delta x \Delta y HT)_y - (\rho C_p \Delta x \Delta y HT)_y + \Delta y
\]

\[
+ (\rho C_p \Delta x \Delta y HT)_z - (\rho C_p \Delta x \Delta y HT)_z + H
\]
It should be noted again that the enthalpy flux leaving the water-level surface is equal to zero. Furthermore, the turbulent thermal diffusion at the water-level surface, is (3)

\[ \rho C_p \frac{\partial T}{\partial z} + H = -K(T - E) \]  

(34)

Considering the above statements, dividing energy equation (33) by \( \rho C_p \Delta x \Delta y \) with \( \rho = \rho_0 \), and letting \( \Delta x, \Delta y \) approach zero, yields

\[ \frac{\partial}{\partial t} (HT) = \frac{\partial}{\partial x} (uHT) - \frac{\partial}{\partial y} (vHT) + wT + \frac{D_x}{\rho C_p} \frac{\partial T}{\partial x} + \frac{D_y}{\rho C_p} \frac{\partial T}{\partial y} + \frac{-D_z}{\rho C_p} \frac{\partial T}{\partial z} - \frac{K(T - E)}{\rho C_p} \]  

(35)

Performing the differentiations indicated by the first three terms of the above, and employing equation (30) gives

\[ \frac{\partial T}{\partial t} = -u \frac{\partial T}{\partial x} - v \frac{\partial T}{\partial y} + \frac{D_x}{H} \frac{\partial (H \frac{\partial T}{\partial x})}{\partial x} + \frac{D_y}{H} \frac{\partial (H \frac{\partial T}{\partial y})}{\partial y} + \frac{-D_z}{H} \frac{\partial T}{\partial z} - \frac{K(T - E)}{\rho C_p} \]
\[ + \frac{D_y}{H} \frac{\partial}{\partial y} (H \frac{\partial T}{\partial y}) - \frac{D_z}{H} \frac{\partial T}{\partial z} - \frac{K (T - E)}{\rho_0 C_p H} \]  

(36)

By a similar reasoning, after employing the surface boundary conditions, to be derived later, the velocity components in the water-level element is found to be

\[ \frac{\partial u}{\partial t} = - u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - \frac{1}{\rho_0 H} \frac{\partial P}{\partial x} + \frac{\nu}{H} \frac{\partial u}{\partial x} (H \frac{\partial u}{\partial x}) \]

\[ + \frac{\nu}{H} \frac{\partial}{\partial y} (H \frac{\partial u}{\partial y}) - \frac{\nu}{H} \frac{\partial u}{\partial z} + \frac{1}{2} \frac{C_{f x} \rho Ho^2}{H} \]

(37)

\[ \frac{\partial v}{\partial t} = - u \frac{\partial v}{\partial x} - v \frac{\partial v}{\partial y} - \frac{1}{\rho_0 H} \frac{\partial P}{\partial y} + \frac{\nu}{H} \frac{\partial v}{\partial y} (H \frac{\partial v}{\partial y}) \]

\[ + \frac{\nu}{H} \frac{\partial}{\partial y} (H \frac{\partial v}{\partial y}) - \frac{\nu}{H} \frac{\partial v}{\partial z} + \frac{1}{2} \frac{C_{f y} \rho Ho^2}{H} \]

(38)

After setting \( D_x = D_y = D_h \) and \( D_z = D_v \), equations (36), (37), and (38) in non-dimensional form become

\[ \frac{\partial T^*}{\partial t} = - u^* \frac{\partial T^*}{\partial x} - v^* \frac{\partial T^*}{\partial y} + \frac{D_h}{\nu R_0 H} \frac{1}{H} \frac{\partial T^*}{\partial x} (H^* \frac{\partial T^*}{\partial x}) \]

\[ + \frac{\nu}{\nu_0 R_0 H} \frac{\partial T^*}{\partial y} \frac{1}{H} \frac{\partial T^*}{\partial y} - \frac{S_0}{H} (T^* - E^*) \]

(39)

\[ \frac{\partial u^*}{\partial t} = - u^* \frac{\partial u^*}{\partial x} - v^* \frac{\partial u^*}{\partial y} - \frac{1}{F_0^2 H} \frac{\partial P^*}{\partial x} \]
\[
\frac{u^*}{v^*} = -u^* \frac{\partial u^*}{\partial x} - v^* \frac{\partial u^*}{\partial y} - \frac{1}{F_0^2 \rho^* H^*} \frac{\partial H^* p^*}{\partial y}
\]

\[
+ \frac{v^*}{v_0} \frac{1}{R_0^H} \left( \frac{\partial}{\partial x} \left( H^* \frac{\partial v^*}{\partial x} \right) + \frac{\partial}{\partial y} \left( H^* \frac{\partial v^*}{\partial y} \right) \right)
\]

\[
- \frac{v^*}{v_0} \frac{1}{R_0^H} \frac{\partial v^*}{\partial z} + \frac{1}{2} \frac{C_f \mu a \omega^* W_x^* W_y^*}{H^*}
\]

where \(S_0\), the Stanton number, is defined by

\[
S_0 = \frac{K}{\mu_0 C \rho_0 U_0}
\]

and

\[
\rho^* = \frac{\rho a}{\rho_0}
\]

\[
W_x^* = \frac{W_x}{U_0} \quad W_y^* = \frac{W_y}{U_0}
\]

The pressure at the air-water interface is always atmospheric and can be considered equal to zero no matter where the water-level may be at any instant of time. For this reason, the pressure at the center of the water-level element can be considered to be hydrostatic,
i.e., for the water-level element

\[ P^* = \rho^* H^* \]  \hspace{1cm} (45)

This equation may also be derived by applying a momentum balance, in the vertical direction, on the water-level flow region and noting that the water vertical velocity in this region is relatively small and, therefore, negligible. If the pressure distribution given by equation (45) is substituted into the horizontal momentum equations, the terms involving pressure gradient would simplify as follows

\[ \frac{1}{2} \frac{\partial H^* p^*}{\partial x} = \frac{1}{2} \frac{\partial}{\partial x} \left( H^* \frac{\partial \rho^*}{\partial x} + 2\rho^* \frac{\partial H^*}{\partial x} \right) \]  \hspace{1cm} (46)

\[ \frac{1}{2} \frac{\partial H^* p^*}{\partial y} = \frac{1}{2} \frac{\partial}{\partial y} \left( H^* \frac{\partial \rho^*}{\partial y} + 2\rho^* \frac{\partial H^*}{\partial y} \right) \]  \hspace{1cm} (47)

It is important to realize the difference between the flow patterns in the sub-water-level and the water-level elements. In the sub-water-level elements, an increase in the fluid boundary velocity will induce horizontal velocity components. However, the time constant associated with the momentum equations will not permit these equations to respond instantaneously to the boundary velocity disturbance at the river or thermal discharge inlets. Because of the flow incompressibility and since the fluid vertical inertia is considerably smaller than the horizontal inertias, the mass imbalance will result in a positive vertical fluid motion designated as "upwelling". This
vertical motion will continue upward until it reaches the water-level element where by virtue of equation (30) will cause the water-level to rise. This rise is large for elements close to the boundary disturbance and small for elements farther away. This spatial change affects the horizontal momentum equations (40) and (41) and induce a horizontal flow in the water-level elements which in turn tends to reduce the water-level through equation (30). A reverse situation takes place when the boundary velocity is decreased. The mass imbalance results in a negative vertical fluid motion, designated as "downwelling". This will cause the lowering of the water-level with an attendant horizontal flow into the element to increase the water-level.

2.3 Boundary Conditions

The water body boundary surfaces may be classified as follows:

Type (1): The surface boundary located at the water-level free surface.

Type (2): The lateral solid-fluid boundary located at the interface between the impoundment wall and the water body.

Type (3): The bottom boundary located at the bottom of the impoundment.

Type (4): The fluid-fluid boundary located at the interface between the water body in the region of interest and surrounding waters.

Implementation of the boundary conditions for the above boundary
surfaces involves calculation of the first spatial derivative of the system variables at the boundary. Two methods have been in usage for the calculation of the derivative:

1) Application of the reflection principle and usage of fictitious fluid nodes behind the solid boundary;

2) Calculation of the derivative at the boundary on the basis of three internal nodes adjacent to the boundary node.

The latter technique requires less computer storage space and is employed in the present study as detailed in the Appendix.

Pressure boundary conditions. For the surface boundary, the pressure is considered atmospheric at the air-water interface and is set equal to zero.

For the lateral solid-fluid and bottom boundaries, application of the momentum equation in the direction normal to the boundaries gives the following equations respectively:

\[
\frac{\partial p^*}{\partial y} = F_0 \frac{\nu h}{\nu_0 R_0} \frac{\partial^2 v^*}{\partial y^2}
\]

\[
\frac{\partial p^*}{\partial z} = -\rho^* + F_0 \frac{\nu}{\nu_0 R_0} \frac{\partial^2 w^*}{\partial z^2}
\]

For the fluid-fluid boundaries, the pressure distribution at the inflow interfaces is considered to be static. The pressure distribution at any outflow interface node is set equal to that of the adjacent upstream node in the flow field. The surrounding waters
entering a water body is characterized by a uniform channel flow. The pressure variation at any cross section of a uniform channel flow may be proved to be hydrostatic by applying the Bernoulli's equation. This equation is applied to two streamlines, one at the channel top and the other at an arbitrary depth. Since, in steady, frictionless, one-dimensional, uniform channel flow, the constants in the Bernoulli's equation are the same for all streamlines, it can be easily shown that the pressure distribution across the channel, as well as the interface between the channel and the impoundment is hydrostatic.

**Velocity boundary conditions.** For the surface boundary, the wind effect produces shear at the water surface which is equal to the shear stress in the fluid near the surface.

\[ \rho_0 \gamma_{v} \left( \frac{\partial u}{\partial z} \right)_{z = z_s} = \frac{1}{2} \rho_a C_{fx} W_x |W_x| \]  

(50)

\[ \rho_0 \gamma_{v} \left( \frac{\partial v}{\partial z} \right)_{z = z_s} = \frac{1}{2} \rho_a C_{fy} W_y |W_y| \]  

(51)

where \( C_{fx} \) and \( C_{fy} \) are skin coefficients along \( x \) and \( y \) axis respectively; \( \rho_a \) and \( \rho_0 \) are the air and water densities; \( W_x \) and \( W_y \) are \( x \) and \( y \) components of the wind velocity; \( u \) and \( v \) are the fluid velocity components along \( x \) and \( y \); and \( \gamma_v \) is the vertical eddy viscosity. Equations (50) and (51) are based on the assumption that the wind velocity components are steady and moderate with zero vertical component. Under these conditions, the water surface will remain
smooth and the skin coefficients may be taken from experimental measurements (22, 16). In dimensionless form, the equations (50) and (51) become

\[
\frac{V_v}{\nu_0 R_0} \left( \frac{\partial V}{\partial z} \right)_{*} = \frac{1}{2} C_f \rho_{a} W_{*} \left| W_{*} \right|
\]  \hspace{1cm} (52)

\[
\frac{V_v}{\nu_0 R_0} \left( \frac{\partial V}{\partial z} \right)_{*} = \frac{1}{2} C_f \rho_{a} W_{*} \left| W_{*} \right|
\]  \hspace{1cm} (53)

For the lateral solid-fluid and bottom boundaries, the velocity components normal to the boundaries is always zero. The tangential velocity components is calculated by the application of no-slip condition. With this condition, both tangential velocity components at the solid boundary are set equal to zero. The no-slip boundary condition is ideal for very fine grid spacing. However, computational economy requires a coarse grid spacing for most dynamic three-dimensional hydrothermal analysis. Since, a coarse grid does not afford sufficient resolution at the boundary, a limited amount of numerical error will be induced in the solution.

For the fluid-fluid boundaries, the velocity distribution at the inflow interfaces is set equal to the channel inflow distribution which is considered to be a known function of time. The velocity distribution at any outflow interface node is set equal to that of the adjacent upstream node in the flow field.
Temperature boundary condition. For the surface boundary, the rate of heat dispersion through the water (involving both conduction and turbulent thermal diffusion) is equal to the heat transfer from water to air.

\[ \rho_0 C_p D_v \left( \frac{\partial T}{\partial z} \right) z = z_s = -K (T_s - E) \]  

(54)

where \( \rho_0, C_p, \) and \( D_v \) are the density, specific heat, and eddy diffusivity of heat respectively; \( T_s \) is the water surface temperature; \( K \) is the heat exchange coefficient; and \( E \) is the equilibrium water temperature. Value of \( K \) and \( E \) may be estimated in terms of meteorological conditions (3). In dimensionless form, equation (54) becomes

\[ \frac{D_v}{\nu_0 R_0} \left( \frac{\partial T^*}{\partial z^*} \right) z^* = z_s^* = -S_0 (T_s^* - E^*) \]  

(55)

For the lateral solid-fluid and bottom boundaries, the heat transfer is considered to be negligible leading to zero temperature gradient at the boundary, i.e.,

\[ \frac{\partial T^*}{\partial n} = 0 \]  

(56)

The temperature distribution at the fluid-fluid boundaries is treated similar to the velocity distribution. The temperature at the inflow interface is set equal to the channel inflow temperature
which is considered to be a known function of time. The temperature at any outflow interface node is set equal to that of the adjacent upstream node in the flow field.

**Water-level boundary condition.** The water-level at the fluid-fluid boundaries is treated similar to the temperature distribution. The water-level at the inflow interface is set equal to the channel inflow which is considered to be known. The water-level at outflow interface nodes is set equal to that of the adjacent upstream nodes in the flow field. For the solid-fluid, the water level is calculated by simplifying the continuity equation for the water-level-region considering no-slip boundary conditions.

Boundary conditions on \( Q_x^* \), \( Q_y^* \) and \( Q_z^* \). In the development of pressure equation, values of \( \frac{\partial Q_x^*}{\partial x} \), \( \frac{\partial Q_y^*}{\partial y} \), and \( \frac{\partial Q_z^*}{\partial z} \) should be evaluated in the flow field. It was shown earlier that variables \( Q_x^* \), \( Q_y^* \), and \( Q_z^* \), are related to the velocity field by equations (24), (25), and (26). It should be realized that the only spatial derivatives of \( Q_x^* \) needed in the present analysis is \( \frac{\partial Q_x^*}{\partial x} \). Spatial derivatives \( \frac{\partial Q_x^*}{\partial y} \) and \( \frac{\partial Q_x^*}{\partial z} \) are not required. This situation simplifies the analysis. Calculation of \( \frac{\partial Q_x^*}{\partial x} \) in the fluid cell next to the boundary requires the calculation of \( Q_x^* \) at the boundaries normal to the \( x \) axis. Similarly, calculation of \( \frac{\partial Q_y^*}{\partial y} \) and \( \frac{\partial Q_z^*}{\partial z} \) require the calculation of \( Q_y^* \) and \( Q_z^* \) at the boundaries normal to the \( y \) and \( z \) axes respectively.

For the surface boundary, only \( Q_z^* \) is needed. As stated earlier,
since the vertical velocity component in the water-level element is small, equation (26) gives

\[ (Q^*_z)_{z = z_s} = -\frac{\rho^*}{F_0} \]

(57)

For the lateral solid-fluid boundaries, only the component of \( Q^* \) normal to the solid boundary is required. Since the velocity component normal to the solid boundary is zero, the component \( Q^* \) normal to the solid boundary can be obtained after simplifying equation (25) as follows:

\[
(Q^*_y)_{y = y_{\text{max}}} = 0
\]

\[ = \frac{\nu h}{\nu_0 R_0} \frac{1}{a_y a^* \nu^*} \]

(58)

For the bottom boundary only \( Q^*_z \) is needed. Simplification of equation (26) at the bottom boundary gives

\[
(Q^*_z)_{z = 0} = 0
\]

\[ = -\frac{\rho^*}{F_0} + \frac{\nu_0}{\nu_0 R_0} \frac{1}{a_z a^* \nu^*} \]

(59)

For the fluid-fluid boundary, the surrounding waters entering the water body are characterized by a uniform channel flow, with a uniform velocity at any cross section and zero axial pressure gradient. As an example, application of the momentum equation along the channel flowing in the \( x \)-direction gives

\[
(Q^*_x)_{x = 0} = 0
\]

(60)
Furthermore, at the outflow interface nodes, the values of $Q^*_x$ are set equal to that of the adjacent upstream nodes in the flow field.
3. NUMERICAL SOLUTION

The numerical scheme, used in this study, is mainly based on the application of spatial and temporal finite difference technique to the governing equation described in the previous section. This scheme is divided into the following steps:

1) Initialization and setting of boundary conditions;
2) Calculation of vertical velocity components;
3) Calculation of pressure distribution;
4) Calculation of the time derivatives for the horizontal velocity components and temperature in the elements of the sub-water-level region;
5) Calculation of the time derivative for the water-level, horizontal velocity components and temperature in the elements of the water-level region;
6) Time integration and updating of the horizontal velocity components and temperature in the sub-water-level region;
7) Calculation of the water-level, velocity components, and temperature in the water-level region;
8) Calculation of density distribution.

After the completion of the last step, the problem time is incremented, the computational procedure is repeated starting at the second step, and the integration cycle is continued until the final problem time is reached.

The initial conditions of the problem consist of the initial
values of temperature, velocity components, and pressure at time $t = 0$. These quantities are most easily obtained by the application of the present dynamic analysis. To demonstrate this application, let us consider that it is desired to determine the dynamic response of a water body when it is subjected to a prescribed thermal discharge. The solution to the problem can be obtained in two steps. In the first step, the dynamic response is obtained by setting the thermal discharge temperature equal to the river inflow temperature. Maintaining the atmospheric and the inflow conditions unchanged, the water body will reach equilibrium conditions and the pressure, velocity and temperature distribution will be obtained. In the second step, these equilibrium conditions are entered as the initial conditions and the thermal discharge temperature is set equal to the prescribed value. Again, if the atmospheric and inflow conditions are held unchanged, the water body will reach new equilibrium conditions. An examination of this dynamic response will enable the program user to predict the effects of the thermal discharge on the state of the water body. The results of this study could then be used to determine if the state's allowable temperature standards have been violated. These standards generally vary from state to state. Many states impose a maximum allowable water temperature, a temperature rise, and a rate of temperature rise on the use of water bodies.

From the temporal viewpoint, the boundary conditions stated earlier are of two types -- time-independent and time-dependent. The
setting of the time-independent boundary conditions are undertaken prior to the beginning of the dynamic solution. The time-dependent boundary conditions are satisfied during the execution of the integration cycle. For example, the time-dependent boundary conditions on vertical velocity and pressure are satisfied during the execution of steps 2 and 3 stated above respectively.

3.1 Calculation of the Vertical Velocity Components

As discussed earlier, the vertical component of velocity \( w^* \) is calculated from the continuity equation (27). This equation can be used to determine the vertical velocity component \( w^* \), once the horizontal velocity components \( u^* \) and \( v^* \) are known throughout the flow field. For the first step of numerical integration, the latter quantities as initialized will be used to calculate the velocity component \( w^* \). For the numerical integration steps higher than the first, the updated values of \( u^* \) and \( v^* \) will be available by the time integration of the momentum equations in the x and y directions to be discussed later.

To facilitate the writing of finite difference equations, to be presented in this section, it is convenient to introduce the following conventions:

1) Variables defining any quantity within an element \( i,j,k \) will be written without subscripts, for example, \( u^*_{i,j,k} \) will be written as \( u^* \).

2) Variables defining any quantity within an element adjacent
to element \( i, j, k \) will be written with the subscript that is different from \( i, j, k \), for example \( u_{i+1} \), \( j, k \) or \( u_{i, j-1, k} \) will be written as \( u_{i+1} \) or \( u_{j-1} \) respectively.

The finite difference form of \( \frac{\partial u}{\partial x} \) and \( \frac{\partial v}{\partial y} \), using a parabolic representation (three points with unequal spacing), are given in the Appendix 3.

Hence, expressing the right hand side equation (27) in finite difference form, yields:

\[
\begin{align*}
\frac{\partial w}{\partial z} &= a_x \frac{u_{i+1} - u_i}{x_{i+1} - x_i} + b_x \frac{u_i - u_{i-1}}{x_i - x_{i-1}} \\
&+ a_y \frac{v_{j+1} - v_j}{y_{j+1} - y_j} + b_y \frac{v_j - v_{j-1}}{y_j - y_{j-1}} \\
\end{align*}
\]

(61)

where

\[
\begin{align*}
a_x &= \frac{x_i - x_{i-1}}{x_{i+1} - x_i} \\
b_x &= \frac{x_{i+1} - x_i}{x_{i+1} - x_{i-1}} \\
a_y &= \frac{y_j - y_{j-1}}{y_{j+1} - y_j} \\
b_y &= \frac{y_{j+1} - y_j}{y_{j+1} - y_{j-1}}
\end{align*}
\]

(62) \hspace{1cm} (63) \hspace{1cm} (64)
Starting from the bottom boundary, equation (59) is then integrated along \( z \) to obtain the vertical velocity component in each element.

### 3.2 Calculation of Pressure Distribution

The updated values of pressure in the flow field are found from equations (24), (25), (26), and (28) by applying a centered three-point finite difference for the first spatial derivatives of \( u^*, v^*, w^*, Q_x^*, Q_y^* \), and \( Q_z^* \) as well as for the second spatial derivatives of \( p^*, u^*, v^*, \) and \( w^* \). A variable mesh size finite difference scheme is used as detailed in Appendix 3. Equation (28) becomes

\[
\begin{align*}
\frac{c_x}{x_{i+1} - x} (\frac{p_{i+1}^* - p}{x} - \frac{p}{x} - \frac{p_{i-1}^*}{x}) + c_y (\frac{p_{j+1}^* - p}{y} - \frac{p}{y} - \frac{p_{j-1}^*}{y}) + c_z (\frac{p_{k+1}^* - p}{z} - \frac{p}{z} - \frac{p_{k-1}^*}{z}) = \xi
\end{align*}
\]  

(66a)

where

\[
\begin{align*}
\xi &= \pi_0^2 \left( a_x \frac{(Q_x^*)_{i+1} - Q_x^*}{x_{i+1} - x} + b_x \frac{Q_x^* - (Q_x^*)_{i-1}}{x - x_{i-1}} \right) \\
&\quad + a_y \frac{(Q_y^*)_{j+1} - Q_y^*}{y_{j+1} - y} + b_y \frac{Q_y^* - (Q_y^*)_{j-1}}{y - y_{j-1}} \\
\end{align*}
\]  

(66b)
\[ a_x = \frac{(x^* - x_{i-1}^*)}{(x_{i+1}^* - x_{i-1}^*)} \]

\[ b_x = \frac{(x_{i+1}^* - x^*)}{(x_{i+1}^* - x_{i-1}^*)} \]

\[ c_x = \frac{2}{(x_{i+1}^* - x_{i-1}^*)} \]

\[ a_y = \frac{(y^* - y_{j-1}^*)}{(y_{j+1}^* - y_{j-1}^*)} \]

\[ b_y = \frac{(y_{j+1}^* - y^*)}{(y_{j+1}^* - y_{j-1}^*)} \]

\[ c_y = \frac{2}{(y_{j+1}^* - y_{j-1}^*)} \]

\[ a_z = \frac{(z^* - z_{k-1}^*)}{(z_{k+1}^* - z_{k-1}^*)} \]

\[ b_z = \frac{(z_{k+1}^* - z^*)}{(z_{k+1}^* - z_{k-1}^*)} \]

\[ c_z = \frac{2}{(z_{k+1}^* - z_{k-1}^*)} \]

\[ Q_x^* = -u^* \left( a_x \frac{u_{i+1}^* - u^*}{x_{i+1}^* - x^*} + b_x \frac{u^* - u_{i-1}^*}{x^* - x_{i-1}^*} \right) \]
\[- v^* \left( a_y \frac{u_{i+1} - u^*}{y_{j+1} - y} + b_y \frac{u^* - u_{j-1}}{y - y_{j-1}} \right) \]

\[- w^* \left( a_z \frac{u_{k+1} - u^*}{z_{k+1} - z} + b_z \frac{u^* - u_{k-1}}{z - z_{k-1}} \right) \]

\[+ \frac{\nu_h}{\nu_0} \frac{1}{R_0} (c_x \left( \frac{u_{i+1} - u^*}{x_{i+1} - x} - \frac{u^* - u_{i-1}}{x - x_{i-1}} \right) \]

\[+ c_y \left( \frac{u_{j+1} - u^*}{y_{j+1} - y} - \frac{u^* - u_{j-1}}{y - y_{j-1}} \right) \]

\[+ \frac{\nu_v}{\nu_0 R_0} c_z \left( \frac{u_{k+1} - u^*}{z_{k+1} - z} - \frac{u^* - u_{k-1}}{z - z_{k-1}} \right) \]

\[Q_y^* = - u^* \left( a_x \frac{v_{i+1} - v^*}{x_{i+1} - x} + b_x \frac{v^* - v_{i-1}}{x - x_{i-1}} \right) \]

\[- v^* \left( a_y \frac{v_{j+1} - v^*}{y_{j+1} - y} + b_y \frac{v^* - v_{j-1}}{y - y_{j-1}} \right) \]

\[- w^* \left( a_z \frac{v_{k+1} - v^*}{z_{k+1} - z} + b_z \frac{v^* - v_{k-1}}{z - z_{k-1}} \right) \]

\[+ \frac{\nu_h}{\nu_0} \frac{1}{R_0} (c_x \left( \frac{v_{i+1} - v^*}{x_{i+1} - x} - \frac{v^* - v_{i-1}}{x - x_{i-1}} \right) \]

(68)
\[ + c_y \left( \frac{v_{j+1} - v}{y_{j+1} - y} - \frac{v - v_{j-1}}{y - y_{j-1}} \right) \]

\[ + \frac{v_y}{v_0 R_0} \ c_z \left( \frac{v_{k+1} - v}{z_{k+1} - z} - \frac{v - v_{k-1}}{z - z_{k-1}} \right) \]

\[ = - u^* \left( a_x \frac{w_{i+1} - w}{x_{i+1} - x} + b_x \frac{w - w_{i-1}}{x - x_{i-1}} \right) \]

\[ - v^* \left( a_y \frac{w_{j+1} - w}{y_{j+1} - y} + b_y \frac{w - w_{j-1}}{y - y_{j-1}} \right) \]

\[ - w^* \left( a_z \frac{w_{k+1} - w}{z_{k+1} - z} + b_z \frac{w - w_{k-1}}{z - z_{k-1}} \right) \]

\[ + \frac{v_h}{v_0 R_0} \ (c_x \left( \frac{w_{i+1} - w}{x_{i+1} - w} - \frac{w - w_{i-1}}{x - x_{i-1}} \right) \]

\[ + c_y \left( \frac{w_{j+1} - w}{y_{j+1} - y} - \frac{w - w_{j-1}}{y - y_{j-1}} \right) \]

\[ + \frac{v_y}{v_0 R_0} \ c_z \left( \frac{w_{k+1} - w}{z_{k+1} - z} - \frac{w - w_{k-1}}{z - z_{k-1}} \right) - \frac{1}{F_0^2} \rho^* \]

(69)
Numerical values for $Q_x^*$, $Q_y^*$, and $Q_z^*$ at any element $i$, $j$, $k$ needed for the right hand side of equation (66b) are provided by equation (68), (69), and (70) respectively. Numerical values for $(Q_x^*)_{i+1}$, $(Q_x^*)_{i-1}$, $(Q_y^*)_{j+1}$, $(Q_y^*)_{j-1}$, $(Q_z^*)_{k+1}$, and $(Q_z^*)_{k-1}$, also needed for the right hand side of equation (66b) are provided by the same equations, after applying these equations to the entire flow field from the first cell ($i=1$, $j=1$, $k=1$) to the last ($i=i_{max}$, $j=j_{max}$, $k=k_{max}$).

Defining

$$e_x = \frac{c_x}{x_{i+1} - x_i}$$

$$f_x = \frac{c_x}{x - x_{i-1}}$$

$$e_y = \frac{c_y}{y_{j+1} - y_j}$$

$$f_y = \frac{c_y}{y - y_{j-1}}$$

$$e_z = \frac{c_z}{z_{k+1} - z_k}$$

$$f_z = \frac{c_z}{z - z_{k-1}}$$

(71)

Equation (66a) becomes

$$e_x (p_{i+1}^* - p^*) - f_x (p^* - p_{i-1}^*) +$$

$$e_y (p_{j+1}^* - p^*) - f_y (p^* - p_{j-1}^*) +$$

$$e_z (p_{k+1}^* - p^*) - f_z (p^* - p_{k-1}^*) = \xi$$

(72)
Within one small time increment, values $e_x, e_y, e_z, f_x, f_y, f_z$, and $\xi$ are constants and equation (72) constitutes a system of linear algebraic equations of $p^*$ which can be solved simultaneously. An iterative method, based on a modified Gauss-Seidel iteration technique, called successive over-relaxation is employed to solve the above system of equations. This method basically consists of solving equation (72) for $p^*$ and designating that by $p_t^*$.

$$p_t^* = \frac{e_x p_{i+1} + f_x p_{i+1} + e_y p_{j+1} + f_y p_{j-1} + e_z p_{k+1} + f_z p_{k-1} - \xi}{e_x + f_x + e_y + f_y + e_z + f_z}$$

(73)

and then calculating the actual pressure distribution employing a weighted average of this value and the value of pressure found from the preceding iteration such that

$$p_{l+1}^* = \omega(p_t^*)_{l+1} + (1 - \omega) p_l^*$$

(74)

where

- $p_l^*$ = pressure in cell $i, j, k$ at the preceding iteration $l$
- $(p_t^*)_{l+1}$ = pressure in cell $i, j, k$ calculated from equation (73) at the present iteration $l+1$
- $p_{l+1}^*$ = actual pressure in cell $i, j, k$ calculated from equation (74) at the present iteration $l+1$
- $\omega$ = relaxation factor to be discussed later in this section
The pressure calculation, employing equation (74) is performed for every cell in the flow field and old values of \( p_x^* \) are replaced by \( p_x^{*+1} \) as soon as the latter value is calculated for any cell. The iterative procedure, involving the calculation of the pressure for the entire flow field, is repeated until values of pressure at every cell converge within a prescribed error. It should be noted that for \( \omega = 1 \), this approach becomes identical with the Gauss-Seidel iterative method. The quantity \( \omega \), designated as relaxation factor varies between 0 and 2. For values of \( \omega < 1 \), the iterative method is referred to as underrelaxation and is usually employed to make a nonconvergent iterative process converge. For values of \( 1 < \omega < 2 \), the iterative method is referred to as overrelaxation and is ordinarily used to accelerate an already convergent iterative process. In this study, successive overrelaxation procedure is used and value of \( \omega \) is chosen between one and two.

The optimum value of \( \omega \) for finite difference method applied to the elliptic differential equation, involved in this study, is found (11, 6, 37) from

\[
\omega_{\text{opt}} = \frac{2}{1 + \left(1 - \lambda^2\right)^{1/2}} \tag{75}
\]

with

\[
\lambda^2 = \lim_{y \rightarrow x} \left| \frac{y^* + 1}{y^*} \right| \tag{76}
\]

and
where
\[ \omega_{\text{opt}} \]
\[ \lambda^2 \]
\[ || Y_{\varepsilon} + 1 || = \sum_{i=1}^{i_{\text{max}}} \sum_{j=1}^{j_{\text{max}}} \sum_{k=1}^{k_{\text{max}}} \left| (p^*_i)_{\varepsilon} + 1 - (p^*_{i,j})_{\varepsilon,j} \right| \]  

The above procedure indicates that the calculation of the optimum value of overrelaxation factor is based on the ratio of norms at the present iteration \( \varepsilon + 1 \) and the preceding iteration as given by equations (76) and (77) using Gauss-Seidel iterative scheme \( (\omega = 1) \).

After having found the optimum value of \( \omega \), the numerical scheme employs the modified Gauss-Seidel technique with \( \omega \) set equal to the optimum value. A question now arises as to how many Gauss-Seidel iterations should be performed before switching to the modified Gauss-Seidel scheme. If the number of iterations \( \varepsilon \) is too few, the value of \( \lambda^2 \) may become larger than unity which will not result in an optimum value for \( \omega \). On the other hand, if the number of iterations \( \varepsilon \) are too many, the value of \( \lambda^2 \) will approach unity \( (\omega_{\text{opt}} = 2) \). In this case, the converged values of pressure would be obtained by using the uneconomical Gauss-Seidel iterations. One effective procedure to solve this problem is to take enough Gauss-Seidel iterations to ensure that the estimate of \( \lambda^2 \) has settled down to a reasonably constant level less than one.
and then employ equation (75) to estimate $\omega_{opt}$. This estimate of $\omega_{opt}$ is then used in equation (74) and the numerical solution iterated to convergence using this overrelaxation factor.

From the foregoing analysis, it is evident that the optimum value of $\omega$ depends on the pressure distribution which is a time-dependent function. For this reason, the optimum value of $\omega$ also becomes time-dependent and should be calculated once during each integration step. However, since values of $\omega$ will only affect the speed of convergence and not the accuracy of the solution, it is not mandatory to compute the optimum value of $\omega$ at each integration step. A considerable amount of computer time will be saved if one calculates the optimum value of $\omega$ for the first integration step and use that value for the following integration steps. Experience with this numerical approach has indicated that since the time variation of pressure in the flow field is not excessively large, the optimum value of $\omega$ found in the first integration step provides a near optimum value for the following steps and the calculation of $\omega_{opt}$ for all integration steps may be eliminated at a considerable cost saving.

3.3 Calculation of the Time Derivative for the Horizontal Velocity Components and Temperature

Time derivatives of horizontal velocity components and temperature in the flow field are found from equations (21), (22), and (14) by applying a centered three point finite difference for the first spatial derivative of $p^*$ and $T^*$ as well as the second spatial deriva-
tive of $T^*$. A variable mesh size finite difference scheme is used as
detailed in Appendix 3.

$$\frac{\partial u^*}{\partial t} = Q_x^* - \frac{1}{F_0^2} \left( a_x \frac{p_{i+1}^* - p_i^*}{x_{i+1}^* - x_i^*} + b_x \frac{p_i^* - p_{i-1}^*}{x_i^* - x_{i-1}^*} \right)$$  (78)

$$\frac{\partial v^*}{\partial t} = Q_y^* - \frac{1}{F_0^2} \left( a_y \frac{p_{j+1}^* - p_j^*}{y_{j+1}^* - y_j^*} + b_y \frac{p_j^* - p_{j-1}^*}{y_j^* - y_{j-1}^*} \right)$$  (79)

$$\frac{\partial T^*}{\partial t} = - u^* \left( a_x \frac{T_{i+1}^* - T_i^*}{x_{i+1}^* - x_i^*} + b_x \frac{T_i^* - T_{i-1}^*}{x_i^* - x_{i-1}^*} \right)$$

$$- v^* \left( a_y \frac{T_{j+1}^* - T_j^*}{y_{j+1}^* - y_j^*} + b_y \frac{T_j^* - T_{j-1}^*}{y_j^* - y_{j-1}^*} \right)$$

$$- w^* \left( a_z \frac{T_{k+1}^* - T_k^*}{z_{k+1}^* - z_k^*} + b_z \frac{T_k^* - T_{k-1}^*}{z_k^* - z_{k-1}^*} \right)$$

$$+ \frac{D_h}{v_0^2 R_0} \left( c_x \left( \frac{T_{i+1}^* - T_i^*}{x_{i+1}^* - x_i^*} - \frac{T_i^* - T_{i-1}^*}{x_i^* - x_{i-1}^*} \right) \right)$$

$$+ \frac{D_y}{v_0^2 R_0} \left( c_y \left( \frac{T_{j+1}^* - T_j^*}{y_{j+1}^* - y_j^*} - \frac{T_j^* - T_{j-1}^*}{y_j^* - y_{j-1}^*} \right) \right)$$

$$+ \frac{D_z}{v_0^2 R_0} \left( c_z \left( \frac{T_{k+1}^* - T_k^*}{z_{k+1}^* - z_k^*} - \frac{T_k^* - T_{k-1}^*}{z_k^* - z_{k-1}^*} \right) \right)$$  (80)

where $Q_x^*$, $Q_y^*$, $a_x$, $b_x$, $a_y$, $b_y$, $a_z$, $b_z$, $c_x$, $c_y$, and $c_z$, are previously
defined by equations (68), (69), and (67).

3.4 Calculation of the Time Derivative for the Water-Level, Horizontal Velocity Components and Temperature in the Water-Level Elements

Time-derivatives of the water-level horizontal velocity components and temperature in the water-level region are found from equations (31), (41), (40), and (39), respectively, by applying a centered three-point finite difference for the first and second spatial derivatives of $H^*, u^*, v^*$, and $T^*$. A variable mesh size finite difference scheme is used as detailed in Appendix 3.

\[
\frac{\partial u^*}{\partial t} = -u^* \left( a_x \frac{u_{i+1}^* - u_i^*}{x_{i+1}^* - x_i^*} + b_x \frac{u_i^* - u_{i-1}^*}{x_i^* - x_{i-1}^*} \right) \\
- v^* \left( a_y \frac{u_{j+1}^* - u_j^*}{y_{j+1}^* - y_j^*} + b_y \frac{u_j^* - u_{j-1}^*}{y_j^* - y_{j-1}^*} \right) \\
- \frac{H^*}{F_0} \left( a_x \frac{\rho_{i+1}^* - \rho_i^*}{x_{i+1}^* - x_i^*} + b_x \frac{\rho_i^* - \rho_{i-1}^*}{x_i^* - x_{i-1}^*} \right) \\
+ \left( a_x \frac{H_{i+1}^* - H^*}{x_{i+1}^* - x_i^*} + b_x \frac{H^* - H_{i-1}^*}{x_i^* - x_{i-1}^*} \right) \\
+ \left( \nu_h \frac{1}{\nu_0 R_0^H} \left( a_x \frac{u_{i+1}^* - u_i^*}{x_{i+1}^* - x_i^*} + b_x \frac{u_i^* - u_{i-1}^*}{x_i^* - x_{i-1}^*} \right) - \frac{2\rho^*}{F_0} \right)
\]
\[ \frac{v_h}{v_0 R_0} c_x \left( \frac{u_{i+1}^* - u_i^*}{x_{i+1}^* - x_i^*} - \frac{u_i^* - u_{i-1}^*}{x_i^* - x_{i-1}^*} \right) \]

\[ + \frac{v_h}{v_0 R_0 H^*} \left( a_y \frac{H_{j+1}^* - H_j^*}{y_{j+1}^* - y_j^*} + b_y \frac{H_j^* - H_{j-1}^*}{y_j^* - y_{j-1}^*} \right) \]

\[ (a_y \frac{u_{j+1}^* - u_j^*}{y_{j+1}^* - y_j^*} + b_y \frac{u_j^* - u_{j-1}^*}{y_j^* - y_{j-1}^*}) \]

\[ + \frac{v_h}{v_0 R_0} c_y \left( \frac{u_{j+1}^* - u_j^*}{y_{j+1}^* - y_j^*} - \frac{u_j^* - u_{j-1}^*}{y_j^* - y_{j-1}^*} \right) \]

\[ - \frac{v_v}{v_0 R_0 H^*} \left( \frac{u_i^* - u_{k-1}^*}{z_i^* - z_{k-1}^*} \right) + \frac{1}{2H} c_f \rho_a \omega^2 \]

\[ \frac{\partial v_i^*}{\partial t} = -u_i^* \left( a_x \frac{v_{i+1}^* - v_i^*}{x_{i+1}^* - x_i^*} + b_x \frac{v_i^* - v_{i-1}^*}{x_i^* - x_{i-1}^*} \right) \]

\[ - v_i^* \left( a_y \frac{v_{j+1}^* - v_j^*}{y_{j+1}^* - y_j^*} + b_y \frac{v_j^* - v_{j-1}^*}{y_j^* - y_{j-1}^*} \right) \]

\[ - \frac{H_{j+1}^*}{F_0^2} \left( a_y \frac{\rho_{j+1}^* - \rho_j^*}{y_{j+1}^* - y_j^*} + b_y \frac{\rho_j^* - \rho_{j-1}^*}{y_j^* - y_{j-1}^*} \right) \]

\[ + \left( a_y \frac{H_{j+1}^* - H_j^*}{y_{j+1}^* - y_j^*} + b_y \frac{H_j^* - H_{j-1}^*}{y_j^* - y_{j-1}^*} \right) . \]
\[
\frac{v_h}{v_0 R_0^H} \left( a_y \frac{v_{i+1} - v}{y_{j+1} - y} + b_y \frac{v - v_{j-1}}{y - y_{j-1}} \right) - \frac{2 \rho^*}{F_0}
\]

+ \frac{v_h}{v_0 R_0^H} c_y \left( \frac{v_{i+1} - v}{y_{j+1} - y} - \frac{v - v_{j-1}}{y - y_{j-1}} \right)

+ \frac{v_h}{v_0 R_0^H} \left( a_x \frac{H_{i+1} - H}{x_{i+1} - x} + b_x \frac{H - H_{i-1}}{x - x_{i-1}} \right)

\]

\[
\left( a_x \frac{v_{i+1} - v}{x_{i+1} - x} + b_x \frac{v - v_{i-1}}{x - x_{i-1}} \right)
\]

+ \frac{v_h}{v_0 R_0^H} c_y \left( \frac{v_{i+1} - v}{y_{j+1} - y} - \frac{v - v_{j-1}}{y - y_{j-1}} \right)

- \frac{v_v}{v_0 R_0^H} \left( \frac{v - v_{k-1}}{z - z_{k-1}} \right) + \frac{1}{2H} c_{fy} \rho \frac{W^2}{y^2}
\]

(82)

\[
\frac{\partial T^*}{\partial t^*} = - u^* \left( a_x \frac{T_{i+1} - T}{x_{i+1} - x} + b_x \frac{T - T_{i-1}}{x - x_{i-1}} \right)
\]

- \[
- v^* \left( a_y \frac{T_{j+1} - T}{y_{j+1} - y} + b_y \frac{T - T_{j-1}}{y - y_{j-1}} \right)
\]
\[ + \frac{D_h}{v_0 R_0} \left( a_x \frac{H_{i+1}^* - H_i^*}{x_{i+1} - x} + b_y \frac{H_i^* - H_{i-1}^*}{x - x_{i-1}} \right) \]

\[ + \frac{D_h}{v_0 R_0} \left( a_x \frac{T_{i+1}^* - T_i^*}{x_{i+1} - x} + b_x \frac{T_i^* - T_{i-1}^*}{x - x_{i-1}} \right) \]

\[ + \frac{D_h}{v_0 R_0} \left( a_y \frac{T_{j+1}^* - T_j^*}{y_{j+1} - y} + b_y \frac{T_j^* - T_{j-1}^*}{y - y_{j-1}} \right) \]

\[ + \frac{D_h}{v_0 R_0} \left( a_y \frac{T_{j+1}^* - T_j^*}{y_{j+1} - y} + b_y \frac{T_j^* - T_{j-1}^*}{y - y_{j-1}} \right) \]

\[ + \frac{D_v}{v_0 R_0} \left( \frac{T_i^* - T_{k-1}^*}{z - z_{k-1}} - \frac{S_0}{H} (T^* - E^*) \right) \]  

\[ \frac{\partial H^*}{\partial t^*} = - u^* \frac{H^* - H_{i-1}^*}{x - x_{i-1}} - v^* \frac{H_i^* - H_{j-1}^*}{y - y_{j-1}} \]

\[ - H^* \left( \frac{u^* - u_{i-1}^*}{x - x_{i-1}} + \frac{v^* - v_{j-1}^*}{y - y_{j-1}} + v_{k-1}^* \right) \]  

(84)

It should be noted that a backward spatial differencing method is employed in equation (84) to ensure a conditional stability of this equation.
3.5 Calculations of the Horizontal Velocity Components and Temperature in the Sub-Water-Level-Region

The updated values of horizontal velocity components and temperature in the sub-water-level region are found by applying a forward two-point finite difference for the first temporal derivatives.

\[
\begin{align*}
    u^n &= u^* + \Delta t^* \left( \frac{\partial u^*}{\partial t} \right) \\
    v^n &= v^* + \Delta t^* \left( \frac{\partial v^*}{\partial t} \right) \\
    T^n &= T^* + \Delta t^* \left( \frac{\partial T^*}{\partial t} \right)
\end{align*}
\]

where superscript \( n \) represents the updated values, \( \Delta t^* \) is time integration step, and values \( \frac{\partial u^*}{\partial t^*} \), \( \frac{\partial v^*}{\partial t^*} \), and \( \frac{\partial T^*}{\partial t^*} \) are calculated from equations (78), (79), and (80).

3.6 Calculation of the Water-Level, Velocity Components and Temperature in the Water-Level Element

The updated values of the water-level, velocity components and temperature in the water-level region are found by applying a forward two-point finite difference for the first temporal derivatives.

\[
\begin{align*}
    u^n &= u^* + \Delta t^* \left( \frac{\partial u^*}{\partial t} \right) \\
    v^n &= v^* + \Delta t^* \left( \frac{\partial v^*}{\partial t} \right)
\end{align*}
\]
The values of \( \frac{\partial u^*}{\partial t^*} \), \( \frac{\partial v^*}{\partial t^*} \), \( \frac{\partial T^*}{\partial t^*} \), and \( \frac{\partial H^*}{\partial t^*} \)
are calculated from equations (81), (82), (83), and (84).

3.7 Calculation of Density Distribution

The fluid density is considered to be a function of temperature only. The dependence of density on pressure, within the range of the variation of pressure in this study, is extremely weak and is, therefore, neglected. The functional relationship between the density and temperature is defined in tabular form as shown in columns 1 and 2 of Table 1 and is designated by

\[ \rho = \rho (T) \]  \hspace{1cm} (92)

in which \( T \) is the fluid temperature in degrees Fahrenheit, and \( \rho \) is the density in lbm/ft\(^3\). In non-dimensional form, equation (92) becomes

\[ \rho^* = \rho^* (T^*) \]  \hspace{1cm} (93)

as exemplified in columns 3 and 4 of Table 1 with temperatures normalized to 90\(^\circ\)F.
### TABLE 1. DENSITY VERSUS TEMPERATURE

<table>
<thead>
<tr>
<th>No.</th>
<th>T °F</th>
<th>$\rho\left(\frac{\text{lbm}}{\text{ft}^3}\right)$</th>
<th>Dimensionless $\frac{T}{\rho}$</th>
</tr>
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<td>1</td>
<td>70</td>
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<td>0.7777777</td>
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<td>0.8222222</td>
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<td>76</td>
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<td>0.8444444</td>
</tr>
<tr>
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<td>0.8666666</td>
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</tr>
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</tr>
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<td>84</td>
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<tr>
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</tr>
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<td>26</td>
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4. NUMERICAL STABILITY AND CONVERGENCE

The numerical procedure, presented in the previous section, involves spatial integrations for the calculation of the vertical velocity component and pressure, as well as time integrations for the calculation of the horizontal velocity components, temperature and water-level. These spatial and time integration procedures are plagued by numerical stability problems caused by round-off errors in the numerical solution and convergence problems caused by the truncation error due to the finite magnitude of the space and time increments. The numerical solution is said to be stable if round-off errors remain either constant or decrease as the solution to the difference equations is carried out. The numerical solution is termed convergent if, as the space or time increment approaches zero, the numerical solution approaches the exact solution to the differential equations. Using these definitions, stability is a necessary condition for convergence and in this sense the problems of stability and convergence are interdependent.

To achieve numerical stability in nonlinear numerical solutions, the governing system of differential equations must first be linearized and the absolute value of the largest eigenvalue of the amplification matrix for the system of differential equations should be made less or equal to unity (1, 26). This condition will provide an upper bound on the integration step size that ensures the numerical stability of the solution. The computation of the eigenvalues of a large
system of differential equations is very laborious and time consuming. It constitutes a problem of the same order of difficulty as the problem to be solved. Fortunately, an exact calculation of the upper bound of integration step size is not necessary. A conservative estimate of the upper bound of integration step size is all that is required. The calculation of a conservative estimate for spatial and time steps will be based on the following arguments:

1) In general, numerical instabilities develop locally in one variable, grow with increasing time and adversely affect other variables through coupling terms in the governing differential equations. If the differential equation for every variable is numerically stable in any small region of the domain, without feedback from the others, instability will not have its origin within the domain and, therefore, will not occur. Employing this hypothesis, each of the differential equations involved in the analysis will be examined independently and their stability criterion will be determined.

2) Experience with the computational properties of various numerical schemes for the solution of the heat and mass transport phenomena has led to a simple rule (3) based on the fact that water temperature is generally a non-negative quantity. This rule, which applies to an explicit computational scheme, is that the coefficients multiplying each of the temperature values should be positive quantities. This rule, used herein for the numerical stability analysis of the differential equations for the horizontal velocity components and temperature, is substantiated in Appendix 2.
4.1 Stability Analysis of Horizontal Momentum and Energy Equations

As discussed earlier, the horizontal momentum and energy equations are solved explicitly.

Using the above-mentioned rule on these explicit solutions, one obtains a set of stability conditions which must be satisfied, otherwise the solutions will not converge. The stability analysis is demonstrated for energy equation only. In view of similarity between the momentum and energy equations, the same results are applicable to the horizontal momentum equations. After linearizing and expressing the energy equation in an explicit finite difference form in terms of a small variation in temperature, as exemplified in Appendix 2, one obtains:

\[
\dot{T} = T_{i+1} \left( -a_x \frac{u \Delta t}{x_{i+1} - x} + c_x \frac{D_h \Delta t}{x_{i+1} - x} \right)
\]

\[
+ T \left( 1 + a_x \frac{u \Delta t}{x_{i+1} - x} - b_x \frac{u \Delta t}{x - x_{i-1}} + a_y \frac{v \Delta t}{y_{j+1} - y} - b_y \frac{v \Delta t}{y - y_j} \right)
\]

\[
+ a_z \frac{w \Delta t}{z_{k+1} - z} - b_z \frac{w \Delta t}{z - z_k} - c_x \frac{D_h \Delta t}{x_{i+1} - x} - c_y \frac{D_h \Delta t}{y_{j+1} - y} - c_z \frac{D_h \Delta t}{z_{k+1} - z} - c_z \frac{D_h \Delta t}{z_k - z_{k-1}}
\]

\[
+ T_{i-1} \left( b_x \frac{u \Delta t}{x - x_{i-1}} + c_x \frac{D_h \Delta t}{x - x_{i-1}} \right)
\]
\[ + \dot{T}_{j+1} \left( -a_y \frac{vDelta t}{y_{j+1} - y} - c_y \frac{D_h Delta t}{y_{j+1} - y} \right) \]

\[ + \dot{T}_{j-1} \left( b_y \frac{vDelta t}{y_{j-1} - y} + c_y \frac{D_h Delta t}{y_{j-1} - y} \right) \]

\[ + \dot{T}_{k+1} \left( -a_z \frac{wDelta t}{z_{k+1} - z} + c_z \frac{D_v Delta t}{z_{k+1} - z} \right) \]

\[ + \dot{T}_{k-1} \left( b_z \frac{wDelta t}{z_{k-1} - z} + c_z \frac{D_v Delta t}{z_{k-1} - z} \right) \]

where superscript (\( \cdot \)) indicates a small variation in temperature.

Setting the temperature coefficients equal to positive value and substituting for \( a_x, b_x, c_x, a_y, b_y, c_y, a_z, b_z, \) and \( c_z \) from equations (67) yields:

\[ x - x_{i-1} < \frac{2D_h}{|u|} \]

\[ x_{i+1} - x < \frac{2D_h}{|u|} \]

\[ y - y_{j-1} < \frac{2D_h}{|v|} \]

\[ y_{j+1} - y < \frac{2D_h}{|v|} \]
\[ z-z_{k-1} \leq \frac{2D_v}{|w|} \]
\[ z_{k+1}-z \leq \frac{2D_v}{|w|} \] \hspace{1cm} (95)

and

\[ \Delta t \leq \frac{1}{\frac{2D_h}{(x-x_{i-1})^2} + \frac{2D_h}{(y-y_{j-1})^2} + \frac{2D_v}{(z-z_{k-1})^2}} \] \hspace{1cm} (96)

The increments of the nodal coordinates may be expressed in terms of the elements mesh size \( \Delta x, \Delta y, \) and \( \Delta z \) by

\[ x-x_{i-1} = \frac{1}{2} (\Delta x + \Delta x_{i-1}) \]
\[ y-y_{j-1} = \frac{1}{2} (\Delta y + \Delta y_{j-1}) \]
\[ z-z_{k-1} = \frac{1}{2} (\Delta z + \Delta z_{k-1}) \] \hspace{1cm} (97)

Substituting equations (97) into equations (95) and (96) results in

\[ \Delta x + \Delta x_{i-1} \leq \frac{4D_h}{|u|} \]
\[ \Delta y + \Delta y_{j-1} \leq \frac{4D_h}{|v|} \]
\[ \Delta z + \Delta z_{k-1} \leq \frac{4D_v}{|w|} \]
\[ \Delta t \leq \frac{8D_h}{(\Delta x + \Delta x_{i-1})^2} + \frac{8D_h}{(\Delta y + \Delta y_{j-1})^2} + \frac{8D_v}{(\Delta z + \Delta z_{k-1})^2} \]  

(98)

Since the energy equation has the same form as the momentum equations, equations (97) and (98) constitute the numerical stability criteria for the energy and horizontal momentum equations.

4.2 Convergence of Pressure Equation

As discussed in Section 3.2, the pressure equation provides a system of linear equations, which is solved by the modified Gauss-Seidel iterative method designated as successive over-relaxation. A sufficient condition for the convergence of this iterative process is given by Isaacson and Keller (12) and Williams (43) as follows:

\[ \Delta = \frac{\sum_{m=1}^{m'} |a_{rr}|}{\sum_{m=1}^{m'} |a_{rm}|} \leq 1 \]  

(99)

where \( a_{rr} \) is the value of the diagonal terms of matrix and \( a_{rm} \) is the value of the off-diagonal terms, with \( m' \) notation signifying that the value of \( a_{rr} \) is omitted from the summation. The convergence of the numerical solution is ensured if \( \Delta < 1 \) for at least one equation in the system of linear pressure equations, given by equations (72). An examination of the later equations shows that for nodal points within the fluid field \( \Delta = 1 \) except for nodal points just below the water-level region or near other boundaries for which pressure is known. The known value of pressure causes a transfer of the corre-
sponding term to the right hand side of equation (72) leading to
\( \Delta < 1 \) which is a sufficient condition for convergence.

4.3 Stability Analysis of Water-Level Equation

The stability criterion for the water-level equation is obtained
by linearizing equation (31), as exemplified in Appendix 2 for the
energy equation, as follows:

\[
\frac{\partial H}{\partial t} = -u \frac{\partial H}{\partial x} - v \frac{\partial H}{\partial y} \quad (100)
\]

where superscript (•) indicates small variation about steady-state.
In this study, equation (100) is expressed in a finite difference
form, by applying a forward two point finite difference approximation
to the temporal derivative and backward finite difference approxima­
tion to first spatial derivatives respectively. Applying Von-Neumann
stability analysis, shown in Appendix 2, to the discretized equation
accounting for variable mesh size, one obtains:

\[
e^{\delta \Delta t} = 1 - \frac{u \Delta t}{x-x_{i-1}} (1 - e^{-i \alpha \Delta x_{i-1}})
- \frac{v \Delta t}{y-y_{j-1}} (1 - e^{-i \beta \Delta y_{j-1}}) \quad (101)
\]

The necessary and sufficient condition for equation (101) to be
stable is that the absolute value of \( e^{\delta \Delta t} \) must be less than or equal
to one.
After expressing the exponential terms in trigonometric forms and calculating the absolute value of $e^{\delta \Delta t}$, equation (101) reduces to

$$|e^{\delta \Delta t}| = (1 - 4 \left(1 - \frac{u \Delta t}{x-x_{i-1}} - \frac{v \Delta t}{y-y_{j-1}}\right) \frac{u \Delta t}{x-x_{i-1}} \sin^2 \frac{a \Delta x_{i-1}}{2}$$

$$- 4 \left(1 - \frac{u \Delta t}{x-x_{i-1}} - \frac{v \Delta t}{y-y_{j-1}}\right) \frac{v \Delta t}{y-y_{j-1}} \sin^2 \frac{\beta \Delta y_{j-1}}{2}$$

$$- 4 \left(\frac{u \Delta t}{x-x_{i-1}}\right) \left(\frac{v \Delta t}{y-y_{j-1}}\right) \sin^2 \frac{a \Delta x_{i-1} - \beta \Delta y_{j-1}}{2} \leq 1 \quad (102)$$

In the above inequality, the maximum values of the trigonometric terms are equal to one. Therefore, further examination shows that this inequality will be satisfied if

$$1 - \frac{u \Delta t}{x-x_{i-1}} - \frac{v \Delta t}{y-y_{j-1}} \geq 0 \quad (103)$$

To satisfy the above stability conditions for negative values of the velocity components, one must have

$$\Delta t \leq \frac{1}{(|u|/(x-x_{i-1}) + |v|/(y-y_{j-1}))} \quad (104)$$

Expressing equation (104) in terms of $\Delta x$ and $\Delta y$ as shown in Section (4.1), one obtains

$$\Delta t \leq \frac{1}{(2|u|/(\Delta x + \Delta x_{i-1}) + 2|v|/(\Delta y + \Delta y_{j-1}))} \quad (105)$$

It should be noted that estimates of the largest values of the velocity components would suffice for the calculation of conservative values of space and time increments.
5. PRESENTATION OF RESULTS

The mathematical formulation and the computer program, developed in this study, is verified by applying the dynamic analysis to a number of cases where the final steady-state solution is either known experimentally or the final steady-state flow pattern has been determined by other investigators. Employing the present program, the fluid flow problem considered is brought to steady-state condition through a dynamic analysis. The initial values of thermal and hydraulic variables in the flow field, needed for this purpose, were selected in such a way to facilitate the entry of input data. Starting from these rough initial values and maintaining the appropriate boundary conditions, the present program is employed to bring the flow problem to steady-state. These steady-state solutions are then compared either with the experimental work or the known flow patterns. These verification studies are considered in the next section.

5.1 Verification Studies

The following problems were selected for verification purposes:

1) Experimental studies on the laminar flow development in a square duct (7)

2) Natural circulation in a pond partially heated from the bottom or the side.

The first study consists of a duct with a square cross section with a spatially and temporally uniform input velocity at the duct
entrance. Since the geometric dimensions of the laboratory model used in the experimental studies were small and our interest here is in water bodies of appreciable size, the similarity between the present and the experimental study is based on two dimensionless parameters: 1) the quotient of aspectation ration \( x/D \) to Reynolds number; and 2) the ratio of the fluid velocity to the average inlet velocity. The geometric and hydraulic input data for this case is given in Table 2 and Fig. 2a. The computational grid used in this dynamic analysis consists of 12 x 5 x 5 mesh points with non-uniform spacing in the longitudinal direction and uniform spacing in the square cross section as shown in Table 3. The initial values of the velocity components in the flow field were selected uniform in the longitudinal direction equal to the inlet velocity with zero transversal and vertical velocities. The dynamic solution was continued until the flow pattern in the duct reached steady-state. Typical time histories for a number of upstream and a downstream points along the duct axial central plane are obtained and used to plot the final steady-state centerline and vertical velocity profiles in the square duct. These velocity profiles, in non-dimensional form, are compared in Figs. 2b and 2c with the experimental results of Goldstein and Kreid (37) who measured the laminar flow development in a square duct using laser-doppler flowmeter. This comparison shows that the results of this study are in good agreement with the experimental data. This confirms that the present formulation can predict the three dimensional flow behavior of water bodies.

The second study consists of natural circulation in a pond par-
TABLE 2. THE GEOMETRIC AND HYDRAULIC INPUT DATA FOR LAMINAR FLOW IN A SQUARE DUCT

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<th>Specifications</th>
<th>Dimensions</th>
<th>British Unit</th>
<th>SI Unit</th>
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<td>85.34 m</td>
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<td>Water Body Width</td>
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<td>21.946 m</td>
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<tr>
<td>Water Body Depth</td>
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<td>21.946 m</td>
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<tr>
<td>Inlet Water Velocity</td>
<td></td>
<td>1 ft/sec</td>
<td>0.3048 m/sec</td>
</tr>
<tr>
<td>Water Temperature</td>
<td></td>
<td>75°F</td>
<td>23.89 °C</td>
</tr>
<tr>
<td>Reynolds Number, $R_0$</td>
<td></td>
<td>20.83</td>
<td>20.83</td>
</tr>
<tr>
<td>Hydraulic Diameter of Square Duct, $D$</td>
<td></td>
<td>72 ft.</td>
<td>21.946 m</td>
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</table>
(a) THE SQUARE DUCT CONFIGURATION

(b) CENTER-LINE VELOCITY DEVELOPMENT

(c) DEVELOPMENT OF VELOCITY PROFILE
   CENTRAL PLANE

FIG 2. VELOCITY DEVELOPMENT IN A SQUARE DUCT
### TABLE 3. GRID DIMENSIONS FOR THE LAMINAR FLOW IN A SQUARE DUCT

<table>
<thead>
<tr>
<th>Element No.</th>
<th>X (ft)</th>
<th>Y (ft)</th>
<th>Z (ft)</th>
<th>X (m)</th>
<th>Y (m)</th>
<th>Z (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>3.048</td>
<td>18</td>
<td>5.486</td>
<td>18</td>
<td>5.486</td>
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<td>5.486</td>
<td>18</td>
<td>5.486</td>
</tr>
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<td>5.486</td>
<td>18</td>
<td>5.486</td>
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<td>9.144</td>
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<td>9.144</td>
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<td>30</td>
<td>9.144</td>
<td></td>
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</tr>
</tbody>
</table>
tially heated from the bottom or the side with uniform initial
temperature, zero initial velocities and zero inlet and outlet mass
flux. The geometric and hydraulic input data for this case is given
in Table 4. The computational grid used in the dynamic analysis con­
sists of 10 x 6 x 6 mesh points with uniform spacing in the longitu­
dinal and transversal directions and non-uniform spacing in the verti­
cal direction as shown in Table 5. Two cases were examined. In these
cases, a step temperature is applied to a portion of side wall or the
bottom surface of the pond. The dynamic response of the pond is ob­
tained for each case and the natural circulation flows developed are
compared with expected flow patterns. Typical natural circulation
flow patterns for partially heated side wall and bottom surface are
shown in Figs. 3, 4, 5, and 6 at times t = 40 and t = 25 seconds,
respectively, into the transient. An examination of these flow
patterns demonstrates that for the case of partially heated side
wall one natural circulation vortex in transversal direction and two
symmetric natural circulation vortices in the longitudinal direction
are developed which lose their strength as the distance from the
heated wall increases (see Figs. 3 and 4). For the case of partially
heated bottom surface two symmetric natural vortices in vertical
longitudinal and vertical diagonal planes are developed which lose
their strength with increasing distance from the heated surface (see
Figs. 5 and 6). In both cases, described above, the natural circula­
tion vortices formed and the resultant mixing are caused by density
differences in the water body. These results establish that the
present formulation can predict the three dimensional flow and thermal
### TABLE 4. GEOMETRIC AND HYDRAULIC INPUT DATA FOR PARTIALLY HEATED POND

<table>
<thead>
<tr>
<th>Specifications</th>
<th>British Unit</th>
<th>SI Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Body Length</td>
<td>540 ft</td>
<td>164.592 m</td>
</tr>
<tr>
<td>Water Body Width</td>
<td>250 ft</td>
<td>76.2 m</td>
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<tr>
<td>Water Body Depth</td>
<td>20.5 ft</td>
<td>6.248 m</td>
</tr>
<tr>
<td>Water Body Temperature</td>
<td>75 °F</td>
<td>23.89 °C</td>
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<td>74 °F</td>
<td>23.33 °C</td>
</tr>
<tr>
<td>Temperature of Heated Area</td>
<td>100 °F</td>
<td>37.77 °C</td>
</tr>
<tr>
<td>Reference Velocity, ( U_0 )</td>
<td>1 ft/sec</td>
<td>0.3048 m/sec</td>
</tr>
<tr>
<td>Reference Length, ( d_0 )</td>
<td>50 ft</td>
<td>15.24 m</td>
</tr>
<tr>
<td>Reference Temperature, ( T_0 )</td>
<td>100 °F</td>
<td>37.77 °C</td>
</tr>
<tr>
<td>Reference Time, ( t_0 )</td>
<td>50 sec</td>
<td>50 sec</td>
</tr>
<tr>
<td>Reference Density, ( \rho_0 )</td>
<td>61.9963 lbm/ft(^3)</td>
<td>960.590 kg m(^{-3})</td>
</tr>
<tr>
<td>Reference Pressure, ( p_0 )</td>
<td>21.52 lbf in(^{-2})</td>
<td>14639.4 kg m(^{-2})</td>
</tr>
<tr>
<td>Reynolds Number, ( R_0 )</td>
<td>0.6061 x 10(^7)</td>
<td>0.6061 x 10(^7)</td>
</tr>
<tr>
<td>Heat Exchange Coefficient, ( K )</td>
<td>100 Btu/ft(^2) day °F</td>
<td>23.64 W/m(^2) °C</td>
</tr>
</tbody>
</table>
TABLE 5. GRID DIMENSIONS FOR PARTIALLY HEATED POND

<table>
<thead>
<tr>
<th>Element No.</th>
<th>X (ft)</th>
<th>X (m)</th>
<th>Y (ft)</th>
<th>Y (m)</th>
<th>Z (ft)</th>
<th>Z (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
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<td>18.29</td>
<td>50</td>
<td>15.24</td>
<td>3.5</td>
<td>1.07</td>
</tr>
<tr>
<td>2</td>
<td>60</td>
<td>18.29</td>
<td>50</td>
<td>15.24</td>
<td>4.0</td>
<td>1.22</td>
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<td>18.29</td>
<td>50</td>
<td>15.24</td>
<td>5.0</td>
<td>1.52</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
<td>18.29</td>
<td>50</td>
<td>15.24</td>
<td>4.0</td>
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<td>50</td>
<td>15.24</td>
<td>4.0</td>
<td>1.22</td>
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<td>60</td>
<td>18.29</td>
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<td>15.24</td>
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<td>1.07</td>
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<tr>
<td>8</td>
<td>60</td>
<td>18.29</td>
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<td></td>
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<td></td>
</tr>
<tr>
<td>9</td>
<td>60</td>
<td>18.29</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td>60</td>
<td>18.29</td>
<td></td>
<td></td>
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<td></td>
</tr>
</tbody>
</table>
FIG 3. NATURAL CIRCULATION AT y = 100 FT. AND t = 40 SEC IN A POND PARTIALLY HEATED FROM SIDE.
FIG 4. NATURAL CIRCULATION AT X = 240 FT. AND t = 40 SEC. IN A POND PARTIALLY HEATED FROM SIDE.
FIG 5. NATURAL CIRCULATION AT y = 100 FT. AND t = 25 SEC. IN A POND PARTIALLY HEATED FROM BOTTOM.
FIG 6. NATURAL CIRCULATION AT t=25 SEC IN VERTICAL DIAGONAL PLANE IN A POND PARTIALLY HEATED FROM BOTTOM
aspects of water bodies.

5.2 Circulation and Stratification in Water Bodies

The mathematical formulation, presented in this study, is applied to the study of circulation and stratification of large water bodies. The water body is considered to be flowing initially at a uniform velocity and temperature and suddenly exposed to a 90° angle jet with higher velocity but the same temperature. This problem is referred to as three-dimensional non-buoyant jet in a cross current. When the dynamic problem reaches steady-state, the water jet temperature is suddenly raised to simulate a thermal discharge. This problem is referred to as three-dimensional buoyant jet in a cross current. To facilitate the understanding of the results, the two problems indicated above (i.e., non-buoyant and buoyant jets) are discussed separately in the following sections but the time histories of the results are shown on the same plots for easy reference.

5.3 Three Dimensional Non-Buoyant Jet in a Cross Current

The problem of a three-dimensional non-buoyant jet in a cross current, as modeled in Fig. 1, was first analyzed. The geometric and hydraulic input data for the case studied are given in Table 6.

The water body is assumed to be flowing initially at a uniform velocity of 0.40 ft/sec when suddenly exposed to a jet velocity of 2.00 ft/sec at a 90 degree angle. The surface wind velocity are considered to be zero. Furthermore, the temperature of the thermal
TABLE 6. GEOMETRIC AND HYDRAULIC INPUT DATA FOR
THREE-DIMENSIONAL, NON-BUOYANT AND BUOYANT JETS
IN A CROSS CURRENT

<table>
<thead>
<tr>
<th>Specifications</th>
<th>British Unit</th>
<th>SI Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water Body Length</td>
<td>600 ft</td>
<td>182.88 m</td>
</tr>
<tr>
<td>Water Body Width</td>
<td>360 ft</td>
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<td>Water Body Depth</td>
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<tr>
<td>Jet Width, $2d_0$</td>
<td>100 ft</td>
<td>30.48 m</td>
</tr>
<tr>
<td>Jet Depth</td>
<td>5.75 ft</td>
<td>1.75 m</td>
</tr>
<tr>
<td>Jet Velocity, $U_0$</td>
<td>2 ft/sec</td>
<td>0.61 m/sec</td>
</tr>
<tr>
<td>River Velocity</td>
<td>0.4 ft/sec</td>
<td>0.12 m/sec</td>
</tr>
<tr>
<td>Water Body Temperature</td>
<td>75 °F</td>
<td>23.89 °C</td>
</tr>
<tr>
<td>Water Body Equilibrium Temperature</td>
<td>74 °F</td>
<td>23.33 °C</td>
</tr>
<tr>
<td>Thermal Discharge Temperature, $T_0$</td>
<td>90 °F</td>
<td>32.22 °C</td>
</tr>
<tr>
<td>Reference Time, $t_0$</td>
<td>25 sec</td>
<td>25 sec</td>
</tr>
<tr>
<td>Reference Density, $\rho_0$</td>
<td>62.1156 lbm ft$^{-3}$</td>
<td>962.730 kg m$^{-3}$</td>
</tr>
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<td>14672.0 kg m$^{-2}$</td>
</tr>
<tr>
<td>Reynolds Number, $R_0$</td>
<td>$0.1212 \times 10^8$</td>
<td>$0.1212 \times 10^8$</td>
</tr>
<tr>
<td>Heat Exchange, Coefficient, $k$</td>
<td>100 Btu/ft$^2$ day °F</td>
<td>23.64 W/m$^2$ °C</td>
</tr>
</tbody>
</table>
discharge is made equal to the water body temperature to simulate a non-buoyant jet. A three-dimensional grid was superimposed on the flow field as detailed in Table 7. The boundaries of this grid coincide with the physical river boundaries. The nodal points, where hydrothermal variables are defined, are located at the point \( i,j,k \) of each grid and are shown in Fig. 7.

The time histories of the dynamic variables are shown, for points A, B, C, D, E, F, G, and H marked on Fig. 7, as follows:

1) Velocity components and water-level for two nodes A and B in front of the incoming jet at the free surface shown in Figs. 8 and 9.

2) Velocity components for two nodes C and D in front of the incoming jet at 12.75 feet below the free surface shown in Figs. 10 and 11.

3) Velocity components and water-level for two nodes E and F located upstream and downstream respectively at the free surface shown in Figs. 12 and 13.

4) Velocity components of two nodes G and H located upstream and downstream respectively at 12.75 feet below the free surface shown in Figs. 14 and 15.

Examination of the above plots shows the following features:

Considering the flow at the water surface, the transversal velocities at points A and B, in front of the incoming jet, rise initially with a subsequent rise in the vertical velocity leading to a rise in water-level in the neighborhood of the incoming jet which
<table>
<thead>
<tr>
<th>Element No.</th>
<th>X (ft/m)</th>
<th>Y (ft/m)</th>
<th>Z (ft/m)</th>
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</thead>
<tbody>
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<td>65/19.81</td>
<td>50/15.24</td>
<td>3.5/1.07</td>
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<td>60/18.29</td>
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<td>55/16.76</td>
<td>55/16.76</td>
<td>5.0/1.52</td>
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<td>55/16.76</td>
<td>70/21.34</td>
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<td></td>
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<tr>
<td>9</td>
<td>70/21.34</td>
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</tr>
<tr>
<td>10</td>
<td>70/21.34</td>
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</table>
FIG 7. GRID WORK WITH VARIABLE MESH SIZE SUPERIMPOSED ON THE WATER BODY
FIG 8. TIME HISTORIES OF VARIABLES AT POINT "A" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS.
FIG 9. TIME HISTORIES OF VARIABLES AT POINT "B" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS
FIG 10. TIME HISTORIES OF VARIABLES AT POINT "C" IN THE WATER BODY SUBJECT TO NON-BUOYANT AND BUOYANT JETS
FIG II. TIME HISTORIES OF VARIABLES AT POINT "D" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS
FIG 12. TIME HISTORIES OF VARIABLES AT POINT "E" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS
Figure 13. Time histories of variables at point "F" in the water body subjected to non-buoyant and buoyant jets.
FIG 14. Time histories of variables at point "G" in the water body subjected to non-buoyant and buoyant jets.
FIG 15. TIME HISTORIES OF VARIABLES AT POINT "H" IN THE WATER BODY SUBJECTED TO NON-BUOYANT AND BUOYANT JETS
in turn affects the field flow conditions as time progresses. Since the total head associated with the jet and river flow entries are constant on a short term basis, an increase in the water-level must be accomplished with a corresponding decrease in the field velocity. The disturbance caused by the incoming jet travel from the discharge point both upstream and downstream of the river. This demonstrates the propagation of the surface gravity waves (surge waves). As time progresses, these disturbances reach the river exit and are reflected back upstream. This demonstrates the reflection of the surface gravity waves from the river exit. However, on a long term basis, a further increase in the water-level in the neighborhood of the incoming jet causes additional horizontal velocity components for further downstream points which will gradually affect the downstream flow. These flow patterns are observed both in axial and transversal direction as discussed hereunder:

a) The increase in the water-level initially decelerates the axial flow on a short term basis. With a further increase in the water-level, the axial flow at various points accelerates to their final steady state values which are larger for the downstream points and smaller for the upstream points from the incoming jet position as expected. These effects can be clearly observed in \( \frac{u}{u_0} \) curves for points E, B, and F, in Figs. 12, 9, and 13, where the transient start with a small dip in the flow rate followed by a steep rise settling to its final steady state value.

b) Similarly, the transversal flow generated by the incoming jet, increases the water-level, which in turn, decelerates the trans-
verseal flow. However, since, unlike the axial flow patterns, the boundary condition in the transversal direction (solid wall) does not accommodate flow, the transversal flow reaches its final steady state value without undergoing large swings observed in the axial flow curves. These effects can be clearly observed in $v/U_0$ curves for points A and B in Figs. 8 and 9, where the transversal flow after the initial rise undergoes a reduction in magnitude followed by a mild rise settling to its final steady state values.

At the end of transients discussed above, the increase in transversal flow caused by the incoming jet is accommodated by an increase in the axial flow. For this reason, the water-level reaches a steady state value which in turn results in zero vertical velocity and constant axial and transversal flow, observed in Figs. 8 and 9, as the end of non-buoyant analysis is approached.

Examining the flow at 12.75 feet below the water surface, the axial flow exhibit a pattern similar to that of the water surface except that the results are further accentuated due to the bottom boundary condition. Figs. 11, 14 and 15 show that the flow transients $u/U_0$ start with a relatively large dip followed by relatively smaller rise as it approaches its final steady state value. The effect of the non-slip bottom boundary condition is to reduce the final steady state value to a quantity below that of the surface and to enlarge the initial dip. Furthermore, the shear initiated by the incoming jet creates transversal flow components $v/U_0$ at this level.
as observed in Figs. 10 and 11. The transversal flow reduces from the top to the bottom. At the end of transients discussed above, the increase in transversal flow caused by the incoming jet cannot be fully accommodated by an increase in the axial flow in cells near the solids boundaries. For this reason, a downward velocity component develops which induces a vertical downward velocity in the upper cells, observed in $w/U_0$ curve in Fig. 10. This downward flow extends in a few cells from the jet entrance and is changed to an upwards flow in cells in the center of the flow field.

5.4 Three Dimensional Buoyant Jet in a Cross Current

The problem of three-dimensional buoyant jet in a cross current is next analyzed. The geometric and hydraulic input data for this case is exactly similar to the non-buoyant jet case detailed in Fig. 7 and Table 6. The water body is assumed to be initially under the steady state conditions reached in the non-buoyant case discussed earlier when the thermal discharge temperature is suddenly increased from 75° to 90°F.

The dynamic buoyant jet problem is analyzed in a manner similar to the non-buoyant jet case. However, in view of the introduction of thermal effects, the water body becomes stratified as discussed hereunder. The time histories of the dynamic variables are shown, for points A, B, C, D, E, F, G, and H marked in Fig. 7, as a continuation of the non-buoyant time history plots in Figs. 8 to 15. Examination
of these plots shows the following features:

From the circulation viewpoint, the effect of the heated thermal discharge entering the flow field can be decomposed into two components: 1) unheated discharge entering the flow field studied in the non-buoyant case problem; and 2) thermal effect of the discharge considered as a heated wall studied earlier under the topic of ponds partially heated from the side. According to the above decomposition, the velocity time history plots in Figs. 8 to 15 for buoyant case should be the sum of the non-buoyant case and the corresponding heated wall problem.

Examining the flow at the water surface at points A and B, in front of the incoming jet, the transversal velocity for the buoyant case shows a small increase as compared with the non-buoyant results due to the natural circulation caused by the heated wall. This additional transversal velocity induces an axial velocity transient similar in nature to the non-buoyant case. However, since the magnitude of the transversal velocity transient is small, the resultant axial velocity transient would also be small. This behavior can be clearly observed in \( \frac{u}{U_0} \) and \( \frac{v}{U_0} \) curves in Figs. 8 and 9 which also show the temperature transients \( \frac{T}{T_0} \) for both points A and B. A similar pattern can also be observed at the surface points E and F in Figs. 12 and 13 as well as the points C, G, and H located at 12.75 feet below the water surface in Figs. 10, 14 and 15.

At the end of the transients, the coldest and the hottest regions are at the river and the thermal discharge entrances respectively.
For this reason, the strongest natural circulation patterns would develop mainly between these coldest and hottest regions. A comparison of the surface velocity vectors at points A and E and the velocity vectors at 12.75 feet depth for buoyant and non-buoyant cases confirm the existence of this natural circulation pattern. The surface velocity field at the end of the transient is shown in Fig. 16. The slowing down of the inlet flow near the thermal discharge entrance and the turning of the thermal discharge and incoming flows as a result of their interaction can be clearly seen in this figure.

The thermal plume effect is shown in Figs. 17 to 22. Isotherms are plotted for the water surface as well as the vertical planes in the longitudinal and transversal direction at times 60 and 500 seconds after the initiation of the heated discharge. An examination of these plots indicates the following features:

1) The stratification pattern is three dimensional in nature and shows the expansion of the thermal plume in all directions. In view of the prevailing advective currents this expansion is much more intense toward the downstream as compared to the other directions (vertical and upstream) as expected.

2) The three distinctive regions which constitute the characteristic behavior of a stratified water body (epilimnion, thermocline and hypolimnion) can be clearly observed in Fig. 22.

3) These results provide the water temperature rise and the rate of temperature rise needed for the assessment of the extent of thermal pollution in the water body.
FIG 16. SURFACE VELOCITY FIELD AT t=500 SEC. IN THE WATER SUBJECTED TO A BUOYANT JET
FIG 17. SURFACE ISOTHERMS AT $t = 60$ SEC. IN THE WATER BODY SUBJECTED TO A BUOYANT JET
FIG 18. VERTICAL ISOOTHERMS AT $y = 100$ FT. AND $t = 60$ SEC. IN THE WATER BODY SUBJECTED TO A BUOYANT JET
FIG 19. VERTICAL ISOHERMS AT X = 222.5 FT. AND
 t = 60 SEC. IN THE WATER BODY SUBJECTED
 TO A BUOYANT JET
FIG 20. SURFACE ISOTHERMS AT $t = 500$ SEC. IN THE WATER BODY SUBJECTED TO A BUOYANT JET.
FIG 21. VERTICAL ISOThERMS AT y = 100 FT. AND t = 500 SEC. IN THE WATER BODY SUBJECTED TO A BUOYANT JET
FIG 22. VERTICAL ISOTHERMS AT X=222.5 FT. AND t=500 SEC. IN THE WATER BODY SUBJECT TO A BUOYANT JET.
As a further verification of the validity of the results, a mass and energy balance was performed on the water body as shown in Table 8. This table shows that conservation equations are satisfied with a good accuracy. The small magnitude of the surface heat transfer verifies the often used assumption that, at high values of equilibrium temperature used herein for the purpose of near field studies, the contribution of heat exchange to the atmosphere is insignificant and may be neglected in simplified analysis.
<table>
<thead>
<tr>
<th>Mass and Energy Balance</th>
<th>Incoming Flow</th>
<th>Thermal Discharge</th>
<th>Outgoing Flow</th>
<th>Surface Flow</th>
<th>% Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass Balance,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>in $10^6$ lbm/sec</td>
<td>0.12252</td>
<td>0.08464</td>
<td>0.20748</td>
<td>0.0</td>
<td>0.15</td>
</tr>
<tr>
<td>Energy Balance,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>in $10^8$ Btu/sec</td>
<td>0.09189</td>
<td>0.07454</td>
<td>0.16204</td>
<td>0.15096</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$x 10^{-4}$</td>
<td></td>
</tr>
</tbody>
</table>
6. CONCLUSIONS AND RECOMMENDATIONS

An improved three-dimensional analytical model for the mathematical description of a large rectangular water body subjected to a thermal discharge has been developed and presented herein. The improvement results from the facts that:

A) Compliance with the conservation of mass and momentum is obtained by the introduction of two flow regions in the entire flow field: a) the water-level region containing a portion of water near the free surface in which the water-level rises or falls, as the case may be, during the dynamic solution; and b) the sub-water-level region located under the water-level region which remains totally filled with fluid at all times during the transient.

B) A different set of differential equations for the conservation of mass, momentum, and energy is applied to the cells located in the water-level and sub-water-level flow regions.

C) The pressure distribution is obtained accurately by combining the momentum and continuity equations and not by the hydrostatic approximation.

D) A detailed numerical stability analysis is performed which provides accurate criteria for the selection of the space and time increments.

The analysis, programmed on a digital computer, is generally capable of predicting velocity, pressure and temperature distribution
in water bodies subjected to thermal discharge. Particularly, the present study can be employed to determine the temperature rise and rate of temperature rise needed for the assessment of the extent of thermal pollution in water bodies.

Further studies should be undertaken with the following objectives:

a) To develop a three-dimensional analytical model for the mathematical description of actual rivers with arbitrary bottom and side configurations.

b) To develop accurate correlations for momentum and thermal eddy diffusivities and to incorporate these correlations in the digital program.

c) To provide an experimental verification for the thermal discharge problem.
a. Terms of matrix involved in the solution of the pressure equations

$a_x, a_y, a_z, b_x, b_y, b_z, c_x, c_y, c_z$  Weighting factors for derivatives

$C_p$  Specific heat of water

$C_{fx}$  Skin coefficient along x-axis

$C_{fy}$  Skin coefficient along y-axis

$d_0$  Half width of thermal discharge

$d_x, d_y, d_z$  Thermal eddy diffusivities

$D_x, D_y, D_z$  Sum of thermal conductivity plus thermal eddy diffusivity as defined by equations (A-19)

$D_h$  Horizontal eddy diffusivity of heat

$D_v$  Vertical eddy diffusivity of heat

$e_x, e_y, e_z, f_x, f_y, f_z$  Weighting factors for pressure

$E$  Equilibrium temperature of water

$F_0$  Froude number

$g$  Gravitational acceleration

$H$  Water-level height

$K$  Heat exchange coefficient for water-air interface

$n$  Normal to the solid boundary

$p$  Pressure

$R_0$  Reynolds number

$S_0$  Stanton number

$T_0$  Thermal discharge temperature

$T$  Local water temperature

$t$  Time
NOMENCLATURE (Cont'd)

\( U_0 \)  Thermal discharge velocity
\( u \)  Horizontal velocity components in x-direction
\( v \)  Horizontal velocity components in y-direction
\( w \)  Vertical velocity components in z-direction
\( W_x \)  Wind velocity components in x-direction
\( W_y \)  Wind velocity components in y-direction
\( x, y, z \)  Cartesian coordinate system
\( \Delta x_i \)  Dimension of element \( i \) in x-direction
\( \Delta y_j \)  Dimension of element \( j \) in y-direction
\( \Delta z_k \)  Dimension of element \( k \) in z-direction

Greek Symbols
\( \alpha_x, \alpha_y, \alpha_z \)  Momentum eddy diffusivities
\( \alpha, \beta, \gamma, \delta \)  Coefficients in Von-Neumann stability analysis
\( \Delta \)  A quantity defined by equation (99)
\( \xi \)  A quantity defined by equation (66b)
\( \lambda^2 \)  Norm ratio
\( \nu \)  Viscosity
\( \nu_0 \)  Kinematic viscosity of thermal discharge
\( \nu_x, \nu_y, \nu_z \)  Sum of viscosity and momentum eddy diffusivity as defined by equation (A-19)
\( \nu_h \)  Horizontal eddy viscosity
\( \nu_v \)  Vertical eddy viscosity
\( \rho \)  Local density of water
Greek Symbols (Cont'd)

\( \rho_a \)  Air density
\( \rho_0 \)  Thermal discharge density
\( \omega \)  Relaxation factor
\( \omega_{opt} \)  Optimum value of relaxation factor

\( \nabla^2 \)  Laplacian operator \( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \)

Superscripts

*  Refers to non-dimensional quantities
n  Refers to updated value
.  Refers to a small variation
-  Refers to time average
'  Refers to fluctuating components used in Appendix 1 and to steady-state components used in Appendix 2

Subscripts

0  Refers to thermal discharge
i,j,k  Refers to element numbers
max  Refers to maximum
s  Refers to surface
rm  Refers to components of matrix "a"


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APPENDIX 1. Derivation of Governing Equation (1) through (15)

To derive the governing equations for the sub-water-level region, the Navier-Stokes equations for incompressible flow (31) are used in conjunction with the Boussinesq's approximation which neglects the fluid density variation in all terms involved in the momentum equations except in the body force term as follows:

Momentum Equations

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = - \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \nu \nabla^2 u \tag{A-1}
\]

\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = - \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \nu \nabla^2 v \tag{A-2}
\]

\[
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = - \frac{1}{\rho_0} \frac{\partial p}{\partial z} + \nu \nabla^2 w - \frac{\rho g}{\rho_0} \tag{A-3}
\]

Energy Equation

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = k \nabla^2 T \tag{A-4}
\]

Continuity Equation

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \tag{A-5}
\]

In describing a turbulent flow in mathematical terms, it is convenient to separate the flow into a mean motion and a fluctuating, or eddying motion. Denoting the time-average of the u-component of
velocity by $\bar{u}$ and its velocity fluctuation by $u'$ and using a similar notation for the other variables, one can express the following relations for the velocity components, temperature and pressure:

$$u=\bar{u} + u'; \; v=\bar{v} + v'; \; w=\bar{w} + w'; \; T=\bar{T} + T'; \; p=\bar{p} + p' \quad (A-6)$$

By definition, the time-average of all fluctuating quantities is equal to zero. For example,

$$\bar{u}' = 0, \; \bar{v}' = 0, \; \bar{w}' = 0, \; \bar{T}' = 0, \; \bar{p}' = 0; \; \frac{\partial \bar{u}'}{\partial x} = 0, \quad (A-7)$$

Upon forming the time-average of equations (A-1) through (A-5), one obtains

$$\frac{\partial \bar{u}}{\partial t} + \bar{u} \frac{\partial \bar{u}}{\partial x} + \bar{v} \frac{\partial \bar{u}}{\partial y} + \bar{w} \frac{\partial \bar{u}}{\partial z} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial x} + \nu \frac{\partial^2 \bar{u}}{\partial y^2} - \left( \frac{\partial \bar{u}^2}{\partial x} + \frac{\partial \bar{v} \bar{u}'}{\partial y} + \frac{\partial \bar{w} \bar{u}'}{\partial z} \right) \quad (A-8)$$

$$\frac{\partial \bar{v}}{\partial t} + \bar{u} \frac{\partial \bar{v}}{\partial x} + \bar{v} \frac{\partial \bar{v}}{\partial y} + \bar{w} \frac{\partial \bar{v}}{\partial z} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial y} + \nu \frac{\partial^2 \bar{v}}{\partial z^2} - \left( \frac{\partial \bar{u} \bar{v}'}{\partial x} + \frac{\partial \bar{v}^2}{\partial y} + \frac{\partial \bar{v} \bar{v}'}{\partial z} \right) \quad (A-9)$$

$$\frac{\partial \bar{w}}{\partial t} + \bar{u} \frac{\partial \bar{w}}{\partial x} + \bar{v} \frac{\partial \bar{w}}{\partial y} + \bar{w} \frac{\partial \bar{w}}{\partial z} = -\frac{1}{\rho_0} \frac{\partial \bar{p}}{\partial z} + \nu \frac{\partial^2 \bar{w}}{\partial x^2} - \left( \frac{\partial \bar{u} \bar{w}'}{\partial x} + \frac{\partial \bar{v} \bar{w}'}{\partial y} + \frac{\partial \bar{w}^2}{\partial z} \right) - \frac{\rho g}{\rho_0} \quad (A-10)$$

$$\frac{\partial \bar{T}}{\partial t} + \bar{u} \frac{\partial \bar{T}}{\partial x} + \bar{v} \frac{\partial \bar{T}}{\partial y} + \bar{w} \frac{\partial \bar{T}}{\partial z} = k \nu \frac{\partial^2 \bar{T}}{\partial x^2} - \left( \frac{\partial \bar{u} \bar{T}'}{\partial x} + \frac{\partial \bar{v} \bar{T}'}{\partial y} + \frac{\partial \bar{w} \bar{T}'}{\partial z} \right) \quad (A-11)$$

$$\frac{\partial \bar{u}}{\partial x} + \frac{\partial \bar{v}}{\partial y} + \frac{\partial \bar{w}}{\partial z} = 0 \quad (A-12)$$
Employing the eddy diffusivity concept,

\[ -\ddot{u}^2 = \alpha_x \frac{\partial \ddot{u}}{\partial x} + \alpha_x \frac{\partial \ddot{u}}{\partial x} \]

\[ -\ddot{u}' \ddot{v}' = \alpha_y \frac{\partial \ddot{u}}{\partial y} + \alpha_y \frac{\partial \ddot{v}}{\partial y} \]

\[ -\ddot{u}' \ddot{w}' = \alpha_z \frac{\partial \ddot{u}}{\partial z} + \alpha_z \frac{\partial \ddot{w}}{\partial z} \]

\[ -\ddot{v}^2 = \alpha_y \frac{\partial \ddot{v}}{\partial y} + \alpha_y \frac{\partial \ddot{v}}{\partial y} \]

\[ -\ddot{v}' \ddot{u}' = \alpha_x \frac{\partial \ddot{v}}{\partial x} + \alpha_x \frac{\partial \ddot{u}}{\partial x} \]

\[ -\ddot{v}' \ddot{w}' = \alpha_z \frac{\partial \ddot{v}}{\partial z} + \alpha_y \frac{\partial \ddot{w}}{\partial y} \]

\[ -\ddot{w}^2 = \alpha_z \frac{\partial \ddot{w}}{\partial z} + \alpha_z \frac{\partial \ddot{w}}{\partial z} \]

\[ -\ddot{w}' \ddot{u}' = \alpha_x \frac{\partial \ddot{w}}{\partial x} + \alpha_z \frac{\partial \ddot{u}}{\partial z} \]

\[ -\ddot{w}' \ddot{v}' = \alpha_y \frac{\partial \ddot{w}}{\partial y} + \alpha_z \frac{\partial \ddot{v}}{\partial z} \]

\[ -\ddot{w}' \ddot{w}' = \alpha_z \frac{\partial \ddot{w}}{\partial z} + \alpha_z \frac{\partial \ddot{w}}{\partial z} \]

\[ -\ddot{u}' \dddot{T}' = d_x \frac{\partial \dddot{T}}{\partial x} \]

\[ -\ddot{v}' \dddot{T}' = d_y \frac{\partial \dddot{T}}{\partial y} \]

\[ -\ddot{w}' \dddot{T}' = d_z \frac{\partial \dddot{T}}{\partial z} \]

(A-13)

after substituting equations (A-13) into equations (A-8) through (A-11), one obtains the following equations for incompressible turbulent flow:
\[
\begin{align*}
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} &= - \frac{1}{\rho_0} \frac{\partial p}{\partial x} + \nu_x \frac{\partial^2 u}{\partial x^2} + \nu_y \frac{\partial^2 u}{\partial y^2} + \nu_z \frac{\partial^2 u}{\partial z^2} \\
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} &= - \frac{1}{\rho_0} \frac{\partial p}{\partial y} + \nu_x \frac{\partial^2 v}{\partial x^2} + \nu_y \frac{\partial^2 v}{\partial y^2} + \nu_z \frac{\partial^2 v}{\partial z^2} \\
\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial x} + v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} &= - \frac{1}{\rho_0} \frac{\partial p}{\partial z} + \nu_x \frac{\partial^2 w}{\partial x^2} + \nu_y \frac{\partial^2 w}{\partial y^2} + \nu_z \frac{\partial^2 w}{\partial z^2} - \frac{\rho 
abla \times \mathbf{v}}{\rho_0} \\
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} &= D_x \frac{\partial^2 T}{\partial x^2} + D_y \frac{\partial^2 T}{\partial y^2} + D_z \frac{\partial^2 T}{\partial z^2} \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0
\end{align*}

where

\[\nu_x = v + \alpha_x\]
\[\nu_y = v + \alpha_y\]
\[\nu_z = v + \alpha_z\]
\[D_x = k + \alpha_x\]
\[D_y = k + \alpha_y\]
\[D_z = k + \alpha_z\] (A-19)
In the present analysis, the horizontal components of diffusivities are considered to be equal, i.e.,

\[ \nu_h = \nu_x = \nu_y \]

\[ \nu_v = \nu_z \]

\[ D_h = D_x = D_y \]

\[ D_v = D_z \] \hspace{1cm} (A-20)

The magnitude of momentum eddy viscosities and thermal eddy diffusivities \((\alpha_x, \alpha_y, \alpha_z, d_x, d_y, d_z)\) are usually so large in comparison with their molecular counterparts \((\nu, k)\) that the contributions of the latter are neglected. For practical calculations, it is satisfactory to assume that the horizontal and vertical momentum eddy viscosities are equal to the horizontal and vertical thermal eddy diffusivities respectively such that \((14, 5, 13)\)

\[ \nu_h = D_h \]

\[ \nu_v = D_v \] \hspace{1cm} (A-21)

Liggett \((19)\), Wada \((39)\), and Lerman \((17)\) are among the recent investigators of eddy diffusivity concept. Based on their studies, the following average values for the eddy viscosities in the horizontal and vertical directions, \(\nu_h\) and \(\nu_v\), are used in this analysis

\[ \nu_h = 1350 \text{ ft}^2/\text{sec} \]

\[ \nu_v = .4 \text{ ft}^2/\text{sec} \]
Considering the above simplifications, equations (A-14) through (A-18) become identical with equations (1) through (5) except that, for convenience, the superscript (-) has been dropped in the latter equations.
APPENDIX 2. Verification of Stability Analysis

The purpose of this appendix is to demonstrate the validity of the rule which applies to an explicit computational scheme as discussed in Section 4. This rule is verified by developing the stability criteria for the energy equation (5) using the two following methods:

a) Setting the coefficients multiplying each of the temperature values equal to a positive quantity (3)

b) Von-Neumann stability analysis (1)

In order to apply stability analysis to a nonlinear equation, the equation must be first linearized. To linearize the equation, one assumes that the temperature $T$ consists of a steady-state component $T'$ and a small variation about steady-state $\dot{T}$ such that

$$T = T' + \dot{T}$$

By substituting these quantities in the energy equation, and considering the velocity components constant over the integration time step, one will obtain the following linearized equation:

$$\frac{\dot{T}}{\partial t} = -u \frac{\partial \dot{T}}{\partial x} - v \frac{\partial \dot{T}}{\partial y} - w \frac{\partial \dot{T}}{\partial z} + D_h \left( \frac{\partial^2 \dot{T}}{\partial x^2} + \frac{\partial^2 \dot{T}}{\partial y^2} \right) + D_v \frac{\partial^2 \dot{T}}{\partial z^2} \quad (B-1)$$

where superscript dot (·) indicates linearized quantities. By applying a forward two points finite difference approximation to the temperature time derivative and centered two and three points finite difference approximations to the first and second spatial differences respectively, and denoting $\dot{T}_{i,j,k}$ by $\ddot{T}$ for convenience, one obtains
\[
\frac{\dot{T}_n - \dot{T}}{\Delta t} = -u \frac{\dot{T}_{i+1} - \dot{T}_{i-1}}{2\Delta x} - v \frac{\dot{T}_{j+1} - \dot{T}_{j-1}}{2\Delta y} - w \frac{\dot{T}_{k+1} - \dot{T}_{k-1}}{2\Delta z} + \dot{T}_{i+1} - 2\dot{T}_{i} + \dot{T}_{i-1} \\
+ D_h \frac{\dot{T}_{i+1} - 2\dot{T}_{i} + \dot{T}_{i-1}}{\Delta x^2} + D_v \frac{\dot{T}_{k+1} - 2\dot{T}_{k} + \dot{T}_{k-1}}{\Delta x^2}
\]

(B-2)

Solving for \( \dot{T} \)

\[
\dot{T} = \dot{T}_{i-1} \left( \frac{u \Delta t}{2\Delta x} + \frac{D_h \Delta t}{\Delta x^2} \right) + \dot{T} \left( 1 - \frac{2\Delta t D_h}{\Delta x^2} - \frac{2\Delta t D_v}{\Delta y^2} - \frac{2\Delta t D_v}{\Delta z^2} \right) \\
+ \dot{T}_{i+1} \left( - \frac{u \Delta t}{2\Delta x} + \frac{D_h \Delta t}{\Delta x^2} \right) + \dot{T}_{j-1} \left( \frac{v \Delta t}{2\Delta y} + \frac{D_v \Delta t}{\Delta y^2} \right) \\
+ \dot{T}_{j+1} \left( - \frac{v \Delta t}{2\Delta y} + \frac{D_v \Delta t}{\Delta y^2} \right) + \dot{T}_{k-1} \left( \frac{w \Delta t}{2\Delta z} + \frac{D_w \Delta t}{\Delta z^2} \right) + \dot{T}_{k+1} \left( - \frac{w \Delta t}{2\Delta z} + \frac{D_w \Delta t}{\Delta z^2} \right)
\]

(B-3)

a) Employing the rule of positive coefficients, which applies to explicit computational scheme discussed previously, the following stability criteria will result from coefficient of \( \dot{T}_{i+1}, \dot{T}_{i-1} \)

\[
\Delta x \leq \frac{2D_h}{|u|} \quad \text{(B-4)}
\]

Similarly, from \( \dot{T}_{j+1}, \dot{T}_{j-1}, \dot{T}_{k+1}, \dot{T}_{k-1} \) coefficients, one obtains

\[
\Delta y \leq \frac{2D_h}{|v|} \quad \Delta z \leq \frac{2D_v}{|w|} \quad \text{(B-5)}
\]
Finally, from the coefficient of \( \dot{T} \), the following temporal condition results

\[
\Delta t \leq \frac{1}{(2D_h/\Delta x^2 + 2D_h/\Delta y^2 + 2D_v/\Delta z^2)} \tag{B-6}
\]

If \( D_h = D_v \) and \( \Delta x = \Delta y = \Delta z \), the expression for \( \Delta t \) simplifies to

\[
\Delta t \leq \frac{\Delta x^2}{6D_h} \tag{B-7}
\]

b) Von-Neumann stability analysis.

According to the Von-Neumann stability analysis approach (1)

\[
T^n = e^{\delta t} e^{i\alpha x} e^{i\beta y} e^{i\gamma z} \tag{B-8}
\]

where

\[
x = \sum_{x=1}^{\lambda} \Delta x_x
\]

\[
y = \sum_{m=1}^{m} \Delta y_m
\]

\[
z = \sum_{p=1}^{p} \Delta z_p
\]

\[
t = n\Delta t \tag{B-9}
\]

Substituting the above quantities into equation (B-3), and dividing by \( e^{\delta t} e^{i\alpha x} e^{i\beta y} e^{i\gamma z} \), the following results:

\[
e^{\delta \Delta t} = \left( \frac{\dot{u} x + \frac{D_h \Delta t}{\Delta x^2}}{2} \right) e^{i\alpha \Delta t} + \left( - \frac{\dot{u} x + \frac{D_v \Delta t}{\Delta x^2}}{2} \right) e^{i\alpha \Delta x}
\]
\[ + \left( \frac{\Delta t}{2\Delta y} + \frac{D_h \Delta t}{\Delta y^2} \right) e^{i\beta \Delta y} + \left( - \frac{\Delta t}{2\Delta y} + \frac{D_v \Delta t}{\Delta y^2} \right) e^{i\beta \Delta y} \]

\[ + \left( \frac{\Delta t}{2\Delta z} + \frac{D_v \Delta t}{\Delta z^2} \right) e^{i\gamma \Delta z} + \left( - \frac{\Delta t}{2\Delta z} + \frac{D_h \Delta t}{\Delta z^2} \right) e^{i\gamma \Delta z} \]

\[ + \left( 1 - \frac{2\Delta t D_h}{\Delta x^2} - \frac{2\Delta t D_h}{\Delta y^2} - \frac{2\Delta t D_v}{\Delta z^2} \right) \]  

\[ (B-10) \]

For the above equation to be numerically stable, the absolute value of \( e^{\Delta t} \) must be set less than or equal to one. This leads to

\[ (1 - 4 \frac{\Delta t D_h}{\Delta x^2} \sin^2 \frac{\alpha \Delta x}{2} - 4 \frac{\Delta t D_h}{\Delta y^2} \sin^2 \frac{\beta \Delta y}{2} - 4 \frac{\Delta t D_v}{\Delta z^2} \sin^2 \frac{\alpha \Delta z}{2})^2 \leq 1 \]  

\[ (B-11) \]

\[ \left( \frac{\Delta t}{\Delta x} \sin \alpha \Delta x + \frac{\Delta t}{\Delta y} \sin \alpha \Delta y + \frac{\Delta t}{\Delta z} \sin \alpha \Delta z \right)^2 \leq 1 \]  

\[ (B-12) \]

Expanding and assuming that \( \Delta x = \Delta y = \Delta z \), and \( D_h = D_v \), the following will be obtained:

\[ 1 + \left( 144 \frac{\Delta t^2 D_h^2}{\Delta x^4} - 36 \frac{u^2 \Delta t^2}{\Delta x^2} \sin^4 \frac{\alpha \Delta x}{2} + (36 \frac{u^2 \Delta t^2}{\Delta x^2} - 24 \frac{\Delta t D_h}{\Delta x^2}) \sin^2 \frac{\alpha \Delta x}{2} \right) \leq 1 \]

\[ (B-13) \]

In order for the above inequality to be less or equal one, the following inequalities must hold.

\[ 144 \frac{\Delta t^2 D_h}{\Delta x^4} - 36 \frac{u^2 \Delta t^2}{\Delta x^2} \leq \epsilon^2 \]  

\[ (B-14) \]
\[ 36 \frac{u^2 \Delta t^2}{\Delta x^2} - 24 \frac{\Delta D_h}{\Delta x^2} = - \epsilon^2 \]  

where \( \epsilon^2 \) is very small number. From equation (B-14), the spatial stability criterion may be written as

\[ 144 \frac{\Delta t^2 D_h^2}{\Delta x^4} \geq 36 \frac{u^2 \Delta t^2}{\Delta x^2} \]

or

\[ \Delta x \leq \frac{2D_h}{|u|} \]  

Adding equations (B-14) and (B-15), the upper limit for \( \Delta t \) becomes:

\[ \Delta t \leq \frac{\Delta x^2}{6D_h} \]  

These results are in agreement with those obtained in part (a) of this appendix.
APPENDIX 3. Calculation of the First and Second Derivatives at the Boundary and in the Field

The calculation of the first derivative in the flow field is based on a quadratic approximation. Let \( q(x) \) be defined by a parabolic equation

\[
q(x) = ax^2 + bx + c \quad (C-1)
\]

shown in Fig. 23. Values of the function at nodal points \( x_1, x_2, \) and \( x_3 \), are \( q_1, q_2, \) and \( q_3 \), respectively. Therefore

\[
\begin{pmatrix}
q_1 \\
q_2 \\
q_3
\end{pmatrix}
= \begin{pmatrix}
x_1^2 & x_1 & 1 \\
x_2^2 & x_2 & 1 \\
x_3^2 & x_3 & 1
\end{pmatrix}
\begin{pmatrix}
a \\
b \\
c
\end{pmatrix}
\quad (C-2)
\]

combining equations (C-1) and (C-2) yields

\[
q(x) = \{x^2 \ x \ 1\} \begin{pmatrix}
x_1^2 & x_1 & 1 \\
x_2^2 & x_2 & 1 \\
x_3^2 & x_3 & 1
\end{pmatrix}^{-1}
\begin{pmatrix}
q_1 \\
q_2 \\
q_3
\end{pmatrix}
\quad (C-3)
\]

\[\text{Nomenclature used in this Appendix is defined within the text and not in Section 7}\]
FIG. 23 CALCULATION OF FIRST AND SECOND DERIVATIVES
First Derivative at $x_1$

The derivative at $x_1$ is given by

$$\frac{dq(x)}{dx} \bigg|_{x=x_1} = \{2x_1 \ 1 \ 0\} \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix}^{-1} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \quad \text{(C-4)}$$

Inverting the matrix, yields

$$\frac{dq(x)}{dx} \bigg|_{x=x_1} = \{2x_1 \ 1 \ 0\} \begin{bmatrix} x_2 - x_3 & x_3 - x_1 & x_1 - x_2 \\ x_3^2 - x_2^2 & x_1^2 - x_3^2 & x_2^2 - x_1^2 \\ x_3x_2(x_2-x_3) & x_3x_1(x_3-x_1) & x_1x_2(x_1-x_2) \end{bmatrix} \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \quad \text{(C-5)}$$

After multiplication, one obtains

$$\frac{dq(x)}{dx} \bigg|_{x=x_1} = \frac{(x_3-x_2) (x_2+x_3-2x_1) q_1 - (x_3-x_1)^2 q_2 + (x_1-x_2)^2 q_3}{(x_2-x_1)(x_3-x_2)(x_1-x_3)} \quad \text{(C-6)}$$

In many instances the value of the first derivative is known at the boundary, i.e.,
\[
\frac{dq(x)}{dx} \bigg|_{x=x_1} = C \tag{C-7}
\]

For this boundary condition, a relationship between \( q_1 \), \( q_2 \), and \( q_3 \) may be obtained by first defining the element boundaries (dotted lines on Fig. 23) and observing that each nodal point is located at the midpoint between its boundary lines. Thus,

\[
x_2-x_1 = \frac{1}{2} (\Delta n_1 + \Delta n_2) \tag{C-8}
\]

\[
x_3-x_2 = \frac{1}{2} (\Delta n_2 + \Delta n_3) \tag{C-9}
\]

\[
x_1-x_3 = -\frac{1}{2} (\Delta n_1 + 2\Delta n_2 + \Delta n_3) \tag{C-10}
\]

Defining

\[
\Delta n_{12} = \Delta n_1 + \Delta n_2 \tag{C-11}
\]

and

\[
\sigma = \frac{\Delta n_1 + \Delta n_2}{\Delta n_2 + \Delta n_3} \tag{C-12}
\]

Combining the equations (C-8) through (C-12) gives the value of the derivative at the boundary.
\[
\frac{dq(x)}{dx} \bigg|_{x=x_1} = - \frac{2}{\Delta n_{12}} \left(1 + 2\sigma\right) q_1 - \left(1+\sigma\right)^2 q_2 + \sigma^2 q_3 \tag{C-13}
\]

or

\[
\frac{dq(x)}{dx} \bigg|_{x=x_1} = \frac{q_2 - q_1}{x_2 - x_1} \left(1 + 2\sigma\right) + \frac{q_3 - q_2}{x_3 - x_2} \left(-\frac{\sigma}{1+\sigma}\right) \tag{C-14}
\]

which means that the slope at \(x = x_1\) is equal to the weighted average of the slopes in intervals \(x_2 - x_1\) and \(x_3 - x_2\). It should be noted that for equally spaced increments, the slope in the interval \(x_2 - x_1\) is weighted by 1.5, while the slope in the interval \(x_3 - x_2\) is weighted by -0.5.

Using equations (C-7) and (C-14), yields a relationship between \(q_1\), \(q_2\), and \(q_3\)

\[
- \frac{2}{\Delta n_{12}} \left(1 + 2\sigma\right) q_1 - \left(1+\sigma\right)^2 q_2 + \sigma^2 q_3 = C \tag{C-15}
\]

from which the value of the variable at the boundary can be determined in terms of the values within the flow region.

\[
q_1 = \frac{1}{1+2\sigma} \left(\left(1+\sigma\right)^2 q_2 - \sigma^2 q_3 - \frac{1}{2} \Delta n_{12} \left(1+\sigma\right) C\right) \tag{C-16}
\]

First Derivative at \(x_2\)
The value of the first derivative in the field is calculated by differentiating equation (C-3) and setting \( x = x_2 \)

\[
\frac{dq(x)}{dx} \bigg|_{x=x_2} = (2x_2 \ 1 \ 0) \begin{bmatrix} x_1^2 & x_1 & 1 \\ x_2^2 & x_2 & 1 \\ x_3^2 & x_3 & 1 \end{bmatrix}^{-1} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}
\]

(C-17)

Inverting the matrix yields,

\[
\frac{dq(x)}{dx} \bigg|_{x=x_2} = (2x_2 \ 1 \ 0) \begin{bmatrix} x_2 - x_3 & x_3 - x_1 & x_1 - x_2 \\ x_2^2 - x_2^2 & x_1^2 - x_3^2 & x_2^2 - x_1^2 \\ x_3x_2(x_2-x_3) & x_3x_1(x_3-x_1) & x_1x_2(x_1-x_2) \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}
\]

(C-18)

After multiplication, one obtains

\[
\frac{dq(x)}{dx} \bigg|_{x=x_2} = \frac{(x_3-x_2)^2 q_1 + (x_3-x_1)(2x_2-x_3-x_1) q_2 - (x_2-x_1)^2 q_3}{(x_2-x_1)(x_3-x_2)(x_1-x_3)}
\]

(C-19)

Using relationships (C-8), (C-9), and (C-10), the above equation becomes
\[
\frac{dq(x)}{dx} \bigg|_{x=x_2} = -\frac{2}{\Delta n_{12}(1+\sigma)} \begin{bmatrix}
q_1 + (\sigma^2 - 1) q_2 - \sigma^2 q_3
\end{bmatrix}
\] (C-20)

or

\[
\frac{dq(x)}{dx} \bigg|_{x=x_2} = \frac{q_2 - q_1}{x_2 - x_1} \left(\frac{1}{1+\sigma}\right) + \frac{q_3 - q_2}{x_3 - x_2} \left(\frac{\sigma}{1+\sigma}\right)
\] (C-21)

which means that the slope at \(x=x_2\) is equal to the weighted average of the slopes in intervals \(x_2-x_1\) and \(x_3-x_2\). It should be noted that for equally spaced increments, the slope at \(x_2\) becomes equal to the mean value of the slopes in intervals \(x_2-x_1\) and \(x_3-x_2\). Furthermore, using equations (C-12), (C-10), (C-9), and (C-8), it can easily be shown that the weighting function can be expressed in terms of spatial differences as follows:

\[
\frac{1}{1+\sigma} = \frac{x_3-x_2}{x_3-x_1}
\] (C-22)

\[
\frac{\sigma}{1+\sigma} = \frac{x_2-x_1}{x_3-x_1}
\] (C-23)

First Derivative at \(x_3\)

The derivative at \(x_3\) is given by
\[ \frac{dq(x)}{dx} \bigg|_{x=x_3} = \begin{pmatrix} 2x_3 & 1 & 0 \\ x_1 & x_1 & 1 \\ x_2 & x_1 & 1 \\ x_3 & x_3 & 1 \end{pmatrix} \begin{bmatrix} x_2 - x_3 & x_3 - x_1 & x_1 - x_2 \\ x_3 - x_2 & x_1 - x_3 & x_2 - x_1 \\ x_3 x_2(x_2-x_3) & x_3 x_1(x_3-x_1) & x_1 x_2(x_1-x_2) \end{bmatrix} \begin{pmatrix} q_1 \\ q_2 \\ q_3 \end{pmatrix} \]

(C-24)

After multiplication, one obtains

\[ \frac{dq(x)}{dx} \bigg|_{x=x_3} = \frac{-(x_3-x_2)^2 q_1 + (x_3-x_1)^2 q_2 + (x_1-x_2) (x_3-x_2+x_3-x_1) q_3}{(x_2-x_3) (x_1-x_3) (x_1-x_2)} \]

(C-25)

Using relationships (C-8), (C-9), and (C-10) and redefining

\[ \Delta n_{23} = \Delta n_2 + \Delta n_3 \]

(C-26)

and

\[ \sigma' = \frac{\Delta n_3 + \Delta n_2}{\Delta n_1 + \Delta n_2} = \frac{1}{\sigma} \]

(C-27)

equation (C-25) becomes

\[ \frac{dq(x)}{dx} \bigg|_{x=x_3} = \frac{2}{\Delta n_{23}(1+\sigma')} \begin{bmatrix} (1+2\sigma') q_3 - (1+\sigma')^2 q_2 + \sigma'^2 q_1 \end{bmatrix} \]

(C-28)
or

\[
\frac{dq(x)}{dx} \bigg|_{x=x_3} = \frac{q_2-q_1}{x_2-x_1} \left( -\frac{\Delta n}{1+\Delta n} \right) + \frac{q_3-q_2}{x_3-x_2} \left( \frac{1+2\Delta n}{1+\Delta n} \right) \quad (C-29)
\]

**Second Derivative**

In a similar manner, the second derivative can be calculated by differentiating equation (C-1) twice with respect to x to obtain

\[
\frac{d^2 q(x)}{dx^2} = \frac{(x_2-x_3)q_1 + (x_3-x_1)q_2 + (x_1-x_2)q_3}{(x_2-x_1)(x_3-x_2)(x_1-x_3)} \quad (C-30)
\]

or

\[
\frac{d^2 q(x)}{dx^2} = \frac{1}{4(\Delta n_{12}+\Delta n_{23})} \left( \frac{q_3-q_2}{x_3-x_2} - \frac{q_2-q_1}{x_2-x_1} \right) \quad (C-31)
\]

It should be noted that:

1) The above equation is exactly identical with the central finite difference form for the second derivative. This is in contrast with the first derivative which introduced weighting factors.

2) The value of the second derivative is equal at positions \(x_1, x_2, \text{ or } x_3\).

Furthermore, the coefficient of the finite difference quotients on the right hand side of equation (C-31) in terms of spatial differences becomes
\[
\frac{1}{4 (\Delta n_{12} + \Delta n_{23})} = \frac{2}{x_3 - x_1}
\]  
\(\text{C-32}\)
PART TWO

USER'S MANUAL
1. DESCRIPTION OF THERMA DIGITAL COMPUTER PROGRAM

The program THERMA, developed for the analysis of three dimensional thermal stratification and circulation in water bodies subjected to thermal discharge, consists of a MAIN program and several subroutines as shown in Figs. 24 to 28. A tabular outline of the MAIN program and its subroutines together with their functions are presented below and followed by a more detailed description.

<table>
<thead>
<tr>
<th>Program</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAIN</td>
<td>The MAIN program reads the input data in, initializes the problem, calls the subroutines needed to solve the problem, and controls the termination of program.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subroutine</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>DERIV</td>
<td>This subroutine calculates spatial and time derivatives of variables as needed.</td>
</tr>
<tr>
<td>PRESS</td>
<td>This subroutine calculates the updated values of pressure.</td>
</tr>
<tr>
<td>SUM</td>
<td>This subroutine performs the time integration.</td>
</tr>
</tbody>
</table>
READ INPUT DATA

READ AND PRINT NON-SUBSCRIPTED VARIABLES
ENRUN, TBEG, TEND, ENITER, ENPRNT, UI, VI, TI, UO,
ANGLE, TO, DO, XNUZ, WINDX, WINDY, RHOA, DH, DV, XNUH,
XNUV, CP, EIMAX, EJMAX, EKMAX, CFX, CFY, E, XK, DEBUG1,
DEBUG2, DEBUG3, DEBUG4, EIRUN, EINDEX, XIIP, SLIP, HYD,
HEAT, EIM, EML, EMK

IMAX=EIMAX        JMAX=EJMAX
KMAX=EKMAX        NITER=ENITER
NPRNT=ENPRNT      XIIP=XIIP
IM=EIM            MK=EMK
ML=EML

FIG 24a FLOW CHART OF MAIN PROGRAM
READ INPUT DATA

READ AND PRINT SUBSCRIPTED VARIABLES

\[ \text{DELX}(i) \quad i=1, I_{\text{MAX}} \quad \text{DELY}(j) \quad j=1, J_{\text{MAX}} \]
\[ \text{DELZ}(k) \quad k=1, K_{\text{MAX}} \quad \text{TTBL}(l) \quad l=1, 26 \]
\[ \text{RHOSTBL}(l) \quad l=1, 26 \]

FIG 24b FLOW CHART OF MAIN PROGRAM
FIG 24c FLOW CHART OF MAIN PROGRAM
CALCULATE $\rho_0$

$\text{RHOZ} = S1(TTBL, RHOTBL, T0, 26)$

PRINT
$I, XIM1(I), XN(I), XM(I)$
$J, YJM1(J), YN(J), YM(J)$
$K, ZKM1(K), ZN(K), ZM(K)$
$\text{RHOZ}$

DO 706 J = 1, JMAX
DO 706 I = 1, IMAX

INITIALIZE WATER LEVEL

$H(I, J) = \text{DELZ(KMAX)} / 2.$

FIG 24d FLOW CHART OF MAIN PROGRAM
DO 613 K=2,K1
DO 613 J=2,J1
DO 613 1=2,11

CALCULATE ALLOWABLE DT FOR FLOW REGION

\[ DT = \frac{1}{2 \times \frac{x_n(i)}{X_{M1}(i)} + \frac{y_n(j)}{Y_{M1}(j)} + \frac{z_n(k)}{Z_{M1}(k)}} \]

\[ DTT = \frac{1}{2 \times \frac{x_m(i)}{X_{M1}(i)} + \frac{y_m(j)}{Y_{M1}(j)} + \frac{z_m(k)}{Z_{M1}(k)}} \]

PRINT I,J,K,DT,DTT

GO TO DT.LT.DTMIN

FIG 24e FLOW CHART OF MAIN PROGRAM
FIG 24F FLOW CHART OF MAIN PROGRAM
DO 707 I=2,11
DO 707 J=2,J1
CALCULATE ALLOWABLE DT FOR WATER LEVEL REGION

\[
DTS = \frac{1}{2 \times (X_N(1) / X_{M1}(1) + Y_N(J) / Y_{M1}(J) + X_{NUV} / (H(I,J) \times H(I,J) + \text{DELZ}(K1))))}
\]

\[
DTTS = \frac{1}{2 \times (X_M(I) / X_{M1}(I) + Y_M(J) / Y_{M1}(J) + D_V / (H(I,J) \times H(I,J) + \text{DELZ}(K1))))}
\]

PRINT I, J, KMAX, DTS, DTTS

FIG 24g FLOW CHART OF MAIN PROGRAM
FIG 24h FLOW CHART OF MAIN PROGRAM
CHOOSE SMALLEST DT

\[ DT = \text{AMINI}(DT_{MIN}, DT, DT_{MIN}, DT_{SH}, DT_{TSM}) \]

BEGIN NON-DIMENSIONALIZATION

\[ TBEG = TBEG \times U_O / D_O \]
\[ DT = (TEND - TBEG) / \text{ENITER} \]
\[ VI = VI / U_O \]
\[ WINDX = WINDX / U_O \]
\[ RH\_OA = RH\_OA / RH\_OZ \]
\[ DV = DV / XNUZ \]
\[ XNUV = XNUV / XNUZ \]

BEGIN NON-DIMENSIONALIZE AND PRINT NON SUBSCRIPTED VARIABLES

\[ TEND = TEND \times U_O / D_O \]
\[ UI = UI / U_O \]
\[ TI = TI / U_O \]
\[ WINDY = WINDY / U_O \]
\[ DH = DH / XNUZ \]
\[ XNUH = XNUH / XNUZ \]

FIG 24i FLOW CHART OF MAIN PROGRAM
DO 13 J=1,JMAX
DO 13 J=1,JMAX

CALCULATE REYNOLDS AND FROID NUMBER

H(I,J)=H(I,J)/D0

FO=UO/((G*DO)**.5)

CONTINUE NON-DIMENSIONALIZATION

TTBL(L)=TTBL(L)/TO

R=DO*UO/XNUZ

L=1,26

RHOTB(L)=RHOTBL(L)/RHOZ

F0=U0/((G*DO)**.5)

L=1,26

DELX(I)=DELX(I)/DO

I=1,IMAX

DELY(J)=DELY(J)/DO

J=1,JMAX

DELZ(K)=DELZ(K)/DO

K=1,KMAX

END NON DIMENSIONALIZATION

FIG 24j FLOW CHART OF MAIN PROGRAM
DO 33 K = 2, K1

ZKP1(K) = DELZ(K+1) + DELZ(K)
ZKM1(K) = DELZ(K-1) + DELZ(K)
ZKPM1(K) = ZKP1(K) + ZKM1(K)
ZK123(K) = ZKP1(K) * ZKM1(K) * ZKPM1(K)
ZKPM(K) = DELZ(K-1) - DELZ(K+1)

DO 34 J = 2, J1

YJP1(J) = DELY(J+1) + DELY(J)
YJM1(J) = DELY(J-1) + DELY(J)
YJPM1(J) = YJP1(J) + YJM1(J)
YJPM1(J) = YJP1(J) * YJM1(J) * YJPM1(J)

DO 35 I = 2, I1

XIP1(I) = DELX(I+1) + DELX(I)
XIM1(I) = DELX(I-1) + DELX(I)
XIPM1(I) = XIP1(I) + XIM1(I)
XI123(I) = XIP1(I) * XIM1(I) * XIPM1(I)

FIG 24k FLOW CHART OF MAIN PROGRAM
COEFFICIENT OF FIRST DERIVATIVES AT THE BOUNDARIES

\[ \text{SIGX} = \frac{\text{DELX}(1) + \text{DELX}(2)}{\text{DELX}(2) + \text{DELX}(3)} \]
\[ \text{SIGX}_1 = 1 + 2 \times \text{SIGX} \]
\[ \text{SIGX}_2 = (1 + \text{SIGX}) \times (1 + \text{SIGX}) \]
\[ \text{SIGX}_3 = \text{SIGX} \times \text{SIGX} \]
\[ \text{SIGX}_4 = \frac{(\text{DELX}(1) + \text{DELX}(2)) \times (1 + \text{SIGX})}{2} \]

\[ \text{SIGY} = \frac{\text{DELY}(1) + \text{DELY}(2)}{\text{DELY}(2) + \text{DELY}(3)} \]
\[ \text{SIGY}_1 = 1 + 2 \times \text{SIGY} \]
\[ \text{SIGY}_2 = (1 + \text{SIGY}) \times (1 + \text{SIGY}) \]
\[ \text{SIGY}_3 = \text{SIGY} \times \text{SIGY} \]
\[ \text{SIGY}_4 = \frac{(\text{DELY}(1) + \text{DELY}(2)) \times (1 + \text{SIGY})}{2} \]
\[ \text{SIGPY} = \frac{\text{DELY}(\text{JMAX}) + \text{DELY}(\text{J1})}{\text{DELY}(\text{J1}) + \text{DELY}(\text{J2})} \]
\[ \text{SIGPY}_1 = 1 + 2 \times \text{SIGPY} \]
\[ \text{SIGPY}_2 = (1 + \text{SIGPY}) \times (1 + \text{SIGPY}) \]
\[ \text{SIGPY}_3 = \text{SIGPY} \times \text{SIGPY} \]
\[ \text{SIGPY}_4 = \frac{(\text{DELY}(\text{JMAX}) + \text{DELY}(\text{J1})) \times (1 + \text{SIGPY})}{2} \]
\[ \text{SIGZ} = \frac{\text{DELZ}(1) + \text{DELZ}(2)}{\text{DELZ}(2) + \text{DELZ}(3)} \]
\[ \text{SIGZ}_1 = 1 + 2 \times \text{SIGZ} \]
\[ \text{SIGZ}_2 = (1 + \text{SIGZ}) \times (1 + \text{SIGZ}) \]
\[ \text{SIGZ}_3 = \text{SIGZ} \times \text{SIGZ} \]
\[ \text{SIGZ}_4 = \frac{(\text{DELZ}(1) + \text{DELZ}(2)) \times (1 + \text{SIGZ})}{2} \]

FIG 241 FLOW CHART OF MAIN PROGRAM
PRINT
SIGX , SIGX1, SIGX2, SIGX3, SIGX4, SIGY, SIGY1, SIGY2, SIGY3, SIGY4, SIGPY, SIGPY1, SIGPY2, SIGPY3, SIGPY4, SIGZ, SIGZ1, SIGZ2, SIGZ3, SIGZ4

DO 17 K=2, K1
DO 17 J=2, J1
DO 17 I=2, I1

PRINT
K, ZKP1(K), ZKM1(K), ZKPM1(K), ZK123(K), J, YJP1(J), YJM1(J), YJPM1(J), YJ123(J), I, XIP1(I), XIM1(I), XIPM1(I), XI123(I)

FIG 24m FLOW CHART OF MAIN PROGRAM
PRINT XNUR, ZNUR, XDHR, XDVR

Z = 0

DD = 2 × DT
F01 = F0 × F0

DO 110: L = 1, K1

Z = Z - ZKP1(K) / 2
PRINT Z

DO 313: J = 2, 11

CALCULATE COEFFICIENT OF FIRST DERIVATIVES

SIGMA1(1) = 2 × XM1(1) / XIPM1(1)
SIGMA2(1) = 2 × XIP1(1) / XIPM1(1)

FIG 24n FLOW CHART OF MAIN PROGRAM
CONTINUE TO CALCULATE COEFFICIENT OF FIRST DERIVATIVES

SIGMA3(J) = 2*YJM1(J)/YJPM1(J)
SIGMA4(J) = 2*YJP1(J)/YJPM1(J)

DO 312 J=2, J1 CONTINUE TO CALCULATE COEFFICIENT OF FIRST DERIVATIVES

SIGMA5(K) = 2*ZKM1(K)/ZKPM1(K)
SIGMA6(K) = 2*ZKP1(K)/ZKPM1(K)

PRINTOUT COEFFICIENT OF FIRST DERIVATIVES

PRINT
1, SIGMA1(I), SIGMA2(I)  I=2, I1
J, SIGMA3(J), SIGMA4(J)  J=2, J1
K, SIGMA5(K), SIGMA6(K)

FIG 240 FLOW CHART OF MAIN PROGRAM
\text{DMH} = \text{AMAX1}(XDR, XNR)
\text{DMV} = \text{AMAX1}(XDR, ZNR)

\text{DO 718 } 1 = 2, 11
\text{DO 718 } J = 2, J1

CLEVEL(1, J) = \left(\frac{1}{\text{DT}}\right) - 8 \frac{\text{DMH}}{XIM1(I) XIM1(I)} - 8 \frac{\text{DMV}}{YJM1(J) YJM1(J)}

\text{IF}\ EIRUN.EQ.1
\text{GO TO} 75
\text{N}

\text{Y}

\text{FIG 24p FLOW CHART OF MAIN PROGRAM}
DO 5 K = 1, KMAX
DO 5 J = 1, JMAX
DO 5 I = 1, IMAX

CLEAR THE ARRAYS

U(I, J, K) = 0
V(I, J, K) = 0
W(I, J, K) = 0
T(I, J, K) = 0
RHS(I, J, K) = 0
DUDT(I, J, K) = 0
DVDT(I, J, K) = 0
DWDT(I, J, K) = 0
DTDT(I, J, K) = 0
DHD(T(I, J) = 0

DO 25 K = 1, KMAX
DO 25 J = 1, JMAX
DO 25 I = 1, IMAX

INITIALIZE VELOCITIES

U(I, J, K) = UI
V(I, J, K) = VI

FIG 24q FLOW CHART OF MAIN PROGRAM
DO 502 K=1,KMAX
DO 506 J=1,JMAX
DO 506 I=1,I1MAX

INITIALIZE TEMPERATURE, DENSITY AND PRESSURE

Z

T(I,J,K)=T1
RH0(I,J,K)=SI(TTBL, RHTBL, T(I,J,K), 26)
P(I,J,K)+-RHO(I,J,K) * Z

506
K.EQ.KMAX

502
GO TO

Z

PRINT Z

Z=-DELZ(KMAX)/4
PRINT Z

Z=Z+ZKP1(K)/2

502
K.EQ.K1

FIG 24r FLOW CHART OF MAIN PROGRAM
DO K J=2, J1
DO 4 I=2, I1

SET

RIVER VELOCITY

U(I,J,K)=U1

502

Z=Z1

4

DO 63 K=2, KMAX
DO 63 I=ML, MK

CALCULATE

THERMAL DISCHARGE VELOCITIES

UI,1,K)=U0*COS(ANGLE)
V)1,1,K)=U0*SIN(ANGLE)

4

63

GO TO 79

75

FIG 24s FLOW CHART OF MAIN PROGRAM
READ FROM DISC

READ(24)
TIME, DT, IP, KMAX, JMAX, IMAX, IPRINT

CONTINUE READING FROM DISC

READ AND PRINT FROM DISC
READ(24)

FIG 24t FLOW CHART OF MAIN PROGRAM
\begin{itemize}
\item DO 635 I=1, 11
  \begin{align*}
  XN(1) &= 4*XNUR/XIM1(1) \\
  XM(1) &= 4*XDHR/XIM1(1) \\
  XL(1) &= AMIN1(XN(1), XM(1))
  \end{align*}

\item DO 636 J=2, J1
  \begin{align*}
  YN(J) &= 4*XNUR/YJM1(J) \\
  YM(J) &= 4*XDHR/YJM1(J) \\
  YL(J) &= AMIN1(YN(J), YM(J))
  \end{align*}

\item DO 637 K=2, K1
  \begin{align*}
  ZN(K) &= 4*ZNUR/ZKM1(K) \\
  ZM(K) &= 4*XDVR/ZKM1(K) \\
  ZL(K) &= AMIN1(ZN(K), ZM(K))
  \end{align*}

\item GO TO 654
\end{itemize}

**FIG 24u FLOW CHART OF MAIN PROGRAM**
DO 61 K=2,KMAX
DO 61 L=ML,MK
SET THERMAL DISCHARGE TEMPERATURE HEAT.EQ.O
Z1=Z
GO TO J,YN(J),YM(J),YL(J)
K,ZN(K),ZM(K),ZL(K)
K=2,K1

FIG 24v FLOW CHART OF MAIN PROGRAM
DO 43 K=IM,KMAX
DO 42 I=ML,MK

SET THERMAL DISCHARGE PRESSURE

P(I,J,K)=- Z

Z=Z+ZP1(K)/2

K.EQ.K1

Y

N

Z=DELZ(KMAX)/4

GO TO 618

DO 617 K=2,KMAX
DO 617 I=2,11
DO 617 J=2,J1

Z=Z1

ABS(U(I,J,K)).GT.XL(I))

CHECK STABILITY FOR VELOCITY COMPONENTS

GO TO 618

FIG 24W FLOW CHART OF MAIN PROGRAM
FIG 24x FLOWCHART OF MAIN PROGRAM
FIG 24: FLOW CHART OF MAIN PROGRAM
DO 719 I=2,11
DO 719 J=2,J1

CHECK STABILITY FOR H

HLEVEL = 2*DMV/(H(I,H)*
H(I,J)+DELZ(K1)))

GO TO 719

PRINT
CALL
CALL
EXIT

GO TO 719

EE

PRINT I,J,HLEVEL,
CLEVEL(I,J)

IPRINT=NPRNT

CALL OUTP

CALL EXIT

719
SET BOUNDARY CONDITION ON VELOCITY COMPONENT U.

SET OUTGOING BOUNDARY CONDITION

U(IMAX, J, K) = U(I1, J, K)

V(IMAX, J, K) = V(I1, J, K)

SET BOTTOM BOUNDARY CONDITION

U(I, J, 1) = 0

V(I, J, 1) = 0

SLIP BOUNDARY CONDITION ON U

U(I, JMAX, K) = (SIGPY2 * U(I, J1, K) - SIGPY3 * U(I, J2, K)) / SIGPY1

SLIP.EQ.0

N

GO TO 950

Y

236

6

FIG 24aa FLOW CHART OF MAIN PROGRAM
DO 135 K=2,kmax
DO 137 I=1,IMAX
K.LT.IM
 I.LT.MH.AND.I.GE.ML
GO TO 137
END SLIP CONDITION ON U.

FIG 24bb FLOW CHART OF MAIN PROGRAM
SET BOUNDARY CONDITION ON TEMPERATURE

T(I,JMAX,K) = (SIGPY2*T(I,J1,K) - SIGPY3*T(I,J2,K))/SIGPY1

FIG 24cc FLOW CHART OF MAIN PROGRAM
DO 7 I=1,IMAX
DO 7 J=1,JMAX

T(I,J,1)=(SIGZ2*T(I,J,2)-SIGZ3*T(I,J,3))/SIGZ1

END BOUNDARY CONDITION ON TEMPERATURE

DO 73 I=1,IMAX
DO 73 J=1,JMAX

T(I,J,K)=T(I1,J,K)

FIG 24dd FLOW CHART OF MAIN PROGRAM
CALCULATE DHDT (TIME DERIVATIVE OF WATER-LEVEL H)

\[ DHDT(I,J) = \frac{-2(H(I,J) \cdot U(I,J,KMAX) - U(IM1,J,KMAX)) + U(I,J,KMAX) \cdot H(IM1,U))}{XIM1(I) - 2}\]

\[ +H(I,J) \cdot V(I,J,KMAX) - V(1,JM1,KMAX) + V(I,J,KMAX) \cdot (H(I,J) - H(1,JM1))/YJM1(J) \]

FIG 24ee FLOW CHART OF MAIN PROGRAM
CALCULATE VERTICAL VELOCITY COMPONENT FROM CONTINUITY EQUATION

\[ KM1 = K - 1 \]
\[ DUDX = \frac{(U(I+1,J,K) - U(I,J,K)) \cdot \sigma_1(I)}{\xi_{I+1}(I)} + \frac{(U(I,J,K) - U(I-1,J,K)) \cdot \sigma_2(I)}{\xi_{I-1}(I)} \]
\[ DVDY = \frac{(V(I,J+1,K) - V(I,J,K)) \cdot \sigma_3(J)}{\eta_{J+1}(J)} + \frac{(V(I,J,K) - V(I,J-1,K)) \cdot \sigma_4(J)}{\eta_{J-1}(J)} \]
\[ W(I,J,K) = W(I,J,K-1) - (\text{dudx} + \text{dvy}) \cdot z_{K-1}(K) / 2 \]

DO 72 K = 2, KMAX
DO 72 J = 2, J1

SET BOUNDARY CONDITION ON \( W(\text{OUTGOING}) \)

\[ W(I_{MAX},J,K) = W(I_{MIN},J,K) \]

FIG 24ff FLOW CHART OF MAIN PROGRAM
SLIP BOUNDARY CONDITION ON W

D) 232 I=2,11
    DO 232 J=2,J1

    72
    \( W(I,J,KMAX) = W(I,J,K1) + DHDT(I,J) \)

    232
    \( SLIP.EQ.O \)
    HH

    GO TO 940

\[ \text{FIG 24gg FLOW CHART OF MAIN PROGRAM} \]
\[ W(1,1,K) = (\text{SIGY}_2 \times W(1,2,K) - \text{SIGY}_3 \times W(1,3))/\text{SIGY}_1 \]

**END BOUNDARY CONDITION ON W**

\[ W(1,J_{\text{MAX}},K) = (\text{SIGPY}_2 \times W(1,J_1,K) - \text{SIGPY}_3 \times W(1,J_2,K))/\text{SIGPY}_1 \]

**CALCULATE DHDT**

\[ \text{DHDT}(1,J) = W(1,J,K_{\text{MAX}}) \]
DO 12 J=1,JMAX
DO 12 I=1,IMAX

SET BOUNDARY CONDITION ON DENSITY

RHO(I,J,1)=SI(TTBL,RHOTBL,
T(I,J,1),26)
RHO(I,J,KMAX)=SI(TTBL,RHOTBL,
T(I,J,KMAX),26)

CALL DERIV
CALL SUM
CALL OUTP
IPRINT=IPRINT+1
IPRINT-NPRNT

FIG 2411 FLOW CHART OF MAIN PROGRAM
DO 40 K=1,KMAX
DO 40 I=2,11
DO 40 J=2,J1

CALCULATE DENSITY

RHO(I,J,K)=SI(TTBL,RHOTBL,T(I,J,K),26)

INCREMENT TIME

TIME=TIME+DT
IPRINT=IPRINT+1

GO TO 2002
GO TO 2001
GO TO 20

FIG 24JJ FLOW CHART OF MAIN PROGRAM
CALL GO TO LL
   IP=IP+1
   GO TO 1
   CALL OUTP

WRITE ONTO A DISC

WRITE(25)
TIME,DT,IP,KMAX,
JMAX,IMAX,IPRINT

PRINT
TIME,DT,IP,KMAX,
JMAX,IMAX,IPRINT

REWIND 25

DO 2003 K=1,KMAX
   DO 2003 J=1,JMAX
      DO 2003 I=1,IMAX
         WRITE(25)
         U(I,J,K),V(I,J,K),W(I,J,K),
         T(I,J,K),P(I,J,K),H(I,J),
         RHO(I,J,K)

CONTINUE WRITING ONTO A DISC

WRITE(25)
MM

CONTINUE WRITING ONTO A DISC

WRITE(25)
U(I,J,K),V(I,J,K),W(I,J,K),
T(I,J,K),P(I,J,K),H(I,J),
RHO(I,J,K)

FIG 24kk FLOW CHART OF MAIN PROGRAM
FIG2411 FLOW CHART OF MAIN PROGRAM

PRINT
U(I,J,K), V(I,J,K), W(I,J,K),
T(I,J,K), P(I,J,K), H(I,J),
RHO(I,J,K)

NN -> 2003 -> STOP -> END
CALCULATE DERIVATIVES OF WATER LEVEL ELEMENTS

\( IP1 = I + 1 \)
\( IM1 = I - 1 \)
\( JP1 = J + 1 \)
\( JM1 = J - 1 \)
\( K = K \)\( \text{MAX} \)
\( ZKP1(K1) = \text{DELZ}(K1) + H(I, J) \)
\( \text{DUDXR} = (U(IP1, J, K) - U(I, J, K))/XIP1(I) \)
\( \text{DUDXL} = (U(I, J, K) - U(IM1, J, K))/XIM1(I) \)
\( \text{DUDYR} = (U(I, JP1, K) - U(I, J, K))/YJP1(J) \)
\( \text{DUDYL} = (U(I, J, K) - U(I, JM1, K))/YJM1(J) \)
\( \text{DUDZL} = 2 \times (U(I, J, K) - U(I, J, K1))/ZKP1(K1) \)
\( \text{DHDXR} = (H(IP1, J) - H(I, J))/XIP1(I) \)
\( \text{DHDXL} = (H(I, J) - H(IM1, J))/XIM1(I) \)
\( \text{DHDYR} = (H(IP1, J) - H(I, J))/YJP1(J) \)
\( \text{DHDYL} = (H(I, J) - H(IM1, J))/YJM1(J) \)
\( \text{DRHDXR} = (\rho(IP1, J, K) - \rho(I, J, K))/XIP1(I) \)
\( \text{DRHDXL} = (\rho(I, J, K) - \rho(IM1, J, K))/XIM1(I) \)
\( \text{DRHDYR} = (\rho(IP1, J, K) - \rho(I, J, K))/YJP1(J) \)
\( \text{DRHDL} = (\rho(I, J, K) - \rho(IM1, J, K))/YJM1(J) \)

FIG 25a FLOW CHART OF SUBROUTINE DERIV
CALCULATE TIME DERIVATIVE OF U FOR WATER LEVEL ELEMENTS

\[ DUDT(I,J,K) = -U(I,J,K) \]
\[ \times (\Sigma_1(I) \times DUDXR + \Sigma_2(I) \times DUDXL) \]
\[ -V(I,J,K) \times (\Sigma_3(J) \times DUDYR + \Sigma_4(J) \times DUDYL) \]
\[ -H(I,J) \times (\Sigma_1(I) \times DRHDXR + \Sigma_2(I) \times DRHDXL) / F01 \]
\[ +((\Sigma_1(I) \times DHDXR + \Sigma_2(I) \times DHDYL) \times XNUR \times (\Sigma_1(I) \times DUDXR + \Sigma_2(I) \times DUDXL) / H(I,J) \]
\[ -2 \times RHO(I,J,K) / F01) \]
\[ +XNUR \times 8 \times ((DUDXR - DUDXL) / XPM1(I) \]
\[ +((DUDYR - DUDYL) / YPM1(J)) \]
\[ +XNUR \times ((\Sigma_3(J) \times DUDYR + \Sigma_4(J) \times DUDYL) \times (\Sigma_3(J) \times DHDYR + \Sigma_4(J) \times DHDYL)) / H(I,J) \]
\[ -ZNUR \times DUDZL / H(I,J) \]
\[ +CFX \times RHOA \times WINDX \times WINDX / (2 \times H(I,J)) \]

**FIG 25b FLOW CHART OF SUBROUTINE DERIV**
CALCULATE TIME DERIVATIVE OF V FOR WATER LEVEL ELEMENTS

\[ DVDT(i,j,k) = -u(i,j,k) \]

\[ \times (\sigma_1(i) \times DVDXR + \sigma_2(i) \times DVDXL) \]

\[ -v(i,j,k) \times (\sigma_3(j) \times DVDYR + \sigma_4(j) \times DVDYL) \]

\[ -h(i,j) \times (\sigma_3(j) \times DRHDYR + \sigma_4(j) \times DRHDYL) / f_01 \]

\[ \times x_{nur} \times (\sigma_1(i) \times DHDXR + \sigma_2(i) \times DHDXL) \]

\[ \times (\sigma_1(i) \times DVDXR + \sigma_2(i) \times DVDXL) / h(i,j) \]

\[ \times (x_{nur} \times (\sigma_3(j) \times DVDYR + \sigma_4(j) \times DVDYL) / h(i,j) \]

\[ -2 \times \rho_0(i,j,k) / f_01 \]

\[ + x_{nur} \times 8 \times (DVXR - DVDXL) / x_{1pm1}(i) \]

\[ + (DVYR - DVDYL) / y_{jpm1}(j) \]

\[ - z_{nur} \times DVDZL / h(i,j) \]

\[ + C_F \times \rho_{0a} \times \text{WINDY} \times \text{WINDY} / (2 \times h(i,j)) \]

**FIG 25d FLOW CHART OF SUBROUTINE DERIV**
CALCULATE TIME DERIVATIVE OF T FOR WATER LEVEL ELEMENTS

\[ DTDT(I,J,K) = -u(I,J,K) \]

\[ \times (\Sigma_1(I) \times DTDXR + \Sigma_2(I) \times DTDXL) \]

\[ - (v(I,J,K) \times (\Sigma_3(J) \times DTDYR + \Sigma_4(J) \times DTDYL) \]

\[ + XDHR \times ((\Sigma_1(I) \times DHDXR + \Sigma_2(I) \times DHDXL) \times H(I,J) \]

\[ \times ((\Sigma_1(I) \times DTDXR + \Sigma_2(I) \times DTDXL) / H(I,J) \]

\[ + XDHR \times 8 \times ((DTDXR - DTDYL) / XIPM1(I) \]

\[ + (DTDXR - DTDYL) / YJPM1(J) \]

\[ + XDHR \times ((\Sigma_3(J) \times DHDR + \Sigma_4(J) \times DHDYL) \]

\[ + (\Sigma_3(J) \times DTDXR + \Sigma_4(J) \times DTDYL) / H(I,J) \]

\[ - XDVR \times DTDZL / H(I,J) \]

\[ - S0 \times (T(I,J,K) - E) / H(I,J) \]

FIG 25e FLOW CHART OF SUBROUTINE DERIV
DO 7 K = 2, K1
DO 6 J = 2, J1
DO 5 I = 2, I1

ADJUST WATER LEVEL VARIABLES

ZKP1(K) = DELZ(K) + H(I, J)
ZKPM1(K) = ZKP1(K) + ZKM1(K)
SIGMA5(K) = 2*ZKM1(K)/ZKPM1(K)
SIGMA6(K) = 2*ZKP1(K)/ZKPM1(K)

FIG 25f FLOW CHART OF SUBROUTINE DERIV
CALCULATE SPATIAL DERIVATIVE OF U AND TIME DERIVATIVE OF $Q_x$

$$DUDXR = \frac{U(IP1,J,K) - U(I,J,K)}{XIP1(I)}$$
$$DUDXL = \frac{U(I,J,K) - U(IM1,J,K)}{XIIM1(I)}$$
$$DUDYR = \frac{U(I,JP1,K) - U(I,J,K)}{YJP1(J)}$$
$$DUDYL = \frac{U(I,J,K) - U(I,JM1,K)}{YJM1(J)}$$
$$DUDZR = \frac{U(I,J,KP1) - U(I,J,K)}{ZKP1(K)}$$
$$DUDZL = \frac{U(I,J,K) - U(I,J,KM1)}{ZKM1(K)}$$

$$DUDT(I,J,K) = -U(I,J,K) \times (SIGMA1(I) \times DUDXR + SIGMA2(I) \times DUDXL)$$
$$-V(I,J,K) \times (SIGMA3(J) \times DUDYR + SIGMA4(J) \times DUDYL)$$
$$-W(I,J,K) \times (SIGMA5(K) \times DUDZR + SIGMA6(K) \times DUDZL)$$
$$+ 8 \times XNUR \times ((DUDXR - DUDXL) / XIPM1(I))$$
$$+(DUDYL - DUDYR) / YJPM1(J))$$
$$+ZNUR \times (DUDZR - DUDZL) / ZKPM1(K)$$

FIG 25g FLOW CHART OF SUBROUTINE DERIV
CALCULATE SPATIAL DERIVATIVE OF $V$ AND TIME DERIVATIVE OF $Q_y$

$$\begin{align*}
DVDXR &= \frac{V(IP1,J,K) - V(I,J,K)}{XIP1(I)} \\
DVDXL &= \frac{V(I,J,K) - V(IM1,J,K)}{XIM1(I)} \\
DVYR &= \frac{V(I,JP1,K) - V(I,J,K)}{YJP1(J)} \\
DVYL &= \frac{V(I,J,K) - V(I,JM1,K)}{YJM1(J)} \\
DVZR &= \frac{V(I,J,KP1) - V(I,J,K)}{ZKP1(K)} \\
DVZL &= \frac{V(I,J,K) - V(I,J,KM1)}{ZKM1(K)} \\
DVDT(I,J,K) &= -U(I,J,K) \\
&\quad \times (\Sigma_1(I) \times DVDXR + \Sigma_2(I) \times DVDXL) \\
&\quad - V(I,J,K) \times (\Sigma_3(J) \times DVYR + \Sigma_4(J) \times DVYL) \\
&\quad - W(I,J,K) \times (\Sigma_5(K) \times DVZR + \Sigma_6(K) \times DVZL) \\
&\quad + 8 \times (XNUR \times (DVYR - DVYR) / XIP1(I)) \\
&\quad + (DVYR - DVYR) / YJP1(J) \\
&\quad + ZNUR \times (DVZL - DVZL) / ZKP1(K))
\end{align*}$$

FIG 25h FLOW CHART OF SUBROUTINE DERIV
CALCULATE SPATIAL DERIVATIVE OF W AND TIME DERIVATIVE OF $Q_z$

\[
\begin{align*}
DWDXR &= \frac{W(IP1,J,K) - W(I,J,K)}{XIP1(I)} \\
DWDXL &= \frac{W(I,J,K) - W(IM1,J,K)}{XIM1(I)} \\
DWDYR &= \frac{W(I,JP1,K) - W(I,J,K)}{YJP1(J)} \\
DWDYL &= \frac{W(I,J,K) - W(I,JM1,K)}{YJM1(J)} \\
DWDZR &= \frac{W(I,J,KP1) - W(I,J,K)}{ZKP1(K)} \\
DWDZL &= \frac{W(I,J,K) - W(I,J,KM1)}{ZKM1(K)} \\
DWDT(I,J,K) &= -U(I,J,K) \\
& \quad \times \sigma_1(I) DWDXR + \sigma_2(I) DWDXL \\
& \quad - V(I,J,K) \sigma_3(J) DWDYR + \sigma_4(J) DWDYL \\
& \quad - W(I,J,K) \times 2 DWDZL \\
& \quad + 8 \times (XNUR ((DWDXR - DWDXL) / XIPM1(I) \\
& \quad + (DWDYR - DWDYL) / YJPM1(J)) \\
& \quad + ZNUR ((DWDZR - DWDZL) / ZKPM1(K)) \\
& \quad - RHO(I,J,K) / F01
\end{align*}
\]

FIG 25: FLOW CHART OF SUBROUTINE DERIV
CALCULATE SPATIAL DERIVATIVE OF T AND TIME DERIVATIVE OF T

\[
\begin{align*}
    DTDXR &= (T(I+1,J,K) - T(I,J,K))/X(I+1) \\
    DTDXL &= (T(I+1,J,K) - T(I-1,J,K))/X(I-1) \\
    DTDXR &= (T(I,J+1,K) - T(I,J,K))/Y(J+1) \\
    DTDXL &= (T(I,J+1,K) - T(I,J,K))/Y(J-1) \\
    DTDXR &= (T(I,J,K+1) - T(I,J,K))/Z(K+1) \\
    DTDXL &= (T(I,J,K+1) - T(I,J,K))/Z(K-1) \\
    DTDT(J,K) &= U(J,K) \\
    \text{fig 25j flow chart of subroutine deriv}
\end{align*}
\]
GO TO 90

I P NE NPRNT

***CALCULATE DIVERGENCE***

\[
\text{DIV} = (\text{SIGMA}_1(I) \times \text{DUDXR} + \text{SIGMA}_2(I) \times \text{DUDXL}) + (\text{SIGMA}_3(J) \times \text{DVDRYR} + \text{SIGMA}_4(J) \times \text{DVDLY}) + 2 \times \text{DWDZL}
\]

PRINT I,J,K,DU(I,J,K),DV(I,J,K),DW(I,J,K),DIV

FIG 25k FLOW CHART OF SUBROUTINE DERIV
SET BOUNDARY CONDITION ON $Q_x$

$DUDT(1,J,K) = 0$
$DUDT(1MAX,J,K) = DUDT(11,J,K)$

SET BOUNDARY CONDITION ON $Q_y$

$\text{DO } 200 \text{ J=2; J1 DO 210 K=2; K1}$

$\text{DVDYRO} = (V(1,3,K) - V(1,2,K))/YJP1(J)$
$\text{DVDYLO} = (V(1,2,K) - V(1,1,K))/YJM1(J)$
$\text{DVDYRM} = (V(1,JMAX,K) - V(1,J1,K))/YJP1(J1)$
$\text{DVDTLM} = (V(1,J1,K) - V(1,J2,K))/YJM1(J1)$
$\text{DVDT}(1,JMAX,K) = 8*XNUR*(DVDYRM - DVDYLM)/YJPM1(J1)$

**FIG 251** FLOW CHART OF SUBROUTINE DERIV
FIG 25m FLOW CHART OF SUBROUTINE DERIV

JJ

K.LT.IH

I.LT.MH.AND.I.GE.ML

205

Y

N

210

205

DVDT(I,1,K)=8*XNUR*(DVYR0-DVDYLO)/YJPM1(2)

GO TO 210

220
END BOUNDARY CONDITION ON $Q_Y$

220

DVDT(1,1,K)=0

210

DO 230 I=2,11  DO 230 J=2,J1

DVDT(I,J,1)=0

DWDZTO=(W(I,J,3)-W(I,J,2))/ZKP1(2)

DWDZL0=(W(I,J,2)-W(I,J,1))/ZKM1(2)

DVDT(I,J,KMAX)=-RHO(I,J,KMAX)/F01

FIG 25n FLOW CHART OF SUBROUTINE DERIV
ADJUST WATER LEVEL VARIABLES

\[
\begin{align*}
ZKP1(K) &= \text{DELZ}(k) + H(I,J) \\
ZKPM1(K) &= ZKP1(K) + ZKM1(K) \\
\Sigma5(K) &= 2 \cdot ZKM1(K) / ZKPM1(K) \\
\Sigma6(K) &= 2 \cdot ZKP1(K) / ZKPM1(K)
\end{align*}
\]

FIG 25o FLOW CHART OF SUBROUTINE DERIV
CALCULATE RHS AND SPATIAL DERIVATIVES OF $Q_x, Q_y$, AND $Q_z$

$$DQDZ = (DWDT(I, J, KP1) - DWDT(I, J, K)) \times \frac{SIGMA5(K)}{ZKP1(K)} + (DWDT(I, J, K) - DWDT(I, J, KM1)) \times \frac{SIGMA6(K)}{ZKM1(K)}$$

$$RHS(I, J, K) = DQDZ$$

$$DQDY = (DVDT(I, JP1, K) - DVDT(I, J, K)) \times \frac{SIGMA3(J)}{YJP1(J)} + (DVDT(I, J, K) - DVDT(I, JM1, K)) \times \frac{SIGMA4(J)}{YJM1(J)}$$

$$RHS(I, J, K) = RHS(I, J, K) + DQDY$$

$$DQDX = (DUDT(IP1, J, K) - DUDT(I, J, K)) \times \frac{SIGMA1(I)}{XIP1(I)} + (DUDT(I, J, K) - DUDT(IM1, J, K)) \times \frac{SIGMA2(I)}{XIM1(I)}$$

$$RHS(I, J, K) = RHS(I, J, K) + DQDX \times F_0$$

**FIG 25p FLOW CHART OF SUBROUTINE DERIV**
CALCULATE SPATIAL DERIVATIVE OF $P$ AND TIME DERIVATIVES OF $U$ AND $V$

$$\begin{align*}
\text{DPDX} &= (P(IP1,J,K) - P(I,J,K)) \times \text{SIGMA1}(I) / \text{XIP1}(I) \\
&\quad + (P(I,J,K) - P(IM1,J,K)) \times \text{SIGMA2}(I) / \text{XIM1}(I) \\
\text{DUDT}(I,J,K) &= \text{DUDT}(I,J,K) - \text{DPDX} / \text{FO1} \\
\text{DPDY} &= (P(I,JP1,K) - P(I,J,K)) \times \text{SIGMA3}(J) / \text{YJP1}(J) \\
&\quad + (P(I,J,K) - P(I,JM1,K)) \times \text{SIGMA4}(J) / \text{YJM1}(J) \\
\text{DVDT}(I,J,K) &= \text{DVDT}(I,J,K) - \text{DPDY} / \text{FO1}
\end{align*}$$

FIG 25q FLOW CHART OF SUBROUTINE DERIV
FIG 25r FLOW CHART OF SUBROUTINE DERIV
CALCULATE PRESSURE FROM HYDROSTATIC APPROXIMATION

START

HYD.EQ.0

GO TO 300

DO 110 L=1,KMAX

Z3=ZKP1(K1)

K=KMAX+1-L

KP1=K+1

IF MH.AND.I.GE.ML YN

GO TO 110

GO TO 300

K.NE.KMAX

GO TO 110

P(1,J,K)=RHO(1,J,K)*H(I,J)/2

GO TO 110

K.NE.K1 YN

GO TO 130

ZKP1(K1)=DELZ(K1)+H(I,J)

GO TO 130

FIG 26a FLOW CHART OF SUBROUTINE PRESS
END HYDROSTATIC APPROXIMATION ON PRESSURE

\[ P(I,J,K) = P(I,J,K) + RHO(I,J,K) \times ZKP1(K)/2 \]

GO TO 110

ZKP1(K1)=Z3

GO TO 100

130

110

300

300

ITN=1
Y1=1
OMEGA=1
LAMB1=1

IP.NE.0

Y

N

GO TO 10

GO TO 10

NOT.(DEBUG4.GE.2)

N

C

FIG 26b FLOW CHART OF SUBROUTINE PRESS
DO 6 K=2,K1
DO 6 J=2,J1
DO 6 I=2,I1

PRINT
K,ZK123(K),ZKP1(K),
ZKM1(K),
J,YJ123(J),YJP1(J),
YJM1(J),
I,XI123(I),XIP1(I),
XIM1(I)

DO 200 L=2,J1 DO 200 K=2,K1

FIG 26c FLOW CHART OF SUBROUTINE PRESS
SET BOUNDARY CONDITION ON PRESSURE

\[
P(1,1,K) = (\text{SIGY2}\cdot P(1,2,K) - \text{SIGY3}\cdot P(1,3,K) - \text{SIGY4}\cdot DVDT(1,1,K)) \cdot F01) / \text{SIGY1}
\]

\[
\text{DO 55 K=2,KMAX}
\]

\[
\text{DO 55 J=2,JMAX}
\]

\[
P(1,JMAX,K) = (\text{SIGPY2}\cdot P(1,J1,K) - \text{SIGPY3}\cdot P(1,J2,K) + \text{SIGPY4}\cdot DVDT(1,JMAX,K) \cdot F01) / \text{SIGPY1}
\]

\[
P(\text{IMAX},J,K) = P(11,J,K)
\]

**FIG 26d FLOW CHART OF SUBROUTINE PRESS**
END BOUNDARY CONDITION ON PRESSURE

DO 50 I = 2, 11
DO 50 J = 2, J1

DO 220 I = 2, 11
DO 220 J = 2, J1

P(I, J, KMAX) = H(I, J) \times \frac{\rho_0(I, J, KMAX)}{2}

50

P(I, J, 1) = \frac{\text{SIGZ2} \times P(I, J, 2) - \text{SIGZ3} \times P(I, J, 3) - \text{SIGZ4} \times \Delta P(I, J, 1) - \text{F01}}{\text{SIGZ1}}

220

DO 5 K = 2, K1
DO 5 J = 2, J1
DO 5 I = 2, 11

ADJUST WATER LEVEL VARIABLES

Y

ZKP1(K) = \Delta Z(K) + H(I, J)
ZKPM1(K) = ZKP1(K) + ZKM1(K)
ZK123(K) = ZKP1(K) \times ZKM1(K) \times ZKPM1(K)

N

18

FIG 26e FLOW CHART OF SUBROUTINE PRESS
CALCULATE PRESSURE IN FLOW REGION

\[ X_1 = YJ123(J) \times ZK123(K) \times XIM1(I) \]
\[ X_2 = YJ123(J) \times ZK123(K) \times XIP1(I) \]
\[ X_3 = XI123(I) \times ZK123(K) \times YJP1(J) \]
\[ X_4 = XI123(I) \times ZK123(K) \times YJM1(J) \]
\[ X_5 = XI123(I) \times YJ123(J) \times ZKP1(K) \]
\[ X_6 = XI123(I) \times YJ123(J) \times ZKM1(K) \]
\[ X_7 = XI123(I) \times YJ123(J) \times ZKI123(K) \]
\[ X = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 \]
\[ A = \text{OMEGA}/X \]
\[ P_{\text{NEW}} = A \times (X_1 \times P(I+1,J,K) + X_2 \times P(I-1,J,K) + X_3 \times P(I,J+1,K) + X_4 \times P(I,J-1,K) + X_5 \times P(I,J,K+1) + X_6 \times P(I,J,K-1) - X_7 \times \text{RHS}(I,J,K) / 8) + B \times P(I,J,K) \]

FIG 26F FLOW CHART OF SUBROUTINE PRESS
FIG 26g FLOW CHART OF SUBROUTINE PRESS

E

RESID = ABS(PNEW - P(I,JK))

RESID < D

GO TO 15

GO TO 16

GO TO 9

16

D = RESID

15

P(I,J,K) = PNEW

OMEGA . NE. 1

Y

N

F
FIG 26h FLOW CHART OF SUBROUTINE PRESS

Y2 = Y2 + RESID

GO TO 5

NOT.(DEBUG4.GE.1)

PRINT
X1, X2, X3, X4,
X5, X6, X7, X, A

LAMB2 = Y2 / Y1

GO TO 28

OMEGA.NE.1

N

G

H
CALCULATE RELAXATION FACTOR

\[ \Omega = \frac{2}{1 + \sqrt{1 - \lambda_2}} \]

FIG 261 FLOW CHART OF SUBROUTINE PRESS
ITN = ITN + 1

LAMB1 = LAMB2

GO TO 14

D - EPS

12

ITN = ITN + 1

J

GO TO 10

J

ITN - ITMAX

PRINT

FAIL TO CONVERGE

CALL EXIT

14

FIG 26j FLOW CHART OF SUBROUTINE PRESS
FIG 26k FLOW CHART OF SUBROUTINE PRESS
DO 22 K=2,K1
DO 21 J=2,J1
DO 20 I=2,11

INTEGRATE FOR VELOCITY COMPONENTS AND TEMPERATURE

U(I,J,K)=U(I,J,K)+DT*DUDT(I,J,K)
V(I,J,K)=V(I,J,K)+DT*DVDT(I,J,K)
T(I,J,K)=T(I,J,K)+DT*DTDT(I,J,K)

START -> 20 -> 21

DO 23 I=1,IMAX
DO 23 J=1,JMAX

INTEGRATE FOR WATER LEVEL H

H(I,J)=H(I,J)+DT*DHT(I,J)

21 -> 22 -> 23

FIG 27a FLOW CHART OF SUBROUTINE SUM
FIG 27b FLOW CHART OF SUBROUTINE SUM
FIG 28a FLOW CHART OF SUBROUTINE OUTP
FIG 28b FLOW CHART OF SUBROUTINE OUTP

PRINT
(I,J,K,U(I,J,K),V(I,J,K),
W(I,J,K),T(I,J,K),P(I,J,K),
I=MJ,MJJ)

GO TO
MJ=MJ+2
MJJ.EQ.IMAX

GO TO
15

GO TO
20
25
MJ=1
30
MJJ=MJ+1
FIG 28c FLOW CHART OF SUBROUTINE OUTP
Subroutine | Function
--- | ---
SI Function | This subroutine performs linear interpolation.
OUTP | This subroutine prints out all the important variables.

### 1.1 MAIN Program

A detailed description of the MAIN program is presented in the flow charts in the Figs. 24a to 24m. This program is employed to perform the following three main functions.

1) Reading in the input data. This part of the program reads in and prints out the input data to maintain a permanent record of each run.

2) Initialization of the system. This part of the program performs the following functions:
   a) Converts input data from dimensional values to non dimensional values; and
   b) Sets time independent boundary conditions.

3) Iterative operations performed after each time increment. This part of the program performs the following tasks:
   a) Checks the stability conditions, if these conditions are not satisfied then OUTP subroutine is called and program terminated.
b) Sets time-dependent boundary conditions.

c) Calculates vertical velocity components by a spatial integration of equation (61) using updated values of horizontal velocity components.

d) Calls subroutine OUTP to print out updated values of the important variables.

e) Calls subroutine DERIV and SUM in the process of finding the updated values.

f) Compares the current time TIME with the final problem time TEND, in order to stop the problem if TEND is exceeded.

1.2 DERIV Subroutine

A detailed description of the subroutine DERIV is shown in Figs. 25a to 25r. This subroutine calculates the first and the second spatial derivatives of the variables. These spatial derivatives are used along with the pressure gradients calculated in subroutine PRESS to find the time derivatives of the horizontal velocity components, temperature, and level according to equation (78) through (84). Communication among DERIV, MAIN and other subroutine take place by means of a COMMON block. The calling sequence of this subroutine is: CALL DERIV.

1.3 PRESS Subroutine

A detailed description of the subroutine PRESS is shown in Figs. 26a to 26k. This subroutine solves for the pressure according to
equations (73) and (74) by an iterative method, based on a modified Gauss-Seidel iteration technique called successive overrelaxation technique. The calculation of the pressure for the entire flow field is repeated until the values of pressure at every grid point converge within a prescribed error. This subroutine further sets the boundary condition on pressure and calculates the overrelaxation factor $\omega$ according to equations (75) through (77). This factor expedites the convergence of the pressure calculations. The variables are passed into and returned from PRESS by means of a COMMON block. The calling sequence of this subroutine is: CALL PRESS.

1.4 SUM Subroutine

A detailed description of the subroutine SUM is shown in Figs. 27a to 27b. This subroutine performs the time integration for the horizontal velocity components, temperature, and water-level. The communications among SUM, MAIN and other subroutines take place by means of a COMMON block. The calling sequence of this subroutine is: CALL SUM.

1.5 SI Function

This function is used to interpolate values from the table of temperature versus density. It is a standard subroutine:

$$\text{SI}(\text{XTBL}, \text{YTBL}, \text{X}, \text{N})$$

$\text{XTBL}=\text{Tabular values of the independent variable.}$

$\text{YTBL}=\text{Tabular values of the dependent variable.}$
X = Value of independent variable.
N = Number of points in the table.

1.6 OUTP Subroutine

A detailed description of subroutine OUTP is shown in Figs. 28a to 28c. This subroutine prints out all variables encompassing the problem solution at any desired multiple of integration time step. These variables are passed into OUTP from the other subroutines, including the MAIN program, by means of a COMMON block. The calling sequence of this subroutine is: CALL OUTP.
2. DESCRIPTION OF INPUT DATA

The input data can be divided into two groups:

1) Non-subscripted variables
2) Subscripted variables.

A brief description of each input variable appears in the program listing to aid in entering the data. All input data are entered using FORMAT E10.3.

2.1 Non-Subscripted Variables

These variables are floating point numbers and are entered into the computer storage by means of cards as specified hereunder:

Card No. 1

1) ENRUN, Run designation number. This number is used to identify each run for record keeping purposes.

Card No. 2

1) TBEG, Starting time, sec. This quantity specifies the initial problem time at which the dynamic problem starts.

2) TEND, End of time, sec. This variable defines the final problem time at which the dynamic problem stops automatically. The dynamic analysis may be aborted, prior to TEND, due to the occurrence of computational errors.

3) ENITER, Number of integration steps. This quantity defines the number of integration steps to be used between starting time
TBEG, and end of time TEND. The program calculates the integration time increment by
\[ \Delta t = \frac{(TEND-TBEG)}{ENITER} \]

4) ENPRNT, Frequency of printout. This variable sets the number of integration steps between two successive printouts.

Card No. 3
1) UI, Initial velocity component in x-direction, ft/sec. This is the initial value of the x-component of the fluid velocity assumed constant throughout the water body. It is used to obtain the dynamic response of the water body from a set of uniform velocity distributions as discussed in Part I Section 5.

2) VI, Initial velocity component in y-direction, ft/sec. This is the initial value of the y-component of the fluid velocity assumed constant throughout the water body. The purpose of the introduction of this variable is described under variable UI above.

3) TI, Initial temperature of water body and inflow, degrees Fahrenheit. This variable defines the initial value of the fluid temperature assumed constant throughout the water body. The purpose of the introduction of this variable is described under variable UI above. Furthermore, this variable defines the inflow temperature into the water body assumed constant during the dynamic analysis.

Card No. 4
1) UO, Thermal discharge velocity, ft/sec. This quantity specifies the water velocity in the thermal discharge channel from the power plant entering the water body. This variable is used to non-
dimensionalize all velocity components involved in the program.

2) ANGLE, Thermal discharge angle, radian. This variable defines the angle at which the thermal discharge enters the water body measured from the x-axis. The $x$ and $y$ velocities of the thermal discharge entering the water body are calculated by $U_0 \cos(\text{ANGLE})$ and $U_0 \sin(\text{ANGLE})$ respectively.

3) $T_0$, Thermal discharge temperature, degrees Fahrenheit. This variable specifies the water temperature in the thermal channel from the power plant entering the water body. This variable is used to nondimensionalize all temperature quantities involved in the program and should be always entered regardless of the value of $\text{HEAT on Card No. 9}$ to be discussed later.

4) $D_0$, The half width of thermal discharge channel, ft. This variable is the half width of the thermal discharge channel. This variable is used to nondimensionalize all geometric dimensions involved in the program.

5) $\nu_0$, Kinematic viscosity of thermal discharge, ft$^2$/sec. This variable defines the kinematic viscosity of the thermal discharge water. It is used to nondimensionalize the thermal eddy diffusivities and momentum eddy viscosities.

Card No. 5

1) $WINDX$, $x$-component of wind velocity, ft/sec. This quantity is the wind velocity component in $x$-direction assumed to be blowing horizontally over the water body.

2) $WINDY$, $y$-component of wind velocity, ft/sec. This quantity
is the wind velocity component in y-direction. This variable together with WINDX provides the means of analyzing wind effects blowing at an oblique angle with respect to the Cartesian coordinate system.

3) **RHOA**, Air density, lbm/ft³. This variable specifies the air density above the water body. It affects the water body dynamics because the wind shear effect at the water surface is proportional to air density as described in Part 1, Section 2.3.

4) **E**, Water body equilibrium temperature, degrees Fahrenheit. This variable specifies the water equilibrium temperature. It is used in the calculation of heat dissipation from water body surface to atmosphere by employing equation (54).

5) **XK**, Heat exchange coefficient, Btu/(ft²-day-°F). This variable specifies the coefficient of heat transfer between water body surface and atmosphere. It is used in the calculation of heat dissipation from water body to atmosphere by employing equation (54).

6) **CFX**, The skin coefficient in x-direction. This variable specifies the skin coefficient in x-direction. It is used to calculate the wind shear effect in x-direction from equation (52).

7) **CFY**, The skin coefficient in y-direction. This variable specifies the skin coefficient in y-direction. It is used to calculate the wind shear effect in y-direction from equation (53).

**Card No. 6**

1) **DH**, Horizontal eddy diffusivity of heat, ft²/sec. This variable is the eddy diffusivity of heat for the water body in the
horizontal direction. It is used to calculate the turbulent heat exchange in the water in both x and y directions.

2) \( D_v \), Vertical eddy diffusivity of heat, \( ft^2/sec \). This variable is the eddy diffusivity of heat for the water body in the vertical direction. It is used to calculate the turbulent heat exchange in the water in the z direction.

3) \( XNUH \), Horizontal eddy viscosity, \( ft^2/sec \). This variable defines the eddy diffusivity of momentum for the water body in the horizontal direction. It is used to calculate the turbulent momentum transfer in the water in both x and y directions.

4) \( XNUV \), Vertical eddy viscosity, \( ft^2/sec \). This variable defines the eddy diffusivity of momentum for the water body in the vertical direction. It is used to calculate the turbulent momentum transfer in the water in the z-direction.

5) \( CP \), Specific heat of water, \( Btu/lbm^\circ F \). This quantity is the specific heat of water in the water body and is used in the calculation of heat transfer from the water to the atmosphere from equation (54).

Card No. 7

1) \( EIMAX \), Number of cells in x-direction. This variable specifies the number of space increments, in the flow field, along x-axis.

2) \( EJMAX \), Number of cells in y-direction. This variable represents the number of space increments, in the flow field, along the y-axis.

3) \( EKMAX \), Number of cells in z-direction. This variable defines the number of space increments, in the flow field, along the z-axis.
Card No. 8

1) **DEBUG1**, Debugging printout controller in the MAIN program. This quantity controls the printout of the calculated variable in the MAIN program for debugging purposes. Calculated results are printed out when DEBUG1 = 1. No printout occurs when DEBUG1 = 0.

2) **DEBUG2**, Debugging printout controller in subroutine DERIV. This quantity controls the printout of the calculated variable in subroutine DERIV for debugging purposes. Calculated results are printed out when DEBUG2 = 1. No printout occurs when DEBUG2 = 0.

3) **DEBUG3**, Unused.

4) **DEBUG4**, Debugging printout controller in subroutine PRESS. This quantity controls the printout of the calculated variables in subroutine PRESS for debugging purposes. Calculated results are printed out when DEBUG4 = 1. No printout occurs when DEBUG4 = 0.

5) **EIRUN**, Restart switch. This quantity controls the initialization of the program. When EIRUN = 0, program will start from initial condition described earlier in this section. When EIRUN = 1, program will skip the initialization and will use previous output already stored on tape as initial values. The latter tape storage is achieved by setting EIEND = 1 in the previous run as discussed next.

6) **EIEND**, Restart controller. This quantity controls the output at the completion of a dynamic run. When EIEND = 1, the final program output will be stored on tape for restarting the run at a later date. When EIEND = 0, no storage of final program output will be made.

Card No. 9

1) **EIM**, Number of thermal discharge cells in vertical direction.
This parameter indicates the number of grid cells in the vertical
direction in the thermal discharge channel.

2) **EML**, Starting location of thermal discharge. This parameter
indicates the grid cell number in the x-direction where the thermal
discharge channel begins.

3) **EMK**, End location of thermal discharge. This parameter
indicates the grid cell number in the x-direction where the thermal
discharge channel ends.

4) **SLIP**, Slip condition controller. This quantity indicates
whether boundary conditions on velocity components are calculated
using slip or non-slip conditions. When SLIP = 1., the boundary
conditions are calculated using slip conditions. When SLIP = 0., the
boundary conditions are calculated with non-slip conditions.

5) **HYD**, Hydrostatic pressure controller. This quantity indi­
cates whether pressure is calculated by hydrostatic approximation or
from equations of motion. When HYD = 1., the pressure is calculated
by hydrostatic approximation. When HYD = 0., the pressure is not
hydrostatic and is calculated from the equations of motion indicated
by equations (73) and (74).

6) **HEAT**, Temperature controller. This quantity controls whether
the thermal discharge is heated or not. When HEAT = 1., it indicates
that the thermal discharge has a higher temperature than surrounding
fluid. When HEAT = 0., it means that the thermal discharge is at same
temperature as the surrounding fluid.

7) **XIIP**, Off line storage controller. This parameter controls
the off-line storage of the output data for subsequent restart. At
the appropriate iteration, indicated by XIIP, the output are stored on a disc, and recalled in the restart mode.

2.2 Subscripted Variables

These variables involve fixed size arrays and are entered into the computer storage using FORMAT 7E10.3.

1) $\text{DELY}(I)$, $x$-increments, ft. This array defines the spatial increments, in the flow field, along the $x$-axis. The number of cards needed to enter this array is equal to $\text{EIMAX}/7$.

2) $\text{DELY}(J)$, $y$-increments. This array defines the spatial increments, in the flow field, along the $y$-axis. The number of cards needed to enter this array is equal to $\text{EJMAX}/7$.

3) $\text{DELY}(K)$, $z$-increments. This array defines the spatial increments, in the flow field, along the $z$-axis. The number of cards needed to enter this array is equal to $\text{EKMAX}/7$.

4) Density versus Temperature, lbm/ft$^3$ and degree Fahrenheit respectively. The densities and their corresponding temperatures, given in Table 1, are entered in a BLOCK DATA routine. The following format is used in entering the data:

```
BLOCK DATA
COMMON/TABLE/TTBL,RHOTBL
REAL * 4 TTBL(26)/26 values of temperature/
REAL * 4 RHOTBL(26)/26 values of density/
```
3. DESCRIPTION OF OUTPUT DATA

The output data can be conveniently divided into the following groups:

1) **Input Data Printout**
   All input data are printed out and labeled using the variable names defined in the description of input data. The temperature and density relation, entered by BLOCK DATA, are printed in a tabular form with one column labeled "Temperature" and the other column labeled "Density", followed by the appropriate values.

2) **Debugging Printout**
   At the user's option, under the control of DEBUG1, DEBUG2, DEBUG3, and DEBUG4 as described in the input data, the intermediate calculations may be printed out for debugging or checking purposes. Labels used for these printouts are described in Part 2, Section 5 in program nomenclature.

   When DEBUG1 = 1., the following variables will be printed out:
   I,J,K,DT,DTT,DTMIN,DTTMIN,DTS,DTTS,DTSM,DTTSM,XIP1,XIM1,XIPM1,XI123, XIPM,YJP1,YJM1,YJPM1,YJ123,YJPM,ZKP1,ZKM1,ZKPM1,ZK123,SIGX,SIGX1,SIGX2, SIGX3,SIGX4,SIGY1,SIGY2,SIGY3,SIGY4,SIGZ,SIGZ1,SIGZ2,SIGZ3,SIGZ4, SIGMA1,SIGMA2,SIGMA3,SIGMA4,SIGMA5,SIGMA6,XN,XM,XL,YN,YL,ZN,ZL,DUDX, DUDY.

   When DEBUG2 = 1., the following variables are printed out:
   I,J,K,IP1,IM1,JP1,JM1,KP1,KM1,DUDT,DVDT,DWDT,DVT,DIV,DQDX,DQDY,DQDZ, RHS.

   DEBUG3 is presently unused and is reserved for future applications.
For DEBUG4 = 1., the following variables are printed out:
I,J,K,ZK123,ZKP1,ZKM1,YJ123,YJP1,YJM1,XI123,XIP1,XIM1,X,X1,X2,X3,X4,
X5,X6,X7,Y1,Y2,LAMB2,OMEGA,ITN

3) Variables Defining the Problem Solution

At user's option, under the control of ENPRNT described in the input data, the solution variables can be printed out at any integration time increment or multiple thereof. These variables are printed in two columns, each column containing the variables:
4. OPERATING PROCEDURE

To employ the THERMA computer program effectively, the user should first familiarize himself with Part 1, Section 2, particularly with the mathematical formulation. The user should then draw a sketch of the receiving water similar to Fig. 7, and determine the space and time increments to be used in the analysis as indicated in Section 4. The input data is then prepared in accordance with the format and description given in Section 2 of this part. These input data are then punched on cards and assembled with the THERMA program deck and the necessary control cards. The control cards vary from one computer organization to the other. A sample of the deck assembly for the UNIVAC SERIES 70 is given below:

```
/LOGONbuser.id,acct.no,bTIME=in second
/PARAMb LIST=YES,MAP=YES,DEBUG=YES
/EXECbBGFOR

PROGRAM Name

(Fortran Source Statement)

END

/FILE THERMA.RESTART1,LINK=DSFT24,FCBTYPE=SAM
/FILE THERMA.RESTART2,LINK=DSFT25,FCBTYPE=SAM

/EXECb *

(Data)

/LOGOFF
```

1RESTART 1 designates the input offline storage
2RESTART 2 designates the output offline storage
The computer program is then run for several integration steps and the user should then verify the following points:

1) Correctness of input data. The input data printed out should be verified for possible error in punching.

2) Correctness of integration results. The integration results should be verified for consistency and the appropriateness of the initial conditions.

The program could then be run for maximum allowable computer time, at the end of which the results may be written on disc. This off-line storage may be used as input data in a later run to continue the computations. This procedure may be repeated until the results have reached steady state. After the program has been fully run, the user should verify the convergence of the solution by halving the time step $\Delta t$ and the space increments $\Delta x$, $\Delta y$, and $\Delta z$. The program is then resubmitted and results are checked to verify the agreement between the two solutions. If the solutions agree, the convergence in time and space has been established. If the solutions do not agree, it would then be necessary to choose a smaller time step or further refine the mesh, and perform the above operations again until convergence is established.
5. PROGRAM NOMENCLATURE

To assist the user in the revision of the THERMA program, a list of some key variables with their corresponding analysis notations and definitions is presented in this section.

<table>
<thead>
<tr>
<th>Program Notation</th>
<th>Analysis Notation</th>
<th>DESCRIPTION</th>
</tr>
</thead>
<tbody>
<tr>
<td>CFX</td>
<td>$C_{fx}$</td>
<td>Skin friction in x-direction</td>
</tr>
<tr>
<td>CFY</td>
<td>$C_{fy}$</td>
<td>Skin friction in y-direction</td>
</tr>
<tr>
<td>CP</td>
<td>$C_p$</td>
<td>Specific heat of water</td>
</tr>
<tr>
<td>DH</td>
<td>$D_h$</td>
<td>Horizontal eddy diffusivity of heat</td>
</tr>
<tr>
<td>DV</td>
<td>$D_v$</td>
<td>Vertical eddy diffusivity of heat</td>
</tr>
<tr>
<td>DO</td>
<td>$D_0$</td>
<td>Half width of thermal discharge channel</td>
</tr>
<tr>
<td>DT</td>
<td>$\Delta t$</td>
<td>Time increment</td>
</tr>
<tr>
<td>DELX(I)</td>
<td>$\Delta x_i$</td>
<td>Space increment in x-direction</td>
</tr>
<tr>
<td>DELY(J)</td>
<td>$\Delta y_j$</td>
<td>Space increment in y-direction</td>
</tr>
<tr>
<td>DELZ(K)</td>
<td>$\Delta z_k$</td>
<td>Space increment in z-direction</td>
</tr>
<tr>
<td>DEBUG1</td>
<td>-</td>
<td>Control for debugging of MAIN program</td>
</tr>
<tr>
<td>DEBUG2</td>
<td>-</td>
<td>Control for debugging of subroutine DERIV</td>
</tr>
<tr>
<td>DEBUG3</td>
<td>-</td>
<td>Unused</td>
</tr>
<tr>
<td>DEBUG4</td>
<td>-</td>
<td>Control for debugging of subroutine PRESS</td>
</tr>
<tr>
<td>Program</td>
<td>Analysis</td>
<td>Notation</td>
</tr>
<tr>
<td>---------</td>
<td>----------</td>
<td>----------</td>
</tr>
<tr>
<td>DUDT</td>
<td>Time derivative of u component of velocity</td>
<td></td>
</tr>
<tr>
<td>DVDT</td>
<td>Time derivative of v component of velocity</td>
<td></td>
</tr>
<tr>
<td>DWDT</td>
<td>Time derivative of w component of velocity</td>
<td></td>
</tr>
<tr>
<td>DTDT</td>
<td>Time derivative of temperature</td>
<td></td>
</tr>
<tr>
<td>DHDT</td>
<td>Time derivative of water-level</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>Equilibrium temperature of water body</td>
<td></td>
</tr>
<tr>
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<td>Description</td>
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<td>Analysis</td>
<td>Description</td>
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<td>Analysis Notation</td>
<td>Description</td>
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<td>ZKPM1(K)</td>
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6. PROGRAM LISTING AND SAMPLE RUN

In this section, the FORTRAN program listing of the THERMA computer program is presented. This listing is followed by a sample run for non-buoyant jet discussed in Part 1, Section 5.3. To reduce the bulk of computer printout, the output data are supplied for iterations number 0, 1 and 50 corresponding to times 0, 0.2 and 10 seconds respectively.
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<td>HYDROSTATIC PRESSURE CONTROLLER</td>
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<td>OFF LINE STORAGE CONTROLLER</td>
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ORTRAN IV (VER 45) SOURCE LISTING:
03/31/77 22:00:10 PAGE 0002

51 C
53 C SUBSCRIPTED VARIABLES
54 C VARIABLE NAME DESCRIPTION
55 C DELX(I)  DIMENSION OF ELEMENT I IN X-DIRECTION  FT
56 C DELY(J)  DIMENSION OF ELEMENT J IN Y-DIRECTION  FT
57 C DELZ(K)  DIMENSION OF ELEMENT K IN Z-DIRECTION  FT
58 C V(I,J,K)  Y-COMPONENT OF VELOCITY
59 C W(I,J,K)  Z-COMPONENT OF VELOCITY
60 C T(I,J,K)  TEMPERATURE
61 C H(I,J)  WATER LEVEL HEIGHT
62 C P(I,J,K)  PRESSURE
63 C RHO(I,J,K)  DENSITY
64 C IMAX  NUMBER OF CELLS IN X-DIRECTION
65 C JMAX  NUMBER OF CELLS IN Y-DIRECTION
66 C KMAX  NUMBER OF CELLS IN Z-DIRECTION
67 C X  COEFFICIENT OF P(I+1,J+K)
68 C X1  COEFFICIENT OF P(I,J+K+1)
69 C X2  COEFFICIENT OF P(I,J+K-1)
70 C X3  COEFFICIENT OF P(I+1,J,K)
71 C X4  COEFFICIENT OF P(I+1,J+1,K)
72 C X5  COEFFICIENT OF P(I+1,J-K+1)
73 C X6  COEFFICIENT OF P(I+1,J-K-1)
74 C X7  COEFFICIENT OF RHS(I,J,K)
75 C COMMON P(20,14,10),DELX(20),DELY(14),DELZ(10),RHO(20,14,10),
76 C U(20,14,10),V(20,14,10),W(20,14,10),T(20,14,10),
77 C 2DUDT(20,14,10),DVDT(20,14,10),BDHDT(20,14,10),DTDT(20,14,10),
78 C 3XNUH,3XNUM,RO,F,SO,DH,ON,ERUN,TBEG,TEND,IP,J,1,F,
79 C 4INDEX,WINDY,E,RHA,A,IMAX,JMAX,KMAX,OMEGA,1,2,3,4,
80 C 5DEBUG,DEBUG2,DEBUG3,DEBUG4,DEBUG5,2NUR2NUR,2XDR,2XVR,2G,2DT,2,
81 C 6NITEST,PRINT,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18,19,20,21,22,23,24,25,26,27,28,29,30,31,32,33,34,35,36,37,38,39,40,41,42,43,44,45,46,47,48,49,50,51,52,53,54,55,56,57,58,59,60,61,62,63,64,65,66,67,68,69,70,71,72,73,74,75,76,77,78,79,80,81,82,83,84,85,86,87,88,89,90,91,92,93,94,95,96,97,98,99,100

1001 FORMAT(1E10.3)
1002 FORMAT(2F10.3)
1003 FORMAT(3F10.3)
1004 FORMAT(4F10.3)
1005 FORMAT(5F10.3)
1006 FORMAT(6F10.3)
1007 FORMAT(7F10.3)
1008 FORMAT(8F10.3)
FORTRAN IV (VER 45) SOURCE LISTING: THERMA PROGRAM

03/31/77 22:00:10 PAGE 0001

101  604 FORMAT(1H0,10X,'II',4X,'XN',15X,'XM')
102  608 FORMAT(1H0,10X,3(I3,2X,E11.4))
103  609 FORMAT(1H0,10X,'J',4X,'YN(J)',15X,'YM(J)')
104  610 FORMAT(1H0,10X,'K',4X,'ZN(K)',15X,'ZM(K)')
105  614 FORMAT(1H0,10X,'I',2X,E11.4,2X)
106  615 FORMAT(1H0,10X,'J',2X,E11.4,2X,'DT',5X,'DTT')
107  616 FORMAT(1H0,10X,3(I2,2X),E11.4,12X,3(I2,2X),E11.4)
108  621 FORMAT(1H0,10X,3(I2,2X),'STABILITY CONDITION FAILS IN X-DIRECTION')
109  617 FORMAT(1H0,10X,'I',2X,5(12,2X,E11.4))
110  622 FORMAT(1H0,10X,3(I2,2X),'STABILITY CONDITION FAILS IN Y-DIRECTION')
111  623 FORMAT(1H0,10X,'I',2X,5(12,2X,E11.4))
112  624 FORMAT(1H0,10X,3(I2,2X),'STABILITY CONDITION FAILS IN Z-DIRECTION')
113  625 FORMAT(1H0,10X,3(I2,2X),E11.4,12X,3(I2,2X),E11.4)
114  626 FORMAT(1H0,10X,6(I2,2X,E11.4,2X))
115  627 FORMAT(1H0,10X,'I',4X,'XH(I)')
116  630 FORMAT(1H0,10X,'J',4X,'YH(J)')
117  631 FORMAT(1H0,10X,'K',4X,'ZH(K)')
118  640 FORMAT(1H0,10X,3(12,2X),E11.4)
119  641 FORMAT(1H0,10X,3(12,2X),E11.4)
120  642 FORMAT(1H0,10X,'I',2X,'YL(J)')
121  643 FORMAT(1H0,10X,'I',2X,'ZL(K)')
122  644 FORMAT(1H0,10X,'I',2X,'SIGMA1',5X,'SIGMA2')
123  645 FORMAT(1H0,10X,'I',2X,5X,E11.4,2X)
124  646 FORMAT(1H0,10X,'J',5X,'SIGMA3',5X,'SIGMA4')
125  647 FORMAT(1H0,10X,'K',5X,'SIGMA5',5X,'SIGMA6')
126  650 FORMAT(1H0,10X,3(I2,2X),E11.4,12X,3(I2,2X),E11.4)
127  651 FORMAT(1H0,10X,3(I2,2X),E11.4,12X,3(I2,2X),E11.4)
128  652 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DT',10X)
129  653 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
130  654 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTS',10X)
131  655 FORMAT(1H0,10X,3(I2,2X),E11.4,12X,3(I2,2X),E11.4)
132  656 FORMAT(1H0,10X,3(I2,2X),E11.4,12X,3(I2,2X),E11.4)
133  657 FORMAT(1H0,10X,'DT BECAME TOO SMALL AT',2X,'I=','%12.2X,'J=',%12.2X)
134  658 FORMAT(1H0,10X,'HLEVEL=','%12.2X,'CLEVEL(I,J)=','%12.4X)
135  659 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
136  660 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTS',10X)
137  661 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
138  662 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
139  663 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
140  664 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
141  665 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
142  666 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
143  667 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
144  668 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
145  669 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
146  670 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
147  671 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
148  672 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
149  673 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
150  674 FORMAT(1H0,10X,'I',2X,'J',2X,'KMAX',2X,'SUGGESTED DTTS',10X)
FORTRAN IV (VER 45) SOURCE LISTING: THERMA PROGRAM 03/31/77 22:00:10 PAGE 0004

151 1E11,4,5X,'DEBUG4=',E11,4,5X,'E11,4,5X,'E11,4,5X,'E11,4,5X,'E11,4,5X
152 1034 FORMAT(1HO,10X,'I\HAX '='I5,5X,'JMAX '='I5,5X,'KMAX '='I5
153 1,15 )
154 1035 FORMAT(1HO,10X,'I1 '='I5,5X,'J1 '='I5,5X,'K1 '='I5
155 1,15 )
156 1036 FORMAT(1HO,10X,'I2 '='I5,5X,'J2 '='I5,5X,'K2 '='I5
157 1,15 )
158 1037 FORMAT(1HO,10X,'G '='E11,4,5X,'DT '='E11,4,5X,'E '='E14,7)
159 1038 FORMAT(1HO,10X,'I1,3X,'DELX(I)=''
160 1039 FORMAT(2(IHO,10X,4(I2,2X,E11,4,5X))/)
161 1040 FORMAT(1HO,10X,'I1,3X,'DELY(J)=''
162 1041 FORMAT(1HO,10X,'I1,3X,'DELZ(K)=''
163 1042 FORMAT(1HO,10X,'XNUR '='E11,4,5X,'ZNUR '='E11,4,5X,'XDHR '='
164 1043 FORMAT(I15)
165 1044 FORMAT(1HO,10X,'I3 '='I5,5X,'J3 '='I5,5X,'K3 '='I5
166 1045 FORMAT(1HO,10X,'K='I2,1X,'ZKP1(K)=''=E11,4,1X,
167 1046 FORMAT(1HO,'K=I2,1X,I12,1X)'ZKP1(K)=''=E11,4,1X,
168 1047 FORMAT(1HO,'K=I2,1X,I12,1X)'ZKP1(K)=''=E11,4,1X,
169 1048 FORMAT(1HO,10X,'I='I2,1X,'ZKP1(K)=''=E11,4,1X,
170 1049 FORMAT(1HO,'K=I2,1X,I12,1X)'ZKP1(K)=''=E11,4,1X,
171 1050 FORMAT(1HO,10X,'I='I2,1X,'ZKP1(K)=''=E11,4,1X,
172 1051 FORMAT(1HO,10X,'I='I2,1X,'ZKP1(K)=''=E11,4,1X,
173 1052 FORMAT(1HO,20X,20('*')),'NON-DIMENSIONAL INPUT VALUES ',20('*'))
174 1053 FORMAT(1HO,20X,20('*')),'END OF NON-DIMENSIONAL INPUT VALUES',20('*'))
175 1054 FORMAT(1HO,20X,20('*')),'NON-DIMENSIONAL INPUT VALUES ',20('*'))
176 1055 FORMAT(I20,5(E14,7,2X))
177 1056 FORMAT(I20,5(E14,7,2X))
178 1057 FORMAT(I20,5(E14,7,2X))
179 1058 FORMAT(I20,5(E14,7,2X))
180 1059 FORMAT(I20,5(E14,7,2X))
181 1060 FORMAT(I20,5(E14,7,2X))
182 1061 FORMAT(I20,5(E14,7,2X))
183 1062 FORMAT(I20,5(E14,7,2X))
184 1063 FORMAT(I20,5(E14,7,2X))
185 1064 FORMAT(I20,5(E14,7,2X))
186 1065 FORMAT(I20,5(E14,7,2X))
187 1066 FORMAT(I20,5(E14,7,2X))
188 1067 FORMAT(I20,5(E14,7,2X))
189 1068 FORMAT(I20,5(E14,7,2X))
190 1069 FORMAT(I20,5(E14,7,2X))
191 1070 FORMAT(I20,5(E14,7,2X))
192 1071 FORMAT(I20,5(E14,7,2X))
193 1072 FORMAT(I20,5(E14,7,2X))
194 1073 FORMAT(I20,5(E14,7,2X))
195 1074 FORMAT(I20,5(E14,7,2X))
196 1075 FORMAT(I20,5(E14,7,2X))
197 1076 FORMAT(I20,5(E14,7,2X))
198 1077 FORMAT(I20,5(E14,7,2X))
199 1078 FORMAT(I20,5(E14,7,2X))
200 C READ IN NON SUBSCRIPTED VARIABLES
201 READ(5,1001) ENRUN
202 WRITE(6,1026) ENRUN
203 READ(5,1004) TBEG,TEND,ENITER,ENPRINT
204 WRITE(6,1027) TBEG,TEND,ENITER,ENPRINT
205 READ(5,1003) UI,VI,FI
206 WRITE(6,1028) UI,VI,FI
207 READ(5,1005) U0,ANGLE,T0,D0,XNUZ
208 WRITE(6,1058) U0,ANGLE,T0,D0,XNUZ
209 READ(5,1007) WINDX,WINDY,RH0A,E,XK,CFX,CFY
210 WRITE(6,1029) WINDX,WINDY,RH0A,E,XK,CFX,CFY
211 READ(5,1005) DH,DV,XNUH,XNUV,CP
212 WRITE(6,1030) DH,DV,XNUH,XNUV,CP
213 READ(5,1003) EIMAX,EMAX,EMAX
214 WRITE(6,1031) EIMAX,EMAX,EMAX
215 READ(5,1006) DEBUG1,DEBUG2,DEBUG3,DEBUG4,EMAIL,END
216 WRITE(6,1032) DEBUG1,DEBUG2,DEBUG3,DEBUG4,EMAIL,END
217 READ(5,1007) EIM,EML,EMK,SILP,HEAT,X11P
218 WRITE(6,1071) EIM,EML,EMK,SILP,HEAT,X11P
219 C READ IN SUBSCRIPTED VARIABLES
220 READ(5,1007) (DELX(I),I=1,IMAX)
221 WRITE(6,1038)
222 WRITE(6,1039) (I,DELX(I),I=1,IMAX)
223 READ(5,1007) (DELY(J),J=1,JMAX)
224 WRITE(6,1040)
225 WRITE(6,1039) (J,DELY(J),J=1,JMAX)
226 READ(5,1007) (DELZ(K),K=1,KMAX)
227 WRITE(6,1041)
228 WRITE(6,1039) (K,DELZ(K),K=1,KMAX)
229 WRITE(6,1065)
230 DO 505 L=1,26
231 WRITE(6,1064) TBL(L),RHTBL(L)
232 505 CONTINUE
233 IM=EIM
234 ML=EML
235 MK=EMK
236 MH=MK+1
237 IMAX=EIMAX
238 JMAX=EMAX
239 KMAX=EMAK
240 CONTINUE
K3=KMAX-3
ITNMAX=500
EPS=.00001
TIME=0.
IP=0
IPRINT=0
G=32.2
DSM=1000.
DTSM=1000.
DMIN=1000.
DTMIN=1000.
WRITE(6,1034) IMAX,JMAX,KMAX
WRITE(6,1035) I,J,K
WRITE(6,1046) I,J,K
DO 624 I=2,IMAX
   XM(I)=4.*XNUH/XM1(I)
   XM(I)=4.*DH/XM1(I)
624 CONTINUE
DO 625 J=2,JMAX
   YJ=2,J1
   YM(J)=4.*XNUH/YJM1(J)
   YM(J)=4.*DH/YJM1(J)
625 CONTINUE
DO 626 K=2,KMAX
   ZK=2,K1
   ZH(K)=4.*DV/ZK1(K)
626 CONTINUE
WRITE(6,1066) RHOZ
WRITE(6,623) (I,X(I),I=2,IMAX)
WRITE(6,624) (J,YJ(J),J=2,JMAX)
WRITE(6,625) (K,ZK(K),K=2,KMAX)
C INITIALIZE WATER LEVEL HEIGHT
DO 706 I=1,IMAX
DO 706 J = 1, JMAX
H(I, J) = DELZ(KMAX)/2.
DO 706 CONTINUE
C
CALCULATE DT
IF (DEBUG1, EQ, 0.) GO TO 655
WRITE (6, 615)
CONTINUE
DO 613 K = 2, K1
DO 613 J = 2, J1
DO 613 I = 2, I1
DT = 1/(2.*XN(I)/XIM1(I)*YN(J)/YJM1(J)*ZN(K)/ZKM1(K))
DTT = 1/(2.*XH(I)/XIM1(I)*YM(J)/YJM1(J)*ZM(K)/ZKM1(K))
IF (DEBUG1, EQ, 0.) GO TO 656
WRITE (6, 614) I, J, K, DT, DTT
CONTINUE
IF (DT .LT. DTMIN) GO TO 701
GO TO 702
701 DTMIN = DT
IIH = I
JJH = J
KKH = K
GO TO 703
702 IF (DTT .LT. DTTHMIN) GO TO 703
GO TO 613
703 DTTHMIN = DTT
IIT = I
JJT = J
KKT = K
CONTINUE
WRITE (6, 705)
C**********************
WRITE (6, 616) IIH, JJH, KKH, DTTHMIN, IIT, JJT, KKT, DTTHMIN
IF (DEBUG1, EQ, 0.) GO TO 717
WRITE (6, 709)
CONTINUE
DO 707 I = 2, I1
DO 707 J = 2, J1
DTS = 1/(2.*XN(I)/XIM1(I)*YN(J)/YJM1(J)*XNUV/H(I,J)*H(I,J) *
  1DELZ(K1)))
DTTS = 1/(2.*XH(I)/XIM1(I)*YM(J)/YJM1(J)*DV/H(I,J)*H(I,J) *
  1DELZ(K1)))
IF (DEBUG1, EQ, 0.) GO TO 708
WRITE (6, 710) I, J, KMAX, DTS, DTTS
GO TO 711
708 CONTINUE
IF (DTS, LT, DTTHMIN) GO TO 711
GO TO 712
711 DTSH = DTS
GO TO 707
FORTRAN IV (VER 45) SOURCE LISTING: THERMA PROGRAM  03/31/77  22:00:10  PAGE 0008

351    713  DTTSM=DTTS
352    714  IIT=I
353    715  JJT=J
354    707  CONTINUE
355     WRITE(6,714)  IIM, JJM, KMAX, DTSM, IIT, JJT, KMAX, DTTSM
356    715  DT=DTHN
357
358  C*****BEGIN NONDIMENSIONALIZATION
359  C
360  WRITE(6,1062)
361     TEBG=TEBG*U0/DO
362     TEND=TEND*U0/DO
363    1062    DT=(TEND-TEBG)/ENITER
364     UI=UI/U0
365     VI=VI/U0
366     TI=TI/T0
367     WINDX=WINDX/U0
368     WINDY=WINDY/U0
369     RHOA=RHOA/RHOZ
370     TD=TD/T0
371     DH=DH/XNUZ
372     DV=DV/XNUZ
373     XNUH=XNUH/XNUZ
374     XNUV=XNUV/XNUZ
375     DO 13 1=1,IMAX
376     DO 13 J=1,JMAX
377  13     H(I,J)=H(I,J)/DO
378     CONTINUE
379     DO 504 LL=1,26
380     TTBL(LL)=TTBL(LL)/T0
381     RHTBL(LL)=RHTBL(LL)/RHOZ
382     CONTINUE
383     DO 601 I=1,IMAX
384     DELX(I)=DELX(I)/DO
385     CONTINUE
386     DO 602 J=1,JMAX
387     DELY(J)=DELY(J)/DO
388     CONTINUE
389     DO 603 K=1,KMAX
390     DELZ(K)=DELZ(K)/DO
391     CONTINUE
392     RO=DO*U0/XNUZ
393     FO=U0/((G*DO)**.5)
394     E=E/T0
395     XK=XK/(3600*24)
396     SD=XK/(RHOZ*CP*U0)
397     U0=U0/U0
398     T0=T0/T0
399     XNUZ=XNUZ/XNUZ
400
FORTRAN IV (VER 45) SOURCE LISTING: THERMA PROGRAM

401 WRITE(6,1026) ENRUN
402 WRITE(6,1067) TBEG,TEND,NITER,NPRINT
403 WRITE(6,1028) UI,VI,TI
404 WRITE(6,1058) U0,ANGLE,T0,DO,XNUZ
405 WRITE(6,1069) WINDX,WINDY,RHOA
406 WRITE(6,1068) DH,DV,XNUH,XNUV
407 WRITE(6,1037) G,DT,E
408 WRITE(6,1038)
409 WRITE(6,1039) (1,DELX(I),I=1,IMAX)
410 WRITE(6,1040) (1,DELY(J),J=1,IMAX)
411 WRITE(6,1041) (1,DELY(J),J=1,IMAX)
412 WRITE(6,1042) (1,DELY(J),J=1,IMAX)
413 WRITE(6,1043) (1,DELY(J),J=1,IMAX)
414 WRITE(6,1044) (1,DELY(J),J=1,IMAX)
415 WRITE(6,1045) (1,DELY(J),J=1,IMAX)
416 DO 507 L=1,26
417 WRITE(6,1064) TTBL(L),RHOTBL(L)
418 CONTINUE
419 WRITE(6,1063)
420 DO 33 K=2,K1
421 ZKP1(K) = DELZ(K-1) + DELZ(K)
422 ZKM1(K) = DELZ(K) + DELZ(K-1)
423 ZKPM1(K) = ZKPI(K) * ZKMI(K)
424 ZK123(K) = ZKPI(K) * ZKMI(K) * ZKPM1(K)
425 CONTINUE
426 DO 34 J=2,J1
427 YJP1(J) = DELY(J-1) + DELY(J)
428 YJM1(J) = DELY(J) + DELY(J-1)
429 YJP1(J) = YJP1(J) * YJM1(J)
430 YJ123(J) = YJP1(J) * YJM1(J) * YJP1(J)
431 CONTINUE
432 DO 35 I=2,I1
433 XIP1(I) = DELX(I-1) + DELX(I)
434 XIM1(I) = DELX(I) + DELX(I-1)
435 XIP1(I) = XIP1(I) * XIM1(I)
436 X1123(I) = XIP1(I) * XIM1(I) * XIPM1(I)
437 CONTINUE
438 IF(DEBUG1,EQ.0.) GO TO 18
439 DO 17 J=2,J1
440 DO 17 K=2,K1
441 DO 17 I=2,I1
442 WRITE(6,1048) K,ZKP1(K),ZKMI(K),ZKPM1(K),ZK123(K),
443 1J,YJP1(J),YJM1(J),YJP1(J),YJ123(J),
444 2I,XIP1(I),XIM1(I),XIPM1(I),X1123(I)
445 CONTINUE
446 CONTINUE
447 CONTINUE
448 CONTINUE
449 CONTINUE
450 CONTINUE
451 C  COEFFICIENT OF VISCOS TERMS
452 XNUR = XNUH/RC
453 ZNUR = XNUV/RC
454 XDHR = DH/RC
455 XDVR = DV/RC
456 WRITE(6,1042) XNUR,ZNUR,XDHR,XDVR
457 DD=2.*DT
458 26 FORMAT(1HO,3(I2),9(I2,E11.4))
459 F01=F0**2
460 C
461 C  Z=TOTAL HEIGHT OF LAKE
462 Z=0.
463 DO 110 L=1,K1
464 Z=Z-ZKPL(L)/2.
465 IF(DEBUG1.EQ.0.) GO TO 58
466 WRITE(6,1072) Z
467 58 CONTINUE
468 110 CONTINUE
469 59 FORMAT(1H5, 'Z=',F5.2)
470 C  FIRST DERIVATIVE CORRECTION
471 DO 311 K=2,K1
472 SIGMA5(K)=2,*ZKM1(K)/ZKPM1(K)
473 SIGMA6(K)=2,*ZKPM1(K)/ZKPM1(K)
474 311 CONTINUE
475 DO 312 J=2,J1
476 SIGMA3(J)=2,*YJM1(J)/YJPM1(J)
477 SIGMA4(J)=2,*YJPM1(J)/YJPM1(J)
478 312 CONTINUE
479 DO 313 I=2,I1
480 SIGMA1(I)=2,*XIM1(I)/XIPM1(I)
481 SIGMA2(I)=2,*XIP1(I)/XIPM1(I)
482 313 CONTINUE
483 C  IF (DEBUG1.EQ.0.) GO TO 92
484 WRITE(6,644)
485 C************
486 DO 657 I=2,I1
487 WRITE(6,645) I,SIGMA1(I),SIGMA2(I)
488 657 CONTINUE
490 C************
491 DO 658 J=2,J1
492 WRITE(6,646) J,SIGMA3(J),SIGMA4(J)
493 658 CONTINUE
495 C************
496 WRITE(6,647)
497 DO 659 K=2,K1
498 WRITE(6,645) K,SIGMA5(K),SIGMA6(K)
499 659 CONTINUE
500 C************
501 648 CONTINUE
502 52 CONTINUE
503 C*************
504 C CALCULATE COEFFICIENT OF FIRST DERIVATIVE AT BOUNDARY
505 C*************
506 SIGY=(DELY(1)+DELY(2))/(DELY(2)+DELY(3))
507 SIGY1=(1.+SIGY)
508 SIGY2=(1.+SIGY)*(1.+SIGY)
509 SIGY3=SIGY+SIGY
510 SIGY4=(DELY(1)+DELY(2))*(1.+SIGY)/2.
511 SIGPY=(DELY(JMAX)+DELY(J1))/(DELY(J1)+DELY(J2))
512 SIGPY1=(1.+SIGPY)
513 SIGPY2=(1.+SIGPY)*(1.+SIGPY)
514 SIGPY3=SIGPY+SIGPY
515 SIGPY4=(DELY(JMAX)+DELY(J1))*(1.+SIGPY)/2.
516 IF(DEBUG1.EQ.0.) GC TO 53
517 WRITE(6,110) SIGY,SIGY1,SIGY2,SIGY3,SIGY4,SIGPY,SIGPY1,SIGPY2,
518 2SIGPY3,SIGPY4
519 53 CONTINUE
520 SIGX=(DELX(1)+DELX(2))/(DELX(2)+DELX(3))
521 SIGX1=(1.+SIGX)
522 SIGX2=(1.+SIGX)*(1.+SIGX)
523 SIGX3=SIGX+SIGX
524 SIGX4=(DELX(1)+DELX(2))*(1.+SIGX)/2.
525 IF(DEBUG1.EQ.0.) GC TO 54
526 WRITE(6,110) SIGX,SIGX1,SIGX2,SIGX3,SIGX4
527 54 CONTINUE
528 SIGZ=(DELZ(1)+DELZ(2))/(DELZ(2)+DELZ(3))
529 SIGZ1=(1.+SIGZ)
530 SIGZ2=(1.+SIGZ)*(1.+SIGZ)
531 SIGZ3=SIGZ+SIGZ
532 SIGZ4=(DELZ(1)+DELZ(2))*(1.+SIGZ)/2.
533 IF(DEBUG1.EQ.0.) GC TO 56
534 WRITE(6,110) SIGZ,SIGZ1,SIGZ2,SIGZ3,SIGZ4
535 56 CONTINUE
536 C END CALCULATION OF COEFFICIENT OF FIRST DERIVATIVE
537 DMH=MAXI(XDRX,XNUR)
538 DMV=MAXI(XDRV,ZNUR)
539 DO 718 I=2,11
540 DO 718 J=2,J1
541 CLEVEL(I,J)=(1./DT)-8.*DMH/(XIM1(I)*XIM1(J))-8.*DMH/(YJM1(J)*
542 1YJM1(J))
543 718 CONTINUE
544 C*************
545 C CHECK FOR RESTART ENTRY FOR DYNAMIC ANALYSIS
546 C*************
547 C BEGIN INITIALIZATION
548 IF(EIRUN.EQ.1.) GO TO 75
549 C*************
550 DO 5 K=1,KMAX

The FORTRAN code calculates the coefficients of the first derivative at the boundary. It involves calculations for SIGY, SIGPY, SIGX, SIGPY, SIGZ, and SIGZ4, and then proceeds to initialize variables for subsequent calculations.
DO 5 J=1,JMAX
5 CONTINUE
DO 5 I=1,IMAX
5 CONTINUE
U(I,J,K)=0,
V(I,J,K)=0,
W(I,J,K)=0,
T(I,J,K)=0,
RHS(I,J,K)=0,
DUDT(I,J,K)=0,
DVDT(I,J,K)=0,
DWDT(I,J,K)=0,
DTD(I,J,K)=0,
DMDT(I,J,K)=0,
5 CONTINUE

C-----INITIALIZATION OF HORIZONTAL VELOCITY COMPONENTS U AND V
DO 25 K=2,KMAX
DO 25 J=1,J1
DO 25 I=1,IMAX
U(I,J,K)=UI
V(I,J,K)=VI
25 CONTINUE
Z1=Z

C-----INITIALIZATION OF TEMPERATURE
DO 502 K=1,KMAX
DO 506 J=1,JMAX
DO 506 I=1,IMAX
T(I,J,K)=TI
RHO(I,J,K)=SI(TTBL,RHOTBL,T(I,J,K),26)
P(I,J,K)=-RHO(I,J,K)*Z
506 CONTINUE
IF(K.EQ.KMAX) GO TO 502
Z=Z+ZKP1(K)/2.
IF(K.EQ.K1) Z=-DELZ(KMAX)/4.
IF(DBG1.EQ.0.) GO TO 57
WRITE(6,1072) Z
57 CONTINUE
502 CONTINUE
Z = Z1

C END INITIALIZATION
C SET THE BOUNDARY CONDITIONS
C FIXED BOUNDARY CONDITIONS
C-----FIXED BOUNDARY CONDITION ON TEMPERATURE IS BEEN SET DURING INITIALIZATION.
C FIXED VELOCITY BOUNDARY CONDITION
C
DO 4 K=2,KMAX
DO 4 J=2,J1
U(I,J,K)=UI
4 CONTINUE
C-----SET VELOCITY OF THERMAL DISCHARGE
FORTRAN IV (VER 45) SOURCE LISTING: THERMA PROGRAM 03/31/77 22:00:10 PAGE 0013

601 DO 63 I=ML,MK
602 DO 63 K=IM,KMAX
603 U(I,1,K)=U0*COS(ANGLE)
604 V(I,1,K)=U0*SIN(ANGLE)
605 63 CONTINUE
606 GO TO 79
607 75 CONTINUE

608 C*****READING FROM DISC*******************************
609 WRITE(6,2006) TIME,DT,IP,KMAX,JMAX,IMAX,IPRINT
610 READ(24) TIME,DT,IP,KMAX,JMAX,IMAX,IPRINT
611 C******DISC OUTPUT*******************************
612 WRITE(6,2006) TIME,DT,IP,KMAX,JMAX,IMAX,IPRINT
613 WRITE(6,2007)
614 DO 76 K=1,KMAX
615 DO 76 J=1,JMAX
616 DO 76 I=1,IMAX
618 IY(I,J,K)
620 IY(I,J,K),H(I,J)
621 76 CONTINUE
622 C********END OF DISC OUTPUT**********
623 IPRINT=0
624 79 CONTINUE
625 C
626 DO 635 I=2,II
627 XN(I)=4.*XNUR/XM1(I)
628 XM(I)=4.*XDHR/XM1(I)
629 XL(I)=AMIN1(XN(I),XM(I))
630 635 CONTINUE
631 DO 636 J=2,J1
632 YN(J)=4.*XNUR/YM1(J)
633 YM(J)=4.*XDHR/YM1(J)
634 YL(J)=AMIN1(YN(J),YM(J))
635 636 CONTINUE
636 DO 637 K=2,K1
637 ZN(K)=4.*ZNUR/ZKM1(K)
638 ZM(K)=4.*ZDHR/ZKM1(K)
639 ZL(K)=AMIN1(ZN(K),ZM(K))
640 637 CONTINUE
641 IF(DEBUG1.EQ,0.) GC TO 654
642 WRITE(6,604)
643 WRITE(6,608)(I,XN(I),XM(I),I=2,II)
644 WRITE(6,609)
645 WRITE(6,608)(J,YN(J),YM(J),J=2,J1)
646 WRITE(6,610)
647 WRITE(6,608)(K,ZN(K),ZM(K),K=2,K1)
648 WRITE(6,640)
649 WRITE(6,641)(I,XL(I),I=2,II)
650 WRITE(6,642)
C FORTRAN IV (VER 45) SOURCE LISTING: THERMA PROGRAM 03/31/77 22:00:10 PAGE 0014

651 WRITE(6,641) (J,YL(J),J=2,JI)
652 WRITE(6,643)
653 WRITE(6,641) (K,ZL(K),K=2,K1)
654 654 CONTINUE
655 WRITE(6,1025)
656 IF(HEAT.EQ.0.) GO TO 1
657 C SET BOUNDARY AT THERMAL DISCHARGE INLET
658 C TEMPERATURE
660 DO 61 K=1,KMAX
661 DO 61 I=ML,MK
662 T(I,1,K)=1.
663 61 CONTINUE
664 C PRESSURE
665 C
666 DO 43 K=1,KMAX
667 DO 42 I=M1,MK
668 P(I,1,K)=-Z
669 42 CONTINUE
670 Z=Z+ZP(K)/2.
671 43 CONTINUE
672 IF(K.EQ.K1) Z=-DELZ(KMAX)/4,
673 Z = Z1
674 43 CONTINUE
675 C ********** BEGINNING OF THE DYNAMIC LOOP **********
676 C VARIABLE BOUNDARY CONDITIONS
677 C
678 1 CONTINUE
679 C CHECK STABILITY FOR U,V, AND W
680 DO 617 K=2,KMAX
681 DO 617 I=M1,MK
682 IF(Abs(U(I,J,K)).GT.XL(I)) GO TO 618
683 IF(Abs(V(I,J,K)).GT.YL(J)) GO TO 619
684 IF(K.EQ.KMAX) GO TO 716
685 IF(Abs(W(I,J,K)).GT.ZL(K)) GO TO 620
686 617 CONTINUE
687 716 CONTINUE
688 GO TO 617
689 618 WRITE(6,621) I,J,K
690 IPRINT=NPRINT
691 CALL OUTP
692 CALL EXIT
693 619 WRITE(6,622) I,J,K
694 IPRINT=NPRINT
695 CALL OUTP
696 CALL EXIT
697 620 WRITE(6,623) I,J,K
698 IPRINT=NPRINT
CALL OUTP
CALL EXIT
CONTINUE
DO 719 I=2,J1
    DO 719 J=2,J1
        HLEVEL = 2.*DMV/(H(I,J)+(H(I,J)+DELZ(K1)))
        IF(HLEVEL.LT.CLEVEL(I,J)) GO TO 719
        WRITE(6,720) I,J,HLEVEL,CLEVEL(I,J)
        IPRINT=NPRINT
        CALL OUTP
        CALL EXIT
    CONTINUE
END STABILITY ANALYSIS

OUTGOING BOUNDARY ON VELOCITY
DO 70 J=1,JMAX
    DO 70 K=2,KMAX
        U(IMAX,J,K)=U(I1,J,K)
        V(IMAX,J,K)=V(I1,J,K)
CONTINUE

BOTTOM BOUNDARY ON HORIZONTAL VELOCITY COMPONENTS
DO 236 I=1(IMAX
    DO 236 J=1,JMAX
        V(I,J,1)=0.
        U(I,J,1)=0.
CONTINUE

SOLID-FLUID BOUNDARY ON HORIZONTAL VELOCITY COMPONENT U
IF(SLIP.EQ.0.) GO TO 950
    DO 6 K=2,KMAX
        DO 6 I=1,IMAX
            U(I,JMAX,K)=(SIGPY2*U(I,J1,K)-SIGPY3*U(I,J2,K))/SIGPY1
        CONTINUE
        IF(K.LT.IM) GO TO 134.
        IF(I.LT.MH .AND. I.GE.ML ) GO TO 137
CONTINUE

SOLID-FLUID BOUNDARY ON TEMPERATURE
DO 130 I=1,IMAX
    DO 130 K=2,KMAX
        T(I,JMAX,K)=(SIGPY2*T(I,J1,K)-SIGPY3*T(I,J2,K))/SIGPY1
    CONTINUE
    IF(K.LT.IM) GO TO 124
    IF(I.LT.MH .AND. I.GE.ML ) GO TO 126
CONTINUE
CONTINUE
T(I,J,K)=SIGY2*T(I,J,2)-SJGY3*T(I,J,3)/SIGY1
CONTINUE
CONTINUE
SURFACE BOUNDARY ON VERTICAL VELOCITY COMPONENT IS ZEROED DURING INITIALIZATION
SURFACE BOUNDARY ON HORIZONTAL VELOCITY COMPONENTS U AND V
DO 7 I=1,IMAX
DO 7 J=1,IMAX
T(I,J,K)=T(I,J,1)/SIGZ1
CONTINUE
DO 73 I=1,IMAX
DO 73 J=1,IMAX
T(IMAX,J,K)=T(I1,J,K)
CONTINUE
CALCULATION OF DHDT(I,J) FOR THE TOP ELEMENTS
DO 661 I=2,II
IM1=I-1
DO 661 J=2,J1
JM1=J-1
DHDT(I,J)=-2.*(H(I,J)*(U(I,J,KMAX)-U(IM1,J,KMAX))*
1*(U(I,J,KMAX)-W(IM1,J,K))/XIM1(I)*
2-2.*(H(I,J)*(V(I,J,KMAX)-V(IM1,J,KMAX))*
3*(V(I,J,KMAX)-W(I,J,K))/YJM1(J)
CONTINUE
CALCULATE VERTICAL VELOCITY COMPONENT FROM CONTINUITY
DO 805 I=2,II
IP1=I-1
IM1=I-1
DO 805 J=2,J1
JDef=J-1
DO 805 K=2,K1
KM1=K-1
DUDX=(U(IP1,J,K)-U(1,J,K))*SIGMA1(I)/XIP1(I)*(U(I,J,K)-U(IM1,J,K))
1*SIGMA2(I)/XIM1(I)
DVDY=(V(I,J,K)-V(IP1,J,K))*SIGMA3(J)/YJP1(J)+(V(I,J,K)-V(IM1,J,K))
1*SIGMA4(J)/YJM1(J)
W(I,J,K)=W(I,J,KM1)-(DUDX+DVDY)*ZKM1(K)/2.
CONTINUE
DO 72 K=2,KMAX
DO 72 J=2,J1
W(IMAX,J,K)=W(I1,J,K)
CONTINUE
DO 232 I=2,II
DO 232 J=2,J1
M(I,J,KMAX)=W(I,J,K1)*DHDT(I,J)
CONTINUE
END OF W CALCULATION
SET THE BOUNDARY ON W
SOLID-FLUID BOUNDARY ON VERTICAL VELOCITY COMPONENT W
IF(SLIP.EQ.0.) GO TO 940
DO 145 K=2,KMAX
DO 146 I=1,IMAX
IF(K.LT.IM) GO TO 144
IF(I.LT.MH .AND. I.GE.ML) GO TO 146
CONTINUE
W(I,1,K)=(SIGY2*W(I,2,K)-SIGY3*M(I,3,K))/SIGY1
CONTINUE
CONTINUE
DO 147 K=2,KMAX
DO 147 I=1,IMAX
W(I,JMAX,K)=SIGPY2*W(I,J2,K)/SIGPY1
CONTINUE
END OF BOUNDARY CONDITIONS
IF(TIME.GT.DQ} GO TO 32
IPRINT=NPRINT
CALL OUTP
IPRINT=IPRINT*2
IJ=IJ+2
CONTINUE
CALL OUTP
IJ=IJ+1
CALL DERIV
CALL SUM
DENSITY CALCULATION
DO 40 K=K1,KMAX
DO 40 J=2,JMAX
RH0(I,J,1)=SI(TTBL,RHOTBL,T(I,J,1),26)
RH0(I,J,KMAX)=SI(TTBL,RHOTBL,T(I,J,KMAX),26)
CONTINUE
END
DO 40 J=2,J1
651 RHO(I,J,K)=SI(TTBL,RHOTBL,T(I,J,K),26)
652 40 CONTINUE
653 TIME=TIME+DT
654 IPRINT=IPRINT+1
655 IF(EIEND.EQ.0,) GO TO 2002
656 IF(IP.EQ.IIP) GO TO 2001
657 2002 CONTINUE
658 IF(NITER.EQ.IP) GO TO 20
659 IP=IP+1
660 GO TO 1
661 20 CALL OUTP
662 2001 CONTINUE
663 C**WRITING OUTPUT ON THE DISC****
664 WRITE(6,1025)
665 REWIND 25.
666 WRITE(25) TIME,DT,IP,KMAX,JMAX,IMAX,IPRINT
667 WRITE(6,2006) TIME,DT,IP,KMAX,JMAX,IMAX,IPRINT
668 WRITE(6,2007)
669 DO 2003 K=1,KMAX
670 DO 2003 J=1,JMAX
671 DO 2003 I=1,IMAX
673 1RHO(I,J,K)
675 1RHO(I,J,K),H(I,J)
676 2003 CONTINUE
677 STOP
678 END
FORTRAN IV (VER 45) SOURCE LISTING: OUTP SUBROUTINE 03/31/77 22:00:10 PAGE 0020

51 1'P',7X,'H',5X)
52 RETURN
53 END
SUBROUTINE SUM

COMMON P(20,14,10),DELX(20),DELY(14),DELZ(10),RHO(20,14,10),
1U(20,14,10),V(20,14,10),W(20,14,10),T(20,14,10),
2DUDT(20,14,10),DVDT(20,14,10),DWDT(20,14,10),DTDT(20,14,10),
3XNUH, XNUV, R0, F0, S0, DH, DV, ENRUN, TBEQ, TEND, IP, IJ, CFX, CFY, Z,
4WINDX, WINDY, E, RHOA, IMAX, JMAX, KMAX, OMEGA, IM, IH, HL, HK,
5DEBUG1, DEBUG2, DEBUG3, DEBUG4, DEBUG5, XNUR, XNUR, XNUR, XDVR, G, DT,
6NITER, NPRINT, I1, I2, J1, J2, K1, K2, TIME, IPRINT, RHST(20,14,10),
7XIP1(20), XIM1(20), XIPM1(20), XI123(20), YJP1(14), XIP2(20),
8YJM1(14), YM1(14), YM1(14), Y123(14), Y123(14), DZ, YJP2(14),
9ZKIP1(10), ZKMP1(10), ZKMP1(10), ZKMP1(10), ZK123(10),
10A, I, JJ, KK, I3, J3, K3, F0, IIT, B, EPS, ITNMAX, SIGMA1(20), SIGMA2(20),
11BSIGMA3(14), SIGMA4(14), SIGMA5(10), SIGMA6(10), DHDT(20,14),
12COMMON SIGX1, SIGX2, SIGX3, SIGX4, SIGY1, SIGY2, SIGY3, SIGY4, SIGY5,
13SIGY6, SIGY7, SIGY8, SIGY9, SIGZ1, SIGZ2, SIGZ3, SIGZ4,
14COMMON/TABLE, TBLR, RHOTBL
15DIMENSION TBLR(26), RHOTBL(26)
16C INTEGRATE FOR HORIZONTAL VELOCITY COMPONENTS AND TEMPERATURE
17DO 22 K=2,KMAX
18DO 20 J=2,J1
19DO 20 I=2,I1
20U(I,J,K)=U(I,J,K)+DT*DUDT(I,J,K)
21V(I,J,K)=V(I,J,K)+DT*DVDT(I,J,K)
22T(I,J,K)=T(I,J,K)+DT*DTDT(I,J,K)
23CONTINUE
24C INTEGRATE FOR WATER LEVEL HEIGHT H
25DO 23 I=1,IMAX
26DO 23 J=1,JMAX
27H(I,J)=H(I,J)+DT*DHDT(I,J)
28CONTINUE
29RETURN
30END
SUBROUTINE DERIV
COMMON p(20,14,10),DELX(20),DELY(14),DELZ(10),RH0(20,14,10),
U(20,14,10),V(20,14,10),W(20,14,10),T(20,14,10),
DUDT(20,14,10),DVT(20,14,10),DDTD(20,14,10),DTDT(20,14,10),
XNUH,XNUV,RO,F0,SO,DH,DV,ENRUN,TBEG,TEND,IP,IJ,CFX,CFY,CZ,
4INDX,NINDY,RHOA,IMAX,OMEGA,IM,MH,ML,MK,
5DEBUG1,DEBUG2,DEBUG3,DEBUG4,DEBUG5,XNUX,ZNUX,ZDXR,DXVR,G,DT,
6NITER,IPRINT,II,JJ,JI,K1,K2,TIME,IPRINT,RHS(20,14,10),
7XIP1(20),XIM1(20),XJP1(14),XJM1(14),XJP2(14),XJM2(14),
8YJP1(14),YJMP1(14),YJP2(14),YJM2(14),
9ZKP1(10),ZKP2(10),ZKM1(10),ZKM2(10),ZKPM1(10),ZK123(10),
A,II,JI,K1,JK,K2,K3,F01,II+1,T-BEPS,ITNMAX,SIGMA1(20),SIGMA2(20),
BSIGMA3(14),SIGMA4(14),SIGMA5(10),SIGMA6(10),DHD(20,14),
14COMMON SIGX1,SIGX2,SIGX3,SIGY1,SIGY2,SIGY3,SIGY4,SIGY5,
15SIGY6,SIGY7,SIGY8,SIGY9,SIGY10,
16COMMON/TABLE/TBL,RH0TBL,
17DIMENSION TT3LC(26),RHOTBL(26),
19CALCULATE DERIVATIVES FOR WATER LEVEL ELEMENTS
20DO 100 I=2,I1
DO 100 J=2,J1
IP1=I*1
IM1=I-1
JP1=J+1
JM1=J-1
ADJUST WATER LEVEL VARIABLE
ZKP1(K1)=DELZ(K1)+H(I,J)
DUDXR=(U(IP1,J,KMAX)-U(I,J,KMAX))/XIP1(I)
DUDXL=(U(I,J,KMAX)-U(IM1,J,KMAX))/XIM1(I)
DUDYR=(U(I,JP1,J,KMAX)-U(I,J,KMAX))/YJP1(J)
DUDYL=(U(I,J,KMAX)-U(IP1,J,KMAX))/YIP1(J)
DUDZL=(U(I,J,KMAX)-U(I,J,KM1))/ZKPM1(K1)
DHDXR=(H(IP1,J)-H(I,J))/XIP1(I)
DHDXL=(H(I,J)-H(IM1,J))/XIM1(I)
DHDYL=(H(I,J)-H(IP1,J))/YIP1(J)
DHDYR=(H(IP1,J)-H(I,J))/YJP1(J)
DRHDXR=(RHO(IP1,J,KMAX)-RHO(I,J,KMAX))/XIP1(I)
DRHDXL=(RHO(I,J,KMAX)-RHO(IM1,J,KMAX))/XIM1(I)
DRHDYR=(RHO(I,JP1,J,KMAX)-RHO(I,J,KMAX))/YJP1(J)
DRHDYL=(RHO(I,J,KMAX)-RHO(I,JP1,J,KMAX))/YJP1(J)
CALCULATE HORIZONTAL MOMENTUM IN X-DIRECTION FOR WATER LEVEL ELEMENTS
DUDT(I,J,KMAX)=-U(I,J,KMAX)*{(SIGMA1(I)*DUDXR+SIGMA2(I)*DUDXL)
+V(I,J,KMAX)*(SIGMA3(I)*DUDYR+SIGMA4(J)*DUDYL)
+H(I,J)*(SIGMA1(I)*DRHDXR+SIGMA2(I)*DRHDXL)/F01
3*{(SIGMA1(I)*DHDXR+SIGMA2(I)*DHDXL)/F01}
FORTRAN IV (VER 45) SOURCE LISTING: DERIV SUBROUTINE 03/31/77 22:00:10 PAGE 0023

51 4*(XNUR*(SIGMA1(I)*DUDXR+SIGMA2(I)*DUDXL)/H(I,J)
52 5-2.*RHO(I,J,KMAX)/FOI)
53 6*XNUR*6.*(DUDXR-DUDXL)/XIPM1(I)+(DUDYR-DUDYL)/YJPM1(J))
54 7*XNUR*((SIGMA3(J)*DUDYR+SIGMA4(J)*DUDYL)
55 8*(SIGMA3(J)*DHDYR+SIGMA4(J)*DHDYL)/H(I,J)
56 4-ZNUR*DDZL/H(I,J)
57 B*CFX*RHOA*WINDX*WINDY)/(2.*H(I,J))
58 C
CALCULATE VELOCITY GRADIENT FOR Y-COMPONENT OF VELOCITY FOR WATER LEVEL
59 C
60 DVXR=(V(IP1,I,J,KMAX)-V(I,J,KMAX))/XIPM1(I)
61 DVXL=V(I,J,KMAX)-V(I,J,KMAX-1)
62 DUDXR=(V(IP1,I,J,KMAX)-V(I,J,KMAX-1))/XIPM1(I)
63 DUVYR=(V(IP1,I,J,KMAX)-V(I,J,KMAX))/YJPM1(J)
64 DUDYL=V(I,J,KMAX)-V(I,J,KMAX-1)/YJPM1(J)
65 DVDZL=2.*(V(I,J,KMAX)-V(I,J,KMAX-1))/ZKPL(K1)
66 C
67 C
68 C
CALCULATE HORIZONTAL MOMENTUM IN Y-DIRECTION FOR WATER LEVEL ELEMENTS
69 C
70 DVDT(I,J,KMAX)=U(I,J,KMAX)*(SIGMA1(I)*DUDXR+SIGMA2(I)*DUDXL)
71 1-V(I,J,KMAX)*)((SIGMA3(J)*DHDYR+SIGMA4(J)*DHDYL)/FOI
72 3*XNUR*(((SIGMA1(I)*DUDXR+SIGMA2(I)*DUDXL)/H(I,J)
73 5*(((SIGMA3(J)*DHDYR+SIGMA4(J)*DHDYL)/FOI)
74 6*(XNUR*((SIGMA3(J)*DUDXR+SIGMA2(I)*DUDXL))//H(I,J)
75 7-2.*RHO(I,J,KMAX)/FOI)
76 8*B*XMUR*H(I,J,KMAX)/(2.*H(I,J))
77 A-ZNUR*DDZL/H(I,J)
78 B*CFY*RHOA*WINDY*WINDY)/(2.*H(I,J))
79 C
80 C
81 C
CALCULATE TEMPERATURE GRADIENT FOR WATER LEVEL ELEMENTS
82 C
83 DTXR=(T(IP1,I,J,KMAX)-T(I,J,KMAX))/XIPM1(I)
84 DTDXL=T(I,J,KMAX)-T(I,J,KMAX-1)
85 DTDYR=(T(IP1,I,J,KMAX)-T(I,J,KMAX))/YJPM1(J)
86 DTDYL=T(I,J,KMAX)-T(I,J,KMAX-1)/YJPM1(J)
87 DTDZL=2.*(T(I,J,KMAX)-T(I,J,KMAX-1))/ZKPL(K1)
88 C
89 C
90 C
CALCULATE ENERGY EQUATION FOR WATER LEVEL ELEMENTS
91 C
92 DTDT(I,J,KMAX)=-U(I,J,KMAX)*(SIGMA1(I)*DTDXR+SIGMA2(I)*DTDXL)
93 1-V(I,J,KMAX)*)((SIGMA3(J)*DTDYR+SIGMA4(J)*DTDYL)
94 2*XDHR*(((SIGMA1(I)*DTDXR+SIGMA2(I)*DTDXL)/H(I,J)
95 3*(((SIGMA3(J)*DTDYR+SIGMA4(J)*DTDYL)/H(I,J)
96 4*XDHR*8.*(DTDXR-DTDXL)/XIPM1(I)+(DTDYR-DTDYL)/YJPM1(J))
97 5*XDHR*(((SIGMA3(J)*DHDYR+SIGMA4(J)*DHDYL)
98 6*(((SIGMA3(J)*DHDYR+SIGMA4(J)*DHDYL)/H(I,J)
99 8*XDVR*DDZL/H(I,J)
100 9-SO*(T(IP1,I,J,KMAX)-E)/H(I,J)
100 CONTINUE
101  DO 7 K=2,K1
102  DO 6 J=2,J1
103  DO 5 I=2,I1
104   IP1=I+1
105   IM1=I-1
106   JP1=J+1
107   JM1=J-1
108   KP1=K+1
109   KM1=K-1
110   C
111   C
112   C
113   IF(K.NE.K1) GO TO 506
114   ZKP1(K1)=DELZ(K1)*H(I,J)
115   ZKPM1(K1)=ZKP1(K1)*ZKM1(K1)
116   SIGMA5(K)=2.*ZKM1(K)/ZKPM1(K)
117   SIGMA6(K)=2.*ZKP1(K)/ZKPM1(K)
118  506 CONTINUE
119   C
120   C
121   C
122   IF(DEBUG2.EQ.0.) GO TO 11
123   WRITE(6,12) I,J,K
124  12 FORMAT(1H0,2X,'I=',I3,2X,'J=',I3,2X,'K=',I3,2X,)/
125   WRITE(6,10) IP1,IM1,JP1,JM1,KP1,KM1
126  10 FORMAT(1H ,2X,'IP1 =',I3,2X,'IM1 =',I3,2X,'JP1 =',I3,2X,,'JM1 =',I3,2X,/'
127   11 CONTINUE
128   C
129   C
130   C
131   DUDXR=(U(IP1,J,K)-U(I,J,K))/XIP1(I)
132   DUDXL=(U(I,J,K)-U(IM1,J,K))/XJM1(J)
133   DUDYR=(V(IP1,J,K)-V(I,J,K))/YIP1(J)
134   DUDYL=(V(I,J,K)-V(IM1,J,K))/YJM1(J)
135   DUDZR=(W(IP1,J,K)-W(I,J,K))/ZKP1(K)
136   DUDZL=(W(I,J,K)-W(IM1,J,K))/ZKM1(K)
137   C
138   C
139   C
140   DVDT(I,J,K)=-U(I,J,K)*(DUDXR*SIGMA1(T)+DUDYR*SIGMA2(J)+
141   1-V(I,J,K)*(DVDXR+SMA3(J)*DVDYL+SMA4(J))
142   2-W(I,J,K)*(DUDZR*SIGMA5(K)+DUDZL*SIGMA6(K))
143   3*XNUR*((DUDXR-DUDXL)/XIP1(I)+(DUDYR-DUDYL)/YJM1(J))
144   4*ZNUR*(DUDZR-DUDZL)/ZKP1(K))
145   C
146   C
147   C
148   DVDT(I,J,K)=-U(I,J,K)*(DVDTX+SMA1(T)+DVDTY+SMA2(J)+
149   1-V(I,J,K)*(DVDTX+SMA3(J)*DVDTY+SMA4(J))
150   2-W(I,J,K)*(DVDTZ+SMA5(K)+DVDTL+SMA6(K))
151   3*XNUR*((DVDTX-DVDXL)/XIP1(I)+(DVDTY-DVDYL)/YJM1(J))
152   4*ZNUR*(DVDTZ-DVDZL)/ZKP1(K)
CALCULATION OF VERTICAL MOMENTUM IN Z-DIRECTION

\[ DwXR = \frac{(W(I+1,J,K) - W(I,J,K))}{X(I+1,J)} \]
\[ DwXL = \frac{(W(I,J,K) - W(I-1,J,K))}{X(I-1,J)} \]
\[ DwYR = \frac{(W(I,J+1,K) - W(I,J,K))}{Y(J+1)} \]
\[ DwYL = \frac{(W(I,J,K) - W(I,J-1,K))}{Y(J-1)} \]
\[ DwZR = \frac{(W(I,J,K+1) - W(I,J,K))}{Z(K+1)} \]
\[ DwZL = \frac{(W(I,J,K) - W(I,J,K-1))}{Z(K-1)} \]

\[ DwDT(I,J,K) = -U(I,J,K) \cdot (DwDXR \cdot \text{SIGMA1}(I) \cdot DwDXL \cdot \text{SIGMA2}(I)) \]
\[ \quad - V(I,J,K) \cdot (DwDYR \cdot \text{SIGMA3}(J) \cdot DwDYL \cdot \text{SIGMA4}(J)) \]
\[ \quad + W(I,J,K) \cdot (DwDZR \cdot \text{SIGMA5}(K) \cdot DwDZL \cdot \text{SIGMA6}(K)) \]
\[ \quad + 3 \cdot XNUR \cdot ((DwDXR - DwDXL)/X(I+1,J)) \]
\[ \quad + (DwDYR - DwDYL)/Y(J+1) \]
\[ \quad + ZNUR \cdot ((DwDZR - DwDZL)/Z(K+1)) \]

CALCULATION OF ENERGY EQUATION

\[ DTDXR = \frac{(T(I+1,J,K) - T(I,J,K))}{X(I+1,J)} \]
\[ DTDXL = \frac{(T(I,J,K) - T(I-1,J,K))}{X(I-1,J)} \]
\[ DTDYR = \frac{(T(I,J+1,K) - T(I,J,K))}{Y(J+1)} \]
\[ DTDYL = \frac{(T(I,J,K) - T(I,J-1,K))}{Y(J-1)} \]
\[ DTDZR = \frac{(T(I,J,K+1) - T(I,J,K))}{Z(K+1)} \]
\[ DTDZL = \frac{(T(I,J,K) - T(I,J,K-1))}{Z(K-1)} \]

\[ DTDT(I,J,K) = -U(I,J,K) \cdot (DTDXR \cdot \text{SIGMA1}(I) \cdot DTDXL \cdot \text{SIGMA2}(I)) \]
\[ \quad - V(I,J,K) \cdot (DTDYR \cdot \text{SIGMA3}(J) \cdot DTDYL \cdot \text{SIGMA4}(J)) \]
\[ \quad + W(I,J,K) \cdot (DTDZR \cdot \text{SIGMA5}(K) \cdot DTDZL \cdot \text{SIGMA6}(K)) \]
\[ \quad + 3 \cdot XDHR \cdot ((DTDXR - DTDXL)/X(I+1,J)) \]
\[ \quad - (DTDYR - DTDYL)/Y(J+1) \]
\[ \quad + ZDVR \cdot ((DTDZR - DTDZL)/Z(K+1)) \]

CALCULATION OF DIVERGENCE

\[ IF (\text{DEBUG2,EQ,0.}) \\text{GO TO 90} \]
\[ IF (I,J,NE,NPRINT) \\text{GO TO 90} \]
\[ \text{DIV} = (\text{DUDXR} \cdot \text{SIGMA1}(I) \cdot \text{DUDXL} \cdot \text{SIGMA2}(I)) + (\text{DUDYR} \cdot \text{SIGMA3}(J) \cdot \text{DUDYL} \cdot \text{SIGMA4}(J)) + \text{DUDZL} \cdot 2 \]
\[ \text{WRITE(6,80)} \ I,J,K,UDT(I,J,K),DVT(I,J,K),DIV \]

CONTINUE

CONTINUE

CONTINUE

CONTINUE

CONTINUE

SET DX AT THE BOUNDARIES
201 DO 200 J=2, JJ
202 DO 200 K=2, K1
203 DUDT(I,J,K)=0.
204 DUDT(IMAX,J,K)=DUDT(I1,J,K)
205 200 CONTINUE
206 C SET QY AT THE BOUNDARIES
207 DO 210 I=2, II
208 DO 210 K=2, K1
209 DVDYR0=(V(I,1,K)-V(I,2,K))/YJP1(2)
210 DVDYLO=(V(I,2,K)-V(I,1,K))/YJM1(2)
211 DVDYRM=(V(I,JMAX,K)-V(I,J1,K))/YJP1(J1)
212 DVDYL0=(V(I,J1,K)-V(I,J2,K))/YJM1(J1)
213 DVDT(I,JMAX,K)=A*KXNU*(DVDYRM-DVDYLO)/YJM1(J1)
214 IF(K.LT.IM) GO TO 205
215 IF(I.LT.MH .AND. I.GE. ML ) GO TO 220
216 205 CONTINUE
217 DVDT(I,1,K)=A*KXNU*(DVDYR0-DVDYLO)/YJM1(2)
218 GO TO 210
219 220 CONTINUE
220 DVDT(I,1,K)=0.
221 210 CONTINUE
222 C SET QZ AT THE BOUNDARIES
223 DO 230 I=2, II
224 DO 230 J=2, J1
225 DWDZT0=(W(I,J,1)-W(I,J,2))/ZKP1(2)
226 DWDZLO=(W(I,J,2)-W(I,J,1))/ZKPM1(2)
227 DWDT(I,J,1)=RHO(I,J,1)/ZKPM1(2)*ZKPM1(2)
228 DWDT(I,J,KMAX)=-RHC(I,J,KMAX)/ZKPM1(2)
229 230 CONTINUE
230 DO 504 K=2, K1
231 KP1=K+1
232 KM1=K-1
233 DO 504 J=2, J1
234 JP1=J+1
235 JM1=J-1
236 DO 504 I=2, II
237 IP1=I+1
238 IM1=I-1
239 C ADJUST WATER LEVEL VARIABLE
240 C
241 C
242 IF(K.NE.K1) GO TO 505
243 ZKP1(K1)=DELZ(K1)+K(I,J)
244 ZKPM1(K1)=ZKP1(K1)+ZKMI(K1)
245 SIGMA5(K)=2.*ZKPM1(K)/ZKPM1(K)
246 SIGMA6(K)=2.*ZKPM1(K)/ZKPM1(K)
247 505 CONTINUE
248 DODD=(DWDT(I,J,KP1)-DWDT(I,J,K))**SIGMA5(K)/ZKP1(K)*DWDT(I,J,K)-
249 (DWDT(I,J,KM1))**SIGMA6(K)/ZKPM1(K)
250 RHS(I,J,K)=DODD
FORTRAN IV (VER 45) SOURCE LISTING: DERIV SUBROUTINE 03/31/77 22:00:10 PAGE 0027

DQDY=(DVDT(I,JP1,K)-DVDT(I,J,K))*SIGMA3(J)/YJP1(J)*(DVDT(I,J,K)-
1DVDT(I,JM1,K))*SIGMA4(J)/YJM1(J)
RHS(I,J,K)=RHS(I,J,K)+DQDY
DQDX=(DUDT(IJP1,K),DUDT(I,J,K))*SIGMA1(I)/XIP1(I)*(DUDT(I,J,K)-
1DUDT(IJM1,K))*SIGMA2(I)/XIM1(I)
RHS(I,J,K)=(RHS(I,J,K)+DQDX)*F01
IF(DEBUG.EQ.0.) GO TO 504
WRITE(6,1000, I, J, K, DQDX, DQDY, DQDZ
1000 FORMAT(1H, 2X, 'I=', I2, 2X, 'J=', I2, 2X, 'K=', I2, 2X, 'DQDX=', E14.7, 2X,
1'DQDY=', E14.7, 2X, 'DQDZ=', E14.7)
504 CONTINUE
600 CONTINUE
CALL PRESS
DO 15 K=2,K1
DO 15 J=2,J1
DO 15 I=2,I1
IP1=I+1
IM1=I-1
JP1=J+1
JM1=J-1
KP1=K+1
KM1=K-1
C CALCULATE PRESSURE GRADIENT IN X-DIRECTION
DPCX=(P(IP1,J,K)-P(I,J,K))*SIGMA1(I)/XIP1(I)+(P(I,JM1,K)-P(I,J,K))
1SIGMA2(I)/XIM1(I)
DUDT(I,J,K)=DUDT(I,J,K)-DPCX/F01
C CALCULATE PRESSURE GRADIENT IN Y-DIRECTION
DPDY=(P(I,JP1,K)-P(I,J,K))*SIGMA3(J)/YJP1(J)+(P(I,JM1,K)-P(I,J,K))
1SIGMA4(J)/YJM1(J)
DVDT(I,J,K)=DVDT(I,J,K)-DPDY/F01
15 CONTINUE
RETURN
END
SUBROUTINE PRESS
REAL LAMB1, LAMB2
COMMON P(20,14,10), DELX(20), DELY(14), DELZ(10), RHO(20,14,10),
V(20,14,10), W(20,14,10), DUDT(20,14,10), DVDT(20,14,10), DWDT(20,14,10),
3XNUH, XNUV, RC, FO, SO, DH, DV, ENRUN, TBEG, TEND, IP, IJ, CFX, CFY, Z,
4XINDEX, WINDX, E, RHOA, IMAK, JMAK, KMAK, OMEGA, IM, MH, NL, NH,
5DEBUG1, DEBUG2, DEBUG3, DEBUG4, DEBUG5, XNUH, XNUV, DXHR, DXVR, DXT,
6NITER, NPRINT, I1, I2, J1, J2, K1, K2, TIME, IPFINT, RMS(20,14,10),
7XIPl(20), XIPl(20), XIPl(20), XIPl(20), XIPl(20), XIPl(14), XIPl(14),
8YJPI(14), YJPI(14), XlPP(14), YJPI(14), XIPl(14), XIPl(14),
9ZKPI(10), ZKPI(10), ZKPI(10), ZKPI(10), ZKPI(10), ZKPI(10),
A, I, J, K, I3, J3, K3, F01, IF, B, EPS, ITMAX, SIGMA1(20), SIGMA2(20),
2SIGMA3(14), SIGMA4(14), SIGMA5(10), SIGMA6(10), DHDT(20,14),
3DEBUG1, DEBUG2, DEBUG3, DEBUG4, DEBUG5, XNUH, XNUV, DXHR, DXVR, DXT,
4NITER, NPRINT, I1, I2, J1, J2, K1, K2, TIME, IPFINT, RMS(20,14,10),
5XIPl(20), XIPl(20), XIPl(20), XIPl(20), XIPl(20), XIPl(14), XIPl(14),
6YJPI(14), YJPI(14), XlPP(14), YJPI(14), XIPl(14), XIPl(14),
7ZKPI(10), ZKPI(10), ZKPI(10), ZKPI(10), ZKPI(10), ZKPI(10),
COMMON SIGX1, SIGX2, SIGX3, SIGX4, SIGX5, SIGX6, SIGX7, SIGX8, SIGX9,
2SIGY1, SIGY2, SIGY3, SIGY4, SIGY5, SIGY6, SIGY7, SIGY8, SIGY9,
3SIGZ1, SIGZ2, SIGZ3, SIGZ4, SIGZ5, SIGZ6, SIGZ7, SIGZ8, SIGZ9,
4COMMON/TABLE/TTBL, RHTBL
5DIMENSION TTBL(26), RHTBL(26)
6IF(HYD.EQ.0.) GO TO 300
7C CALCULATE PRESSURE FROM HYDROSTATIC APPROXIMATION
8DO 110 L=1, KMAX
9K=KMAX+1-L
10KP1=K1
11DO 110 J=1, JMAX
12DO 110 I=1, IMAX
13IF( I .LT. MH .AND. I .GE. ML ) GO TO 110
14IF(K.NE.KMAX) GO TO 120
15P(I,J,K)=RHO(I,J,K)*K1/(1,K)/2.
16110 CONTINUE
17GO TO 100
18300 CONTINUE
19IF(K.NE.K1) GO TO 130
20ZKPI(K1)=DELZ(K1)*K1(I,J)
21130 CONTINUE
22P(I,J,K)=P(I,J,K)+RHO(I,J,K)*ZKPI(K)/2.
23110 CONTINUE
24GO TO 100
25700 CONTINUE
26C END PRESSURE CALCULATION FROM HYDROSTATIC APPROXIMATION
27FORMAT(1H0,2X, 'OMEGA=', E14.6, /)
28C CALCULATE FROM PRESSURE EQUATION
29ITN =1
30Y1=1
31LAMB1=1.
32OMEGA1=1.
33IF(IP.NE.0) GO TO 10
34IF(NOT.(DEBUG4.GE.2,)) GO TO 10
35WRITE(6,33) OMEGA
36DO 6 K=2,K1
37DO 6 J=2,J1
38DO 6 I=2,I1
FORTRAN IV (VER 45) SOURCE LISTING: PRESS SUBROUTINE 03/31/77 22:00:10 PAGE 029

WRITE(6,23) K, ZK123(K), ZKP1(K), ZKP2(K), ZKM1(K), J, YJ123(J),
1YJP1(J), YJM1(J), I, XI123(I), XI141(I), XI1M1(I)
CONTINUE
D = 0.
B = 1.0, -OMEGA
Y2 = 0.
PRESSURE BOUNDARY CONDITIONS
SOLID-FLUID BOUNDARY ON PRESSURE
DO 200 I = 2, I1
DO 200 K = 2, K1
IF(K, LT, [LT], I) GO TO 225
IF(I, LT, [LT], [MP], [MP], [MI], [MI]) GO TO 210
CONTINUE
PCF[L, K] = [SIGY2*PCF[L, K] - SIGY3*PCF[L, J, 3] - SIGY4*DVDT(I, K)*FO1]/SIG
AY1
CONTINUE
P(I, J, K) = [SIGZ2*P(I, J, 2) - SIGZ3*P(I, J, 3) - SIGZ4*DVDT(I, J, K)*FO1]/SIG
AZ1
CONTINUE
CALCULATION OF PRESSURE IN THE WATER LEVEL ELEMENTS
DO 50 I = 2, I1
DO 50 J = 2, J1
CONTINUE
FLUID-FLUID BOUNDARY CONDITION IS SET DURING INITIALIZATION
BOTTOM BOUNDARY ON PRESSURE
DO 220 I = 2, I1
DO 220 J = 2, J1
P(I, J, 1) = [SIGZ2*P(I, J, 2) - SIGZ3*P(I, J, 3) - SIGZ4*DVDT(I, J, 1)*FO1]/SIG
AZ1
CONTINUE
OUTGOING BOUNDARY ON PRESSURE
DO 55 K = 2, KMAX
DO 55 J = 2, J1
P(I, J, K) = P(I, J, K)
CONTINUE
CALCULATE PRESSURE IN FLOW REGION
DO 5 K = 2, K1
DO 5 J = 2, J1
DO 5 I = 2, I1
ADJUST WATER LEVEL VARIABLE

IF(K.NE.K1) GO TO 18
ZKP1(K1)=DELZ(K1)+H(I,J)
ZKM1(K1)=ZKP1(K1)*ZKM1(K1)
ZK123(K1)=ZKP1(K1)*ZK121(K1)*ZK122(K1)
CONTINUE

X1 =YJ123(J)*ZK123(K)@X[I1(1)
X2 =YJ123(J)*ZK123(K)@X[I1(1)
X3 =X123(I)*ZK123(K)@YJM1(J)
X4 =X123(I)*ZK123(K)@YJP1(J)
X5 =X123(I)*YJ123(J)@ZKM1(K)
X6 =X123(I)*YJ123(J)@ZKP1(K)
X7 =X123(I)*YJ123(J)@ZK123(K)
X =X1+X2+X3+X4+X5+X6
A =@OMEGA/X
RESID =ABS(PNEW-P(I,J,K))
IF(RESID-D) 15,15,16
D =RESID
P(I,J,K) =PNEW
IF(OMEGA,NE,1,) GO TO 9
Y2=Y2*RESID
CONTINUE
IF(I.P,NE,0) GO TO 5
IF(.NOT.,(DEBUG4. GE,1,)) GO TO 5
WRITE(6,20) X,X1,X2,X3,X4,X5,X6,X7,A
CONTINUE
CALCULATE RELAXATION FACTOR OMEGA

IF(OMEGA,NE,1,) GO TO 28
LAMB2=Y2/Y1
IF(DEBUG4,GE,0,) GO TO 60
WRITE(6,42)Y1,Y2,LAMB2
CONTINUE
IF(LAMB2,GE,1,) GO TO 28
IF(ABS(LAMB2-LAMB1).GT, .G1) GO TO 28
OMEGA=2./1+SQRT(1-LAMB2))
IF(DEBUG4,GE,0,) GO TO 28
WRITE(6,33) OMEGA
CONTINUE

IF(D-EPS) 14,12,12
ITN=ITN+1
FORTRAN IV (VER 45) SOURCE LISTING: PRESS SUBROUTINE 03/31/77 22:00:10 PAGE 0031

151 IF(ITN-ITNMAX) 10,10,21
152 21 WRITE(6,22)
153 CALL EXIT
154 22 FORMAT(1HO,4X,' FAILS TO CONVERGE IN GIVEN ITNMAX !,/
155 26 FORMAT(1HO,2X,'I',2X,'J',3X,'K',6X,'X',6X,'X1',6X,'X2',6X,'X3',
156 16X,'X4',6X,'X5',6X,'X6',6X,'X7',6X,'A')
157 14 CONTINUE
158 IF(DEBUG4.EQ,0.) GO TO 40
159 WRITE(6,20) ITN
160 20 FORMAT(1HO,10X,'ITN=',I3)
161 23 FORMAT(1HO,'K=',12,2X,'ZK123(K)=',E11,4,2X,'ZKP1(K)=',E11,4,2X,
162 1'ZKP2(K)=',E11,4,2X,'ZK213(K)=',E11,4,2X,'J=',12,2X,'YJ123(J)=',
163 2E11,4,2X,'YJP1(J)=',E11,4,2X,'YJM1(J)=',E11,4,2X,'I=',12,
164 32X,'XI123(I)=',E11,4,2X,'XIP1(I)=',E11,4,2X,'XIM1(I)=',E11,4)
165 24 FORMAT(1HO,'I=',12,2X,'J=',12,2X,'K=',12,2X,
166 1'P(I,J,K)=',E11,4,1X,'P(I-1,J,K)=',E11,4,1X,'P(I,J-1,K)=',E11,4,
167 21X,'P(I,J,K+1)=',E11,4,1X,'P(I+1,J,K)=',E11,4,1X,'P(I,J,K)=',
168 3E11,4,1X,'P(I,J,K)=',E11,4,1X,'P(I,J,K)=',E11,4,1X,'P(I,J,K)=',
169 4,E11,4,7X,'B=',E11,4,2X,'D=',E11,4)
170 100 CONTINUE
171 40 RETURN
172 END
FUNCTION SI (XTBL,YTBL,X,N)
C LINEAR INTERPOLATION OR EXTRAPOLATION OF SINGLE VARIABLE FUNCTION
C XTB L = INDEPENDENT VARIABLE TABLE
C YTBL = DEPENDENT VARIABLE TABLE
C X = VALUE OF THE INDEPENDENT VARIABLE
C N = NO. OF POINTS IN TABLE
C 0=NO EXTRAPOLATION, 1=LOWER EXTRAPOLATION, 2=UPPER EXTRAPOLATION
C DIMENSION XTB L(30),YTB L(30)
C CHECK TO SEE IF EXTRAPOLATION IS NEEDED
10 IF(X<XTBL(1)) 120,130,150
11 120 WRITE(6,202)
12 202 FORMAT(1H0,5X,'DENSITY TABLE OUT OF RANGE AT THE LOWER END')
13 130 11=2
14 GO TO 254
15 160 IF(XTB L(N)<X) 160,180,210
16 180 WRITE(6,201)
17 201 FORMAT(1H0,5X,'DENSITY TABLE OUT OF RANGE AT THE UPPER END')
18 130 II = N
19 GO TO 254
C FIND X IN TABLE
21 210 DO 220 IK=2,N
22 220 XI = XI-1
23 220 CONTINUE
24 254 XI = XTB L(I I-1)
25 X2 = XTB L(I I)
26 Y1 = YTBL(I I-1)
27 Y2 = YTBL(I I)
28 SI = Y1*(Y2-Y1)*(X-X1)/(X2-X1)
30 RETURN
31 END
BLOCK DATA
COMMON/TABLE/TTBL,RHOTBL
REAL 4 TTBL(26)/70,72,74,76,78,80,82,84,86,88,90,92,
*94, *
96,98,100,102,104,106,108,110,112,114,116,118,120,/
REAL 4 RHOTBL(26)/62,3029595,62,28977202,62,27425562,62,25487144
*62,23937263,62,22009969,62,20065933,62,18132073,62,16199416,
*62,13881812,62,11565936,62,09251785,62,06939358,62,04628653,
62,02319666,61,99628022,61,96938712,61,94251734,61,91567086,
*61,88884763,61,86204763,61,83144747,61,80469716,61,77415369,
*61,74364041,61,71315729/
END
ENRUN = 0.2000 E 01
TBEG = 0.0000 E 00 TEND = 0.1000 E 04 ENITER = 0.5000 E 04 NPRINT = 0.2500 E 02
UI = 0.4000 E 00 VI = 0.0000 E 00 TI = 0.7500 E 02
UD = 0.2000 E 01 ANGLE = 0.1571 E 01 TD = 0.9000 E 02 DD = 0.5000 E 02 XNUZ = 0.6250 E 05
WINDX = 0.0000 E 00 WINDY = 0.0000 E 00 RHOA = 0.7630 E-01 E = 0.7400 E 02 XK = 0.1000 E 03
CFX = 0.0000 E 00 CFY = 0.0000 E 00
DH = 0.1350 E 04 DV = 0.4000 E 00 XNUH = 0.1350 E 04 XNUV = 0.4000 E 00 CP = 0.1000 E 01
EIMAX = 0.1000 E 02 EJMAX = 0.7000 E 01 EKMAX = 0.5000 E 01
DEBUG1 = 0.0000 E 00 DEBUG2 = 0.0000 E 00 DEBUG3 = 0.0000 E 00 DEBUG4 = 0.0000 E 00
EIRUN = 0.0000 E 00 E END = 0.1000 E 01
EIM = 0.4000 E 01 EML = 0.4000 E 01 EMK = 0.5000 E 01 SLIP = 0.0000 E 00 HYD = 0.0000 E 00
HEAT = 0.0000 E 00 XIIP = 0.5200 E 02
I DELX(I)
  1 0.6500 E 02 2 0.6000 E 02 3 0.5500 E 02 4 0.5000 E 02
  5 0.5000 E 02 6 0.5500 E 02 7 0.6000 E 02 8 0.6500 E 02
9 0.7000 E 02 10 0.7000 E 02
J DELY(J)
  1 0.5000 E 02 2 0.5000 E 02 3 0.5500 E 02 4 0.6000 E 02
  5 0.6500 E 02 6 0.7000 E 02 7 0.7000 E 02
K DELZ(K)
  1 0.3500 E 01 2 0.4000 E 01 3 0.5000 E 01 4 0.4000 E 01
  5 0.3500 E 01
TITL     RHOTL
0.7000000E 02 0.6230296E 02
0.7200000E 02 0.6229766E 02
0.7400000E 02 0.6229286E 02
0.7600000E 02 0.6228872E 02
0.7800000E 02 0.6228454E 02
0.8000000E 02 0.6228020E 02
0.8200000E 02 0.6227572E 02
0.8400000E 02 0.6227114E 02
0.8600000E 02 0.6226643E 02

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IMAX SUGGESTED DT: 5 2 2 0.465E-00
JHAX SUGGESTED DT: 5 2 2 0.465E-00
KMAX SUGGESTED DT: 5 2 2 0.465E-00

I J K MAX SUGGESTED DT: 5 2 2 0.465E-00
I J K SUGGESTED DT: 5 2 2 0.465E-00

0.4511E+00
0.4465E+00
### Non-Dimensional Input Values

<p>| ENRUN= | 0.2000E 01 |
| TBEG= | 0.0000E 00 |
| TEND= | 0.4000E 02 |
| NITER= | 5000 |
| NPRINT= | 25 |
| UI= | 0.2000E 00 |
| VI= | 0.0000E 00 |
| TD= | 0.8333E 00 |
| TD= | 0.1000E 01 |
| DOE= | 0.5000E 02 |
| XNUZ= | 0.1000E 01 |
| WINDX= | 0.0000E 00 |
| WINDY= | 0.0000E 00 |
| RH0A= | 0.1228E 02 |
| DH= | 0.1636E 09 |
| DV= | 0.4848E 05 |
| XNUH= | 0.1636E 09 |
| XNUV= | 0.4848E 05 |
| G= | 0.3220E 02 |
| DT= | 0.6222222E 00 |
| DELX(I)= |
| 1 | 0.1300E 01 |
| 2 | 0.1200E 01 |
| 3 | 0.1100E 01 |
| 4 | 0.1000E 01 |
| 5 | 0.1000E 01 |
| 6 | 0.1100E 01 |
| 7 | 0.1200E 01 |
| 8 | 0.1300E 01 |
| 9 | 0.1400E 01 |
| 10 | 0.1400E 01 |
| J DELY(J)= |
| 1 | 0.1000E 01 |
| 2 | 0.1000E 01 |
| 3 | 0.1100E 01 |
| 4 | 0.1200E 01 |
| 5 | 0.1300E 01 |
| 6 | 0.1400E 01 |
| 7 | 0.1400E 01 |
| K DELZ(K)= |
| 1 | 0.7000E-01 |
| 2 | 0.8000E-01 |
| 3 | 0.1000E 00 |
| 4 | 0.8000E-01 |
| 5 | 0.7000E-01 |
| RO= | 0.1212E 08 |
| FO= | 0.4968E-01 |
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VITA

The author was born in Iran. He began attending Fairleigh Dickinson University, Teaneck, New Jersey in 1966 obtaining the B.S.M.E. degree from that university in 1969. He continued at Fairleigh Dickinson University for the master’s degree in Mechanical Engineering, which was completed in 1971. He was accepted into the doctoral program at Newark College of Engineering the same year. The development of this dissertation took place between April 1974 and April 1977 at New Jersey Institute of Technology.