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Asynchronous multiuser decorrelating detector for AWGN channel

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ABSTRACT

ASYNCHRONOUS MULTIUSER DECORRELATING DETECTOR FOR AWGN CHANNEL

by
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Decorrelating detector is one of the sub-optimum detectors for CDMA applications that generally has better performance than the conventional detector and is near-far resistant. Synchronous decorrelating detector employing DPSK modulation is considered and its simulation results are presented.

Attention is focussed on one-shot decorrelating detector for a two user case for asynchronous transmission, where each symbol interval is considered seperately. Initially the performance of the detector with exact estimation of the relative delay is analyzed and then the effect of inaccurate estimation on the probability of error is discussed.

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by
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CHAPTER 1

INTRODUCTION

Given the rapid and continuing growth of cellular systems throughout the world, CDMA digital cellular systems will probably be the widest deployed form of spread-spectrum system for voice communication. In CDMA, each user is assigned a distinct signature sequence (or waveform) which the user employs to modulate and spread the information bearing signal. The signature sequences also allow the receiver to demodulate the message transmitted by multiple users of the channel. Several packets of information are transmitted simultaneously over a common channel using pre-assigned signature code waveform, unlike other cellular systems, where signals from different users either transmit in different frequency band (Frequency Division Multiple Access i.e. FDMA) or different time (Time Division Multiple Access i.e. TDMA). The information sent by each user is demodulated by correlating the received signal with each of signature waveform.

CDMA digital cellular systems have some unique attributes. They were designed to offer improvements over analog cellular system designs in the following areas:

An improvement in the capacity/ spectral efficiency over analog cellular systems.

An improvement in quality, measured in terms of improved speech quality and/or improved system reliability, which in turn is terms of number of "dropped" calls and interference rejection capabilities.

Since CDMA transmits signals in the same time and in the same frequency band, its capacity is essentially limited by the interference (unlike FDMA and TDMA where capacities are primarily bandwidth limited). Any reduction in the interference convert directly into an increase in its capacity. Another important phenomenon

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that arises in a CDMA system is the partial cross-correlation in [1] of signature sequence, which manifests itself as a result of cross-talk between any two users sharing a common environment. Ideally, the codes or the signature waveforms used should be completely orthogonal to each other but practically that is not the case. Some of the efforts to improve the performance of multiuser CDMA detectors were directed toward using conventional and suboptimum detectors. A conventional single-user detector implemented in CDMA system consists of a bank of filters, each one matched to the signature sequence of a particular user. The sampled output of each matched filter, besides the desired signal, contains the residual interference from all the users. The performance of the conventional single-user detector is acceptable provided that the energies of the received signals are not too dissimilar and that the signature waveforms are designed so that their crosscorelations are low enough (this depends on the desired maximum number of simultaneous users). In practice, low crosscorelation are usually achieved employing spread-spectrum pseudonoise sequences in[4] of long period. If the received signal energies are indeed dissimilar, i.e., some users are very weak in comparison to others, then the conventional single-user detector is unable to recover the messages of the weak users reliably, even if the signature waveforms have very low crosscorelations. This is known as the near-far problem. Stringent power control and/or low cross-correlation codes have been used to solve the problem. However, power control comes at the price of increased complexity of the system and low cross-correlation between given number of signals can be achieved only at the expense of an increased bandwidth.

Acknowledging the fact that multiuser interference can not be modeled as an additive white Gaussian process, in recent years, a lot of work has been done by defining a detector whose performance is superior to the performance of the conventional detector. In [8], an optimum CDMA multiuser detector based on the Viterbi algorithm as a decision system has been studied. The complexity of this optimum

detector grows exponentially with the number of users and thus it is impractical to use unless there are small number of users. Sub-optimum detector like decorrelating detector was studied.

In this thesis we have proposed a differentially-coherent version of synchronous sub-optimum decorrelating detector using DPSK where carrier phase synchronization is not required, but it does involve somewhat more hardware. It can be seen in [1, 2] that from the performance curves DPSK has a 3-dB energy advantage over non-coherent FSK for the same value of signal-to-noise ratio. We also studied asynchronous one-shot decorrelation detector for two users proposed by Verdu and analyzed its performance for exact and inaccurate estimation of τ which is the relative delay due to asynchronous transmission. Chapter 2 gives a brief background of some of the CDMA detectors, it also includes the synchronous DPSK decorrelating detector. Chapter 3 of this thesis will give a thorough description of the asynchronous one-shot decorrelating detector. Chapter 4, we present the output error performance of the synchronous and asynchronous decorrelating detector.

User 1

$b1(t-\tau)$

Transmitter of
user 1

$\otimes$ ← $c1(t-\tau)$

User 2

$b2(t)$

$c2(t)$ → $\otimes$

Transmitter
of user 2

Channel

Base Station

$\otimes$ ← $c1(t-\tau)$

$c2(t)$ → $\otimes$

Receiver of

$\int_0^{Tb} dt$ ← user 1

user 2 → $\int_0^{Tb} dt$

$z1$

$z2$

Decision device

Threshold of zero

Decision device

Say 1
if z1>0

Say -1
if z1<0

Say 1
if z2>0

Say -1
if z2<0
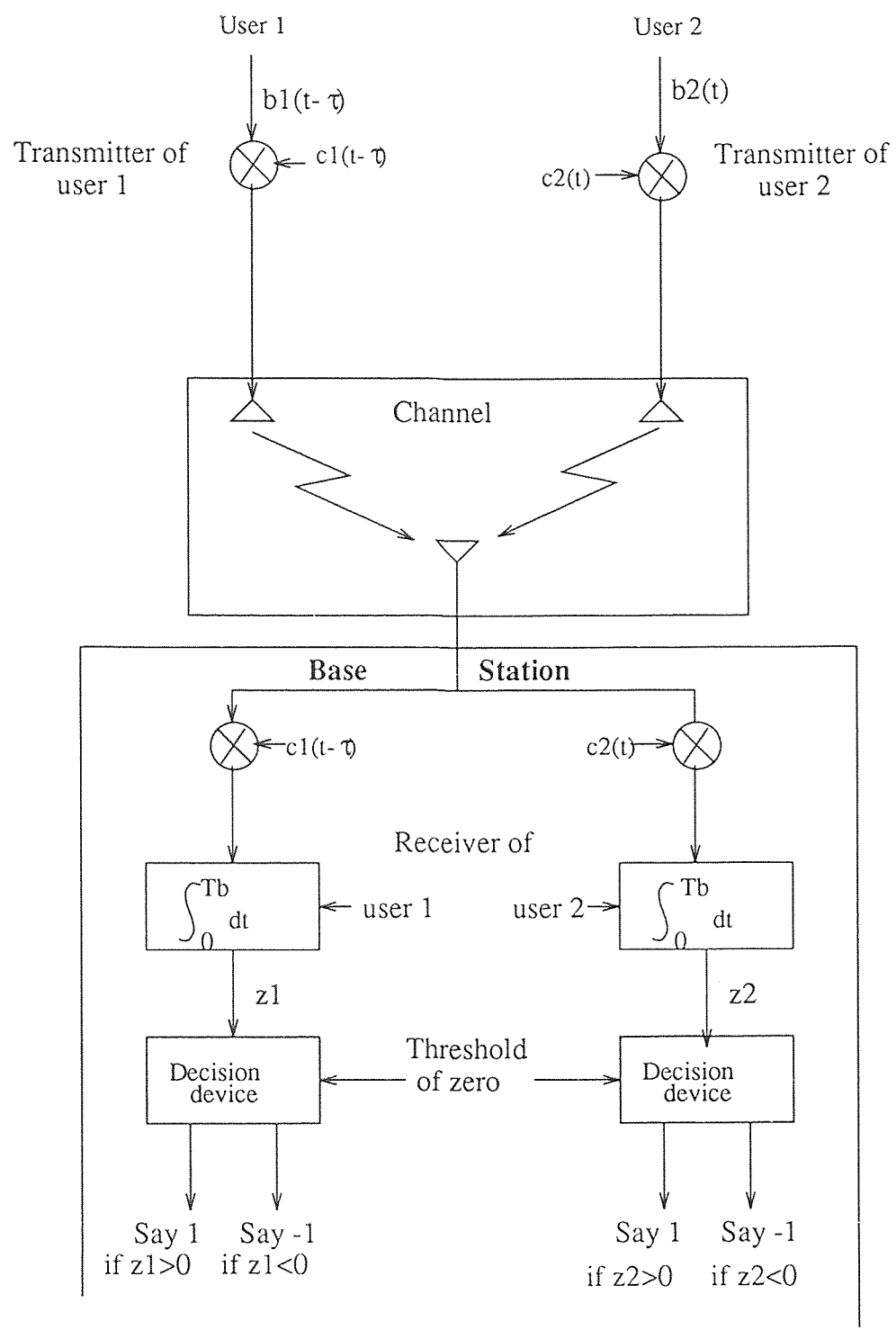
Figure 1.1 Partial Cross-correlation Between Two Users

CHAPTER 2

MULTIUSER DETECTORS

In this chapter, several types of synchronous CDMA detectors are introduced and their error performances are also presented.

2.1 CDMA System

In CDMA, the users access the channel in an asynchronous manner. Each user is assigned a unique code sequence that allows the user to spread the information signal across the assigned frequency band. Thus signals from the various users are separated at the receiver by cross-correlation of the received signal with each of the possible user signature sequences. By designing these code sequences to have relatively small cross-correlations, the crosstalk inherent in the demodulation of the signals received from multiple transmitters is minimized. A multiple access system in which a large number of users share a AWGN channel is shown in Figure 2.1 A set of K users transmit the information simultaneously over a CDMA channel. Each user (transmitter) produces its own data bit $b_k(j) = \pm 1, k = 1, 2, ..., K$, for symbol interval i of duration T. The data sequence is multiplied by the unit energy signature sequence $s_k(t)$ of the same duration T, which is known to the receiver, thus spreading the spectrum of the data sequence. These encoded signals superimpose asynchronously in the channel. The baseband equivalent waveform $r(t)$ at the input of the detector is given by,

$$r(t) = \sum_{k=1}^{K} \sum_{j} b_k(j)\sqrt{a_k}s_k(t - jT - \tau_k) + n(t) \tag{2.1}$$

where $n(t)$ is the additive, zero mean white Gaussian noise with two sided power spectral density $N_0/2$. The received energy of the k-th user signal, unknown to the receiver is denoted by a_k. In realty it is slowly time varying but it is assumed to remain unchanged throughout the whole transmission.
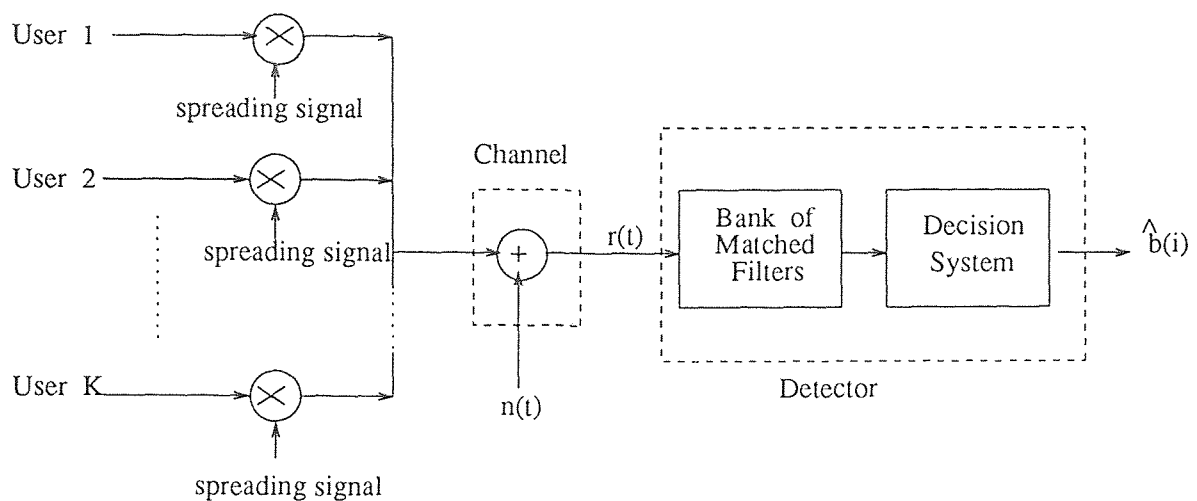
Figure 2.1 Multiple Access Model

2.2   The Conventional Detector

The conventional approach to a multiuser detector is to treat all other users as noise [6]. A conventional single-user detector implemented in the CDMA system consists of a bank of filters, each one matched to the signature sequence of the particular user. The sampled output of each matched filter, besides the desired signal, contains the residual interference from all other users. To obtain desired information from a specific user, a matched filter correlates the received signal with the same signature sequence that was originally assigned to that user in the transmitter. Undesired signals of other users are affected by the signature sequence in the same manner the original data sequence was affected at the transmitter. However, since the cross-correlations between signature sequences are non-zero, not only the desired information is received but also the interference of all other users.

2.3   The Optimum Multiuser Detector

The optimum receiver is defined as the receiver that selects the most probable sequence of bits $b_k(n), 1 \leq n \leq N, 1 \leq k \leq K$ given the received signal $r(t)$ observed over
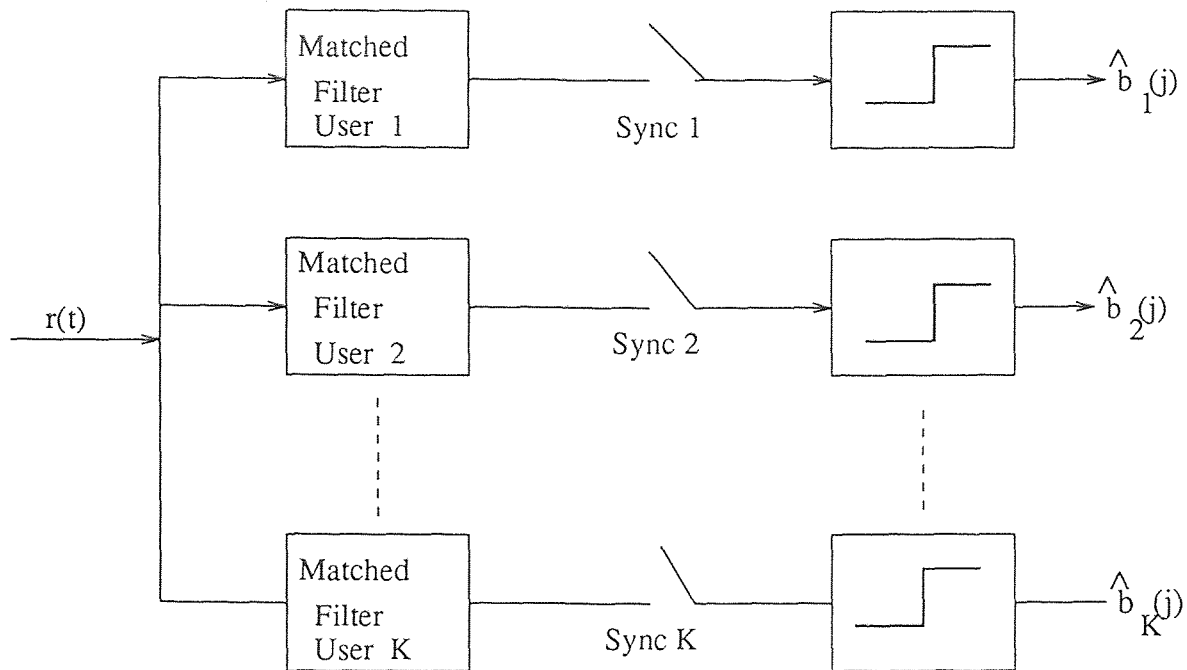
Figure 2.2 Conventional Single User Detector

the time interval $0 \leq t \leq NT + 2T$. It consists of a matched filter front-end followed by a decision system employing the Viterbi algorithm. Figure 2.3 shows an optimum detector, proposed in [1], for the multiuser interference environment and it is shown to obtain minimum probability of error. The structure of the optimum multiuser receiver consists of a bank of matched filters followed by a Viterbi algorithm. The optimum detector affords important performance gain over the conventional single-user detector, and, in particular, it solves the near-far problem. However, the price for this is exponential complexity in the number of users. The interference can be alleviated by using signature sequences with lower cross-correlations or by using power control. Lower cross-correlations are usually achieved by employing pseudo-noise sequences of long periodicity, which reduces the bandwidth efficiency.
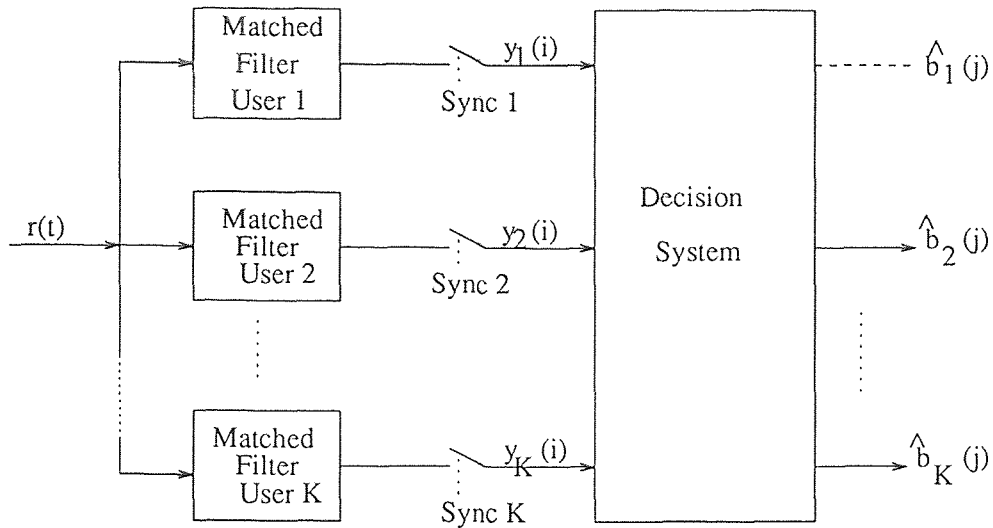
Figure 2.3 Optimum Multiuser Detector

2.4 Suboptimum Multiuser Detector

Th optimum detector for the K CDMA users has a computational complexity which increases exponentially with number of users but in case of suboptimum detectors the complexities grow linearly with number of users and the performance is slightly worse than the optimum detector, but better than the conventional detector. The suboptimum detector discussed here is the decorrelating detector.

2.4.1 The Synchronous Decorrelating Detector

The aim is find a multi-user detector with low complexity and which gives a good performance i.e. it should be near-far resistant. If the receiver has no information about the energies of the users and in order to obtain its maximum likelihood estimations and also the decisions on the transmitted stream, the receiver has to multiply the output of matched filter (y) with P^{-1}. The most likely transmitted bit can be obtained from the sign of $P^{-1}y$ and their energies can be estimated from the absolute value. In K user case, $sgn(P^{-1}y)$ essentially means that the receiver correlates the recieved signal with projection of $s_k(t)$ on the subspace orthogonal to

that spanned by other signature waveforms. This eliminates multiuser interference and thus it is called decorrelating detector.

The decorrelating detector shown in Figure 2.4 consists of a bank of matched filter
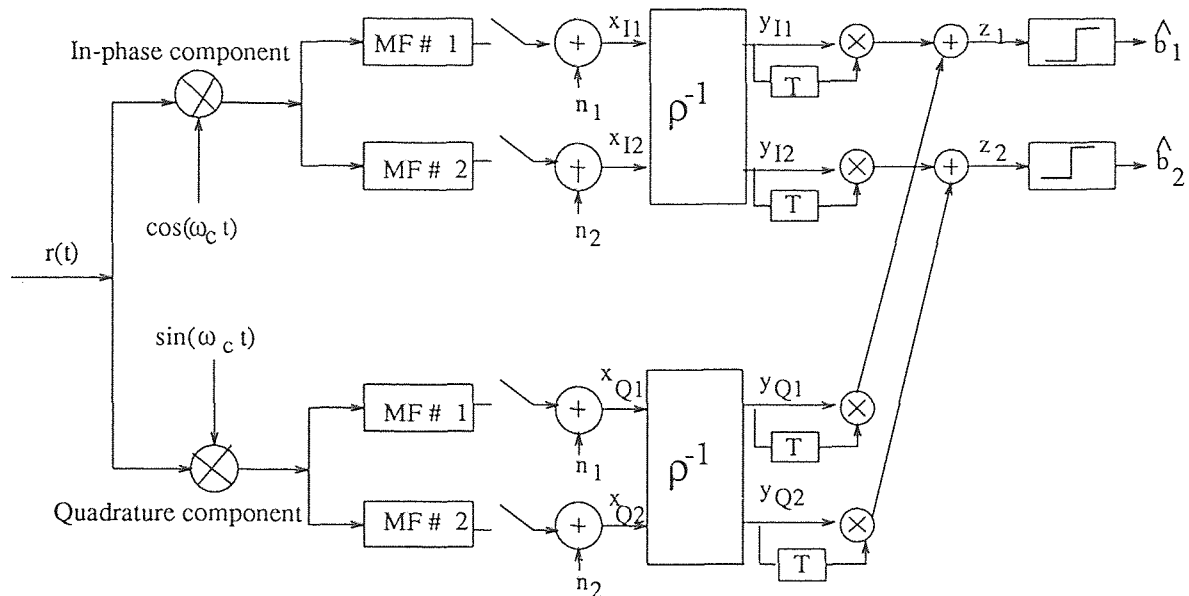
Figure 2.4 Synchronous Decorrelating Detector

and a decorrelator. The decorrelator is actually a linear transformation using the inverse of code cross-correlation matrix. The received signal of the decorrelating detector using DPSK modulation is given by,

$$r(t) = \sum_{k=1}^{K} \sum_{j} \sqrt{a_k} d_k(j) s_k(t - iT) e^{j\phi_k} + n(t) \qquad (2.2)$$

where $d_k(j) \in (\pm 1)$ is the k-th user's differentially encoded information bit (antipodal) in the j-th time interval, ϕ_k is the carrier phase of k-th user. As Differential Phase Shift Keying can be viewed as non-coherent version of PSK, it eliminates the need of a coherent reference signal at the receiver. The receiver is equipped with a storage capability, so that it can measure the relative phase difference between the waveforms received during two successive bit intervals. Provided that the unknown phase contained in the received wave varies slowly (that is, slow enough for it to be considered essentially constant over two bit interval).

The sampled output of the matched filter is given by,

In-phase component :-

$$x_I(j) = PA\phi_I d(j) + n_I(j) \tag{2.3}$$

Quadrature component:-

$$x_Q(j) = PA\phi_Q d(j) + n_Q(j) \tag{2.4}$$

For the sake of convenience the index j will be omitted elsewhere in the text. In Equation 2.3, $x_I = [x_{I1}, x_{I2}, \cdots, x_{IK}]^T$, $d = [d_1, d_2, \cdots, d_K]^T$, $A = diag[\sqrt{a_1}, \sqrt{a_2}, \cdots, \sqrt{a_K}]$, $\phi_I = diag[\cos\phi_1, \cos\phi_2, \cdots, \cos\phi_K]$, $\phi_Q = diag[\sin\phi_1, \sin\phi_2, \cdots, \sin\phi_K]$ and $n_I = [n_{I1}, n_{I2}, \cdots, n_{IK}]^T$. The (k, j)-th element of the symmetric cross-correlation matrix P is defined as;

$$\rho_{ki} = \int_0^T s_k(t)s_i(t)\,dt \quad k, i = 1, 2, \cdots, K \tag{2.5}$$

with $\rho_{kk} = 1$. The covariance matrix of a zero mean Gaussian noise vector n_I is,

$$E\{n_I n_I^T\} = \frac{N_0}{2} P \tag{2.6}$$

The output of the decorrelator for a two user case (in phase component) is given as,

$$y_I(j) = P^{-1}x_I(j) = A\phi_I d(j) + P^{-1}n_I(j) \tag{2.7}$$

Similarly, the in-phase output of the decorrelator in the preceding signaling interval is given by,

$$y_I(j-1) = P^{-1}x_I(j-1) = A\phi_I d(j-1) + P^{-1}n_I(j-1) \tag{2.8}$$

The decoded in-phase component at the output of the decorrelator is given by:

$$z_I(j) = y_I(j) \cdot y_I(j-1) \tag{2.9}$$

The decoded quadrature component at the output of the decorrelator is given by:

$$z_Q(j) = y_Q(j) \cdot y_Q(j-1) \tag{2.10}$$

where · represents element-by-element multiplication, refer Appendix A for details. Combining the in-phase and the quadrature component, the decision variable is given by:

$$z = z_I + z_Q,$$

$$\hat{b} = sgn(z) \tag{2.11}$$

The probability of error for user 1 is obtained by simulation which is shown in Figure 4.2.

CHAPTER 3

THE ASYNCHRONOUS ONE-SHOT DECORRELATING DETECTOR

In asynchronous transmission, there are exactly two consecutive symbols from each interferer that overlap a desired symbol. It is not possible to design signature sequences for any pair of users that are orthogonal for all time offsets. Consequently, interference from other users is unavoidable. In this case, the near-far problem resulting from unequal power in the signals transmitted by the various users is particularly serious.

3.1 General Case (K Users)

The detector which is presented here can be obtained by taking a one-shot approach where each symbol interval is considered separately thus, each user's contribution to the interference can be clearly seen.

Attention is focussed on bit 0 of user 1, which occupies the interval $[0, T]$ (assuming without loss of generality $\tau_1 = 0$). From Figure 3.1 it can be seen that bit 0 of user 1 overlaps with bit -1 of k where $k = 2, 3, ..., K$ over interval $[0, \tau_k]$ and bit 0 of the same user k in the interval $[\tau_k, T]$. This situation can be viewed in [7] as $(2K - 1)$ user synchronous channel with unit energy signature waveform

$$\bar{s}_1(t) = s_1(t), \quad \bar{s}_k(t) = e_k^{-1/2} s_k^L(t), \quad \bar{s}_k(t) = (1 - e_k)^{-1/2} s_k^R(t), \qquad (3.1)$$

where

$$s_{2k-2}^L(t) = \begin{cases} s_k(t + T - \tau_k) & 0 \le t \le \tau_k \\ 0 & \tau_k \le t \le T \end{cases} \qquad (3.2)$$

$$s_{2k-1}^R(t) = \begin{cases} 0 & 0 \le t \le \tau_k \\ s_k(t - \tau_k) & \tau_k \le t \le T \end{cases} \qquad (3.3)$$

and

$$e_k = \int_0^{\tau_k} s_k^2(t + T - \tau_k) \, dt \qquad (3.4)$$
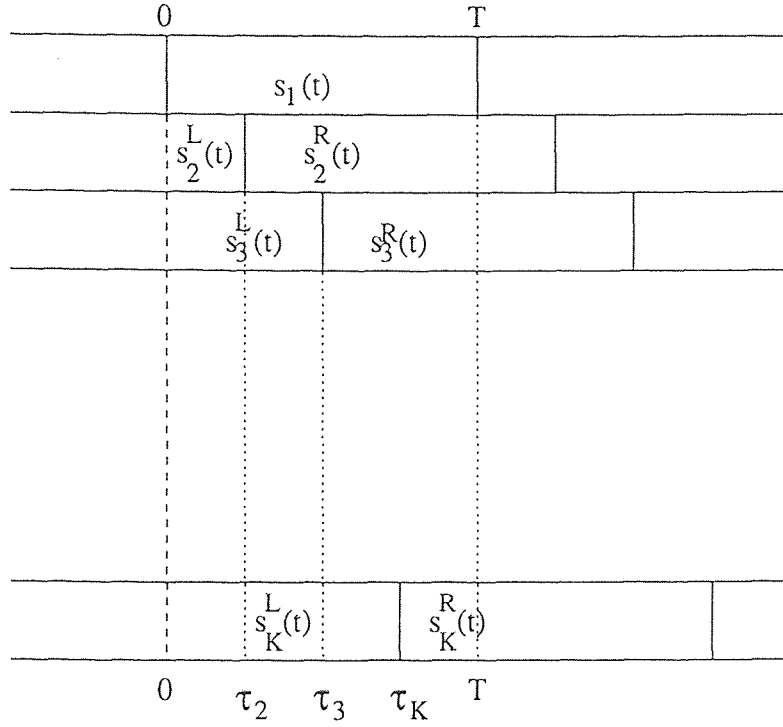
Figure 3.1 Asynchronous Signals (For K Users)

and $(0 < e_k < 1)$ is the partial energy of the interfering signal over the left overlapping interval. The cross-correlation matrix between the synchronous users is given as follows:

$$H = \begin{bmatrix} 1 & \varphi_2 & \varphi_3 & \cdots & \varphi_K \\ \varphi_2^T & I & \phi_3^{(2)} & \cdots & \phi_K^{(2)} \\ \varphi_3^T & \phi_3^{(2)^T} & I & \cdots & \phi_K^{(3)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \varphi_K^T & \phi_K^{(2)^T} & \phi_K^{(2)^T} & \cdots & I \end{bmatrix}$$ (3.5)

where

$$\varphi_i = \left[\frac{\rho_{1i}}{\sqrt{e_i}}, \frac{\rho_{i1}}{\sqrt{1-e_i}} \right], \quad i = 2, 3, \cdots, K$$ (3.6)

and

$$\phi_i^{(j)} = \begin{bmatrix} \Psi_{ji}^L & 0 \\ \frac{\rho_{ij}}{\sqrt{e_i(1-e_j)}} & \Psi_{ji}^R \end{bmatrix}$$ (3.7)

$j = 2, 3, \cdots, K-1$ and $i = j+1, \cdots, K$. The cross-correlation coefficients are

$$\rho_{i1} = \int_0^T s_1(t) s_k^L(t)\, dt$$ (3.8)

$$\rho_{1i} = \int_0^T s_1(t) s_k^R(t) \, dt \tag{3.9}$$

$$\rho_{ij} = \int_{\tau_j}^{\tau_i} s_i^L(t) s_j^R(t) \, dt \tag{3.10}$$

$$\Psi_{ij}^L = \int_0^{\tau_j} \bar{s}_{2j-2}(t) \bar{s}_{2i-2}(t) \, dt \tag{3.11}$$

$$\Psi_{ij}^R = \int_{\tau_i}^T \bar{s}_{2j-1}(t) \bar{s}_{2i-1}(t) \, dt \tag{3.12}$$

The output of the matched filter is

$$x_s = H A_s b_s + n_s \tag{3.13}$$

where $x_s = [x_1, x_2, \cdots, x_{2K-1}]^T$, $A = diag[\sqrt{a_1}, \sqrt{a_2}, \sqrt{a_2}, \sqrt{a_3}\sqrt{a_3}, \cdots, \sqrt{a_K}, \sqrt{a_K}]$, $b_s = [b_1(0), b_2(-1), b_2(0), \cdots, b_K(-1), b_K(0)]^T$, and $n_s = [\bar{n}_1, \bar{n}_2, \cdots, \bar{n}_{2K-1}]^T$ with

$$\bar{n}_m = \int_0^T n(t) \bar{s}_m(t) \, dt, \quad m = 1, 2, \cdots, 2K - 1 \tag{3.14}$$

are zero-mean Gaussian noise with variance of $N_0/2$. The covariance matrix of the noise vector n_s is

$$E\{n_s n_s^T\} = \frac{N_0}{2} H \tag{3.15}$$

The resulting output of the decorrelator is:

$$z_s = H^{-1} x_s = A_s b_s + H^{-1} n_s = A_s b_s + \xi_s, \tag{3.16}$$

where $\xi_s = H^{-1} n_s$. The covariance matrix of the zero mean gaussian noise vector ξ_s is :

$$E\{\xi_s \xi_s^T\} = \frac{N_0}{2} H^{-1} \tag{3.17}$$

Let $\hat{b}_s = [\hat{b}_1, \hat{b}_2, \hat{b}_3, \cdots, \hat{b}_K]$ be the decision output of the decorrelator; thus $\hat{b}_s$ is defined as :
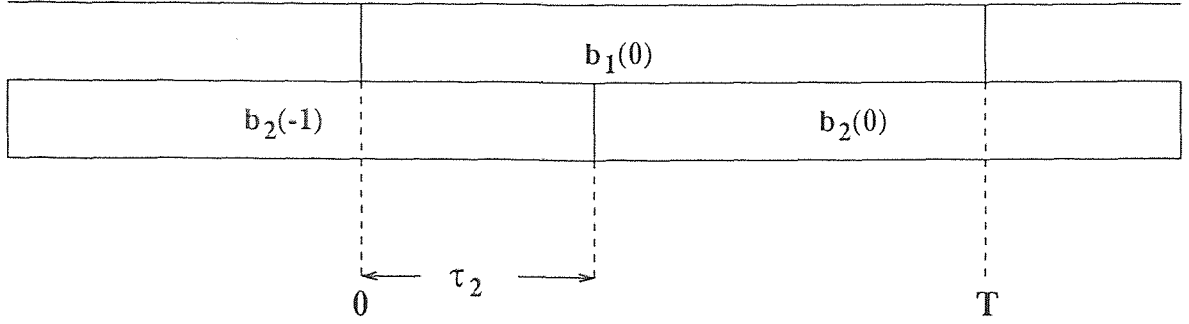
$$\hat{b}_s = sgn(z_s) \tag{3.18}$$

Figure 3.2 Received Asynchronous Signals, Two User Case

3.2 Two User Case

To illustrate the above general case, the derivations of the statistics for the two-user asynchronous detector are given here in detail. White gaussian asynchronous multiple access channel is given by:

$$y(t) = \sum_{i=-M}^{M} \sqrt{a_1} b_1(i) s_1(t - iT - \tau_1) + \sum_{i=-M}^{M} \sqrt{a_2} b_2(i) s_2(t - iT - \tau_2) + n(t) \quad (3.19)$$

From Figure 3.2 bit 0 of user 1 overlaps with two bits of user 2, bit -1 over the interval $[0, \tau_2]$ and bit 0 over the interval $[\tau_2, T]$. The situation can be viewed as 3 user synchronous channel. The unit energy signature waveforms of the synchronous users can be derived from the waveforms of the original asynchronous users;

$$\bar{s}_1(t) = s_1(t), \quad \bar{s}_2(t) = e_2^{-1/2} s_2^L(t), \quad \bar{s}_2(t) = (1 - e_2^{-1/2}) s_2^R(t) \quad (3.20)$$

where

$$s_2^L(t) = \begin{cases} s_2(t + T - \tau_2) & 0 \le t \le \tau_2 \\ 0 & \tau_2 \le t \le T \end{cases} \quad s_2^R(t) = \begin{cases} 0 & 0 \le t \le \tau_2 \\ s_2(t - \tau_2) & \tau_2 \le t \le T \end{cases} \quad (3.21)$$

and

$$e_2 = \int_0^{\tau_2} s_2^2(t + T - \tau_2) \, dt \quad (3.22)$$

where $(0 < e_2 < 1)$ is the partial energy of user 2 that overlaps the bit 0 of the user 1. The 3-user decorrelating detector can be solved with the cross-correlation
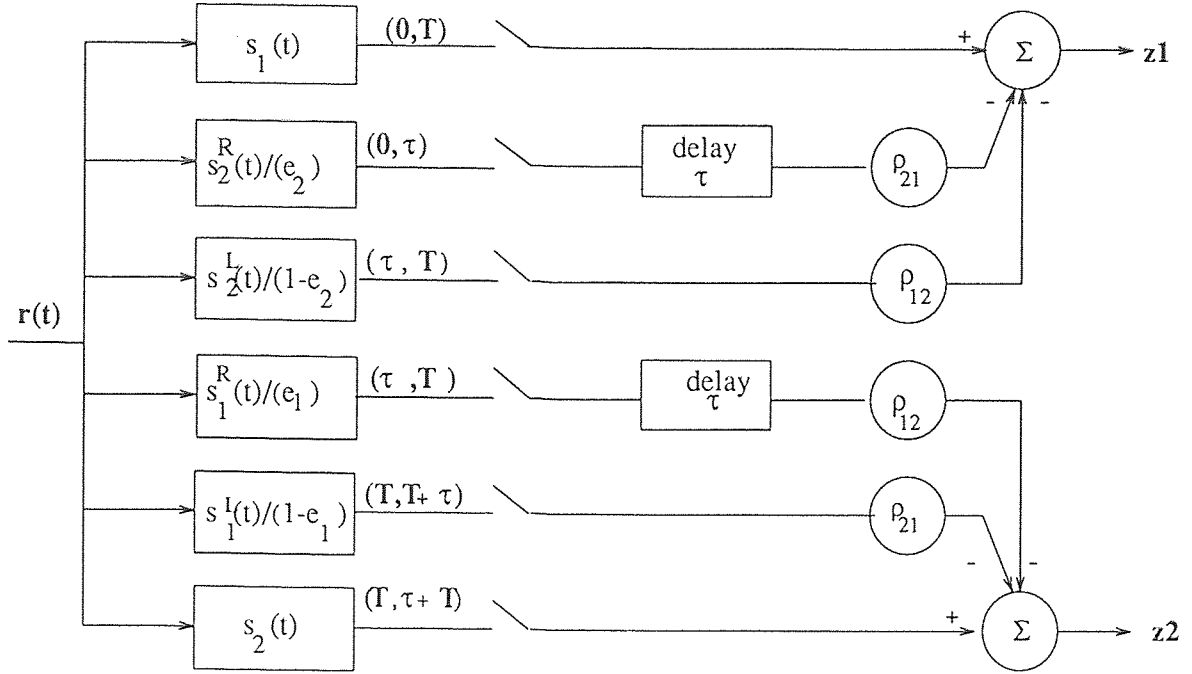
Figure 3.3 Asynchronous Decorrelator

matrix :

$$H = \begin{bmatrix} 1 & \rho_{21}/\sqrt{e_2} & \rho_{12}/\sqrt{1-e_2} \\ \rho_{21}/\sqrt{e_2} & 1 & 0 \\ \rho_{12}/\sqrt{1-e_2} & 0 & 1 \end{bmatrix} \tag{3.23}$$

The sampled output of the matched filter is

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = H \begin{bmatrix} \sqrt{a_1} & & \\ & \sqrt{a_2} & \\ & & \sqrt{a_2} \end{bmatrix} \begin{bmatrix} b_1(0) \\ b_2(-1) \\ b_2(0) \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix} \tag{3.24}$$

where

$$n_1 = \int_0^T n(t)s_1(t)\,dt; \quad n_2 = \int_0^{\tau_2} n(t)s_2(t)\,dt; \quad n_3 = \int_{\tau_2}^T n(t)s_2(t)\,dt \tag{3.25}$$

thus, n_1, n_2 and n_3 are the noise vectors with zero mean but they are correlated. These Gaussian random variables have variance of $N_0/2$. The output of the decorrelator for user 1 shows that there is no contribution from user 2 i.e. interference free. The output of the decorrelating detector for two users looks like this:

$$z_i = Ab + H^{-1}n = Ab + \xi \tag{3.26}$$

where $b = [b_1(0), b_2(-1), b_2(0)]$ and $\xi = [\xi_{s1}, \xi_{s2}, \xi_{s3}]$, ξ_{s1} is a zero mean gaussian random variable with variance:

$$E\{\xi_{s1}^2\} = E\{\xi_1^2(0)\} = \left[\frac{N_0/2}{1 - \rho_{21}^2/e_2 - \rho_{12}^2/(1 - e_2)} \right] \qquad (3.27)$$

The decorrelator is followed by the hard limiter, thus the decision variable for user 1 is given by

$$b_1 = sgn(z_1)$$

The output probability of error for user 1 is given by :

$$P_{e1} = Q(\sqrt{a_1}/var\{\xi_1\}) \qquad (3.28)$$

3.3 Sensitivity of One-shot Decorrelating Detector to Inaccurate Estimation of τ

In asynchronous transmission there is misalignment between different users in a multiuser detector. When the delay τ factor comes into picture, it is important for the receiver to estimate this delay. Depending upon the delay which is estimated by the receiver the matrix H^{-1} is calculated. Till now it was assumed that the delay estimated by the receiver was correct (so $HH^{-1} = I$). Here in this section we are assuming that the delay estimated by the receiver is not correct so, our new delay is $\hat{\tau} = \tau \pm \Delta\tau$. With $\hat{\tau}$ the decorrelator of the receiver will change. Here $H^{-1} \neq \hat{H}^{-1}$ thus, $H\hat{H}^{-1} \neq I$. The effect of inaccurate τ on the performance of the decorrelator is evaluated. Thus $\hat{H}$ is given by

$$\hat{H} = \begin{bmatrix} 1 & \hat{\rho}_{21}/\sqrt{\hat{e}_2} & \hat{\rho}_{12}/\sqrt{1 - \hat{e}_2} \\ \hat{\rho}_{21}/\sqrt{\hat{e}_2} & 1 & 0 \\ \hat{\rho}_{12}/\sqrt{1 - \hat{e}_2} & 0 & 1 \end{bmatrix} \qquad (3.29)$$

where

$$\hat{e}_2 = \int_0^{\hat{\tau}} s_2^2(t + T - \hat{\tau})\, dt \qquad (3.30)$$

$$\hat{\rho}_{21} = \int_0^{\hat{\tau}} s_1(t)\hat{s}_2^L(t)\, dt, \qquad \hat{\rho}_{12} = \int_{\hat{\tau}}^T s_1(t)\hat{s}_2^R(t)\, dt \qquad (3.31)$$
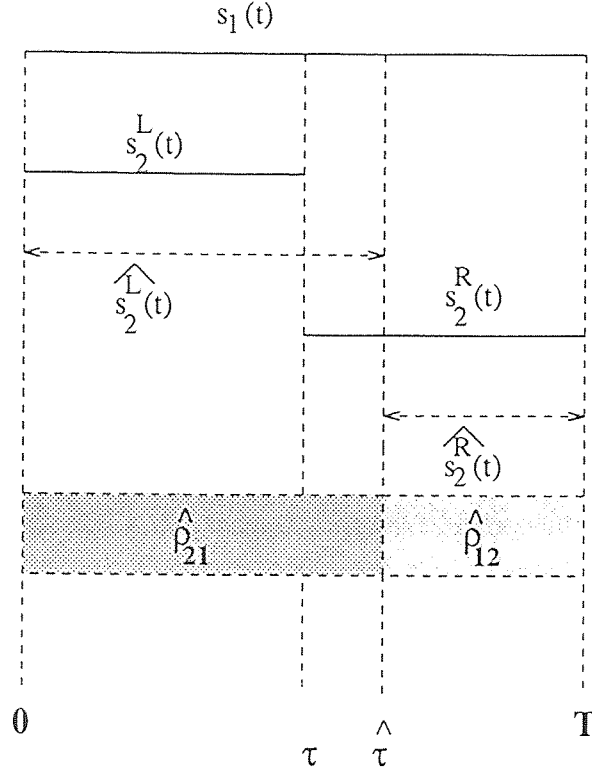
Figure 3.4 Asynchronous Signals with inaccurate estimation of τ

and $\hat{H}^{-1}$ is given by :

$$\hat{H}^{-1} = \frac{1}{det(\hat{H})} \begin{bmatrix} 1 & -\hat{\rho}_{21}/\sqrt{\hat{e}_2} & -\hat{\rho}_{12}/\sqrt{1-\hat{e}_2} \\ \hat{\rho}_{21}/\sqrt{\hat{e}_2} & 1-\hat{\rho}_{12}^2/(1-\hat{e}_2) & -(\hat{\rho}_{21}/\sqrt{\hat{e}_2})(\hat{\rho}_{12}/\sqrt{1-\hat{e}_2}) \\ \hat{\rho}_{12}/\sqrt{1-\hat{e}_2} & (\hat{\rho}_{21}/\sqrt{\hat{e}_2})(\hat{\rho}_{12}/\sqrt{1-\hat{e}_2}) & (1-\hat{\rho}_{21}^2/\hat{e}_2) \end{bmatrix}$$

$$(3.32)$$

The output of the decorrelator for user 1 will have contributions from bit -1 and bit 0 of user 2 which can be seen from the following equation:

$$z = \hat{H}^{-1} H A b + \hat{H}^{-1} n \qquad (3.33)$$

$$\begin{aligned} z_1(0) &= \frac{1}{det(H)} \{ [1 - \frac{\hat{\rho}_{21}\rho_{21}}{\sqrt{\hat{e}_2 e_2}} - \frac{\hat{\rho}_{12}\rho_{12}}{\sqrt{(1-\hat{e}_2)(1-e_2)}}] b_1(0) \\ &+ [\frac{\rho_{21}}{\sqrt{e_2}} - \frac{\hat{\rho}_{21}}{\sqrt{\hat{e}_2}}] b_2(-1) + [\frac{\rho_{12}}{\sqrt{1-e_2}} - \frac{\hat{\rho}_{12}}{\sqrt{1-\hat{e}_2}}] b_2(0) + \hat{\xi}_1 \} \quad (3.34) \end{aligned}$$

where

$$\hat{\xi}_1 = [n_1(0) - \frac{\hat{\rho}_{21}}{\sqrt{\hat{e}_2}}n_2(-1) - \frac{\hat{\rho}_{12}}{\sqrt{1 - \hat{e}_2}}n_2(0)]$$

$$= [n_1(0) - \frac{\hat{\rho}_{21}}{\sqrt{\hat{e}_2}}\frac{n_2^L(-1)}{\sqrt{\hat{e}_2}} - \frac{\hat{\rho}_{12}}{\sqrt{1 - \hat{e}_2}}\frac{n_2^R(0)}{\sqrt{1 - \hat{e}_2}}] \qquad (3.35)$$

The variance of $\hat{\xi}_1$ is given by :

$$var\{\hat{\xi}_1\} = E\{\hat{\xi}_1^2\} = \frac{N_0/2}{[1 - \hat{\rho}_{21}^2/e_2 - \hat{\rho}_{12}^2/(1 - e_2)]} \qquad (3.36)$$

The decision variable for user 1 is given by: $b_1(0) = sgn(z_1(0))$

The output probability of error for user 1 can be evaluated as:

$$P_{e1} = \frac{1}{4}[\quad Q \quad (\frac{A\sqrt{a_1} + B\sqrt{a_2} + C\sqrt{a_2}}{var\{\hat{\xi}_1\}})$$

$$+ \quad Q(\frac{A\sqrt{a_1} + B\sqrt{a_2} - C\sqrt{a_2}}{var\{\hat{\xi}_1\}})$$

$$+ \quad Q(\frac{A\sqrt{a_1} - B\sqrt{a_2} + C\sqrt{a_2}}{var\{\hat{\xi}_1\}})$$

$$+ \quad Q(\frac{A\sqrt{a_1} - B\sqrt{a_2} - C\sqrt{a_2}}{var\{\hat{\xi}_1\}})] \qquad (3.37)$$

where

$A = [1 - \hat{\rho_{21}}\rho_{21}/\sqrt{\hat{e_2}e_2} - \hat{\rho_{12}}\rho_{12}/\sqrt{(1 - \hat{e}_2)(1 - e_2)}]$,

$B = [\rho_{21}/\sqrt{e_2} - \hat{\rho}_{21}/\sqrt{\hat{e}_2}]$ and

$C = [\rho_{12}/\sqrt{1 - e_2} - \hat{\rho}_{12}/\sqrt{1 - \hat{e}_2}]$

The proof of the P_{e1} is given in detail in Appendix B

CHAPTER 4

ANALYSIS OF RESULTS AND CONCLUSION

This chapter includes the simulation and analytical results of synchronous and asynchronous decorrelating detector.

4.1 Synchronous Decorrelating Detector
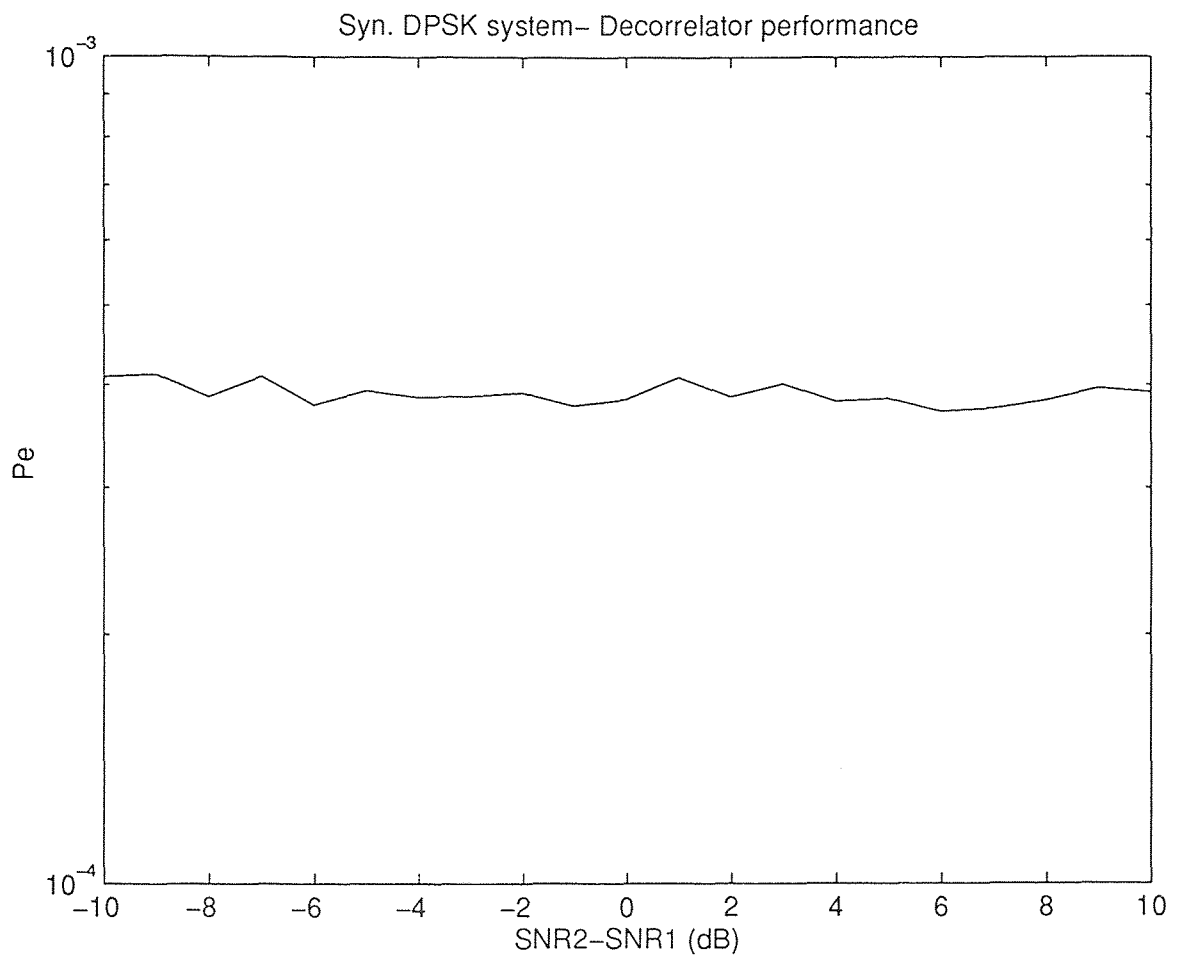

Figure 4.1 Error Performance for $K = 2$, $SNR1 = 11dB$, $\rho = 0.7$

A synchronous decorrelating detector for two users with additive white Gaussian channel for a CDMA system was proposed and analysed. The decorrelator using Differentially encoded PSK (DPSK) modulation where the estimation of carrier

phase is not required thus, a non-coherent communication technique. Instead, the received signal in any given signaling interval is compared to the phase of the received signal from the preceding signaling interval. The encoding is performed by a relatively simple logic circuit preceding the modulator. Demodulating the signal is done in the similar manner.

4.2   Asynchronous One-shot Decorrelating Detector

In this section two possibilities are considered. Initially the performance of the detector with correct estimation of τ is analised and then the possibility of incorrect estimation of τ is discussed. These two possibilities check the robustness of the detector.
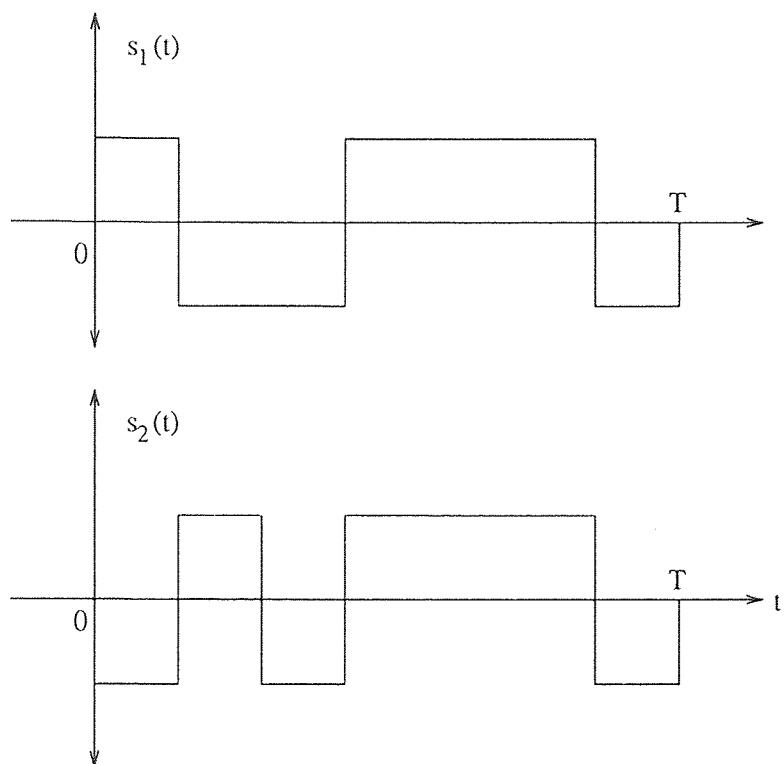

Figure 4.2 Pair of Gold codes used

4.2.1 Exact Estimation of τ

As the numerical analysis of the error probability for the K-user case (where K is considerably large number) is computationally intensive, so we restricted ourselves to the two-user case.For the given pair of Gold codes as shown in the follwing figure attention was focused on the bit-error-rate in the region of low σ^2, because that quantifies the inherent performance degradation due to the presence of other users in the channel.
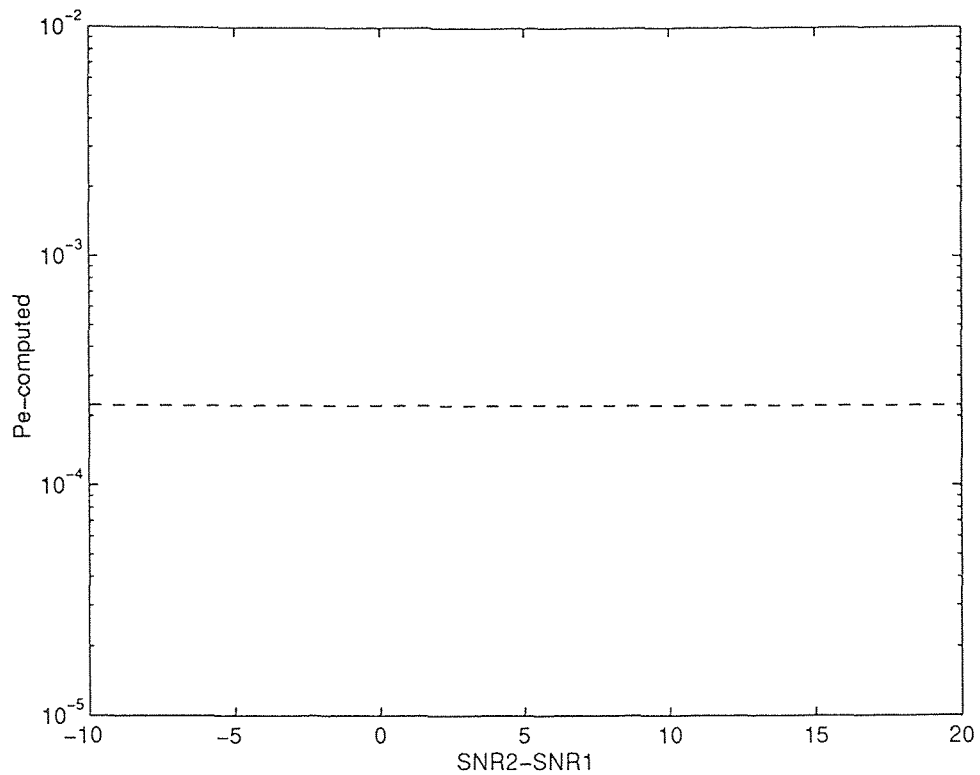
Figure 4.3 Error Performance for $K = 2$, $SNR1 = 8dB$

In case of asynchronous transmission, as one bit of user 1 overlaps with two consecutive bits of user 2 in order to cancel out their contribution two terms should be subtracted. Therefore, at the output of each stage in the conventional detector there is a need to store a sliding window of decisions. For example, if there is only one stage there is need to store two consecutive outputs of each conventional single-user-detector, where as if there are two stages, two consecutive output of first stage,

and hence, a window of three consecutive conventional decisions at the begining. In general, if there are m stages, the k-th stage needs to store $m - k + 1$ consecutive outputs for each user. But in case of one-shot decorrelating detector, it can be simplified using (partial) correlations with $s_2^L(t)$ and $s_2^R(t)$

It can be seen from the Figure 4.3 that the decorrelating detector eliminates the multiuser interference from the decision statistics. Therefore, its bit-error-rate has the very desirable property that it is independent of the energy of the interfering users.

4.2.2 Inaccurate Estimation of τ
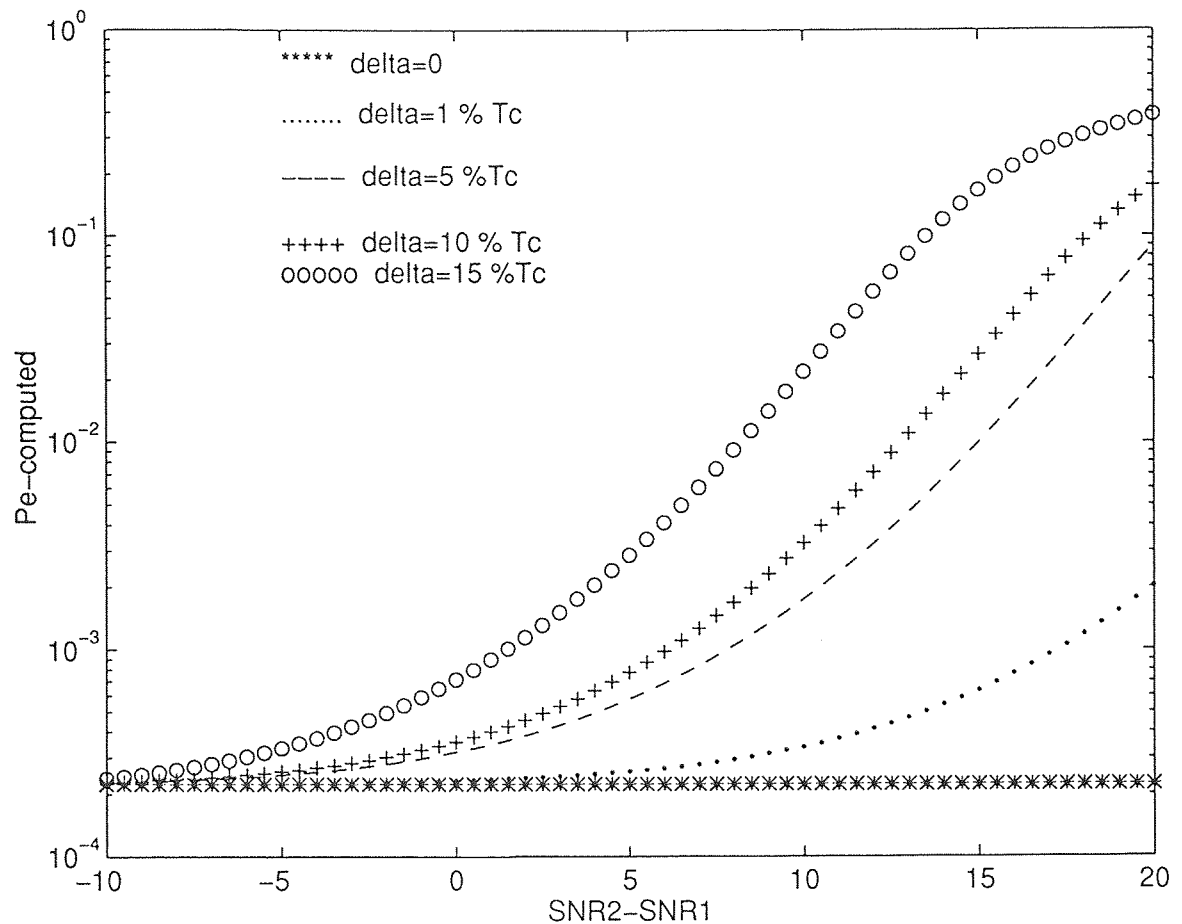
Figure 4.4 Error Performance with inaccurate τ

Due to inaccurate estimation of τ i.e. $\hat{\tau} = \tau + \Delta\tau$, for a given pair of Gold codes, the crosscorrelation matrix used for decorrelation is different from H. Figure 4.4 shows that as $\Delta\tau$ increases the performance of the detector deteriorates.

Figure 4.5 Error Performance as $\Delta\tau$ increases

Figure 4.5 shows the performance of the detector as $\Delta\tau$ increases with respect to the chip time T_c of the signature waveform. As the effect of user 2 starts dominating the error probability of the detector is affected significantly.

APPENDIX A

TWO USER SYNCHRONOUS DPSK DETECTOR

The computation of the error probability for two-user case is derived here. Without loss of generality consider only user 1.

The received signal is given by:

$$r(t) = \sum_{k=1}^{2} \sum_{j} \sqrt{a_k} d_k(j) s_k(t - jT) e^{j\phi_k} + n(t) \qquad (A.1)$$
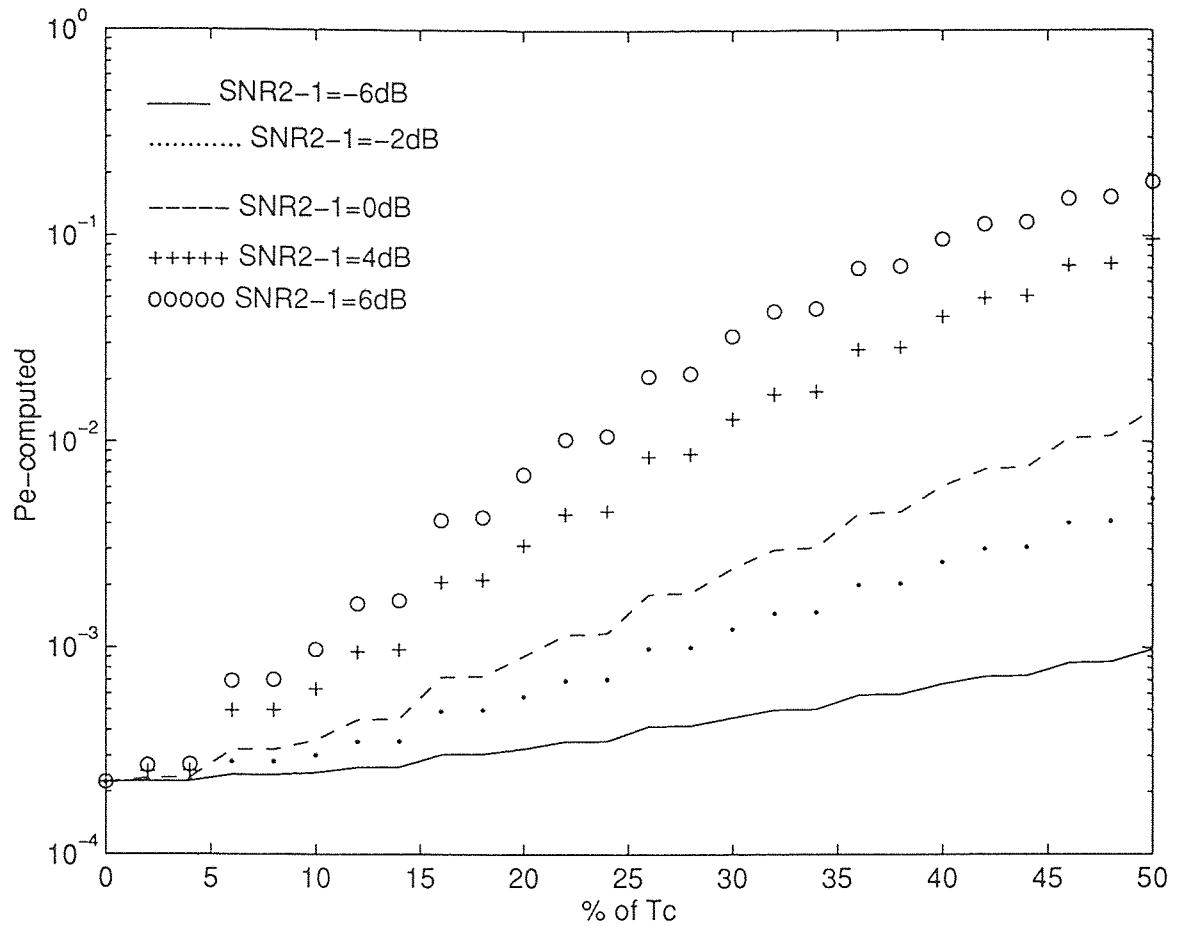
By seperating the in-phase and the quadrature componenets for a two user case the received signal can be written as;

$$r(t) = \sum_{i} \sqrt{a_1} d_1(j) s_1(t - jT) \cos(\phi_1) + \rho \sqrt{a_2} d_2(j) s_2(t - jT) \cos(\phi_2) + n(t) \quad (A.2)$$

where

a_1: Received energy of user 1;

$d_1(j)$: Differentially encoded information bit $d_1(j) \in \{\pm 1\}$;

$s_1(t)$: signature waveform of user (normalized);

$n(t)$: Additive White Gaussian noise, with zero-mean and power spectral density equal to σ^2

$$\int_0^T s_j^2(t) dt = 1 \quad j = 1, 2; \qquad \rho = \int_0^T s_1(t) s_2(t) dt < 1 \qquad (A.3)$$

The sampled output of the matched filter is with time index j omitted:

In-phase component:-

$$x_{I1} = \sqrt{a_1} \cos \phi_1 d_1 + \rho \sqrt{a_2} \cos \phi_2 d_2 + n_1 \qquad (A.4)$$

Quadrature component:-

$$x_{Q1} = \sqrt{a_1} \sin \phi_1 d_1 + \rho \sqrt{a_2} \sin \phi_2 d_2 + n_1 \qquad (A.5)$$

where cross-correlation between user 1 and 2 is $\rho=0.7$ Signal-to-noise ratio of user 1 i.e. SNR1 = 8 dB SNR2 is relative to user 1 and varies from -10 dB to 8 dB

$$n_1 = \int_{jT}^{(j+1)T} n(t)s_1(t-jT)dt \qquad (A.6)$$

$E\{n_1\} = 0$ and

$$E\{n_1^2\} = \sigma_{n1}^2 = \rho\frac{N_0}{2} \qquad (A.7)$$

The decorrelated output for user 1 and 2 (in-phase component) are given by:

$$y_{I1} = x_{I1} - \rho x_{I2} = (1-\rho^2)\sqrt{a_1}d_1\cos\phi_1 + n_{I1} - \rho n_2$$

$$y_{I2} = x_{I2} - \rho x_{I1} = (1-\rho^2)\sqrt{a_2}d_2\cos\phi_2 + n_{I2} - \rho n_1 \qquad (A.8)$$

In order to decode the information bits the receiver has to store the relative phase difference between the waveforms received during two successive it intervals, in-phase component for user 1 can be given as,

$$z_I = \begin{bmatrix} z_{I1}(j) \\ z_{I2}(j) \end{bmatrix} = \begin{bmatrix} a_1\cos^2\phi_1 d_1(j)d_1(j-1) \\ a_2\cos^2\phi_2 d_2(j)d_2(j-1) \end{bmatrix} + n_{bI}(j) \qquad (A.9)$$

while the quadrature component can be given as,

$$z_Q = \begin{bmatrix} z_{Q1}(j) \\ z_{Q2}(j) \end{bmatrix} = \begin{bmatrix} a_1\sin^2\phi_1 d_1(j)d_1(j-1) \\ a_2\sin^2\phi_2 d_2(j)d_2(j-1) \end{bmatrix} + n_{bQ}(j) \qquad (A.10)$$

where $n_{bI}(j)$ and $n_{bQ}(j)$ include three terms: two signal×noise terms and one noise×noise term. The statistics of this noise are quite complex.

For binary DPSK, $d_k(j) = b_k(j)d_k(j-1)$, where $d_k(j)$ and $b_k(j) \in \{\pm 1\}$

thus, combining the in-phase and quadrature component

$$z_1 = z_{I1} + z_{Q1}$$
$$= \begin{bmatrix} a_1 b_1(j) \\ a_2 b_2(j) \end{bmatrix} + n_z(j) \qquad (A.11)$$

the decision variable $\hat{b}_1$ is given by:

$$\hat{b}_1 = sgn(z_1) \qquad (A.12)$$

The performance if this decorrelating detector is determined from the simulation results.

APPENDIX B

ERROR PERFORMANCE OF ASYNCRONOUS DECORRELATING DETECTOR

The performance analysis of the asynchronous one-shot decorrelating detector where the receiver estimates the delay τ incorrectly. The received signal is given by for K users :

$$r(t) = \sum_{k=1}^{K} \sum_{i} \sqrt{a_k} b_k(i) s_k(t - iT - \tau_k) + n(t) \tag{B.1}$$

where

$$\int_0^T s_1^2(t) dt = 1 \tag{B.2}$$

The determination of the output error probability involved analytical approach. As one bit of user 1 $b_1(0)$ overlaps two consecutive bits of user 2, $b_2(-1)$ and $b_2(0)$

The two user asynchronous white Gaussian CDMA channel considered here is given by:

$$y(t) = \sum_{i=-M}^{M} \sqrt{a_1} b_1(i) s_1(t - iT - \tau_1) + \sum_{i=-M}^{M} \sqrt{a_2} b_2(i) s_2(t - iT - \tau_2) + n(t) \tag{B.3}$$

Without loss of generality that $\tau_1 = 0$, the sampled output of the matched filter for two users with time index i ommited is given by

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & \rho_{21}/\sqrt{e_2} & \rho_{12}/\sqrt{1-e_2} \\ \rho_{21}/\sqrt{e_2} & 1 & 0 \\ \rho 12/\sqrt{1-e_2} & 0 & 1 \end{bmatrix} \begin{bmatrix} \sqrt{a_1} & & \\ & \sqrt{a_2} & \\ & & \sqrt{a_2} \end{bmatrix} \times$$

$$\begin{bmatrix} b_1(0) \\ b_2(-1) \\ b_2(0) \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

where

$$\rho_{12} = \int_0^T s_1(t) s_2(t - \tau_2) dt,$$

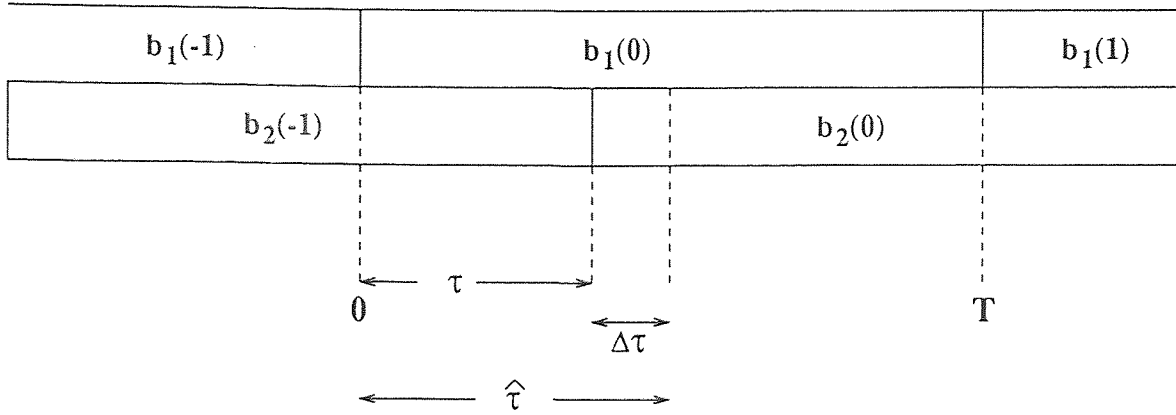$$\rho_{21} = \int_0^T s_1(t) s_2(t + T - \tau_2) dt, \tag{B.4}$$

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Figure B.1 Asynchronous Decorrelating Detector with inaccurate τ

are the crosscorrelations between the signature waveforms. The discrete-time random process $\{[n_1(i)n_2(i)]^T\}$ which is given in Equation B.4 is Gaussian with zero mean and the covariance matrix in [3] is given by:

$$E = \left[\begin{bmatrix} n_1(i) \\ n_2(i) \end{bmatrix} \begin{bmatrix} n_1(j)n_2(j) \end{bmatrix} \right] = \sigma^2 H\{i-j\} \tag{B.5}$$

Output of the decorrelator where $H^{-1} \neq \hat{H}^{-1}$ and thus, $H\hat{H}^{-1} \neq I$ is given by:

$$\begin{bmatrix} z_1(0) \\ z_2(-1) \\ z_2(0) \end{bmatrix} = H\hat{H}^{-1} \begin{bmatrix} \sqrt{a_1} & & \\ & \sqrt{a_2} & \\ & & \sqrt{a_2} \end{bmatrix} \begin{bmatrix} b_1(0) \\ b_2(-1) \\ b_2(0) \end{bmatrix} + \hat{H}^{-1} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

where

$$H\hat{H}^{-1} = \frac{1}{det(\hat{H})} \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} \tag{B.6}$$

and the elements of $H\hat{H}^{-1}$ are given by:

$A_{11} = \left[1 - \hat{\rho_{21}}\rho_{21}/\sqrt{\hat{e_2}e_2} - \hat{\rho_{12}}\rho_{12}/\sqrt{(1-\hat{e_2})(1-e_2)} \right]$

$A_{12} = [\rho_{21}/\sqrt{e_2} - \hat{\rho}_{21}/\sqrt{\hat{e_2}}]$

$A_{13} = [\rho_{12}/\sqrt{1-e_2} - \hat{\rho}_{12}/\sqrt{1-\hat{e_2}}]$

$A_{21} = \left[\rho_{21}/\sqrt{e_2} - \hat{\rho}_{21}/\sqrt{\hat{e_2}} - \rho_{21}\hat{\rho}_{12}^2/\sqrt{e_2}(1-\hat{e_2}) + \rho_{12}\hat{\rho}_{12}\hat{\rho}_{21}/\sqrt{(1-e_2)(1-\hat{e_2})\hat{e_2}} \right]$

$$A_{22} = [-\rho_{21}\hat{\rho}_{21}/\sqrt{e_2\hat{e}_2} + 1 - \hat{\rho}_{12}^2/(1-\hat{e}_2)]$$

$$A_{23} = [\hat{\rho}_{21}\rho_{12}/\sqrt{\hat{e}_2(1-e_2)} - \hat{\rho}_{21}\rho_{12}\sqrt{\hat{e}_2(1-\hat{e}_2)}]$$

$$A_{31} = \left[\rho_{12}/\sqrt{1-e_2} - \hat{\rho}_{12}/\sqrt{1-\hat{e}_2} - \rho_{12}\hat{\rho}_{21}^2/(\hat{e}_2)\sqrt{1-e_2} + \rho_{21}\hat{\rho}_{12}\hat{\rho}_{21}/\sqrt{e_2\hat{e}_2}\sqrt{1-\hat{e}_2}\right]$$

$$A_{32} = [-\rho_{21}\hat{\rho}_{21}/\sqrt{e_2\hat{e}_2} + \hat{\rho}_{21}\hat{\rho}_{21}/\hat{e}_2]$$

$$A_{33} = [-\rho_{12}\hat{\rho}_{12}/\sqrt{(1-e_2)(1-\hat{e}_2)} + 1 - \hat{\rho}_{21}^2/\hat{e}_2]$$

where

$$det(\hat{H}) = \left[1 - \frac{\hat{\rho}_{21}^2}{\hat{e}_2} - \frac{\hat{\rho}_{12}^2}{(1-\hat{e}_2)}\right] \tag{B.7}$$

It can be seen that if the estimation of τ is correct then $H\hat{H}^{-1}$ will collapse to I

Derivation of the output probability of error for user 1 P_{e1} The decorrelated tentative decisions are given by:

$$z = \hat{H}^{-1}HAb + \hat{H}^{-1}n_1 \tag{B.8}$$

$$z_1(0) = \frac{1}{det(H)}\left\{\left[1 - \frac{\hat{\rho}_{21}\rho_{21}}{\sqrt{\hat{e}_2 e_2}} - \frac{\hat{\rho}_{12}\rho_{12}}{\sqrt{(1-\hat{e}_2)(1-e_2)}}\right]b_1(0)\right.$$
$$\left. + \left[\frac{\rho_{21}}{\sqrt{e_2}} - \frac{\hat{\rho}_{21}}{\sqrt{\hat{e}_2}}\right]b_2(-1) + \left[\frac{\rho_{12}}{\sqrt{1-e_2}} - \frac{\hat{\rho}_{12}}{\sqrt{1-\hat{e}_2}}\right]b_2(0) + \hat{\xi}_1\right. \tag{B.9}$$

where

$$\hat{\xi}_1 = \frac{1}{det(H)}\left[n_1(0) - \frac{\hat{\rho}_{21}}{\sqrt{\hat{e}_2}}n_2(-1) - \frac{\hat{\rho}_{12}}{\sqrt{1-\hat{e}_2}}n_2(0)\right]$$
$$= \frac{1}{det(H)}\left[n_1(0) - \frac{\hat{\rho}_{21}}{\sqrt{\hat{e}_2}}\frac{n_2^L(-1)}{\sqrt{\hat{e}_2}} - \frac{\hat{\rho}_{12}}{\sqrt{1-\hat{e}_2}}\frac{n_2^R(0)}{\sqrt{1-\hat{e}_2}}\right] \tag{B.10}$$

The variance of $\hat{\xi}_1$ is given by :

$$var\{\hat{\xi}_1\} = E\{\hat{\xi}_1^2\} = \frac{N_0/2}{[1 - \hat{\rho}_{21}^2/e_2 - \hat{\rho}_{12}^2/(1-e_2)]} \tag{B.11}$$

The output of the decorrelator for bit 0 of user 1 is given by:

$$z_1(0) = A\sqrt{a_1}b_1(0) + B\sqrt{a_2}b_2(-1) + C\sqrt{a_2}b_2(0) + \xi_1 \tag{B.12}$$

where

$$A = [1 - \hat{\rho_{21}}\rho_{21}/\sqrt{\hat{e_2}e_2} - \hat{\rho_{12}}\rho_{12}/\sqrt{(1-\hat{e_2})(1-e_2)}], \quad B = [\rho_{21}/\sqrt{e_2} - \hat{\rho}_{21}/\sqrt{\hat{e_2}}] \quad \text{and}$$

$$C = [\rho_{12}/\sqrt{1-e_2} - \hat{\rho}_{12}/\sqrt{1-\hat{e_2}}]$$

The error probability for user 1 is given by:

$$
\begin{aligned}
P_{e1} &= \frac{1}{4} \sum_{b_2(-1),b_2(0)} [\frac{1}{2}P(e/b_1(0)=1) + \frac{1}{2}P(e/b_1(0)=-1)] \\
&= \frac{1}{4} \sum_{b_2(-1),b_2(0)} \frac{1}{2}[P(z_1(0)<0/b_1(0)=1) + P(z_1(0)>0/b_1(0)=-1)] \\
&= \frac{1}{8}\{P(\xi_1 < 0/b_1(0)=1, b_2(-1)=1, b_2(0)=1) \\
&\quad + P(\xi_1 < 0/b_1(0)=1, b_2(-1)=-1, b_2(0)=1) \\
&\quad + P(\xi_1 < 0/b_1(0)=1, b_2(-1)=1, b_2(0)=-1) \\
&\quad + P(\xi_1 < 0/b_1(0)=1, b_2(-1)=-1, b_2(0)=-1) \\
&\quad + P(\xi_1 > 0/b_1(0)=-1, b_2(-1)=1, b_2(0)=1) \\
&\quad + P(\xi_1 > 0/b_1(0)=-1, b_2(-1)=-1, b_2(0)=1) \\
&\quad + P(\xi_1 > 0/b_1(0)=-1, b_2(-1)=1, b_2(0)=-1) \\
&\quad + P(\xi_1 > 0/b_1(0)=-1, b_2(-1)=-1, b_2(0)=-1)\}
\end{aligned}
$$

$$
\begin{aligned}
P_{e1} &= \frac{1}{4}\{Q(\frac{A\sqrt{a_1}+B\sqrt{a_2}+C\sqrt{a_2}}{var\{\hat{\xi}_1\}}) + Q(\frac{A\sqrt{a_1}-B\sqrt{a_2}+C\sqrt{a_2}}{var\{\hat{\xi}_1\}}) \\
&\quad + Q(\frac{A\sqrt{a_1}+B\sqrt{a_2}-C\sqrt{a_2}}{var\{\hat{\xi}_1\}}) + Q(\frac{A\sqrt{a_1}-B\sqrt{a_2}-C\sqrt{a_2}}{var\{\hat{\xi}_1\}})\} \quad \text{(B.13)}
\end{aligned}
$$

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