Dynamic friction of UHMWPE angular compliant bearing for joint implants and machinery

Max Roman
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ABSTRACT

DYNAMIC FRICTION OF UHMWPE ANGULAR COMPLIANT BEARING FOR JOINT IMPLANTS AND MACHINERY

by

Max Roman

The feasibility of a unique design of orthopedic joint implant is investigated. The work focuses on the dynamic friction characteristics of a new joint. This unique design can be also useful in machinery involving oscillating motion similar to that of the hip joint during walking. An elastomeric layer bonded behind the rigid acetubular cup can be beneficial in improving the durability of prosthetic implants. Angular compliance offered by the elastomeric layer reduces relative sliding between the surfaces, lowers contact stresses by distributing pressure more evenly, and reduces the possibility of incongruent loading conditions caused by misalignment. The application of a compliant bearing is not limited to prosthetic implants. Like a working biological joint, machinery operating with frequent start-stop sinusoidal motion is characterized by high friction at the start-up and point of velocity reversal. Contact between the surfaces, which results in wear, is most likely to occur when the velocity passes through the zero velocity region. In the presence of lubrication, angular compliance allows a hydrodynamic film to build up before there is a sliding motion with direct contact between surfaces. A short sliding motion at the start of the cycle is replaced by rolling motion.

A friction measuring apparatus was constructed to study the dynamic friction properties of both the current and proposed bearing in order to establish the feasibility of such a design. Dynamic friction measurements were conducted without lubrication and with LW 104, 5W, and 10W viscosity oils. In addition, tests were conducted with varied
loading, frequency of oscillations, and sinusoidal velocity for both the rigid and compliant design. Although only a small amount of angular compliance was introduced to the bearing, the results indicated a significant improvement in performance. It was found that when lubricated, use of the compliant bearing resulted in reduced start-up friction. Increasing load, viscosity, and frequency of oscillation resulted in a more significant reduction of the start-up friction.
DYNAMIC FRICTION OF UHMWPE ANGULAR COMPLIANT BEARING FOR JOINT IMPLANTS AND MACHINERY

by
Max Roman

A Thesis
Submitted to the Faculty of
New Jersey Institute of Technology
in Partial Fulfilment of the Requirements for the Degree of
Master of Science in Mechanical Engineering

Department of Mechanical Engineering

September 2001
DYNAMIC FRICTION OF UHMWPE ANGULAR COMPLIANT BEARING FOR
JOINT IMPLANTS AND MACHINERY

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To my mother, whose sacrifice has made all of this possible
ACKNOWLEDGEMENT

I would like to thank my advisor, Dr. Avraham Harnoy, for all his guidance and support in previous courses and throughout the research of this work. His enthusiasm of the subject contained in this thesis has been contagious. Most importantly, I would like to thank Dr. Harnoy for the key role in encouraging me to achieve all that I have.

I would like to extend my sincere gratitude to the members of my committee, Dr. Ernest Geskin and Dr. Edward L. Dreyzin. I am particularly appreciative of their effort in reviewing my work on such short notice.

I am also grateful to the New Jersey Center for Biomaterials Research. The stipend that I was awarded by the center helped initiate the work that culminated into this thesis.

None of the work presented here would have been possible without the help of John S. Hoinowski, chief research machinist. His willingness to help and to expedite my requests when I came calling at the most impromptu times was invaluable in keeping the research on course.

I could not end this list without thanking all my friends and family for all their support, both moral and physical. I could not have done any of this without your help.
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CHAPTER 1
INTRODUCTION, OBJECTIVES, AND OUTLINE

1.1 Introduction

The scientific study of friction can be traced at least as far back as Leonardo da Vinci and to the works of Newton in the 17th century. Newton's first law states that an object in motion will remain in motion unless acted upon by external forces. Friction is a force that is always present and acts on every object in motion. Credit for pioneering the investigation of friction is often given to Amontons (1699), Coulomb (1785), and Morin (1833). These early investigators hypothesized and sought explanations for friction phenomena, which was thought to be either the result of adhesive forces acting between contacting surfaces or to the interlocking of surface asperities (Rabinowicz 1995). Since their time many books and papers have been published on the subject, and, today, the adhesion hypothesis tends to prevail.

The important role that friction plays in our daily lives cannot be understated. The energy resources lost to friction in the railway and steel industries in the 19th century are staggering by modern standards (Hersey 1966). Advances in material science and lubrication theory in recent years have helped to substantially reduce frictional losses. Machines are now fitted with hydrodynamic, hydrostatic, or rolling bearing elements that have helped increase the life of machines and reduce maintenance costs by orders of magnitude. Nevertheless, even in the best designs as much as 10-30% of the energy input is still consumed in simply overcoming friction.
The additional energy burden imposed is not the only negative effect of friction. The additional energy that must be put into a system is primarily dissipated as heat. Ancient man used this to his advantage to start fires by rubbing sticks, and we of the modern world use the same principle to light a match. However, in most instances the additional heat is highly undesirable for some obvious reasons. Namely, the heat must be removed to avoid damage to the machinery being operated. Most often this is the bottleneck to achieving higher operating speeds. Additionally, some of the energy is dissipated in various deformation processes. This results in wear of the sliding surfaces and to their eventual degradation to the point where replacement of whole components becomes necessary. This imposes yet another economic liability because without sliding friction these surfaces would not wear and would not require replacement and maintenance.

1.2 Objectives

In this work, several objectives have been set. As this thesis will concentrate on frictional properties of ultra-high molecular weight polyethylene as used in artificial orthopedic joints, the first objective is to give the reader a thorough account of the development of artificial joints. The initial attempts of replicating nature’s joints were certainly crude. Nevertheless, from the very beginning the initial pioneers understood the necessity of reducing friction and of creating a “slippery” interface if the joint was to succeed. The early designs evolved as new biocompatible materials became available and principles of friction, wear, and lubrication were incorporated.
Secondly, a compliant journal bearing is introduced as an alternative to the current rigid bearing used in artificial joint implants. The objective is to show that such a bearing can be beneficial at reducing friction and the resulting wear. The spherical geometry of a joint has been simplified to cylindrical 2-D geometry (fig. 1.1). The dynamic friction of a compliant bearing and a rigid bearing are compared. Each bearing is tested with different degrees of lubrication, loads, maximum velocity and frequency. The friction is measured using an apparatus that has been designed specifically for this purpose.

**Figure 1.1** Angular compliant and rigid bearings as tested.
A very important phenomenon that is observed in dynamic friction studies is the presence of hysteresis. The third objective is to investigate the presence and development of hysteresis using high viscosity 5W and 10W oils. Hysteresis is investigated as a function of frequency for both the rigid and compliant bearing.

In addition to dynamic friction experiments of a journal bearing, friction studies of a fixed roller sliding on a flat steel plate were conducted. Five material specimens of plastics and metals were used for the linear friction studies. The fourth objective is to compare the dynamic friction properties of all five materials. The experiments were conducted for both lubricated and unlubricated contact.

1.3 Outline

Chapter 2 defines friction and the difference between static and dynamic friction. It also introduces the reader to ultra-high molecular weight polyethylene as a friction reducing bearing material. An account of the development of artificial joints closes off the chapter. The account is necessary in order to establish the motive for the experiments conducted in this thesis.

In chapter 3, the principles of hydrodynamic lubrication are discussed. The Striebeck curve, which plots friction coefficient as a function of viscosity, speed, and load, is introduced. Some models that have been used to explain the state of lubrication in natural joints are also discussed in this chapter.

Chapter 4 deals with the subject of compliant bearings. Previous attempts of introducing compliant materials to artificial joints are discussed. The advantages of using a compliant bearing outlined.
In chapter 5, the design of the apparatus used for the experiments is covered in detail. The chapter covers the design of the apparatus itself, as well as the controlling software and hardware that allow for the data acquisition and control. An analysis of the calculation of the friction torque is included.

The test protocol is completely outlined in chapter 6. Justification is given as to the frequency, lubrication, velocity, and load chosen for the measurements. The reliability and repeatability of the results are also covered in this chapter.

The results are presented in chapter 7. The rigid and compliant bearing are compared. The chapter is organized according to the lubricant used. The chapter begins by presenting the results when no lubricant was used, then proceeds to show results for 104 oil, 5W, and 10W oils.

In chapter 8, the presence of hysteresis is explored. The rigid and compliant bearing are tested with 5W and 10W oils. Three sets of results are presented for low frequency, medium frequency, and high frequency. The graphs show the development of hysteresis as frequency is increased. Overall comparisons are presented in chapter 9.

In chapter 10, measurements of dynamic sliding friction of a fixed roller on a steel plate are shown. Ultra-high molecular weight polyethylene, nylon 6,6, bronze, aluminum, and steel were tested with and without lubricant. The design of the apparatus used is also discussed.

The entire work presented here is summarized in the final chapter. In chapter 11, conclusions are also given. Several thoughts on future study are included. The work concludes with design and material issues that were not addressed in the paper, which deserve mention.
CHAPTER 2
FRICION AND THE DEVELOPMENT OF THE ARTIFICIAL ORTHOPEDIC JOINT

2.1 Coefficient of Friction

The coefficient of friction, $f$, is commonly used to quantify friction. The coefficient of friction is the ratio between the friction force, $F$, and the load, $N$:

$$f = \frac{F}{N} \quad (2.1)$$

![Figure 2.1 Load on a horizontal surface with horizontal force, $F$, applied.](image)

The friction force is the tangential force that must be overcome so that one solid contacting body will slide over another. It acts in the opposite direction of the relative velocity of the surfaces.

It is important to note that the coefficient of friction is not a material property. There are many mechanisms involved in generating the friction force. The friction force depends on such factors as the measuring conditions, surface roughness, presence or absence of oxides or absorbed films, and loading, as well as many other factors (ASM 1992). Therefore, values for the coefficient of friction are given for certain material pairs.
under specified conditions. It is generally accepted that values measured by different laboratories will differ by 20 – 30 % of each other (ASM 1992).

2.2 Static and Dynamic Friction

Early studies of friction were mainly concerned with reducing friction and wear of machine components operating at steady velocities. Once started, such components operate at a relatively constant velocity until the unit is shut down. For such static conditions, it suffices to identify a friction coefficient for the particular operating condition of the component. However, there are many practical applications in which the sliding speed and load periodically vary with time, and the static friction models as described above are inadequate. Under dynamic conditions the frequency of oscillation, velocity, and acceleration affect the friction. These effects manifest into hysteresis and friction induced vibrations. In addition, a discontinuity is observed at the point of velocity reversal. This is of particular importance in applications where precise motion control is desired. In particular, at low velocities stick-slip friction is highly undesirable. All of these factors cause errors that throw control systems off-course and reduce precision.

Various dynamic friction models have been proposed that can take into account all these effects. However, relative to static friction, studies into dynamic friction are sparse. Most experimental studies of dynamic friction vary velocity with time, but do not include velocity reversals. Ting (1993) studied the friction of an automobile piston ring and accounted for velocity reversal. Benabdallah (1997) considered sliding friction in reciprocating dry planer contact between steel and three thermoplastics. Harnoy et al
(1994) designed a dynamic friction measuring apparatus, and Cohn (1998) studied dynamic friction for precise motion control. A review of the literature has revealed that a study of dynamic friction in a lubricated journal bearing, as conducted in this thesis, is warranted.

2.3 Friction Reducing Bearing Materials

Until the advent of plastics, the study of friction and bearing materials comprised largely of metal alloys and brake liners. More recently this study has been extended to include thermoplastics. Thermoplastics are well suited to bearing and gear applications for several reasons. Using injection molding techniques complex parts can be easily and economically manufactured. Additionally, thermoplastic composites are able to absorb shock and vibration and operate with less power and noise, and, in certain applications, offer the benefit of greater weight reduction when compared to metal alloys. (ASM 1992)

Of special interest in this group is ultra high molecular weight polyethylene (UHMWPE). A major application of UHMWPE as a bearing material is in orthopedic implants (fig. 2.2). Well known for its low friction and wear resistance characteristics, UHMWPE is the dominant polymeric material in total joint replacement such as knees and hips. Its biocompatibility makes it ideally suited for use inside the body.
2.4 History and Development of the Modern Total Hip Replacement Joint

The earliest known replacement of a joint was performed in 1891 by the German surgeon Theophilus Gluck. He replaced a diseased hip joint with an ivory ball and socket held in place with cement and screws. Two years later, a French surgeon named Emile Pean replaced a shoulder joint with an artificial joint made of platinum shafts joined by a hard rubber ball. It is not surprising that these early attempts quickly failed. Gluck's hip popped out due to the load it had to support, and Pean's shoulder joint soon failed due to infection (Pinchot 1985). These early attempts marked the beginning of man's efforts to replace diseased joints with artificial implants designed to replicate the functions of those given to us by nature.
It was quickly realized that a metal capable of withstanding the forces encountered in motion would have to be found for artificial joints to become feasible. In addition, the metal would have to be biocompatible to avoid infection. In 1938, the American surgeons W.G. Stuck and C.S. Venable developed an alloy of cobalt, chromium and molybdenum, which they called Vitallium, that met the requirements. Today, Vitallium, stainless steel, and titanium alloys are the primary metal compounds used in artificial joints (Pinchot 1985).

That same year, Phillip Wiles designed and introduced the stainless steel total replacement hip joint. Early designs of replacement used a metal-on-metal prosthesis in which both the femoral head and acetabular cup were made of stainless steel. This close fitting metal-on-metal design of cup and head was characterized by high friction. The friction played a major roll in loosening the cup, which was screwed into the pelvis. Failure usually occurred within one year. Use of Vitallium in place of stainless steel by G.K. McKee, improved the success rates to 50% between the years of 1956 - 1960. The introduction of methyl-methacrylate as a cement by G.K. McKee and his colleague Watson-Farrar really pushed the success rates of such implants. (ASM 1992)

The durability of implants essentially depends on the components remaining tight and fixed in place. The use of cement as a bonding agent ensured a secure fit that McKee believed could be expected to last 10 to 20 years. Despite the apparent initial success, McKee recognized that use of identical metals in metal-on-metal designs is unsound from a tribological point of view. However, the use of identical metals is essential in order to avoid electrochemical corrosion.
McKee pioneered the use of metal-on-metal. However, it was the work of Professor Sir John Charnley that led to the success of the modern replacement joint. When one of his patients, who had been fitted with a metal-on-metal replacement joint, came to him with a squeaky joint he realized the importance of reducing the friction at the interface. The frictional torque generated in metal-on-metal designs played a large role in loosening the components. Charnley studied lubrication in natural and in total replacement synovial joints and concluded that synovial fluid was a remarkable lubricant in natural joints, but was not at all suited at reducing friction and wear on any materials being used at the time for total hip replacement. In his own words:

"... the only chance of success in lubricating an artificial animal joint would be by using surfaces which were intrinsically slippery on each other; in other words, self-lubricating irrespective of whether tissue fluid were present or not." (Quoted in ASM 1992)

To employ his belief, in 1959 Charnley turned to polytetrafluoroethylene (PTFE, or Teflon) as the cup material and stainless steel for the femoral head. The very low friction properties of this material led to remarkable initial success. However, PTFE's very poor wear characteristics inevitably caused long term failures as the femoral head wore into the softer acetubular cup. Total failure occurred within 2 or 3 years. In 1961, Charnley replaced PTFE with then newly available ultrahigh molecular weight polyethylene (UHMWPE). His new design has proven very successful and still remains the primary design of choice. (ASM 1992)

Ceramics have also been used in prosthetic implants. Aluminum oxide ceramic femoral heads in combination with polyethylene cups have found increasing application. In both laboratory tests and in vivo a decrease of 50% in the wear rate of polyethylene
over steel has been reported (Dowson 1980). The apparent success of the ceramic head design lead to the introduction of a ceramic-on-ceramic joint, where the UHMWPE cup is replaced with a ceramic cup. However, failure due to fatigue and surface fracture has raised concerns. Ceramic-on-ceramic designs require an exceptional surface finish and precise manufacturing to ensure that critical tolerances are met. Surgical implantation of the all-ceramic joint is made more difficult by the necessity to maintain proper alignment. In addition, the strength requirements must be carefully considered during the design. (Walter and Plitz 1984) (Mahoney and Dimon 1990)

The use of cement in place of screws, UHMWPE, ceramics, and metal alloys with super fine surface finishes have led to the remarkable success of orthopedic joint implants and is, undoubtedly, one of the medical wonders of the 20th century. However, while these materials do alleviate the suffering of patients who receive them, increase their mobility, and are reliable for quite some time, there are still problems associated with these implants. Over time, wear debris generated by the articulating motion is released into the area surrounding the implant. Although UHMWPE is biocompatible, a foreign body response is initiated against the wear debris. Awakened by the presence of the debris, in time the body begins to also attack the cement used to hold the joint fixed in place. The end result is loosening of the joint and total failure, requiring replacement (Pappas et al, 1996). Younger recipients can almost certainly be expected to outlive the life of the implant. Additionally, with the average life span increasing and the increased likely-hood that recipients will outlive the working life span of the joint there is no doubt that in the future many patients will find themselves needing multiple surgeries. An obvious reason for this is that implants as they are today do a very poor job of mimicking
the joints given to us by nature. Replacement joints are very rigid. Certainly, the lubrication is inferior, and there is nothing to absorb load and cushion impact. There is no doubt that if substantial improvements are to be made future implants will have to be more like the natural joint.
CHAPTER 3
UHMWPE AS A FRICTION REDUCING BEARING SURFACE

3.1 Lubrication Regimes

Machine components operating under sinusoidal conditions in which motion reverses direction and the velocity passes through zero are subject to higher wear than components operating under steady unidirectional velocity. A certain minimum velocity is required to build up a hydrodynamic film capable of supporting loads that keep the asperities of the articulating surfaces separated. At the onset of motion, when velocity is lowest, the friction and wear between surfaces is determined by the properties of the material surfaces and by the properties of the lubricant rather than bulk viscosity (fig. 3.1a). It is in this region that the highest potential for wear can be found. After a certain velocity is reached, thin-film lubrication ensues. Friction and wear in this region is determined by the viscosity of the fluid, as well as the properties of the material surfaces as in boundary lubrication (fig. 3.1b). If velocity is further increased, full hydrodynamic lubrication is achieved (fig. 3.1c). It is most desirable to operate in this region since the surface asperities are fully separated by the fluid film, and wear will, therefore, be lowest.

![Lubrication regimes](image)


**Figure 3.1** Lubrication regimes.
A graphical depiction of these distinct regions is shown in the Stribeck curve (fig. 3.2), which plots friction coefficient versus the dimensionless number $\mu n/P$, where $\mu$ is the dynamic viscosity, $n$ is the speed (RPM for journal bearings), and $P$ is the load per unit of projected area. Generating a Stribeck curves aids in determining the state of lubrication in a system in which there is relative sliding between surfaces. It also provides a way of seeing how the friction coefficient changes as a function of velocity, load, or viscosity.

![Graph of friction coefficient versus $\mu n/P$](image)

**Figure 3.2** Plot of friction coefficient, $f$, versus dimensionless variable $\mu n/P$. (From Harnoy 2002)

### 3.2 Dry Friction of Ultrahigh Molecular Weight Polyethylene

Polymers are unique in that the static friction is lower than dynamic friction in the absence of lubrication when sliding against a hard surface such as steel. As the velocity
increases, so does the friction coefficient. Metals operating under similar conditions experience the opposite effect (figures 3.3 and 3.4).

**Figure 3.3** Steel on steel without lubrication. (From Cohn 1998)

**Figure 3.4** Steel on UHMWPE without lubrication.

An interfacial film, commonly called a third body, forms between the sliding surface and the interface. The formation and transfer of these films results in a lower coefficient of friction and reduced wear. The details of the film formation process have
yet to be well defined. However, it is apparent that the initial surface roughness of the metal interface plays an important role in the wear characteristics (Benabdallah 1997).

3.3 Lubrication in Natural and Artificial Prosthetic Joints

From a tribological perspective, the operating conditions of artificial prostheses are conducive to accelerated wear. The reciprocating swinging motion of the leg means that the velocity will pass through zero, where friction is highest, with each cycle. The degree of wear and probability of failure is largely dependent on the state of lubrication present in the joint. In present designs, the maximum velocity reached in an artificial joint is not sufficient or sustained long enough to support full hydrodynamic lubrication. Various theories have been suggested in an attempt to explain the fluid film regime present in the natural joint under strokes of such low velocity and duration (Higginson, 1978 and Dowson, et al, 1986). However, the state of lubrication in artificial joints is yet to be fully understood. Under normal activity, much of the motion associated with joints is of low velocity and frequency. In artificial joints, this means that lubrication is characterized mostly by boundary or at best mixed-film lubrication. In contrast, natural, synovial joints are characterized by a mixture of fluid-film and mixed lubrication, attributing to the low friction coefficient found in natural joints. The compliant cartilage coating on the bone is largely responsible for extending the fluid regime.

The synovial fluid provides lubrication in the natural joint. It is highly non-Newtonian, exhibiting very high viscosity at low shear rates, but is only slightly more viscous than water at high shear rates (Dowson 1986). A more complete discussion on synovial fluid may be found in chapter 5.
Dowson and Jin (1986) have attempted to explain by analytical methods the state of lubrication in nature's bearings. In their work, they couple overall elastohydrodynamic theory with a study of the local, microelastohydrodynamic action associated with surface asperities. For the purpose of analysis, the synovial joint is modeled as a cylinder on a flat plate (fig. 3.5). Elastohydrodynamic theory considers that under the pressures induced in the fluid, the bearing surfaces deform elastically and enhance the bearing’s ability to maintain a fluid film between the surfaces and hold them apart. Their analysis indicates clearly that microelastohydrodynamic action smoothes out the initial roughness of cartilage surfaces in the loaded junctions in articulating synovial joints.

Higginson (1978) offered a similar analysis to explain the phenomenal load bearing ability and low friction properties of synovial joints. He concluded that it is
highly unlikely that a fluid film could be generated by conventional rolling/sliding elastohydrodynamic lubrication. Instead, under fluctuating loads a squeeze film mechanism separates the layers of healthy cartilage. The most important factor in this mechanism is the flexibility of the cartilage. During the cycle, there is a period in which the load is very close to zero. In this period a thick film of lubricant finds its way between the surfaces.

Figure 3.6 Diagram of a typical natural joint

In natural joints, articular cartilage is attached to the bone surfaces. This cartilage is elastic and porous. The elastic properties of the cartilage allow for some compliance that can aid in extending the fluid film regime. This is in contrast with artificial joints, which are very rigid and consequently exhibit poor lubrication in which ideal separation of the surfaces does not occur. Contact of the plastic-metal surfaces increases the friction and leads to wear. The problem is compounded by the fact that synovial fluid in implants is much less viscous than that which is found in healthy natural joints (Cooke et al., 1978). Even the best artificial joints exhibit a friction coefficient that is 4 to 5 times higher than a healthy natural joint (Pearcy 1988). As previously stated, this is largely due to the natural joint operating in the fluid film regime. Therefore, extending the fluid film regime in artificial joints should have the effect of reducing friction and minimizing wear.
The use of compliant bearings is not a new idea. The most common compliant-surface bearing is the foil bearing (fig. 4.1). Foil bearings were first introduced in 1953 by Block and Van Rossum. Compliant bearings can offer several advantages over standard rigid bearings. Precise machining and maintenance of tight tolerances is not as essential for satisfactory operation. In addition, compliant bearings are able to tolerate misalignment and are capable of maintaining a fluid-film at low speeds.

![Angular compliant-foil bearing. A flexible foil can stretch around bearing.](Harnoy 2002)

**Figure 4.1** Angular compliant-foil bearing. A flexible foil can stretch around bearing. (Harnoy 2002)

### 4.1 Previous Work Using Compliant Bearings for Prosthetic Implants

The notion of adding some compliant layer to the surfaces of artificial joints dates at least as far back as the first prosthetic designs. Early in the 20th century, Finnish surgeon K.E. Kellio attempted to reproduce the nearly frictionless surface of natural joints by drawing skin over the head of an implanted femur. At the same time, others were striving for the
same effect by adding sheets of fat, fibrous tissue called fascia, gold foil, nylon and glass (Pinchot 1985).

More recently, an attempt to better copy the natural joint was undertaken by the Bioengineering Research Group of the University of Durham. They introduced elasticity by adding polyurethane compliant surface linings at the contact, thus imitating the cartilage present in natural joints. Initial testing revealed that the design proved useful in improving artificial joints. Namely, the friction coefficient was lowered to values very close to that of natural joints. This clearly showed that introducing some elasticity in the joint allows the fluid film to develop sufficiently to separate the material surfaces. An issue of concern in the Durham design was the prospect of damage and wear to the surface linings immediately after surgery when lubrication is known to be minimal. Operating under dry non-lubricated conditions could subject the linings to high shear stresses that over time could tear the lining away from the substrata (Pearcy 1988). As the squeaky metal-on-metal joint witnessed by Charnley demonstrated, lubrication by synovial fluid is not necessarily guaranteed once the natural joint has been removed. More than ten years have passed since the Durham investigation, yet the design has not been implemented in artificial joints. This might be attributable to the difficulty in attaching the layers to the substrata and durability issues that arise under lack of lubrication.

4.2 Proposed Use of Compliant Bearing

An alternative approach that has been the subject of study for this research is the addition of a compliant layer behind a UHMWPE lining (fig. 4.1). This design would combine
the desirable properties of UHMWPE of durability and low friction at the contact with the flexibility of an elastic backing that can provide some angular compliance capable of extending the fluid film regime. The advantages of compliant or “hybrid” bearings in machinery operating at high speeds have already been established. A thorough investigation of an angular-compliant bearing subject to dynamic loading has been covered by Harnoy and Rachoor (1993).

With angular compliance the surface of the socket can have small elastic angular flexibility due to elastic shear deformation. (Harnoy, Technical Proposal 1999) This is

![Proposed design of artificial hip joint with UHMWPE and compliant backing.](image)

**Figure 4.2** Proposed design of artificial hip joint with UHMWPE and compliant backing.

particularly significant for low speed oscillating motion. In fact, relative sliding motion between surfaces of very small angle limb movements, such as “twitches,” could be completely eliminated. It is such small motion that largely contributes to wear because
the motion does not allow a fluid film capable offering some degree of separation of the surfaces to develop. The angular compliance of the elastic substrate would completely absorb the motion and minimize wear incurred under dry conditions.

The principle can be demonstrated using one’s hands by making a fist with one hand and then cupping it with the other. To simulate the motion in the current design, the cup hand is held stationary while the fist hand is slightly rotated. It will be noticed that sliding contact between the surfaces is experienced. On the other hand, if the cup hand is allowed to roll or rotate with the fist hand it will be noticed that there is no longer sliding between the surfaces. It is a logical conclusion that given such motion at least part of the motion between surfaces can be completely eliminated, and consequently wear can be reduced.

4.2.1 Reduction of Stresses

There are many mechanisms that lead to failure of artificial joints. Among them are abrasive wear, adhesive wear, third body wear, and fatigue. Abrasive wear results when there is direct contact between surfaces. A weld can form, resulting in adhesive wear, when similar materials sliding against each other are used. Third body wear is the result of hard debris material wedged between articulating surfaces. However, it is wear caused by fatigue that is a major mode of failure of joint prostheses. Incongruent alignment of components results in a relatively small contact area between cup and ball, which leads to higher contact stresses (Canonaco and Pappas 1993). The rubber bed has the effect of allowing the UHMWPE to better conform to the ball surface, resulting in some measure of “self-alignment,” thus enlarging the contact area and lowering stresses.
Elevated contact stresses have been linked to damage of the articulating surfaces. The highest stresses are incurred about one millimeter below the surface near the center of area of contact. During rotation, the highest point of stress moves along the bearing’s inner surface. Cracks will begin forming just below the surface if the peak stress is higher than the fatigue strength of the bearing material. The cracks can lead to pitting or splitting of the bearing surface into several layers. (Canonaco and Pappas 1993)

It has been shown that contact stresses are a function of the cup insert thickness as well as the condition of the contact surfaces (Kurtz 1997). Current prostheses commonly use a metal backing with a UHMWPE insert as the acetabular component. A metal backing offers a more biocompatible surface for bone ingrowth. Clinical reports show favorable results of the use of metal backings. Analytical studies indicate reduction of cement stresses and even distribution of load in the acetabulum in metal-backed acetabular components. However, other reports show increased wear of metal-backed cups compared to all-polyethylene cemented components. Using three-dimensional, planar-symmetric finite element models, Kurtz (1997) has studied the effects of backside polishing, cup angle, and polyethylene thickness on the contact stress. His work revealed that there is a strong association between contact stresses and polyethylene thickness. The results indicate unequivocally that contact stresses are reduced as the thickness of polyethylene is increased. This is due to the polyethylene’s considerably lower elastic modulus (974 MPa for PE compared to 200 Gpa). The equations for contact stress are a function of the modulus of elasticity. As E is decreased the contact stresses also decrease. The softer UHMWPE allows for some deformation, spreading the load over a larger area. As thickness is increased, the area available for the distribution of load
increases. A more compliant elastomeric layer than PE should amplify this effect, reducing stresses much like a soft mattress reduces high pressure points. Kurtz’ work also showed that improving the surface finish and lowering friction coefficient had relatively very little effect on lowering contact stresses when compared to changing thickness and load angle.
CHAPTER 5
DYNAMIC FRICTION MEASURING APPARATUS

5.1 Previous Studies of Friction of UHMWPE
As previously mentioned, the friction coefficient is dependent on the particular system being tested. Many different configurations have been applied to study the tribological behavior, and to evaluate prosthetic implant materials and lubricants. Among these are the pendulum and pin-on-plate setups. Pin-on-plate configurations are commonly chosen, primarily for their simplicity. However, these rigs cannot take into account the rolling motion present in rotating systems such as the hip. Many friction studies also do not account for the oscillatory reciprocating motion found in many practical applications. In such setups, it is difficult to study the static to dynamic friction regime. In reciprocating components, this is the most crucial region of the cycle since it is at the start-up that the highest friction magnitude is found. The step function encountered at zero velocity also poses problems for systems in which precise motion control is desired, as the high friction throws the control off course.

Hip joint pendulums that measure decay in amplitude or pendulum acceleration have been used to indirectly determine friction. Other pendulums that measure the friction torque directly have also been used (O’Kelly et al 1977). Some of these systems allow for reciprocating motion and dynamic loads to be applied that more closely resemble the actual physiological state. Nevertheless, a survey of the literature reveals that many experiments are conducted using pin-on-disc or similar setups. Pendulum machines with actual artificial joints mounted in place are ideal for evaluating final
designs. However, in such systems it is difficult, if not impractical, to quickly study new concepts and materials that have yet to be incorporated into a final prosthetic design.

5.2 Hardware of Dynamic Friction Measuring Apparatus

To test the feasibility and to study any potential benefits that could be gained by using a compliant bearing, a test apparatus capable of measuring dynamic friction in a journal sleeve bearing was constructed (fig. 5.1). A simple journal bearing design allows the critical rolling motion to be studied. Human joints are subjected to oscillatory sliding conditions with frequent start-ups. The reciprocating motion of the shaft closely resembles the swinging motion of the leg.

The experimental test system consists of a mechanical test apparatus, a personal computer, a data acquisition and control interface board, and analog signal amplifiers and DC power supplies. A DC servo-motor is used to drive the shaft of the mechanical apparatus by a no-slip cogged timing belt and pulleys. An IBM compatible, 486DX-33, computer controls the motion of the servo-motor. This is accomplished through a C program written by Amin (1996) with a LabWindows graphical user interface that allows the user to specifically define the control parameters. The interface allows the user to vary maximum and minimum velocities, frequency, velocity wave, and time of run. The program precisely monitors and controls shaft velocity through information sent and received by an encoder mounted on the motor. Changes in velocity caused by friction are sensed and adjusted by increasing or decreasing the motor’s torque by adjusting the voltage to the motor. A more detailed description of the control used can be found in Amin (1996, 1997) and Cohn (1998).
An IBM Data Acquisition and Control Adapter (DACA) board routes all input and output from the computer. The shaft velocity, voltage signal from the load cell, time, and shaft position are stored in a data file. The data is later retrieved for conversion and plotting (see appendix A).

![Diagram](image)

**Figure 5.1** Schematic of friction measuring components
5.3 Design of Friction Measuring Apparatus

In the design of the apparatus, consideration was given to minimizing the effects of inertial forces and to isolating friction sources. Under dynamic conditions inertial forces introduce errors by adding an unaccounted force to the measurement. Also, it is necessary to isolate the friction at the journal bearing from the friction in the roller bearings that support the shaft.

![Exploded view of friction measuring test apparatus.](image-url)

Figure 5.2 Exploded view of friction measuring test apparatus.

The test apparatus includes a main support frame, shaft-bearing assembly, bearing loading rod, DC servo-motor with encoder, load cell, and a lubrication reservoir. A one-inch diameter steel shaft is supported on each end by rolling bearings, which are mounted to the main support frame. The shaft passes through two sleeve bearings that are pressed
Figure 5.3 Cross section of apparatus.
into two bearing carriers. The sleeves can be pressed out for replacement. The servomotor drives the test shaft with a pre-determined voltage waveform, which induces a specific shaft motion.

For this test, the sleeves where machined from a rod of ultrahigh molecular weight polyethylene. The outer sleeve diameter is 1 1/4 inches with an interior diameter of one inch and a length of 0.6 inches. The bearing carriers are fixed inside an outer bearing housing with setscrews. Load is applied and adjusted by hanging weights from a rod passed through the bearing housing. The bearing-housing assembly is, therefore, pulled down. The shaft supports the housing, dividing the load between the two bearings. Some damping by means of rubber washers is provided to the rod to eliminate vibrations that could be transmitted to the bearing housing by motion of the loaded rod. The total friction torque in the two bearings is measured by a load cell mounted parallel to the direction of the radial load applied on the bearings. The load cell also prevents the rotation of the bearing housing. In this manner the friction in the journal bearings is isolated from the friction of the rolling bearing. The signal from the load cell is filtered to reduce noise. A lubricating reservoir is mounted above the mechanical apparatus in order to supply lubricant by gravity directly into the two bearings through segments of flexible tubing. The lubricant squeezed out of the bearing is collected below the housing.

5.4 Mounting of Rigid Bearing

In order to simulate the acetubular cup of current bearing designs of orthopedic implants, a UHMWPE sleeve bearing is pressed into the bearing carrier (fig. 5.4). This rigidly fixes the bearing in place. The rotating shaft passed through the bearing simulates the
motion of the femoral head. This model simplifies the 3-D spherical geometry of the hip with 2-D cylindrical geometry. Unlike pin on disk setups, this model makes it possible to study the important rolling and sliding contact motion present in the hip.

![Diagram of Hub carrier configuration with UHMWPE bearing.

**Figure 5.4** Hub carrier configuration with UHMWPE bearing.

### 5.5 Mounting of Compliant Bearing

The compliant bearing was fabricated by molding a layer of silicone rubber in place in a second set of bearing carriers. Silicone was chosen because it is readily available and easily molded. When dry, the silicone will give the desired flexibility. It is realized that silicone rubber is not suited for use in the body; however, with the use of silicone the concept of a flexible compliant bearing can be easily tested.

As the silicone rubber dried, it bonded to the steel carrier. A smaller steel ring machined with grooves on the backside was used to mount the sleeve through which the
shaft would pass (fig. 5.5). Initially the silicon was bonded directly to the polyethylene sleeve. However, the silicone rubber soon peeled away from the polyethylene. The use of a thin, grooved steel outer sleeve around the polyethylene provided a more ideal surface for the silicone to bond to. By using the outer metal sleeve, it is also possible to remove the bearing without requiring a new silicone rubber layer to be cast each time. A replacement bearing can be simply pressed into the bearing carrier.

![Diagram of bearing carrier and compliant UHMWPE bearing](image)

**Figure 5.5** Bearing carrier and compliant UHMWPE bearing

### 5.6 Calculation of Frictional Torque

The following analysis is used to determine the frictional torque. The frictional force, $F_p$, is measured at a distance, $R_p$ from the center of the shaft to the center of the load cell. The tangential force, $F_t$, can be calculated from the following relation
The calibration value used to convert the voltage signal from the piezoelectric load cell to force is 0.875v/lbs. The following equation is used to calculate the coefficient of friction.

\[ f = \frac{2F_t}{W} \]  

(5.2)

where \( W \) is the total weight of the load, housing, and bearing carriers. The tangential force, \( F_t \), is multiplied by two, since the total load is shared by each of the two bearings.
CHAPTER 6

TEST PROTOCOL FOR EXPERIMENTAL MEASUREMENT
OF FRICTION COEFFICIENT

Most often, dynamic friction is not what is shown in the literature. Most friction plots found in the literature fail to show friction characteristics during the deceleration portion of the cycle through the reversal of velocity. Instead, the friction is measured at a constant velocity. The friction coefficient at that particular velocity is obtained and recorded. This procedure is repeated for a series of velocities, and the friction coefficient is then plotted versus velocity to obtain a curve. The graphs shown here are unique in that the graph reveals the friction curve for the entire cycle, which can reveal the presence or absence of hysteresis.

6.1 Repeatability of Results

It has been stated throughout the literature that measurement of the friction coefficient is not exactly repeatable. Plots of friction curves often show scattered points through which a curve has been fitted. Most often, the friction coefficient is a statistical average of multiple runs. However, in this study it was found that measurement of the friction coefficient is indeed quite repeatable.

A test protocol was established in order to verify that friction values measured with the equipment were reliable, accurate, and repeatable. It was recognised that the shaft is not perfectly straight. The friction curve can vary from run to run depending on where the cycle is begun relative to shaft and bearing alignment. To overcome this, the shaft and bearing were precisely aligned at the start of each run. A minimum of five runs
for each set of parameters (velocity, frequency, load, and lubricant) was conducted. Each plot was superimposed on the same graph. It was found that each plot was very nearly identical.

![Graph showing friction coefficient versus velocity for UHMWPE](image)

**Figure 6.1** High frequency plots taken one week apart. Dots and crosses depict testing on different days.

UHMWPE is quite sensitive to changes in temperature. Even under small changes in temperature, the bearing sleeves can expand or contract significantly. This can cause changes in the clearance and change the total contact area between shaft and bearing. This can also affect the fit between bearing and bearing carrier. The same set of tests was repeated on different days to determine whether variations in temperature and humidity in the room incurred from day to day could cause discrepancies in the measurements.

Lastly, the apparatus was completely disassembled and reassembled to determine if this would alter the measurements. In all cases, except for disassembly of the
apparatus, it was found that the friction coefficient and friction curves were not affected by any of the aforementioned conditions as long as all components were precisely aligned for each set of tests. When the apparatus was disassembled and reassembled, slight variations of the friction coefficient were sometimes observed. To overcome this limitation, great care was taken during reassembly to ensure that components were aligned.

The measured friction was found to be within the range of values found in the literature, ranging from a high just above 0.3 to a low of less than 0.05. Exact values of the friction coefficient are difficult to compare with published values since the measured friction coefficient will vary with the system used for testing. Also, such factors as degree of lubrication, surface finish, loading, and sliding velocity will affect the measurements. All measured values compare well with published values.

6.2 Lubrication

Many different lubricants have been used in friction and wear tests of prosthetic joint materials. Among these are distilled water, synovial fluids from diseased and healthy joints, saline, bovine fluids, and synthetic silicone oils. All of these lubricants have been used in an attempt to simulate the lubrication found in joints.

In this study, tests were conducted under three modes of lubrication: dry, low viscosity, and high viscosity oil. Dry experiments simulate a worst case scenario, in which the joint is deprived of lubricant. Lack of lubrication has been reported to occur immediately following implantation (Pearcy 1988). These tests also reveal the static and kinetic friction characteristics of UHMWPE. In addition, dry coefficient of friction
values are more easily compared to published values because it eliminates the effects of viscosity and can be applied more universally.

Low viscosity, lightweight 104 oil was used to simulate synovial fluid. It is recognized that 104 oil is not suitable for use in the body. This oil was chosen because it has a viscosity (table 6.1) in the range of bovine synovial fluid and synovial fluid removed from joints that have undergone arthroplasty. Its low viscosity offers very little benefit in building a fluid film. In fact, it is only a marginal improvement over water alone. With respect to synovial fluid, 104 oil is readily available and no special precautions are necessary for handling or storing.

Studies of synovial fluid removed from patients who have had an artificial joint implanted reveal a viscosity similar to that of synovial fluid taken from patients with rheumatoid arthritic joints. Synovial fluid removed from rheumatoid arthritic joints exhibits a lower viscosity than healthy and osteoarthrosic joints. The viscosity of fluid removed from osteoarthrosic joints falls between that of rheumatoid and healthy joints (Cooke et al, 1978). Table 6.1 summarizes the viscosity of synovial fluids as measured by Cooke et al (1978). The table also includes other lubricants that have been used in published experiments. When subjected to shear rates between $10^{-1} - 10^3 \text{ s}^{-1}$, the listed ranges of viscosity are obtained. It should be noted that no universal agreement of viscosity values has been established in the literature. Recorded values can vary by at least an order of magnitude depending on the source. As such, it is a reasonable assumption that the use of the lightweight oil will facilitate the analysis of the two bearings in a system that resembles the joint with a fluid of similar viscosity.
In addition to the lightweight oil, higher viscosity 5W and 10W oil was also used. The higher viscosity of the 5W and 10W oils also falls within values of lubricants that have been used in published studies. The heavier weight oil is of similar viscosity of healthy synovial fluid. It is expected that the greatest reduction of friction at the startup will be noticed when the compliant bearing is used in combination with the higher viscosity oil. With higher viscosity oil, the fluid film builds up faster than with low viscosity oil.

Table 6.1 Viscosity of different joint lubricants. Values for synovial fluids taken from graphs of Cook et al (1978).

<table>
<thead>
<tr>
<th>Lubricant</th>
<th>Viscosity (Pa-s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Healthy Synovial Fluid</td>
<td>0.05 - 5</td>
</tr>
<tr>
<td>Osteoarthritis Arthritis Synovial fluid</td>
<td>0.05 - 0.5</td>
</tr>
<tr>
<td>Rheumatoid Arthritis Synovial fluid</td>
<td>0.01 - 0.05</td>
</tr>
<tr>
<td>Bovine Synovial Fluid</td>
<td>0.001 - 0.05</td>
</tr>
<tr>
<td>Synovial Fluid From Joint Arthroplasty</td>
<td>0.008 - 0.5</td>
</tr>
<tr>
<td>Silicone Fluid (F111)</td>
<td>0.5</td>
</tr>
<tr>
<td>LW 104 @ 25 °C</td>
<td>0.007</td>
</tr>
<tr>
<td>5W @ 25 °C</td>
<td>0.03</td>
</tr>
<tr>
<td>10W @ 25 °C</td>
<td>0.07</td>
</tr>
<tr>
<td>Water @ 25 °C</td>
<td>0.0009</td>
</tr>
</tbody>
</table>

According to hydrodynamic lubrication theory, for a short bearing, the pressure generated is a function of both viscosity and velocity (Harnoy 2002):

\[
p(\theta, z) = \frac{3\mu U}{RC^2} \left( \frac{L^2}{4} - z^2 \right) \frac{\varepsilon \sin \theta}{(1 + \varepsilon \cos \theta)^3}
\]  \hspace{1cm} (6.1)
where $\varepsilon$, the dimensionless eccentricity ratio, is defined as the ratio of the journal eccentricity $e$ to the circumferential clearance, $C$, between the shaft and bearing:

$$\varepsilon = \frac{e}{C}$$  \hspace{1cm} (6.2)

Figure 6.2 is used for reference to define the variables. When all other variables are held constant, from equation 6.2 it can be seen that the higher viscosity will generate a greater pressure.

6.3 Frequency Settings

The hip, like the reciprocating engine and other common machine components, is subject to oscillatory motion. The oscillations cause the sliding components to have a zero relative velocity at the point of velocity reversal. Studies have been conducted to capture the range of motion in the hip. Though the range of motion can vary greatly depending on the activity, studies show a total of 40° of flexion-extension is experienced by a
healthy person while walking (Stevens et al. 1979 and Broome 1978). A shaft frequency that would give the required 40° of flexion-extension was used to simulate walking.

Frequencies below and above that experienced during walking were also tested. A low frequency of 0.25 rads/s was used to determine the friction characteristics under full hydrodynamic lubrication. At this low frequency, the presence or absence of hysteresis could also be noted. Although this low frequency does not have much relevance to hip motion, it is applicable to many machine components.

At low frequency, it is not expected that there will be any significant decrease of the friction coefficient with the compliant bearing. The greatest difference is expected when there is maximum compliance of the silicone rubber behind the UHMWPE. There should be maximum compliance when the torque exerted by the shaft is highest and when the friction coefficient is highest, so that there is no slip between shaft and sleeve. The torque will be highest when the shaft acceleration is greatest. Additionally, at higher velocity there will be greater friction between the shaft and sleeve. The increase in friction will cause the sleeve and the silicone rubber layer behind it to rotate with the shaft. A frequency of 12 rads/s was used to test this principle.

A limitation of the test device is that at high frequency there is a lag between the velocity measured at the encoder and the actual velocity of the shaft. This is due to the elasticity of the rubber belt used to drive the shaft. This effect is seen on the graphs of plots taken at high frequency. In figure 6.3, an example of this effect is shown. In these plots, it can be seen that the zero is shifted to the right and left of the vertical axis. To
overcome this limitation, the friction coefficient at high frequency is also plotted as a function of time rather than velocity. In the future, the velocity will be measured directly on the shaft passing through the bearings rather than on the motor.

### 6.4 Velocity

The shaft velocity used was determined based on the frequency of oscillations. For low frequencies, a high velocity of 0.58 m/s was chosen. The high velocity ensured that full hydrodynamic lubrication was achieved. At higher frequency of oscillations a much lower velocity was used. This was done to give the desired displacement angle between shaft rotation and bearing. The minimum velocity used to simulate hip motion was 0.07 m/s.
6.5 Load Settings

Published values of contact stresses in a typical hip prosthesis estimate an average contact stress of 3.45 MPa and maximum contact stresses of 6.90 to 10.35 MPa (McKelpp et al 1981). The maximum pressure applied for this study was approximately 0.5 MPa when a static load of 215 N was applied to each of the two bearings (see appendix B). With minor modifications, the apparatus could be made to handle greater loads more comparable to those found in the hip. The reduction in the coefficient of friction of UHMWPE with increasing load has been reported (Benabdallah 1997). However, Unsworth et al (1975) reported that the effect of varying the load between 150-1500 N had very little effect on the dry friction coefficient (0.08 – 0.098) of a Charnley joint. Varying loads up to 215 N were applied to measure the friction coefficient as a function of load. It is expected that the effect of the compliant layer will be greater under higher loading. The increase in compliance will act to reduce friction and wear.
CHAPTER 7

EXPERIMENTAL RESULTS OF DYNAMIC FRICTION OF UHMWPE JOURNAL BEARING

7.1 Dry Friction

7.1.1 Rigid Bearing

Initially, a series of tests were conducted to establish the dynamic friction characteristics of a reciprocating UHMWPE journal bearing subjected to a sinusoidal velocity wave. It is well documented in the literature (Benabdallah 1997) that some thermoplastic components under dry conditions exhibit a lower coefficient of friction at the start-up than at higher velocities. The experiments conducted here confirm this.

For a rigid bearing under a constant load of 215 N, and constant stress of 0.5 MPA, the curve in figure 7.1 was obtained in the absence of lubrication. Each point on the graph represents an instant of time for which friction torque and velocity were recorded by the computer. The minimum friction is seen to occur shortly after startup with a coefficient of approximately 0.2. As sliding velocity increases, the friction coefficient also increases. A maximum friction coefficient of 0.31 is measured at the point when the maximum velocity is reached. As the shaft begins decelerating, the friction decreases. A mirror image of the plot is produced when the shaft reverses direction. The high velocity chosen extends the curve so that a more complete understanding of the friction trend can be obtained.

The friction plot in figure 7.2 shows the friction coefficient as a function of time for the same conditions as in figure 7.1. The maximum friction measured during each
cycle is approximately 0.31. The maximum friction is reached at the point of maximum velocity.

Figure 7.1 Friction plot as a function of velocity for rigid bearing at low frequency without lubrication. Frequency = 0.25 rads/s, max velocity = ± 0.57 m/s, load = 215 N.

Figure 7.2 Friction plot as a function of time for rigid bearing at low frequency without lubrication. Frequency = 0.25 rads/s, max velocity = ± 0.57 m/s, load = 215 N.
Comparing figures 7.3 and 7.4, it can be seen that changing the frequency has no apparent effect on the maximum friction coefficient. For both 0.25 and 4 rads/s, the maximum friction coefficient reached is approximately 0.28.

**Figure 7.3** Friction plot as a function of time for rigid bearing at frequency of 4 rads/s without lubrication. Max velocity = ± 0.285 m/s, load = 215 N.

**Figure 7.4** Friction plot as a function of time for rigid bearing at frequency of 0.25 rads/s without lubrication. Max velocity = ± 0.285 m/s, load = 215 N.
Figures 7.5 and 7.6 plot the friction coefficient for the same parameters as in figures 7.3 and 7.4 as a function of velocity. Here, it can be seen that the friction coefficient is lower when the shaft is decelerating than when the shaft is accelerating. A drift to the right and left of zero is now present. The first cycle starts at zero, but on the return it is shifted to the right and left. The drift is caused by the difference in measurement of the velocity on the encoder and the velocity of the shaft. At high frequency, the elasticity of the drive belt causes the discrepancy in measurement.

![Friction plot as a function of velocity for rigid bearing at frequency of 4 rads/s without lubrication. Max velocity = ± 0.285 m/s, load = 215 N.](image)

**Figure 7.5** Friction plot as a function of velocity for rigid bearing at frequency of 4 rads/s without lubrication. Max velocity = ± 0.285 m/s, load = 215 N.
Figure 7.6 Friction plot as a function of velocity for rigid bearing at frequency of 0.25 rads/s without lubrication. Max velocity $= \pm 0.285$ m/s, load $= 215$ N.

In figures 7.7 – 7.9, the maximum shaft velocity has been reduced to 0.07 m/s. At a frequency of 4 rads/s, this velocity gives a shaft displacement of 90° in each direction. As expected, the maximum friction coefficient has decreased to approximately 0.25. Looking at figure 7.8, it will be noticed that the zero drift to the right and left of zero on the x-axis has been greatly reduced by reducing the velocity. Hysteresis is also observed. Whether this is caused by the elasticity present in the system or is an actual characteristic of the friction is not clear.

The plot in figure 7.9 reveals the effects of reducing the load to 110 N. Compared to 215 N, a slight increase in the friction coefficient is observed. Additionally, the drift to the right and left of the zero is more pronounced with the lighter load. It also appears that
there is less slope to the curve as the shaft accelerates. The maximum friction is reached at a lower velocity.

**Figure 7.7** Friction plot as a function of velocity for rigid bearing at frequency of 4 rads/s without lubrication. Max velocity = ± 0.07 m/s, load = 215 N.

**Figure 7.8** Friction plot as a function of time for rigid bearing at frequency of 4 rads/s without lubrication. Max velocity = ± 0.07 m/s, load = 215 N.
Figure 7.9 Friction plot as a function of velocity for rigid bearing at frequency of 4 rads/s without lubrication. Max velocity = ± 0.07 m/s, load = 110 N.

7.1.2 Compliant Bearing

Testing at low frequency of oscillation indicates that the friction coefficient without lubrication of the two designs is very nearly the same. There is no apparent change in friction coefficient between figures 7.1 and 7.10. The only noticeable difference in the two plots is the presence of hysteresis in the compliant bearing. Figure 7.11 shows the comparison between the rigid and compliant bearing at a frequency of 12 rads/s and a max velocity of ± 0.07 m/s. Similarly, there is no significant difference in friction between the two designs. The conclusion is that the compliant bearing offers no significant reduction in friction when unlubricated. These results are as expected in the absence of lubrication.
Figure 7.10 Friction plot as a function of velocity for compliant bearing at low frequency without lubrication. Frequency = 0.25 rads/s, max velocity = ± 0.57 m/s, load = 215 N.

Figure 7.11 Comparison between compliant and rigid bearing at high frequency. Frequency = 12 rads/s, max velocity = ± 0.07 m/s, load = 215 N.
7.2 Dynamic Friction Characteristics with Low Viscosity 104 Oil

Under dry conditions, it was found that there was little reduction of the friction coefficient in the use of a compliant bearing for all frequencies and velocities tested. However, it was not expected that in dry conditions there would be a significant reduction of the friction coefficient. The greatest reduction should occur in the presence of a lubricant. The compliant layer allows the bearing sleeve to move with the shaft at the start of the shaft motion, when, in the presence of a lubricant, the friction is highest. This gives time for the fluid film to develop and separate the surfaces before there is relative sliding between surfaces. When the shaft does begin to slide relative to the sleeve, it does so with a greater velocity than it does when it starts from rest for the rigid bearing. Furthermore, the effect should be greater yet for higher viscosity lubricants.

7.2.1 Rigid Bearing

For the next series of tests, lightweight 104 oil was introduced to the rubbing surfaces. Figure 7.12 shows the effect of adding the lubricant. As expected, the coefficient of friction is high at the start-up and then begins decreasing as velocity is increased. At the beginning of the cycle, a peak in friction is occurs. This peak value reaches 0.25. This effect, called sticktion, was noticed when the load was applied for a period of time before any motion of the shaft was induced. With time, the fluid is squeezed out of the clearance. Adhesion forces take effect and the two surfaces tend to stick together. For the rest of the cycles, this peak is no longer present. The friction coefficient ranges from a high of 0.15 at the start-up to a low of 0.075 when maximum velocity is reached. Figure 7.12 can be compared to figure 7.6 to see the effects of adding lubricant.
Figure 7.12  Rigid bearing with 104 oil. Frequency = 0.25 rads/s, max velocity = ±0.285 m/s, load = 215 N.

Figure 7.13  Rigid bearing with 104 oil. Frequency = 4 rads/s, max velocity = ±0.285 m/s, load = 215 N.
In figure 7.13, the velocity and load have the same values as in figure 7.12, but the frequency has been increased from 0.25 to 4. The maximum and minimum values are nearly the same, but the graph has been pulled open near the zero. It can be concluded that frequency has little effect on the maximum and minimum friction values.

In figure 7.14, the load and frequency have been held constant as in figure 7.13, but the maximum velocity has been decreased to $\frac{1}{4}$ that of figure 7.13. The plot has closed in near the zero. The maximum friction coefficient has increased to nearly 0.2. This could be the result of decreased acceleration at the start-up. The minimum friction is comparable to that reached at the same velocity in figure 7.13. There also appears to be some hysteresis present. The friction on deceleration is lower than the friction on acceleration. The friction coefficient holds the minimum value reached, 0.015, until the shaft reaches zero velocity and reverses direction.

Figure 7.14 Rigid bearing with 104 oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 215 N.
In figures 7.15 – 7.17, the effect of decreasing load has been examined. Varying only the load, figures 7.15 – 7.17 have the same frequency and velocity as figure 7.14. It appears that a decrease in load results in slightly higher friction. In addition, the graph opens up at the zero as the load is decreased. The initial start-up friction is most greatly affected by the decrease in load. At the lowest load, 71 N, the initial friction upon starting is approximately 0.32. It also appears that the curve has a greater slope as load is increased. That is, the friction coefficient reduces more quickly as the load is increased.

Figure 7.15 Friction plot of rigid bearing during simulated walking velocity and frequency with 104 oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 71 N.
**Figure 7.16** Friction plot of rigid bearing during simulated walking velocity and frequency with 104 oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 124 N.

**Figure 7.17** Friction plot of rigid bearing during simulated walking velocity and frequency with 104 oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 159 N.
In figure 7.18, the effects of high frequency have been explored. The limitations of the test apparatus of accurately measuring shaft velocity at high frequency become evident in this plot. There is a large shift to the left and right of the zero. It is difficult to draw conclusions about the friction characteristics throughout the cycle. However, the maximum and minimum values are clearly evident. At such high frequency of oscillation, lubrication is poor. This is reflected in the high values. Nevertheless, it appears that the maximum and minimum values are not too different than those recorded for lower frequency and same velocity. When plotted versus time (fig. 7.28), a more accurate depiction of the friction is obtained.

![Figure 7.18](image)

**Figure 7.18** Rigid bearing with 104 oil. Frequency = 12 rads/s, max velocity = ± 0.06 m/s, load = 215 N.
7.2.2 Compliant Bearing with 104 Oil

The plot in figure 7.19 was obtained with the compliant bearing. Compared to the rigid bearing (fig. 7.12), the maximum and minimum friction coefficients decreased to 0.125 and 0.05, respectively. The initial friction has also decreased from 0.25 to 0.18.

![Graph showing friction coefficient vs. velocity](image)

**Figure 7.19** Compliant bearing with 104 oil. Frequency = 0.25 rads/s, max velocity = ± 0.285 m/s, load = 215 N.

In figure 7.20, the frequency has been increased to 4. Like the rigid bearing, it appears that increasing the frequency has no effect on the maximum friction coefficient. However, the minimum friction at the maximum velocity has increased with respect to figure 7.19. At higher frequency, the minimum friction has increased from 0.05 to 0.09. As in the rigid bearing, the graph has been pulled open near the zero.
Figure 7.20 Compliant bearing with 104 oil. Frequency = 4 rads/s, max velocity = ± 0.285 m/s, load = 215 N.

In figure 7.21, the maximum velocity has been reduced to \( \frac{1}{4} \) that of figure 7.20. The zero drift has been eliminated. The maximum friction is slightly higher than in figure 7.20. The minimum friction coefficient has increased to 0.125. For the maximum velocity reached in figure 7.21, 0.07 m/s, the friction in figure 7.20 at 0.07 m/s is equal.

In figure 7.22, a single cycle of figure 7.21 has been plotted. This plot emphasizes the high start-up friction at the initial onset of motion. The profile of the first cycle is quite different than the rest of the cycles. This is exactly as expected. The first cycle is very similar to the cycles in the rigid bearing (fig. 7.14). The lubricant is completely squeezed out, so the effect of the compliant layer does not manifest. Once the lubricant is brought to the surfaces by the rotating shaft, the effect of the compliant bearing comes into play. The initial peaks at the start-up of each cycle have been
eliminated. This observation is more important than the actual difference in friction coefficient, for it shows the effect of the compliant bearing.

**Figure 7.21** Friction plot of compliant bearing during simulated walking velocity and frequency with 104 oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 215 N.

**Figure 7.22** Friction plot of first cycle of compliant bearing during simulated walking velocity and frequency showing high start-up friction on first cycle with 104 oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 215 N.
When the load is decreased (fig. 7.23), it can be seen from the graph that the compliant bearing acts more like a rigid bearing. Again, this is as expected. Under light load, there is very little compliance of the rubber backing. As the load is increased, there is more compliance by the rubber backing. This effect lowers the initial high start-up friction. The result is that the overall friction coefficient is lower throughout the cycle, although the magnitude of maximum friction might be similar. This effect is expected to be amplified as load is further increased.

**Figure 7.23** Friction plot of compliant bearing during simulated walking velocity and frequency with 104 oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 71 N.
Figure 7.24 Friction plot of compliant bearing during simulated walking velocity and frequency with 104 oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 124 N.

Figure 7.25 Friction plot of compliant bearing during simulated walking velocity and frequency with 104 oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 159 N.
Figure 7.26 has frequency and velocity set to values that simulate walking. As for the rigid bearing, this plot is useful primarily to show the peak and minimum friction coefficient experienced at high frequency.

![Friction plot of compliant bearing during simulated walking velocity and frequency with 104 oil. Frequency = 12 rads/s, max velocity = ± 0.07 m/s, load = 215 N.](image)

**Figure 7.26** Friction plot of compliant bearing during simulated walking velocity and frequency with 104 oil. Frequency = 12 rads/s, max velocity = ± 0.07 m/s, load = 215 N.

### 7.2.3 Comparison Between Compliant and Rigid Bearing With 104 Oil

A comparison between the rigid and compliant bearing is more clearly made when the two bearings are plotted together versus time. Figure 7.27 is a combination of figures 7.12 and 7.19 plotted versus time. From the figure, it can be seen that with a lubricant of low viscosity the compliant bearing can significantly reduce the friction coefficient under high frequency of oscillation. On deceleration, in particular, the friction is reduced by as much as 30 percent.
A significant reduction in friction is also evident at higher frequency of oscillation. Figure 7.28 is generated when figures 7.18 and 7.26 are combined and plotted versus time. Again, the decrease is greater on deceleration.

**Figure 7.27** Friction vs Time plots for compliant and rigid bearings with 104 oil. Load = 215 N, Frequency = 0.25 rads/sec, max velocity = ± 0.285 m/s.

**Figure 7.28** $f$ vs time for rigid and compliant bearings with 104 oil. Load = 215 N, Frequency = 12, max velocity = ± 0.07 m/s.
Figure 7.29 f vs time for rigid and compliant bearings with 104 oil. Load = 215 N, Frequency = 4, max velocity = ± 0.07 m/s.

Figure 7.30 Compliant bearing Comparison between a load of 71N and 215N. Frequency = 4 rad/s, max velocity = ± 0.07 m/s.
7.3 Dynamic Friction Characteristics with 5W Oil

7.3.1 Rigid Bearing

In the third set of experiments, 5W oil was used. Using heavy weight, high viscosity oil the fluid film develops more quickly. Figure 7.31 reveals that compared to when the 104 oil was used (fig. 7.12), the friction of the rigid bearing drops off more rapidly and at a lower velocity with the 5W oil. In addition, the minimum friction reached is considerably lower with the 5W oil. These results are as expected. An interesting effect is the hysteresis loops that appear below 1 m/s. At speeds above 1 m/s, the hysteresis loops disappear and the friction curve becomes nearly a straight line. The hysteresis loops are well defined and symmetrical with respect to the velocity axis. The loops are not as pronounced when the lightweight oil was used for the same frequency and velocity.

At higher frequency and velocity, it was not possible to obtain meaningful plots. Therefore, the plot at a frequency = 4 and velocity = 10 are not included. When the frequency was increased and the velocity decreased (fig. 7.32), the minimum friction increased to 0.06, compared to 0.015 at the higher velocity and lower frequency (fig. 7.28). The start-up friction, however, remained the same at 0.11. It can also be seen that the first positive cycle is quite different in figures 7.31 and 7.32.

The effects of increasing torque are realized in figure 7.33 by increasing the frequency to 12. The plot looks very similar to the plots obtained at high frequency with lightweight and no oil. The characteristic drift caused by error in velocity measurement is clearly evident. At this frequency, the plot is very similar to that obtained with the lightweight 104 oil. The one discernable difference is that the slope of the friction on acceleration is greater with the heavier oil.
Figure 7.31 Rigid bearing with 5W oil. Frequency = 0.25 rads/s, max velocity = ± 0.285 m/s, load = 215 N.

Figure 7.32 Rigid bearing with 5W oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 215 N.
Figure 7.33 Rigid bearing with 5W oil. Frequency = 12 rads/s, max velocity = ± 0.07 m/s, load = 215 N.

Figure 7.34 Compliant bearing with 5W oil. Frequency = 0.25 rads/s, max velocity = ± 0.285 m/s, load = 215 N.
7.3.2 Compliant Bearing

The effect of the compliant backing is most clearly visible with the heavy oil. As expected, the peak friction is reduced by the compliant backing. At low frequency (fig. 7.34), a plateau appears at low velocity. This plateau is the effect of the compliant layer. Beyond the plateau, the friction is more or less equal to the rigid bearing (fig. 7.31).

When the frequency is increased (fig. 7.35), there is no longer any slope to the curve. The maximum velocity is reached shortly after the start-up and remains constant until the shaft begins decelerating. The maximum friction reached is approximately equal to the minimum friction of the rigid bearing, 0.07. The hysteresis loops are also more clearly evident in the compliant bearing. Of particular interest is that the curve of the first cycle of figure 7.35, where the fluid film has been completely squeezed out, is very similar to the first cycle of figure 7.32. This is further evidence that the compliant bearing can reduce friction only if there is sufficient lubrication present.

At higher frequency (fig. 7.37), the compliant bearing has been especially beneficial in reducing friction. A plateau is quickly reached after start-up and maintained until the shaft begins decelerating. With the rigid bearing (fig. 7.33), the minimum friction is reached at the maximum velocity. The friction starts at approximately 0.15 and slopes downward to 0.09. When the compliant bearing was used, the minimum friction was reached very quickly after the start-up. Figure 7.33 is plotted again in figure 7.38. Comparing figures 7.37 and 7.38, the reduction can be clearly seen. The figures are plotted together as a function of time in figure 7.39.
Figure 7.35 Compliant bearing with 5W oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 215 N.

Figure 7.36 $f$ vs time for rigid and compliant bearings with 5W oil at walking frequency. Load = 215N, Frequency = 4 rads/s, max velocity = ± 0.07 m/s
Figure 7.37 Compliant bearing with 5W oil. Frequency = 12 rads/s, max velocity $= \pm 0.07$ m/s, load $= 215$ N.

Figure 7.38 Rigid bearing with 5W oil. Frequency = 12 rads/s, max velocity $= \pm 0.07$ m/s, load $= 215$ N.
Figure 7.39 $f$ vs time for rigid and compliant bearings with 5W oil at high frequency. Load = 215N, Frequency = 12 rads/s, max velocity = ± 0.07 m/s.
CHAPTER 8

Hysteresis with 5W and 10W Oil

In traditional static friction models for lubricated surfaces, the friction is represented as a continuous curve. The friction is high near zero velocity, quickly drops and then levels off again as velocity increases. As stated previously, dynamic friction experiments can reveal that the friction is actually higher during acceleration than on deceleration. Generally, this phenomena, called hysteresis, can be observed if the lubricant viscosity and frequency are high enough. Hysteresis is a result of the time delay associated with the changing thickness of the hydrodynamic fluid film. The presence of hysteresis has been dealt with theoretically in dynamic models, such as that of Harnoy and Friedland (1993) and Rachoor and Harnoy (1996). The following chapter deals with the presence of hysteresis with changing frequency for a constant load and equal velocity.

8.1 Tests with 5W and 10W Oil

The experiments were conducted using 5W and 10W oil for both the rigid and compliant bearings. The tests were conducted at three different frequencies. A low frequency of 0.01 rads/s was chosen so that a 'quasi-static' condition could be achieved. A higher frequency of 0.25 rads/s was chosen so that dynamic friction and a fully developed hydrodynamic fluid film were ensured. Lastly, a frequency of 1 rad/s was used to study the effects of increased acceleration and deceleration. Higher frequencies than 1 were not used so that the zero drift experienced at higher frequency would not complicate the comparisons.
8.1.1 Rigid Bearing with 5W Oil

As expected, at low frequency and quasi-static conditions hysteresis is not observed (fig. 8.1). The peak friction occurs at start-up at zero velocity. The friction quickly slopes downward as the shaft accelerates and a fluid film builds. At approximately 0.12 m/s, the curve levels out and the friction coefficient remains steady at approximately 0.02 until maximum velocity is reached. As the shaft decelerates, the friction follows exactly the same path it took while accelerating. Peak friction is again reached at zero velocity. A large step is observed as the shaft reverses direction. A mirror image is obtained as the shaft accelerates in the opposite direction.

In figure 8.2, the frequency has been increased to 0.25 rads/s. Hysteresis is now clearly visible as large loops in the sloped region of the curve. On acceleration, the friction curve takes very nearly the same path as in figure 8.1. However, it can be seen that at approximately 0.8 m/s the curve breaks away, depicting a lower coefficient of friction.
friction as the velocity decreases. Again, a nearly identical mirror image is produced as the shaft motion reverses direction. The very high start-up friction is a result of the fluid film having been squeezed out while the shaft was static prior to testing.

Figure 8.2 Rigid bearing with 5W oil at frequency = 0.25 rads/s.

Figure 8.3 Rigid bearing with 5W oil at frequency = 1 rad/s.
In figure 8.3, the frequency of oscillations has been increased further to 1 rad/s. The hysteresis loops are more prevalent than at 0.25 rads/s. The downward slope during acceleration is more gradual, leveling out at 0.2 m/s, very close to the maximum velocity. On deceleration the friction curve is nearly flat. The friction coefficient maintains about the minimum value of 0.02 until it decelerates to almost 0 m/s. The trend is repeated in the opposite direction. It is also noteworthy that the maximum friction coefficient has decreased from approximately 0.15 in the quasi-static case to 0.07 at higher frequency.

### 8.1.2 Compliant Bearing with 5W Oil

The friction curve in figure 8.4 for the compliant bearing at low frequency is very similar to the curve for the rigid bearing at 0.01 rads/s (fig. 8.1). This was expected. As was previously described, the compliant bearing offers no advantage at low frequency. The one peculiarity is the two distorted loops that appear at approximately 0.1 m/s. These loops are most likely the result of damage that has occurred to the bearing that manifests at the tested frequency.

The friction curve in figure 8.5 for the compliant bearing is very similar to the curve in figure 8.2 for the rigid bearing. As shown in chapter 7, the compliant bearing offers no advantage over the rigid bearing at a frequency of 0.25. The hysteresis loops are present below 0.1 m/s. The peak friction after the initial start-up for the compliant bearing is equal to that of the rigid bearing.

In figure 8.6, as frequency is increased, a difference between the rigid bearing and compliant bearing begins to emerge. Like the rigid bearing, the hysteresis loops are larger than for the lower frequency plots. The peak friction after the initial start-up has
been decreased to 0.05 from 0.07 in figure 8.3. Like the rigid bearing in figure 8.3, the hysteresis loops begin to form at approximately 0.15 m/s on deceleration.

**Figure 8.4** Compliant bearing with 5W oil at frequency = 0.01 rads/s.

**Figure 8.5** Compliant bearing with 5W oil at frequency = 0.25 rads/s.
8.1.3 Rigid Bearing with 10W Oil

As with the 5W oil, it can be seen in figure 8.7 that at low frequency no hysteresis is present. In fact, the plot for 10W (fig. 8.7) oil is very similar to the plot for 5W oil in figure 8.1. As frequency is increased, the hysteresis loops become very prevalent (fig. 8.8). Compared to the 5W oil in figure 8.2, the hysteresis loops are much more pronounced. The friction is also observed to be lower.

In figure 8.9, the frequency has been increased to 1 rad/s. On deceleration it appears that the friction remains nearly constant at 0.02 until the shaft stops and reverses direction. The start-up friction has been reduced to a 0.05. In addition, the hysteresis loops extend further to the maximum velocity. Hysteresis begins to be seen at 0.18 m/s. The step at direction reversal has reduced considerably, and the discontinuity is no longer present.
Figure 8.7 Rigid bearing with 10W oil at frequency = 0.01 rads/s.

Figure 8.8 Rigid bearing with 10W oil at frequency = 0.25 rads/s.
8.1.4 Compliant Bearing with 10W Oil

Not surprisingly, the friction plot in figure 8.10 for the compliant bearing at low frequency does not look too different than the friction plots for the other low frequency plots. No hysteresis is observed. It is interesting that the small loops seen in figure 8.4 are not present in 8.10.

Comparing the rigid bearing in figure 8.8 and the compliant bearing in figure 8.11, it is evident that the start-up friction very near the zero velocity has been cut off. The compliant bearing exhibits a gradual upward slope very near zero that extends to ± 0.05 m/s. As the shaft velocity approaches ± 0.05 m/s, the maximum friction is reached. As the shaft velocity increases past ± 0.05 m/s, the friction begins to resemble that for the rigid bearing in figure 8.8. The friction slope decreases sharply until a shaft speed of ± 0.10 m/s, after which the friction remains constant. On the return, the friction breaks away from the accelerating friction at ± 0.10 m/s and hysteresis is observed.
The friction plot in figure 8.12 is very different from the same plot for the rigid bearing (fig. 8.9). The hysteresis loops are not as evident. However, the friction throughout the cycle is considerably lower than that of the rigid bearing. It appears that the friction curve for the compliant bearing takes the path of the friction curve for the rigid bearing on deceleration. The step at the start up has been greatly reduced and spread apart. The plot is also very different from the plot of the compliant bearing with the lighter viscosity 5W oil of figure 8.6. The plot of the compliant bearing with 5W oil resembles more the plot of the rigid bearing with 10W oil. It seems that the greatest effect of the compliant bearing has been achieved at the highest frequency tested with the highest viscosity oil.

![Friction vs. Velocity Plot]

**Figure 8.10** Compliant bearing with 10W oil at frequency = 0.01 rads/s.
Figure 8.11 Compliant bearing with 10W oil at frequency = 0.25 rads/s.

Figure 8.12 Compliant bearing with 10W oil at frequency = 1 rad/s.
Figure 8.13 f vs time for rigid and compliant bearings with 10W oil. Load = 215N, Frequency = 1 rad/s, max velocity = ± 0.285 m/s.
CHAPTER 9

COMPARISON OF RIGID AND ANGULAR COMPLIANT BEARING

The experimental results in chapters 7 and 8 were presented as a function of evolving frequency. In the following chapter, the most notable experimental results are presented again so that a comparison can be made more conveniently. The friction plots presented in this chapter emphasize the reduction of the start-up friction. In all the graphs, it will be noticed that the start-up friction has been cut by application of the compliant backing. On each page are the measured friction plots under identical conditions.

Figure 9.1 represents a fair estimation of the lubrication, frequency, and velocity that might be expected in a hip joint while walking. Even with such low viscosity oil, a substantial reduction of start-up friction has been achieved. Figures 9.2 through 9.4 are for higher viscosity 5W oil. In particular, graph 9.3 shows substantial reduction of friction in simulated walking conditions. The last set of graphs, figures 9.5 and 9.6, are for 10W oil. As expected, the high viscosity oil results in the greatest reduction of start-up friction. No comparisons have been made for conditions without lubrication or very low frequency, as it was previously shown that under such conditions there is no discernible difference in friction plots between the angular compliant bearing and the rigid bearings.
Figure 9.1 Friction plot of rigid and compliant bearings during simulated walking velocity and frequency with 104 oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 215 N.
Figure 9.2 Rigid and compliant bearing with 5W oil. Frequency = 0.25 rads/s, max velocity = ± 0.285 m/s, load = 215 N.
Figure 9.3 Rigid and compliant bearing with 5W oil. Frequency = 4 rads/s, max velocity = ± 0.07 m/s, load = 215 N
Figure 9.4 Rigid and compliant bearing with 5W oil at frequency = 1 rads/s.
Figure 9.5 Rigid and compliant bearing with 10W oil at frequency = 0.25 rads/s.

a. Rigid bearing

b. Compliant bearing
Figure 9.6 Rigid and compliant bearing with 10W oil at frequency = 1 rad/s.

a. Rigid bearing

b. Compliant bearing
CHAPTER 10

FRICITION STUDIES OF SLIDING LINE CONTACT

In addition to the rotating journal bearing studies of the previous chapters, measurements of dynamic friction in sliding line contact were also conducted. Except for the main test apparatus, the controlling hardware is the same as that used for the journal bearings. Reference should be made to chapter 5 for further details. Modifications were made to the Matlab program to calculate the appropriate applied load and to calibrate the sliding velocity. A schematic of the setup is shown in figure 10.1. In figure 10.2, the test apparatus

![Diagram](image)

**Figure 10.1** Schematic of friction compensating velocity control and dynamic linear friction measuring apparatus.
Figure 10.2 Linear dynamic friction measuring apparatus.

Figure 10.3 Detail of load cell and cylindrical test specimen location.
itself is displayed. A detail of the location and configuration of the test specimen and load cell is shown in figure 10.3.

The test cylinder measures two inches long and has a diameter of 1 inch. The cylinder is removable to allow for testing of different material specimens. A removable steel plate is fixed to a 21-inch long bed. A belt driven worm gear below the bed moves the bed linearly from right to left. The maximum achievable stroke length is 16 inches. The load is placed on top of the cylinder housing. The sliding bed can be filled with a lubricant or left dry. A load cell measures the friction force. In this set-up the friction force at the sliding interface is isolated from other friction sources in the system, such as the friction at the worm gear. One of the unique features of this apparatus is that through its gear reduction extremely stable low velocities can be achieved. This allows for studies to be made in the boundary and mixed film regime.

10.1 Evaluation of Bearing Materials

In this study, five bearing materials sliding on steel were evaluated. Cylinders of UHMWPE, Nylon 6,6, bronze, aluminum, and steel were tested without lubrication and with 5W oil. A maximum load of 25 lbs. was applied. For the UHMWPE specimen this results in a maximum pressure of 0.39 MPa (see Appendix C). As was done for the journal bearing, the motor was driven by a sinusoidal velocity wave, so that the bed accelerated, decelerated, and reversed direction with each cycle.
10.2 Plastics

10.2.1 Friction Measurements of UHMWPE

The curve in figure 10.4 shows the friction for a UHMWPE cylinder sliding on a fixed steel plate without lubrication and a light load of 33 N. Predictably, we see from the plot that the friction rises as sliding velocity increases. However, it is unexpected that under light load a high degree of hysteresis can be observed. As the sliding speed decreases the friction is lower than that on acceleration. The friction continues decreasing until zero velocity is reached and the direction reversed.

When the load is increased (fig. 10.5), hysteresis is no longer prevalent. The maximum friction appears to be approximately the same, 0.1, for both the light and heavy load. Under the heavier load, the gap between accelerating and decelerating friction has closed.

In figure 10.6, the effect of adding 5W oil is seen. As expected, in contrast to dry sliding, as in figures 10.4 and 10.5, the dynamic friction is lower than the static friction. The maximum friction is still approximately 0.1; however, with oil the maximum friction occurs at the beginning of the cycle rather than at the point of maximum acceleration and velocity as in unlubricated sliding. On deceleration, it appears that the friction remains relatively constant until minimum velocity is reached.

It is interesting that in the positive direction the friction on deceleration is higher than on acceleration. This could possibly be caused by the lubricant having been ‘plowed’ out of the way on acceleration. As the cylinder begins decelerating, the oil ahead of the cylinder has not yet had time to fall and make its way back to the contact.
The lack of a fully developed fluid film leads to an increase in the friction relative to acceleration. As will be seen later in the chapter, this phenomenon seems to manifest in many of the other tests performed with lubrication.

**Figure 10.4** UHMWPE on steel sliding friction plot without lubrication and a 33N load applied.

**Figure 10.5** UHMWPE on steel sliding friction plot without lubrication and a 122 N load applied.
10.2.2 Friction Measurements of Nylon 6,6

Figures 10.7-10.9 show friction curves of nylon 6,6 cylinder sliding on a steel plate. Nylon, like UHMWPE, is found as a bearing and friction reducing material in many applications. Compared to figures 10.4 and 10.5, it is seen immediately that nylon exhibits a higher coefficient of friction than UHMWPE. Additionally, it is evident that there is no discernable hysteresis exhibited by nylon under light load (fig. 10.7). The friction at the low velocity tested tends to be rather linear after start-up, reaching a maximum value of 0.15.

In figure 10.8, 5W oil has been added. The maximum friction is achieved shortly after start-up. As expected for lubricated surfaces with a high viscosity oil, the friction
In figure 10.9, the effect of increasing the load is realized. On acceleration, the friction gradually decreases as expected. However, on deceleration, the nearly linear friction as seen in figure 10.6 on deceleration is seen again. A noticeable difference from figure 10.8 is the drift to the right and left of zero velocity at direction reversal.

**Figure 10.7** Nylon 6,6 on steel sliding friction plot without lubrication and a 122 N load applied.
Figure 10.8 Nylon 6,6 on steel sliding friction plot with 5W oil and a 55 N load applied.

Figure 10.9 Nylon 6,6 on steel sliding friction plot with 5W oil and a 122 N load applied.
10.3 Metals

10.3.1 Friction Measurements of Bronze

Bronze is another material that is widely used as a bearing material. Nevertheless, dynamic friction studies of bronze are scarce in the literature. In this study, metals are presented largely for comparison. A more exhaustive future study of metals alone is certainly warranted.

The friction curve for bronze in dry sliding on steel is shown in figure 10.10. At the velocity tested, the friction appears to be linear throughout the cycle after start-up. The maximum friction is quickly reached shortly after start-up and maintained through the cycle of acceleration and deceleration. The friction on deceleration is only marginally less than that on acceleration. It is no surprise that the measured friction is much higher than the plastics.

When 5W oil is added, figure 10.11, the friction is reduced considerably from 0.23 without oil to approximately 0.15 with oil. Some reduction of friction with sliding velocity is expected, but it does not appear to occur. The friction remains relatively linear throughout the cycle. Some hysteresis is present in the positive direction. The friction is higher on deceleration than on acceleration. This is probably caused by the ‘plowing’ effect as explained in the last paragraph of 10.2.1.

It should be noted that the maximum load that could be applied to the metal specimens was considerably less than what could be applied to the plastic specimens. The measured voltage by the load cell was much greater than the ± 5 volts tolerance of the load cell used. For the aluminum and steel used in the following sections this limitation was even more pronounced.
**Figure 10.10** Bronze on steel sliding friction plot without lubrication and a 55 N load applied.

**Figure 10.11** Bronze on steel sliding friction plot with 5W oil and a 100 N load applied.
10.3.2 Friction Measurements of Aluminum

In comparing the plot for aluminum in figure 10.12 and the plot for bronze in figure 10.10, many of the same observations can be made. The curve profile looks very similar to that of the bronze. The sliding friction coefficient also remains relatively constant and is only slightly higher than that of bronze. The constant friction contrasts with that of the plastics, which tends to rise as sliding velocity increases.

When the 5W oil is added, the friction decreases as expected. Some decrease of friction with sliding velocity can be noticed. Lubricated, the sliding friction coefficient of the aluminum is 25% higher than the bronze. The plowing effect previously noted is noticeable when the bed moves in the positive direction.

Figure 10.12 Aluminum on steel sliding friction plot without lubrication and a 55 N load applied.
10.3.3 Friction Measurements of Steel

The final metal to be included in this study was steel. The problem of the load cell signal described in section 10.3.1 was particularly pronounced in the unlubricated steel on steel measurements. Even with no external load applied other than the specimen housing, it was not possible to obtain any curves of the dry steel on steel. Therefore, no graphs for unlubricated steel on steel are included here.

In figure 10.14 5W oil has been added. A decrease in friction as velocity increases is immediately observable. As with the other materials tested with lubrication, the friction on deceleration remains constant. A surprisingly low value of 0.12 is obtained on the return.

In the final figure, 10.15, a very low frequency and velocity has been used. It can be seen that even at such low velocity the curve slopes downward as velocity increases.
As the plate reaches maximum velocity at ± 0.0065 m/s, the friction has decreased to approximately 0.13. As the steel plate decelerates the friction increases, yet it is slightly lower than on acceleration.

**Figure 10.14** Steel on steel sliding friction plot with 5W oil and a 100 N load applied.

**Figure 10.15** Steel on steel sliding friction plot with 5W oil and a 100 N load applied at low velocity
CHAPTER 11
CONCLUSION AND FUTURE STUDY

11.1 Conclusion

The dynamic frictional properties of a rigid and compliant bearing have been established. Published values of the coefficient of friction for prosthetic implants vary greatly. Reported values range from 0.02 to 0.25. The values recorded in this study fall well within this range, depending on the state of lubrication. The coefficient of friction itself does not determine the success of an implant. However, it is apparent that high friction can cause loosening and result in catastrophic failure of the implant.

The apparatus designed for this study is not intended to be a full joint simulator. However, it does make it possible to evaluate a new concept for artificial hip designs. The apparatus allows for measurement of friction with reciprocating motion. In addition, with the journal bearing design it is possible to simulate the critical rolling motion present between ball and socket. The degree of lubrication, frequency of oscillations, and applied load have been varied to study the effect of each on start-up friction.

Reduction of friction during the critical start-up and reversal of motion by the use of a compliant bearing has been clearly established. As expected, a higher viscosity lubricant results in a greater reduction of the start-up friction by the compliant bearing. It was also found that increasing the frequency of oscillations leads to more compliance and to lower start-up friction. Increasing the applied load yields similar positive results.

Today, the use of metal-on-metal implants is again gaining popularity. The results of this study indicate that a compliant bearing might actually be even more beneficial on metal-on-metal applications than on UHMWPE-on-metal. The start-up friction of metal-
on-metal applications is higher and could be more effectively reduced, resulting in more desirable wear characteristics.

11.2 Future Study

Clearly, friction is not the only determining factor that leads to wear. In this study, only friction has been measured. Future work should focus on the degree of wear of both the compliant and rigid bearing. It is believed that the compliant bearing will have an even greater effect on reducing wear than on reducing friction coefficient compared to the rigid bearing. The current test apparatus can be used with only minor modification to test for wear. The most feasible method of measuring wear would be to conduct a volumetric comparison between the bearing of the rigid and compliant designs. Each bearing should be weighed with a sensitive scale before testing and then weighed again after testing. The difference in total weight will reveal the amount of wear that has occurred.

In addition to conducting wear tests, the current apparatus should be modified to yield more accurate friction plots at higher frequencies. This can be achieved by measuring the velocity directly on the driven shaft rather than on the motor as in the current set-up. This would eliminate the elastic effects of the drive belt. The apparatus should also be modified to accept larger loads, comparable to those found in the body.

Future testing could also focus on comparing a compliant metal bearing and a compliant UHMWPE bearing. As previously stated, the compliant layer could be more effective at reducing the start-up friction of a steel bearing.
11.3 Design and Material Issues

Although the feasibility of a compliant bearing has been established, there are design and material issues that would need to be addressed. The problem of attaching the compliant layer to the polyethylene has been partially resolved in this study by adding an intermediate grooved steel ring around the polyethylene. However, a more robust and reliable method of attachment capable of withstanding the cyclical loading that such a bearing would demand is undoubtedly required. Wear tests could help evaluate the effectiveness of attachment methods.

It has also not been established what the thickness of the compliant layer should be. Clearly, a thicker compliant layer would offer more compliance and greater stress reduction. However, dimensional constraints would have to be considered in determining an ideal thickness.

In this study, silicone rubber has been used to create the compliant layer. It is recognized that this material is not suited for biological use due to biocompatibility issues. Although silicone may work well in non biological applications, a material that would offer the required compliance as well as being biocompatible must be found before it could be considered for actual use inside the body.
APPENDIX A
MATLAB CODE FOR CALCULATION AND PLOT OF FRICTION COEFFICIENT

The following matlab code interprets the acquired data and plots the friction coefficient versus time or velocity, as defined by the user.

% control3
% Used for the compliant bearing apparatus
% Separates the variables from the control experiment data
% Plots one graph of friction coeff. vs. velocity
% Program can be modified on line 133 to plot friction coeff. vs. time
% dir *.frc % Lets user know which files are available for loading

if (~exist('datafile')) datafile= 'none'; end; % Sets default filename
prevdatafile= datafile; % Previous entry (if exists)
datafile= input(["Enter the data file name (in single quotes): (', prevdatafile, ') :"]);

if isempty(datafile) % Loads the entered data
    datafile= prevdatafile; % file if not yet done
elseif (~exist(datafile(1,1:size(datafile,2)-4))) % so.
    load(datafile);
end

data=eval(datafile(1,1:size(datafile,2)-4)); % Makes the var. data = to
% the var. created when data
% file was loaded

[m,n]=size(data); % Print the # of samples

if (~exist('cutoff')) % Initialized cutoff to 0 first time control is run
    cutoff=200;
end;
prevcutoff=cutoff;
cutoff=input(["How many steps would you like to cut off? ",num2str(prevcutoff)," ? "]);
if isempty(cutoff) % Makes cutoff=prevcutoff if enter is pressed
    cutoff=prevcutoff; % with no value entered
end;

if (~exist('endcutoff')) % Initialized endcutoff to 0
    endcutoff=0; % first time control is run
end;
prevendcutoff=endcutoff;
endcutoff=input(["How many steps would you like to cut off the end? ",num2str(prevendcutoff)," ? "]);
if isempty(endcutoff) % Makes endcutoff=prevendcutoff if enter is pressed
    endcutoff=prevendcutoff; % with no value entered
end;
if (exist('coeff_offset')) coeff_offset = 0; end;
prevcoeff_offset = coeff_offset;
coeff_offset = input(['Friction Coeff. offset: ', num2str(prevcoeff_offset), ': ']);
if isempty(coeff_offset) coeff_offset = prevcoeff_offset; end;
sizestep = data(m,1)/m; % sample time = total time / # samples
time = data(cutoff:m-endcutoff,1); % All the variables are read in with cutoff
ref = data(cutoff:m-endcutoff,2); % # of steps lopped off the beginning
pos = data(cutoff:m-endcutoff,3);
velrad = data(cutoff:m-endcutoff,4);
u = data(cutoff:m-endcutoff,5);
error = data(cutoff:m-endcutoff,6);
fric = data(cutoff:m-endcutoff,7);
if(n>7) mfric = data(cutoff:m-endcutoff,8); end
[m,n] = size(time);

if (exist('counts')) counts = 1; end; % Prompts for Load Cell Count
prevcounts = counts;
counts = input(['Load in pounds? ', num2str(prevcounts), '? ']);
% total weight added on top of roller housing
if isempty(counts)
counts = prevcounts;
end

if (exist('frequency')) frequency = 0.25; end; % Prompts for frequency
prevfrequency = frequency;
frequency = input(['Frequency in rad/sec? ', num2str(prevfrequency), '? ']);
if isempty(frequency)
frequency = prevfrequency;
end

if (exist('color')) color = 'k'; end;
prevcolor = color;
color = input(['Enter plot color in single quotes - r y g b k - :(', prevcolor, ') :']);
if isempty(color)
color = prevcolor;
end

%%% NOTE: pulley constant not needed in linear friction test
% if (exist('pulleycomp')) pulleycomp = 1; end; % Prompts whether or not to
% prevpulleycomp = pulleycomp;
% pulleycomp = input(['Compensate for new pulleys (1=y; 0=n)? ', num2str(prevpulleycomp), '? ']);
% if isempty(pulleycomp)
% pulleycomp = prevpulleycomp;
% end

% if (pulleycomp == 1) pulley = (3.75/3.6);
% else pulley = 1;
% end;

%%% CALIBRATION DATA %%%%
piezocal = 0.845; % Piezoelectric calibration in [Volts/lb].
arm = 3.6; % Distance from center of journal to piezoelectric cell in inches

velms = velrad * 0.027; % (*pulley / 100) changes velocity (velrad) from [rad/s] to [m/s]

Wo = 3; % weight of outer hub [lbs] Wbc = 5; % weight of the TWO bearing carriers [lbs]
Wr = 2; % weight of rod, nut, washer, and U-clamp [lbs] Wt = Wo + Wbc + Wr; % Combined weight on shaft [lbs]

%Wrh= 2.51; % Weight of silicone housing + weight of rod [lbs]
Nave = counts + Wt; % Total load on shaft [lbs.] Nave = Nave * (9.81/2.2)/2; % Converted to Newtons

mfric = (mfric-2048) * 5 / 2048; % Changes mfric (meas. friction) into a voltage
mfricvoltage = mfric; % Stores meas fric as read at piezo [volts]
mfric = -mfric/piezocal; % Changes mfric to [lbs] (meas. at piezo)
mfric = (arm/0.5) * mfric;

mfcoeff = mfric / (counts + Wt); % Calculates measured friction coeff. f
mfcoeff = mfcoeff + coeff_offset;
convert = ((-piezocal)*(counts+Wo)*coeff_offset);
disp('If you would like to compensate for the coefficient offset at the');
disp([' piezoelectric coupler, move ', num2str(convert), ' Volts']);
disp(');

% mfcoeff = -(mfric/piezocal)*arm/(2*0.5*(counts/loadringcal)); % Calculates the measured friction coefficient

figure(2);
figure(figsize=[8,6]);
h = plot(velms,mfcoeff,'.'); % Plots vel (s) vs. friction coeff. f
h = plot(time,mfcoeff,'.'); % use this to plot friction coeff vs. time (dots)
set(h,'color', color);

axis([-0.15 .15 -.24 .24]); % scale for bi directional
axis([0 .48 0 .24]); % scale for uni directional
grid on;
title(sprintf('f vs. Velocity; Ave. Load =%6.0f N; Freq. = %2.2f rad/sec', Nave, frequency));
title(['Time vs. f; Ave. Load = ',num2str(Nave), ' N; Freq. = ',num2str(frequency), ' rad/sec']);
xlabel('Velocity [m/s]');
ylabel('Friction Coefficient, f');

%%%% Averages the friction data for several cycles for control.exe
%%%% Double %% means that the line is commented out in original program
%%%% This part of the program has been edited out since the average
%%%% friction plot over n cycle is not being used for analysis
%period=2*pi/frequency; % seconds/cycle
%cyclesteps=round(period/sizestep); % (sec/cycle) / (sec/step) = (# steps/ cycle)
%cycles=m/cyclesteps; % # steps/ (# steps/cycle)= # cycles
%if cycles > 2
% wholecycles=fix(cycles); % # whole cycles

% clear mfriccoef; % Clears the var. mfriccoef. Just in case 2nd time

% for i = 1:wholecycles
% mfriccoef(:,i)=mfcoeff((1+(i-1)*cyclesteps):(cyclesteps+(i-1)*cyclesteps),1);
% end % for loop

% mfriccoef=mfriccoef;
% mfcoeffave=mean(mfriccoef);
% mfcoefficient=mfcoefficient;

% figure(3);
% plot(velms(1:cyclesteps),mfcoefficient);
    % axis([-15 .15 -.24 .24]); % scale for bi directional
    % axis([0 .48 .15]); % scale for uni directional
% grid on;

% title(['Velocity vs. Average f for ',num2str(wholecycles), ' cycles. Ave. Load = ',num2str(Nave), ' N; Freq. = ',num2str(frequency), ' rad/sec']);
% xlabel('Velocity [m/s]');
% ylabel('Friction Coefficient, f');

% end %if

% stribek
APPENDIX B

CALCULATION OF HERTZ STRESS FOR RIGID BEARING

The following analysis was used to calculate the maximum stress for the rigid bearing.

Material properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>V</th>
<th>E</th>
</tr>
</thead>
<tbody>
<tr>
<td>UHMWPE</td>
<td>0.46</td>
<td>1.2 Gpa</td>
</tr>
<tr>
<td>Steel</td>
<td>0.3</td>
<td>200 GPa</td>
</tr>
</tbody>
</table>

Equivalent radius:

\[
\frac{1}{R_e} = \frac{1}{R_2} - \frac{1}{R_1}
\]

\[
\frac{1}{R_e} = \frac{1}{25.4} - \frac{1}{25.4254}
\]

\[
\frac{1}{R_e} = 3.933 \times 10^{-5}
\]

\[R_e = 25425.4 \text{ mm}\]

Equivalent modulus of elasticity:

\[
\frac{2}{E_{eq}} = \frac{1 - \nu_1^2}{E_1} + \frac{1 - \nu_2^2}{E_2}
\]

\[
\frac{2}{E_{eq}} = \frac{1 - 0.3^2}{200 \times 10^9} + \frac{1 - 0.46^2}{1.2 \times 10^9}
\]

\[E_{eq} = 3.02 \times 10^9 \text{ Pa}\]
Dimensionless load:

\[ \bar{W} = \frac{W}{LE_{eq} R_x} \]

\[ \bar{W} = \frac{215N}{(0.0152m) \times \left(3.02 \times 10^9 \frac{N}{m^2}\right) \times (25.4254m)} \]

\[ \bar{W} = 184.21 \times 10^{-9} \]

Maximum contact pressure:

\[ P_{\text{max}} = E_{eq} \left( \frac{\bar{W}}{2\pi} \right)^{\frac{1}{2}} \]

\[ P_{\text{max}} = (3.02 \times 10^9 \ Pa) \times \left( \frac{184.21 \times 10^{-9}}{2\pi} \right)^{\frac{1}{2}} \]

\[ P_{\text{max}} = 5.17 \times 10^5 \ Pa = 0.52 \text{ MPa} \]
The following analysis was used to calculate the maximum stress for line contact stress of the UHMWPE roller on the steel plate.

Dimensionless load:

\[ \overline{W} = \frac{W}{LE_{eq}R_x} \]

\[ \overline{W} = \frac{122N}{(0.0152m) \times \left(3.02 \times 10^9 \frac{N}{m^2}\right) \times (25.4254m)} \]

\[ \overline{W} = 104.5 \times 10^{-9} \]
Maximum contact pressure:

\[ P_{\text{max}} = E_{eq} \left( \frac{W}{2\pi} \right)^{\frac{1}{2}} \]

\[ P_{\text{max}} = (3.02 \times 10^9 \text{ Pa}) \times \left( \frac{104.5 \times 10^{-9}}{2\pi} \right)^{\frac{1}{2}} \]

\[ P_{\text{max}} = 0.39 \text{ MPa} \]
REFERENCES


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