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Kinematic analysis of a swash-plate mechanism using transformation matrices

David Edgar Evans
New Jersey Institute of Technology

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ABSTRACT

KINEMATIC ANALYSIS OF A SWASH-PLATE MECHANISM USING TRANSFORMATION MATRICES

by
David Edgar Evans

The swash-plate mechanism is used to transfer rotational motion into translational motion and vice versa. Using transformation matrices to describe each joint, the displacements of the swash-plate mechanism were analyzed. The general transformation matrix model for the plane joint, not previously developed using transformation matrices, was presented. The solution was tested using a computer program. Examples of three different swash-plate angles; 10, 45, and 25° were considered. The solution was found to agree with previous solutions while offering a more complete description of the mechanism. Future work in this area could include a complete static, velocity, and/or dynamic analysis of the swash-plate mechanism based on the solution presented.
KINEMATIC ANALYSIS OF A SWASH-PLATE MECHANISM USING TRANSFORMATION MATRICES

by

David Edgar Evans

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Submitted to the Faculty of
New Jersey Institute of Technology
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KINEMATIC ANALYSIS OF A SWASH-PLATE MECHANISM USING TRANSFORMATION MATRICES

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This thesis is dedicated to
Lois Speiden, M.D.
and
Edgar Speiden
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CHAPTER 1

INTRODUCTION

The swash-plate mechanism is comprised of four links and four joints: The ground link is connected to the shaft/swash link by a revolute joint (R). The shaft/swash link is mated with the coupler by a plane joint (E). The coupler is connected to the follower (also referred to as a piston) with a ball joint (S), and the follower (piston) is mated with the ground link by a prismatic joint (P). The mechanism is defined by its joints as an RESP mechanism.

The appeal of the swash-plate mechanism is that its pistons and shaft of rotation are parallel, thus facilitating a compact design with low weight and high-energy transfer efficiency. The configuration is robust because it is easily possible to completely balance the mechanism by choosing a thickness of the angled plane to offset the momentum of the pistons when there are three or more odd number of pistons as observed by Galin and Harris [3].

The swash-plate mechanism has been described previously by Galin and Harris [3] and Mitchell [11] using an old style methodology. While these previous solutions were effective in finding a specific solution for an output translation given an input rotation, they did not fully describe the displacements and/or rotations of all the links in the mechanism before proceeding with the dynamic and/or force analysis.

This work completely describes the mechanism using dual number transformation matrices. In considering the kinematics of this mechanism, a plane (E) joint is modeled for
the first time using transformation matrices. Transformation matrix solutions can now be utilized to describe other mechanisms with plane and/or spherical joints, such as the Rzeppa constant-velocity coupling, which may have no other current means of exhaustive solution.
CHAPTER 2

PREVIOUS WORK AND SCOPE

James Watt invented the swash-plate mechanism in the late nineteenth century. The purpose of the mechanism was to translate rotating motion into reciprocating motion or reciprocating motion into rotating motion. It’s compact and simple design made the swash-plate appealing to designers at the time it was invented. However, Maki and DeHart report that it was not until the 1920’s that the mechanism was rendered operational enough to be considered a viable alternative to the more widely used reciprocating/rotating mechanisms.[1]

In the early 1920’s, A. G. M. Mitchell made the first application of the swash-plate mechanism that could compete with the crank and connecting rod mechanism. [11] According to Galin and Harris, Mitchell’s first application for the swash-plate mechanism was an air compressor. [2] However, the swash-plate was soon used in an internal combustion engine for airplanes. Other applications for the swash plate include gas compressors and air motors. Its most notable application (because of shear volume of production) is in the automotive air-conditioning compressor. Weibel and Mantey provide an elegant description of the swash-plate mechanism, as it is used in the automotive air-conditioning compressor. [6] Because of the mechanism’s wide range of utilization, it is important that its kinematics be well defined for reference of machine designers.

Using transformation matrices, the equation that describes the position of the piston in a swash-plate mechanism will be derived. The equation should be of such form
that the output displacement $s_4$ is a sole function of the input variable $\theta_i$ and the mechanism's dimensions.

To properly design a balanced swash-plate type mechanism, the thickness of the swash plate can be calculated to produce a balancing moment. The balancing moment counteracts the moment produced by the cyclical force of the pistons reciprocating at a distance from the center of the plate. This configuration of swash-plate/piston balance only works for a three or more piston mechanism. Only one piston is shown in Figure 1.

Before the moment produced by the reciprocating pistons can be found, the position equation for one piston assembly must be calculated relative to the input angle of the shaft. Let us call that the first position equation. Using the first position equation, a general expression can be made to describe the position of any piston in the assembly. Also, the velocity and change of momentum equations can be found by taking the first and second derivatives of the position equation respectively.
The position equation describes the displacement of a designated piston from a defined origin based on a given shaft angle of rotation. The only variable in the first displacement equation should be the shaft angle of rotation $\theta_1$.

The kinematic solution for a swash-plate mechanism will be developed as follows:

In chapter three, the transformation matrix method of solving the mechanism will be explained (including a full solution of a generalized plane joint). In the same chapter, the variables used in the calculation and model of the mechanism will also be presented and explained. In chapter four, the solution of the basic model will be presented. Multiple iterations of data output will be listed and graphed. Finally, in chapter five, the results of the analysis will be discussed, recommendations for the use of the data in mechanism design will be made, future work using the general solution will be highlighted and conclusions based on the presented solution will be drawn.
3.1 Description of Methodology

The motion of the swash plate mechanism is modeled using coordinate transformation matrices. Therefore, the essential elements of the Transformation Method will now be discussed.

Clifford introduced the concept of the dual number in 1873 and the name of “dual number” was coined by Study in 1903. The definition of the dual number ($\varepsilon$) is:

\[ \varepsilon \neq 0 \]
\[ \varepsilon^2 = 0 \]  

(1)

In analogy with complex numbers, a duplex number can be written as:

\[ \hat{a} = a + \varepsilon(a_o) \]  

(2)

Where:

- $a$ = primary (or real) component of $\hat{a}$
- $a_o$ = dual component of $\hat{a}$

Dual numbers follow all the rules of regular algebra keeping in mind that a dual number raised to any power is equal to zero.

A dual angle can be used to specify the relative displacement and orientation between two lines in space.

The dual angle is written as:

\[ \hat{\theta} = \theta + \varepsilon(s) \]  

(3)
Where $\theta$ is the primary rotational part (projected angle between the lines) and $s$ is the dual translation part (shortest distance between the lines) of the dual angle $\hat{\theta}$.

The trigonometric functions of dual angles are obtained by using Taylor Series expansions:

$$
\sin \hat{\theta} = \sin \theta + \varepsilon(s) \cos \theta \\
\cos \hat{\theta} = \cos \theta - \varepsilon(s) \sin \theta \\
\tan \hat{\theta} = \tan \theta + \varepsilon(s) \sec^2 \theta
$$

(4)

All trigonometric identities hold for dual trigonometric functions.

Using coordinate transformation matrices, dual numbers can be used to describe a mechanism. This is accomplished by describing the succession of joints and links found in the mechanism with the descriptive transformation matrix of each joint. Then, the joint parameters used are determined and shown in a Denavit-Hartenberg table. Finally, the mechanism displacements are evaluated by performing a series of transformations through the joints and links specific to the mechanism. In this problem, the mechanism being considered is an RESP (Revolute, Plane, Spherical, and Prismatic) swash-plate mechanism. There are four joints and four links in the swash-plate mechanism.

For the solution presented, all transformations are relative to moving coordinate systems. Therefore the order of multiplication will be to the right.

The first joint encountered in the swash-plate mechanism is the revolute joint. The revolute, cylindrical and prismatic joints can each be described in general as a transformation through the dual angle $\hat{\theta}_n = \theta_n + \varepsilon(s_n)$ about the z-axis followed by a dual angle transformation about the x-axis through the angle $\hat{\alpha}_n = \alpha_n + \varepsilon(\alpha_n)$. In the case of the revolute joint, there is one degree of rotational freedom. Also, in the prismatic joint there is one translational degree of freedom along the shaft.
Note that expression (5) represents the relation of frame \{n\} fixed on the distal end of link “n-1” to frame \{n+1\} fixed on the distal end of link “n”. Letting a matrix \( \mathbf{M} \) represent the general form of the revolute/prismatic transformation, this is expressed as follows:

Note: \( s = \text{sine}, c = \text{cosine} \)

\[
\begin{pmatrix}
    c\theta_n & -s\theta_n & 0 \\
s\theta_n & c\theta_n & 0 \\
0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
    1 & 0 & 0 \\
0 & c\alpha_n & -s\alpha_n \\
0 & s\alpha_n & c\alpha_n
\end{pmatrix}
\]

(5)
Multiplying these matrices and separating the primary and dual components gives:

\[
\mathbf{M}(\hat{\theta}_n, \hat{\theta}_n) = \begin{bmatrix}
    c\theta_n & -c\alpha_n s\theta_n & s\alpha_n s\theta_n \\
    s\theta_n & c\alpha_n c\theta_n & -s\alpha_n c\theta_n \\
    0 & s\alpha_n & c\alpha_n \\
\end{bmatrix}
\]

\[
= + E
\begin{bmatrix}
    -s_n s\theta_n & a_n s\alpha_n s\theta_n - s_n c\theta_n & a_n c\alpha_n s\theta_n + s_n s\alpha_n c\theta_n \\
    s_n c\theta_n & -a_n s\alpha_n c\theta_n - s_n c\alpha_n s\theta_n & -a_n c\alpha_n c\theta_n + s_n s\alpha_n s\theta_n \\
    0 & a_n c\alpha_n & -a_n s\alpha_n \\
\end{bmatrix}
\]

(6)

The next joint encountered in the swash-plate is the plane joint denoted by E after the German word for plane, ebene (pronounced ehb-ih-nah). This joint has three degrees of freedom; two in translation and one in rotation. The transformation that describes the general form of the plane joint consists of a translation \( \hat{r}_n = 0 + \epsilon(r_n) \) about the x-axis followed by a translation \( \hat{s}_n = 0 + \epsilon(s_n) \) along the y-axis followed by a rotation \( \hat{\theta}_n = \theta_n + \epsilon(0) \) about the z-axis, and concluding with a translation \( \hat{a}_n = 0 + \epsilon(a_n) \) about the x-axis.
Start at system \{i\} fixed on the distal end of link n-1. Translate distance \( r_n \) along the \( i_n \)-axis. Translate distance \( s_n \) in the direction of the \( j_n \)-axis. Then rotate through \( \theta_n \) about \( k_n \)-axis into alignment with system \{n'\}. Then make a screw motion through translation \( a_n \) and rotation \( \alpha_n \) into alignment with frame \{n+1\} on the distal end of link n.

Let \( \hat{\mathbf{P}} \) represent the general form of the plane joint:

Note: \( c = \cos \), \( s = \sin \)

\[
\begin{align*}
_{n+1}^n \hat{\mathbf{P}} &= \hat{\mathbf{X}}(\hat{r}_n)\hat{\mathbf{Y}}(\hat{s}_n)\hat{\mathbf{Z}}(\hat{\theta}_n)\hat{\mathbf{X}}(\hat{\alpha}_n) \\
&= \begin{bmatrix}
1 & 0 & 0 & c\hat{s}_n & 0 & s\hat{s}_n & c\hat{\theta}_n - s\hat{\theta}_n & 0 & 1 & 0 & 0 \\
0 & c\hat{r}_n - s\hat{r}_n & 0 & 1 & 0 & s\hat{\theta}_n & c\hat{\theta}_n & 0 & 0 & c\hat{\alpha}_n - s\hat{\alpha}_n \\
0 & s\hat{r}_n & c\hat{r}_n & -s\hat{s}_n & 0 & c\hat{s}_n & 0 & 0 & 1 & 0 & s\hat{\alpha}_n & c\hat{\alpha}_n
\end{bmatrix}
\end{align*}
\] (7)
Multiplying these matrices and separating the primary from the dual components gives:

\[
\begin{bmatrix}
    c\theta_n & -s\theta_n & 0 \\
    s\theta_n & c\theta_n & 0 \\
    0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
    0 & 0 & a_n s\theta_n + s_n \\
    0 & 0 & -a_n c\theta_n - r_n \\
    r_n s\theta_n - s_n c\theta_n & r_n c\theta_n + s_n s\theta_n + a_n & 0
\end{bmatrix}
\]

(8)

The final joint in need of description for this mechanism is a spherical or ball joint. This joint has three degrees of rotational freedom about all three axes. The general form of transformations for this mechanism will be denoted as \( \hat{L} \). Because there are no natural axes, the motions of the ball joint are expressed with respect to arbitrarily chosen axis. The order of motion is:

First, a screw motion about the z-axis \( \hat{\theta}_n = \theta_n + \varepsilon(s_n) \) followed by a screw motion about the y-axis \( \hat{\eta}_n = \eta_n + \varepsilon(N_n) \), then a screw motion about the x-axis \( \hat{\zeta}_n = \zeta_n + \varepsilon(Z_n) \).
The expression represents transformation from frame \{n\} fixed on link "n-1" to frame \{n+1\} fixed on the distal end of link "n".

Let \( \hat{L} \) represent the general form of the plane joint:

Note: \( c = \text{cosine}, \ s = \text{sine} \)

\[
\begin{align*}
\begin{bmatrix}
    c\hat{\theta}_n & -s\hat{\theta}_n \\
s\hat{\theta}_n & c\hat{\theta}_n \\
0 & 1
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
    c\hat{\eta}_n & 0 & s\hat{\eta}_n \\
0 & 1 & 0 \\
-s\hat{\eta}_n & 0 & c\hat{\eta}_n
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
    1 & 0 & 0 \\
0 & c\hat{\xi}_n & -s\hat{\xi}_n \\
0 & s\hat{\xi}_n & c\hat{\xi}_n
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
    1 & 0 & 0 \\
0 & c\hat{\gamma}_n & -s\hat{\gamma}_n \\
0 & s\hat{\gamma}_n & c\hat{\gamma}_n
\end{bmatrix}
\end{align*}
\]

\[
\begin{align*}
(9)
\end{align*}
\]
Multiplying these matrices and separating the primary from the dual components gives:

\[
\begin{bmatrix}
\hat{L}(\hat{\theta}_n, \hat{\Psi}_n, \hat{\zeta}_n, \hat{\alpha}_n) \\
\end{bmatrix}
\begin{bmatrix}
\eta\epsilon
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
c\eta_n c\theta_n & s\eta_n c\theta_n c\zeta_n - s\theta_n c\zeta_n & s\eta_n c\theta_n c\zeta_n + s\theta_n s\zeta_n \\
c\eta_n s\theta_n & s\eta_n s\theta_n s\zeta_n + c\theta_n c\zeta_n & s\eta_n s\theta_n c\zeta_n - c\theta_n s\zeta_n \\
-s\eta_n & c\eta_n s\zeta_n & c\eta_n c\zeta_n \\
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
e\epsilon c\eta_n c\theta_n s\zeta_n \\
+ s\eta_n c\theta_n (z_n + a_n) c\zeta_n \\
-s\eta_n s\theta_n s\zeta_n \\
-e_n s\eta_n c\theta_n \\
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
e\epsilon c\eta_n s\theta_n s\zeta_n \\
+ s\eta_n s\theta_n s\zeta_n \\
-s\eta_n s\theta_n c\zeta_n \\
-c\theta_n (z_n + a_n) s\zeta_n \\
-s\eta_n s\theta_n c\zeta_n \\
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
e\epsilon c\eta_n (z_n + a_n) c\zeta_n \\
-c\theta_n (z_n + a_n) s\zeta_n \\
-s\eta_n s\zeta_n \\
-e_n s\eta_n s\zeta_n \\
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
e\epsilon c\eta_n s\theta_n c\zeta_n \\
+ s\eta_n s\theta_n c\zeta_n \\
-s\eta_n s\theta_n c\zeta_n \\
-c\theta_n (z_n + a_n) c\zeta_n \\
-s\eta_n s\theta_n s\zeta_n \\
\end{bmatrix}
\end{bmatrix}
\begin{bmatrix}
\begin{bmatrix}
e\epsilon c\eta_n (z_n + a_n) s\zeta_n \\
-c\theta_n (z_n + a_n) s\zeta_n \\
-e_n s\eta_n s\zeta_n \\
-e_n s\eta_n c\zeta_n \\
\end{bmatrix}
\end{bmatrix}
\]
3.2 Definition of Variables

Table 1 Variables

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<tr>
<th>VARIABLE</th>
<th>DESCRIPTION OF VARIABLES</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>Input shaft rotation in revolute joint</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>swash-plate twist about $i_1$-axis</td>
</tr>
<tr>
<td>$r_2$</td>
<td>plane joint translation in $j_2$-axis direction</td>
</tr>
<tr>
<td>$s_2$</td>
<td>plane joint translation in $i_2$-axis direction</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>plane-joint Rotation about $k_2$-axis</td>
</tr>
<tr>
<td>$\eta_3$</td>
<td>ball-joint rotation about $j_3$-axis</td>
</tr>
<tr>
<td>$\zeta_3$</td>
<td>ball-joint rotation about $i_3$-axis</td>
</tr>
<tr>
<td>$s_4$</td>
<td>cylindrical sliding along $k_4$-axis (piston oscillation)</td>
</tr>
<tr>
<td>$c_4$</td>
<td>frame offset along $i_4$-axis (perpendicular distance from geometric axis of the swash plate to the center of the piston)</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>frame twist about $i_4$-axis</td>
</tr>
</tbody>
</table>
3.3 Formulation of Solution

The first step in the transformation matrix solution process is defining and arranging the parameters that define the swash-plate mechanism. The arrangement of the joint variables and link dimensions in a table of parameters as suggested by Denavit and Hartenberg [13] is shown in Table 1.

<table>
<thead>
<tr>
<th>Index</th>
<th>Name</th>
<th>( \theta_i )</th>
<th>( s_i )</th>
<th>( a_i )</th>
<th>( r_i )</th>
<th>( c_i )</th>
<th>( \eta_i )</th>
<th>( N_i )</th>
<th>( \zeta_i )</th>
<th>( Z_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Input Shaft</td>
<td>( \theta_1 )</td>
<td>0</td>
<td>( a_1 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>Swash-Plate</td>
<td>( \theta_2 )</td>
<td>( s_2 )</td>
<td>0</td>
<td>( r_2 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>Ball coupler</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \eta_3 )</td>
<td>0</td>
<td>( \zeta_3 )</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>Follower</td>
<td>0</td>
<td>( s_4 )</td>
<td>( a_4 )</td>
<td>( c_4 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

In the solution of this problem, a pictorial representation mixed with a mathematical description will be used. The order of solution will be as follows: First, the position of the coordinate system on the swash plate at each transformation will be shown with the corresponding transformation matrix. Second, the transformation matrices will be arranged according to their order of multiplication. Thirdly, the matrices will be rearranged for the solution. Fourthly and finally, mathematical manipulation software (Derive) will be used to multiply the matrices together and then set their components equal to each other for the final solution equation.

The starting point of the swash plate transformations lies on the neutral axis. The neutral axis can be thought of as a line lying on the face of the swash-plate that is
perpendicular to the input shaft regardless of the swash-plate angle. The piston/follower assembly is at mid-stroke when the center of the coupler contacts the swash-plate at the neutral axis. See figures 5 and 6.

**Figure 5**
Top View of Swash-Plate Mechanism Showing Neutral Axis as a Point

**Figure 6**
Side View of Swash-Plate Mechanism Showing Neutral Axis
3.4 Swash Plate Transformations

3.4.1 Transformation #1

For this transformation, the input variable $\theta_i$ is shown as a rotation about the k-axis. Note the intermediate position of the coordinate axis denoted $i'$, $j'$ and $k'$. The constant swash-angle $\alpha_i$ about the i-axis is also shown. These transformation matrices will be represented as:

$$\hat{M}(\hat{\theta}_i, \hat{\alpha}_i) = \hat{Z}(\theta_i)\hat{X}(\alpha_i)$$

(11)

Figure 7
Swash-Plate Showing Transformation #1
3.4.2 Transformation #2

For this transformation, the plane joint is described with the intermediate variable translations of $r_2$ and $s_2$ along the i-axis and j-axis respectively. Also described is the intermediate variable transformation $\theta_2$ acting about the k-axis. This transformation is shown as follows:

$$ \hat{P}(\hat{r}_2, \hat{s}_2, \hat{\theta}_2) = \hat{X}(r_2)\hat{Y}(s_2)\hat{Z}(\theta_2) $$

(12)

Figure 8
Swash-Plate Showing Transformation #2
3.4.3 Transformation #3

For this transformation, the ball or spherical joint will be described using the intermediate rotational variables $\eta_3$ and $\zeta_3$ about the j-axis and i-axis respectively. The transformation is shown as follows:

$$\hat{L}(\zeta_3, \hat{\eta}_3) = \hat{Y}(\eta_3)\hat{X}(\zeta_3)$$

(13)

Figure 9
Swash-Plate Showing Transformation #3
3.4.4 Transformation #4

In this transformation, the prismatic joint is described by a translation through the output variable \( s_4 \) along the k-axis. There is also a screw motion through intermediate variable \( \alpha_4 \) and constant \( c_4 \) about the i-axis. The coordinate transformation is as follows:

\[
\hat{M}(\alpha_4, c_4, s_4) = \hat{Z}(s_4)\hat{X}(\alpha_4, c_4)
\]  

(14)

Figure 10
Swash-Plate Showing Transformation #4

Multiplying these four transformations together forms one transformation equation:

\[
\hat{M}(\theta_1, \alpha_1)\hat{P}(r_2, s_2, \theta_2)\hat{L}(\eta_3, \zeta_3)\hat{M}(\alpha_4, c_4, s_4) = \hat{I}
\]  

(15)

For purposes of the solution, the input variables from the output variables are segregated before performing the multiplication:

\[
\hat{P}(r_2, s_2, \theta_2)\hat{L}(\eta_3, \zeta_3) = \hat{M}^{-1}(\theta_1, \alpha_1)\hat{M}^{-1}(\alpha_4, c_4, s_4)
\]  

(16)
3.5 Solution of the Swash-Plate Mechanism

3.5.1 Initial Results

Multiplying the matrices described in section 3.4 together with some simplification gives:

\[
\begin{bmatrix}
  c \eta_3 c \theta_2 & - s \theta_2 c \zeta_3 \\
  - c \eta_3 s \theta_2 & + c \theta_2 c \zeta_3 \\
  + \varepsilon s_3 \eta_3 & + s s_3 c \eta_3 s \zeta_3 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  c \eta_3 s \zeta_3 \\
  - s \eta_3 \\
  - c \eta_3 c \theta_2 - s c \theta_2 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  s \eta_3 s \theta_2 s \zeta_3 \\
  - s \theta_2 s \zeta_3 \\
  + s s_2 c \eta_3 s \zeta_3 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  s \eta_3 c \theta_2 c \zeta_3 \\
  - s \theta_2 s \zeta_3 \\
  + s s_2 c \eta_3 c \zeta_3 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  - c \eta_3 s \theta_2 c \zeta_3 \\
  + c \theta_2 c \zeta_3 \\
  - c \eta_3 c \theta_2 c \zeta_3 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  - \varepsilon c \eta_3 s \theta_2 s \zeta_3 \\
  s \eta_3 s \theta_2 c \zeta_3 \\
  + s s_2 c \eta_3 s \zeta_3 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  c \alpha_4 s \theta_1 \\
  - \varepsilon \left( c_4 s a_4 s \theta_1 + s_4 c \theta_1 \right) \\
  - \varepsilon \left( c_4 a_4 s \alpha_1 s \alpha_1 + s_4 c \theta_1 \right) \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  s a_4 s \theta_1 \\
  + c_4 c a_4 s \theta_1 c a_1 \\
  + c_4 c a_4 c \theta_1 c a_1 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  c \alpha_1 c \alpha_4 c \theta_1 \\
  - s \alpha_4 c \alpha_1 \\
  - \varepsilon \left( c_4 c a_4 s \alpha_1 s \alpha_1 + s_4 s c \theta_1 c a_1 \right) \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  c \alpha_4 s \alpha_1 \\
  + s \alpha_4 c \alpha_1 \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  - \varepsilon \left( c_4 c a_4 s \alpha_1 s \alpha_1 + s_4 s c \theta_1 c a_1 \right) \\
\end{bmatrix}
\]

\[
= \begin{bmatrix}
  c_4 c a_4 c \theta_1 s a_1 \\
  - s_4 s c \theta_1 s a_1 \\
\end{bmatrix}
\]

The solution matrix primary and dual elements are set equal to each other. This gives eighteen equations that can be used to solve for six unknown variables \((r_2, s_2, a_4, \theta_2, \eta_3, \zeta_3)\).
3.5.2 Primary Elements

Equation of the primary components of the elements in Equation (15) yield

primary matrix element 1-1:

\[ c\eta_3 c\theta_2 = c\theta_1 \]  (18)

primary matrix element 1-2:

\[ s\eta_3 c\theta_2 s\zeta_3 - s\theta_2 c\zeta_3 = c\alpha_4 s\theta_1 \]  (19)

primary matrix element 1-3:

\[ s\eta_3 c\theta_2 c\zeta_3 + s\theta_2 s\zeta_3 = s\alpha_4 s\theta_1 \]  (20)

primary matrix element 2-1:

\[ c\eta_3 s\theta_2 = -c\alpha_1 s\theta_1 \]  (21)

primary matrix element 2-2:

\[ s\eta_3 s\theta_2 s\zeta_3 + c\theta_2 c\zeta_3 = c\alpha_1 c\alpha_4 c\theta_1 - s\alpha_1 s\alpha_4 \]  (22)

primary matrix element 2-3:

\[ s\eta_3 s\theta_2 c\zeta_3 - c\theta_2 s\zeta_3 = c\alpha_1 s\alpha_4 c\theta_1 + s\alpha_1 c\alpha_4 \]  (23)

primary matrix element 3-1:

\[ -s\eta_3 = s\alpha_1 s\theta_1 \]  (24)

primary matrix element 3-2:

\[ c\eta_3 s\zeta_3 = -c\alpha_1 s\alpha_4 - s\alpha_1 c\alpha_4 c\theta_1 \]  (25)

and primary matrix element 3-3:

\[ c\eta_3 c\zeta_3 = c\alpha_1 c\alpha_4 - s\alpha_1 s\alpha_4 c\theta_1 \]  (26)
3.5.3 Dual Elements

Equation of the dual components of the elements of Equation (15) yield

dual matrix element 1-1:

\[-s_2 s_\eta_3 = -s_4 c_\alpha_4 s_\theta_1\]

(27)

dual matrix element 1-2:

\[s_2 c_\eta_3 s_\zeta = -c_4 s_\alpha_4 s_\theta_1 + s_4 c_\theta_1\]

(28)

dual matrix element 1-3:

\[s_2 c_\eta_3 c_\zeta = c_4 c_\alpha_4 s_\theta_1\]

(29)

dual matrix element 2-1:

\[r_2 s_\eta_3 = s_4 s_\alpha_1 s_\alpha_4 - c_\alpha_1 s_4 c_\alpha_4 c_\theta_1\]

(30)

dual matrix element 2-2:

\[-r_2 c_\eta_3 s_\zeta = -c_\alpha_1 c_4 s_\alpha_4 c_\theta_1 - c_\alpha_1 s_4 s_\theta_1 - s_\alpha_1 c_4 c_\alpha_4\]

(31)

dual matrix element 2-3:

\[-r_2 c_\eta_3 c_\zeta = c_\alpha_1 c_4 c_\alpha_4 c_\theta_1 - s_\alpha_1 c_4 s_\alpha_4\]

(32)

dual matrix element 3-1:

\[c_\eta_3 r_2 s_\theta_2 - c_\eta_3 s_2 c_\theta_2 = s_4 c_\alpha_4 s_\alpha_4 + s_\alpha_1 s_4 c_\alpha_4 c_\theta_1\]

(33)

dual matrix element 3-2:

\[s_\eta_3 \left(r_2 s_\theta_2 s_\zeta_3 - s_2 c_\theta_2 s_\zeta_3\right) + r_2 c_\theta_2 c_\zeta_3 + s_2 s_\theta_2 c_\zeta_3\]

\[= -c_\alpha_1 c_4 c_\alpha_4 + s_\alpha_1 \left(c_4 c_\alpha_4 c_\theta_1 + s_4 s_\theta_1\right)\]

(34)

and dual matrix element 3-3:

\[s_\eta_3 r_2 s_\theta_2 c_\zeta_3 - s_\eta_3 s_2 c_\theta_2 c_\zeta_3 - r_2 c_\theta_2 s_\zeta_3 - s_2 s_\theta_2 s_\zeta_3 = c_4 s_\alpha_4 - s_\alpha_1 c_4 c_\alpha_4 c_\theta_1\]

(35)
3.5.4 Derivation of the Solution in its Final Form

Using equation (19):
\[ c \eta_3 = \frac{-c\alpha_1 s\theta_1}{s\theta_2} \quad (36) \]

Using equation (22):
\[ s \eta_3 = -s\alpha_1 s\theta_1 \quad (37) \]

Using equations (34) and (35), rotation angle \( \eta_3 \) can be determined in its proper quadrant.

Substituting equation (34) into equation (16) gives:
\[ \frac{-c\alpha_1 s\theta_1 c\theta_2}{s\theta_2} = c\theta_1 \]
\[ \frac{s\theta_2}{c\theta_2} = -c\alpha_1 s\theta_1 \quad (38) \]

This reveals angle \( \theta_2 \) as per its proper quadrant.

Substituting equation (34) into equation (23) gives:
\[ \frac{-c\alpha_1 s\theta_1}{s\theta_2} s\zeta_3 = -c\alpha_1 s\alpha_4 - s\alpha_1 c\alpha_4 c\theta_1 \]
\[ s\zeta_3 = \frac{s\theta_2 (c\alpha_1 s\alpha_4 + s\alpha_1 c\alpha_4 c\theta_1)}{c\alpha_1 s\theta_1} \quad (39) \]

Substituting equation (34) into equation (24) gives:
\[ \frac{-c\alpha_1 s\theta_1}{s\theta_2} c\zeta_3 = c\alpha_1 c\alpha_4 - s\alpha_1 s\alpha_4 c\theta_1 \]
\[ c\zeta_3 = -\frac{s\theta_2 (c\alpha_1 c\alpha_4 - s\alpha_1 s\alpha_4 c\theta_1)}{c\alpha_1 s\theta_1} \quad (40) \]

Rotation \( \zeta_3 \) can now be determined in its proper quadrant using equations (37) and (38).
Using equation (27):

\[ s_2 = \frac{c_4 c \alpha_4 s \theta_1}{c \eta_3 c \zeta_3} \]  

(41)

Using equation (30):

\[ r_2 = \frac{-c \alpha_4 c c_4 c \alpha_4 c \theta_1 + s \alpha_4 c s \alpha_4}{c \eta_3 c \zeta_3} \]  

(42)

Using equation (25):

\[ s_4 = \frac{s_2 s \eta_3}{c \alpha_4 s \theta_1} \]  

(43)

These equations represent a full solution for the displacements and orientations of the swash-plate mechanism’s links.
CHAPTER 4

RESULTS

To represent the kinematic behavior of the swash-plate mechanism according to the solution developed in chapter 3, the variation of the output variable \( s_4 \) with respect to the input variable \( \theta_1 \) will be shown and explained first. Secondly, the variation of the intermediate variable \( \theta_2 \) will be considered. Thirdly, the intermediate variables \( r_2 \) and \( s_2 \) will be described and explained. Finally, the intermediate variables \( \zeta_3 \) and \( \eta_3 \) will each be described, explained and graphed with respect to the input angle \( \theta_1 \).

Several standard dimensions were used in the example of the results. These include a frame offset \( c_4 \) of 1" and a standard frame twist angle \( \alpha_4 \) of 180°. To show the displacement behavior of the swash-plate mechanism, the equations (34) through (41) derived in chapter 3 were incorporated in a FORTRAN computer program called “SWASHP” (see Appendix A). The step increment of 10° was used for the input angle \( \theta_1 \).

The input for the first data set is shown in Table 2 and the output of the program “SWASHP” with a swash angle \( \alpha_i \) of 10° and an input angle \( \theta_1 \) step increment of 10° can be found in Table 3.
Table 3 Input for First Data Set ($\alpha_1 = 10^\circ$)

<table>
<thead>
<tr>
<th>INPUT VARIABLE</th>
<th>VALUE</th>
<th>DESCRIPTION</th>
<th>UNITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_i$ initial value</td>
<td>$0^\circ$</td>
<td>Initial input angle value</td>
<td>degrees</td>
</tr>
<tr>
<td>$\theta_i$ step increment</td>
<td>$10^\circ$</td>
<td>The step increment at which the program SWASHP will evaluate all intermediate and output variables</td>
<td>degrees</td>
</tr>
<tr>
<td>$\alpha_i$</td>
<td>$10^\circ$</td>
<td>The swash-plate angle</td>
<td>degrees</td>
</tr>
<tr>
<td>$\alpha_4$</td>
<td>$180^\circ$</td>
<td>The angle between the input shaft axis of rotation and the output axis of translation</td>
<td>degrees</td>
</tr>
<tr>
<td>$c_4$</td>
<td>$1$</td>
<td>Perpendicular distance (or offset) between the input shaft axis of rotation and the output axis of translation</td>
<td>inches</td>
</tr>
</tbody>
</table>
Table 4 FORTRAN Program “SWASHP” Output for First Data Set ($\alpha_1 = 10^\circ$)

<table>
<thead>
<tr>
<th>THETA1</th>
<th>THETA2</th>
<th>ETA3</th>
<th>ZETA3</th>
<th>S2</th>
<th>R2</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>360.0000</td>
<td>180.0000</td>
<td>190.0000</td>
<td>0</td>
<td>1.0000</td>
<td>0</td>
</tr>
<tr>
<td>10</td>
<td>350.1489</td>
<td>181.7280</td>
<td>189.8511</td>
<td>-0.17633</td>
<td>0.98481</td>
<td>-0.03062</td>
</tr>
<tr>
<td>20</td>
<td>340.2803</td>
<td>183.4049</td>
<td>189.4081</td>
<td>-0.34730</td>
<td>0.93969</td>
<td>-0.06031</td>
</tr>
<tr>
<td>30</td>
<td>330.3784</td>
<td>184.9809</td>
<td>188.6822</td>
<td>-0.50771</td>
<td>0.86603</td>
<td>-0.08816</td>
</tr>
<tr>
<td>40</td>
<td>320.4313</td>
<td>186.4087</td>
<td>187.6926</td>
<td>-0.65270</td>
<td>0.76604</td>
<td>-0.11334</td>
</tr>
<tr>
<td>50</td>
<td>310.4325</td>
<td>187.6443</td>
<td>186.4664</td>
<td>-0.77786</td>
<td>0.64279</td>
<td>-0.13507</td>
</tr>
<tr>
<td>60</td>
<td>300.3813</td>
<td>188.6492</td>
<td>185.0384</td>
<td>-0.87939</td>
<td>0.50000</td>
<td>-0.15270</td>
</tr>
<tr>
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<td>290.2836</td>
<td>189.3913</td>
<td>183.4512</td>
<td>-0.95419</td>
<td>0.34202</td>
<td>-0.16569</td>
</tr>
<tr>
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<td>181.7538</td>
<td>-1.00000</td>
<td>0.17365</td>
<td>-0.17365</td>
</tr>
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<td>180.0000</td>
<td>-1.01543</td>
<td>0</td>
<td>-0.17633</td>
</tr>
<tr>
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<td>178.2462</td>
<td>-1.00000</td>
<td>-0.17365</td>
<td>-0.17365</td>
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<tr>
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<td>176.5488</td>
<td>-0.95419</td>
<td>-0.34202</td>
<td>-0.16569</td>
</tr>
<tr>
<td>120</td>
<td>239.6188</td>
<td>188.6492</td>
<td>174.9616</td>
<td>-0.87939</td>
<td>-0.50000</td>
<td>-0.15270</td>
</tr>
<tr>
<td>130</td>
<td>229.5676</td>
<td>187.6443</td>
<td>173.5336</td>
<td>-0.77786</td>
<td>-0.64279</td>
<td>-0.13507</td>
</tr>
<tr>
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<td>172.3074</td>
<td>-0.65270</td>
<td>-0.76604</td>
<td>-0.11334</td>
</tr>
<tr>
<td>150</td>
<td>209.6217</td>
<td>184.9809</td>
<td>171.3178</td>
<td>-0.50771</td>
<td>-0.86603</td>
<td>-0.08816</td>
</tr>
<tr>
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<td>199.7198</td>
<td>183.4049</td>
<td>170.5919</td>
<td>-0.34730</td>
<td>-0.93969</td>
<td>-0.06301</td>
</tr>
<tr>
<td>170</td>
<td>189.8511</td>
<td>181.7280</td>
<td>170.1489</td>
<td>-0.17633</td>
<td>-0.98481</td>
<td>-0.03062</td>
</tr>
<tr>
<td>180</td>
<td>170.0000</td>
<td>180.0000</td>
<td>170.0000</td>
<td>0</td>
<td>-1.00000</td>
<td>0</td>
</tr>
<tr>
<td>190</td>
<td>160.2803</td>
<td>176.5951</td>
<td>170.5919</td>
<td>0.34730</td>
<td>-0.39369</td>
<td>0.06301</td>
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<tr>
<td>200</td>
<td>150.3784</td>
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<td>171.3178</td>
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<td>0.08816</td>
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<tr>
<td>210</td>
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<td>173.5914</td>
<td>172.3074</td>
<td>0.65270</td>
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<td>0.77786</td>
<td>-0.64279</td>
<td>0.13507</td>
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<td>171.3508</td>
<td>174.9616</td>
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Output Translation as a Function of Input Rotation ($\alpha_i = 10^\circ$)

Figure 11 indicates that the output translation is periodic in nature given a constant input shaft rotation. This graph represents the physical oscillation of the piston as a result of input shaft rotation.
Figure 12 represents the relationship between input rotation $\theta_1$ and intermediate rotation $\theta_2$. While not immediately apparent, upon close examination, it is observed that the relationship is not linear. This will be accentuated with the second data set with a larger value for $\alpha_i$. 

Intermediate Angle $\theta_2$ and Input Rotation $\theta_1$ ($\alpha_i = 10^\circ$)
Figure 13 represents the path of the coupler traced on the face of the swash plate. Upon close examination, it is observed that the path is elliptical. This shape can be accentuated with a larger swash angle $\alpha_i$. This accentuated shape will be shown in the second and following data sets.
The physical meaning of the curve graphed in Figure 14 is a representation of the forward rocking motion or "pitch" of the spherical joint given the input rotation $\theta_1$. A single frequency fluctuation of small amplitude is observed.
The side-to-side rotation or "yaw" of the spherical joint on the end of the follower is described by angle $\zeta_3$. It is noted that the behavior of angle $\zeta_3$ is similar in nature to the behavior of angle $\eta_3$. The functions that describe the single frequency fluctuation evident in these two variables are identical except for a $90^\circ$ shift. This corresponds to the angle between the coordinate axis from which each is measured.
Figure 16

Intermediate Variable $\zeta_3$ and Intermediate Variable $\eta_3$ ($\alpha_1 = 10^\circ$)

Figure 16 is a graph describing the pattern traced by the center of the coupler pad in two-dimensional space as the swash-plate rotates. It is a perfect circle as expected.
For the second and third data sets, the swash angle $\alpha_i$ is varied. In the second data set it is set at $45^\circ$ to represent an extreme value thus accentuating the behavior of the intermediate variables for observation. In the third data set, $\alpha_i$ is set $25^\circ$ to show a trend of $\alpha_i$ between $10^\circ$ and $45^\circ$. The results of all three examples are combined in figures 17-22.

**Table 5 Input for Second Data Set ($\alpha_i = 45^\circ$)**

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<td>The angle between the input shaft axis of rotation and the output axis of translation</td>
<td>degrees</td>
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### Table 6: FORTRAN Program “SWASHP” Output for Second Data Set ($\alpha_i = 45^\circ$)

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Table 7 Input for Third Data Set ($\alpha_i = 25^\circ$)

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Table 8 FORTRAN Program “SWASHP” Output for Third Data Set ($\alpha_i = 25^\circ$)

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Figure 17

Output Translation $s_4$ and Input Rotation $\theta_1$
Intermediate Angle $\theta_2$ and Input Rotation $\theta_1$.

Figure 18
Figure 19

Intermediate Translations \( s_2 \) and \( r_2 \)
Figure 20

Intermediate Variable $\eta_3$ and Input Variable $\theta_1$
Intermediate Variable $\phi_3$ and Input Variable $\theta_i$. 

Figure 21
Intermediate Variable $\eta_3$ and Intermediate Variable $\zeta_3$
CHAPTER 5

DISCUSSION AND CONCLUSION

In the previous chapters, a study to analyze the kinematic behavior of the swash-plate mechanism was presented. Upon examination of the data provided in Chapter 4, the solution presented seemed to be exhaustive and complete. The position and/or orientation of any link in the swash-plate mechanism was found given any input angle. This solution can be expanded to describe the position and/or orientation of any number of followers simply by adding the corresponding phase angle describing the location of that follower with respect to the reference follower at which the solution originated.

Furthermore, it was proven that the general transformation matrix solutions for the ebene and spherical joints provided accurate results in the solution of the swash-plate mechanism.

In figure 17, the trend over the single frequency produced in $360^\circ$ of rotation is a non-linear increase in magnitude. This can be confirmed by checking the data. Figure 18 is perhaps the most interesting graph in chapter 4. It shows how input angle $\theta_1$ and intermediate angle $\theta_2$ are related with a double frequency. The variation in magnitude is observed as non-linear. Figure 19 is perhaps the most dramatic example of the nonlinear relationship between the data sets. It is obvious that the magnitude of deviation from a circle for $\alpha_1 = 10^\circ$ is not just under a fourth of the magnitude of deviation from a circle when $\alpha_1 = 45^\circ$. The single frequency relationships of intermediate angles $\eta_3$ and $\zeta_3$ and

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input angle $\theta_1$ in figures 20 and 21 show the direct relationship between the swash-plate angle and magnitude of those intermediate variables. The relationship is 1:1 with a 90° phase shift because of the axis used to measure the angles being different. In figure 22, the trend for intermediate variables $\zeta_3$ and $\eta_3$ is to increase in magnitude proportionally to the swash-plate angle $\alpha_1$. This trend is shown by the larger circles corresponding to the larger swash-plate angles.

The figures in chapter 4 showing the relationships of the intermediate, input and output variables indicate that there may be some new information available to machine designers. This solution could assist in optimizing the swash-plate mechanism as it is configured in the automobile air-conditioning compressor. For example, figures 12 and 18 show a double frequency relationship between $\theta_1$ and $\theta_2$. This could explain a critical vibration that might affect the fatigue life of the mechanism. Cursory observation indicates that the critical area affected by this double frequency vibration is the junction between the input rotation shaft and the swash-plate plane.

Future work with this solution could be extensive. The equations derived in chapter 3 offer much more potential information than was presented in chapter 4. For example, the swash angle $\alpha_1$ could be held at zero and the angle between the input axis and the piston $\alpha_4$ could be varied. This would show a different configuration for the mechanism that could be compared to the original configuration already presented. A combination of varying the swash-plate angle $\alpha_1$, the angle between the input shaft and the output follower/piston assembly $\alpha_4$ could also be studied and used for a possible mechanism configuration optimization in the future. Another possibility for future work is
a dynamic analysis of the swash-plate mechanism. By taking the first and second derivatives, the velocity and change in momentum of each link in the mechanism could be studied in detail.

In conclusion, the aforementioned work presents a reasonable, accurate and complete solution for the swash-plate mechanism. The solution is consistent with previous geometric solutions in its explanation of the extreme positions of the swash plate and yet it provides a simple way to understand the motion of the mechanism more completely. This information could prove useful to mechanism designers interested in optimizing the behavior of the swash-plate mechanism for any of its current applications or possibly for any new application being explored.
APPENDIX A

FORTRAN Computer Program “SWASP”

PROGRAM SWASP

C This program is designed to solve the equations produced
C by a transformation matrix analysis of the swash-plate mechanism.
C Also, a data file will be produced for plotting the function
described by the last equation.

PARAMETER (PI=3.14159265359)

CHARACTER *1 ANSWER
CHARACTER *10 FILNAM

INTEGER K,K2

REAL ZTA3CT,ZTA3CB
REAL THETA2S,THETA2C,ETA3S,ETA3C,ZETA3S,ZETA3C
REAL ALPH1,ALPHA1, ALPH4,ALPHA4, C4, R2, S2
REAL THETA1, THETA2, ETA3, ZETA3, S4

C This statement is asking for the swash-angle as defined by the
C user

PRINT *,'Enter the swash-angle, in degrees:'
READ*, ALPH1

C ALPH1 is converted to radians and changed into its true name
"ALPHA1"

ALPHA1=(ALPH1/180.0)*PI

C Swash-Plate offset is now entered. This value is in inches

PRINT *, 'Enter the offset of the piston from the center of the
+ swash-plate axis, in inches:'
READ *, C4

C The angle between the piston axis and the axis of the swash-plate
is entered here

PRINT *,'Angle between piston and swash-plate axis, in degrees:'
READ*,ALPH4

C ALPHA4 is derived through the conversion of ALPH4 to radians

ALPHA4=(ALPH4/180.0)*PI

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This statement initializes the angle \( \theta_1 \)
\[
\theta_1 = \left( \frac{0.000001}{180} \right) \pi
\]

This statement determines the input step (\# of data points) between 0 and 360 degrees of rotation.

\[
\text{PRINT*, 'Enter input angle step increment:'}
\]
\[
\text{READ *,K2}
\]

This statement asks the user if an output file should be created. It also prompts the user for the name of the output file.

\[
\text{PRINT *, 'Output file? y/n (Surround answer with parenthesis)?'}
\]
\[
\text{READ *,ANSWER}
\]

This statement is used to open the output file in which the data for the graph is placed.

\[
\text{IF(ANSWER .EQ. 'y') THEN}
\]
\[
\text{PRINT *,'Name of Output file? (In parenthesis please)'}
\]
\[
\text{READ *,FILNAM}
\]
\[
\text{OPEN(UNIT=2,FILE=FILNAM,STATUS='NEW')}
\]
\[
\text{WRITE(2,40)'THETA1','THETA2','ETA3','ZETA3','S2','R2','S4'}
\]
\[
\text{ENDIF}
\]
\[
\text{PRINT 40, 'THETA1',' THETA2', 'ETA.3,1', 'ZETA3', 'S2','R2','S4'}
\]

This loop is used to actually perform the calculations of the output formulas for the swash-plate transform.

\[
\text{DO 100 K = 0,360,K2}
\]
\[
\text{THTA2S=-COS(ALPHA1)*SIN(THETA1)}
\]
\[
\text{THTA2C= COS(THETA1)}
\]
\[
\text{THETA2= ATAN2(THTA2S,THTA2C)}
\]
\[
\text{IF (THETA2 .LT. 0) THEN}
\]
\[
\text{THETA2 = THETA2 + 2*PI}
\]
\[
\text{ENDIF}
\]
\[
\text{ETA3S=-SIN(ALPHA1)*SIN(THETA1)}
\]
\[
\text{ETA3C= COS(ALPHA1)*SIN(THETA1)/SIN(THETA2)}
\]
\[
\text{ETA3= ATAN2(ETA3S,ETA3C)}
\]
IF (ETA3 .LT. 0) THEN
    ETA3 = ETA3 + 2*PI
ENDIF

ZETA3S = (SIN (THETA2) * (COS (ALPHA1) * SIN (ALPHA4) +
          SIN (ALPHA1) * COS (ALPHA4) * COS (THETA1)))
       + / (COS (ALPHA1) * SIN (THETA1))
ZTA3CT = -SIN (THETA2) * (COS (ALPHA1) * COS (ALPHA4) -
           SIN (ALPHA1) * SIN (ALPHA4) * COS (THETA1))
ZTA3CB = COS (ALPHA1) * SIN (THETA1)
ZETA3C = ZTA3CT / ZTA3CB
ZETA3 = ATAN2 (ZETA3S, ZETA3C)
IF (ZETA3 .LT. 0) THEN
    ZETA3 = ZETA3 + 2*PI
ENDIF

S2 = (C4 * COS (ALPHA4) * SIN (THETA1)) / (COS (ETA3) * COS (ZETA3))
R2 = (C4 *
     (SIN (ALPHA1) * SIN (ALPHA4) -
      COS (ALPHA1) * COS (ALPHA4) * COS (THETA1)) )
     + / (COS (ETA3) * COS (ZETA3))
S4 = R2 * SIN (ETA3) / (SIN (ALPHA1) * SIN (ALPHA4) -
                      COS (ALPHA1) * COS (ALPHA4) * COS (THETA1))

IF (ANSWER .EQ. 'y') THEN
   "This statement is used to write the program's output to the"
   "data files for use later by the graphing package."
   WRITE (2, 50) THETA1*(180.0/PI), THETA2*(180.0/PI),
          ETA3*(180.0/PI), ZETA3*(180.0/PI), S2, R2, S4
ENDIF

PRINT 50, THETA1*(180.0/PI), THETA2*(180.0/PI), ETA3*(180.0/PI),
       + ZETA3*(180.0/PI), S2, R2, S4
THETA1 = THETA1 + (K2/180.0)*PI
100 CONTINUE
STOP

C End the Program

END
BIBLIOGRAPHY


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