Decomposing non-product form queueing lattices through autorouting with A* algorithm

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Decomposing Non-Product Form Queueing Lattices Through Autorouting With A * Algorithm

BY CHUN-CHANG YU

Thesis submitted to the faculty of the graduate school of the New Jersey Institute of Technology in partial fulfillment of the requirements for the degree of Master of Science in Electrical Engineering

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Direct solution techniques are expensive for the state transition lattice of a class of non-product form queueing modes. In this thesis, autorouting with A* algorithm is adapted to decompose it as solvable subsets which can be solved sequentially and independently. Autorouting with A* algorithm is heuristics in problem solving. It also is one of a class of global optimization problems that are difficult to solve. Some queueing modes of type A will be decomposed in this thesis through a Autorouting with A* algorithm.
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Approval Sheet

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Chapter 1
Introduction

In studying the behavior of computer system, queueing network modes are a useful tool for evaluating performance and for studying the interaction between resources, software and workload [1]. The product form solution of the balance equations, when it exists, plays an important role in analyzing such queueing network modes. Product form solutions are of great interest because the direct numerical solution of balance equations is computationally expensive. Typically the existence of product form solutions has been characterized in terms of certain classes of queueing networks. In [2,3,4] it was characterized in terms of the algebraic topology of the state transition lattice. It was shown that the existence of the product form solution corresponds to a decomposition of the state transition lattice - complex - into elementary geometric building blocks - cells.

Non-product form queueing networks are far less likely to have a closed form solution for the equilibrium probabilities. Direction solution techniques are prohibitively expensive. In this paper autorouting with A* algorithm will be submitted to decompose the non-product queueing lattice.

In chapter 2, the product form and the non-product will be reviewed, while at the same time sequential decomposition will be discussed. In chapter 3, the autorouting with A* algorithm is described. In chapter 4, the algorithm discussed in chapter 3 is implemented to decompose the non-product form queueing lattice for A type. Chapter 5 shows the results of simulation.
Chapter 2
Product and Non-Product Form

2.1 General Review

In [4] it is shown that specific structure in the state transition lattice leads to a sequential method of solution. The method is sequential either in terms of individual state or in terms of groups of states. In both cases, substantial computational savings are possible. A great deal of research has been performed on the subject of product form solutions. Originally, Jackson described the equilibrium state probabilities for open networks of queues with a single class of jobs, Poisson arrival statistics, and exponential service time[5]. These probabilities were of a characteristic “product form”:

\[ P(n_1n_2,\ldots,n_m) = P_1(n_1)P_2(n_2)\ldots P_m(n_m) \quad (1.1) \]

Later, Gordon and Newell analyzed closed queueing networks with each station having an exponential service time distribution[7]. The equilibrium states probability as:

\[ P(n_1,n_2,\ldots,n_m) = \frac{1}{G(N)} \prod_{i=1}^{m} f_i(n_i) \quad (1.2) \]

Where \( G(N) \) is a normalization constant chosen to make all the feasible state probabilities sum to one. \( N \) is the total number of jobs. The \( f_i \) are akin to the marginal probabilities of (1.1).

There is similarity between the balance equation of queueing network state transition lattices and current conservation equations of resistant circuits. Naturally, the flow in the former involves probability flux[6] rather than current. Transition rates three significant differences though. There is a scaling of equilibrium probabilities
to unity which induces probability flux flow in place of voltage sources. The flow directions are predetermined from the transition directions. Finally, the transition rates are labeled in a patterned manner from the queueing scheme. The existence of product form solutions has been characterized in terms of certain types of queueing networks. That is the algebraic topology of the state transition lattice as shown in fig. 2.1.

Non-product is shown in fig 2.2. Direct solution techniques are prohibitively expensive. Techniques analogous to that of the z-transform can sometimes be used to determine distributions of interest[4,7]. In[7] the sequential decomposition technique was described.
Figure 2.1 Product Form
Figure 2.2 Non-Product Form
2.2 Sequential Decomposition

The exact computation of the state probabilities of a class of non-product form queueing networks is discussed in this section. It is shown that specific structure in the state transition lattice leads to a sequential method of solution. The method is sequential either in terms of individual state or in terms of groups of states.

In [4] a class of non-product form networks is described whose state transition lattices can be shown to be equivalent to a lattice tree of simplexes. In this "flow redirection" method the lattice geometry is manipulated by equivalence transformations. Sequential decomposition refers to the related process of solving one subset of states at a time for the equilibrium probabilities:

Definition 1.1: a solvable subset of queueing network states is a subset of states for which the equilibrium probabilities can be determined without regard to the equilibrium probabilities of the remaining unknown states. Here probabilities are determined with respect to a reference probability.

A simple example is presented in figure 2.3. States S1 and S2 form a solvable subset. The equilibrium probabilities can be determined without regard to the values associated with the other states. This is done through the following global balance equations:

\[(a+\beta) P_0 = \gamma P_1 + \beta P_2\]
\[a P_2 = \gamma P_1\]

These equations can be solved for the probabilities \(P_1\) and \(P_2\) as a function of reference probability \(P_0\). In fact, a more general principle can be established. The method of sequential decomposition is applicable to queueing systems whose states have the geometry shown in figure 2.4. Here each circular cluster represents a state
or a group of states. Consider the ith cluster, the rule is that there must be only one state, with unknown probability, external to the cluster form which a transition(s) entering the cluster originates.

The clusters are solved sequentially, starting from the first cluster to the second and so on. Note that there is no restriction on the number of transitions which may leave the ith cluster for destination in the $j = i+1, i+2, \ldots, \text{cluster}$. The solution equations may not be unique.

Note also that the direct solution of linear equations takes time proportional to the cube of the number of equations. If $N$ states can be solved as $M$ states then the computational effort is proportional to $(N/M)^3 M$ rather than $N^3$.

Two types of the structure which allows the state transition lattice to be decomposed into solvable subsets were obtained in [7]. The first type of structure is illustrated in Figure 2.4 and Figure 2.6. Here each circular subset represents a state or a group of states. For the ith subset the rule is that there must be only one state, with unknown probability, external to the subset from which a transition(s) entering the subset to the second and so on. There is no restriction on the number of transitions which may leave the ith subset for destinations in the $j = i+1, i+2, \ldots$, subsets. This type of structure is type A structure.

The second type of structure is illustrated in Figure 2.5. Here the first subset consists of a single state. The remaining subsets each consist of a state or a group of states. They are arranged in a tree of configuration with the flow between subsets from the top of the diagram to the bottom and a return flow from the bottom level back to the top. The subsets may be solved from the top to the bottom. Transitions may traverse several levels as long as the direction of flow is downward. This type is referred to as type B structure.
Figure 2.3 A Solvable Subset
Figure 2.4 The Basic Structure of A Type A
Figure 2.5 The Basic Structure of Type B
Figure 2.6 A Typical Example of Type A Lattices
Chapter 3
Autorouting with the A* Algorithm

Autorouting is one of a class of global optimization problems that are difficult to solve. A good decomposition for the non-product form queueing networks, for example, minimizes things like:

- the number of groups of states
- the time to search the group

At the same time, the decomposition maximizes things like reliability and ease of debugging. The overall value of a decomposition for a non-product form queueing network design is a function of these often conflicting variables. It is usually acceptable to find a solution for all but the most trivial problems. In this chapter, we would describe the autorouting with the A* algorithm about how it works in the two phases and analysis of its complexity.

3.1 Autorouting Operation

Autorouting search algorithms typically operate in two phase[9], treating the state-transition lattice as a matrix of cells. The first phase starts at the source cell and searches for the target cell, usually by going in several directions at the same time.

The algorithms build in an auxiliary data structure to keep track of how each cell was reached. The first phase ends when the target cell has been found, and the second phase begins. If the first phase exhausts all possibilities without reaching the target cell, then no route exists between them, and there is no reason to do the second phase.
The second phase uses the auxiliary data structure to trace the route from the target cell back to the source cell.

The second phase is identical for the breadth-first [10][11] and A* search[12] algorithms. The first phase is different, and it is this phase that gives these algorithms different behaviors.

### 3.1.1 Data Structure for the First Phase

The main data structures used in the first phase are the OPEN QUEUE and the CLOSED SET, which hold cell coordinates. Because a cell's coordinates uniquely identifies it, we'll say that the OPEN QUEUE and CLOSED SET contain cells. Cell coordinate will be represented as r5c5 for the cell at row 5, column 5.

To remind ourselves that OPEN is a queue and CLOSED is a set, when we talk about adding cell to them, we will put the cells " on " the queue and " in " the set. Initially, the OPEN QUEUE contains the source cell and the CLOSED SET is empty.

The first phase is a loop which removes an element from the head of the OPEN QUEUE, puts it in the CLOSED SET ( which indicates that it has been searched ) and checks to see if it is the target cell. If so, the first phase is done, otherwise, the neighbors of the cell ( those adjacent to it ) are placed on the OPEN QUEUE, and the loop continues. As we'll see later, the essential difference in the breadth-first and A* search algorithms is placed on the OPEN QUEUE.

### 3.2 Breadth-First Search

Breadth-first search looks for the target among all elements of the matrix from the source cell.
Here is the procedure with proper terminating condition

To conduct a breadth-first search:

1. From a one cell set consisting of the source.

2. Until the QUEUE is empty or the goal has been reached, determine if the cell(s) in the set is(are) the target.
   
   (a) If the set is the target, do nothing.
   
   (b) If the set is not the target, put its neighbors to the SET and add the neighbors of their neighbors to the QUEUE. Go back to 2a.

3. If target has been found, announce success; otherwise announce failure.
Breadth-first search works by processing a first in first out (FIFO) queue of open cells, that is, cells that have been reached, but not yet searched. Initially, the OPEN QUEUE contains only the source cell. A cell is removed from the head of the OPEN QUEUE, placed in the SET of closed cells (cells that have been searched) and checked to see if it is the target cell. If not, its neighbors are placed at the tail of the OPEN QUEUE. Neighboring cells that have already been reached are ignored (If a cell's coordinates is on the OPEN QUEUE or in the CLOSED SET, then it has been reached, otherwise it has not). This continues until one of two things happens:

- The goal cell has been found.
- The OPEN QUEUE is empty in which case the goal cannot be reached from the source cell.

A version of breadth-first search known as Lee's algorithm[11] has served as the basis for some autorouters since the early 1960s.
Figure 3.1a
Target Cell: \( r_8c_2 \)

\[
\begin{array}{cccccccc}
\text{T} & \text{} & \text{} & \text{} & \text{} & \text{} & \text{} & \text{} \\
\text{} & \leftarrow & \leftarrow & \text{} & \text{} & \text{} & \text{} & \text{} \\
\text{} & \text{} & \downarrow & \text{} & \text{} & \text{} & \text{} & \text{} \\
\text{} & \text{} & \text{} & \text{} & \text{} & \text{} & \text{} & \text{} \\
\rightarrow & \rightarrow & \text{S} & \leftarrow & \leftarrow & \text{} & \text{} & \text{} \\
\text{} & \text{} & \text{} & \uparrow & \text{} & \text{} & \text{} & \text{} \\
\text{} & \text{} & \text{} & \text{} & \uparrow & \text{} & \text{} & \text{} \\
\end{array}
\]

CLOSED \hspace{1cm} OPEN

\begin{align*}
\text{r}_5\text{c}_5 & \quad \text{r}_5\text{c}_4 & \quad \text{r}_5\text{c}_3 & \quad \text{r}_6\text{c}_3 & \quad \text{r}_7\text{c}_3 \\
\text{r}_6\text{c}_5 & \quad \text{r}_5\text{c}_6 & \quad \text{r}_7\text{c}_4 & \quad \text{r}_7\text{c}_5 & \quad \text{r}_7\text{c}_6 \\
\text{r}_4\text{c}_5 & \quad \text{r}_6\text{c}_4 & \quad \text{r}_7\text{c}_7 & \quad \text{r}_6\text{c}_7 & \quad \text{r}_5\text{c}_7 \\
\text{r}_6\text{c}_6 & \quad \text{r}_4\text{c}_6 & \quad \text{r}_4\text{c}_7 & \quad \text{r}_3\text{c}_7 & \quad \text{r}_3\text{c}_6 \\
\text{r}_4\text{c}_4 & \quad \text{r}_3\text{c}_5 & \quad \text{r}_3\text{c}_3 & \quad \text{r}_3\text{c}_4 \\
\text{r}_4\text{c}_3 & \quad & \quad & \quad & \\
\end{align*}

Figure 3.1b
Target Cell: \( r_8c_2 \)

CLOSED      OPEN

\[ \begin{array}{cccccccc}
  r_5c_5 & r_5c_4 & r_6c_5 & r_5c_2 & r_6c_2 & r_7c_2 & r_8c_2 \\
  r_5c_6 & r_4c_5 & r_6c_4 & r_8c_3 & r_8c_4 & r_8c_5 & r_8c_6 \\
  r_6c_6 & r_4c_6 & r_4c_4 & r_8c_7 & r_8c_8 & r_7c_8 & r_6c_8 \\
  r_5c_3 & r_6c_3 & r_7c_3 & r_5c_8 & r_4c_8 & r_3c_8 & r_2c_8 \\
  r_7c_4 & r_7c_5 & r_7c_6 & r_2c_7 & r_2c_6 & r_2c_5 & r_2c_4 \\
  r_7c_7 & r_6c_7 & r_5c_7 & r_2c_3 & r_2c_2 & r_3c_2 & r_4c_2 \\
  r_4c_7 & r_3c_7 & r_3c_6 & r_3c_5 & r_3c_4 & r_3c_3 \\
  r_4c_3 \\
\end{array} \]

Figure 3.1c
Lee’s algorithm searching for a path
In Figure 3.1a, the source cell (r5c5) has been searched, and its eight neighbors have been placed on the OPEN QUEUE. The arrows indicate the direction from which each cell was reached, and correspond to first phase data structure. After the first eight cells on the OPEN QUEUE have been reached and moved to the CLOSED SET, the algorithm searches the configuration in Figure 3.1b, where there are sixteen cells on the OPEN QUEUE. Once these sixteen cells have been searched, the configuration in Figure 3.1c is reached. Now the goal cell (r8c2) is fourth from the end on the OPEN QUEUE, and a solution is imminent.

3.3 A* Algorithm Search

The A* procedure is branch-and-bound search, with an estimate of remaining distance, combined with the dynamic-programming principle [10]. It is a priority queue, which means cells are inserted according to the estimated distance to the target [10], not just at the end.

Cells that are on the shortest estimated path from source to target go to the head of the queue. The A* algorithm removes the cell from the head of the OPEN QUEUE and checks to see if it's the target. If not, the neighboring cells are put on the OPEN QUEUE at the proper position. The algorithm checks neighboring cells that have already been searched to see if the new path between them and the source is shorter than the previous one. If it is, they are repositioned on the OPEN QUEUE according to the new estimated path length from source to goal. As in Breadth-First search, this continues until the target cell has been found or until the OPEN QUEUR is empty.
A* depends on being able to estimate the distance between a cell and the target cell. In the case of autorouting, a simple measure of this distance is available, and this helps A* to concentrate the search on the direction most likely to succeed. The more accurate the estimate, the faster the search.

Figure 3.2 shows the behavior of the A* search algorithm. A* does not specify whether new cells go in front of or behind cells already on the OPEN QUEUE that evaluate to identical estimated path lengths. We use the convention that they are placed in front. This minimizes the time to insert a cell on the OPEN QUEUE.
Target Cell: $r_8c_2$

$F(x)=3$

CLOSED OPEN

$\begin{array}{cccccc}
& & & & & \\
& & & & & \\
r_5c_5 & r_5c_4(3) & r_6c_5(3) & r_5c_6(4) & r_4c_5(4) & \\
r_6c_4(2) & r_6c_6(4) & r_4c_6(4) & r_4c_4(4) & & \\
& & & & & \\
& & & & & \\
\end{array}$

Figure 3.2a
Target Cell: \( r_5c_2 \)

\[ F(x) = 2 \]

Figure 3.2b
Target Cell: $r_8c_2$

$F(x) = 1$

CLOSED  OPEN

\begin{align*}
  r_5c_5 & \quad r_6c_4 & \quad r_5c_4(3) & \quad r_6c_5(3) & \quad r_5c_6(4) \\
  r_7c_3 & \quad r_4c_5(4) & \quad r_6c_6(4) & \quad r_4c_6(4) \\
           & \quad r_4c_4(4) & \quad r_6c_3(2) & \quad r_7c_4(2) \\
           & \quad r_7c_5(3) & \quad r_5c_3(3) & \quad r_7c_2(1) \\
           & \quad r_8c_3(1) & \quad r_8c_2(0) & \quad r_8c_4(2) \\
           & \quad r_6c_2(2)
\end{align*}

Figure 3.2c

Behavior of the A* search algorithm
In Figure 3.2a, the source cell (r5c5) has been searched, and its neighbors are on the OPEN QUEUE. Each cell on the OPEN QUEUE also includes the estimated length of the shortest path from S to T that goes through that cell (we set the estimated length of each grid to be one). After the first cell (r6c4) has been searched and moved to the CLOSED SET, the configuration in Figure 2.2b is reached, where there are 12 cells on the OPEN QUEUE. After searching the next cell (r7c3), the algorithm reaches the configuration in Figure 3.2c. Now the goal cell (r8c2) is at the head of the OPEN QUEUE, and solution will be found on the next iteration of the loop. Searching r8c2, A* recognizes it as the target and uses the first phase data structure to construct a trace back to the source cell.

### 3.4 Discussion

Lee’s algorithm suffers from a behavior inherent in the breadth-first search technique, which limits its application to problems of relatively small size. As the distance between the source and target cells increases by a factor of N, the number of cells processed by Lee’s algorithm—and therefore processing time—increases by the square of N.

A* algorithm search process, as seen in the Figure 3.2, goes more directly toward the goal cell. The target draws the search much as the earth’s gravity pulls objects toward the center of mass. If we double the size of the problem, the search will process about twice as many cells, and if we triple its size, the search will run through three times as many. This linear characteristic makes A* more attractive for autorouting than the quadratic Lee’s algorithm.
Chapter 4
Adapted Autorouting with A* Algorithm to Non-Product Form Queueing Mode

In chapter 3 the autorouting with A* algorithm operation is introduced. In this chapter it will be implemented to decompose the non-product form queueing lattices. The non-product form queueing lattices will be seen as a matrix. From a given cell, we will discuss how the target(group) will be found in this chapter. The pseudocode will also be supplied.

4.1 Type A

Type A structure is illustrated in Figure 2.4. Here each circular subset represents a state or a group of states. For the ith subset there must be only one state with unknown probability, external to the subset from which a transition(s) entering the subset originates. Our target is to find a small subgroup of cells that only one cell, not contained in the target group, can enter this subgroup from the source cell.

From Figure 2.4 we can find that, at most, only one node enters the subgroup from other subgroups. For convenience we call this node the bridge node. Now we give every node an estimate. The rule is that the estimates of the cells of the former subset should be less than that of the cells of the latter subset, i.e. the estimates of the nodes of the second subset should be greater than those of the nodes of the first subset. After each subset has been searched, i.e. after the bridge node for the subset has been found, the every bridge node will be seen as the source cell for the search of the other subgroups. Repeat this until we have done with each node.
If a source cell is a subset we have just finished with, set this subset's neighbors as the bridge cell and search again. If not, put the source cell in CLOSED SET, add its neighboring nodes that have path and compare their estimates to search for the cell with largest estimate as a bridge node until there is only one node i.e. the bridge cell, left on the OPEN QUEUE. Search the other subgroups again until no node can be found.
Pseudo code for searched subgroup from type A

( * S is the start node * )
( * OPEN is an ordered list of nodes, CLOSED is a set of nodes (order contain the lower F value). In general, nodes that need to be searched are put on OPEN (at the proper position). As they are searched, they are removed from OPEN and put in CLOSED *)
( * $F(x)$ is the estimated of $x$ * )
( * $G(x)$ is the number of nodes that enter the $x$, from its neighbors * )

1. OPEN $\leftarrow$ { S } ( * a list of one element * )
CLOSED $\leftarrow$ { } ( * the empty set * )

while OPEN $\neq$ { } and not found do

x $\leftarrow$ the first node on OPEN ( * node with largest F value * )
OPEN $\leftarrow$ - {x} ( * remove x from OPEN * )
CLOSED $\leftarrow$ CLOSED + {X} ( * put x in CLOSED * )

5. IF $G(X) == 1$ ( * x itself is a subset * )

Let N be the set of neighboring nodes of x

For each y on N do

IF( $G(x) == \{ \}$ ) do nothing

IF ( $G(X) == 1$ ) ( * x itself is subset * )

S $\leftarrow$ y
$F(x) \leftarrow F(y)$
$G(x) \leftarrow G(y)$
Print CLOSED ( * searched finished * )

go to 1
ELSE
IF y not on OPEN or CLOSED THEN
IF F(y) / F(x) x ← y
F(x) ← F(y)
ELSE
put y in CLOSED
ELSE
remove y from OPEN ( * each node search one time * )
until T is reached
else T can not reached from S

Starting with the source cell, the process stops if the cell is the last element. If
the cell is a subset itself, record it, take it as a subset and puts it neighboring cells on
OPEN, as they are treated as the source in this case. Compare F(y) with F(x). Add
y to CLOSED if F(x) ≥F(y). If F(x)<F(y), put y into OPEN and make F(x)←F(y).
Continue the comparison between F(x) and F(y). When the neighboring cells of the x
have all gone through this comparison procedure, take the estimate of their neighbors
to do the comparison. Keep on until the largest F(x) is found.

4.2 Estimate Calculation

The A* search algorithm use a heuristic to estimate the distance between the
current cell and the target cell. As implemented in the autorouting program, the
heuristic is a simple geometric distance approximation. Figure 4.1 illustrates all the
possible cell types used to construct a trace, grouped by type.
For each group, the estimate of that cell type is also given. These estimates are calculated based on an infinite cell size. If we take the example of Figure 4.1, the estimated of $r_{4c0}$ equals to

$$2 \times \text{group C} + 2 \times \text{group D} + 1 \times \text{group A}$$

$$= 2 + 2 + 0$$

$$= 4$$

and the estimate of $r_{4c1}$ equals to

$$1 \times \text{group A} + 2 \times \text{group C} + 2 \times \text{group D} + 1 \times \text{group E}$$

$$= 0 + 2 + (5 \times 1) + 2$$

$$= 11$$

According to the estimated we can easily find the target cell. The estimate design based on Figure 2.4. Every solvable subset has only one bridge node that has incoming path to this subset. So we design the estimate of bridge node is larger than that of all nodes in the subset that it enters. Every time when we search the bridge node we compare the estimates. Finally, we can get a node with largest estimate that is bridge node and the subset search is finished. The bridge node will be seen as a source cell to search the other subset. Until OPEN QUEUE is empty, all solvable subsets will be searched.
Figure 4.1 Definition of Estimate for Type A

<table>
<thead>
<tr>
<th>GROUP</th>
<th>ESTIMATE</th>
<th>MEMBER</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>D</td>
<td>1</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>E</td>
<td>5*i</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>F</td>
<td>2</td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

i: the number of column
Chapter 5
Decomposition Results

The simulation program is written in C code [appendix]. The author tested the program on the lattices of type A, Figure 5.1, on an IBM 286AT Personal Computer with source cell(r4c2). The results are as following:

<table>
<thead>
<tr>
<th>Source cell : r4c2(14)</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 32 34 34 34 34 36 34 34 32</td>
</tr>
<tr>
<td>5 27 29 29 31 29 35 29 29 27</td>
</tr>
<tr>
<td>4 22 24 24 30 24 26 24 24 22</td>
</tr>
<tr>
<td>3 17 21 19 19 19 25 19 19 17</td>
</tr>
<tr>
<td>2 14 20 14 14 14 14 14 14 12</td>
</tr>
<tr>
<td>1 13 9 9 9 11 9 9 9 7</td>
</tr>
<tr>
<td>0 2 4 4 4 10 4 4 4 2</td>
</tr>
<tr>
<td>0 1 2 3 4 5 6 7 8</td>
</tr>
</tbody>
</table>

$F(X)$
Figure 5.1 Type A Lattice
Source Cell : r4c2(14)

\[
F(x) = 14
\]

<table>
<thead>
<tr>
<th>CLOSED</th>
<th>OPEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>r4c2</td>
<td>r3c2(14)</td>
</tr>
<tr>
<td></td>
<td>r4c1(11)</td>
</tr>
<tr>
<td></td>
<td>r5c2(14)</td>
</tr>
</tbody>
</table>

First Search

Source Cell : r4c2(14)

\[
F(x) = 14
\]

<table>
<thead>
<tr>
<th>CLOSED</th>
<th>OPEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>r4c2</td>
<td>r2c2(14)</td>
</tr>
<tr>
<td>r3c2</td>
<td>r3c1(9)</td>
</tr>
<tr>
<td>r4c1</td>
<td>r4c0(10)</td>
</tr>
<tr>
<td>r5c2</td>
<td>r5c1(9)</td>
</tr>
<tr>
<td></td>
<td>r6c2(14)</td>
</tr>
</tbody>
</table>

Second Search
Source Cell : r4c2(14)
F(x) = 14

CLOSED        OPEN
r4c2  r2c2  r1c2(20)
r3c2  r3c1  r2c1(9)
r4c1  r4c0  r3c0(4)
r5c2  r5c1  r5c0(4)
r6c2  r6c1(9)
r7c2(14)

Third Search

Source Cell : r4c2(14)
F(x) = 20

CLOSED        OPEN
r4c2  r2c2  r3c2  r1c2(20)
r3c1  r4c1  r4c0  r1c1(9)
r5c2  r5c1  r6c2  r2c0(4)
r2c1  r3c0  r5c0  r6c0(4)
r6c1  r7c2  r7c1(9)
r8c2(12)

Fourth Search
Source Cell : r4c2(14)

F(x) = 20

<table>
<thead>
<tr>
<th>CLOSED</th>
<th>OPEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>r4c2</td>
<td>r2c2</td>
</tr>
<tr>
<td>r3c1</td>
<td>r4c1</td>
</tr>
<tr>
<td>r5c2</td>
<td>r5c1</td>
</tr>
<tr>
<td>r2c1</td>
<td>r3c0</td>
</tr>
<tr>
<td>r6c1</td>
<td>r7c2</td>
</tr>
<tr>
<td>r4c0</td>
<td>r4c0</td>
</tr>
<tr>
<td>r6c2</td>
<td>r6c0</td>
</tr>
<tr>
<td>r5c0</td>
<td>r5c0</td>
</tr>
<tr>
<td>r8c2</td>
<td>r7c1</td>
</tr>
<tr>
<td>r0c2</td>
<td>r0c0</td>
</tr>
<tr>
<td>r2c0</td>
<td>r2c0</td>
</tr>
<tr>
<td>r6c0</td>
<td>r6c0</td>
</tr>
<tr>
<td>r0c1(13)</td>
<td>r0c0(4)</td>
</tr>
<tr>
<td>r1c0(4)</td>
<td>r1c0(4)</td>
</tr>
<tr>
<td>r7c0(4)</td>
<td>r7c0(4)</td>
</tr>
<tr>
<td>r8c0(2)</td>
<td></td>
</tr>
<tr>
<td>r8c1(7)</td>
<td></td>
</tr>
</tbody>
</table>

Fifth Search

Source Cell : r4c2(14)

F(x) = 20

<table>
<thead>
<tr>
<th>CLOSED</th>
<th>OPEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>r4c2</td>
<td>r2c2</td>
</tr>
<tr>
<td>r3c1</td>
<td>r4c1</td>
</tr>
<tr>
<td>r5c2</td>
<td>r5c1</td>
</tr>
<tr>
<td>r2c1</td>
<td>r3c0</td>
</tr>
<tr>
<td>r6c1</td>
<td>r7c2</td>
</tr>
<tr>
<td>r4c0</td>
<td>r4c0</td>
</tr>
<tr>
<td>r6c2</td>
<td>r6c0</td>
</tr>
<tr>
<td>r5c0</td>
<td>r5c0</td>
</tr>
<tr>
<td>r8c2</td>
<td>r7c0</td>
</tr>
<tr>
<td>r0c2</td>
<td>r0c0</td>
</tr>
<tr>
<td>r2c0</td>
<td>r2c0</td>
</tr>
<tr>
<td>r6c0</td>
<td>r6c0</td>
</tr>
<tr>
<td>r0c1(13)</td>
<td>r0c0</td>
</tr>
<tr>
<td>r1c0(4)</td>
<td>r1c0(4)</td>
</tr>
<tr>
<td>r7c0(4)</td>
<td>r7c0(4)</td>
</tr>
<tr>
<td>r8c0(2)</td>
<td></td>
</tr>
<tr>
<td>r8c1(7)</td>
<td></td>
</tr>
</tbody>
</table>

Sixth Search
Now, only one element has the largest estimate in OPEN QUEUE. The search of first subset that is the group of the cells on CLOSED SET is finished. We begin to search the second subset. The element in OPEN will be seen as a source cell for second subset.

Source Cell : r1c2(20)

\[ F(x) = 21 \]

Now, only one element has the largest estimate in OPEN QUEUE. The search of second subset that is the group of the cells on CLOSED SET is finished. We begin to search the third subset. The element in OPEN will be seen as a source cell for third subset.

Source Cell : r1c3(21)

\[ F(x) = 21 \]
Source Cell : r1c3(21)
F(x) = 21

CLOSED     OPEN
r1c3      r0c3      r3c3(19)
r2c3

Second Search

Source Cell : r1c3(21)
F(x) = 21

CLOSED     OPEN
r1c3      r0c3      r4c3(19)
r3c3      r2c3

Third Search

Source Cell : r1c3(21)
F(x) = 25

CLOSED     OPEN
r1c3      r0c3      r5c3(25)
r3c3      r2c3

Fourth Search
Now, same as before there is only one cell in OPEN QUEUE, the third subset search is finished. We begin to search the fourth subset. The element in OPEN will be seen as a source cell for third subset.

Source Cell : r5c3(25)
\[ F(x) = 26 \]

<table>
<thead>
<tr>
<th>CLOSED</th>
<th>OPEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>r5c3</td>
<td>r5c4(26)</td>
</tr>
<tr>
<td></td>
<td>r6c3(19)</td>
</tr>
</tbody>
</table>

First Search

Source Cell : r5c3(25)
\[ F(x) = 26 \]

<table>
<thead>
<tr>
<th>CLOSED</th>
<th>OPEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>r5c3</td>
<td>r5c4(26)</td>
</tr>
<tr>
<td>r6c3</td>
<td>r7c3(19)</td>
</tr>
</tbody>
</table>

Second Search
Source Cell :\(r_5c_3(25)\)
\[ F(x) = 26 \]

CLOSED OPEN
r5c3 r5c4(26)
r6c3 r8c3(17)
r7c3

Third Search

Source Cell :\(r_5c_3(25)\)
\[ F(x) = 26 \]

CLOSED OPEN
r5c3 r6c3 r5c4(26)
r8c3 r7c3

Fourth Search

Fourth subset search is finished. Now, start the next subset. The cell \(r_5c_4\) will be the source cell of the next subset.
Source Cell : r5c4(26)
F(x) = 26

CLOSED   OPEN
r5c4     r4c4(24)
r6c4(24)

First Search

Source Cell : r5c4(26)
F(x) = 30

CLOSED   OPEN
r5c3     r5c4     r3c4(30)
r6c3     r7c4(24)

Second Search

Source Cell : r5c4(26)
F(x) = 26

CLOSED   OPEN
r5c4     r4c4     r6c4     r3c4(26)
r7c4     r8c4

Third Search
Fifth subset search is finished. Now, start the next subset. The cell r3c4 will be the source cell of next subset.

Source Cell : r3c4(30)

\[ F(x) = 31 \]

CLOSED OPEN
r3c4 r3c5(31) r2c4(24)

First Search

Source Cell : r3c4(30)

\[ F(x) = 31 \]

CLOSED OPEN
r3c4 r2c4 r3c5(31)
 r1c4(24)

Second Search
Sixth subset search is finished. Now, start the next subset. The cell r3c5 will be the source cell of next subset.

Source Cell : r3c5(31)

\[ F(x) = 31 \]

**CLOSED**  **OPEN**

r3c5  r4c5(31)

**First Search**
Seventh subset search is finished. Now, start the next subset. The cell r5c5 will be the source cell of next subset.
Source Cell : r5c5(35)

\[ F(x) = 36 \]

CLOSED  OPEN

r5c5  r5c6(36)
r6c5(29)

First Search

Source Cell : r5c5(35)

\[ F(x) = 36 \]

CLOSED  OPEN

r5c5  r6c5  r5c6(36)
r7c5

Second Search

Source Cell : r5c5(35)

\[ F(x) = 36 \]

CLOSED  OPEN

r5c5  r6c5  r5c6(36)
r7c5  r8c5

Third Search
Eighth subset search is finished. Now, start the next subset. The cell r5c6 will be the source cell of next subset.

**Source Cell : r5c6(36)**  
\[ F(x) = 36 \]

<table>
<thead>
<tr>
<th>CLOSED</th>
<th>OPEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>r5c6</td>
<td>r4c6(34)</td>
</tr>
<tr>
<td></td>
<td>r6c6(34)</td>
</tr>
</tbody>
</table>

**First Search**

**Source Cell : r5c6(36)**  
\[ F(x) = 36 \]

<table>
<thead>
<tr>
<th>CLOSED</th>
<th>OPEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>r5c6</td>
<td>r4c6</td>
</tr>
<tr>
<td>r3c6(34)</td>
<td>r6c6</td>
</tr>
<tr>
<td>r7c6(34)</td>
<td></td>
</tr>
</tbody>
</table>

**Second Search**
Source Cell : r5c6(36)

\[ F(x) = 36 \]

**CLOSED**  **OPEN**

- r5c6  r4c6  r2c6(34)
- r3c6  r6c6  r8c6(32)
- r7c6

**Third Search**

Source Cell : r5c6(36)

\[ F(x) = 36 \]

**CLOSED**  **OPEN**

- r5c6  r4c6  r2c6
- r3c6  r6c6  r8c6
- r7c6  r1c6  r0c6

**Fourth Search**

All solvable subsets are searched. The list follows:
First subset

r4c2  r2c2  r3c2  r1c2  r0c1  r3c1  r4c1  r4c0
r2c0  r0c0  r5c2  r5c1  r6c2  r6c0  r1c0  r2c1
r3c0  r5c0  r8c2  r7c0  r6c1  r7c2  r7c1  r7c0
r8c0  r8c1

Second subset

r1c2

Third subset

r1c3  r0c3  r3c3  r2c3

Fourth subset

r5c3  r6c3  r7c3  r8c3

Fifth subset

r5c4  r4c4  r6c4  r7c4  r8c4

Sixth subset

r3c4  r2c4  r1c4  r0c4

Seventh subset

r3c5  r2c5  r1c5  r0c5  r4c5

Eighth subset

r5c5  r6c5  r7c5  r8c5

Ninth subset

r0c6  r1c6  r2c6  r3c6  r4c6  r5c6  r6c6  r7c6  r8c6
Chapter 6
Discussion and Conclusion

6.1 Discussion

The lattice is represented as a two-dimensional array with one dimension for X axis and the other for y axis in which the dimensions are the number of rows and columns of cells. There are eight possible cell contents, which can be represented with 3 bits per cell. On a matrix of N rows and M columns, we will need N*M*3 bits in total.

The preprocessor structure is also a two-dimensional array, where an entry must be able to represent one of the four compass directions. This takes 2 bits per cell, or N*M*2 bits.

The CLOSED can be represented by a pair of two dimensional, single-bit array, where a bit is one if the corresponding cell has been searched, and zero otherwise. This will take N*M*2 bits in total.

F(x) and G(x) will be similar to the lattice arrays, but they must have a 2 bit integer for each cell, requiring N*M*4 bits in total. Note that if memory usage needs to be minimized at the cost of increased processing time, we could omit the F(x) arrays, and calculate the F values as they are needed from G(x) arrays.

6.2 Conclusion

A decomposition algorithm has been developed to solve the state transition lattice of Type A structure. By dividing the lattice into several solvable subsets which can be solved sequentially, a large amount of computation time can be saved.
References


Appendix
```c
#include <stdio.h>
#include <dos.h>
#include <process.h>
#define point 1
#define xmax 9
#define ymax 7
#define n (xmax*ymax)

struct {
    int x;
    int y;
} buffer[n];

struct neighbor {
    int link;
};

struct {
    struct neighbor north;
    struct neighbor east;
    struct neighbor south;
    struct neighbor west;
} state[xmax][ymax];

int temp[n];

int main()
{
    unsigned before, after;
    int count = 0;
    int x1, y1;
    int num_cont = 0;
    before = biostime(0, 0);
    read_data();
    clrscr();
    printf("\n\n\n\n\n\n\n");
    printf("Please Enter Initial State !\n");
    scanf("%d %d", &x1, &y1);
    if (x1 < 0 || x1 >= xmax || y1 < 0 || y1 >= ymax)
    {
        printf("Please Enter X From 0 to %d\n", (xmax - 1));
        printf("Please Enter Y From 0 to %d\n", (ymax - 1));
        printf("Please Try Again !\n");
        scanf("%d %d", &x1, &y1);
        if (x1 < 0 || x1 >= xmax || y1 < 0 || y1 >= ymax)
            exit(0);
    }
    while (x1 < xmax && y1 < ymax && x1 >= 0 && y1 >= 0)
    {
        switch ((state[x1][y1].north.link + 
               state[x1][y1].east.link + 
               state[x1][y1].south.link + 
               state[x1][y1].west.link))
        { case 0 :
```
c_comp_0(&x1,&y1,&num_cont,count);
break;
case 1 :
c_comp_1(&x1,&y1,&num_cont,count);
break;
case 2 :
c_comp_2(&x1,&y1,&num_cont,count);
break;
case 3 :
c_comp_3(&x1,&y1,&num_cont,count);
break;
case 4 :
c_comp_4(&x1,&y1,&num_cont,count);
break;
}
count++;
out_print();
after=biostime( 0, 0);
printf("\n\n This program use %f second to execute ",
(unsigned float)(after-before)/18.204);
}
out_print()
{
int i=0;
int x=0;
int j;
FILE *out;
out=fopen("y_0.out","w");
while (i < n) {
    fprintf(out,"THE GROUP\n");
    for(j=0;j<temp[k];j++)
    {
        fprintf(out,"( %d , %d )\n",buffer[i].x,buffer[i].y);
        i++;
    }
k++;
}fclose(out);
}
read_data()
{
int i,j;
FILE *in;
if((in=fopen("isdn.dat","r")) == NULL)
{
    puts("Can't open data file !\n");
    exit(0); }
else
{
    for ( j = 0 ; j < ymax ; j++ )
        for ( i = 0 ; i < xmax ; i++ )
        {
            fscanf( in,"%d %d %d %d",\
            &state[i][j].north.link,\
&state[i][j].east.link,
&state[i][j].south.link,
&state[i][j].west.link);
}

close(in);

c_comp_0(a,b,c,d)
int *a,*b,*c;
int d;
{
buffer[*c].x=*a;
buffer[*c].y=*b;
(*c)++;
(*a)++;
(*b)++;
temp[d]=1;
}
c_comp_1(a,b,c,d)
int *a,*b,*c;
int d;
{
buffer[*c].x=*a;
buffer[*c].y=*b;
(*c)++;

if(state[*a][*b].north.link == point )
{
 (*b)++;
 state[*a][*b].south.link = 0 ;
}
else if (state[*a][*b].east.link == point )
{
(*a)++;
 state[*a][*b].west.link = 0 ;
 if(*b<(ymax-1))
  state[(*a)-1][(*b)+1].south.link=0;
}
else if (state[*a][*b].south.link == point )
{
(*b)--;
 state[*a][*b].north.link = 0 ;
}
else
{
 (*a)--;
 state[*a][*b].east.link = 0 ;
 if(*b < (ymax-1))
  state[(*a)+1][(*b)+1].south.link=0;
}
}
c_comp_4(a,b,c,d)
int *a,*b,*c;
int d;
{int i,j;
int sum=0;
*a=*a;
(*b)++;
for(j=0;j<*b;j++)
  for(i=0;i<xmax;i++)
  {
    buffer[*c].x=i;
    buffer[*c].y=j;
    (*c)++;
    sum++;
  }
  for(i=0;i<xmax;i++)
    state[i][*b].south.link=0;
  temp[d]=sum;
}

c_comp_2(a,b,c,d)
int *a,*b,*c;
int d;
{
  int i,j,_value,t_value;
  int sum=0;
  if(state[*a][*b].north.link==point)
  {
    if(state[*a][*b].east.link==0)
      for(i=0;i<=*a;i++)
      {
        buffer[*c].x=i;
        buffer[*c].y=*b;
        (*c)++;
        sum++;
        state[i][(*b)+1].south.link=0;
      }
    else
      for(i=*a;i<xmax;i++)
      {
        buffer[*c].x=i;
        buffer[*c].y=*b;
        (*c)++;
        sum++;
        state[i][(*b)+1].south.link=0;
      }
      (*b)++;
      }
  else /* state[*a][*b].north.link=0 */
  {
    if(state[*a][*b].south.link==point)
    {
      if(state[*a][*b].east.link==0)
        for(i=0;i<=*a;i++)
        {
          buffer[*c].x=i;
          buffer[*c].y=*b;
          (*c)++;
          sum++;
          state[i][(*b)+1].south.link=0;
        }
      else
        for(i=*a;i<xmax;i++)
        {
          buffer[*c].x=i;
          buffer[*c].y=*b;
          (*c)++;
          sum++;
          state[i][(*b)+1].south.link=0;
        }
        (*b)++;
        }
    else /* state[*a][*b].north.link=0 */
    {
      if(state[*a][*b].south.link==0)
      {
        if(*b!=(ymax-1))
        {
          }
for(i=0;i<xmax;i++)
if(state[i][b].north.link==point)
{
    _value=i;
    break;
}
if(_value < *a)
{
    (*a)--;
    state[*a][b].east.link=0;
    for(i=(*a)+1;i<xmax;i++)
    {
        buffer[*c].x=i;
        buffer[*c].y=b;
        (*c)++;
        sum++;
        state[i][(*b)+1].south.link=0;
    }
}
else /* _value > a */
{
    (*a)++;
    state[*a][b].west.link=0;
    for(i=0;i<*a;i++)
    {
        buffer[*c].x=i;
        buffer[*c].y=b;
        (*c)++;
        sum++;
        state[i][(*b)+1].south.link=0;
    }
}
else /* b=ymax */
{
    if(*a>xmax/2)
    {
        for(i=*a;i<xmax;i++)
        {
            buffer[*c].x=i;
            buffer[*c].y=b;
            (*c)++;
            sum++;
        }
        (*a)--;
        state[*a][b].east.link=0;
    }
    else
    {
        (*a)++;
        for(i=0;i<*a;i++)
        {
            buffer[*c].x=i;
buffer[*c].x=i;
buffer[*c].y=*b;
(*c)++;
sum++;

} state[*a][*b].west.link=0;
}
}
}
}
}
else /* south=1 */
{
for(j=0;j<*b;j++)
for(i=0;i<xmax;i++)
{
 buffer[*c].x=i;
 buffer[*c].y=j;
 (*c)++;
 sum++;
}
if((*b)!=(ymax-1))
{
for(i=0;i<xmax;i++)
if(state[i][*b].north.link==point)
{
 value=i;
 break;
}
for(i=0;i<xmax;i++)
if(state[i][(*b)-1].north.link==point)
{
 t_value=i;
 break;
}
if(state[*a][*b].east.link==point)
{
if( (t_value==(*a)) && (t_value==_value) )
{
 (*b)++;
 for(i=*a;i<xmax;i++)
 {
 buffer[*c].x=i;
 buffer[*c].y=(*b)-1;
 (*c)++;
 sum++;
 state[i][*b].south.link=0;


}
else if( t_value == *a)
state[*a][(*b)+1].south.link=0;
buffer[*c].x=*a;
buffer[*c].y=*b;
(*c)++;
sum++;
(*a)++;
for(i=*a;i<xmax;i++)
    state[i][(*b)].south.link=0;
state[*a][*b].west.link=0;

else if(t_value<_value)
{
    for(i=*a;i<=t_value;i++)
    {
        buffer[*c].x=i;
        buffer[*c].y=*b;
        (*c)++;
        sum++;
        state[i][(*b)+1].south.link=0;
    }
    *a=t_value+1;
    for(i=*a;i<xmax;i++)
        state[i][*b].south.link=0;
    state[*a][*b].west.link=0;
}
else /* t_value>_value */
{
    for(i=*a;i<_value;i++)
    {
        buffer[*c].x=i;
        buffer[*c].y=*b;
        (*c)++;
        sum++;
        state[i][(*b)+1].south.link=0;
    }
    for(i=_value+1;i<xmax;i++)
    {
        buffer[*c].x=i;
        buffer[*c].y=*b;
        (*c)++;
        sum++;
        state[i][(*b)+1].south.link=0;
    }
    *a=value;
    state[*a][*b].south.link=0;
    state[*a][*b].east.link=0;
    state[*a][*b].west.link=0;
}
if ((t_value==*a) && (t_value==_value))
{
    for(i=0;i<=(*a);i++)
    {
        buffer[*c].x=i;
        buffer[*c].y=*b;
        (*c)++;
        sum++;
        state[i][(*b)+1].south.link=0;
    }
    (*b)++;
}
else if(t_value == *a)
{
    state[*a][(*b)+1].south.link=0;
    buffer[*c].x=*a;
    buffer[*c].y=*b;
    (*c)++;
    sum++;
    (*a)--;
    for(i=0;i<*a;i++)
    {
        state[i][*b].south.link=0;
        state[*a][*b].east.link=0;
    }
    if(t_value>_value)
    {
        for(i=t_value;i<*a;i++)
        {
            buffer[*c].x=i;
            buffer[*c].y=*b;
            (*c)++;
            sum++;
            state[i][(*b)+1].south.link=0;
        }
        for(i=0;i<t_value;i++)
        {
            state[i][*b].south.link=0;
            *a=_value-1;
            state[*a][*b].east.link=0;
        }
    }
    else /* <= */
    {
        for(i=0;i<= *a;i++)
        {
            buffer[*c].x=i;
            buffer[*c].y=*b;
            (*c)++;
            sum++;
            state[i][(*b)+1].south.link=0;
        }
    }
}
else /* east=0 */
{
    if((t_value==*a) && (t_value==_value))
    {
        for(i=0;i<=(*a);i++)
        {
            buffer[*c].x=i;
            buffer[*c].y=*b;
            (*c)++;
            sum++;
            state[i][(*b)+1].south.link=0;
        }
        (*b)++;
    }
    if(t_value>_value)
    {
        for(i=t_value;i<*a;i++)
        {
            buffer[*c].x=i;
            buffer[*c].y=*b;
            (*c)++;
            sum++;
            state[i][(*b)+1].south.link=0;
        }
        for(i=0;i<t_value;i++)
        {
            state[i][*b].south.link=0;
            *a=_value-1;
            state[*a][*b].east.link=0;
        }
    }
    else /* <= */
    {
        for(i=0;i<= *a;i++)
        {
            buffer[*c].x=i;
            buffer[*c].y=*b;
            (*c)++;
            sum++;
            state[i][(*b)+1].south.link=0;
        }
    }
}
*a= _value;
state[*a][*b].west.link=0;
state[*a][*b].south.link=0;
state[*a][*b].east.link=0;
}
}
} /* b= ymax */

for(i=0;i<xmax;i++)
  if(state[i][(*b)-1].north.link==point)
    {
    t_value=i;
    break;
    }
if(state[*a][*b].east.link==point)
  {
    if(t_value  <= (*a))
    {
    for(i=0;i<=*a;i++)
      {
        buffer[*c].x=i;
        buffer[*c].y=*b;
        (*c)++;
        sum++;
      }
    (*a)++;
    for(i=*a;i<xmax;i++)
      state[i][*b].south.link=0;
      state[*a][*b].west.link=0;
    }
  else
  for(i=*a;i<xmax;i++)
    {
    buffer[*c].x=i;
    buffer[*c].y=*b;
    (*c)++;
    sum++;
  }
  (*a)--; 
  for(i=0 ;i<=*a;i++)
    state[i][*b].south.link=0;
    state[*a][*b].east.link=0;
  }

/*  */
else
  {
  for(i=0;i<xmax;i++)
    {
    buffer[*c].x=i;
    buffer[*c].y=*b;
    (*c)++;
    sum++;
  }
temp[d]=sum;
}
c_comp_3(a,b,c,d)
int *a,*b,*c;
int d;
{
int i,j,_valud,_valup;
int sum=0;
if(state[*a][*b].north.link==point)
{
    (*b)++;
    if(state[*a][(*b)-1].south.link==point)
    {
        for(j=0;j<=*b;j++)
            for(i=0;i<xmax;i++)
                if (j == *b )
                    state[i][*b].south.link=0;
            else
            {
                buffer[*c].x=i;
                buffer[*c].y=j;
                (*c)++;
                sum++;
            }
    }
    else /* state[*a][*b].south.link != point */
    {
        for(i=0;i<xmax;i++)
        {
            buffer[*c].x=i;
            buffer[*c].y=(*b)-1;
            (*c)++;
            sum++;
            state[i][*b].south.link=0;
        }
    }
}
else /*north=0 */
{
    if((*b) < (ymax-1))
    {
        for(j=0;j<=(*b)-1;j++)
            for(i=0;i<xmax;i++)
                {buffer[*c].x=i;
                buffer[*c].y=j;
                (*c)++;
                }
sum++; 
}
for(i=0;i<xmax;i++)
if(state[i][*b].north.link==point)
{
    _valup=i;
    break;
}
for(i=0;i<xmax;i++)
if(state[i][(*b)-1].north.link==point)
{
    _valud=i;
    break;
}

if(_valup < *a )
{
    if(_valud >= *a )
    {
        for(i=*a;i<xmax;i++)
        {
            state[i][(*b)+1].south.link=0;
            buffer[*c].x=i;
            buffer[*c].y=*b;
            (*c)++;
            sum++; 
        }
        for(i=0;i<*a;i++)
        state[i][*b].south.link=0;
        (*a)--;
        state[*a][*b].east.link=0;
    }
    else if ( _valud > _valup )
    {
        for(i= _valud+1 ; i < xmax ; i++)
        {
            state[i][(*b)+1].south.link=0;
            buffer[*c].x=i;
            buffer[*c].y=*b;
            (*c)++;
            sum++; 
        }
        *a = _valud;
        state[*a][*b].east.link=0;
        for(i=0 ;i<=*a ;i++)
        state[i][*b].south.link=0;
    }
}
else /* _valud <= _valup */
{
    *a = _valup;
    for(i=0;i<xmax;i++)
    if( i != *a )
{   
    state[i][(*b)+1].south.link=0;
    buffer[*c].x=i;
    buffer[*c].y=*b;
    (*c)++;
    sum++;
}
state[*a][*b].east.link=0;
state[*a][*b].south.link=0;
state[*a][*b].west.link=0;

}

else /* _valup > a */
{
    
    if(_valud<=*a)
    {
        for(i=0 ;i<= *a;i++)
        {
            state[i][(*b)+1].south.link=0;
            buffer[*c].x=i;
            buffer[*c].y=*b;
            (*c)++;
            sum++;
        }
        (*a)++;
        for(i=*a;i<xmax;i++)
        state[i][*b].south.link=0;
        state[*a][*b].west.link=0;
    }
    else if ( _valud < _valup )
    {   
        *a= _valud ;
        for(i=0 ;i<*a ;i++)
        {
            state[i][(*b)+1].south.link=0;
            buffer[*c].x=i;
            buffer[*c].y=*b;
            (*c)++;
            sum++;
        }
        for(i=*a;i<xmax;i++)
        state[i][*b].south.link=0;
        state[*a][*b].west.link=0;
    }
    else /* _valud >= _valup */
    {
        *a=_valup;
        for(i=0;i<xmax;i++)
        if( i != *a )
        {
            state[i][(*b)+1].south.link=0;
        }


buffer[*c].x=i;
buffer[*c].y=*b;
(*c)++;
sum++;
}
state[*a][*b].east.link=0;
state[*a][*b].west.link=0;
state[*a][*b].south.link=0;
}
}
else /* b=ymax */
{
    for(j=0;j< (*b) ;j++)
    for(i=0;i<xmax;i++)
    {
        buffer[*c].x=i;
        buffer[*c].y=j;
        (*c)++;
        sum++;
    }
    for(i=0;i<xmax;i++)
    if(state[i][(*b)-1].north.link==point)
    {
        _valup = i ;
        Break;
    }
    if( _valup < (*a) )
    {
        (*a)++;
        for(i=0 ; i < xmax ; i++)
        {
            if(i >= (*a) )
                state[i][*b].south.link=0;
            else
            {
                buffer[*c].x=i;
                buffer[*c].y=*b;
                (*c)++;
                sum++;
            }
        }
        state[*a][*b].west.link=0;
    }
    else if(_valup ==(*a))
    {
        if(*a > xmax/2)
        {
            (*a)--;
            for(i=0 ; i < xmax ; i++)
            {
                if(i <= *a )
                    state[i][*b].south.link=0;
            }
        }
        else
        {
            (*a)++;
            for(i=0 ; i < xmax ; i++)
            {
                if(i > (*a) )
                    state[i][*b].south.link=0;
                else
                {
                    buffer[*c].x=i;
                    buffer[*c].y=*b;
                    (*c)++;
                    sum++;
                }
            }
            state[*a][*b].west.link=0;
        }
    }
}
{ buffer[*c].x=i; buffer[*c].y=*b; (*c)++; sum++; }
}
state[*a][*b].east.link=0;
}
else /* a < xmax/2 */
{ (*a)++; for(i=0 ; i< xmax; i++)
{ if(i >=(*a))
    state[i][*b].south.link=0;
else
{ buffer[*c].x=i; buffer[*c].y=*b; (*c)++; sum++; }
 }
state[*a][*b].west.link=0;
}
}
else /* _valup > a */
{ (*a)--; for(i=0 ; i< xmax; i++)
if(i <= *a )
    state[i][*b].south.link=0;
else
{ buffer[*c].x=i; buffer[*c].y=*b; (*c)++; sum++; }
    state[*a][*b].east.link=0;
}
}
temp[d]=sum;