Local stresses on lateral pipe-nozzle with 45 degree angle intersection

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ABSTRACT

LOCAL STRESSES ON LATERAL PIPE-NOZZLE
WITH 45° DEGREE ANGLE INTERSECTION

by
James Jin Xu

This dissertation presents a comprehensive study of local stresses, due to internal pressure around a pipe-nozzle with 45 degree angle intersection. The resulting circumferential and longitudinal stresses on the pipe around the pipe-nozzle region are normalized as local stress factors and plotted as function of beta, \( \beta \), (the radius of the nozzle/the radius of the pipe) and gamma, \( \gamma \), (the radius of the pipe/the thickness of the pipe) through the finite element method. The range of beta, \( \beta \), is from 0.1, to 1.0, and gamma, \( \gamma \), from 10 to 300. Comprehensive studies were made for the boundary parameters, such as \( \alpha_p \) (pipe length / pipe mean radius) and \( \alpha_n \) (nozzle length / nozzle mean radius), the optimized numbers of nodes around the pipe-nozzle juncture and total elements of the model. To justify a wide range of application of the 45° degree pipe-nozzle angles, extensive studies and a set of plots are provided to show that local stress factors vary with the pipe-nozzle intersection angle, from 90° to 30°.

An approximate theoretical analysis, which is based on thin-shell theory together with stress multipliers for the peak stresses at the both inside and outside crotch points, has derived to compare the data from 3D finite element models.

This study concludes that the maximum local stress is in the circumferential direction and occurs at the inside crotch point. The 45° intersecting angle yields relatively less local
stresses when the pipe-nozzle intersecting angle other than 90° must be used for operational purposes. The local pressure stresses in the pipe-nozzle juncture are mostly in tension except on the inside surface of pipe in longitudinal direction. For certain combinations of \( \beta \) and \( \gamma \), however, the longitudinal stress at point C (see Figure 2) on the outside surface of pipe may be compressive also.

Twelve (12) plots of local pressure stress factors are provided in this thesis allows design engineers of pressure vessel to compute local stress on both the outside and the inside shell of pipe when the pipe-nozzle intersecting angle is 45°. A numerical example is given.
LOCAL STRESSES ON LATERAL PIPE-NOZZLE 
WITH 45° DEGREE ANGLE INTERSECTION

by
James Jin Xu

A Dissertation
Submitted to the Faculty of 
New Jersey Institute of Technology 
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Doctor of Philosophy

Department of Mechanical Engineering

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WITH 45° DEGREE ANGLE INTERSECTION

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This Dissertation
is dedicated to my parents, my wife and
all my family members.
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$\alpha_p = $ pipe length/pipe mean radius

$\alpha_n = $ nozzle length/nozzle mean radius

$\beta = $ nozzle radius/pipe mean radius

$\beta_6 = $ see equation (6),

$\beta_p = $ see equation (25)

$\beta_a = $ see equation (29)

$\gamma = $ pipe mean radius/pipe thickness

$\varepsilon_\phi, \varepsilon_\theta = $ circumferential strain

$\nu = $ Poisson's ratio

$\sigma_{ap}, \sigma_{an} = $ meridional membrane stress

$\sigma_{bp}, \sigma_{bn} = $ meridional bending stress

$\sigma_{cp}, \sigma_{cn} = $ circumferential membrane stress

$\theta = $ angle between pipe and nozzle on the symmetric plan alone the pipe axis

$a = $ shell radius

$E = $ Young's modules

$D = $ see equation (5)

$D_p = $ see equation (24)

$D_n = $ see equation (28)

$h = $ shell thickness

$K_r = $ radial nozzle stress factor

$K_{nr} = $ non-radial nozzle stress factor
NOMENCLATURES  
(Continued)

\[ L_p = \text{length of pipe} \]
\[ L_n = \text{length of nozzle} \]
\[ M_0 = \text{bending moments at the edge of shell, lb-in./in.} \]
\[ M_{wp}, M_{wn} = \text{shell bending moments, lb-in./in.} \]
\[ M_x = \text{shell moment resultants, lb-in./in.} \]
\[ N_x = \text{meridional direct stress resultants, lb/in.} \]
\[ N_{op}, N_{on} = \text{circumferential direct stress resultants, lb/in.} \]
\[ N_{\phi} = \text{circumferential direct stress resultants, lb/in.} \]
\[ p = \text{internal pressure} \]
\[ Q_0 = \text{shear force at the edge of pipe, lb/in.} \]
\[ Q_{wp}, Q_{wn} = \text{transverse shear stress resultants, lb/in} \]
\[ R_p = \text{pipe mean radius} \]
\[ R_n = \text{nozzle mean radius} \]
\[ t_p = \text{pipe thickness} \]
\[ t_n = \text{nozzle thickness} \]
\[ u = \text{displacement in x direction} \]
\[ v = \text{displacement in } \phi \text{ direction} \]
\[ w = \text{displacement in r direction} \]
\[ \frac{\partial w_p}{\partial x_p}, \frac{\partial w_n}{\partial x_n} = \text{rotations} \]
\[ x_p, x_n = \text{coordinates along shell meridians for pipe and nozzle respectively} \]
NOMENCLATURES
(Continued)

$x, \phi, r =$ coordinates for general cylindrical shell

$X, Y, Z =$ globe coordinates for 3D modelling

$Z =$ intensity of load

Subscripts

$p =$ pipe, main shell

$n =$ nozzle, branch
CHAPTER 1

INTRODUCTION

Pipe tees and lateral connections are essential components in process and power generation facilities for functional purposes. The lateral tee or nozzle makes an elliptical opening in the pipe or vessel which cause the higher stress concentration than the standard 90° nozzle. The high local stresses at the juncture of these connections cause major safety concerns especially in nuclear power design.

The commonly used laterals have the intersection angles of 30°, 45°, and 60°, respectively. Although considerable investigations have been available on nozzle junctions and branches under internal pressure and other external loadings by experimental, analytical and numerical methods, they are limited to the 90° intersection. The fundamental difficulty in the analysis of pipe-nozzle is that they are not axisymmetric and the curve of intersection is a nongeodesic curve. This thesis investigates the effect of these lateral connection variations in angle $\theta$ on the local stresses due to internal pressure, and it also presents a comprehensive data of local stress factors for the 45° pipe-nozzle connection, shown in Figure 1. In the figure, points A and B are designated as outside and inside crotch points, respectively.

Using ALGOR finite element analysis package, the pipe-nozzle juncture is simulated by using a full pipe-nozzle model. To ensure proper convergence of the numerical results on the local stresses, comprehensive studies are made to optimize the models with 96...
Figure 1: Pipe-nozzle configuration
nodes on the pipe-nozzle junction, and the values for the geometry parameter are 10.0 for $\alpha_p$ (pipe length / pipe mean radius) and 5.0 for $\alpha_n$ (nozzle length / nozzle mean radius). These values ensure that boundary conditions at the end of the pipe and nozzle will not affect the accuracy of the numerical results. Assuming that the membrane pressure stresses in the pipe and the nozzle are identical, the nozzle thickness is proportional to the pipe thickness by a factor beta, $\beta$, i.e. $t_n = \beta t_p$.

To provide a comprehensive range of local stress results for design engineers and stress analysts, this thesis presents twelve plots of local stress factors for both the circumferential and longitudinal stresses at points A, B, and C, respectively, as shown in Figure 2, of which six plots are for the stresses on the outside surface of pipe and the remaining six are for the inside surface. In these plots, the geometrical parameter $\beta$ (nozzle mean radius / pipe mean radius) range from 0.1 to 1.0 with an increment of 0.1 and the $\gamma$ (pipe mean radius / pipe thickness) range from 10 to 300 in ten random selected intervals. The local stress factors are defined by normalizing the resulting local stresses by the applied internal pressure value.
CHAPTER 2

LITERATURE SURVEY

There exist many theoretical analyses, experimental data and finite element analysis on the local stresses of the pipe-nozzle intersection since the 1950s. However, they are either mainly concerned about the pipe-nozzle intersection with 90 degree angle or have various limitations which can only be applied to certain special cases.

K.R. Wichman, A.G. Hopper and J.L. Mershon [1] published WRC Bulletin No. 107 in 1965. It suggested a method to calculate the local stresses of spherical and cylindrical shells with a nozzle due to external loading. The theory of this bulletin is based on a study by Bijlaard published in 1955 [2]. His work is based on the thin-shell theory and double Fourier series solutions. The latest revision of Bulletin No. 107 was published in March 1979. Due to the mathematical limitation of Bijlaard's work, Bulletin No. 107 can only apply to problems of lug or a solid trunnion at 90° intersection angle with the vessel. It does not recommend any specific method in analyzing an actual nozzle connection to a pressure vessel, either cylindrical or spherical. The induced normal stresses were reported as membrane and bending stress factors in biaxial directions. The shear stresses due to external shear forces and torsional moments are obtained through approximated formulas. Finally, stresses from various nozzle loads are summed in their respective directions before the principal stresses and stress intensity are calculated.
Mirza and Gupgupoglu [3] [4] in 1988 introduced a 17-node doubly curved shell finite element model to simulate the case of longitudinal moments applied at discrete points around the circumference of the vessel. The results from the finite element method were in agreement with WRC 107 [1], but were not applicable to pipe-nozzle other than 90° intersection.

J.L. Mershon et al. published the WRC Bulletin No. 297 in August 1984 [5]. It is a supplement to WRC 107 and is specifically applicable to round nozzles on cylindrical vessels. This bulletin was based on Professor Steele's theoretical work [6] for larger γ (radius/thickness) values than what is provided in WRC 107 [1]. Steele's theoretical work considers an opening on the shell together with restraining effect of nozzle wall. The β values are limited to 0.5.

Sadd and Avent [7] in 1982 studied a trunnion pipe anchor by the finite element method. The model is analyzed for the case of internal pressure and various end moment loadings. With Georgia Tech ICES STRUDL finite element package, a quadrilateral element with six degrees of freedom at each of the four corner nodes was utilized. The α value is taken as 8.0 for their models. Data are provided for a beta, β, (trunnion mean radius/pipe mean radius) range from 0.5 to 1.0 and gamma, γ, (pipe radius/pipe thickness) range from 5 to 20.

Tabone and Mallett [8] in 1987 established a finite element model of a nozzle in a cylindrical shell subjected to internal pressure, and out-of-plane moment. This model used ANSYS 3-D finite elements and the analysis considered the elastic behavior at small
displacements. Two elements along the thickness direction of the nozzle and vessel were employed in this study. It resulted in an estimation of limited loads based on extrapolation of the load-versus-inverse-displacement curves. An expression is given for the effect of the combined loadings for a case in which the internal pressure reduces the moment capability of the nozzle by approximately 35 percent.

H. Sun, B.C. Sun and H. Herman [9] [10] in 1991 published comprehensive results of studies on local pipe stresses using the finite element method. These papers reported a series bending and membrane stress factors for local circumferential and longitudinal stresses on the pipe region of the pipe-nozzle intersection due to all six external loading components.

J. Ha, B.C. Sun and B. Koplik [11] in 1994 presented a comprehensive study of local stresses around a pipe-nozzle due to internal pressure using the finite element method. In this paper, the local pressure stresses for both the pipe and nozzle around the pipe-nozzle juncture are normalized into pressure stress factors which are then plotted as functions of geometrical parameters, $\beta$ and $\gamma$. The ranges of these stress factors cover $\beta$ from 0.1 to 1.0, $\gamma$ from 10 to 300. To ensure proper convergence, the optimized numbers is 8 for $\alpha_p$, 4 for $\alpha_n$, respectively. Their model contains 96 nodes around the pipe-nozzle juncture, and 3000 ~ 4000 elements for the full model. For accuracy and faster convergence, ten separate finite element models were established, one for each $\beta$ value.

From above literature survey, it is obvious that in the past decades few studies were available on the local stresses around pipe-nozzle with 45 degree angle. The theoretical
analysis of local stresses around pipe-nozzle intersection involve tremendous mathematical
difficulties due to the absence of axial symmetry. Instead of an ordinary differential
equation for the pipe-nozzle stress field, partial differential equations with various
non-symmetrical terms are needed for the pipe-nozzle geometry which led to extreme
difficulties in obtaining the exact equilibrium equations of force and moment at the
 juncture of the pipe-nozzle. The approximate solution are restricted to a fairly small range
of the intersection curvatures since the mid-surfaces of the pipe-nozzle intersection is
generally not a geodesic curve. There exists neither analytical nor experimental data for
pressure vessel designer to analyze the stresses on pipe-nozzle with 45 degree angle with
neither external loading, nor internal pressure. A comprehensive database to calculate
these local stresses are needed by industry.
CHAPTER 3

THEORETICAL ANALYSIS

Analytical methods for local stresses around the pipe-nozzle under internal pressure involve tremendous mathematical difficulties caused by the absence of axial symmetry and non-geodesic curve on either the pipe or the nozzle. Several researchers have achieved some approximate solutions for certain locations on the juncture of pipe-nozzle based on certain special geometrical configurations and assumptions [12] [13] [14] [15] [16]. The linear stress distribution through the thickness of the pipe-nozzle intersection and the continuity conditions of axial membrane stress, circumferential strain, rotation of normal, and bending moment at the intersection of pipe-nozzle connection, are commonly assumed in most of the approximated theoretical solutions. However, their studies are limited by the location and special geometry configuration and most of them only discussed the 90° pipe-nozzle intersection.

3.1 General Thin-shell Theory

In terms of the components of displacement and their partial derivatives, the basic thin shell theory equation was established by Timoshenko [17][18] for a radial loading per unit surface. Let the components of displacement be u, v, and w respectively in the direction of x, φ, and r (see Figure 3a). The equilibrium equations are as following:
\[
\frac{\partial^2 u}{\partial x^2} + \frac{1 - v}{2a^2} \frac{\partial^2 u}{\partial \phi^2} + \frac{1 + v}{2a} \frac{\partial^2 v}{\partial x \partial \phi} - \frac{v}{a} \frac{\partial w}{\partial x} = 0
\]  

(1)

\[
\frac{1 + v}{2} \frac{\partial^2 u}{\partial x \partial \phi} + \frac{1}{a} \frac{\partial^2 v}{\partial \phi^2} + a \frac{1 - v}{2} \frac{\partial^2 v}{\partial x^2} - \frac{1}{a} \frac{\partial w}{\partial \phi} = 0
\]  

(2)

\[
\frac{v}{a} \frac{\partial u}{\partial x} + \frac{1}{a} \frac{\partial v}{\partial \phi} - \frac{w}{a} - \frac{h}{12} \left[ a \frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{a \partial \phi^2} + \frac{\partial^4 w}{a^3 \partial \phi^4} \right]
\]  

\[
= - \frac{ap(1 - v^2)}{Eh}
\]  

(3)

### 3.2 Cylindrical Shell with Axial Symmetric Loadings

When a cylindrical shell is loaded symmetrically with respect to its axis and the thickness of shell is constant, a fourth order differential equation is obtained [17][18]:

\[
\frac{\partial^4 w}{\partial x^4} + 4 \beta \frac{\partial^4 w}{\partial z^4} = \frac{Z}{D}
\]  

(4)

where

\[
D = \frac{Eh^3}{12(1 - v^2)}
\]  

(5)
\( \beta_g = \frac{Eh}{4a^2D} = \frac{3(1-v^2)}{a^2h^2} \) (6)

For a long circular pipe submitted to the action of bending moment \( M_0 \) and shearing forces \( Q_o \), both uniformly distributed along the edge \( x = 0 \) shown in Figure 3b, the expression for \( w \) is given by solving equation (4) in the case there is no pressure \( Z \) distributed over the surface of the shell:

\[
\begin{align*}
    w &= \frac{e^{-\beta_x}}{2 \beta_g^3D} \left[ \beta_g M_0 (\sin \beta_g x - \cos \beta_g x) - Q_o \cos \beta_g x \right] \\
    &= \frac{e^{-\beta_x}}{2 \beta_g^3D} \left[ \beta_g M_0 (\sin \beta_g x - \cos \beta_g x) - Q_o \cos \beta_g x \right] \\
\end{align*}
\] (7)

At the load end, \( x = 0 \)

\[
(w)_{x=0} = -\frac{1}{2 \beta_g^3D} (\beta_g M_0 + Q_o) \\
\] (8)

\[
\left( \frac{dw}{dx} \right)_{x=0} = -\frac{1}{2 \beta_g^2D} (2 \beta_g M_0 + Q_o) \\
\] (9)

\[
\left( \varepsilon_r \right)_{x=0} = \left( \frac{w}{a} \right)_{x=0} = \frac{1}{2 \beta_g^3Da} (\beta_g M_0 + Q_o) \\
\] (10)

### 3.3 Approximated Solution for 45° Lateral Tee or Nozzle

An approximate method for the elastic stresses at the crotch of a branch pipe connection under internal pressure is developed based on thin-shell theory together with stress...
(a) Coordinate for general cylindrical shell.

(b) Edge loadings

(c) Presentative cylinder panels

Figure 3: General cylindrical shell configuration
Figure 4: Section used for calculating forces in overall equilibrium equation
(a). Forces at point A

(b). Forces at point B

Figure 5: Edge loading on cylinder strip due to internal pressure
multipliers for the peak stress at the crotch point. Lind [19] developed an overall equilibrium equation for a tee branch connection with 90° degree angle. Lind's equation represents a balance of forces across the mid-plane of the structure. In the case of pipe-nozzle with 45° degree intersection, referring to Figure 4, the area to balance the tensile forces along the shell is modified in this study.

The tensile forces on the cross section $C_nB_nBB_pC_pJK$ of the structure are set equal to the resultant force of the pressure acting on area $C_nB_nBB_pC_pJKD$. If it is assumed that both the main shell and the branch are long and that stress along $KJ$ is the nominal hoop stress, then the pressure times area, $EC_pJK$, is balanced by the tensile force along $C_pJK$. Then, this requires that the tensile force on $C_nB_nBB_pC_p$ balance the pressure times area $C_nB_nBB_pC_pED$, which includes sub-areas $C_nB_nBD (R_n \cdot L_n)$, $BB_pC_pD_p (R_p \cdot L_p)$, $DBD_pE (R_p \cdot R_n \cdot \cos 45°)$ and $D_nBD (R_n \cdot R_n/2)$. The force balance equation then becomes

$$\int_{C_nB_nBB_pC_p} N_\phi \, dx = p \left( R_p L_p + R_n L_n + \frac{R_n^2}{2} + R_p R_n \sqrt{2} \right)$$

(11)

Expressions for the edge shearing forces $Q_{xp}$ and $Q_{xm}$ acting on the cylindrical panels $BB_pM_pL_p$ and $BB_nM_nL_n$ at the junction point $B$ (Figure 3c) are derived by Updike [20] to be

$$\int_{BB_pC_p} N_\phi \, dx = Q_{xp} R_p + pR_p L_p$$

(12)

$$\int_{C_nB_nB} N_\phi \, dx = Q_{xm} R_n + pR_n L_n$$

(13)
Summing equations (12) and (13) and invoking (11) results in

\[ Q_{zp} R_p + Q_{xn} R_n = p \left( \frac{R_x^2}{2} + R_p R_n \sqrt{2} \right) \]  

(14)

The rotation of the shell normal and the circumferential strain of each of the strips meeting at point B of Figure 5 are matched by satisfying the equations due to the continuity conditions

\[ \frac{dw_p}{dx_p} + \frac{dw_n}{dx_n} = 0 \]  

(15)

\[ \varepsilon_{zp} = \varepsilon_{xn} \]  

(16)

Equilibrium of a shell element at the junction point B of the strips is satisfied by means of the equations:

\[ M_{zn} = M_{zp} \]  

(17)

\[ \sqrt{2}(N_{xn} - Q_{xn}) / 2 = Q_{zp} \]  

(18)

\[ \sqrt{2}(N_{xn} + Q_{xn}) / 2 = N_{zp} \]  

(19)

where the edge loads \( M_{zp} \), \( M_{zp} \), \( N_{zp} \), \( N_{zp} \), \( Q_{zp} \), and \( Q_{zp} \) are shown in Figure 5. Equations (18) and (19), are solved for \( N_{zp} \) and \( N_{zp} \) yield the expressions

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If the derivatives in the circumferential direction of stress and the deformation variable are neglected and stresses due to ovaling deformation are ignored, the equation governing the stresses in the cylindrical strip $BB_p$ reduce to those of a complete circular cylindrical shell of radius, $R_p$, subjected to axisymmetric loading. In terms of the internal pressure, $p$, and the edge loads, $M_{sp}$, $Q_{sp}$, and $N_{sp}$ of Figure 5, the flexibility relations for semi-infinite cylindrical shell are determined by equations (9) and (10) as

\[ N_{sp} = O_{sp} + \sqrt{2}Q_{sn} \]  
\[ N_{sn} = O_{sn} + \sqrt{2}Q_{sp} \]  
\[ dw_p / dx_p = M_{sp} / \beta_p D_p - Q_{sp} / 2 \beta_p^2 D_p \]  
\[ \varepsilon_{sp} = Q_{sp} / 2 \beta_p^2 D_p R_p - M_{sp} / 2 \beta_p^2 D_p R_p + pR_p / E_t - vN_{sp} / E_t \]

where
\[ D_p = E_t^2 / 12(1 - v^2) \]  
\[ \beta_p^4 = 3(1 - v^2) / R_p^2 t_p^2 \]  

Likewise, the governing equations for nozzle $B_nB$ reduce to equations for finite cylindrical shell of radius $R_n$ subjected to axisymmetric loading. With section $BB_nC_nD_n$ as a plane of symmetry the flexibility relations in terms of the internal pressure, $p$, and the edge loads, $M_{nsp}$, $Q_{nsp}$, and $N_{nsp}$ of Figure 5 are determined by equations (9) and (10) as

\[ N_{nsp} = O_{nsp} + \sqrt{2}Q_{sn} \]  
\[ N_{sn} = O_{sn} + \sqrt{2}Q_{sp} \]  
\[ dw_p / dx_p = M_{sp} / \beta_p D_p - Q_{sp} / 2 \beta_p^2 D_p \]  
\[ \varepsilon_{sp} = Q_{sp} / 2 \beta_p^2 D_p R_p - M_{sp} / 2 \beta_p^2 D_p R_p + pR_p / E_t - vN_{sp} / E_t \]

where
\[ D_p = E_t^2 / 12(1 - v^2) \]  
\[ \beta_p^4 = 3(1 - v^2) / R_p^2 t_p^2 \]  

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loads, $M_{zn}$, $Q_{zn}$, and $N_{zn}$ of Figure 5, the flexibility relations for semi-infinite cylindrical shell are determined by equations (9) and (10) as

\[
\frac{dw_n}{dx_n} = \frac{M_{zn}}{\beta_nD_n} - \frac{Q_{zn}}{\beta_n^2D_n} \tag{26}
\]

\[
\varepsilon_{zn} = \frac{Q_{zn}}{2\beta_n^3D_nR_n} - \frac{M_{zn}}{2\beta_n^2D_nR_n} + \frac{pR_n}{Et_n} - \frac{vN_{zn}}{Et_n} \tag{27}
\]

where

\[
D_n = \frac{Et_n^3}{12(1 - v^2)} \tag{28}
\]

\[
\beta_n^4 = 3(1 - v^2) / R_n^2t_n^2 \tag{29}
\]

Equations (20) and (21) is used to eliminate $N_{xp}$ and $N_{zn}$ from equations (23) and (27). Equations (20) and (24) are then used to eliminate $dw_p/dx_p$ and $dw_n/dx_n$ from equation (20). Therefore, $Q_{xp}$, $Q_{zn}$ $M_{xp}$, and $M_{zn}$ may be obtained from the reduced equations (14), (15), (16), and (17).

Equations for meridional and circumferential direction stresses are

\[
\sigma_n = \frac{N_x}{t} \tag{30}
\]
and

$$\sigma_c = \frac{N_s}{t} = E\varepsilon_s + v\sigma_x$$  \hspace{1cm} (31)$$

The meridional bending stresses is obtained from

$$\sigma_y = \frac{6M_z}{t^2}$$  \hspace{1cm} (32)$$

while the circumferential bending stresses due to local bending are obtained by multiplying the meridional stresses by Poisson's ratio.

The stresses at point A may be obtained by the same method as for point B except that the equation (14) is substituted by

$$Q_{xp} R_p + Q_{x\gamma} R_\gamma = p \left[ \frac{R_p R_\gamma \sqrt{2}}{2} - \frac{R_\gamma^2}{2} \right]$$  \hspace{1cm} (33)$$

From equations (14) and (33), one can observe the local stresses at the inside crotch point B, are larger than those at outside crotch point A. Based on this approximate mathematic method, the local stresses at crotch points A and B on the pipe-nozzle junction have been computed for several case. The approximate solutions in a range of $\beta$ from 0.6 to 0.8 and $\gamma$ larger than 100 are close to the results from the 3-D finite element models in this thesis with 5% to 30% difference (see numerical example in Chapter 7).
CHAPTER 4

THREE DIMENSIONAL FINITE ELEMENT MODEL

4.1 General

The finite element method for stress analysis fully utilizes the advantages of computer capacity in performing speedy and reliable calculations for a wide range of engineering design problem. This is particularly true when the problem is difficult to solve by a traditional mathematic model or when the geometrical model is too complex. In the study of the problem of the pipe-nozzle connection, a gamma, $\gamma$, value (the ratio of radius to the thickness of pipe) of 10 is often considered as a lower bound for the applicability of thin shell theory. Since pipe is considered as a thin shell for most cases, the numerical analysis in this study is based on the quadrilateral thin shell element models.

3D finite element models are generated by a well developed finite element analysis package, ALGOR [21] [22], with each specific $\beta$ value. The pipe-nozzle system here is modeled by using plate/shell elements based on 3-node and 4-nodes. Material properties, such as Young's Modulus, Poisson's ratio, thermal expansion and density, are assigned to the elements. The models are constructed of elements by locating points (nodes) using coordinates in the global coordinate system. The elements are defined by a way in which the nodes are connected. Each node has six potential degrees of freedom. This means that a given node may displace in three translational degrees of freedom, and also in three rotational degrees of freedom. The translation refers to the movement of a node along the
X, Y, or Z axes (or any combination of the three), while rotation refers to the movement of a node about the X, Y, or Z axes (or any combination). Boundary conditions are set by restricting various degrees of freedom.

The Algor system will solve the following equations.

**Static Stress Analysis**

\[
{F} = [K]{D}
\]

where:
- \( {F} \) = force vector
- \([K]\) = stiffness matrix
- \({D}\) = the displacement vector (stresses are back-calculated from this vector)

**Model Analysis**

\[
[K]{D} = [M]{D} [W]^2
\]

where:
- \([D]\) = displacement matrix (mode shapes)
- \([W]^2\) = diagonal matrix containing eigenvalues
- \([M]\) = mass matrix
For the analysis, the following assumptions are made:

1). The material is assumed to be homogeneous and isotropic.

2). The resulting stresses are within the proportional limit of the material and obeys Hook's law.

3). The influences of self-weight are neglected.

4). There are no transitions, fillets, or reinforcing pad at the junction.

5). In the pipe-nozzle model, the boundary conditions in each case does not significantly effect the results of the computation since the parameters \( \alpha_p \) (pipe length / pipe mean radius) is assigned as large as 10.0 and \( \alpha_n \) (nozzle length / nozzle mean radius) is assigned as large as 5.0.

For the convergence requirement of the finite element method, several models with different number of element, node number around the intersection, geometric parameters, and boundary condition have been studied.

To simulate the true pipe-nozzle geometry, full finite element models (see Appendix F for figures) are employed with a symmetric plan (X-Z plan). The number of element is approximately 4000 to 5000 for the whole finite element models (see Table 4.1), which is required to develop large number of elements and generate sufficient meshes to provide sufficient convergence to the stress results.
Table 4.1: The number of nodes and elements in 3D models

<table>
<thead>
<tr>
<th>$\beta$</th>
<th>number of nodes</th>
<th>number of elements</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>10796</td>
<td>5520</td>
</tr>
<tr>
<td>0.2</td>
<td>9298</td>
<td>4750</td>
</tr>
<tr>
<td>0.3</td>
<td>8592</td>
<td>4386</td>
</tr>
<tr>
<td>0.4</td>
<td>9484</td>
<td>4884</td>
</tr>
<tr>
<td>0.5</td>
<td>8472</td>
<td>4286</td>
</tr>
<tr>
<td>0.6</td>
<td>10540</td>
<td>5384</td>
</tr>
<tr>
<td>0.7</td>
<td>9752</td>
<td>5028</td>
</tr>
<tr>
<td>0.8</td>
<td>8886</td>
<td>4554</td>
</tr>
<tr>
<td>0.9</td>
<td>8784</td>
<td>4484</td>
</tr>
<tr>
<td>1.0</td>
<td>10044</td>
<td>5122</td>
</tr>
</tbody>
</table>

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4.2 Convergence Studies

The convergence of the finite element models have been carefully studied which included the following factors:

1). The numbers of the elements for the 3-D finite element models.

2). The numbers of the node points at the juncture of pipe-nozzle.

3). The optimum values of $\alpha_p$ (pipe length/pipe mean radius).

4). The optimum values of $\alpha_n$ (nozzle length/nozzle mean radius).

5). Boundary conditions.

To ensure the accuracy of the finite element model, several samples have been tested, such as the cases of closed end cylinder and small hole on the plate, which completely match the available results from theoretical analysis [17] [18] [23].
4.2.1 The Number of Elements for the Finite Element Models

The more the number of nodes and elements has, the more accuracy of the results for the finite element model is, but the more running time will be required. Table 4.2 indicates that, for model No.3, the stress results for a 4286 elements model has 0.134% in difference of the results from a 6122 elements model (model #5) but the running time is reduced almost 37.5%. Therefore, the optimum number of elements for the models in this work is between 4000 ~ 5500.

<table>
<thead>
<tr>
<th>Model</th>
<th>No. of Element</th>
<th>Max. Stress, psi</th>
<th>Improv., %</th>
<th>Run Time, min</th>
</tr>
</thead>
<tbody>
<tr>
<td>No. 1</td>
<td>2422</td>
<td>78313</td>
<td>----</td>
<td>90</td>
</tr>
<tr>
<td>No. 2</td>
<td>3634</td>
<td>83076</td>
<td>4.53</td>
<td>160</td>
</tr>
<tr>
<td>No. 3</td>
<td>4286</td>
<td>83687</td>
<td>0.73</td>
<td>200</td>
</tr>
<tr>
<td>No. 4</td>
<td>5384</td>
<td>83792</td>
<td>0.125</td>
<td>260</td>
</tr>
<tr>
<td>No. 5</td>
<td>6122</td>
<td>83799</td>
<td>0.01</td>
<td>320</td>
</tr>
</tbody>
</table>

Table 4.2: Comparison of models with different element numbers

The data in Table 4.2 are based on a model with $\beta = 0.5$ and $\gamma = 50$ under internal pressure of 100 psi.
4.2.2 The Number of the Node Points at the Juncture of Pipe-nozzle

Previous researchers [11] [24] have shown as the number of node points on the pipe-nozzle junction increase to 96 the stresses converged asymptotically when the pipe-nozzle intersected with 90 degree angle. For the 45 degree pipe-nozzle junction, the studies show that the same value is obtained, which is shown in Figures A1 through A12 in Appendix A when a typical model is employed with $\beta = 0.5, \gamma = 50$ and the number of node range from 72 to 112.

4.2.3 The Optimum Values of $\alpha_p$ and $\alpha_n$

The boundary parameters, $\alpha_p$ and $\alpha_n$, should be large enough to obtain a converged solution of various stresses. Previous researchers [11] [24] have shown the values of $\alpha_p$ and $\alpha_n$ are 8 and 4, respectively. These were done for stresses due to internal pressure when the pipe-nozzle intersected with 90 degree angle. For the 45 degree pipe-nozzle juncture, the previous values of $\alpha_p$ and $\alpha_n$ may not be sufficient since there exists only one symmetric plan. Figures B1 to B12 in Appendix B show the percentage of improvement with next larger $\alpha_p$ to the previous $\alpha_p$, and Figures C1 to C12 in Appendix C show the percentage of improvement with larger $\alpha_n$ to the previous $\alpha_n$. It is evident that $\alpha_p = 10$ and $\alpha_n = 5$ are the optimum values that the boundary conditions would not have any significant effect on the solution of the stresses at the pipe-nozzle junction due to internal pressure.
4.2.4 Boundary Conditions

In the real pipe-nozzle system, both pipe and nozzle are considered as closed end system which means the local pressure stresses at the pipe-nozzle juncture are superimposed with the membrane pressure stresses. Meanwhile, in order to prevent the thermal expansion stresses from occurring, the vessel and nozzle are usually modelled with simply supported.

Table 4.3 shows the comparison of data from this study with Ha's data [11], which does not have longitudinal membrane stress contribution since his model is assumed with clamped boundary conditions on the ends of pipe and nozzle, and his pipe-nozzle model is not closed. One can see that the stress factors for both models vary about 9 to 15% at the critical point A or B where the local stresses are mainly caused by the local circumferential stress. The affect to point C or D is from -1.5 to 52%. However, it is believed that the models used in this studies with simply supported boundary condition and closed ends should more closely simulated the real application in engineering.

From the 3D finite element models used in this study, the stresses away from the intersection area approach to \( PR/T \) in circumferential direction and approach to \( PR/2T \) in longitudinal direction when the closed ends are simply supported. This indicate the real situation that the local stresses are no longer affect the stress field at the location away from the nozzle area.
### Table 4.3: Comparison of local stress factors from different boundary conditions

<table>
<thead>
<tr>
<th>Pressure stress factors</th>
<th>Data from this work</th>
<th>Ha's stress* factors</th>
<th>Percentage different %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal stress factor at $A_U$</td>
<td>328</td>
<td>386</td>
<td>15.8</td>
</tr>
<tr>
<td>Longitudinal stress factor at $A_L$</td>
<td>-256</td>
<td>-284</td>
<td>9.8</td>
</tr>
<tr>
<td>Circumferential stress factor at $A_U$</td>
<td>438</td>
<td>517</td>
<td>15.2</td>
</tr>
<tr>
<td>Circumferential stress factor at $A_L$</td>
<td>218</td>
<td>255</td>
<td>14.5</td>
</tr>
<tr>
<td>Longitudinal stress factor at $C_U$</td>
<td>-42.8</td>
<td>-90.4</td>
<td>52.6</td>
</tr>
<tr>
<td>Longitudinal stress factor at $C_L$</td>
<td>-34.3</td>
<td>-40.2</td>
<td>14.6</td>
</tr>
<tr>
<td>Circumferential stress factor at $C_U$</td>
<td>59.1</td>
<td>75.5</td>
<td>21.7</td>
</tr>
<tr>
<td>Circumferential stress factor at $C_L$</td>
<td>-70</td>
<td>-68.9</td>
<td>-1.5</td>
</tr>
</tbody>
</table>

*open ends with clamped boundary conditions.

In the Table 4.3, the models in both work have the same geometric parameters with $\beta = 0.5$, $\gamma = 50$, and the intersecting angle is $90^\circ$ degree.
4.3 Normalization Studies

Normalization studies have verified the validity of using $\beta(\text{R}/\text{R}_\text{p})$ and $\gamma(\text{R}_\text{p}/t_\text{p})$ as the geometric parameters under internal pressure. Several cases of normalization studies have been made as discussed in the following:

1). Two models with the same geometric parameters, i.e. $\beta, \gamma$, but under different internal pressure has been studied. Parameters for these two models are listed in Table 4.4. The local stresses and stress factors from those models are listed in Table 4.5, which has shown that the normalized of pressure stress factor by a randomly selected applied internal pressure is valid.

Table 4.4: Geometric parameters and dimensions of models for normalization study one

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model #1</th>
<th>Model #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>100 psi</td>
<td>125 psi</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$\alpha_n$</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$L_p$</td>
<td>100 in</td>
<td>100 in</td>
</tr>
<tr>
<td>$R_p$</td>
<td>10 in</td>
<td>10 in</td>
</tr>
<tr>
<td>$t_p$</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>$L_n$</td>
<td>20 in</td>
<td>20 in</td>
</tr>
<tr>
<td>$R_n$</td>
<td>4 in</td>
<td>4 in</td>
</tr>
<tr>
<td>$t_n$</td>
<td>0.08</td>
<td>0.08 in</td>
</tr>
</tbody>
</table>
Table 4.5: Local stress comparison of models for normalization study one

<table>
<thead>
<tr>
<th>Model No.</th>
<th>Model #1</th>
<th>Model #2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stress, psi</td>
<td>Stress*, factors</td>
</tr>
<tr>
<td>Longitudinal stress at $A_U$</td>
<td>45,508</td>
<td>455.08</td>
</tr>
<tr>
<td>Longitudinal stress at $A_L$</td>
<td>-29,256</td>
<td>-292.56</td>
</tr>
<tr>
<td>Circumferential stress at $A_U$</td>
<td>50,157</td>
<td>501.57</td>
</tr>
<tr>
<td>Circumferential stress at $A_L$</td>
<td>28,307</td>
<td>283.07</td>
</tr>
<tr>
<td>Longitudinal stress at $B_U$</td>
<td>53,248</td>
<td>532.48</td>
</tr>
<tr>
<td>Longitudinal stress at $B_L$</td>
<td>-41,009</td>
<td>-410.09</td>
</tr>
<tr>
<td>Circumferential stress at $B_U$</td>
<td>79,241</td>
<td>792.41</td>
</tr>
<tr>
<td>Circumferential stress at $B_L$</td>
<td>42,800</td>
<td>428.00</td>
</tr>
<tr>
<td>Longitudinal stress at $C_U$</td>
<td>-3,393</td>
<td>-33.93</td>
</tr>
<tr>
<td>Longitudinal stress at $C_L$</td>
<td>-12,746</td>
<td>-127.46</td>
</tr>
<tr>
<td>Circumferential stress at $C_U$</td>
<td>6,279</td>
<td>62.79</td>
</tr>
<tr>
<td>Circumferential stress at $C_L$</td>
<td>-9,452</td>
<td>-94.52</td>
</tr>
</tbody>
</table>

* Stress factor is local stress normalized by the applied internal pressure for each case.
2). Table 4.7 shows that the local stresses from two models of different size but with the same geometric parameters, such as $\beta$, $\gamma$, $\alpha_p$ and $\alpha_n$ and under the same internal pressure. This verifies the validity of using $\beta$, $\gamma$, $\alpha_p$ and $\alpha_n$ as geometric parameters for this study. The parameters and dimensions for two test models are listed in Table 4.6.

Table 4.6: Geometric parameters and dimensions of models for normalization study two

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Model #1</th>
<th>Model #2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>100 psi</td>
<td>100 psi</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>$\alpha_n$</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.4</td>
<td>0.4</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>$L_p$</td>
<td>100 in</td>
<td>200 in</td>
</tr>
<tr>
<td>$R_p$</td>
<td>10 in</td>
<td>20 in</td>
</tr>
<tr>
<td>$t_p$</td>
<td>0.2</td>
<td>0.4</td>
</tr>
<tr>
<td>$L_n$</td>
<td>20 in</td>
<td>40 in</td>
</tr>
<tr>
<td>$R_n$</td>
<td>4 in</td>
<td>8 in</td>
</tr>
<tr>
<td>$t_n$</td>
<td>0.08</td>
<td>0.16 in</td>
</tr>
</tbody>
</table>
Table 4.7: Local stress comparison of models for normalization study two

<table>
<thead>
<tr>
<th>Model No.</th>
<th>Model #1, psi</th>
<th>Model #2, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal stress at A_{u}</td>
<td>45,508</td>
<td>45,508</td>
</tr>
<tr>
<td>Longitudinal stress at A_{l}</td>
<td>-29,256</td>
<td>-29,256</td>
</tr>
<tr>
<td>Circumferential stress at A_{u}</td>
<td>50,157</td>
<td>50,157</td>
</tr>
<tr>
<td>Circumferential stress at A_{l}</td>
<td>28,307</td>
<td>28,307</td>
</tr>
<tr>
<td>Longitudinal stress at B_{u}</td>
<td>53,248</td>
<td>53,248</td>
</tr>
<tr>
<td>Longitudinal stress at B_{l}</td>
<td>-41,009</td>
<td>-41,009</td>
</tr>
<tr>
<td>Circumferential stress at B_{u}</td>
<td>79,241</td>
<td>79,241</td>
</tr>
<tr>
<td>Circumferential stress at B_{l}</td>
<td>42,800</td>
<td>42,800</td>
</tr>
<tr>
<td>Longitudinal stress at C_{u}</td>
<td>-3,393</td>
<td>-3,393</td>
</tr>
<tr>
<td>Longitudinal stress at C_{l}</td>
<td>-12,746</td>
<td>-12,746</td>
</tr>
<tr>
<td>Circumferential stress at C_{u}</td>
<td>6,279</td>
<td>6,279</td>
</tr>
<tr>
<td>Circumferential stress at C_{l}</td>
<td>-9,452</td>
<td>-9,452</td>
</tr>
</tbody>
</table>
CHAPTER 5

STUDY ON THE EFFECT OF PIPE-NOZZLE INTERSECTING ANGLES

This thesis has studied the local stresses around the pipe-nozzle junction, due to an internal pressure when the angle of intersection varies from 90° to 30° by using a typical model with β value of 0.5 and γ value of 50 under internal pressure, p, of 100 psi. When the angle of intersection is at 90°, it is a typical pipe-nozzle, which many literature exist both in theoretical and numerical approaches [1], [10], [11], [20], [25], [26], etc. However, few results exist in literature for a pipe with lateral connection. Among all the lateral nozzle (tee), the 45° of intersection is the most popular one in industrial applications. Local stress studies for these lateral nozzle are very rare or non-existent due to the difficulties in mathematical modelling of the actual geometries.

This study selects the angle of intersection varying from 90°, 75°, 60°, 45°, 38°, 34° to 30°. Local stresses under this study are both the circumferential and longitudinal stress on both the outside and inside surfaces of the pipe-nozzle juncture. Figure 2 shows these stress points, A_U, A_L, B_U, B_L, etc around the pipe-nozzle juncture. The local stress factors are defined by normalizing these resulting local stresses with the internal pressure. These local stress factors are then plotted as function of the intersection angles as shown in Figure D1 to through D12 in Appendix D.

From these figures, one notes that the 90° intersection exhibits the most favorable local stresses. These stresses increase as the angle of intersection decreases from 90° and
become more severe when the angle of intersection is less than 45°. The inside crotch point has the worst stresses and the local stresses in circumferential direction are generally higher than that in the longitudinal direction. For an intersection angle other than 90°, the inside crotch point, point B, would have higher stress than the opposite side, point A. Points A and B would have the same local stresses if the intersection is orthogonal. The points C and D always yield symmetric local pressure stresses due to symmetry with the pipe axis. From the above, conclusions are drawn to justify why the 45° lateral is the most popular one as far as the local stresses are concerned.

Some analytical and experimental investigations of the stress distribution around non-radial holes in flat plates [27] [28], which may be considered as a pipe with very large radius, have shown that the maximum stresses occur in the vessel on the major axis of the elliptical opening close to the nozzle [29] [30], and are greater with nozzles of increased non-radiality, which is the same conclusion from the plots in Appendix D. According to the ASME Boiler and Pressure Vessel Code [31], the stress concentration factors for non-radial nozzle in spherical and cylindrical vessels can be approximately related to that for the same radial nozzle by following relation:

\[
K_{nr} = K_r \left[ 1 + \left( \tan \theta \right)^{3/4} \right]
\]  

(34)

where

- \(K_{nr}\) = non-radial nozzle stress concentration factor
- \(K_r\) = radial nozzle stress concentration factor
- \(\theta\) = angle the axis of the nozzle makes with the normal to the vessel wall
From equation (34), the stress concentration factor goes up sharply with the angle of non-radiality. As indicated, this equation is especially applicable at the acute internal lip and external crotch where it has been found that the maximum stress occurs and fatigue failure originates, which agrees with the results from the 3-D finite element models in this study. One can get the same conclusion from Figure 6 to Figure 9. The stress concentration factors increase with the intersection angle changing from $90^\circ$ to smaller angles, and goes up sharply when the angle is less than $45^\circ$. The plots of pressure stress factors at point B close to the plots of the equation. The pressure stress factors at point A have less value than point B, and those at the point C have much less values than those at point A and B on both the outside and the inside surfaces of the pipe in both longitudinal and circumferential directions. Therefore, the crotch point B is the critical design point due to internal pressure.

In Figure 6 – 9, the left Y axis shows the local stress factors due to internal pressure, which is the ratio of local pressure stress to applied internal pressure; the right Y axis shows the stress concentration factors, which is the ratio of local pressure stress to membrane stress away from nozzle area in the same direction.

For longitudinal direction:

\[
\text{Stress concentration factors} = \frac{\text{local stress}}{(pR_p / 2t_p)}.
\]

For circumferential direction:

\[
\text{Stress concentration factors} = \frac{\text{local stress}}{(pR_p / t_p)}.
\]
**Figure 6**: Comparison of longitudinal stress factors at outside surface of pipe with ASME code

\[ \beta = 0.5, \gamma = 50 \]

\[ \text{Stress Factors due to Internal Pressure} \]

\[ \text{Local Stress } / (pR_0 / 2t) \]

\[ \text{Angle } \theta \]

Equation from ASME codes

At point B

At point A

At point C

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Angle $\theta$

$\beta = 0.5, \gamma = 50$

**Figure 7:** Comparison of longitudinal stress factors at inside surface of pipe with ASME code.
Figure 8: Comparison of circumferential stress factors at outside surface of pipe with ASME code.
Figure 9: Comparison of circumferential stress factors at inside surface of pipe with ASME code.
CHAPTER 6

THE LOCAL STRESSES
ON THE PIPE NEAR THE PIPE - NOZZLE JUNCTURE

To study the local stresses in the area near the nozzle, a typical model is employed with \( \alpha_p = 10 \), \( \alpha_n = 5 \), \( L_p = 200 \) in., nozzle radius, \( R_n = 10 \) in., the pipe radius \( R_p = 20 \) in. and the pipe thickness is 0.4 in. which yield \( \beta = 0.5 \), \( \gamma = 50 \). The local stress factors are plotted as function of \( x \) which is the distance from the center of nozzle. Since the nozzle and pipe intersect with 45°, the distance from point A or B is \( 1.414R_n \) which is not equal to the radius of the nozzle. From the plots from Figure G1 to Figure G8 in Appendix G, the stress factors from point A or B decrease fast within a range of the half of the distance between the point A or B to the center of the nozzle. The local stresses approach to the membrane stress value when \( x \) approximately reach to the twice of the distance from point A or B to the center of nozzle, which agrees with the theory of reinforced openings for the design of reinforcement in the nozzle area as suggested by Harvey [23].
For the local pressure stresses around pipe-nozzle with 45° degree intersection, one can obtain the data from twelve plots of stress factors in Appendix E.

Example: An 50.25 in. outside diameter, with 0.25 in. thickness, pipe is intersected by a 35.125 in. nozzle with 0.175 in. thickness. The internal pressure is 100 psi. In this example, the mean radius of the pipe, \( R_p = 25 \text{ in.} \), the mean radius of nozzle is \( R_n = 17.5 \text{ in.} \). Assume any other nozzles, trunnion, or pipe bend is at least 250 in. away from this nozzle.

Table 7.1: Geometric parameters and dimensions of the sample model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_p ) = Pipe length / Pipe mean radius</td>
<td>10</td>
</tr>
<tr>
<td>( \alpha_n ) = nozzle length / nozzle mean radius</td>
<td>5</td>
</tr>
<tr>
<td>( \beta ) = Nozzle radius / Pipe mean radius</td>
<td>0.7</td>
</tr>
<tr>
<td>( \gamma ) = Pipe radius / Pipe thickness</td>
<td>100</td>
</tr>
<tr>
<td>( L_p ) = Pipe length</td>
<td>&gt; 250 in</td>
</tr>
<tr>
<td>( R_p ) = Pipe mean radius</td>
<td>25 in</td>
</tr>
<tr>
<td>( L_n ) = Nozzle length</td>
<td>&gt; 176 in</td>
</tr>
<tr>
<td>( R_n ) = nozzle mean radius</td>
<td>17.5 in</td>
</tr>
<tr>
<td>( t_p ) = Pipe thickness</td>
<td>0.25 in</td>
</tr>
<tr>
<td>( t_n ) = nozzle thickness</td>
<td>0.175 in</td>
</tr>
</tbody>
</table>
and the nozzle has a minimum length of 176 in. The detail information is listed in Table 7.1.

The results are listed in Table 7.2.

**Table 7.2: Local stresses from Appendix E for numerical example**

<table>
<thead>
<tr>
<th>Data point</th>
<th>Stress factor</th>
<th>Stress, psi</th>
<th>From figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal stress at $A_U$</td>
<td>867.72</td>
<td>86772</td>
<td>E1</td>
</tr>
<tr>
<td>Longitudinal stress at $A_L$</td>
<td>-640.25</td>
<td>-64025</td>
<td>E2</td>
</tr>
<tr>
<td>Circumferential stress at $A_U$</td>
<td>946.20</td>
<td>94620</td>
<td>E3</td>
</tr>
<tr>
<td>Circumferential stress at $A_L$</td>
<td>499.43</td>
<td>49943</td>
<td>E4</td>
</tr>
<tr>
<td>Longitudinal stress at $B_U$</td>
<td>1706.36</td>
<td>170636</td>
<td>E5</td>
</tr>
<tr>
<td>Longitudinal stress at $B_L$</td>
<td>-1363.34</td>
<td>-136334</td>
<td>E6</td>
</tr>
<tr>
<td>Circumferential stress at $B_U$</td>
<td>2473.91</td>
<td>247391*</td>
<td>E7</td>
</tr>
<tr>
<td>Circumferential stress at $B_L$</td>
<td>1418.72</td>
<td>141872</td>
<td>E8</td>
</tr>
<tr>
<td>Longitudinal stress at $C_U$</td>
<td>-138.20</td>
<td>-13820</td>
<td>E9</td>
</tr>
<tr>
<td>Longitudinal stress at $C_L$</td>
<td>-413.04</td>
<td>-41304</td>
<td>E10</td>
</tr>
<tr>
<td>Circumferential stress at $C_U$</td>
<td>179.62</td>
<td>17962</td>
<td>E11</td>
</tr>
<tr>
<td>Circumferential stress at $C_L$</td>
<td>-209.55</td>
<td>-20955</td>
<td>E12</td>
</tr>
</tbody>
</table>

* maximum local pressure stress
In this example, the circumferential membrane stress under internal pressure away from the pipe-nozzle area is

$$\sigma_c = \frac{pR}{T} = \frac{100 \times 50}{0.25} = 20000 \text{(psi)}$$

which is 13.35 times less than the maximum local stress located at the inside crotch point B.

When the elastic modulus is different from $30 \times 10^6$ psi, new local pressure stress may be obtained by multiplying the ratio of new modulus to $30 \times 10^6$ psi to the factor.

Table 7.3 listed the comparison of data from 3D finite element model with approximate mathematic solution. Since the approximate mathematic solution have more than 20% off, the results from this method are for reference only.

<table>
<thead>
<tr>
<th>Data point</th>
<th>Approximate math. solution</th>
<th>3D FEA Model</th>
<th>Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Longitudinal stress at $B_u$</td>
<td>184000</td>
<td>170636</td>
<td>8.3%</td>
</tr>
<tr>
<td>Longitudinal stress at $B_l$</td>
<td>-145800</td>
<td>-136334</td>
<td>7%</td>
</tr>
<tr>
<td>Circumferential stress at $B_u$</td>
<td>187200</td>
<td>247391</td>
<td>24.5%</td>
</tr>
<tr>
<td>Circumferential stress at $B_l$</td>
<td>98010</td>
<td>140872</td>
<td>29%</td>
</tr>
</tbody>
</table>
CHAPTER 8

CONCLUSIONS

8.1 Conclusions on Lateral Connections with Various Intersecting Angles

From the studies of the effect of the intersection angle, the dimensionless stress factors (local stress/applied internal pressure) have been plotted as a function of angle $\theta$ for the stresses at all critical points. Results from Figure D1 to D12 of Appendix D show that:

1). When $\theta = 90^\circ$, the local pressure stresses at points A and B are identical and the same for points C and D due to symmetry. The stress factors for this case exhibit the most favorable value when compared with other angle of intersection.

2). The local pressure stresses increase when the angle of intersection decrease from $90^\circ$, and the increasing of stresses become more severe when the angle of intersection is smaller than $45^\circ$ degree.

3). When the angle of intersection is less than $90^\circ$, the maximum stress occurs at inside crotch point, B, the circumferential stresses on the outside surface are most critical.

8.2 Conclusions on $45^\circ$ Lateral Connection

From the plots of stress factors for pipe-nozzle connection with $45^\circ$ degree (Figure E1 to E12 of Appendix E), one conclude that:

1). The increase of the parameter, $\gamma$, $(R_p/t_p)$ makes the local pressure stress higher. It is known from WRC Bulletin No. 107 [1] that when $\gamma$ increases, the local bending stress
decreases while the membrane stress increases. One may conclude that the membrane component gives major contribution of the local pressure stresses when the shell is very thin.

2). The highest local pressure stress occurs at the inside crotch point, B, on the outside surface of the pipe in circumferential direction. The stresses increase when $\beta$ increases. Therefore, the point B will be the critical stress point due to internal pressure.

3). At point A, the highest local stress appears to be around 0.4 of $\beta$, i.e., occurring when the nozzle diameter is about 40% of the pipe diameter.

4). The local pressure stresses can be many times higher than the circumferential membrane stress. It is 4.5 to 39.6 times higher on outside surface at point B (see Figure G1 to G8 in Appendix G). Therefore, these results provide significant data base for pressure vessel design.

5). The stress at point C, on the transverse plane of the pipe-nozzle intersection, have less value than the points A and B, no matter it is in longitudinal or circumferential direction.

6). The circumferential stresses at points A and B are always in tension. In the longitudinal direction, these stresses are in tension on the outside surface and in compression on the inside surface.

7). At point C, the local stresses on the inside surface are under compression in both longitudinal and circumferential directions, while on the outside surface of the pipe, the stresses in the circumferential direction are always in tension, but the longitudinal stresses may change from tension to compression when $\gamma$ and $\beta$ increase, which is shown in Figure E9b in Appendix E.
8). The approximate mathematic solution is close to the data from finite element method in a range of $\beta$ from 0.6 ~ 0.8 and $\gamma$ larger than 100 with difference of 5% to 30%, which verified the validity of the data from 3D finite element models.

Since the finite element method is capable of simulating the real geometry of the pipe-nozzle configuration, and meanwhile the convergence of the results are closely monitored through node points, geometric parameters and boundary conditions, the results from the finite element method should be very useful and reliable.
Stress Factors due to Internal Pressure

Figure A1: Study on node points at point $A_y$ of pipe in longitudinal direction

Number of nodes on pipe-nozzle juncture

$\alpha_p = 10, \alpha_n = 5, \beta = 0.5, \gamma = 50$
Figure A2: Study on node points at point $A_1$ of pipe in longitudinal direction

Number of nodes on pipe-nozzle juncture

$\alpha_p = 10$, $\alpha_n = 5$, $\beta = 0.5$, $\gamma = 50$
Figure A3: Study on node points at point $A_{ij}$ of pipe in circumferential direction

Number of nodes on pipe-nozzle juncture

$\alpha_p = 10$, $\alpha_n = 5$, $\beta = 0.5$, $\gamma = 50$
Stress Factors due to Internal Pressure

Figure A4: Study on node points at point A, of pipe in circumferential direction

Number of nodes on pipe-nozzle juncture

\[ \alpha = 10, \, \gamma = 5 \]

\[ \beta = 0.5, \, \gamma = 50 \]
Figure A5: Study on node points at point $B_1$ of pipe in longitudinal direction.

Number of nodes on pipe-nozzle juncture

\[ \alpha_p = 10, \alpha_n = 5, \beta = 0.5, \gamma = 50 \]
Figure A6: Study on node points at point \( B_L \) of pipe in longitudinal direction

Number of nodes on pipe-nozzle juncture
\[ \alpha_p = 10, \ \alpha_n = 5, \ \beta = 0.5, \ \gamma = 50 \]
Figure A7: Study on node points at point $B_u$ of pipe in circumferential direction.

Number of nodes on pipe-nozzle juncture

$\alpha_p = 10, \alpha_n = 5, \beta = 0.5, \gamma = 50$
Figure A8: Study on node points at point B₁ of pipe in circumferential direction.

Number of nodes on pipe-nozzle juncture

\[ \alpha_p = 10, \ \alpha_n = 5, \ \beta = 0.5, \ \gamma = 50 \]
Figure A9: Study on node points at point $C_U$ of pipe in longitudinal direction
Figure A10: Study on node points at point $C_L$ of pipe in longitudinal direction
Figure A11: Study on node points at point $C_u$ of pipe in circumferential direction.
Figure A12: Study on node points at point $C_4$ of pipe in circumferential direction

Number of nodes on pipe-nozzle juncture

$\alpha_p = 10$, $\alpha_n = 5$, $\beta = 0.5$, $\gamma = 50$
APPENDIX B

FIGURES FOR THE CONVERGENCE STUDY OF $\alpha_p$
Figure B1: Asymptotic study on $\alpha_p$ at point $A_U$ of pipe in longitudinal direction

$\alpha_n = 5, \beta = 0.05, \gamma = 50, 96$ nodes on juncture
Figure B2: Asymptotic study on $\alpha_p$ at point $A_L$ of pipe in longitudinal direction.

- $\alpha_n = 5$, $\beta = 0.5$, $\gamma = 50$, 96 nodes on juncture.
Figure B3: Asymptotic study on $\alpha_p$ at point $A_U$ of pipe in circumferential direction

$\alpha_n = 5$, $\beta = 0.5$, $\gamma = 50$, 96 nodes on juncture
Figure B4: Asymptotic study on $\alpha_{p}$ at point $A_L$ of pipe in circumferential direction.

$\alpha_n = 5$, $\beta = 0.5$, $\gamma = 50$, 96 nodes on juncture.
Figure B5: Asymptotic study on $\alpha_p$ at point $B_U$ of pipe in longitudinal direction

$\alpha_n = 5, \beta = 0.05, \gamma = 50, 96$ nodes on juncture

Percentage improvement (%) of local stress factor for larger $\alpha_p$ to previous $\alpha_p$
Figure B6: Asymptotic study on $\alpha_p$ at point $B_L$ of pipe in longitudinal direction
Figure B7: Asymptotic study on $\alpha_p$ at point $B_U$ of pipe in circumferential direction.
Figure B8: Asymptotic study on $\alpha_p$ at point $B_L$ of pipe in circumferential direction

$\alpha_n = 5$, $\beta = 0.5$, $\gamma = 50$, 96 nodes on juncture
Figure B9: Asymptotic study on $\alpha_p$ at point $C_U$ of pipe in longitudinal direction

$\alpha_n = 5, \beta = 0.5, \gamma = 50, 96$ nodes on juncture

Percentage improvement (\%) of local stress factor for larger $\alpha_p$ to previous $\alpha_p$
Figure B10: Asymptotic study on $\alpha_p$ at point $C_L$ of pipe in longitudinal direction.
Figure B11: Asymptotic study on $\alpha_p$ at point $C_U$ of pipe in circumferential direction

$\alpha_n = 5$, $\beta = 0.5$, $\gamma = 50$, 96 nodes on juncture
Figure B12: Asymptotic study on $\alpha_p$ at point $C_L$ of pipe in circumferential direction

$\alpha_n = 5$, $\beta = 0.5$, $\gamma = 50$, 96 nodes on juncture
APPENDIX C

FIGURES FOR THE CONVERGENCE STUDY OF $\alpha_n$
Figure C1: Asymptotic study on $\alpha_n$ at point $A_U$ of pipe in longitudinal direction
Figure C2: Asymptotic study on $\alpha_n$ at point $A_L$ of pipe in longitudinal direction.
Figure C3: Asymptotic study on $\alpha_n$ at point $A_U$ of pipe in circumferential direction

$\alpha_p = 10, \beta = 0.5, \gamma = 50$, 96 nodes on juncture.
Figure C4: Asymptotic study on $\alpha_n$ at point $A_L$ of pipe in circumferential direction

\[ \alpha_p = 10, \ \beta = 0.5, \ \gamma = 50, \ 96 \text{ nodes on juncture} \]
Figure C5: Asymptotic study on $\alpha_n$ at point $B_U$ of pipe in longitudinal direction.

Diagram: Graph showing the percentage improvement (%) of local stress factor for larger $\alpha_n$ to previous $\alpha_n$.

Equation: $\alpha_p = 10, \beta = 0.5, \gamma = 50, 96$ nodes on juncture.
Figure C6: Asymptotic study on $\alpha_n$ at point $B_L$ of pipe in longitudinal direction

$\alpha_p = 10$, $\beta = 05$, $\gamma = 50$, 96 nodes on juncture
Figure C7: Asymptotic study on $\alpha_n$ at point $B_U$ of pipe in circumferential direction

$\alpha_p = 10$, $\beta = 05$, $\gamma = 50$, 96 nodes on juncture
Figure C8: Asymptotic study on $\alpha_n$ at point $B_L$ of pipe in circumferential direction

$$\alpha_p = 10, \ \beta = 0.5, \ \gamma = 50, \ 96 \text{ nodes on juncture}$$
Figure C9: Asymptotic study on $\alpha_n$ at point $C_U$ of pipe in longitudinal direction.
Percentage improvement (%) of local stress factor for larger $\alpha_n$ to previous $\alpha_n$

Figure C10: Asymptotic study on $\alpha_n$ at point $C_L$ of pipe in longitudinal direction

$\alpha_p = 10$, $\beta = 0.5$, $y = 50$, 96 nodes on juncture
Figure C11: Asymptotic study on $\alpha_n$ at point $C_U$ of pipe in circumferential direction
Figure C12: Asymptotic study on $\alpha_n$ at point $C_L$ of pipe in circumferential direction

$\alpha_p = 10, \beta = 0.5, \gamma = 50$, 96 nodes on juncture
APPENDIX D

FIGURES FOR THE STUDY OF ANGLE EFFECT
Figure D1: Study on intersecting angle at point $A_{ij}$ of pipe in longitudinal direction

Angle $\theta$

$\beta = 0.5, \gamma = 50$
Figure D2: Study on intersecting angle at point $A_k$ of pipe in longitudinal direction.
Figure D3: Study on intersecting angle at point $\Lambda_{11}$ of pipe in circumferential direction.
Figure D4: Study on intersecting angle at point $A_1$ of pipe in circumferential direction
Figure D5: Study on intersecting angle at point $B_{tt}$ of pipe in longitudinal direction
**Figure D6:** Study on intersecting angle at point $B_1$ of pipe in longitudinal direction.

<table>
<thead>
<tr>
<th>Angle $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>40</td>
</tr>
<tr>
<td>60</td>
</tr>
<tr>
<td>80</td>
</tr>
<tr>
<td>100</td>
</tr>
</tbody>
</table>

$\beta = 0.5, \gamma = 50$
Figure D7: Study on intersecting angle at point B_{11} of pipe in circumferential direction

\[
\beta = 0.5, \gamma = 50
\]
Figure D8: Study on intersecting angle at point $B_1$ of pipe in circumferential direction.

$\beta = 0.5, \gamma = 50$

The graph shows the relationship between stress factors due to internal pressure and angle $\theta$. The data points indicate a decrease in stress factors as the angle $\theta$ increases.
Figure D9: Study on intersecting angle at point $C_u$ of pipe in longitudinal direction

$\beta = 0.5, \gamma = 50$
Figure D10: Study on intersecting angle at point $C_1$ of pipe in longitudinal direction

\begin{align*}
\text{Stress Factors due to Internal Pressure} \\
\text{Angle } \theta \\
\beta = 0.5, \gamma = 50
\end{align*}
Figure D11: Study on intersecting angle at point $C_{10}$ of pipe in circumferential direction

Angle $\theta$

$\beta = 0.5, \gamma = 50$

Stress Factors due to Internal Pressure

20 30 40 50 60 70 80 90 100

0 20 40 60 80 100 120
Figure D12: Study on intersecting angle at point $C_1$ of pipe in circumferential direction.

\[ \text{Angle } \theta \]

$\beta = 0.5, \gamma = 50$
APPENDIX E

PLOTS OF LOCAL PRESSURE STRESS FACTORS
Figure E1: Longitudinal pressure stress factors on the pipe at point $A_1$.

Stress Factors due to Internal Pressure
Figure E2: Longitudinal pressure stress factors on the pipe at point $A_l$. 

Stress Factors due to Internal Pressure

$\beta$

$10^4$ $10^3$ $10^2$ $10^1$

$x (-1)$

$\gamma = 300$ $\gamma = 150$ $\gamma = 100$ $\gamma = 75$ $\gamma = 50$ $\gamma = 35$ $\gamma = 25$ $\gamma = 15$ $\gamma = 10$
Figure E3: Circumferential pressure stress factors on the pipe at point A0.
Figure E4: Circumferential pressure stress factors on the pipe at point $A_1$. 
Figure E5: Longitudinal pressure stress factors on the pipe at point B_1.
Figure E6: Longitudinal pressure stress factors on the pipe at point B.

Stress Factors due to Internal Pressure
Figure E7: Circumferential pressure stress factors on the pipe at point $B_1$
Figure E8: Circumferential pressure stress factors on the pipe at point B.
Figure E9a: Longitudinal pressure stress factors on the pipe at point C10

Stress factors due to internal pressure
Figure E9b: Longitudinal pressure stress factors on the pipe at point $C_1$. 
Figure E10: Longitudinal pressure stress factors on the pipe at point $C_1$. 
Figure E11: Circumferential pressure stress factors on the pipe at point $C_\text{CM}$.
Figure E12: Circumferential pressure stress factors on the pipe at point $C_1$. 

Stress Factors due to Internal Pressure

$y = 300$
$y = 200$
$y = 150$
$y = 100$
$y = 75$
$y = 50$
$y = 35$
$y = 25$
$y = 15$
$y = 10$
APPENDIX F

FIGURES OF 3D FINITE ELEMENT MODELS
Figure F1: A full 3D finite element model
Figure F2: A full 3D pipe-nozzle model with 45 degree angle
nozzle free in X and Z directions

Figure F3: The boundary conditions on the nozzle end
a. At the simple supported end  
   (free in X direction)

b. At the clamped end

**Figure F4:** The boundary conditions on the pipe end
number of nodes = 10796
number of elements = 5520
96 node point on the pipe-nozzle juncture

Figure F5: Pipe-nozzle juncture with $\beta = 0.1$
number of nodes = 8472
number of elements = 4286
96 node point on the pipe-nozzle juncture

Figure F6: Pipe-nozzle juncture with $\beta = 0.5$
number of nodes = 8784
number of elements = 4484
96 node point on the pipe-nozzle juncture

Figure F7: Pipe-nozzle juncture with $\beta = 0.9$
Figure F8: Pipe-nozzle juncture with $\beta = 1.0$
96 node point on the pipe-nozzle juncture (top view)

Figure F9: Pipe-nozzle juncture with 30° degree intersection
Figure G1: The variation of stress factor away from point $A_U$ of pipe in longitudinal direction.
Figure G2: The variation of stress factor away from point $A_L$ of pipe in longitudinal direction

$\beta_p / \pi$

$\alpha_p = 10$, $\alpha_n = 5$, $\beta = 0.5$, $\gamma = 50$
Local Stress/ \( ( p R / t _ p ) \)

Figure G3: The variation of stress factor away from point \( A_U \) of pipe in circumferential direction

\[ \alpha_p = 10, \, \alpha_n = 5, \, \beta = 0.5, \, \gamma = 50 \]
Figure G4: The variation of stress factor away from point A₁ of pipe in circumferential direction.

\[ a_p = 10, a_h = 5, \beta = 0.5, \gamma = 50 \]
Figure G5: The variation of stress factor away from point $B_U$ of pipe in longitudinal direction.

Stress Factor due to Internal Pressure

$\beta_0 / \pi$

$\alpha_p = 10, \alpha_n = 5, \beta = 0.5, \gamma = 50$

Local Stress / $(pR / 2t_p)$

point $B_U$

stress factor

longitudinal membrane stress / $p$
Figure G6: The variation of stress factor away from point $B_L$ of pipe in longitudinal direction

$\beta_p / \pi$

$\alpha_p = 10, \alpha_n = 5, \beta = 0.5, \gamma = 50$
Figure G7: The variation of stress factor away from point $B_U$ of pipe in circumferential direction

$\text{Local Stress (} pR_p / t_p \text{)}$

$\text{Stress Factor due to Internal Pressure}$

$\beta_g / \pi$

$\alpha_p = 10, \alpha_n = 5, \beta = 0.5, \gamma = 50$

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Figure G8: The variation of stress factor away from point $B_L$ of pipe in circumferential direction.

Local Stress $\left( \frac{pR_p}{t_p} \right)$

Stress Factor due to Internal Pressure

$\beta_g / \pi$

$\alpha_p = 10, \alpha_n = 5, \beta = 0.5, \gamma = 50$
REFERENCES


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