Investigation of static and dynamic enhancement of laminar natural convection cooling of electronics

Laurie Ann Florio
New Jersey Institute of Technology

Follow this and additional works at: https://digitalcommons.njit.edu/dissertations

Part of the Mechanical Engineering Commons

Recommended Citation
Florio, Laurie Ann, "Investigation of static and dynamic enhancement of laminar natural convection cooling of electronics" (2004).
Dissertations, 671.
https://digitalcommons.njit.edu/dissertations/671

This Dissertation is brought to you for free and open access by the Theses and Dissertations at Digital Commons @ NJIT. It has been accepted for inclusion in Dissertations by an authorized administrator of Digital Commons @ NJIT. For more information, please contact digitalcommons@njit.edu.
Copyright Warning & Restrictions

The copyright law of the United States (Title 17, United States Code) governs the making of photocopies or other reproductions of copyrighted material.

Under certain conditions specified in the law, libraries and archives are authorized to furnish a photocopy or other reproduction. One of these specified conditions is that the photocopy or reproduction is not to be “used for any purpose other than private study, scholarship, or research.” If a user makes a request for, or later uses, a photocopy or reproduction for purposes in excess of “fair use” that user may be liable for copyright infringement.

This institution reserves the right to refuse to accept a copying order if, in its judgment, fulfillment of the order would involve violation of copyright law.

Please Note: The author retains the copyright while the New Jersey Institute of Technology reserves the right to distribute this thesis or dissertation.

Printing note: If you do not wish to print this page, then select “Pages from: first page # to: last page #” on the print dialog screen.
The Van Houten library has removed some of the personal information and all signatures from the approval page and biographical sketches of theses and dissertations in order to protect the identity of NJIT graduates and faculty.
ABSTRACT

INVESTIGATION OF STATIC AND DYNAMIC ENHANCEMENT OF LAMINAR NATURAL CONVECTION COOLING OF ELECTRONICS

by

Laurie Ann Florio

With the proliferation of consumer electronics devices and the increasingly demanding thermal control requirements for such devices, more effective and efficient means of cooling electronics are needed. The feasibility of three alternative approaches to enhancing pure natural convection cooling of electronics is investigated in the current research. The three static or dynamic methods investigated are alternate cross-flow passages, the strategic placement of transversely oscillating plates near the heat sources and a combined method using both the cross-flow passages and the strategically placed oscillating plates. These cooling methods are intended to operate in the regime for which natural convection cooling is inadequate, yet conventional fan driven cooling is inefficient, and, thus, to serve as an intermediary between pure natural convection and forced convective cooling.

Parametric studies of these three cooling methods were carried out through two-dimensional laminar flow numerical simulations. A finite volume program was developed for a simplified model geometry while for more complex geometries and boundary conditions, a finite element package was utilized. The resulting temperature and velocity field data from these studies was used to examine the impact that each cooling method makes on these fields and to determine measures of the cooling effect including average surface temperatures and average and local heat transfer coefficients.
Through comparisons among the results of the parametric studies and to a reference natural convection case the potential cooling effects and the parameter range over which any improvement is possible were estimated. This study provides the information necessary to judge the potential thermal benefits and feasibility of each method.

Based on the results, each of the three methods has the capability to produce a significant cooling effect for viable parameter values with the amount of cooling dependent on the system parameters and the cooling method. Thus each of the three methods has practical potential as a means of enhancing pure natural convection cooling of electronics in a vertically oriented channel arrangement.
INVESTIGATION OF STATIC AND DYNAMIC ENHANCEMENT OF LAMINAR NATURAL CONVECTION COOLING OF ELECTRONICS

Laurie Ann Florio

Dr. Avraham Harnoy, Dissertation Advisor
Professor of Mechanical Engineering, NJIT

Dr. Rong-Yaw Chen, Committee Member
Professor of Mechanical Engineering, NJIT

Dr. Ernest Geskin, Committee Member
Professor of Mechanical Engineering, NJIT

Dr. Boris Khusid, Committee Member
Associate Professor of Mechanical Engineering, NJIT

Dr. John Tavantzis, Committee Member
Professor of Mathematical Sciences, NJIT
BIOGRAPHICAL SKETCH

Author: Laurie Ann Florio
Degree: Doctor of Philosophy

Undergraduate and Graduate Education:

- Doctor of Philosophy in Mechanical Engineering, New Jersey Institute of Technology, Newark, NJ 2005
- Master of Science in Mechanical Engineering, New Jersey Institute of Technology, Newark, NJ 1999
- Bachelor of Science in Mechanical Engineering, New Jersey Institute of Technology, Newark, NJ 1997

Major: Mechanical Engineering

Presentations and Publications:

Accepted for Publication:

“Patience and persistence have a magical effect before which difficulties disappear and obstacles vanish.” John Quincy Adams
ACKNOWLEDGEMENT

I would like to express my appreciation to Dr. Avraham Harnoy for serving as my advisor and for his useful discussions that aided the direction of my work. I would also like to thank Dr. Rong-Yaw Chen, Dr. Ernest Geskin, Dr. Boris Khusid and Dr. John Tavantzis for actively participating in my committee.

In addition I would like to thank Dr. P.J. Florio for insightful guidance and reassurance, Cathie Florio for helping with the data processing and preparation of the manuscript particularly as the work neared completion, and Iolanda Florio for her editing skills and encouragement.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Objective</td>
<td>1</td>
</tr>
<tr>
<td>1.2 Motivation for Study</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Natural Convection</td>
<td>4</td>
</tr>
<tr>
<td>1.4 Placement of Current Work in Existing Knowledge</td>
<td>5</td>
</tr>
<tr>
<td>1.5 Scope of Present Work</td>
<td>20</td>
</tr>
<tr>
<td>2 PROBLEM FORMULATION</td>
<td>24</td>
</tr>
<tr>
<td>2.1 General Problem Description</td>
<td>24</td>
</tr>
<tr>
<td>2.2 General Modeling Assumptions</td>
<td>25</td>
</tr>
<tr>
<td>2.3 General Governing Equations</td>
<td>26</td>
</tr>
<tr>
<td>2.4 Discussion of Channel Inlet and Outlet Boundary Conditions</td>
<td>31</td>
</tr>
<tr>
<td>3 FINITE ELEMENT NUMERICAL INVESTIGATION OF ALTERNATE CROSS-FLOW PASSAGES</td>
<td>33</td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>33</td>
</tr>
<tr>
<td>3.2 Preliminary Investigation of Alternate Cross-Flow Passages</td>
<td>34</td>
</tr>
<tr>
<td>3.2.1 Problem Statement Preliminary Study of Alternate Cross-Flow Passages</td>
<td>35</td>
</tr>
<tr>
<td>3.2.2 Results</td>
<td>38</td>
</tr>
<tr>
<td>3.3 Parametric Studies of Alternate Cross-Flow Passages with Conduction in Solids</td>
<td>45</td>
</tr>
<tr>
<td>3.3.1 Problem Statement Parametric Studies of Alternate Cross-Flow Passages with Conduction in Solids</td>
<td>45</td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>3.3.2 Results</td>
<td>47</td>
</tr>
<tr>
<td>3.4 Parametric Studies with Conduction in Solids and a Modified Opening Arrangement</td>
<td>63</td>
</tr>
<tr>
<td>3.4.1 Problem Description</td>
<td>64</td>
</tr>
<tr>
<td>3.4.2 Results</td>
<td>64</td>
</tr>
<tr>
<td>3.5 Conclusions</td>
<td>67</td>
</tr>
<tr>
<td>4 FINITE VOLUME NUMERICAL INVESTIGATION OF CHANNEL WITH TRANSVERSELY OSCILLATING WALL UNDER A SQUEEZE FILM VELOCITY FIELD ASSUMPTION</td>
<td>69</td>
</tr>
<tr>
<td>4.1 Introduction</td>
<td>69</td>
</tr>
<tr>
<td>4.2 Squeeze Film Model Problem Statement</td>
<td>71</td>
</tr>
<tr>
<td>4.3 Analysis of Oscillating Squeeze Film</td>
<td>74</td>
</tr>
<tr>
<td>4.3.1 Dimensionless Variables</td>
<td>74</td>
</tr>
<tr>
<td>4.3.2 Governing Equations</td>
<td>74</td>
</tr>
<tr>
<td>4.3.3 Initial Conditions and Boundary Conditions</td>
<td>76</td>
</tr>
<tr>
<td>4.3.4 Analytical Determination of Squeeze Film Velocity Field</td>
<td>77</td>
</tr>
<tr>
<td>4.3.5 Numerical Solution of Energy Equation for Temperature Field.</td>
<td>78</td>
</tr>
<tr>
<td>4.3.6 Calculated Parameters</td>
<td>80</td>
</tr>
<tr>
<td>4.4 Squeeze Film Results of Heat Transfer Performance</td>
<td>82</td>
</tr>
<tr>
<td>4.4.1 Time Averaged Transient Steady State Results</td>
<td>83</td>
</tr>
<tr>
<td>4.4.2 Transient Results at Transient Steady State</td>
<td>92</td>
</tr>
<tr>
<td>4.5 Conclusions</td>
<td>99</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS

(Continued)

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>FINITE VOLUME NUMERICAL INVESTIGATION OF CHANNEL WITH TRANSVERSELY OSCILLATING WALL –MORE GENERAL MODEL WITH NATURAL CONVECTION AND INERTIA</td>
</tr>
<tr>
<td>5.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>5.2</td>
<td>Problem Statement for More General Model to Investigate Use of Transverse Oscillations to Enhance Natural Convection</td>
</tr>
<tr>
<td>5.3</td>
<td>Non-Dimensionalization</td>
</tr>
<tr>
<td>5.3.1</td>
<td>Dimensionless Form of Governing Equations</td>
</tr>
<tr>
<td>5.3.2</td>
<td>Initial Conditions and Boundary Conditions</td>
</tr>
<tr>
<td>5.3.3</td>
<td>Numerical Solution Procedure</td>
</tr>
<tr>
<td>5.3.4</td>
<td>Calculated Parameters</td>
</tr>
<tr>
<td>5.4</td>
<td>Analysis of Transverse Oscillation Enhanced Natural Convection</td>
</tr>
<tr>
<td>5.4.1</td>
<td>General Conclusions and Observations</td>
</tr>
<tr>
<td>5.4.2</td>
<td>Effect of Parameter Values</td>
</tr>
<tr>
<td>5.5</td>
<td>Conclusions</td>
</tr>
<tr>
<td>6</td>
<td>FINITE ELEMENT NUMERICAL INVESTIGATIONS OF THE USE OF DISCRETE TRANSVERSE OSCILLATION SOURCES FOR NATURAL CONVECTION ENHANCEMENT WITH VARIOUS GEOMETRIES</td>
</tr>
<tr>
<td>6.1</td>
<td>Introduction</td>
</tr>
<tr>
<td>6.2</td>
<td>General Problem Statement</td>
</tr>
<tr>
<td>6.3</td>
<td>Discrete Transverse Oscillation Sources to Supplement Natural Convection in Plain Channel</td>
</tr>
<tr>
<td>6.3.1</td>
<td>Problem Statement – Plain Channel Geometry</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS
(Continued)

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.3.2</td>
<td>155</td>
</tr>
<tr>
<td>6.3.3</td>
<td>188</td>
</tr>
<tr>
<td>6.4</td>
<td>190</td>
</tr>
<tr>
<td>6.4.1</td>
<td>190</td>
</tr>
<tr>
<td>6.4.2</td>
<td>193</td>
</tr>
<tr>
<td>6.4.3</td>
<td>228</td>
</tr>
<tr>
<td>6.5</td>
<td>229</td>
</tr>
<tr>
<td>6.5.1</td>
<td>229</td>
</tr>
<tr>
<td>6.5.2</td>
<td>235</td>
</tr>
<tr>
<td>6.5.3</td>
<td>240</td>
</tr>
<tr>
<td>6.6</td>
<td>249</td>
</tr>
<tr>
<td>7</td>
<td>251</td>
</tr>
<tr>
<td>7.1</td>
<td>251</td>
</tr>
<tr>
<td>7.2</td>
<td>252</td>
</tr>
<tr>
<td>7.2.1</td>
<td>252</td>
</tr>
<tr>
<td>7.2.2</td>
<td>254</td>
</tr>
<tr>
<td>7.2.3</td>
<td>283</td>
</tr>
<tr>
<td>7.3</td>
<td>284</td>
</tr>
<tr>
<td>7.3.1</td>
<td>285</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS
(Continued)

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.3.2</td>
<td>Study Results — Combined Method with Rectangular Heat Sources</td>
</tr>
<tr>
<td>7.3.3</td>
<td>Conclusions</td>
</tr>
<tr>
<td>7.4</td>
<td>Conclusions</td>
</tr>
<tr>
<td>8</td>
<td>SUMMARY, CONCLUSIONS, AND FUTURE WORK</td>
</tr>
<tr>
<td>8.1</td>
<td>Summary and Conclusions</td>
</tr>
<tr>
<td>8.2</td>
<td>Future Work</td>
</tr>
</tbody>
</table>

## APPENDIX A
VERIFICATION OF FIDAP© SOLUTIONS FOR BUOYANCY DRIVEN FLOWS AND TRANSIENT MOVING BOUNDARY FLOWS | 336 |

<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.1</td>
<td>Buoyancy Driven Flows - Comparison with Theoretical and Experimental Results</td>
<td>337</td>
</tr>
<tr>
<td>A.1.1</td>
<td>Buoyancy Driven Flows-Comparison with Theoretical Approximate Solution by Aung [24]...</td>
<td>337</td>
</tr>
<tr>
<td>A.1.2</td>
<td>Buoyancy Driven Flows- Comparison with Experimental Solutions by Darbhe [87].............</td>
<td>341</td>
</tr>
<tr>
<td>A.2</td>
<td>Transient Moving Boundary Flows – Comparison With Theoretical Results</td>
<td>344</td>
</tr>
<tr>
<td>A.2.1</td>
<td>Case A-Channel Flow - One Plate Fixed One Plate Moving at Constant Velocity-Fluid Starts from Rest</td>
<td>345</td>
</tr>
<tr>
<td>A.2.2</td>
<td>Case B - Channel Flow - One Plate Fixed One Plate Starting From Rest and Velocity Increases Linearly to a Constant Velocity-Fluid From Rest</td>
<td>347</td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>------------------------------------------------------------------------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>A.2.3 Case C-Channel Flow- One Plate Fixed One Plate Moving with Sinusoidal Velocity-Fluid From Rest</td>
<td>349</td>
<td></td>
</tr>
<tr>
<td>APPENDIX B DIMENSIONLESS FORM OF GOVERNING EQUATIONS USED IN FINITE ELEMENT ANALYSES</td>
<td>351</td>
<td></td>
</tr>
<tr>
<td>B.1 Dimensionless Variables and FIDAP©</td>
<td>352</td>
<td></td>
</tr>
<tr>
<td>B.2 Dimensionless Equations Governing Fluid Flow and Heat Transfer</td>
<td>354</td>
<td></td>
</tr>
<tr>
<td>B.3 Dimensionless Energy Equation in Solid Bodies</td>
<td>356</td>
<td></td>
</tr>
<tr>
<td>B.3.1 Constant Heat Flux Heat Source</td>
<td>357</td>
<td></td>
</tr>
<tr>
<td>B.3.2 Constant Volumetric Heat Source Solid</td>
<td>359</td>
<td></td>
</tr>
<tr>
<td>B.4 Non-Dimensionalization of Flow Rate</td>
<td>365</td>
<td></td>
</tr>
<tr>
<td>B.5 Additional Non-Dimensionalization For Oscillating Body.</td>
<td>366</td>
<td></td>
</tr>
<tr>
<td>APPENDIX C FINITE ELEMENT METHOD FOR FLUID FLOW AND HEAT TRANSFER</td>
<td>368</td>
<td></td>
</tr>
<tr>
<td>C.1 Finite Element Procedure</td>
<td>368</td>
<td></td>
</tr>
<tr>
<td>C.2 Development of Finite Element Equations for Heat Transfer and Fluid Flow</td>
<td>371</td>
<td></td>
</tr>
<tr>
<td>C.2.1 Interpolation Functions</td>
<td>372</td>
<td></td>
</tr>
<tr>
<td>C.2.2 Mathematical Theorems</td>
<td>373</td>
<td></td>
</tr>
<tr>
<td>C.2.3 Development of Finite Element Equations</td>
<td>374</td>
<td></td>
</tr>
<tr>
<td>APPENDIX D GOVERNING EQUATIONS AND FINITE VOLUME NUMERICAL METHOD USED IN SQUEEZE FILM MODEL</td>
<td>383</td>
<td></td>
</tr>
<tr>
<td>D.1 Squeeze Film Model Governing Equations</td>
<td>383</td>
<td></td>
</tr>
<tr>
<td>D.2 Finite Volume Numerical Method</td>
<td>385</td>
<td></td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>APPENDIX E</td>
<td>TRANSFORMED GOVERNING EQUATIONS AND APPLICATION OF SIMPLER FINITE VOLUME METHOD FOR SOLUTION OF VELOCITY, PRESSURE, AND TEMPERATURE.................................</td>
<td>391</td>
</tr>
<tr>
<td>E.1 Transformation of Governing Equations</td>
<td>391</td>
<td></td>
</tr>
<tr>
<td>E.2 Finite Volume SIMPLER Numerical Method for Heat and Fluid Flow...............................</td>
<td>393</td>
<td></td>
</tr>
<tr>
<td>E.2.1 SIMPLER Pressure Correction Method for Momentum Equations......................</td>
<td>396</td>
<td></td>
</tr>
<tr>
<td>E.2.2 SIMPLER Pressure Correction Method for Pressure Solution..................</td>
<td>398</td>
<td></td>
</tr>
<tr>
<td>E.2.3 SIMPLER Pressure Correction Method for Solution of the Energy Equation........</td>
<td>403</td>
<td></td>
</tr>
<tr>
<td>E.2.4 Outline of SIMPLER Solution Procedure..........</td>
<td>404</td>
<td></td>
</tr>
<tr>
<td>E.2.5 SIMPLER Method Boundary Conditions........</td>
<td>404</td>
<td></td>
</tr>
<tr>
<td>E.2.6 Specification of Parameters for the Current Investigation........................</td>
<td>408</td>
<td></td>
</tr>
<tr>
<td>E.2.7 Convergence Criteria........................</td>
<td>411</td>
<td></td>
</tr>
<tr>
<td>E.2.8 Flow Chart..................................</td>
<td>412</td>
<td></td>
</tr>
<tr>
<td>E.2.9 Program Verification..........................</td>
<td>412</td>
<td></td>
</tr>
<tr>
<td>APPENDIX F</td>
<td>AUTHOR-WRITTEN FORTRAN PROGRAMS..................................</td>
<td>417</td>
</tr>
<tr>
<td>F.1 Squeeze Film...................................</td>
<td>417</td>
<td></td>
</tr>
<tr>
<td>F.2 Simpler..........................................</td>
<td>417</td>
<td></td>
</tr>
<tr>
<td>APPENDIX G</td>
<td>FIDAP© FINITE ELEMENT RESULTS FOR TRANSVERSE OSCILLATION INVESTIGATIONS...............................</td>
<td>418</td>
</tr>
<tr>
<td>Chapter</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>G.1 Plain Channel Geometry</td>
<td>418</td>
<td></td>
</tr>
<tr>
<td>G.2 Rectangular Heat Source Geometry</td>
<td>427</td>
<td></td>
</tr>
<tr>
<td>G.3 Plain Channel With Opening Geometry</td>
<td>442</td>
<td></td>
</tr>
<tr>
<td>G.4 Modified Heat Source Geometry</td>
<td>451</td>
<td></td>
</tr>
<tr>
<td>G.4.1 Extension Plate</td>
<td>451</td>
<td></td>
</tr>
<tr>
<td>G.4.2 Dummy Block</td>
<td>459</td>
<td></td>
</tr>
<tr>
<td>G.4.3 Upstream Plate</td>
<td>464</td>
<td></td>
</tr>
<tr>
<td>G.5 Two Heating Element Geometry</td>
<td>469</td>
<td></td>
</tr>
<tr>
<td>G.5.1 Upstream Oscillation Source</td>
<td>469</td>
<td></td>
</tr>
<tr>
<td>G.5.2 Plate Between Heating Elements – Top of Heat Source Level</td>
<td>479</td>
<td></td>
</tr>
<tr>
<td>G.5.3 Plate Over First Heat Source</td>
<td>489</td>
<td></td>
</tr>
<tr>
<td>G.5.4 Plate Between Heat Sources - Hole Level</td>
<td>499</td>
<td></td>
</tr>
<tr>
<td>REFERENCES</td>
<td>509</td>
<td></td>
</tr>
</tbody>
</table>
# LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Reference Values Used In Non-Dimensionalizing the Governing Equations For the Cases Indicated</td>
<td>30</td>
</tr>
<tr>
<td>3.1</td>
<td>Data Used in Preliminary Investigation of Alternate Cross-Flow Passages with Conduction in Heating Elements and Board Neglected</td>
<td>37</td>
</tr>
<tr>
<td>3.2</td>
<td>Summary Comparison of Average Heat Transfer Coefficient Results for Preliminary Investigation*</td>
<td>41</td>
</tr>
<tr>
<td>3.3</td>
<td>Data Used in Investigation of Alternate Cross-Flow Passages Including Conduction in Heating Elements and Board</td>
<td>46</td>
</tr>
<tr>
<td>3.4</td>
<td>Average Dimensionless Heat Transfer Coefficients from Right Side of Heat Source 2 (B2R)*</td>
<td>56</td>
</tr>
<tr>
<td>3.5</td>
<td>Average Dimensionless Heat Transfer Coefficients from Left Side of Heat Source 3 (B3L)*</td>
<td>56</td>
</tr>
<tr>
<td>3.6a</td>
<td>Maximum Dimensionless Temperature Heat Source 1</td>
<td>57</td>
</tr>
<tr>
<td>3.6b</td>
<td>Maximum Dimensionless Temperature Heat Source 2</td>
<td>57</td>
</tr>
<tr>
<td>3.6c</td>
<td>Maximum Dimensionless Temperature Heat Source 3</td>
<td>58</td>
</tr>
<tr>
<td>4.1</td>
<td>Squeeze Film Model Input Parameters</td>
<td>82</td>
</tr>
<tr>
<td>4.2</td>
<td>Maximum Percent Increase in Modified Nusslet Number of Natural Convection Reference</td>
<td>84</td>
</tr>
<tr>
<td>4.3</td>
<td>Nusselt Number Correlation Equations For Current Data Nu = c + dω^f.</td>
<td>87</td>
</tr>
<tr>
<td>5.1a</td>
<td>Summary of Results for More General Model Investigation qa=150 W/m^2</td>
<td>129</td>
</tr>
<tr>
<td>5.1b</td>
<td>Summary of Results for More General Model Investigation qa=600 W/m^2</td>
<td>130</td>
</tr>
<tr>
<td>6.1</td>
<td>Data Used in Investigation of Use of Discrete Oscillation Sources To Supplement Natural Convection – Plain Channel Geometry</td>
<td>155</td>
</tr>
</tbody>
</table>
**LIST OF TABLES**  
(Continued)

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.2a</td>
<td>Summary of Average Dimensionless Heat Transfer Coefficients – Plain Channel Geometry C=0.15</td>
<td>165</td>
</tr>
<tr>
<td>6.2b</td>
<td>Summary of Average Local Dimensionless Heat Transfer Coefficients – Plain Channel Geometry C=0.15</td>
<td>165</td>
</tr>
<tr>
<td>6.3</td>
<td>Data Used in Investigation of Use of Discrete Oscillation Sources To Supplement Natural Convection – Rectangular Heat Source Geometry</td>
<td>192</td>
</tr>
<tr>
<td>6.4a</td>
<td>Summary of Average Dimensionless Heat Transfer Coefficients – Rectangular Heat Source Geometry C=0.30</td>
<td>211</td>
</tr>
<tr>
<td>6.4b</td>
<td>Summary of Average Dimensionless Heat Transfer Coefficients – Rectangular Heat Source Geometry C=0.15</td>
<td>211</td>
</tr>
<tr>
<td>6.5a</td>
<td>Summary of Average Local Dimensionless Heat Transfer Coefficients – Rectangular Heat Source Geometry C=0.30</td>
<td>212</td>
</tr>
<tr>
<td>6.5b</td>
<td>Summary of Average Local Dimensionless Heat Transfer Coefficients – Rectangular Heat Source Geometry C=0.15</td>
<td>212</td>
</tr>
<tr>
<td>6.6</td>
<td>Dimensional Parameter Values for Dummy Block Geometry</td>
<td>230</td>
</tr>
<tr>
<td>6.7a</td>
<td>Summary of Average Dimensionless Heat Transfer Coefficients – Dummy Block Geometry C=0.15</td>
<td>231</td>
</tr>
<tr>
<td>6.7b</td>
<td>Summary of Average Local Dimensionless Heat Transfer Coefficients – Dummy Block Geometry C=0.15</td>
<td>232</td>
</tr>
<tr>
<td>6.8</td>
<td>Dimensional Parameter Values for Extension Geometry</td>
<td>236</td>
</tr>
<tr>
<td>6.9a</td>
<td>Summary of Average Dimensionless Heat Transfer Coefficients – Extension Plate Geometry C=0.15</td>
<td>237</td>
</tr>
<tr>
<td>6.9b</td>
<td>Summary of Average Local Dimensionless Heat Transfer Coefficients – Extension Plate Geometry C=0.15</td>
<td>238</td>
</tr>
<tr>
<td>6.10a</td>
<td>Summary of Average Dimensionless Heat Transfer Coefficients – Extension Plate Geometry C=0.30</td>
<td>239</td>
</tr>
</tbody>
</table>
LIST OF TABLES
(Continued)

Table | Page
--- | ---
6.10b | Summary of Average Local Dimensionless Heat Transfer Coefficients — Extension Plate Geometry C=0.30 | 240
6.11 | Dimensional Parameter Values for Upstream Oscillation Source | 244
6.12a | Summary of Average Dimensionless Heat Transfer Coefficients — Extension Plate Geometry C=0.30 | 245
6.12b | Summary of Average Local Dimensionless Heat Transfer Coefficients — Extension Plate Geometry C=0.30 | 245
7.1 | Dimensional Parameters for Combined Method-Plain Channel Geometry* | 254
7.2a | Summary of Average Heat Transfer Coefficient Results — Combined Method - Plain Channel Geometry C=0.15 | 282
7.2b | Summary of Local Heat Transfer Coefficient Results — Combined Method — Plain Channel Geometry C=0.15 | 282
7.3 | Dimensionless Maximum Velocities, Average Temperatures, and Average Surface Heat Transfer Coefficients at Steady State | 288
7.4 | Dimensional Parameter Values for Upstream Oscillation Source | 290
7.5a | Summary of Average Heat Transfer Coefficient Results — Upstream Oscillating Source — Two Block Geometry — Block 1 C=0.30 | 293
7.5b | Summary of Local Heat Transfer Coefficient Results — Upstream Oscillating Source — Two Block Geometry — Block 1 C=0.30 | 294
7.6a | Summary of Average Heat Transfer Coefficient Results — Upstream Oscillating Source — Two Block Geometry — Block 2 C=0.30 | 295
7.6b | Summary of Local Heat Transfer Coefficient Results — Upstream Oscillating Source — Two Block Geometry — Block 2 C=0.30 | 296
7.7 | Dimensional Parameter Values for Oscillation Source Over Board Opening | 299
<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.8a</td>
<td>Summary of Average Heat Transfer Coefficient Results — Oscillating Source Over Board Opening — Two Block Geometry — Block1 C=0.50...</td>
<td>302</td>
</tr>
<tr>
<td>7.8b</td>
<td>Summary of Local Heat Transfer Coefficient Results — Oscillating Source Over Board Opening — Two Block Geometry — Block1 C=0.50...</td>
<td>303</td>
</tr>
<tr>
<td>7.9a</td>
<td>Summary of Average Heat Transfer Coefficient Results — Oscillating Source Over Board Opening — Two Block Geometry — Block2 C=0.50...</td>
<td>304</td>
</tr>
<tr>
<td>7.9b</td>
<td>Summary of Local Heat Transfer Coefficient Results — Oscillating Source Over Board Opening — Two Block Geometry — Block2 C=0.50...</td>
<td>305</td>
</tr>
<tr>
<td>7.10</td>
<td>Dimensional Parameter Values for Oscillation Source Over Board Opening at Level of Top Heat Source Surface</td>
<td>307</td>
</tr>
<tr>
<td>7.11a</td>
<td>Summary of Average Heat Transfer Coefficient Results — Oscillating Source Over Board Opening at Level of Top Heat Source Surface — Two Block Geometry — Block 1 C=0.30...</td>
<td>310</td>
</tr>
<tr>
<td>7.11b</td>
<td>Summary of Local Heat Transfer Coefficient Results — Oscillating Source Over Board Opening at Level of Top Heat Source Surface — Two Block Geometry — Block 1</td>
<td>311</td>
</tr>
<tr>
<td>7.12a</td>
<td>Summary of Average Heat Transfer Coefficient Results — Oscillating Source Over Board Opening at Level of Top Heat Source Surface — Two Block Geometry — Block 2 C=0.30...</td>
<td>312</td>
</tr>
<tr>
<td>7.12b</td>
<td>Summary of Local Heat Transfer Coefficient Results — Oscillating Source Over Board Opening at Level of Top Heat Source Surface — Two Block Geometry — Block 2</td>
<td>313</td>
</tr>
<tr>
<td>7.13</td>
<td>Dimensional Parameter Values for Oscillation Source Over First Heat Source</td>
<td>316</td>
</tr>
<tr>
<td>7.14a</td>
<td>Summary of Local Heat Transfer Coefficient Results — Oscillating Source Over First Heat Source — Two Block Geometry — Block1 C=0.30...</td>
<td>319</td>
</tr>
</tbody>
</table>
# LIST OF TABLES
(Continued)

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.14b</td>
<td>Summary of Average Heat Transfer Coefficient Results – Oscillating Source Over First Heat Source – Two Block Geometry – Block 1 C=0.30</td>
<td>320</td>
</tr>
<tr>
<td>7.15a</td>
<td>Summary of Average Heat Transfer Coefficient Results – Oscillating Source Over First Heat Source – Two Block Geometry – Block 2</td>
<td>321</td>
</tr>
<tr>
<td>7.15b</td>
<td>Summary of Local Heat Transfer Coefficient Results – Oscillating Source Over First Heat Source – Two Block Geometry – Block 2</td>
<td>322</td>
</tr>
<tr>
<td>B.1</td>
<td>Summary of Equivalent Properties For Studies Indicated</td>
<td>365</td>
</tr>
<tr>
<td>B.2</td>
<td>Reference Values for Non-Dimensionalizing Calculated Values for Cases Indicated</td>
<td>366</td>
</tr>
<tr>
<td>B.3</td>
<td>Dimensionless Oscillation Source Parameters</td>
<td>367</td>
</tr>
<tr>
<td>E.1</td>
<td>Parameters Used in Verification of SIMPLER Program</td>
<td>413</td>
</tr>
</tbody>
</table>
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Possible implementation of alternate cross-flow passages</td>
<td>12</td>
</tr>
<tr>
<td>1.2</td>
<td>Alternate cross-flow passage model</td>
<td>12</td>
</tr>
<tr>
<td>1.3</td>
<td>Use of oscillation source placed in immediate vicinity of heat source</td>
<td>18</td>
</tr>
<tr>
<td>1.4</td>
<td>Potential use of piezoelectric fan as vibration source</td>
<td>18</td>
</tr>
<tr>
<td>2.1</td>
<td>Vertically oriented channel system</td>
<td>26</td>
</tr>
<tr>
<td>3.1</td>
<td>Geometry for alternate cross-flow passage model</td>
<td>36</td>
</tr>
<tr>
<td>3.2</td>
<td>Geometry for standard closed board model</td>
<td>36</td>
</tr>
<tr>
<td>3.3</td>
<td>Dimension labels for alternate cross-flow passage models</td>
<td>37</td>
</tr>
<tr>
<td>3.4</td>
<td>Typical pressure contours with alternate cross flow passages</td>
<td>41</td>
</tr>
<tr>
<td>3.5</td>
<td>Velocity field preliminary investigation: (a) alternate cross-flow passage geometry, (b) standard board geometry</td>
<td>42</td>
</tr>
<tr>
<td>3.6</td>
<td>Detailed view of velocity field between heat sources preliminary investigation (third shown): (a) alternate cross-flow passage geometry, (b) standard board geometry</td>
<td>43</td>
</tr>
<tr>
<td>3.7</td>
<td>Temperature contours from preliminary investigation: (a) alternate cross-flow passage geometry, (b) standard board geometry</td>
<td>44</td>
</tr>
<tr>
<td>3.8</td>
<td>Velocity field: (a) alternate cross-flow passages, (b) modified opening arrangement, (c) standard geometry</td>
<td>53</td>
</tr>
<tr>
<td>3.9</td>
<td>Velocity field near 3rd opening: (a) alternate cross-flow passages, (b) modified opening arrangement, (c) standard geometry</td>
<td>54</td>
</tr>
<tr>
<td>3.10</td>
<td>Temperature contours: (a) alternate cross-flow passages, (b) modified opening arrangement, (c) standard geometry</td>
<td>55</td>
</tr>
<tr>
<td>3.11</td>
<td>Comparison of dimensionless heat transfer coefficients on side B2R: (a) geometric parameter study fixed heat rate, (b) heat rate parameter study fixed geometry</td>
<td>59</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES (Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.12</td>
<td>60</td>
</tr>
<tr>
<td>3.13</td>
<td>61</td>
</tr>
<tr>
<td>3.14</td>
<td>62</td>
</tr>
<tr>
<td>3.15</td>
<td>63</td>
</tr>
<tr>
<td>4.1</td>
<td>72</td>
</tr>
<tr>
<td>4.2</td>
<td>78</td>
</tr>
<tr>
<td>4.3</td>
<td>88</td>
</tr>
<tr>
<td>4.4</td>
<td>89</td>
</tr>
<tr>
<td>4.5</td>
<td>90</td>
</tr>
<tr>
<td>4.6</td>
<td>91</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>4.7</td>
<td>Typical squeeze film velocity field stream functions</td>
</tr>
<tr>
<td>4.8</td>
<td>Typical dimensionless temperature contour plots at same position in oscillation for ( L/b_0 = 100 ), ( \omega = 2000 \text{ rad/sec} ): (a) ( a_0/b_0 = 0.25 ), (b) ( a_0/b_0 = 0.50 ), (c) ( a_0/b_0 = 0.75 )</td>
</tr>
<tr>
<td>4.9</td>
<td>Typical local heated surface results for ( L/b_0 = 100 ), ( \omega = 2000 \text{ rad/sec} ): (a) typical dimensionless temperature along heated surface, (b) typical dimensionless heat transfer coefficient along heated surface</td>
</tr>
<tr>
<td>4.10</td>
<td>Dimensionless temperatures at transient steady state vs. time for ( L/b_0 = 100 ), ( \omega = 2000 \text{ rad/sec} ): (a) ( a_0/b_0 = 0.25 ), (b) ( a_0/b_0 = 0.75 )</td>
</tr>
<tr>
<td>5.1</td>
<td>Model used to investigate use of transverse oscillations to supplement natural convection cooling</td>
</tr>
<tr>
<td>5.2</td>
<td>Comparison of dimensionless velocity distributions under squeeze film assumptions and with the inclusion of the inertia terms at ( x=0 ) with ( b_0=0.010 \text{ m} ), ( a_0/b_0=0.10 ), ( L/b_0=20 ), ( \omega=82.2 \text{ rad/s} )</td>
</tr>
<tr>
<td>5.3</td>
<td>Typical dimensionless velocity distributions with the inclusion of the inertia terms over the course of a plate oscillations at ( x_R=1 ), ( b_0=0.010 \text{ m} ), ( a_0/b_0=0.10 ), ( L/b_0=20 ), ( \omega=82.2 \text{ rad/s} ): (a) portion of cycle ( \tilde{V}<em>{wall}&gt;0 ), (b) portion of cycle ( \tilde{V}</em>{wall}&lt;0 )</td>
</tr>
<tr>
<td>5.4</td>
<td>Typical temperature distribution for model with inertia effects and natural convection included ( b_0=0.010 \text{ m} ), ( a_0/b_0=0.10 ), ( L/b_0=20 ), ( \omega=82.2 \text{ rad/s} ), ( q_a=150 \text{ W/m}^2 )</td>
</tr>
<tr>
<td>5.5</td>
<td>Average heated surface temperature as a function of time with inertia effects and with and without natural convection for ( b_0=0.010 \text{ m} ), ( a_0/b_0=0.10 ), ( L/b_0=20 ), ( \omega=82.2 \text{ rad/s} ), ( q_a=150 \text{ W/m}^2 )</td>
</tr>
<tr>
<td>5.6</td>
<td>Velocity distributions at ( x_R=1/2 ) for model with inertia effects and natural convection included for ( b_0=0.010 \text{ m} ), ( a_0/b_0=0.10 ), ( L/b_0=20 ), ( \omega=82.2 \text{ rad/s} ), ( q_a=150 \text{ W/m}^2 )</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>5.7</td>
<td>Heated surface temperature development with and without natural convection $b_0=0.010m$, $a_0/b_0=0.10$, $L/b_0=20$, $\omega=82.2$ rad/s, $q_a=150$ W/m$^2$; (a) $t=6$, (b) $t=12$, (c) $t=24$.</td>
</tr>
<tr>
<td>5.8</td>
<td>Contour plots showing development of temperature field with the inclusion of natural convection $b_0=0.010m$, $a_0/b_0=0.10$, $L/b_0=20$, $\omega=82.2$ rad/s, $q_a=150$ W/m$^2$; (a) $t=11.5$, (b) $t=16.5$, (c) $t=19.5$, (d) $t=35.5$.</td>
</tr>
<tr>
<td>5.9</td>
<td>Comparison of inlet velocity distributions (modified dimensionless) at 30% of cycle for $a_0/b_0=0.25$, $L/b_0=10$, $q_a=600$ W/m$^2$, $\omega=40$ rad/sec, for $b_0=0.010m$ and $b_0=0.005m$ ....</td>
</tr>
<tr>
<td>5.10</td>
<td>Comparison of inlet velocity distributions (modified dimensionless) over a cycle: $a_0/b_0=0.25$, $L/b_0=10$, $q_a=600$ W/m$^2$; (a) $b_0=0.010m$ (b) $b_0=0.005m$.</td>
</tr>
<tr>
<td>5.11</td>
<td>Comparison of mid-length velocity distributions (modified dimensionless): over a cycle for $a_0/b_0=0.25$, $L/b_0=10$, $\omega=40$ rad/sec, $q_a=600$ W/m$^2$; (a) $b_0=0.010m$ (b) $b_0=0.005m$.</td>
</tr>
<tr>
<td>5.12</td>
<td>Variation of heated surface temperature distributions with time (modified dimensionless) for $a_0/b_0=0.25$, $L/b_0=10$, $\omega=40$ rad/sec, $q_a=600$ W/m$^2$; (a) $b_0=0.010m$ (b) $b_0=0.005m$.</td>
</tr>
<tr>
<td>5.13</td>
<td>Variation of inlet velocity distribution over a cycle (modified dimensionless): $a_0/b_0=0.25$, $L/b_0=20$, $b_0=0.010m$, $\omega=40$ rad/sec, $q_a=150$ W/m$^2$.</td>
</tr>
<tr>
<td>5.14</td>
<td>Comparison of inlet velocity distributions at 30% of cycle (modified dimensionless) for $a_0/b_0=0.25$, $L/b_0=20$, $b_0=0.010m$, $\omega=40$ rad/sec for $q_a=150$ W/m$^2$ and $q_a=600$ W/m$^2$.</td>
</tr>
<tr>
<td>5.15</td>
<td>Variation of heated surface temperature distributions with time (modified dimensionless) for $a_0/b_0=0.25$, $L/b_0=20$, $b_0=0.010m$, $\omega=40$ rad/sec, $q_a=150$ W/m$^2$.</td>
</tr>
<tr>
<td>5.16</td>
<td>Variation of inlet velocity distribution over cycle (modified dimensionless) for $a_0/b_0=0.25$, $L/b_0=20$, $b_0=0.010m$, $\omega=25$ rad/sec, $q_a=150$ W/m$^2$.</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>5.17</td>
<td>Comparison of inlet velocity distributions at 30% of cycle (modified dimensionless) for (a_o/b_o=0.25, L/b_o=20, b_o=0.010m, q_a=150W/m^2) for (\omega=40) rad/sec and (\omega=40) rad/sec</td>
</tr>
<tr>
<td>5.18</td>
<td>Comparison of mid-length velocity distributions (modified dimensionless) for (a_o/b_o=0.25, L/b_o=20, b_o=0.010m, q_a=150W/m^2): (a) (\omega=25) rad/sec, (b) (\omega=40) rad/sec</td>
</tr>
<tr>
<td>5.19</td>
<td>Variation of heated surface temperature distributions with time (modified dimensionless): (a_o/b_o=0.25, L/b_o=20, b_o=0.010m, \omega=25) rad/sec, (q_a=150W/m^2)</td>
</tr>
<tr>
<td>5.20</td>
<td>Comparison of inlet velocity distributions at 30% of oscillation (modified dimensionless) for (L/b_o=20, b_o=0.010m, \omega=164) rad/sec for (a_o/b_o=0.10) and (a_o/b_o=0.25)</td>
</tr>
<tr>
<td>5.21</td>
<td>Close-up of mid-channel length region showing flow reversal near the heated wall indicating strong oscillation effects for: (L/b_o=20, a_o/b_o=0.25, b_o=0.010m, \omega=164) rad/sec, (q_a=600W/m^2)</td>
</tr>
<tr>
<td>5.22</td>
<td>Development of temperature field in a channel for (a_o/b_o=0.25, L/b_o=20, b_o=0.010m, \omega=164) rad/sec, (q_a=600W/m^2): (a) (\bar{T}_F=4.70), (b) (\bar{T}_F=6.20), (c) (\bar{T}_F=10.70), (d) (\bar{T}_F=11.70), (e) (\bar{T}_F=12.70)</td>
</tr>
<tr>
<td>5.23</td>
<td>Development of surface temperature distributions (modified dimensionless for: (L/b_o=20, a_o/b_o=0.25, b_o=0.010m, \omega=164) rad/sec, (q_a=600W/m^2)</td>
</tr>
<tr>
<td>5.24</td>
<td>Comparison of temperature distributions along heated surface (modified dimensionless) for different oscillation amplitudes: (L/b_o=20, b_o=0.010m, \omega=164) rad/sec, (q_a=600W/m^2)</td>
</tr>
<tr>
<td>6.1</td>
<td>Specification of oscillation source and heat source positions</td>
</tr>
<tr>
<td>6.2</td>
<td>Plain channel model geometry</td>
</tr>
<tr>
<td>6.3</td>
<td>Dimensioning of plain channel geometry model</td>
</tr>
<tr>
<td>6.4</td>
<td>Steady state velocity distributions – plain channel: (a) no plate, (b) plate fixed in channel</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES
(Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.5</td>
<td>Steady state temperature distributions – plain channel: (a) no plate, (b) plate fixed in channel</td>
<td>158</td>
</tr>
<tr>
<td>6.6</td>
<td>Velocity distributions near moving plate for $d=0.10 \ V=0.20 \ \omega = 2$: (a) $\tilde{t}_F = 9.70$, (b) $\tilde{t}_F =10.2$, (c) $\tilde{t}_F =10.8$, (d) $\tilde{t}_F =11.4$</td>
<td>174</td>
</tr>
<tr>
<td>6.7</td>
<td>Temperature distributions near moving plate for $d=0.10 \ V=0.20 \ \omega = 2$: (a) $\tilde{t}_F = 9.70$, (b) $\tilde{t}_F =10.2$, (c) $\tilde{t}_F =10.8$, (d) $\tilde{t}_F =11.4$</td>
<td>176</td>
</tr>
<tr>
<td>6.8</td>
<td>Velocity distributions near moving plate for $d=0.10 \ V=0.20\pi \ \omega = 2\pi$: (a) $\tilde{t}_F =5.225$, (b) $\tilde{t}_F =5.300$, (c) $\tilde{t}_F =5.55$, (d) $\tilde{t}_F =5.725$</td>
<td>178</td>
</tr>
<tr>
<td>6.9</td>
<td>Temperature distributions near moving plate for $d=0.10 \ V=0.20\pi \ \omega = 2\pi$: (a) $\tilde{t}_F =10.225$, (b) $\tilde{t}_F =10.30$, (c) $\tilde{t}_F =5.50$, (d) $\tilde{t}_F =5.725$</td>
<td>180</td>
</tr>
<tr>
<td>6.10</td>
<td>Velocity distributions near moving plate for $d=0.10 \ V=0.40\pi \ \omega = 4\pi$: (a) $\tilde{t}_F =3.075$, (b) $\tilde{t}_F =3.15$, (c) $\tilde{t}_F =3.325$, (d) $\tilde{t}_F =3.45$</td>
<td>182</td>
</tr>
<tr>
<td>6.11</td>
<td>Temperature distributions near moving plate for $d=0.10 \ V=0.40\pi \ \omega = 4\pi$: (a) $\tilde{t}_F =3.075$, (b) $\tilde{t}_F =3.15$, (c) $\tilde{t}_F =3.325$, (d) $\tilde{t}_F =3.45$</td>
<td>184</td>
</tr>
<tr>
<td>6.12</td>
<td>Time averaged dimensionless local heat transfer coefficient at Point 4 along the heated surface for plain channel geometry: (a) $d=0.05$, (b) $d=0.10$</td>
<td>186</td>
</tr>
<tr>
<td>6.13</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) $d=0.10 \ V=0.20 \ \omega =2$, (b) $d=0.10 \ V=0.20\pi \ \omega =2\pi$, (c) $d=0.10 \ V=0.40\pi \ \omega =4\pi$</td>
<td>187</td>
</tr>
<tr>
<td>6.14</td>
<td>Model geometry for rectangular heat source investigation</td>
<td>191</td>
</tr>
<tr>
<td>6.15</td>
<td>Dimensioning for rectangular heat source investigation</td>
<td>193</td>
</tr>
<tr>
<td>6.16</td>
<td>Typical natural convection induced flow over rectangular heat source: (a) velocity field, (b) stream lines</td>
<td>197</td>
</tr>
<tr>
<td>6.17</td>
<td>Typical flow over rectangular heat source: (a) without plate, (b) with plate</td>
<td>198</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
<td></td>
</tr>
<tr>
<td>6.18</td>
<td>199</td>
<td></td>
</tr>
<tr>
<td>6.19</td>
<td>213</td>
<td></td>
</tr>
<tr>
<td>6.20</td>
<td>215</td>
<td></td>
</tr>
<tr>
<td>6.21</td>
<td>217</td>
<td></td>
</tr>
<tr>
<td>6.22</td>
<td>219</td>
<td></td>
</tr>
<tr>
<td>6.23</td>
<td>221</td>
<td></td>
</tr>
<tr>
<td>6.24</td>
<td>223</td>
<td></td>
</tr>
<tr>
<td>6.25</td>
<td>225</td>
<td></td>
</tr>
<tr>
<td>6.26</td>
<td>226</td>
<td></td>
</tr>
<tr>
<td>6.27</td>
<td>227</td>
<td></td>
</tr>
<tr>
<td>6.28</td>
<td>230</td>
<td></td>
</tr>
<tr>
<td>6.29</td>
<td>230</td>
<td></td>
</tr>
</tbody>
</table>
LIST OF FIGURES
(Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.30</td>
<td>Typical natural convection induced flow over dummy block geometry: (a) velocity field, (b) temperature field. d=0.10 V=0.4π ω=4π C=0.15</td>
<td>234</td>
</tr>
<tr>
<td>6.31</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: d=0.10 V=0.4π ω=4π C=0.15</td>
<td>235</td>
</tr>
<tr>
<td>6.32</td>
<td>Model geometry for extension plate investigation</td>
<td>236</td>
</tr>
<tr>
<td>6.33</td>
<td>Dimensioning for extension plate investigation</td>
<td>236</td>
</tr>
<tr>
<td>6.34</td>
<td>Typical natural convection induced flow over extension plate geometry: (a) velocity field, (b) temperature field. d=0.20 V=0.8π ω=4π C=0.30</td>
<td>242</td>
</tr>
<tr>
<td>6.35</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: d=0.20 V=0.8π ω=4π C=0.30</td>
<td>243</td>
</tr>
<tr>
<td>6.36</td>
<td>Model Geometry for Upstream Oscillation Source Investigation</td>
<td>243</td>
</tr>
<tr>
<td>6.37</td>
<td>Dimensioning for Upstream Oscillation Source Investigation</td>
<td>244</td>
</tr>
<tr>
<td>6.38</td>
<td>Typical natural convection induced flow over upstream oscillating plate geometry: (a) velocity field, d=0.20 V=0.4π ω=2π C=0.30 (b) temperature field d=0.20 V=0.4π ω=2π C=0.30</td>
<td>247</td>
</tr>
<tr>
<td>6.39</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: (b) d=0.20 V=0.4π ω=2π C=0.30</td>
<td>248</td>
</tr>
<tr>
<td>7.1</td>
<td>Combined method – plain channel geometry layout</td>
<td>253</td>
</tr>
<tr>
<td>7.2</td>
<td>Combined method – plain channel geometry definitions</td>
<td>254</td>
</tr>
<tr>
<td>7.3</td>
<td>Typical steady state velocity distributions for plain channel geometry: (a) without plate, (b) with plate</td>
<td>256</td>
</tr>
<tr>
<td>7.4</td>
<td>Typical steady state temperature distributions for plain channel geometry: (a) without plate, (b) with plate</td>
<td>257</td>
</tr>
<tr>
<td>7.5</td>
<td>Velocity distribution for combined method plain channel with opening d=0.10 V=0.20 ω=2: (a) $\bar{t}_F = 11.5$, (b) $\bar{t}_F = 12.6$, (c) $\bar{t}_F = 10.2$, (d) $\bar{t}_F = 10.9$</td>
<td>267</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>7.6</td>
<td>Temperature distribution for combined method plain channel with opening d=0.10 V=0.20 ( \omega =2 ): (a) ( \bar{T}_F =11.5 ), (b) ( \bar{T}_F =12.6 ), (c) ( \bar{T}_F =10.2 ), (d) ( \bar{T}_F =10.9 )</td>
<td>269</td>
</tr>
<tr>
<td>7.7</td>
<td>Velocity distribution for combined method plain channel with opening d=0.10 V=0.40π ( \omega =2\pi ): (a) ( \bar{T}_F =6.75 ), (b) ( \bar{T}_F =6.90 ), (c) ( \bar{T}_F =6.1875 ), (d) ( \bar{T}_F =6.125 )</td>
<td>271</td>
</tr>
<tr>
<td>7.8</td>
<td>Temperature distribution for combined method plain channel with opening d=0.10 V=0.40π ( \omega =2\pi ): (a) ( \bar{T}_F =6.75 ), (b) ( \bar{T}_F =6.90 ), (c) ( \bar{T}_F =6.1875 ), (d) ( \bar{T}_F =6.125 )</td>
<td>273</td>
</tr>
<tr>
<td>7.9</td>
<td>Velocity distribution for combined method plain channel with opening d=0.10 V=0.40π ( \omega =4\pi ): (a) ( \bar{T}_F =3.375 ), (b) ( \bar{T}_F =3.50 ), (c) ( \bar{T}_F =3.125 ), (d) ( \bar{T}_F =3.2875 )</td>
<td>275</td>
</tr>
<tr>
<td>7.10</td>
<td>Temperature distribution for combined method plain channel with opening d=0.10 V=0.40π ( \omega =4\pi ): (a) ( \bar{T}_F =3.375 ), (b) ( \bar{T}_F =3.50 ), (c) ( \bar{T}_F =3.125 ), (d) ( \bar{T}_F =3.2875 )</td>
<td>277</td>
</tr>
<tr>
<td>7.11</td>
<td>Variation in the heat transfer coefficient as a function of time plain channel with opening: (a)d=0.10V=0.20 ( \omega =2 ), (b) d=0.10 V=0.20π ( \omega =2 \pi ), (c) d=0.10 V=0.40π</td>
<td>279</td>
</tr>
<tr>
<td>7.12</td>
<td>Time-averaged local heat transfer coefficients at Point 3: (a)d=0.05, (b) d=0.10</td>
<td>281</td>
</tr>
<tr>
<td>7.13</td>
<td>General configuration for the two block studies</td>
<td>284</td>
</tr>
<tr>
<td>7.14</td>
<td>Temperature distributions at steady state: (a) without plate, (b) without plate or hole</td>
<td>288</td>
</tr>
<tr>
<td>7.15</td>
<td>Oscillating plate upstream of two blocks - geometry layout</td>
<td>289</td>
</tr>
<tr>
<td>7.16</td>
<td>Oscillating plate upstream of two blocks - geometry definitions</td>
<td>290</td>
</tr>
<tr>
<td>7.17</td>
<td>Local dimensionless heat transfer coefficient as a function of time d=0.20, V=1.6π, ( \omega =8\pi ): (a) heat source 1, (b) heat source 2</td>
<td>291</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>7.18</td>
<td>Time-averaged local heat transfer coefficients at heat source 1 Point 1</td>
<td>291</td>
</tr>
<tr>
<td>7.19</td>
<td>Typical velocity distribution and temperature field for upstream oscillation source: $d=0.20$, $V=0.8\pi$, $\omega=4\pi$: (a) velocity, (b) temperature.</td>
<td>292</td>
</tr>
<tr>
<td>7.20</td>
<td>Oscillating source over board opening - geometry layout.</td>
<td>298</td>
</tr>
<tr>
<td>7.21</td>
<td>Oscillating source over board opening - geometry definitions.</td>
<td>299</td>
</tr>
<tr>
<td>7.22</td>
<td>Local dimensionless heat transfer coefficient as a function of time $d=0.20$, $V=0.8\pi$, $\omega=4\pi$: (a) heat source 1, (b) heat source 2.</td>
<td>300</td>
</tr>
<tr>
<td>7.23</td>
<td>Time-averaged local dimensionless heat transfer coefficient Heat Source 2 Point 1.</td>
<td>300</td>
</tr>
<tr>
<td>7.24</td>
<td>Typical velocity distribution and temperature field oscillating source over board opening $d=0.20$, $V=0.8\pi$, $\omega=4\pi$: (a) velocity, (b) temperature.</td>
<td>301</td>
</tr>
<tr>
<td>7.25</td>
<td>Oscillating source over board opening - level of top heat source surface - geometry layout.</td>
<td>307</td>
</tr>
<tr>
<td>7.26</td>
<td>Oscillating source over board opening at level of top heat source surface - geometry definitions.</td>
<td>307</td>
</tr>
<tr>
<td>7.27</td>
<td>Local dimensionless heat transfer coefficient as a function of time $d=0.20$, $V=0.4\pi$, $\omega=2\pi$: (a) heat source 1, (b) heat source 2.</td>
<td>308</td>
</tr>
<tr>
<td>7.28</td>
<td>Time averaged local dimensionless heat transfer coefficient heat source 2 Point 3.</td>
<td>308</td>
</tr>
<tr>
<td>7.29</td>
<td>Typical velocity distribution and temperature field oscillating source over opening level of top block $d=0.20$ $V=0.4\pi$ $\omega=2\pi$: (a) velocity, (b) temperature.</td>
<td>309</td>
</tr>
<tr>
<td>7.30</td>
<td>Oscillating source over first heat source - geometry layout.</td>
<td>315</td>
</tr>
<tr>
<td>7.31</td>
<td>Oscillating source over first heat source - geometry definitions.</td>
<td>316</td>
</tr>
<tr>
<td>7.32</td>
<td>Local dimensionless heat transfer coefficient as a function of time $d=0.20$ $V=0.8\pi$ $\omega=4\pi$: (a) heat source 1, (b) heat source 2.</td>
<td>317</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES
(Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.33</td>
<td>Time averaged local dimensionless heat transfer coefficient heat source 1 Point 3</td>
</tr>
<tr>
<td>7.34</td>
<td>Typical velocity distribution and temperature field oscillating source over first heat source: ( d=0.20 ) ( V=0.8\pi ) ( \omega=4\pi ): (a) velocity, (b) temperature</td>
</tr>
<tr>
<td>A.1</td>
<td>Comparison of theoretical calculations for fully developed flow to FIDAP© results at channel outlet for constant temperature plates</td>
</tr>
<tr>
<td>A.2</td>
<td>Comparison of theoretical calculations for fully developed flow to FIDAP© results at channel outlet for constant heat flux plates</td>
</tr>
<tr>
<td>A.3</td>
<td>Comparison of temperature contours constant surface temperature of ( 60.0^\circ \text{C} ): (a) experimental results Darbhe, (b) FIDAP© results</td>
</tr>
<tr>
<td>A.4</td>
<td>Comparison of temperature contours constant surface temperature of ( 101.0^\circ \text{C} ): (a) experimental results Darbhe, (b) FIDAP© results</td>
</tr>
<tr>
<td>A.5</td>
<td>Sketch of system investigated</td>
</tr>
<tr>
<td>A.6</td>
<td>Sketch of system investigated for Case A</td>
</tr>
<tr>
<td>A.7</td>
<td>Comparison of results for Case A - plate moving at a constant velocity-fluid initially at rest- theoretical and FIDAP© solutions</td>
</tr>
<tr>
<td>A.8</td>
<td>Sketch of system investigated for Case B</td>
</tr>
<tr>
<td>A.9</td>
<td>Comparison of results for Case B - plate moving from rest increasing linearly to constant velocity-fluid initially at rest- theoretical and FIDAP© solutions</td>
</tr>
<tr>
<td>A.10</td>
<td>Sketch of system investigated for Case C</td>
</tr>
<tr>
<td>A.11</td>
<td>Comparison of results for Case C - plate moving with a sinusoidal velocity-fluid initially at rest - theoretical and FIDAP© solutions</td>
</tr>
<tr>
<td>D.1</td>
<td>Typical control volume used in this study as indicated by shaded area</td>
</tr>
<tr>
<td>E.1</td>
<td>Typical staggered grid</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES
(Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>E.2</td>
<td>u control volume</td>
</tr>
<tr>
<td>E.3</td>
<td>v control volume</td>
</tr>
<tr>
<td>E.4</td>
<td>p, T control volume</td>
</tr>
<tr>
<td>E.5</td>
<td>Sketch of channel inlet region $w$, $W$ location coincides with channel inlet $e$ is location of center first interior $u$ control volume</td>
</tr>
<tr>
<td>E.6</td>
<td>SIMPLER program flow chart</td>
</tr>
<tr>
<td>E.7</td>
<td>A representative grid for finite volume studies</td>
</tr>
<tr>
<td>E.8</td>
<td>Comparison of dimensionless $\tilde{u}_R$ component of velocity from SIMPLER program results to those of FIDAP© for $b_o=0.01m$, $L/b_0=20$, $a_o/b_o=0.10$, $\omega=82.25rad/s$, $q=150 \text{ W/m}^2$: (a) $\tilde{t}_R=0.20$, (b) $\tilde{t}_R=1.60$</td>
</tr>
<tr>
<td>G.1</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) $d=0.05$, $V=0.10$, $\omega=2$, (b) $d=0.05$, $V=0.1\pi$, $\omega=2\pi$, (c) $d=0.05$, $V=0.20\pi$, $\omega=4\pi$, (d) $d=0.10$, $V=0.20\pi$, $\omega=2\pi$, (e) $d=0.10$, $V=0.20\pi$, $\omega=2\pi$, (f) $d=0.10$, $V=0.40\pi$, $\omega=4\pi$</td>
</tr>
<tr>
<td>G.2</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 1: (a) $d=0.05$, (b) $d=0.10$</td>
</tr>
<tr>
<td>G.3</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 2: (a) $d=0.05$, (b) $d=0.10$</td>
</tr>
<tr>
<td>G.4</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 3: (a) $d=0.05$, (b) $d=0.10$</td>
</tr>
<tr>
<td>G.5</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 4: (a) $d=0.05$, (b) $d=0.10$</td>
</tr>
<tr>
<td>G.6</td>
<td>Variation of local heat transfer coefficient along heated surface at dimensionless times indicated $d=0.10$, $V=0.4\pi$, $\omega=2\pi$</td>
</tr>
<tr>
<td>G.7</td>
<td>Variation of surface average heat transfer coefficient with dimensionless time $d=0.10$, $V=0.4\pi$, $\omega=2\pi$</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-----------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>G.8</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) $d=0.05$ $V=0.10$ $\omega=2$ $c=0.30$, (b) $d=0.05$ $V=0.10\pi$ $\omega=2\pi$ $c=0.30$, (c) $d=0.10$ $V=0.20$ $\omega=2$ $c=0.30$, (d) $d=0.10$ $V=0.4\pi$ $\omega=4\pi$ $c=0.30$, (e) $d=0.10$ $V=0.2\pi$ $\omega=2\pi$ $c=0.30$, (f) $d=0.15$ $V=0.6\pi$ $\omega=4\pi$ $c=0.30$, (g) $d=0.20$ $V=0.4\pi$ $\omega=2\pi$ $c=0.30$, (h) $d=0.20$ $V=0.8\pi$ $\omega=4\pi$ $c=0.30$, (i) $d=0.10$ $V=0.4\pi$ $\omega=4\pi$ $c=0.30$, (j) $d=0.10$ $V=0.2\pi$ $\omega=2\pi$, $c=0.15$</td>
</tr>
<tr>
<td>G.9</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 1: (a) $d=0.05$ $c=0.30$, (b) $d=0.10$ $c=0.30$, (c) $d=0.15$ and $d=0.20$ $c=0.30$, (d) $d=0.10$ $c=0.15$</td>
</tr>
<tr>
<td>G.10</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 2: (a) $d=0.05$ $c=0.30$, (b) $d=0.10$ $c=0.30$, (c) $d=0.15$ and $d=0.20$ $c=0.30$, (d) $d=0.10$ $c=0.15$</td>
</tr>
<tr>
<td>G.11</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 3: (a) $d=0.05$ $c=0.30$, (b) $d=0.10$ $c=0.30$, (c) $d=0.15$ and $d=0.20$ $c=0.30$, (d) $d=0.10$ $c=0.15$</td>
</tr>
<tr>
<td>G.12</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 4: (a) $d=0.05$ $c=0.30$, (b) $d=0.10$ $c=0.30$, (c) $d=0.15$ and $d=0.20$ $c=0.30$, (d) $d=0.10$ $c=0.15$</td>
</tr>
<tr>
<td>G.13</td>
<td>Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: $d=0.20$ $V=0.8\pi$ $\omega=4\pi$ $c=0.30$</td>
</tr>
<tr>
<td>G.14</td>
<td>Variation surface average heat transfer coefficient with time: $d=0.20$ $V=0.8\pi$ $\omega=4\pi$ $c=0.30$: (a) left, (b) top, (c) right</td>
</tr>
<tr>
<td>G.15</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) $d=0.05$ $V=0.10$ $\omega=2$ $c=0.3$, (b) $d=0.05$ $V=0.10\pi$ $\omega=2\pi$ $c=0.15$, (c) $d=0.10$ $V=0.20$ $\omega=2$ $c=0.15$, (d) $d=0.10$ $V=0.20\pi$ $\omega=2\pi$, $c=0.15$, (e) $d=0.10$ $V=0.4\pi$ $\omega=4\pi$, $c=0.15$</td>
</tr>
<tr>
<td>G.16</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 1: (a) $d=0.05$, (b) $d=0.10$</td>
</tr>
<tr>
<td>G.17</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 2: (a) $d=0.05$, (b) $d=0.10$</td>
</tr>
</tbody>
</table>
LIST OF FIGURES
(Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.18</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 3: (a) d=0.05, (b) d = 0.10</td>
</tr>
<tr>
<td>G.19</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 4: (a) d=0.05, (b) d = 0.10</td>
</tr>
<tr>
<td>G.20</td>
<td>Variation of local heat transfer coefficient over the heated surface at dimensionless times indicated: d=0.10 V=0.4π ω=4π c=0.15</td>
</tr>
<tr>
<td>G.21</td>
<td>Variation surface average heat transfer coefficient with time: d=0.10 V=0.4π ω=4π c=0.15, (a) left, (b) right</td>
</tr>
<tr>
<td>G.22</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) d=0.10 V=0.20π ω=2π c=0.15, (b) d=0.10 V=0.40π ω=4π c=0.15, (c) d=0.10 V=0.80π ω=8π c=0.15, (d) d=0.20 V=0.80π ω=4π c=0.30</td>
</tr>
<tr>
<td>G.23</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 1: (a) c=0.15, (b) c = 0.30</td>
</tr>
<tr>
<td>G.24</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 2: (a) c=0.15, (b) c = 0.30</td>
</tr>
<tr>
<td>G.25</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 3: (a) c=0.15, (b) c = 0.30</td>
</tr>
<tr>
<td>G.26</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 4: (a) c=0.15, (b) c = 0.30</td>
</tr>
<tr>
<td>G.27</td>
<td>Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: d=0.20 V=0.8π ω=4π c=0.30</td>
</tr>
<tr>
<td>G.28</td>
<td>Variation surface average heat transfer coefficient with time: d=0.20 V=0.8π ω=4π c=0.30, (a) left, (b) top, (c) right</td>
</tr>
<tr>
<td>G.29</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) d=0.10 V=0.20π ω=2π c=0.15, (b) d=0.10 V=0.40π ω=4π c=0.15</td>
</tr>
<tr>
<td>G.30</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 1</td>
</tr>
</tbody>
</table>
## LIST OF FIGURES
(Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.31</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 2</td>
<td>460</td>
</tr>
<tr>
<td>G.32</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 3</td>
<td>461</td>
</tr>
<tr>
<td>G.33</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 4</td>
<td>461</td>
</tr>
<tr>
<td>G.34</td>
<td>Variation of local heat transfer coefficient over the top surface of the element at the dimensionless times indicated: d=0.10 V=0.40π ω=4π c=0.15</td>
<td>462</td>
</tr>
<tr>
<td>G.35</td>
<td>Variation of surface average heat transfer coefficient with time: d=0.10 V=0.40π ω=4π c=0.15, (a) left, (b) top, (c) right</td>
<td>462</td>
</tr>
<tr>
<td>G.36</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) d=0.10 V=0.20π ω=2π c=0.30, (b) d=0.10 V=0.40π ω=2π c=0.30</td>
<td>464</td>
</tr>
<tr>
<td>G.37</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 1</td>
<td>465</td>
</tr>
<tr>
<td>G.38</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 2</td>
<td>465</td>
</tr>
<tr>
<td>G.39</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 3</td>
<td>466</td>
</tr>
<tr>
<td>G.40</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 4</td>
<td>466</td>
</tr>
<tr>
<td>G.41</td>
<td>Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: d=0.20 V=0.40π ω=2π c=0.30</td>
<td>467</td>
</tr>
<tr>
<td>G.42</td>
<td>Variation of surface average heat transfer coefficient with time: d=0.20 V=0.40π ω=2π c=0.30: (a) left, (b) right, (c) top</td>
<td>467</td>
</tr>
<tr>
<td>G.43</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time d=0.20V=0.80π ω=4π c=0.30: (a) block 1, (b) block 2</td>
<td>469</td>
</tr>
<tr>
<td>G.44</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time d=0.20 V=1.6π ω=8π c=0.30: (a) block 1 (b) block 2</td>
<td>470</td>
</tr>
<tr>
<td>G.45</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 1: (a) block 1, (b) block 2</td>
<td>471</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td>------</td>
</tr>
<tr>
<td>G.46</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 2: (a) block 1, (b) block 2</td>
<td>472</td>
</tr>
<tr>
<td>G.47</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 3: (a) block 1, (b) block 2</td>
<td>473</td>
</tr>
<tr>
<td>G.48</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 4: (a) block 1, (b) block 2</td>
<td>474</td>
</tr>
<tr>
<td>G.49</td>
<td>Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: ( d=0.20 \ V=1.6\pi \ \omega=8\pi \ c=0.3 ): (a) block 1, (b) block 2</td>
<td>475</td>
</tr>
<tr>
<td>G.50</td>
<td>Variation of surface average heat transfer coefficient with time: ( d=0.20 \ V=1.6\pi \ \omega=8\pi \ c=0.30 ): (a) block 1 left, (b) block 1 top, (c) block 1 right, (d) block 2 left, (e) block 2 top, (f) block 2 right</td>
<td>476</td>
</tr>
<tr>
<td>G.51</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: ( d=0.10 \ V=0.20\pi \ \omega=2\pi \ c=0.30 ): (a) block 1 (b) block 2</td>
<td>479</td>
</tr>
<tr>
<td>G.52</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: ( d=0.20 \ V=0.40\pi \ \omega=2\pi \ c=0.30 ): (a) block 1 (b) block 2</td>
<td>480</td>
</tr>
<tr>
<td>G.53</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 1: (a) block 1, (b) block 2</td>
<td>481</td>
</tr>
<tr>
<td>G.54</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 2: (a) block 1, (b) block 2</td>
<td>482</td>
</tr>
<tr>
<td>G.55</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 3: (a) block 1, (b) block 2</td>
<td>483</td>
</tr>
<tr>
<td>G.56</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 4: (a) block 1, (b) block 2</td>
<td>484</td>
</tr>
<tr>
<td>G.57</td>
<td>Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: ( d=0.20 \ V=0.4\pi \ \omega=2\pi \ c=0.30 ), (a) block 1, (b) block 2</td>
<td>485</td>
</tr>
</tbody>
</table>
LIST OF FIGURES
(Continued)

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.58</td>
<td>Variation of surface average heat transfer coefficient with time: (d=0.20) (V=0.4\pi\ \omega=2\pi \ c=0.30): (a) block 1 left, (b) block 1 top, (c) block 1 right, (d) block 2 left, (e) block 2 top, (f) block 2 right. Page 486</td>
</tr>
<tr>
<td>G.59</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: (d=0.20) (V=0.40\pi\ \omega=2\pi \ c=0.30): (a) block 1, (b) block 2. Page 489</td>
</tr>
<tr>
<td>G.60</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: (d=0.20) (V=0.8\pi\ \omega=4\pi \ c=0.30): (a) block 1, (b) block 2. Page 490</td>
</tr>
<tr>
<td>G.61</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 1: (a) block 1, (b) block 2. Page 491</td>
</tr>
<tr>
<td>G.62</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 2: (a) block 1, (b) block 2. Page 492</td>
</tr>
<tr>
<td>G.63</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 3: (a) block 1, (b) block 2. Page 493</td>
</tr>
<tr>
<td>G.64</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 4: (a) block 1, (b) block 2. Page 494</td>
</tr>
<tr>
<td>G.65</td>
<td>Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: (d=0.20) (V=0.8\pi\ \omega=4\pi \ c=0.30): (a) block 1, (b) block 2. Page 495</td>
</tr>
<tr>
<td>G.66</td>
<td>Variation of surface average heat transfer coefficient with time: (d=0.20) (V=0.8\pi\ \omega=4\pi \ c=0.30): (a) block 1 left, (b) block 1 top, (c) block 1 right, (d) block 2 left, (e) block 2 top, (f) block 2 right. Page 496</td>
</tr>
<tr>
<td>G.67</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time: (d=0.20) (V=0.40\pi\ \omega=2\pi \ c=0.30): (a) block 1, (b) block 2. Page 499</td>
</tr>
<tr>
<td>G.68</td>
<td>Local dimensionless heat transfer coefficient as a function of dimensionless time (d=0.20) (V=0.80\pi\ \omega=4\pi \ c=0.30): (a) block 1, (b) block. Page 500</td>
</tr>
<tr>
<td>G.69</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 1: (a) block 1, (b) block 2. Page 501</td>
</tr>
</tbody>
</table>
### LIST OF FIGURES
*(Continued)*

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>G.70</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 2: (a) block 1, (b) block 2</td>
<td>502</td>
</tr>
<tr>
<td>G.71</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 3: (a) block 1, (b) block 2</td>
<td>503</td>
</tr>
<tr>
<td>G.72</td>
<td>Time averaged local dimensionless heat transfer coefficient at Point 4: (a) block 1, (b) block 2</td>
<td>504</td>
</tr>
<tr>
<td>G.73</td>
<td>Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: $d=0.20$, $V=0.80\pi$, $\omega=4\pi$, $c=0.30$: (a) block 1, (b) block 2</td>
<td>505</td>
</tr>
<tr>
<td>G.74</td>
<td>Variation of surface average heat transfer coefficient with time: $d=0.20$, $V=0.80\pi$, $\omega=4\pi$, $c=0.30$, (a) block 1 left, (b) block 1 top, (c) block 1 right, (d) block 2 left, (e) block 2 top, (f) block 2 right</td>
<td>506</td>
</tr>
</tbody>
</table>
**LIST OF SYMBOLS**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>English</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>constant (Eq. (A.3))</td>
</tr>
<tr>
<td>$a$</td>
<td>coefficients values in finite volume equations (Eq. (D.10) and (D.11))</td>
</tr>
<tr>
<td>$a_{wall}(t)$</td>
<td>acceleration of moving plate (Eq. (5.3))</td>
</tr>
<tr>
<td>$a_o$</td>
<td>displacement amplitude of moving plate (Eq. (4.2))</td>
</tr>
<tr>
<td>$A$</td>
<td>displacement amplitude of moving plate for FIDAP© models (Eq. (B.51), Eq. (6.1))</td>
</tr>
<tr>
<td>$A$</td>
<td>constant (Eq. (A.4))</td>
</tr>
<tr>
<td>$A$</td>
<td>area as in (Eqs. (D.6), (E.16))</td>
</tr>
<tr>
<td>$A$</td>
<td>weighing function for diffusion/convection for power law differencing scheme finite volume (Eq. (D.7))</td>
</tr>
<tr>
<td>$b$</td>
<td>reference length in y direction for FIDAP© models (Table 2.1, Chapters 6 and 7)</td>
</tr>
<tr>
<td>$b$</td>
<td>channel width (Table 6.1)</td>
</tr>
<tr>
<td>$b$</td>
<td>heating element height and width (Table 6.3)</td>
</tr>
<tr>
<td>$b$</td>
<td>constant term in finite volume form of governing equations as in Eq. E.7</td>
</tr>
<tr>
<td>$b(t)$</td>
<td>$y$ reference for finite volume models (Table 2.1)</td>
</tr>
<tr>
<td>$b(t)$</td>
<td>instantaneous channel width (Eq. (4.2))</td>
</tr>
<tr>
<td>$b_{nc}$</td>
<td>natural convection channel width for reference case (Eq. (4.24))</td>
</tr>
<tr>
<td>$b_o$</td>
<td>mean channel width (Figure 4.1, Eq. (4.2))</td>
</tr>
<tr>
<td>$BIL$</td>
<td>left surface of the first heating element (Table 3.2)</td>
</tr>
<tr>
<td>$BIR$</td>
<td>right surface of the first heating element (Table 3.2)</td>
</tr>
<tr>
<td>$BIT$</td>
<td>top surface of the first heating element (Table 3.2)</td>
</tr>
<tr>
<td>$B2L$</td>
<td>left surface of the second heating element (Table 3.2)</td>
</tr>
<tr>
<td>$B2R$</td>
<td>right surface of the second heating element (Table 3.2)</td>
</tr>
<tr>
<td>$B2T$</td>
<td>top surface of the second heating element (Table 3.2)</td>
</tr>
<tr>
<td>$B3L$</td>
<td>left surface of the third heating element (Table 3.2)</td>
</tr>
<tr>
<td>$B3R$</td>
<td>right surface of the third heating element (Table 3.2)</td>
</tr>
<tr>
<td>$B3T$</td>
<td>top surface of the third heating element (Table 3.2)</td>
</tr>
<tr>
<td>$BH$</td>
<td>heating element/block height (Figure 3.3, Table 6.6)</td>
</tr>
<tr>
<td>$BL$</td>
<td>heating element/block width (Table 6.6)</td>
</tr>
<tr>
<td>$BRDH$</td>
<td>thickness of the board (Figure 3.3)</td>
</tr>
<tr>
<td>$BRTH$</td>
<td>board thickness (Table 6.8)</td>
</tr>
<tr>
<td>$BT$</td>
<td>board thickness (Table 6.3)</td>
</tr>
<tr>
<td>$BW$</td>
<td>heating element/block width (Figure 3.3)</td>
</tr>
<tr>
<td>$c$</td>
<td>constant (Table 4.3)</td>
</tr>
<tr>
<td>$cI$</td>
<td>specific heat of the heating elements (Table 6.3)</td>
</tr>
<tr>
<td>$c2$</td>
<td>specific heat of solids without heat generation (Table 6.3)</td>
</tr>
<tr>
<td>$c_p$</td>
<td>specific heat at constant pressure</td>
</tr>
<tr>
<td>$C$</td>
<td>mean position of oscillating source from heat source (Figure 6.1)</td>
</tr>
<tr>
<td>$C$</td>
<td>source term in finite volume equations (Eq. (D.11))</td>
</tr>
<tr>
<td>$CH$</td>
<td>channel height (Table 6.3)</td>
</tr>
<tr>
<td>$CHL$</td>
<td>lower channel height (Table 6.6)</td>
</tr>
</tbody>
</table>
CHU = upper channel height (Table 6.6)
CLB = plate clearance from block (Table 6.6)
CLH = plate clearance from the hole (Table 7.7)
d = displacement amplitude of moving plate in finite element studies
   (Eq. (B.51)), Chapter 6 and 7

\( d \) = parameter used in pressure/pressure correction equations, finite volume
       SIMPLER (Eq. (E.16))

\( d \) = constant (Table 4.3)

\( d_{\text{Vol}} \) = volume of the control volume \( \Delta x \Delta y \) (Eq.(D.5))

\( D \) = a differential operator (Eq. (C.2))

\( D \) = a measure of the diffusion effects (Eq. (D.8))

e = the control volume surface to the right of the center for SIMPLER
   method

\( E \) = the grid point to the right of the center for the control volume in
       SIMPLER method

\( f \) = constant (Table 4.3)

\( f \) = \( (2\pi/\omega) \) oscillating frequency in parametric studies (Chapter 6 and 7)

\( F \) = measure of convective effects,
      mass flow into control volume (Eq. (D.6))

\( g, g_x, g_y \) = gravitational acceleration and components

\( Gr \) or \( GR \) = Grashof number, measure of buoyancy to shear force (Eq.( B.10))

\( h \) = heat transfer coefficient

\( H_2 \) = height of the lower channel (Figure 3.3)

\( H \) = height of the upper channel (Figure 3.3)

\( HB1L \) = heat transfer coefficient of the left side of the first block (Table 7.3)
       notation follows as defined above.

\( HL \) = width of the hole in the board (Table 7.1)

\( HW \) = hole width (Figure 3.3)

\( i \) = finite volume grid location in x direction (Appendix E)

\( j \) = finite volume grid location in y direction (Appendix E)

\( J \) = convection and diffusion flux (Eq. (D.4))

\( k \) = thermal conductivity

\( k_1 \) = thermal conductivity of the heating element (Table 3.3, Table 6.3)

\( k_2 \) = thermal conductivity of solids not generating heat (Table 3.3, Table 6.3)

\( K \) = parameter (Eq. (4.19))

\( l \) = channel width (Figure A.5)

\( L \) = reference length in x direction for finite volume models (Table 2.1)

\( L \) = half the channel width for squeeze film study (Figure 4.1)

\( L \) = entire channel length for more general model study (Figure 5.1)

\( L_{\text{ref}} \) = reference dimension for system (Eq. (B.1))

\( m \) = the number of element nodes for which \( P_{Di} \) are unknown (Eq. (C.9))

\( m \) = finite volume grid count in x direction (Appendix E)

\( n \) = normal direction (Eq. (B.2))

\( n \) = integer (Eq. (A.3))

\( n \) = any spatial direction (Eq. (B.19))
\( n \) = number of element nodes for which \( u_j, v_j, \) and \( T_j \) are unknown (Eq. (C.8))

\( n \) = finite volume grid count in y direction (Appendix E)

\( n \) = the control volume surface to the north of the center for SIMPLER method

\( N \) = the grid point north of the center for the control volume in SIMPLER method

\( Nu \) = Nusselt number

\( p \) = pressure

\( P \) = center point in the control volume for the finite volume method (Appendix D and E)

\( P_D \) = dynamic pressure (Eq. (2.5))

\( P_H \) = hydrostatic pressure (Eq. (2.6))

\( Pe \) = Peclet number measure of convective to diffusive effects

\( Pr \) = Prandtl number, measure of viscous to thermal diffusion effects

\( PL \) = plate length (Table 6.1)

\( PLB \) = plate length (Table 6.6)

\( PLH \) = plate thickness (Table 6.6)

\( PT \) = plate thickness (Table 6.1)

\( q \) = heat flux

\( Q_{flow} \) = flow rate (Eq. (B.4))

\( Q'' \) = volumetric heat generation rate

\( Re \) = Reynolds number, measure of inertia force to shear force

\( Re_{b0} \) = Reynolds number based on mean channel width (Eq. (5.10))

\( Re_L \) = Reynolds number based on channel length (Eq. (5.10))

\( Res \) = local residual for convergence in finite volume investigation (Eq. (E.41))

Residual = average residual for convergence in finite volume investigation (Eq. (E.42))

\( s \) = boundary curve (Eq. (C.11))

\( s \) = the control volume surface to the south of the center for SIMPLER method

\( S \) = the grid point at the south of the center for the control volume in SIMPLER method

\( S \) = surface area (Eq. (C.10))

\( S \) = source term (Eq. (D.3))

\( \bar{S}_\phi, \bar{S}_1 \) = source term averaged over finite control volume for governing equation for \( \phi \) (Eq. (D.7)), continuity respectively (Eq. (D.6))

\( SL \) = starting length location of first heat source (Figure 3.3, Table 6.6)

\( SPACE \) = length between geometric elements (Figure 3.3, Table 6.6)

\( t \) = time

\( T \) = temperature

\( T \) = period of oscillation (Eq. (B.56))

\( T_I \) = constant surface temperature applied at \( \bar{y}_b = 0 \) (Eq. (5.5))

\( TB_1 \) = average temperature of first block (Table 7.3)

\( TB_2 \) = average temperature of second block (Table 7.3)
$TL = \text{ channel length (Table 6.1)}$

$u, v = \text{ components of velocity}$

$u^e = \text{ the finite element approximate solution for } u \text{ over a typical interior element } e \text{ (Eq. (C.1)), similar expressions for other unknowns}$

$u_j = \text{ the finite element approximate solution for } u \text{ at node } j \text{ on element } e \text{ (Eq. (C.1)), similar expressions for other unknowns}$

$U = \text{ reference velocity (Eq. (B.1))}$

$V = \text{ volume (Eq. (C.10))}$

$V = \left( \frac{V}{A} \right) \text{ measure of velocity in parametric studies (Chapters 6 and 7)}$

$V(t) = \text{ velocity of moving plate}$

$w = \text{ the control volume surface to the right of the center for SIMPLER method}$

$W = \text{ the grid point to the left of the center for the control volume in SIMPLER method}$

$W = \text{ channel width (Figure 3.3)}$

$W = \text{ weighing function (Eq. (C.4))}$

$x, y, z = \text{ coordinates}$

$Z = \text{ channel length (Figure A.5)}$

**Vectors**

$F = \text{ a vector (Eq. (C.10))}$

$N = \text{ outwardly directed normal vector to the surface (Eq. (C.10))}$

$\tilde{s} = \text{ surface length vector (Eq. (B.4))}$

$\tilde{v} = \text{ velocity vector (Eq. (B.4))}$

**Matricies**

$[A] = \text{ a coefficient matrix (Eq. (C.30))}$

$[B] = \text{ a coefficient matrix (Eq. (C.30))}$

**Greek letters**

$\alpha = \text{ thermal diffusivity}$

$\alpha = \text{ relaxation factor as in (Eq. (E.13))}$

$\beta = \text{ coefficient of volumetric expansion}$

$\delta x = \text{ distance between the W (Or E) and P grid points in the SIMPLER method (Figure D.1)}$

$\delta y = \text{ distance between the N (Or S) and P grid points in the SIMPLER method (Figure D.1)}$

$\Delta x = \text{ distance between the } e \text{ and } w \text{ control volume surfaces in the SIMPLER method (Figure D.1)}$

$\Delta y = \text{ distance between the } n \text{ and } s \text{ grid points in the SIMPLER method (Figure D.1)}$

$\varepsilon = \text{ residual for finite element approximation (Eq. (C.3))}$

$\Gamma = \text{ diffusion parameter (Eq. (D.3))}$
\( \mu \) = dynamic viscosity
\( \nu \) = kinematic viscosity
\( \omega \) = oscillation frequency
\( \phi \) = \( \phi \) in FIDAP© parametric studies (Chapters 6 and 7)
\( \psi_{ij} \) = interpolation function associated with node \( j \) in element \( e \) finite element method (Eq. (C.1))
\( \rho \) = density
\( \rho_1 \) = density of the heating element (Table 6.3)
\( \rho_2 \) = density of the board (Table 6.3)
\( \theta \) = an unknown vector (Eq. (C.30))

Subscripts

\( (-)_{a} \) = applied heat flux/heat rate
\( (-)_{am} \) = related to atmospheric conditions
\( (-)_{avg} \) = average
\( (-)_{comp} \) = component of the gravitational acceleration (Eq. (B.1))
\( (-)_{e} \) = relating to the \( e \) surface of the finite volume (Eq. (D.5))
\( (-)_{equ} \) = equivalent properties in Appendix B
\( (-)_{equ} \) = related to governing equations as in (Eq. (E.41))
\( (-)_{F} \) = FIDAP© dimensionless parameter (Eq. (B.1))
\( (-)_{M} \) = modified dimensionless variables (Eqs. (4.24, 5.26, 5.27, 5.27a))
\( (-)_{n} \) = relating to the \( n \) surface of the finite volume (Eq. (D.5))
\( (-)_{nb} \) = refers to 4 neighboring finite volume grid points (Eq. (E.9))
\( (-)_{new} \) = modified as in (Eqs. (4.8) and (5.9))
\( (-)_{o} \) = evaluated at local ambient temperature conditions (Eq. (2.6))
\( (-)_{o} \) = oscillation parameter as in (Eqs. (4.1-4.2))
\( (-)_{os} \) = of the oscillation source in FIDAP (Eq. (B.51), Eq. (6.1))
\( (-)_{old} \) = relating to the previous iteration (Eq. (E.11), Eq. (E.12))
\( (-)_{p} \) = relating to control volume point \( P \) as in (Eq. (D.7))
\( (-)_{p} \) = relating to pressure in finite volume method
\( (-)_{pp} \) = relating to pressure correction in finite volume method
\( (-)_{Q} \) = squeeze film dimensionless parameter (Eq. (4.3))
\( (-)_{R} \) = more general model dimensionless parameter (Eq. (5.4))
\( (-)_{ref} \) = reference
\( (-)_{s} \) = relating to the \( s \) surface of finite volume (Eq. (D.5))
\( (-)_{s} \) = solid
\( (\cdot)_s \) = relating to the heated surface (Eq. (4.20), Eq. (5.20))
\( (\cdot)_u \) = relating to \( u \) component of velocity in finite volume method
\( (\cdot)_v \) = relating to \( v \) component of velocity in finite volume method
\( (\cdot)_T \) = relating to temperature in finite volume method
\( (\cdot)_w \) = relating to the \( w \) surface of finite volume (Eq. (D.5))
\( (\cdot)_{wall} \) = at wall for squeeze film study (Eq. (4.1))
\( (\cdot)_{wall} \) = at wall for more general model (Eq. (5.2))
\( (\cdot)_x \) = in the \( x \) direction
\( (\cdot)_y \) = in the \( y \) direction
\( (\cdot)_{hi} \) = solid, heat generating (Eq. (B.30))
\( (\cdot)_{h2} \) = solid, not heat generating (Eq. (B.30))
\( (\cdot)_\phi \) = related to governing equation for unknown \( \phi \) (Eq. (D.3))
\( (\cdot)_{avg} \) = average (Table 6.1)

Superscripts
\( (\cdot)^p \) = of the previous time step (Eq. (D.5))
\( (\cdot)^* \) = of the finite previous element (Eq. (C.1))
\( (\cdot)^+ \) = updated solution values as in (Eq. (E.20))
\( (\cdot)^- \) = correction factor as in (Eq. (E.19))

Notation
\( \overline{\cdot} \) = dimensionless
\( \overline{\overline{\cdot}} \) = quantity averaged over a surface, dimensionless
\( \overline{\overline{\overline{\cdot}}} \) = quantity averaged over a surface and time, dimensionless
\( \overline{\overline{\overline{\overline{\cdot}}}} \) = surface averaged, time averaged
\( \cdot \) = time derivative (Eq. (C.6))
\( \tilde{\cdot} \) = \( \cdot \) \( - \) \( \cdot \) \( o \) (Eq. (C.7))
\( \hat{\cdot} \) = pseudo velocities used in SIMPLER method (Eq. (E.14))
1.1 Objective

The objective of this dissertation is to investigate the feasibility of three static and/or dynamic methods of enhancing pure natural convection cooling of electronics. The three methods investigated are:

1. Alternate cross-flow passages
2. Small transversely oscillating plates strategically placed near the heat sources
3. A combined method using both the cross-flow passages and the strategically placed oscillating plate near the heat sources

The investigations in this work are carried out through a series of numerical experiments. These numerical simulations are used to determine the pressure, velocity, and temperature fields in a system to which the particular cooling method has been applied. A finite volume program was developed for studies of a simplified geometry while a commercially available finite element package was used for more complex models and boundary conditions. Employing these numerical tools, parametric studies varying geometric, heat rate, or oscillation parameters were performed. The velocity and temperature field data obtained was then used to calculate measures of the cooling effect including the average surface temperatures and local and average heat transfer coefficients.

Through analysis and comparisons of the results of the parametric studies and comparisons to the results for reference natural convection cases, the cooling performance of each method was evaluated. Insight into the manner in which each
method may act to enhance natural convection cooling is gained from the examination of the velocity and temperature field results. Analyzing the differences in the results for different parameters provides a better understanding of the roles the various parameters play in establishing any cooling effect and contributes to the knowledge of the basic cooling mechanisms at work. Most importantly, when measures of the cooling effect are compared to those for a reference case, estimations of the amount of cooling possible and the parameter range over which any improvement may occur can be established. Based on this information, the practicality and the potential thermal benefits of each method can be weighed, and a well informed judgment as to the feasibility and viability of the three methods as techniques for enhancing natural convection cooling of electronics can be made.

In the remainder of this chapter, the motivation for the study of such cooling enhancement techniques is explained along with background information, a review of the related literature, and a brief discussion of the scope of the present work.

1.2 Motivation for Study
While the thermal conditions of any system can affect its performance, the thermal conditions of electronic devices are particularly critical. Even a temperature rise as small as 2°C can cause a 10% reduction in the performance of the electronics [1]. Elevated temperatures reduce the speed of operation of the electronic components as well as their life span. In addition, high temperatures can leave electronic devices prone to mechanical failure as a result of thermally induced deformations [2]. The decreasing size of electronic components coupled with their rising power dissipation rates has made the
task of controlling the thermal conditions of an electronic system more demanding, necessitating the development of more effective means of cooling these devices [1-4].

Spurred by the proliferation of increasingly diverse, compact, and portable applications of consumer electronics, there is particular interest in the development of cooling enhancement techniques for such lower power consuming devices. In addition to meeting certain thermal design constraints, the cooling techniques employed must be mindful of both the needs of the human end users as well as the economic considerations of the manufacturers. Hence, the feasibility of a cooling method is dependent not only on its effectiveness, but also on its ability to provide for safe, quiet, reliable, energy efficient, and cost effective operation in a compact form that is easily portable [1-3, 5-11].

Based on these criteria, natural convection appears to be the ideal cooling method [12-15]. However, natural convection cooling is effective over a limited range of heat dissipation rates. When natural convection cooling is insufficient, forced convection cooling with a rotary fan is commonly adopted to meet the cooling requirements for many, lower power consuming consumer electronics. Despite its frequent use, the rotary fan has many drawbacks. Among these are low reliability and energy efficiency, space and noise concerns, as well as a susceptibility to mechanical failure and the detrimental effects of the accumulation of particulate matter [5, 6, 16-20]. While certain higher power dissipating electronics may require the use of forced convection cooling, many of the consumer-type electronics operate where natural convection cooling alone may not be sufficient, yet the use of forced convection is inefficient. In this transitional domain, there is a need to explore alternative intermediary cooling methods that can adequately cool without the use of standard forced convection. In this way, the operating regime of
natural convection can be extended to higher heat rates, allowing for some of its benefits to be retained, and delaying the need to use standard forced convection. More effective cooling can then be achieved in a manner that better meets the additional criteria described above. This thesis undertakes the study of such alternative natural convection enhancement cooling approaches.

1.3 Natural Convection

Natural convection is a simple, safe, cost effective, efficient and reliable cooling method and is the preferred and widely used method of cooling lower power consuming devices. It makes use of no moving parts, requires no additional power or physical components, and produces no additional heat or noise [12, 14, 15]. As a result, there is a great interest in natural convection cooling that has produced an extensive body of related research. Early investigations into fundamental natural convection channel flow and temperature fields including the studies of Elenbaas [21] and Bar-Cohen and Rosenhow [22, 23] as well as the numerical work of Aung [24, 25], Engel and Mueller [26], and Aihara [27] report important information about natural convection channel flow and cooling and its differences from forced convection. Buoyancy effects alone resulting from the temperature differences and the ensuing density and pressure differences drive the flow as there is no external source forcing the flow. Therefore, the flow distribution and flow rate in the channel and thus the subsequent cooling effect are determined solely by the system configuration. Specific flow conditions cannot be independently prescribed as they can be in mainly forced convection flows. Any changes applied to the channel system will alter the flow rate through the channel and, thus, the cooling effect. One of
the more apparent conditions affecting the flow and heat transfer is the heat source distribution. Velocities within the channel are higher near a hotter channel wall than those near a cooler wall. Another factor influencing the natural convection cooling effect is the channel orientation. A vertical orientation of a channel containing heat sources permits through flow along the entire channel length and allows for the development of higher buoyancy forces. (In this work, the terms “heat source” and “heating element” refer to an electronic component.) Therefore, greater flow velocities can be achieved and heat can be more readily carried out of the channel. Also, the channel spacing affects the flow development and temperature field. When the channel width is decreased while holding other parameters fixed, the buoyancy induced channel flow is restricted, decreasing the channel flow rate and hence reducing the cooling effect. Because each enhancement method modifies the standard channel system, each enhancement method alters the channel flow conditions. In order to best capture the potential of natural convection cooling, the natural convection flow characteristics discussed above must be kept in mind when considering methods by which the channel system is to be modified to produce heat transfer enhancement.

1.4 Placement of Current Work in Existing Knowledge

Though extensive research into natural convection cooling exists, the application of enhancement techniques to natural convection is quite limited [12]. Enhancement techniques can be categorized as static methods, which rely on changes in the geometry or material properties to improve thermal conditions, and dynamic methods, which involve the motion of some device or surface. Static methods are more favorable since
they require no energy input and are typically simpler; however, active methods typically have more cooling potential.

To better understand the means by which an enhancement method acts to improve the cooling of electronics, it may be useful to consider the typical flow, temperature, and heat distributions common in electronic devices. The typical electronics system consists of an array of heat sources attached to a solid board in an open-ended enclosure. Studies of the effects of such arrays of heat sources have been carried out for both forced convection in the works of Moffat and Ortega [28], Garimella and Eibeck [17], and Tou [15] and in free convection flows in the works of Heindel and Incropera [29], Choi and Ortega [30], Asako and Faghri [31], Lin and Chen [32], Kwak and Song [33], and Desrayaud and Fichera [34]. Despite the flow source differences, certain common flow and temperature field characteristics are found among these studies since the electronic components tend to obstruct the flow no matter the flow source. Because of the flow obstruction caused by the heat sources, low velocities develop in the regions between and behind the heat sources. High temperatures and low fluid temperature gradients result in these areas due to the fact that the heated fluid is trapped in these regions. Areas of higher velocities are confined to just above the heat source. Consequently, the temperatures are lower and temperature gradients are higher near the top surfaces than near other exposed heat source surfaces and most of the heat is removed from the top surfaces of the heat sources. To improve the thermal conditions of the electronics, the cooling enhancement methods studied in the present investigation and in previous research must strive to increase the fluid velocities near all heat source surfaces, to increase the heat transfer rate to the fluid in the vicinity of the heat source surfaces, and to
more evenly distribute the heat flows among the heat source surfaces. The higher velocities thin the boundary layers, increase the temperature gradients in the fluid nearby the heat source surfaces, improve the interaction of fluid flow streams from different regions in the system, and reduce the size and magnitude of high temperature regions.

Among the simplest static means of cooling enhancement is the alteration of the arrangement or placement of discrete heat sources within the flow channel. A few of these studies are mentioned below. For natural convection flows, the effects of the spacing between the heat sources have been studied by many including Kwak and Song [33] who published correlations relating the Nusselt number to the spacing as well as the array dimensions. Liu [35] studied the optimal location of a three heat source array in a natural convection cooled enclosure and found that with the heat sources at the “optimal location”, a 10% reduction in the maximum temperature could be achieved relative to that occurring with the standard equally spaced arrangement. In forced convection flows, Bazydola [36] found that offsetting heated elements in an array improved the heat transfer from the downstream heat sources. Jurban [37] concluded that removing one element from an array increased the Nusselt numbers at downstream heat sources. While the heat source layout and geometric parameters certainly influence the temperature and flow field, the enhancement potential caused by altering these conditions is limited.

Further enhancement by static means can be achieved by altering the typical channel components so as to alter the overall flow patterns in the channel. Among the few investigations for natural convection heat transfer enhancement is the work by Sparrow and Prakash [12] that indicated the replacement of an array of “continuous parallel plates” by a staggered array of short vertical plates in a vertical channel enhanced
heat transfer significantly. The placement of a fixed body such as a cylinder in the flow channel cooled by forced convection has been shown by Ratts et al. [38], among others, to act as a vortex generator causing increased fluid velocities near the heated surfaces downstream of the body. Kim and Cho [39] demonstrated that altering the standard rectangular channel inlet region geometry can promote better fluid mixing throughout the channel.

Still other static heat transfer enhancement methods aim to change the local velocity and heat flow patterns in the vicinity of the heat source. Such investigations have been largely focused on forced convection dominant flows. Some of the more novel investigations are discussed below. In a 2002 experimental investigation, Herman and Kang [40] showed that the placement of curved vanes above the downstream edge of a heat source can be used as a static means of heat transfer enhancement. The vanes tend to accelerate the flow between the blocks and to deflect the flow into the previously low velocity regions between the heat sources, leading to increased temperature gradients near all exposed heat source surfaces. The results indicate that the vanes can result in heat transfer values 1.5 to 3.5 times greater than those without the vanes. However, the pressure drop in the channel was of some concern.

Ould-Amer et al. [18] numerically investigated the enhancement of forced convection cooling by the placement of porous materials between the heating elements. For Reynolds numbers (Re) from 100 to 1000, a decrease in the maximum operating temperature of up to 34% is reported, most likely as a result of higher heat removal rates from the side heat source surfaces due mainly to conduction through the porous material.
Hence, this method is only of benefit when the conductivity of the material is much greater than that of the fluid.

Another static method of forced convection heat transfer enhancement is the use of perforations or openings in the direction of the main flow in the flow obstructions themselves as was experimentally investigated by Sara et al. [41]. The study demonstrates that the openings create important flow paths through the previously low velocity-high temperature regions between the flow obstructions. The size of each opening rather than the number of openings was found to have more influence on the cooling effect. Openings angled towards the bases of the obstructions provided a maximum of a 60% increase in the heat removal of relative to the standard geometry. However, such a use of openings is not practical for electronic components.

A more practical use of such alternate flow passages for electronic components particularly is the use of cross-flow passages created by openings in the board to which the heat sources have been attached. The use of such passages has been investigated numerically by Kim and Anand [42] and has been reported in a number of publications by Hung [8, 9, 43]. In a numerical investigation, Kim and Anand [42] tested the effects of thin slots placed at different positions between an array of rectangular heat sources in forced convection. The study results showed that under certain conditions the presence of the slots allowed for increased flow in the low velocity regions between the heat sources causing increased temperature gradients in the fluid near the side heat source surfaces and increased side surface Nusselt numbers. Though any cooling effect may be highly dependent on the opening arrangement and the heat source dimensions, Kim and Anand [42] state that even a “small increase in the Nu of the block is very important to
lowering the block temperature and maximum temperature” and, therefore, is very important in controlling the thermal conditions of a system. Based on the results of the study, the use of openings has potential to be a practical cooling method under the proper conditions.

In 1999 Hung [9] numerically investigated the use of the cross-flow passages for a single geometry and heat rate in forced convection dominant flows for openings that extend completely between the heating elements. Again, the results indicate the openings allow for higher velocities in the regions between the heat sources. Hence, higher velocity gradients occur near the exposed heat source surfaces. This leads to increased temperature gradients in the nearby fluid, and thus increased heat removal from the heat source and lower maximum heat source temperatures. Hung concluded that the openings produce the greatest cooling effect at the heat source surface downstream of the openings and that the new flow paths created by the openings can be used to alter the locations of the higher temperature regions. However, no specific quantitative comparisons to results without openings are made.

In a 2001 publication, Hung [8] numerically investigated the use of these openings in pure natural convection, mixed convection, and forced convection for an array of heat sources with four different shapes for a single arrangement and set of dimensions. The channel used in these studies is oriented so that the gravitational force acts perpendicular to the channel length and parallel to the flow path through the opening. In Hung’s study, the effects of conduction in the board are minimized by setting an artificial board thermal conductivity. Hung concluded that the cross-flow method functions best for lower Re flows where buoyancy effects dominate and consequently
there is greater flow through the openings. While the horizontal orientation provides for
greater flow through the openings, the channel orientation parallel to the gravitation force
may result in more favorable overall flow as discussed in Section 1.3. The results also
show that those heat source shapes with the greatest exposed surface area performed best.
However, there is little firm comparison between the results for a standard solid board
and those with the openings.

Though the use of openings in the electronics board or alternate cross-flow
passages appears to be a viable method for enhancing natural convection cooling in a
vertically oriented channel containing heat sources, the present knowledge concerning the
use of alternate cross-flow passages with natural convection cooling in a vertically
oriented channel is insufficient to draw conclusions as to its cooling potential relative to
standard natural convection and the conditions under which it may be beneficial.
Therefore, the current study extends the previous investigations of the alternate cross-
flow passage concept by applying it to natural convection in a vertically oriented channel.
Figure 1.1 depicts a possible arrangement for the actual implementation of the alternate
cross-flow passages. A two-dimensional simplification of flow passages is illustrated in
Figure 1.2, which allows for easier understanding of the workings of this method. With
the vertical channel arrangement, the openings generate alternate cross-flow paths
through the low velocity regions between adjacent heating elements in the following
manner. The high temperature fluid near the base of the heat sources has a lower density
than that of the surrounding fluid. The resulting buoyancy and pressure effects combined
with lower pressures over the tops of the heat sources in the upper channel due to the
higher main channel flow velocities cause the heated fluid nearby the heat source to rise
and join with the main flow along the channel length. Cooler fluid rushes in through the openings to replace the higher temperature fluid as indicated in the figure. While the flow through the openings is of a lower velocity than that of the main channel flow, the velocities near the heat source surfaces in the vicinity of the openings are greater than their values with no flow paths. These higher velocities promote higher temperature gradients in the vicinity of the heat source, increasing the heat removal from the area near the heat source, lowering the maximum temperatures, and increasing the heat transfer coefficients.

**Figure 1.1** Possible implementation of alternate cross-flow passages.

**Figure 1.2** Alternate cross-flow passage model.
In order to be able to properly judge the cooling potential and practicality of the flow passage-natural convection enhancement technique, the following areas need to be addressed. First, because little information is known about the relationship between the various system parameters and the cooling potential of the cross-flow passage enhanced natural convection, parametric studies are needed to estimate the potential cooling effect, to provide an understanding of the role the parameters play in such a cooling effect, and to establish whether beneficial cooling can occur over a feasible range of parameters. Second, Incropera [4], Fugesi [44], Chen [32], and Tou [15] all state that, particularly for natural convection, conjugate heat transfer may have a significant influence on the temperature field, and so conduction effects in the solid bodies should be included in any models. Finally, a more reasonable opening geometry than that in Hung’s [8, 9, 43] study should be used. To allow for proper attachment of the heat source to the board some material needs to be retained to the sides of the heat source. (See Figure 1.2.) Since the opening geometry affects the flow near the opening and thus the cooling effect, investigations with a more realistic opening geometry are important. To facilitate a better understanding of the potential and feasibility of the use of alternate cross-flow passages in a vertically oriented channel, the current research addresses the needs described above.

When the cooling rates for static heat transfer enhancement methods are inadequate to meet a cooling requirement, a greater cooling effect can generally be achieved through the use of active enhancement methods. Hence, the majority of the present work involves investigations of active methods of heat transfer enhancement. While there are many active methods of heat transfer enhancement, one of the most important active cooling enhancement methods involves the use of oscillations to
promote higher velocities near the heat sources, to increase mixing of fluid streams, and to improve heat removal. Cooling enhancement with vibrations has been the subject of numerous investigations. Early studies into the effects of oscillations determined the impact of free stream oscillations or plate oscillations on the flow and thermal conditions in the laminar boundary layers at the heated surface [45, 46]. The placement of a heated part in an acoustical field [47] and the application of mechanical vibrations to the heated part itself [48, 49] as well as the use of flow-induced vibrations in forced convection [7, 50-52] were later investigated and found to promote better fluid mixing and heat transfer, though these methods may not be practical for sensitive electronics.

Numerous studies have also been conducted concerning the effects of imposing oscillating inlet velocities or pressure gradients on an open-ended channel, yielding a significant amount of useful information about the fundamental effects of oscillating flow on heat transfer. These studies conclude that as the heated fluid is repeatedly swept away and replaced by cooler bulk fluid, the oscillations alter the thermal boundary layer near the heated surface and lead to improved heat transfer [53-57]. While such results clearly indicate that under the proper conditions, oscillating flows can produce substantial heat transfer enhancement, the cooling effect produced by such an application of the oscillations is more global in nature and cannot be easily used to target specific high temperature regions.

Instead of imposing oscillations on inlet flow conditions, controlled oscillations and movements of discrete bodies within the channel under forced convection conditions have also been explored as a means of cooling enhancement. In a series of publications, Fu and Tong [58-61] have numerically demonstrated that for forced convection flow in a
channel, the transverse oscillations of a cylinder placed upstream of the heat sources can produce as much as a 116% increase in the overall Nusselt number in a channel. Yang [62] concluded numerically that a transversely oscillating bar upstream of a heat source could result in a 74% increase in the Nusselt Number. Again, while these studies indicate that significant heat transfer enhancement can result from the oscillations of a finite body in forced convection flows, the oscillation sources are used in a manner that relies on altering the general flow pattern throughout the channel downstream of the oscillation source and does not allow for focused cooling of the heat sources.

A more effective use of the oscillations to produce heat transfer enhancement may be the placement of discrete oscillation sources in the immediate vicinity of the heat source for a more localized and focused cooling effect. Studies of the use of oscillations in such a manner are scarcer. Khaled and Vafai [63, 64] analytically and numerically investigated the effects of transverse oscillating of one bearing surface on the flow and temperature field in an incompressible squeeze film bearing and showed that under the proper conditions the oscillations could improve the thermal conditions. Krussing [16] performed a limited experimental investigation of the cooling of a heated plate through the use of a vibrating piezoelectric beam and numerically simulated the cooling effects by adjusting the boundary conditions and material properties in a heat conduction problem. While Hyun et al. [65] suggest the use of oscillating piezoelectric devices to enhance cooling, their results are limited to a discussion of the frequencies, velocities, and displacements capable of being produced by piezoelectric fans. Information more relevant to the cooling potential of piezoelectric oscillation sources can be found in the naphthalene sublimation experiments performed by Schmidt [66] which are used to
determine the local and average mass transfer at a surface in the vicinity of two oscillating piezoelectric fans. The results indicate the greatest mass transfer occurs in the immediate vicinity of the oscillation source. More recently, in a series of publications, Garimella et al. [5,6] investigated the potential use of piezoelectric fans for electronics cooling applications. One area of investigation involves the application of a piezoelectric fan device to a heat source in an enclosure system for two fan orientations; a "vertical orientation" with the fan blade sweeping "side to side" over the heat source with full, partial, and no coverage of the heat source and a "horizontal orientation" with the fan blade moving "up and down" with no coverage of the heat source. The greatest cooling effect was found to occur for the partial coverage vertical orientation, where as much as a 100% increase in the heat transfer coefficient at the heated surface relative to pure natural convection was achieved. In another part of the study, the use of the piezoelectric fans situated at "hot spots" in a conventional fan-cooled laptop was investigated. Thermocouple readings showed that the air exiting the laptop was of higher temperature with piezoelectric fans operating than without, indicating improved heat removal. These investigations have recently been extended to numerical investigations. Despite their limited nature, these investigations illustrate that the use of discrete oscillating sources has potential for heat transfer enhancement.

As seen in the survey of related literature, while there appears to be practical potential for the use of strategically placed oscillation sources as a means of natural convection heat transfer enhancement, the scope of the present knowledge of this method does not allow for a clear assessment as to its viability, particularly for natural convection. The present research builds upon the current knowledge of the use of
oscillation sources to enhance heat transfer and investigates the enhancement of natural convection cooling of an electronic component in a vertically oriented channel through the use of transverse oscillations of a small plate placed in close proximity, but not in contact with, the electronics. (See Figure 1.3.) Because the oscillation source is placed in the immediate vicinity of the electronic component, the higher local fluid velocities, higher velocity and temperature gradients, thinner thermal and momentum boundary layers, and improved fluid and thermal mixing resulting from the oscillations are local to the heat source. The higher temperature gradients that develop in the fluid nearby the heat source as a consequence of the oscillations lead to higher rates of heat removal from the heat source region and lower heat source temperatures. A more effective, focused, and localized cooling of the electronic components as compared to the more global cooling produced by the conventional fan can be achieved in this way. (Previous studies have shown that oscillations may be highly beneficial when applied to flows with lower velocities induced by natural convection [67].) A piezoelectric fan may be a convenient and practical means of producing these contactless fluid vibrations, and the piezoelectric fan holds a number of advantages over cooling with a typical conventional fan. Figure 1.4 shows the possible implementation of the use of the piezoelectric fan for electronic cooling purposes. Piezoelectric fans require lower power input, generate less heat, and are more reliable than conventional fans [5, 6, 19, 65, 68, 69]. In addition, they produce no electromagnetic interference. With the appropriate design and frequency limitations, the possibility of audible noise can be minimized [5, 6]. Thus, the use of the local application of transverse oscillations has practical potential as a means of augmenting natural convection, and its use is further explored in the present work.
In order to properly assess the feasibility of the use of the local oscillation sources in the manner indicated above, the following issues need to be addressed. First, as with the alternate cross flow passage method, the potential cooling enhancement that can be
achieved through this method has not been well established over a viable range of system parameters. Particularly for oscillations used in conjunction with the natural convection cooling, the oscillation amplitude is likely a key parameter. Though higher oscillation displacement amplitudes can displace more fluid and, hence, may cause greater fluid mixing, the higher displacements can also act to constrict the natural convection induced channel flow. Parametric studies are needed to determine the extent to which the two cooling methods can work together to yield a beneficial cooling effect over a range of realistic oscillation and geometric system parameters. Second, most of the previous published studies neglect the effects of conduction in the oscillation source and the heat source/channel system. Again, especially for lower velocity flows, the conduction in the solid bodies may have important effects on the temperature and flow distributions in the channel and needs to be included in the models. Finally, a short, thin rectangular plate oscillation source geometry instead of the rectangular bar or circular cylinder used in many of the previous numerical studies may provide for better flow conditions. The smaller cross sectional area of the plate geometry in the direction perpendicular to the main flow likely causes less flow constriction while allowing for the displacement of a significant amount of fluid. By including these points, the current research aims to provide the information necessary for a better understanding of the potential of the use of transverse oscillations of a discrete short thin plate to enhance natural convection cooling in a vertically oriented channel.

An extension of the two natural convection enhancement techniques described above is the combined use of the strategically placed oscillation sources together with the alternate cross-flow passages. Little information about such a means of enhancement
exists, with the closest related studies investigating the effects of oscillating the velocities of a jet impinging in channel such as the studies of Sert [70] and Taneda [71] among others. Under the transverse oscillation – opening enhancement scheme, the potential for further enhancement exists as the cooler air brought closer to the heat sources through the openings may be better mixed with the heated fluid through the use of locally applied oscillations. Taking into account the issues discussed previously for the individual heat transfer enhancement methods, parametric studies including the conjugate heat transfer problem need to be done in order to determine whether the method can produce a significant cooling effect compared to pure natural convection. The current research investigates the use of this combined heat transfer enhancement method in natural convection in a vertically oriented channel.

1.5 Scope of Present Work

As described above, three static or dynamic methods are investigated as means of extending the regime over which natural convection cooling in a vertical channel can be used to meet cooling requirements. Through the information gathered in this investigation, a more well-informed judgment can be made as to the feasibility of the use of three static or dynamic methods for enhancing natural convection cooling in a vertically oriented channel. Specifically, in order to achieve these goals, the three methods are investigated in the following manner.

The first set of investigations examines the feasibility of the alternate cross-flow passages as a means of natural convection heat transfer enhancement through two-dimensional finite element studies for a three heat source array including the conjugate
heat transfer effects. For two opening arrangements, parametric studies were performed varying heat source, channel, and opening geometric dimensions for fixed heat source heat rates and varying the heat rates for fixed geometric dimensions.

The next set of investigations examines the use of transverse oscillations of a short thin rectangular plate to enhance natural convection in a vertical channel and is divided into two main studies. The fundamental effects of the transverse oscillations were investigated in the first portion of this study through two-dimensional finite volume investigations of the flow and temperature field within a channel with one fixed heated wall and one oscillating insulated wall. As a first approximation, the initial model used the squeeze film assumptions and neglected the natural convection effects. Parametric studies of the oscillation parameters and channel width were involved. In a later model, the inertia terms and the natural convection effects were included and parametric studies of the oscillation parameters, channel width, and applied heat rate were then performed.

The second portion of study of the use of the transverse oscillation involves two dimensional finite element investigations of the effects of the transverse oscillations of a short thin plate two dimensional oscillation source in a vertical channel including natural convection effects. A number of channel geometries and arrangements were studied. The first finite element model set investigated the oscillations of a short plate two dimensional oscillation source placed near a constant heat flux fixed channel wall. With this model, parametric studies varying the oscillation parameters were performed. A more complex geometry was investigated in the second finite element model set which involves the oscillations of a short plate centered above a single rectangular heat source (two dimensional oscillation source and heat source) in a vertically oriented channel.
Conduction in the heat source as well as the board and oscillation source was accounted for. The mean spacing between the oscillation source and heat source top surface was varied as were the oscillation parameters. The third set of finite element models for this portion of the investigation was for a single rectangular heat source in the channel with modified arrangements of the oscillation source. The arrangements modeled include an oscillation source centered above an upstream unheated or “dummy” rectangular solid, an oscillation source centered above a thin unheated heat source plate extension, and an oscillation source located slightly upstream of the heat source. The oscillation parameters and, in some cases, the clearance spacing were varied and conduction in the heat source, oscillation source, and board were included in the third set of models.

The third set of investigations of the current research examines the combined alternate cross-flow passage transverse oscillation method. Initially, a plain channel with an oscillation source above an opening placed in a board in the vertically oriented channel was studied for varied oscillations parameters and with conduction in the oscillation source modeled. Then, finite element two-dimensional studies for a model with an opening between two rectangular heat sources were performed for various oscillation source arrangements. These arrangements include an oscillation source slightly upstream of the first heat source, an oscillation source centered above the opening in the cavity between the heat sources, an oscillation source centered above the opening but at a level above the top of the heat sources, and an oscillation source centered above the first heat source. The oscillation parameters were varied and conduction in the heat source, oscillation source, and board were included in the model.
The governing equations and assumptions used in this investigation, and the results and conclusions of this study follow. Chapter 2 includes the general governing equations used in the current work, along with the general assumptions made and an explanation of the prescribed channel inlet and outlet boundary conditions. Chapter 3 reports the results of the finite element alternate cross-flow passage investigation. The finite volume numerical investigation of the channel with the oscillating wall under the squeeze film velocity field assumptions is discussed in Chapter 4. The finite volume numerical investigation of the channel with the oscillating wall for a more general model including inertia and natural convection effects is discussed in Chapter 5. Chapter 6 presents the finite element numerical investigations of the application of transverse oscillations for the various heat source geometries and system arrangements. The results of the finite element numerical investigations of the combined alternate cross-flow passage transverse oscillation method are discussed in Chapter 7. In each of the Chapters 3 through 7, the specific problem descriptions and assumptions used in each problem are given along with the results. Finally, in Chapter 8, the conclusions of the work are discussed and some recommendations for future work are made.
CHAPTER 2
PROBLEM FORMULATION

2.1 General Problem Description

This work numerically investigates three different natural convection heat transfer enhancement techniques. While the model components, geometry, and specific modeling assumptions used in each individual study may differ, each study is interested in determining the heat and fluid flow that develops in a vertically oriented channel containing heat generating elements. Therefore, certain basic assumptions are common among all studies of this work as is the manner in which the channel inlet and outlet boundary conditions are prescribed.

The general procedure followed for each particular model investigated follows. After the problem specific modeling assumptions and boundary conditions have been applied, numerical parametric studies are performed to determine the temperature, velocity, and pressure fields in the vertically oriented channel. Analysis of these results yields measures of the cooling effect of the various enhancement techniques. Assessment of the effectiveness and viability of the three cooling enhancement techniques is based on comparisons of these measures for different parameter values and for those resulting under pure natural convection conditions.

In this chapter, the general modeling assumptions used in this work are explained. The resulting general set of governing equations that must be solved to establish the potential cooling effect of each method is presented. Finally, the common channel inlet and outlet boundary conditions used in this investigation are stated.
2.2 General Modeling Assumptions

Because the intent of this research is to evaluate the feasibility of the three static or dynamic natural convection enhancement methods, any assumptions used must allow for the effects of the cooling enhancement technique alone to be observed. In response to such a goal the following assumptions are made in the current investigation. The flow and temperature fields are two dimensional. (The channel depth is large relative to the channel length and width.) The flow is laminar. The fluid used is air that is assumed to be a Newtonian, incompressible constant property substance except for the density in the weight force term in the momentum equations. In this term, the Boussinesq approximation is employed to account for changes in the air density with temperature. In comparison to the heat transferred, the viscous dissipation, radiation, and internal heat generation in the fluid are negligible. For simplicity, all geometries used including the flow channel, heating elements, openings, and vibration sources are rectangular. Where appropriate, the heat sources are modeled as rectangular solids with uniform properties and a uniform volumetric heat generation rate, and the solid board and vibration sources are modeled as rectangular solids with constant and uniform properties. All contact between solids and solids and solids and fluids is assumed to be thermally perfect where appropriate.
2.3 General Governing Equations

The general governing equations necessary to determine the heat transfer and fluid flow for the problem under investigation are the continuity equation, the Navier-Stokes equations, and the energy equation. Under the general assumptions described above, the governing equations for the temperature, pressure, and velocity fields in the fluid in the vertically oriented channel and the governing equation for the temperature field of any solid system components are presented below. Because the natural convection effects are included, the governing equations are coupled and the velocity field $u(x,y,t)$, $v(x,y,t)$, the pressure field $p(x,y,t)$, and the temperature field $T(x,y,t)$ can not be determined independently. For a rectangular coordinate system oriented as shown in Figure 2.1, after applying the assumptions described above, the continuity equation takes the form:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

(2.1)

Figure 2.1 Vertically oriented channel system.
The x and y components of the momentum equation or the Navier-Stokes equations are given by:

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + \rho g_x
\]  
(2.2)

\[
\rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right) + \rho g_y
\]  
(2.3)

where the only body force acting is the gravitational force. For the vertically oriented channel with the coordinate system in Figure 2.1, \( g_x = -g \) and \( g_y = 0 \).

The energy equation is given by:

\[
\rho c_v \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)
\]  
(2.4)

In the momentum equation, the local static pressure can be seen as the sum of the hydrostatic pressure, \( P_H \), due to “quiescent” hydrostatic forces, and the dynamic pressure, \( P_D \), due to the motion of the fluid, or

\[
p = P_H + P_D
\]  
(2.5)

For the coordinate system and channel orientation of the current study, the hydrostatic pressure at local “quiescent” ambient conditions, \( P_H \), is given by:

\[
P_H = \rho_0 g_x x + \text{constant}
\]  
(2.6)

where the \( \sigma \) subscript denotes the ambient fluid conditions. For this study the hydrostatic pressure is measured relative to the hydrostatic pressure at \( x = 0 \). With \( g_x = -g \) and \( g_y = 0 \) for the given orientation, the hydrostatic pressure can be expressed as:

\[
P_H = -\rho_0 g_x x \left. + \frac{p_{\text{atm}}}{x=0} \right]
\]  
(2.7)

Consequently, the static pressure gradient in the x direction can be written as:
To simplify this momentum equation, the Boussinesq approximation is often employed when the temperature and therefore property changes are relatively low. In this approximation,

\[
(\rho_o - \rho) \approx \beta \rho (T - T_o) \quad (2.10)
\]

where the coefficient of volumetric expansion, is given by:

\[
\beta = -\frac{1}{\rho} \frac{\partial \rho}{\partial T} \quad (2.11)
\]

Applying this Boussinesq approximation to Eq. (2.9) yields:

\[
\rho \left( \frac{\partial \dot{u}}{\partial t} + u \frac{\partial \dot{u}}{\partial x} + v \frac{\partial \dot{u}}{\partial y} \right) = -\frac{\partial P_D}{\partial x} + \mu \left( \frac{\partial^2 \dot{u}}{\partial x^2} + \frac{\partial^2 \dot{u}}{\partial y^2} \right) + g \beta \rho (T - T_o) \quad (2.12)
\]

Therefore, under the assumption used, the set of governing equations for the fluid is:

\[
\frac{\partial \dot{u}}{\partial x} + \frac{\partial \dot{v}}{\partial y} = 0 \quad (2.1)
\]

\[
\rho \left( \frac{\partial \dot{u}}{\partial t} + u \frac{\partial \dot{u}}{\partial x} + v \frac{\partial \dot{u}}{\partial y} \right) = -\frac{\partial P_D}{\partial x} + \mu \left( \frac{\partial^2 \dot{u}}{\partial x^2} + \frac{\partial^2 \dot{u}}{\partial y^2} \right) + g \beta \rho (T - T_o) \quad (2.12)
\]

\[
\rho \left( \frac{\partial \dot{v}}{\partial t} + u \frac{\partial \dot{v}}{\partial x} + v \frac{\partial \dot{v}}{\partial y} \right) = -\frac{\partial P_D}{\partial y} + \mu \left( \frac{\partial^2 \dot{v}}{\partial x^2} + \frac{\partial^2 \dot{v}}{\partial y^2} \right) \quad (2.13)
\]
For the solid bodies only the temperature field in the solid must be determined, so the energy equation is the only governing equation. Under the assumptions described above, the energy equation in a solid is given by:

$$\rho c_p \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right)$$

(2.7)

For the solid bodies only the temperature field in the solid must be determined, so the energy equation is the only governing equation. Under the assumptions described above, the energy equation in a solid is given by:

$$\rho c_p \left( \frac{\partial T}{\partial t} \right) = k \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q''$$

(2.14)

where $Q''$ represents an externally applied volumetric heat source and $s$ indicates a solid.

The above governing equations were then applied to the problems under investigation in a modified form as they were non-dimensionalized. The presentation of the results of an investigation in dimensionless form allows for the results of a limited investigation under specific conditions to be applicable to a much broader range of conditions and parameters. In addition, the dimensionless computational domain, equations, and related parameters can be more convenient to work with. For these reasons, dimensionless forms of the governing equations are solved and non-dimensional results are presented. Throughout this work, four different sets of dimensionless variables are utilized for the following reasons. The finite element commercial package restricts the forms of the dimensionless variables that can be used. For these finite element studies, one set of dimensionless variables is used for the finite element investigations with a constant heat flux heat source and another for the constant volumetric heat generating heat source. Appendix A contains the verification information for the finite element package, FIDAP©, used in this investigation. The specific dimensionless forms of the governing equations used in FIDAP© are explained in Appendix B. An explanation of the finite element method for heat and fluid flow
problems is provided in Appendix C. Another set of dimensionless variables is used for the squeeze film velocity field finite volume investigation. This set of dimensionless variables and the resulting form of the governing equations are discussed in Chapter 4 and Appendix D. Finally, yet another set is used for the more general finite volume investigation where the acceleration and natural convection terms are included, and details about this set of variables and governing equation forms are provided in Chapter 5 and Appendix E. For dimensionless variables defined in Eq (2.15), Table 2.1 provides a summary of the reference values used for the various dimensionless variables.

\[
\tilde{x} = \frac{x}{x_{ref}}, \quad \tilde{y} = \frac{y}{y_{ref}}, \quad \tilde{u} = \frac{u}{u_{ref}}, \quad \tilde{v} = \frac{v}{v_{ref}},
\]

\[
\tilde{t} = \frac{t}{t_{ref}}, \quad \tilde{p} = \frac{P}{P_{ref}}, \quad \tilde{T} = \frac{T - T_c}{\Delta T_{ref}}.
\]

(2.15)

### Table 2.1 Reference Values Used In Non-Dimensionalizing the Governing Equations

<table>
<thead>
<tr>
<th>Study</th>
<th>(x_{ref})</th>
<th>(y_{ref})</th>
<th>(u_{ref})</th>
<th>(v_{ref})</th>
<th>(p_{ref})</th>
<th>(t_{ref})</th>
<th>(\Delta T_{ref})</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIDAP - Constant Heat Flux Heat Source</td>
<td>(b)</td>
<td>(b)</td>
<td>(\sqrt{bg \beta \Delta T_{ref}})</td>
<td>(\sqrt{bg \beta \Delta T_{ref}})</td>
<td>(bg \beta \rho \Delta T_{ref})</td>
<td>(b/u_{ref})</td>
<td>(\frac{q_o b}{k})</td>
</tr>
<tr>
<td>FIDAP - Constant Volumetric Rate Heat Source Solid**</td>
<td>(b)</td>
<td>(b)</td>
<td>(\sqrt{bg \beta \Delta T_{ref}})</td>
<td>(\sqrt{bg \beta \Delta T_{ref}})</td>
<td>(bg \beta \rho \Delta T_{ref})</td>
<td>(b/u_{ref})</td>
<td>(\left(\frac{Q_{in}}{\rho c_p}\right)^{\frac{2}{3}} \left(\frac{b}{\beta g}\right)^{\frac{1}{3}})</td>
</tr>
<tr>
<td>Finite Volume - Squeeze Film</td>
<td>(L)</td>
<td>(b(t))</td>
<td>(\omega L)</td>
<td>(\omega b_o)</td>
<td>(\mu \omega \left(\frac{L}{b(t)}\right)^2)</td>
<td>(1/\omega)</td>
<td>(\frac{q_o b_o}{k})</td>
</tr>
<tr>
<td>Finite Volume - Complete Equations ***</td>
<td>(L)</td>
<td>(b(t))</td>
<td>(\omega L)</td>
<td>(\omega b_o)</td>
<td>(\rho \omega L^2)</td>
<td>(1/\omega)</td>
<td>(\frac{q_o b_o}{k})</td>
</tr>
</tbody>
</table>

* See Section 3.2, 6.3, and 7.2.
** See Section 3.3, 6.4, 6.5, and 7.3.
*** See Chapter 4.
**** See Chapter 5.
2.4 Discussion of Channel Inlet and Outlet Boundary Conditions

For natural convection studies, the channel inlet and outlet boundary conditions must be carefully selected to ensure that the flow that develops in the channel is a result of the buoyancy effects alone, not the boundary conditions as described in Chapter 1. In order to make certain of this, the boundary conditions described in this section are applied to the various models in this work. This facilitates a fair comparison of the cooling potential and the feasibility of the various cooling methods being investigated. These conditions are based on reasonable engineering assumptions and simplifications. In the discussion in this section, the inlet is taken to be a location where flow is into the channel and the outlet is taken to be a location where flow is out of the channel.

Since the flow at the channel inlet is determined by the buoyancy effects that develop in the channel for natural convective flows, the inlet velocity “cannot be prescribed, either in terms of intensity or in terms of profile” [72]. The flow velocities and rates at the inlet vary with the cooling method used as well as with the particular set of system parameters. Attempts to fix the flow conditions across all studies of the various enhancement methods will fail to capture the effects of the cooling enhancement method. Anticipating the use of the cooling methods for a multiple channel arrangement with a small channel width, a thin wall thickness, and a sharp entrance, it is reasonable to assume that the flow at the inlet has only one component with the specific distribution to be determined. Hence, the y component of velocity at the inlet is set to zero while the x component of velocity at the inlet is solved for from the continuity equation. This inlet flow condition is used by Aung [24, 25], Ortega [28], Amon [73], and others in their natural convection studies.
Since the flow is not known at the inlet (or outlet), the pressure must be prescribed. In this study, the dynamic pressure defined in Section 2.3 is set to zero at both the channel inlet and outlet. This requires that the pressure at the inlet and outlet be equal to the local hydrostatic pressure. Such pressure conditions have been used by Amon [73], Chen [57, 74], Ziskind [75], Aung [24, 25], Bodia [76], and Engel [26] under similar natural convection channel studies.

The temperature at the channel inlet is set to the ambient temperature. Previous research has shown this is a valid boundary condition for Grashof numbers greater than 400 as there is no significant heat transfer from the heat source in the channel towards the channel inlet. [77, 78] This condition is used in many studies including those of Amon [73], Desrayaud and Fichera [34, 79], Aung [24, 25], and Ortega [28] among others.

Finally, a zero gradient outlet boundary condition for the temperature and the \( y \) component of velocity is also used in the present study. This allows for the investigation of the local effects with a reasonably sized computational model while permitting a good approximation of the physical system. The outlet \( x \) component of velocity is determined from the continuity equation. These outlet boundary conditions are widely used in the literature. Among the publications making use of these boundary conditions are those by Ortega [28] and Zebib [80].

By specifying the boundary conditions in this manner, the effects of the natural convection enhancement methods and not the effects of the boundary conditions can be investigated. The discussion in this chapter leads to a clearer understanding of the research undertaken in the current study. Any additional assumptions and specifications necessary are stated in the more detailed descriptions of individual studies.
CHAPTER 3

FINITE ELEMENT NUMERICAL INVESTIGATION OF
ALTERNATE CROSS-FLOW PASSAGES

3.1 Introduction

The static cooling enhancement method of the alternate cross-flow passages is a simple
safe and efficient method that makes use of no additional components and requires no
power input. The sole modification to the standard system is the placement of openings
in the solid board that holds the heat sources. This method can be used to augment
standard natural convection under conditions where natural convection cooling is
insufficient, but the use of the convectional rotating fan is inefficient. The concept behind
the use of these openings was discussed in Section 1.4. The opening of alternate cross-
flow passages between the heating elements as in Figure 1.1 creates flow paths through
the low velocity, high temperature regions that typically develop between the heating
elements with the standard electronics geometry. The flow through these pathways not
only increases the velocities near the heat source side surfaces, but also provides a means
by which heat can be more readily carried away from the vicinity of the heat source.
With more heat leaving the region between the heat sources and the higher velocities and
velocity gradients, higher temperature gradients near the heat source surface and lower
heat source operating temperatures result. However, the extent of the cooling effect and
its relationship to the system parameters has not been well established, particularly with
natural convection. Thus, the viability of the alternate cross-flow passages as a practical
cooling method is not known.
In order to determine the feasibility of the use of alternate cross flow passages for enhancement of natural convection in a vertically oriented channel containing heat sources, the effect this method has on the flow and temperatures fields in such a system needs to be examined. Hence, the primary aims of the current investigations were to establish an estimate of the potential cooling effect offered by the alternate cross-flow passages relative to pure natural convection cooling (the standard closed board geometry), to gain insight into the role the heat source and geometric parameters play in the determination of the level of the cooling effect, and to broaden the understanding of the basic mechanisms involved. To meet these objectives, the current research undertook three major finite element studies named:

1. Preliminary Investigation of Alternate Cross-Flow Passages
2. Parametric Studies of Alternate Cross-Flow Passages with Conduction in Solids
3. Parametric Studies of Alternate Cross-Flow Passages with Conduction in Solids with a Modified Passage Arrangement

In this chapter, the three studies performed are reviewed and the results of these studies are presented. Conclusions as to the feasibility of the method may then be drawn.

3.2 Preliminary Investigation of Alternate Cross-Flow Passages

A brief preliminary finite element investigation of the use of alternate cross-flow passages was performed to establish the merit of the use of alternate cross-flow passages for natural convection enhancement. A discussion of this study and its results follow.
3.2.1 Problem Statement Preliminary Study of Alternate Cross-Flow Passages

In the preliminary study, the alternate cross-flow passage cooling method was applied to a system consisting of a three heat source array attached to a board in a vertically oriented channel. A three heat source array was selected because it is the minimum number of heat sources that can reveal the effects of the flow paths [81]. Figure 3.1 depicts the model geometry for the system with the alternate cross-flow passages where flow passages are created upstream and downstream of each heat source. Figure 3.2 depicts the model geometry for the system with no openings - the standard geometry. Each of these two models was then subjected to the same conditions and assumptions. Where appropriate, the general assumptions discussed in Section 2.2 were applied. In addition, in this preliminary study, conduction in all solid model components was neglected. A constant heat flux was applied to the left, top, and right exposed surfaces of each of the three heat sources. All other fluid-solid boundaries were held thermally insulated. A no slip boundary condition was applied to all fluid-solid boundaries. The inlet/outlet boundary conditions described in Section 2.4 were applied. (A uniform inlet fluid temperature and a zero transverse (y) component of velocity were specified. The gradients of the temperature and the transverse component of velocity were set to zero at the outlet. At the inlet and outlet, the dynamic pressure was set to zero.) The specific dimension, heat flux, and material property parameters used in this study are listed in Table 3.1.

The finite element models of the two systems described above (with and without the openings) were generated using the FIDAP© program. The same program was used to solve for the resulting velocity and temperature fields. A 377 x 137 quadratic
quadrilateral element mesh was used for this finite element investigation with finer mesh grading at the channel inlet and at fluid-solid boundaries, particularly near the heat source surfaces. A sensitivity study of the effect of the mesh on the solution was performed and showed a more dense mesh yielded results with under a 0.5% difference.

The results for the models with and without the flow passages were then compared to investigate the effects that the new flow paths have on the fluid and thermal conditions in the channel system.

Figure 3.1 Geometry for alternate cross-flow passage model.

Figure 3.2 Geometry for standard closed board model.
Table 3.1 Data Used in Preliminary Investigation of Alternate Cross-Flow Passages with Conduction in Heating Elements and Board Neglected

<table>
<thead>
<tr>
<th>Material Property*</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>k (Thermal conductivity of air)</td>
<td>0.027 W/mK</td>
</tr>
<tr>
<td>γ (kinematic viscosity of air)</td>
<td>1.717e-5 m/s²</td>
</tr>
<tr>
<td>ρ (density of air)</td>
<td>1.12492 kg/m³</td>
</tr>
<tr>
<td>c_p (specific heat of air)</td>
<td>1005.93 J/kg K</td>
</tr>
<tr>
<td>β (volumetric expansion)</td>
<td>1/315.5K</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>g (gravitational acceleration)</td>
<td>9.81 m/s²</td>
</tr>
<tr>
<td>T_o (ambient temperature)</td>
<td>25°C</td>
</tr>
<tr>
<td>q_a (applied heat flux)</td>
<td>150 W/m²</td>
</tr>
<tr>
<td>Grashof Number based on BH</td>
<td>15122.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimensional Parameter**</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH (block height) = L_ref (length basis)</td>
<td>0.250 in = 0.00635 m</td>
</tr>
<tr>
<td>BH (block height)</td>
<td>1 BH</td>
</tr>
<tr>
<td>SL (starting length)</td>
<td>4 BH</td>
</tr>
<tr>
<td>BW (block width)</td>
<td>1 BH</td>
</tr>
<tr>
<td>H (upper channel height)</td>
<td>2 BH</td>
</tr>
<tr>
<td>W (channel length)</td>
<td>14 BH</td>
</tr>
<tr>
<td>SPACE (spacing)</td>
<td>0.20 BH</td>
</tr>
<tr>
<td>HW (hole width)</td>
<td>0.60 BH</td>
</tr>
<tr>
<td>BRDH (board thickness)</td>
<td>0.10 BH</td>
</tr>
<tr>
<td>H2 (lower channel height)</td>
<td>0.75 BH</td>
</tr>
</tbody>
</table>

*Properties at T_avg = 42.5°C = 315.5 K
**Dimensions defined in Figure 3.3.

Figure 3.3 Dimension labels for alternate cross-flow passage models.
3.2.2 Results

From the results of this investigation the typical flow and temperature field characteristics that occur with the use of the alternate cross-flow passages can be obtained and from this information the effect that these patterns have on the heat flow in the channel and thus the cooling of the heat sources can be explored. In the results for this chapter the rectangular heating elements may be referred to as "blocks". The "lower channel" refers to the channel portion below the board with no heat sources while the "upper channel" refers to the channel portion containing the heat sources.

3.2.2.1 Velocity Field. The results of the preliminary study showed that significant flow through the openings is possible through this method. Some observations about the alterations in the velocity field produced by the openings are discussed below. The typical pressure contour plot for the geometry with the openings given in Figure 3.4 allows for better understanding of how the pressure in the channel that results from the temperature field and geometry allows for the flow through the openings. Because of the higher flow velocities in the upper channel, the pressures are lower than those in the lower channel. As explained in Chapter 1, the buoyancy effects and corresponding pressure forces resulting from the lower density higher temperature fluid near the heat source side surfaces together with this pressure difference between the lower and upper channels cause fluid motion through the openings. Figure 3.5a and Figure 3.5b depict the velocity distributions with and without the flow paths. At the most upstream opening, just before the first heat source, the flow is diverted from the upper channel to the lower channel. This is a result of the system geometry as well as the small temperature differences in the channel upstream of the heat sources. For the remainder of the
openings, flow occurs from the lower channel to the upper channel with the flow rates and velocities through the openings increasing for openings further downstream where the temperature differences and, therefore, buoyancy effects are higher. Figure 3.6a and Figure 3.6b show a typical velocity field through a new flow path created by an opening and the flow in the corresponding region for the standard geometry, respectively. With the openings, there is substantial flow through the region between the heat sources with a non-uniform distribution across the opening width. The flow enters the openings at an angle such that the majority of the flow impinges on the side surface of the heat source surface that is downstream of the opening. Without the openings low velocity circulations develop between adjacent heat sources. It was also found that since the opening flow must move around the top upstream corner of a heat source, possibly separating from the top heat source surface, some of the main channel flow is diverted away from the top surface of the heat source. This results in slightly lower velocities over a portion of the top surfaces of the heat source relative to the case with no openings. Relative to the effect of the increased velocities in the regions between adjacent heat sources, the effect of the slight decrease in the velocities near the top surfaces of the heat source is minor. Because the opening of the flow paths causes higher velocities in the region where high temperatures tend to develop in the standard geometry and increases the velocity gradients near the heat source surfaces, the openings should prove beneficial to the overall thermal performance of the system.

3.2.2.2 Temperature Results. The changes in the flow patterns resulting from the new flow paths alter the temperature field distributions. The temperature distribution for the alternate cross-flow passage model is shown in Figure 3.7a, while that for the
standard geometry without any openings is given in Figure 3.7b. In the standard geometry, the stagnant nature of the velocity field in the region between the heat sources permits little heat removal from the region. The side surfaces are thermally "dead" with low heat transfer coefficients and high temperatures. With the flow paths, the cooler fluid brought through the openings and directed towards the side surfaces of the heat sources alters the temperature field. The higher velocity gradients and the increased transport of the heat produced by the heat sources away from the heat source region and into the main channel flow results in higher temperature gradients near the side surfaces than those with the standard closed board geometry. Instead of a stagnant higher temperature area occupying most of the region between the heat sources, a much more confined higher temperature region appears in the area along the side surfaces near the tops of the heating elements with significantly lower temperature magnitudes than for the standard geometry. The regions of higher temperatures correspond exactly to the regions of lower velocities seen in Figure 3.5a. For the set of parameters studied, the maximum dimensionless operating temperature with the openings decreased by 40% from that with no openings. Also, the heat transfer coefficients on the side heat source surfaces increase with up to a 165% increase for the parameters investigated. The maximum drop in the heat transfer coefficient at a top heat source surfaces was only 17% as a result of the altered flow pattern. (All dimensionless variables used in FIDAP© are defined in Appendix B.) Therefore, the improvement in the heat transfer coefficients from the side surfaces overcomes the slight decrease at the top surface. The heat transfer coefficient data is presented in Table 3.2.
From the results of the preliminary investigation, the use of the alternate cross-flow passages has potential to be a viable method of effectively enhancing laminar natural convection in a vertically oriented channel and further study is warranted.

**Table 3.2** Summary Comparison of Average Heat Transfer Coefficient Results for Preliminary Investigation

<table>
<thead>
<tr>
<th>Surface</th>
<th>Dimensionless Heat Transfer Coefficient No Openings</th>
<th>Dimensionless Heat Transfer Coefficient Openings</th>
<th>Percentage Difference Openings from No Openings</th>
</tr>
</thead>
<tbody>
<tr>
<td>B1L</td>
<td>0.1010</td>
<td>0.1034</td>
<td>2.41%</td>
</tr>
<tr>
<td>B1T</td>
<td>0.1299</td>
<td>0.1201</td>
<td>-7.52%</td>
</tr>
<tr>
<td>B1R</td>
<td>0.0446</td>
<td>0.0702</td>
<td>57.45%</td>
</tr>
<tr>
<td>B2L</td>
<td>0.0360</td>
<td>0.0656</td>
<td>82.37%</td>
</tr>
<tr>
<td>B2T</td>
<td>0.0737</td>
<td>0.0728</td>
<td>-1.25%</td>
</tr>
<tr>
<td>B2R</td>
<td>0.0366</td>
<td>0.0824</td>
<td>125.24%</td>
</tr>
<tr>
<td>B3L</td>
<td>0.0314</td>
<td>0.0835</td>
<td>165.92%</td>
</tr>
<tr>
<td>B3T</td>
<td>0.0592</td>
<td>0.0694</td>
<td>17.41%</td>
</tr>
<tr>
<td>B3R</td>
<td>0.0404</td>
<td>0.0925</td>
<td>128.95%</td>
</tr>
</tbody>
</table>

*Note: The number in the surface name refers to the heating element number (numbered from left to right) L, T, R stands for the left, top, and right heating element surfaces, respectively.*

**Figure 3.4** Typical pressure contours with alternate cross-flow passages.
Figure 3.5 Velocity field preliminary investigation: (a) alternate cross-flow passage geometry, (b) standard board geometry.
Figure 3.6 Detailed view of velocity field between heat sources preliminary investigation (third shown): (a) alternate cross-flow passage geometry, (b) standard board geometry.
Figure 3.7 Temperature contours from preliminary investigation: (a) alternate cross-flow passage geometry, (b) standard board geometry.
3.3 Parametric Studies of Alternate Cross-Flow Passages with Conduction in Solids

The results of the preliminary investigation indicate that the use of the alternate cross-flow passages has the potential for causing significant improvement in the thermal conditions within a vertically oriented channel; however, conduction in the board and heating elements was neglected. As discussed in Chapter 1, the accompanying conduction in solids may play an important role in the heat transfer for lower velocity flows like natural convection. Because the solid conduction alters the temperature field in the vicinity of the openings, it may have a strong influence on the flow through the openings and thus on the potential flow passage cooling effect. The results of the preliminary investigation may then be looked upon as an "upper limit" of the cooling potential of the flow paths. The second finite element study of the use of alternate cross-flow passages undertakes an investigation of the use of alternate cross-flow passages where conduction in all solid bodies is modeled. The results of such a study should provide a more realistic account of the cooling effect as well as the role the system parameters play in achieving this cooling.

3.3.1 Problem Statement Parametric Studies of Alternate Cross-Flow Passages with Conduction in Solids

In this second investigation, parametric studies of the effects of the alternate cross-flow passages on the flow and thermal conditions in a vertically oriented channel containing a three heat source array cooled by natural convection were performed for models taking into account conduction in all solids. Where appropriate, the assumptions stated in Section 2.2 are applied. A constant volumetric heat generation rate is supplied to each of the three heat sources, shown in Figure 3.1 and Figure 3.2. The overall upper and lower
enclosing channel walls are insulated as well as the active board ends. Again, at all fluid-solid boundaries a no-slip boundary condition is applied. The same inlet/outlet boundary conditions as in the first study are used in this second study. Realistic material properties are applied to the board and heat source solids to allow for the modeling of the effects of conduction in all solids. The material properties used as well as the heat and geometric parameter values used in the first case investigated in the parametric studies are listed in Table 3.3. Under these assumptions and conditions, the values of certain geometric and heat rate parameters were varied in order to better understand the influence of the system parameters on the workings of the alternate cross-flow passage cooling technique. Judgment of the effectiveness of this method can then be made by comparing the thermal conditions in the channel with the flow passages to the equivalent channel with no board openings.

**Table 3.3** Data Used in Investigation of Alternate Cross-Flow Passages Including Conduction in Heating Elements and Board

<table>
<thead>
<tr>
<th>Material Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k ) (Thermal conductivity of air)</td>
<td>0.027 W/mK</td>
</tr>
<tr>
<td>( \nu ) (kinematic viscosity of air)</td>
<td>1.717e-5 m/s²</td>
</tr>
<tr>
<td>( \rho ) (density of air)</td>
<td>1.12492 kg/m³</td>
</tr>
<tr>
<td>( c_p ) (specific heat of air)</td>
<td>1005.93 J/kg K</td>
</tr>
<tr>
<td>( \beta ) (volumetric expansion of air)</td>
<td>1/315.5K</td>
</tr>
<tr>
<td>( k_1 ) (thermal conductivity of heating elements)</td>
<td>186 W/mK</td>
</tr>
<tr>
<td>( k_2 ) (thermal conductivity of board)</td>
<td>0.26 W/mK</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_o ) (ambient temperature)</td>
<td>25°C</td>
</tr>
<tr>
<td>( Q_a^{*\prime} ) (volumetric heat generation)</td>
<td>702.99e3 W/m³</td>
</tr>
<tr>
<td>( Gr ) (Grashof number based on BH)</td>
<td>1157.8</td>
</tr>
</tbody>
</table>
Table 3.3 continued

<table>
<thead>
<tr>
<th>Dimensional Parameter**</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BH (block height)= (length basis)</td>
<td>0.250 in = 0.00635 m</td>
</tr>
<tr>
<td>SL (starting length)</td>
<td>4 BH</td>
</tr>
<tr>
<td>BH (block height)</td>
<td>1 BH</td>
</tr>
<tr>
<td>BW (block width)</td>
<td>1 BH</td>
</tr>
<tr>
<td>H (upper channel height)</td>
<td>2 BH</td>
</tr>
<tr>
<td>W (channel length)</td>
<td>14 BH</td>
</tr>
<tr>
<td>Space (spacing)</td>
<td>0.20 BH</td>
</tr>
<tr>
<td>HW (hole width)</td>
<td>0.60 BH</td>
</tr>
<tr>
<td>BRDH (board thickness)</td>
<td>0.10 BH</td>
</tr>
<tr>
<td>H2 (lower channel height)</td>
<td>0.75 BH</td>
</tr>
</tbody>
</table>

*Properties at $T_{avg} = 42.5^\circ C = 315.5 \, K$  
**Dimensions defined in Figure 3.3.

3.3.2 Results

The results of this study demonstrate that the effects of the conduction in the solids do have a major influence on the velocity and temperature fields and thus the heat flow in the channel. Typical velocity and temperature distributions for the models with and without the flow passages are shown in Figure 3.8 through Figure 3.10. Figure 3.11 through Figure 3.14 contain bar graphs comparing a sample of the heat source heat transfer coefficients as well as the average heat source temperatures from the parametric study results. A summary of the results are presented in Table 3.4 through Table 3.6.

3.3.2.1 General Results. Even with the inclusion of conduction through the heat sources and board, the alternate cross-flow passages continue to have a positive effect on the flow and temperature field in the channel. However, the cooling effect achieved with the openings where conduction in the solids is modeled is slightly lower than that found in the preliminary investigation of Section 3.2. When conduction in the solids is modeled, the temperatures in the board rise as some heat moves from the attached heat sources and possibly from the surrounding heated fluid through the solid board and to the
surrounding cooler fluid further upstream, downstream, or in the lower channel. Consequently, the temperature differences in the vicinity of the openings are lower than those where conduction is neglected, decreasing the magnitude of the buoyancy effects and lowering the flow rates through each opening. However, the flow through the openings that does result continues to alter the flow velocities, particularly in the areas between the heat sources, and thus continues to change the temperature distribution and improve the thermal conditions.

Many of the flow characteristics are similar to those found in the preliminary investigation. The typical velocity field with openings is shown in Figure 3.8a where Figure 3.8c is without openings. Figure 3.9a and Figure 3.9c show typical velocity fields between heat sources that occur for the cases with and without the openings. As in the previous study, the higher velocities in the region between the heat sources occur near the heat source side surface downstream of the opening. Except for the first opening, fluid flows through the openings from the lower channel to the upper channel with the velocities and flow rates at the downstream openings higher than those at the further upstream openings. The greatest flow rate and velocities through any opening occur at the fourth opening. However, because the flow through the fourth opening is diverted away from the side surface of the third heat source, as seen in Figure 3.8a, it produces little cooling effect on the downstream side of the surface of the third heat source. Typically between 40 and 50% of the flow that enters the lower channel flows through the openings to the upper channel resulting in a significant drop in the fluid flowing through the downstream portion of the lower channel. The minimal heat input into the lower channel downstream of the final opening is the likely cause of this effect.
Some observations on the impact this altered velocity field has on the temperature and heat flow distribution in the channel for models including conduction in the solids are described next. Comparing Figure 3.10a and Figure 3.10c, the changes in the temperature field caused by the new flow paths are apparent. Not only are temperature values lower, but also the high temperature region is reduced in size and has been moved away from the bases of the heat sources. As previously stated, the flow through the openings helps to transport heat away from the region between adjacent heat sources and therefore results in higher temperature gradients in the fluid near the heat source surfaces, lower temperatures in the region between the heat sources, and higher heat transfer coefficients and heat fluxes from the sides of the heat sources. (See Table 3.4 and Table 3.5.) Also, as discussed in Section 3.2, compared with heat source surfaces with no openings, heat transfer coefficients (and heat fluxes) from the top surfaces of heat sources with openings decreased slightly as a result of the somewhat lower velocities in the region caused by the movement of the opening flow around the top upstream corner of the heat source. Since there is conduction in the board as well as the heating elements, the opening geometry also alters the flow in the lower channel. Because of the higher velocities, velocity gradients, and thus temperature gradients in the fluid just below the heat sources resulting from the opening flow, more heat is conducted through the bottom heat source surfaces into the board, and, subsequently into the lower channel than in the model with no openings. The increase in the heat transfer from bottom and side surfaces of the heat source more than compensates for the slight decrease in heat transfer from the top heat source surface. With the modified heat flow patterns from the heat source caused by the openings, for the same geometry as in the preliminary study, the maximum dimensionless
heat source temperature resulting from the alternate cross-flow paths drops by only 12% from the value for the standard geometry compared to the 40% decrease obtained in the preliminary investigation. Another important finding is that for most parameter cases studied, the temperature difference between the second and third heat sources is minimal compared with the significant increase in the heat source temperatures proceeding downstream for the no opening geometry. (See Table 3.6.) The third heat source often attains a lower temperature than the second heat source because of the greater opening cooling effect experienced at the third heat source. This is significant because the less the variation in the temperature across the electronic components, the better the operation of the electronics and the lower the potential for thermal stresses that can damage the electronics and their connections to the board.

Based on the findings of these studies, the use of the alternate cross flow passages improves the thermal and fluid mixing in the vicinity of the heat source. The heat flow from the heat sources is more evenly distributed with more heat carried from the side surfaces to the main channel flow and from the bottom heat source surfaces towards the cooler lower channel. The effects of this redistribution of the heat flow caused by the use of the alternate cross-flow passages leads to lower heat source temperatures and improved thermal conditions.

3.3.2.2 Parametric Studies Dimension Parameters. By varying a number of the dimensional parameters of the systems in Figure 3.1 and Figure 3.2 and by comparing the results, an indication of the influence of the dimensional parameters on the characteristics of the flow and temperature fields, and therefore the cooling effect produced by the
alternate cross-flow passages can be obtained. The results are summarized in Table 3.4 through Table 3.6 and Figure 3.11 through Figure 3.14.

From the finite element parametric study results the following conclusions may be made. Decreasing the spacing of the upper or lower channel height (Figure 3.3) causes a flow constriction and results in less flow through each channel leading to higher temperatures regardless of the presence of a cross-flow opening. Reducing the heating element height increases the flow rate in the upper channel but decreases the flow rate through the openings. The increase in the heat transfer coefficients and heat fluxes that results from the smaller channel height can be attributed to the increases in the upper channel flow rates, not the effect of the openings. Sufficient heat source heights are necessary to produce the flow constriction and low velocity regions that lead to the development of the higher temperature, density, and pressure differences required for significant flow through the openings. Similar findings about the influence of the block height were reported by Kim [42] for a forced convection study. It was also found that reducing the opening width slightly increases the flow through the openings and causes a slight decrease in the maximum heat source temperature. The reduction in the opening width allows for the development of greater temperature differences in the region between the openings, inducing greater flow through the openings. Reducing the heating element width significantly increases the maximum temperature because the areas of both the top and bottom heat source surfaces from which a significant amount of heat is transferred are reduced by 50%. Among all the parameter cases investigated, the opening of alternate cross-flow passages for this geometry results in the maximum percentage drop in the maximum temperature (24%) compared with the case with no openings.
Because of the elevated temperatures involved and the decrease in the areas of the top and bottom heat source surfaces, this system may be more sensitive to improvements in the heat transfer from the side surfaces of the elements than other geometry cases.

These geometric parametric studies demonstrate that the system geometry does play an important role in the determination of the effectiveness of the alternate cross-flow passages with the cooling effect being most sensitive to the heating element width followed by the heating element height and opening width.

3.3.2.3 Parametric Studies Heat Generation Rates. To study how the fluid flow, temperature, and heat flow in a system containing the alternate cross-flow passages respond to changes in the applied heat source heat generation rate, the applied heat rate was varied for a fixed geometry and the results compared. The results of these studies are summarized in Table 3.4 through Table 3.6 and Figure 3.11 through Figure 3.14.

Parametric studies of the volumetric heat generation rate allow for the investigation of the relationship between the cooling effect and the heat introduced to the system. With an increase in heat rate or Grashof number, the cooling effect increased and then decreased. For the lower heat rates, the lower temperature differences and pressure difference near the opening, do not promote the development of significant flow through the openings, resulting in a lower cooling effect. For higher heat rates, the temperatures on both sides of the opening become elevated, causing the temperature differences and buoyancy induced flow rates across the opening to drop. Some optimum heat rate must exist for which there is sufficient heat to produce sufficient opening flow to effectively cool the heat source. The parametric heat rate study results show that the alternate cross-flow passages may be most beneficial over a limited range of heat rates.
Figure 3.8 Velocity field: (a) alternate cross-flow passages, (b) modified opening arrangement, (c) standard geometry.
Figure 3.9 Velocity field near 3rd opening: (a) alternate cross-flow passages, (b) modified opening arrangement, (c) standard geometry.
Figure 3.10 Temperature contours: (a) alternate cross-flow passages, (b) modified opening arrangement, (c) standard geometry.
### Table 3.4 Average Dimensionless Heat Transfer Coefficients from Right Side of Heat Source 2 (B2R)*

<table>
<thead>
<tr>
<th>Alterations in Array, Board, and Channel Geometry</th>
<th>No Openings</th>
<th>All Openings</th>
<th>% Diff All Openings from No Openings</th>
<th>Openings 1&amp;4 Closed</th>
<th>% Diff 1&amp;4 Closed from No Openings</th>
<th>%Diff 1&amp;4 Closed from All Openings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Geometry</td>
<td>0.02220</td>
<td>0.04026</td>
<td>81.40%</td>
<td>0.04250</td>
<td>91.48%</td>
<td>5.56%</td>
</tr>
<tr>
<td>H=1.5BH</td>
<td>0.01883</td>
<td>0.03404</td>
<td>80.78%</td>
<td>0.03993</td>
<td>112.02%</td>
<td>17.28%</td>
</tr>
<tr>
<td>H2=1.275BH</td>
<td>0.02215</td>
<td>0.03271</td>
<td>47.67%</td>
<td>0.03902</td>
<td>76.15%</td>
<td>19.28%</td>
</tr>
<tr>
<td>BH=0.5BW</td>
<td>0.03915</td>
<td>0.03997</td>
<td>2.09%</td>
<td>0.03909</td>
<td>-0.13%</td>
<td>-2.18%</td>
</tr>
<tr>
<td>BW=0.5BH</td>
<td>0.02853</td>
<td>0.05962</td>
<td>108.95%</td>
<td>0.06448</td>
<td>126.02%</td>
<td>8.17%</td>
</tr>
<tr>
<td>HW=0.45BH</td>
<td>0.01963</td>
<td>0.04432</td>
<td>125.77%</td>
<td>0.04822</td>
<td>145.61%</td>
<td>8.79%</td>
</tr>
</tbody>
</table>

**Alterations in Heat Rate for a Fixed Geometry**

| GR=1158.29                                       | 0.01110     | 0.02013      | 81.40%                               | 0.02125             | 91.48%                            | 5.56%                               |
| GR=3129.9                                        | 0.01347     | 0.02300      | 70.71%                               | 0.02400             | 78.13%                            | 4.35%                               |
| GR=5765.4                                        | 0.01542     | 0.02704      | 75.43%                               | 0.02966             | 92.38%                            | 9.66%                               |
| GR=7500                                          | 0.01648     | 0.02932      | 77.90%                               | 0.03193             | 93.74%                            | 8.90%                               |

*All dimensionless variables presented use the original geometry and heat rate parameters in Table 3.1 as the basis for non-dimensionalization.

### Table 3.5 Average Dimensionless Heat Transfer Coefficients from Left Side of Heat Source 3 (B3L)*

<table>
<thead>
<tr>
<th>Alterations in Array, Board, and Channel Geometry</th>
<th>No Openings</th>
<th>All Openings</th>
<th>% Diff All Openings from No Openings</th>
<th>Openings 1&amp;4 Closed</th>
<th>% Diff 1&amp;4 Closed from No Openings</th>
<th>%Diff 1&amp;4 Closed from All Openings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Geometry</td>
<td>0.04430</td>
<td>0.07328</td>
<td>65.44%</td>
<td>0.09464</td>
<td>113.65%</td>
<td>29.15%</td>
</tr>
<tr>
<td>H=1.5BH</td>
<td>0.04671</td>
<td>0.05622</td>
<td>20.34%</td>
<td>0.08969</td>
<td>92.00%</td>
<td>59.54%</td>
</tr>
<tr>
<td>H2=1.275BH</td>
<td>0.04478</td>
<td>0.04909</td>
<td>9.63%</td>
<td>0.06916</td>
<td>54.45%</td>
<td>40.88%</td>
</tr>
<tr>
<td>BH=0.5BW</td>
<td>0.07045</td>
<td>0.07564</td>
<td>7.38%</td>
<td>0.06377</td>
<td>-9.49%</td>
<td>-15.70%</td>
</tr>
<tr>
<td>BW=0.5BH</td>
<td>0.04930</td>
<td>0.09320</td>
<td>89.03%</td>
<td>0.13840</td>
<td>180.71%</td>
<td>48.50%</td>
</tr>
<tr>
<td>HW=0.45BH</td>
<td>0.03955</td>
<td>0.06416</td>
<td>62.22%</td>
<td>0.07854</td>
<td>98.60%</td>
<td>22.42%</td>
</tr>
</tbody>
</table>

**Alterations in Heat Rate for a Fixed Geometry**

| GR=1158.29                                       | 0.02215     | 0.03664      | 65.44%                               | 0.04732             | 113.65%                           | 29.15%                              |
| GR=3129.9                                        | 0.02831     | 0.05697      | 101.24%                              | 0.05313             | 87.68%                            | -6.74%                              |
| GR=5765.4                                        | 0.03351     | 0.05401      | 61.15%                               | 0.04903             | 46.30%                            | -9.22%                              |
| GR=7500                                          | 0.03622     | 0.05030      | 38.87%                               | 0.04680             | 29.18%                            | -6.97%                              |

*All dimensionless variables presented use the original geometry and heat rate parameters in Table 3.1 as the basis for non-dimensionalization.
### Table 3.6a Maximum Dimensionless Temperature Heat Source 1*

<table>
<thead>
<tr>
<th>Alterations in Array, Board, and Channel Geometry</th>
<th>No Openings</th>
<th>All Openings</th>
<th>% Diff All from No Openings</th>
<th>Openings 1&amp;4 Closed</th>
<th>% Diff 1&amp;4 Closed from No Openings</th>
<th>% Diff 1&amp;4 Closed from All Openings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Geometry</td>
<td>1.8010</td>
<td>1.7860</td>
<td>-0.83%</td>
<td>1.6903</td>
<td>-6.15%</td>
<td>-5.36%</td>
</tr>
<tr>
<td>H=1.5BH</td>
<td>1.7450</td>
<td>1.9130</td>
<td>9.63%</td>
<td>1.8018</td>
<td>3.26%</td>
<td>-5.81%</td>
</tr>
<tr>
<td>H2=1.275BH</td>
<td>1.7700</td>
<td>1.7316</td>
<td>-2.17%</td>
<td>1.7024</td>
<td>-3.82%</td>
<td>-1.69%</td>
</tr>
<tr>
<td>BH=0.5BW</td>
<td>1.9089</td>
<td>1.9317</td>
<td>1.19%</td>
<td>1.8652</td>
<td>-2.29%</td>
<td>-3.44%</td>
</tr>
<tr>
<td>BW=0.5BH</td>
<td>2.1826</td>
<td>1.7907</td>
<td>-17.95%</td>
<td>1.6840</td>
<td>-22.85%</td>
<td>-5.96%</td>
</tr>
<tr>
<td>HW=0.45BH</td>
<td>1.8360</td>
<td>1.7862</td>
<td>-2.71%</td>
<td>1.6865</td>
<td>-8.14%</td>
<td>-5.58%</td>
</tr>
</tbody>
</table>

### Alterations in Heat Rate for a Fixed Geometry

<table>
<thead>
<tr>
<th>Alterations in Heat Rate for a Fixed Geometry</th>
<th>No Openings</th>
<th>All Openings</th>
<th>% Diff All from No Openings</th>
<th>Openings 1&amp;4 Closed</th>
<th>% Diff 1&amp;4 Closed from No Openings</th>
<th>% Diff 1&amp;4 Closed from All Openings</th>
</tr>
</thead>
<tbody>
<tr>
<td>GR=1158.29</td>
<td>0.2251</td>
<td>0.2233</td>
<td>-0.83%</td>
<td>0.2113</td>
<td>-6.15%</td>
<td>-5.36%</td>
</tr>
<tr>
<td>GR=3129.9</td>
<td>0.8089</td>
<td>0.7915</td>
<td>-2.15%</td>
<td>0.9566</td>
<td>18.27%</td>
<td>20.86%</td>
</tr>
<tr>
<td>GR=5765.4</td>
<td>1.7584</td>
<td>1.7560</td>
<td>-0.14%</td>
<td>1.6674</td>
<td>-5.17%</td>
<td>-5.04%</td>
</tr>
<tr>
<td>GR=7500</td>
<td>2.4473</td>
<td>2.4730</td>
<td>1.05%</td>
<td>2.3363</td>
<td>-4.53%</td>
<td>-5.53%</td>
</tr>
</tbody>
</table>

*All dimensionless variables presented use the original geometry and heat rate parameters in Table 3.1 as the basis for non-dimensionalization.

### Table 3.6b Maximum Dimensionless Temperature Heat Source 2*

<table>
<thead>
<tr>
<th>Alterations in Array, Board, and Channel Geometry</th>
<th>No Openings</th>
<th>All Openings</th>
<th>% Diff All from No Openings</th>
<th>Openings 1&amp;4 Closed</th>
<th>% Diff 1&amp;4 Closed from No Openings</th>
<th>% Diff 1&amp;4 Closed from All Openings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Geometry</td>
<td>2.4470</td>
<td>2.3300</td>
<td>-4.78%</td>
<td>2.1479</td>
<td>-12.22%</td>
<td>-7.82%</td>
</tr>
<tr>
<td>H=1.5BH</td>
<td>2.4230</td>
<td>2.5440</td>
<td>4.99%</td>
<td>2.3528</td>
<td>-2.90%</td>
<td>-7.52%</td>
</tr>
<tr>
<td>H2=1.275BH</td>
<td>2.4216</td>
<td>2.4075</td>
<td>-0.58%</td>
<td>2.2920</td>
<td>-5.35%</td>
<td>-4.80%</td>
</tr>
<tr>
<td>BH=0.5BW</td>
<td>2.5399</td>
<td>2.6070</td>
<td>2.64%</td>
<td>2.5474</td>
<td>0.30%</td>
<td>-2.29%</td>
</tr>
<tr>
<td>BW=0.5BH</td>
<td>2.9835</td>
<td>2.3909</td>
<td>-19.86%</td>
<td>2.1096</td>
<td>-29.29%</td>
<td>-11.76%</td>
</tr>
<tr>
<td>HW=0.45BH</td>
<td>2.4961</td>
<td>2.3044</td>
<td>-7.68%</td>
<td>2.1829</td>
<td>-12.55%</td>
<td>-5.27%</td>
</tr>
</tbody>
</table>

### Alterations in Heat Rate for a Fixed Geometry

<table>
<thead>
<tr>
<th>Alterations in Heat Rate for a Fixed Geometry</th>
<th>No Openings</th>
<th>All Openings</th>
<th>% Diff All from No Openings</th>
<th>Openings 1&amp;4 Closed</th>
<th>% Diff 1&amp;4 Closed from No Openings</th>
<th>% Diff 1&amp;4 Closed from All Openings</th>
</tr>
</thead>
<tbody>
<tr>
<td>GR=1158.29</td>
<td>0.3059</td>
<td>0.2913</td>
<td>-4.78%</td>
<td>0.2682</td>
<td>-12.31%</td>
<td>-7.91%</td>
</tr>
<tr>
<td>GR=3129.9</td>
<td>1.0857</td>
<td>0.9925</td>
<td>-8.59%</td>
<td>0.9568</td>
<td>-11.88%</td>
<td>-3.60%</td>
</tr>
<tr>
<td>GR=5765.4</td>
<td>2.3445</td>
<td>2.2065</td>
<td>-5.89%</td>
<td>2.1327</td>
<td>-9.03%</td>
<td>-3.34%</td>
</tr>
<tr>
<td>GR=7500</td>
<td>3.2529</td>
<td>3.1220</td>
<td>-4.03%</td>
<td>3.0174</td>
<td>-7.24%</td>
<td>-3.35%</td>
</tr>
</tbody>
</table>

*All dimensionless variables presented use the original geometry and heat rate parameters in Table 3.1 as the basis for non-dimensionalization.*
Table 3.6c Maximum Dimensionless Temperature Heat Source 3°

<table>
<thead>
<tr>
<th>Alterations in Array, Board, and Channel Geometry</th>
<th>No Openings</th>
<th>All Openings</th>
<th>% Diff All from No Openings</th>
<th>Openings 1&amp;4 Closed</th>
<th>% Diff 1&amp;4 Closed from No Openings</th>
<th>% Diff 1&amp;4 Closed from All Openings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Original Geometry</td>
<td>2.6600</td>
<td>2.3300</td>
<td>-12.40%</td>
<td>2.0577</td>
<td>-22.64%</td>
<td>-11.69%</td>
</tr>
<tr>
<td>H=1.5BH</td>
<td>2.7659</td>
<td>2.5440</td>
<td>-8.02%</td>
<td>2.1419</td>
<td>-22.56%</td>
<td>-15.81%</td>
</tr>
<tr>
<td>H2=1.275BH</td>
<td>2.6514</td>
<td>2.4075</td>
<td>-9.20%</td>
<td>2.3303</td>
<td>-12.11%</td>
<td>-3.21%</td>
</tr>
<tr>
<td>BW=0.5BH</td>
<td>2.7697</td>
<td>2.6070</td>
<td>-5.87%</td>
<td>2.7389</td>
<td>-1.11%</td>
<td>5.06%</td>
</tr>
<tr>
<td>HW=0.45BH</td>
<td>3.1580</td>
<td>2.3909</td>
<td>-24.29%</td>
<td>1.9003</td>
<td>-39.83%</td>
<td>-20.52%</td>
</tr>
</tbody>
</table>

| Alterations in Heat Rate for a Fixed Geometry    |             |              |                             |                     |                                  |                                  |
| GR=1158.29                                      | 0.3325      | 0.2875       | -13.53%                     | 0.2573              | -22.63%                           | -10.52%                           |
| GR=3129.9                                       | 1.1621      | 0.9578       | -17.56%                     | 0.9315              | -19.84%                           | -2.74%                            |
| GR=5765.4                                       | 2.4945      | 2.2165       | -11.14%                     | 2.1563              | -13.56%                           | -2.72%                            |
| GR=7500                                         | 3.4592      | 3.1880       | -7.84%                      | 3.0914              | -10.63%                           | -3.03%                            |

All dimensionless variables presented use the original geometry and heat rate parameters in Table 3.1 as the basis for non-dimensionalization.
All dimensionless variables presented use the original geometry and heat rate parameters in Table 3.1 as the basis for non-dimensionalization. Note different scales.

**Figure 3.11** Comparison of dimensionless heat transfer coefficients on side B2R: (a) geometric parameter study fixed heat rate, (b) heat rate parameter study fixed geometry.
All dimensionless variables presented use the original geometry and heat rate parameters in Table 3.1 as the basis for non-dimensionalization. Note different scales.

**Figure 3.12** Comparison of dimensionless heat transfer coefficients on side B3L: (a) geometric parameter study fixed heat rate, (b) heat rate parameter study fixed geometry.
All dimensionless variables presented use the original geometry and heat rate parameters in Table 3.1 as the basis for non-dimensionalization.

Figure 3.13 Graphical comparison of the maximum dimensionless temperatures geometric parameter study fixed heat rate: (a) heat source 1, (b) heat source 2, (c) heat source 3.
All dimensionless variables presented use the original geometry and heat rate parameters in Table 3.1 as the basis for non-dimensionalization.

Figure 3.14 Comparison of maximum temperatures heat rate parameter study fixed geometry: (a) heat source 1, (b) heat source 2, (c) heat source 3.
The velocity field results of the previous finite element study suggest that a modified opening arrangement may be able to more effectively capture the cooling potential of the flow through the openings. The velocity field in Figure 3.5a clearly shows that the flow rates through the openings and thus the cooling effects produced by each opening are not uniformly distributed among the openings. The flow through the first opening is nearly negligible causing only a minimal cooling effect on the first heat source. Though the greatest flow rate through any opening occurs through the fourth opening, the flow is directed away from the third and final heat source and so contributes little to the cooling of the third heating element. Therefore, in this third numerical investigation of the cross-flow passages, the first and fourth openings are closed as in Figure 3.15, and the effects of this modified opening arrangement on the thermal conditions in the vertically oriented channel containing a three heat source array are investigated.

![Figure 3.15](image)

Figure 3.15 Modified opening arrangement.
3.4.1 Problem Description

The three heat source array in the vertically oriented channel with the modified opening arrangement is shown in Figure 3.15. The same parametric studies as in the second study (Section 3.3) are performed on this geometric arrangement to determine whether the modified opening can result in any further increase in the cooling effect. The same assumptions, boundary conditions, material properties, and parameters as in the study in Section 3.3 are applied. In an effort to investigate the impact of the opening arrangement on the cooling effect, comparisons of the results of this investigation to the results for the standard arrangement with no board openings as well as the four opening arrangement of the previous investigation are made.

3.4.2 Results

The results of this study reveal that the modification of the opening arrangement by the removal of the first and fourth openings does significantly alter the flow in the channel and therefore significantly alters the thermal conditions in the channel.

3.4.2.1 General Results. The closing of the fourth opening, which was the opening with the greatest flow rate, causes a reduction in the net flow from the lower channel to the upper channel over that with the four openings. As a result, more fluid flows through the lower channel. However, some increase in the through-flow rates is noticed at both the second and third openings. (See Figure 3.8b and Figure 3.9b.) It is suspected that with the closing of the first and fourth flow paths, the balance of the net buoyancy or pressure forces is satisfied with an increase in the flow through the remaining openings. Partially as a result of the higher flow rates in the lower channel, the velocities in the lower channel just under the first and second heat sources are higher than those with the
four opening arrangement. However, since there is now a solid board after the third heat source, the velocities in the lower channel just under the third heat source are much lower than they were with the four opening arrangement. The velocities above the third heat source are higher than they were with all four openings, a result of the greater flow through the remaining openings and also the greater temperature differences due to the accompanying reduction in the heat flow through the bottom heat source surface.

The changes in the velocity field produced by the modified opening arrangement alter the temperature field and redistribute the heat flow from the heat sources. (See Figure 3.10b.) All interior side heat source surfaces experience an increase in the heat fluxes and heat transfer coefficients over their four openings values. Also, the higher velocities just under the first and second heat sources cause more heat to be drawn towards the bottoms of the first and second heat sources than in the four openings case. This heat is then carried away into the lower channel and does not pass over tops of the remaining heat sources. However, the fluid velocities in the lower channel just under the third heat source are lower with the modified arrangement. Consequently, the velocity gradients and therefore temperature gradients near the bottom of the third heat source are lower. This prompts more heat to be transferred from the top surface of this element where the nearby velocities are higher than in previous cases. Because, overall, more heat is drawn to the cooler lower channel and more fluid flows through the remaining openings relative to the four opening arrangement, the modified opening arrangement case does improve the thermal conditions of the system relative to that of the four openings arrangement for most parameters. For the original geometry and heat rate parametric study parameters, the maximum drop in the temperature is 22%, almost
double that obtained for the four openings case. As found in the previous study, the modified heat flow pattern leads to a reduction in the temperature difference between the three heat sources. For this modified arrangement, this effect is more pronounced with the majority of the cases investigated actually having the third heat source temperatures lower than the second heat source temperature. Such a trend may be attributed to the higher opening flow velocities, the greater amount of heat brought into the lower channel and the change in the flow pattern downstream of the third heat source.

The results of this study demonstrate that the closing of the first and fourth openings improves the cooling provided by the alternate cross-flow passages.

3.4.2.2 Parametric Studies Dimensional Parameters. For this modified geometry, the effects of varying the system geometric parameters were also investigated and the results are summarized in Table 3.4 through Table 3.6 and Figure 3.11 through Figure 3.14. For the modified opening geometry, the results for cases with the reduced heating element height and lower channel height show little change from the results with the four opening arrangement. Reducing these parameters causes a drop in the amount of flow through the openings and thus reduces the effectiveness of alternate flow path cooling regardless of the opening arrangement. Contrary to the results of the second study, decreasing the upper channel height improves the cooling of the heat sources. This may be attributed to the fact that the heat flow in the system is more dependent on the flow rate in the lower channel than for the four opening case because of the greater amount of flow in the lower channel. Decreasing the opening width also increases the cooling effect beyond that for the four openings case due to the higher flow rates through the openings that result. As found in the four opening arrangement, the heating element width was
found to be the dimensional parameter most sensitive to the cooling effect with the modified opening arrangement. The slight increase in the flow through the openings with the modified opening arrangement has a significant impact on the velocities and heat transfer near the side heat source surfaces and causes a 40% decrease in the maximum temperature. However, the lower heating element width geometry consistently has the highest operating temperatures and lowest heat transfer coefficients. These results, like those of the second finite element study, indicate that the effectiveness of the openings is highly dependent on the system geometry.

3.4.2.3 Parametric Studies Heat Rate Parameters. Applying the same heat source heat generation rates to the system with the modified opening arrangement for a single system geometry as in the study of Section 3.3, the results show all parameter cases experience some further reduction in the heat source operating temperature over that of the four opening configuration. The maximum drop in the temperature relative to that of the four openings cases occurs for the minimum and maximum heat rates with a smaller drop for the intermediate heat rates. These intermediate heat rates are thought to be near the optimum heat rate values for use with alternate cross-flow passages for the given geometry. Therefore, there appears to be a range of heat rates over which the cross-flow passages have cooling potential.

3.5 Conclusions
The three finite element investigations of the use of the alternate cross-flow passages indicate that under the proper system conditions, the method has practical potential to cause significant enhancement of laminar natural convection in a vertically oriented
channel. By increasing the fluid and thermal mixing in the regions between neighboring heat sources, the additional flow paths created by the openings cause active heat removal from the previously inactive side surfaces of the elements. Heat is more effectively transported away from the vicinity of the heat sources than with the standard geometry by both increased convection as the opening flow carries heat to the main channel flow and by increased conduction through the board to the lower channel. This results from the higher temperature gradients that develop under the heat sources due to the opening flow. For models including the conduction in the solids, a maximum decrease in the average operating temperature of about 40% was achieved with a maximum increase in the average heat transfer coefficient on a side surface of over 180% relative to those achieved for the standard board geometry. The channel, opening, and heat source geometry as well as the heat generation rate were found to influence the magnitude of the cooling effect provided by the alternate cross-flow passages. The general flow and temperature field characteristics resulting from the flow passages found in this study are similar to those reported in the forced convection studies of Hung [8] and Anand [42]. Some trends in the effects of the geometric parameters published by Anand are similar to those found in this work. The temperature drops caused by the flow passages found by Anand ranged between 5 and 30%, comparable to the results of the current work. While the parametric studies performed indicate that the greatest cooling effects may occur over a limited parameter range, the studies have shown that a significant cooling effect can be achieved for realistic geometric parameters, heat source parameters, and material properties. Thus, the use of alternate cross-flow passages is a viable alternative method of improving the laminar natural convection cooling in a vertically oriented channel.
CHAPTER 4

FINITE VOLUME NUMERICAL INVESTIGATION OF CHANNEL WITH TRANSVERSELY OSCILLATING WALL UNDER A SQUEEZE FILM VELOCITY FIELD ASSUMPTION

4.1 Introduction

The previous chapter has shown that significant enhancement of laminar natural convection cooling can be achieved through the use of the static enhancement method of alternate cross-flow passages. However, the results imply that the potential cooling effect of this method may be limited. When further cooling enhancement is required, the implementation of a dynamic enhancement technique may be necessary. In this work, an alternative dynamic natural convection enhancement method, the strategic placement of a small transverse oscillation source in the immediate vicinity of the heat source, is investigated. This alternative approach is to operate between pure natural convection and conventional fan driven forced convection and should provide a greater cooling effect than alternate cross-flow passages at the expense of slightly increased cooling system complexity and the need for some additional power input. However, since this method does not require the use of a conventional rotating fan, this approach has the potential to be more energy efficient and reliable than that of standard fan driven forced convection cooling. As described in Chapter 1, this alternative approach involves the cooling of an electronic component through transverse oscillations of a small plate placed in close proximity to the heat source. (See Figure 1.2.) Because the oscillation source is placed close to the heat source, the fluid motion induced by the oscillation source produces higher local fluid velocities right in the vicinity of the heat source. Hence, the improved
thermal and fluid mixing, the higher fluid velocity gradients and the subsequently thinner thermal and momentum boundary layers develop right at heat source surfaces. The higher temperature gradients in the fluid close to the heat sources resulting from the oscillations lead to higher rates of heat transfer to the fluid near the heat source and lower heat source temperatures. Hence, these oscillations also have the potential for producing a more effective, focused, and localized cooling than the more global cooling effect produced by a conventional fan.

A practical means by which these oscillations can be applied is through the use of a piezoelectric device. A piezoelectric transverse oscillation source generates less heat and, particularly when operated at its resonance frequency, requires less power input than the typical conventional fan. It is more reliable and compact. In addition, the device produces no electromagnetic interference, and the appropriate design and frequency limitations can minimize any audible noise generated [5, 6, 16, 65]. Energy and space efficient, a cooling system utilizing a piezoelectric oscillation source better satisfies the criteria for evaluating cooling methods described in Chapter 1 and offers a cooling alternative to extend the “natural convection regime.”

At present, there exists little information about the potential cooling level that may result when transverse oscillations are used to augment natural convection. The current work undertakes an investigation of the use of the transverse oscillations to obtain estimates of the possible cooling effect and information about the role the geometric, heat rate, and oscillation parameters play in attaining this cooling, and also to gain information about the overall effect these oscillations have on the velocities, temperatures, and heat flow in the channel so that these characteristics can be better utilized for cooling.
Initially, to gather information about the effects of the transverse oscillations on the fluid and thermal conditions in the vicinity of the heat source, numerical finite volume studies were carried out for a simplified model geometry consisting of a parallel plate channel representing the region between the oscillation source and the top heat source surface. To obtain an estimate of the cooling effect that can be achieved, in the first of these investigations, a squeeze film velocity field model was employed so that the effects of the oscillations dominate the effects of the natural convection. This chapter is concerned with the findings of the numerical investigation for this squeeze film model. For the same simplified channel geometry, a more general model is used in a second finite volume investigation where the operating conditions are such that the oscillations act to supplement the natural convection effects. The results from this more general finite volume model are discussed in the following chapter. The use of the oscillation sources with more complex heat source and oscillation source geometries with various arrangements was investigated through finite element techniques. Later chapters will discuss the results of these finite element investigations. With the knowledge gained from these finite volume and finite element studies, well-informed conclusions can be drawn about the conditions under which it may be feasible to implement the transverse oscillation source cooling.

4.2 Squeeze Film Model Problem Statement

The objective of this initial investigation into the effects of the transverse oscillations is to examine the potential thermal benefits, practicality, and feasibility of this method. In order to do so, estimates of the local cooling enhancement that can be achieved by this
method over a range of parameters must be obtained. The model geometry, shown in Figure 4.1, consists of a parallel plate channel where the plate at \( y = 0 \), which simulates the top heat source surface, is fixed and supplies a constant heat flux to the channel. A second parallel plate at \( y = b(t) \) is insulated and oscillates transversely to simulate the oscillation source. The upper surface oscillates in the \( y \) direction with a velocity given by:

\[
V_w(t) = V_o \cos(\omega t)
\]  

(4.1)

The channel width at any time, \( b(t) \), is given by:

\[
b(t) = b_o + a_o \sin(\omega t) \quad \text{where} \quad a_o = \frac{V_o}{\omega}
\]  

(4.2)

![Figure 4.1 Squeeze film model system.](image)

In addition to the appropriate general assumptions discussed in Section 2.2, for the purposes of determining an initial estimate of the oscillation induced cooling effect, the “well-known” squeeze film assumptions are adopted for the velocity field. This assumption is valid for small channel width to length ratios, \( \frac{b_o}{L} \ll 1 \) (small clearance
between the surfaces and thus a small mass) and small oscillation Reynolds numbers
\[ Re = \frac{\omega a^2 b \rho}{\nu} \ll 1 \] (slower oscillation velocities and frequencies and low displacements).

Under the squeeze film assumption conditions, the gradients in the x direction are small relative to those in the y and the inertia and buoyancy forces are small relative to the shear and pressure effects. Therefore, for the squeeze film model, inertia and natural convection effects are neglected. While these assumptions may not be valid for all oscillation and geometric parameters, the inclusion of the inertia effects as well as the natural convection effects results in a greater cooling effect than that achieved under the squeeze film assumptions as shown in the proceeding chapter. (One should also be aware that the constant property assumptions employed place a limit on the possible oscillation parameter values.)

Under the stated modeling assumptions, analytical expressions for the velocity and pressure fields within the channel in Figure 4.1 were obtained and the temperature field is then determined numerically. Parametric studies varying the channel and oscillation parameters were performed. Based on the numerical temperature field results, measures of the cooling effect of the oscillations were obtained including the heated surface temperatures, the heated surface Nusselt numbers, the maximum temperatures, and the system volume averaged temperatures. Correlation equations of the Nusselt number data were also formulated to aid in the interpretation of the parametric study results. To estimate the improvement in the cooling, the comparisons were made to a pure natural convection reference case. By analyzing the subsequent cooling improvement and the parameter ranges for which substantial cooling occurs, the general feasibility of the use of the oscillations in the manner described can be explored.
4.3 Analysis of Oscillating Squeeze Film

In this section the analytical and numerical procedures used to determine the velocity, pressure, and temperature fields in the channel are discussed and any necessary dimensionless design parameters are defined.

4.3.1 Dimensionless Variables

For ease of computation, the following set of dimensionless variables was used in this squeeze film investigation where the $Q$ subscript refers to the squeeze film dimensionless parameters.

\[
\tilde{x}_Q = \frac{x}{L}, \quad \tilde{y}_Q = \frac{y}{b(t)}, \quad \tilde{u}_Q = \frac{u}{\omega L}, \quad \tilde{v}_Q = \frac{v}{\omega b_o}
\]

\[
\tilde{p}_Q = \frac{P}{\mu \left( \frac{L}{b(t)} \right)^2 \omega}, \quad \tilde{t}_Q = t\omega, \quad \tilde{T}_Q = \frac{T - T_o}{q_o b_o k}
\]

(4.3)

The scaling of the $y$-coordinate by $b(t)$, the instantaneous channel width, was employed to “fix” the moving surface in the transformed coordinate system. It does not alter the governing equations, but was used to facilitate the ease of solution.

In addition, the following dimensionless moving wall velocity, $\tilde{V}_{wQ}$, was defined.

\[
\tilde{V}_{wQ} = \frac{V_w}{b_o \omega} \cos(\tilde{r}_Q)
\]

(4.4)

4.3.2 Governing Equations

The continuity equation, the momentum equations, and the energy equation must be utilized to determine the velocity, pressure, and temperature fields in the channel, $\tilde{u}_Q(x_0, \tilde{y}_Q, \tilde{t}_Q)$, $\tilde{v}_Q(x_0, \tilde{y}_Q, \tilde{t}_Q)$, $\tilde{p}_Q(x_0, \tilde{y}_Q, \tilde{t}_Q)$, and $\tilde{T}_Q(x_0, \tilde{y}_Q, \tilde{t}_Q)$, respectively. The
forms of these governing equations under the assumptions made in this investigation follow. More details are given in Appendix D.

The continuity equation for the given system using the set of dimensionless variables listed in Eq. (4.3) is:

\[
\frac{\partial \tilde{u}_Q}{\partial \tilde{x}_Q} + \frac{b_0}{b(t_0)} \frac{\partial \tilde{v}_Q}{\partial \tilde{y}_Q} = 0
\]  

(4.5)

Applying the dimensionless variables in Eq. (4.3) and the squeeze film assumptions discussed in Section 4.2 to the \( x \) component of the momentum equation, Eq. (2.12), yields:

\[
\frac{\partial \tilde{p}_Q}{\partial \tilde{x}_Q} \approx \frac{\partial^2 \tilde{u}_Q}{\partial \tilde{y}_Q^2}
\]  

(4.6)

Similarly, applying the dimensionless variables in Eq. (4.3) along with the squeeze film assumptions to the \( y \) component of the momentum equation, Eq. (2.13), yields:

\[
\frac{\partial \tilde{p}_Q}{\partial \tilde{y}_Q} \approx 0
\]  

(4.7)

Thus, \( \tilde{p}_Q \), the dimensionless dynamic pressure, is dependent on \( \tilde{x}_Q \) and \( \tilde{t}_Q \) alone.

The temperature distribution, \( \tilde{T}_Q (\tilde{x}_Q, \tilde{y}_Q, \tilde{t}_Q) \) is governed by the energy equation.

In the transformed coordinate system, the energy equation, in Eq. (2.7), becomes:

\[
\frac{\omega}{\alpha} \left\{ \frac{\partial \tilde{T}_Q}{\partial \tilde{t}_Q} + \tilde{V}_{newQ} \frac{\partial \tilde{T}_Q}{\partial \tilde{y}_Q} + \tilde{u}_Q \frac{\partial \tilde{T}_Q}{\partial \tilde{x}_Q} \right\} = \left\{ \frac{1}{\alpha} \frac{\partial^2 \tilde{T}_Q}{\partial \tilde{x}_Q^2} + \frac{1}{b_0^2} \frac{\partial^2 \tilde{T}_Q}{\partial \tilde{y}_Q^2} \right\}
\]  

(4.8)

where:

\[
\tilde{V}_{newQ} = \frac{1}{\left( \frac{b(t)}{b_0} \right)} \left( \tilde{V}_Q - \tilde{V}_Q \tilde{V}_{\ast Q} \right) \text{ and } \alpha = \frac{k}{\rho c_p}
\]  

(4.8a,b)
4.3.3 Initial Conditions and Boundary Conditions

The fluid is assumed to initially be at rest at local ambient temperature and hydrostatic pressure. Hence,

\[ \tilde{u}_Q (\tilde{x}_Q, \tilde{y}_Q, 0) = 0 \]  
\[ \tilde{v}_Q (\tilde{x}_Q, \tilde{y}_Q, 0) = 0 \]  
\[ \tilde{p}_Q (\tilde{x}_Q, \tilde{y}_Q, 0) = 0 \]  
\[ \tilde{T}_Q (\tilde{x}_Q, \tilde{y}_Q, 0) = 0 \]

(4.9a)  
(4.9b)  
(4.9c)  
(4.9d)

The dimensionless form of the boundary conditions is listed below:

From the no slip boundary conditions at the walls:

\[ \tilde{y}_Q = 0, \quad \tilde{u}_Q = 0, \quad \tilde{v}_Q = 0 \]  
\[ \tilde{y}_Q = 1, \quad \tilde{u}_Q = 0, \quad \tilde{v}_Q = \tilde{V}_w \]

(4.10a)  
(4.10b)

From symmetry:

\[ \tilde{x}_Q = 0, \quad \tilde{u}_Q = 0 \]  
\[ \tilde{x}_Q = 0, \quad \frac{\partial \tilde{p}_Q}{\partial \tilde{x}_Q} = 0 \]  
\[ \tilde{x}_Q = 0, \quad \frac{\partial \tilde{T}_Q}{\partial \tilde{x}_Q} = 0 \]

(4.11a)  
(4.11b)  
(4.11c)

Since the pressure at the channel inlet/outlet is assumed to be the local hydrostatic pressure, the dynamic pressure at this location is equal to zero.

\[ \tilde{x}_Q = 1, \quad \tilde{p}_Q = 0 \]

(4.12a)

As the fluid is periodically expelled into the surroundings and then drawn into the channel with the amount of fluid per unit depth passing through the boundary equal to
\( V_w(t) \ L \), the fluid at the channel inlet/outlet is assumed to be well mixed with the fluid in the surroundings. Thus:

\[
\tilde{x}_Q = 1, \quad \tilde{T}_Q = 0
\]  

(4.12b)

To simulate the electronic device, a constant heat flux is applied to the surface at \( y = 0 \).

\[
\tilde{y}_Q = 0, \quad \frac{\partial \tilde{T}_Q}{\partial \tilde{y}_Q} = \frac{b(\tilde{r}_Q)}{b_o}
\]  

(4.13a)

A thermally insulated boundary condition is used for the moving surface to provide a "conservative estimate of the maximum temperature." [82]

\[
\tilde{y}_Q = 1, \quad \frac{\partial \tilde{T}_Q}{\partial \tilde{y}_Q} = 0
\]  

(4.13b)

### 4.3.4 Analytical Determination of Squeeze Film Velocity Field

The analytical solutions for the velocity field are obtained from Eqs. (4.5-4.7).

Recalling that \( \tilde{P}_Q \) is a function of \( \tilde{x}_Q \) and \( \tilde{r}_Q \) alone, Eq. (4.6) along with the no-slip boundary conditions yield an expression for the \( x \) component of velocity:

\[
\tilde{u}_Q = \frac{1}{2} \frac{\partial \tilde{P}_Q}{\partial \tilde{x}_Q} \left( \tilde{y}_Q - \tilde{y}_Q \right)
\]  

(4.14)

The dimensionless pressure gradient then can be found by integrating the continuity equation over the domain from \( \tilde{y}_Q = 0 \) to \( \tilde{y}_Q = 1 \) and \( \tilde{x}_Q = 0 \) to any \( \tilde{x}_Q \) and then applying the necessary velocity and pressure boundary conditions, yielding:

\[
\frac{\partial \tilde{P}_Q}{\partial \tilde{x}_Q} = \frac{12 \tilde{T}_w Q \tilde{x}_Q}{\left( b(\tilde{r}_Q)/b_o \right)}
\]  

(4.15)

Using the pressure boundary condition in Eq. (4.12a) along with Eq. (4.15), the pressure distribution in the channel can be expressed as:
A typical pressure distribution is shown in Figure 4.2.

Substituting Eq. (4.16) into Eq. (4.14), the x component of velocity becomes:

\[
\tilde{u}_Q(x_0, \tilde{t}_Q) = \frac{6\tilde{V}_w}{b(t_o) / b_o} \left( \frac{x_0^2}{b(t_o) / b_o} - 1 \right)
\] (4.16)

Through the use of the continuity equation in Eq. (4.5), Eq. (4.17), and the necessary boundary conditions, the y component of velocity, \( \tilde{v}_Q \), becomes:

\[
\tilde{v}_Q(y_0, \tilde{t}_Q) = -6\tilde{V}_w \left( \frac{y_0^3}{3} - \frac{y_0^2}{2} \right)
\] (4.18)

![Dynamic Pressure vs. Dimensionless Time](image-url)

**Figure 4.2** Typical dynamic gage pressure at channel symmetry line as a function of dimensionless time \( \tilde{t} = \omega t \) for \( L/b_o = 100 \), \( a_o/b_o = 0.75 \), \( \omega = 2000 \text{ rad/sec} \).

### 4.3.5 Numerical Solution of Energy Equation for Temperature Field

Under the analytically determined squeeze film velocity field in the channel, the energy equation, Eq. (4.8), is solved numerically using a modified formulation of Patankar's SIMPLER method [83, 84]. The governing equation must be placed into the form used
in the SIMPLER formulation. After some manipulation, the differential energy equation in Eq. (4.8) can be expressed as:

\[ \left\{ \frac{\partial \tilde{T}_Q}{\partial t_Q} + \frac{\partial}{\partial x_Q} \left( \tilde{u}_Q \tilde{T}_Q \right) - K_x \frac{\partial \tilde{\tilde{T}}_Q}{\partial x_Q} \right\} + \frac{\partial}{\partial y_Q} \left( \tilde{v}_Q \tilde{T}_Q \right) - K_T \frac{\partial \tilde{\tilde{T}}_Q}{\partial y_Q} \right\} = - \frac{\tilde{V}_{wQ} \tilde{\tilde{T}}_Q}{b(t_Q)/b_o} \]  

(4.19)

where:

\[ \tilde{V}_{newQ} = \frac{1}{b(t_Q)/b_o} \left( \tilde{V}_Q - \tilde{y}_Q \tilde{\tilde{V}}_{wQ} \right) \]  

(4.19a)

\[ \tilde{\omega}_Q = \frac{\omega b_o^2}{\alpha} \]  

(4.19b)

\[ K_x = \frac{1}{\tilde{\omega}_Q} \left( \frac{b_o}{L} \right)^2 \]  

(4.19c)

\[ K_T = \frac{1}{\tilde{\omega}_Q} \left( \frac{b_o}{b(t_Q)} \right)^2 \]  

(4.19d)

This is the form of the energy equation to which Patankar’s SIMPLER finite volume scheme is then applied [83, 84]. (See Appendix D for more information about this numerical method and Appendix F for program code.) In the current model, a 171 x 101 grid is used. (See Figure E.7 for a representative grid.) Further increases in the number of grid lines produce less than a 0.1% change in the results. To maintain control on convergence, a variable time step is used. For the first two plate oscillations, each oscillation is divided into 600 segments. The number of segments is gradually decreased over successive plate oscillations, and after five plate oscillations, 100 segments are used.

In addition to the grid and time step, other specifications are made in the implementation of the modified SIMPLER method. A backward difference implicit time discretization scheme is used. The Gauss-Seidel iteration method with a relaxation factor
of 0.85 is employed to solve the discretized finite volume equations for the temperature at a given time step. The solution at each time step is defined to have converged when the maximum difference in the temperature values between solution iterations is less than 1.0e-05 over all the grid points. The solution proceeds in time until a periodically repeating solution or "transient steady state" is attained. The periodically repeating solution condition is met when the maximum difference in the temperature field from one plate oscillation to the next at the same point in the plate oscillation as well as the difference in the time averaged heated surface temperature from that of the previous plate oscillation are both under 0.15%. All results presented are those at transient steady state.

4.3.6 Calculated Parameters

The following design parameters are defined to assist in the analysis of the results. First, a local dimensionless heat transfer coefficient is defined as follows:

\[
\tilde{h}_Q = \frac{1}{\tilde{T}_{s,Q}}
\]

(4.20)

where \(\tilde{T}_{s,Q}\) is the local dimensionless surface temperature and \(h\) is made dimensionless by \(k/b_o\). The average dimensionless temperature of the heated surface at any time, \(\tilde{T}\), is defined as:

\[
\tilde{T}_{s,Q} = \frac{1}{\tilde{T}_{s,Q}} \int_0^1 \tilde{T}_{s,Q} \, d\tilde{x}_Q
\]

(4.21a)

The time average of this average heated surface dimensionless temperature over one period of oscillation, \(\tilde{T}_{s,Q}\), is given by:
In this investigation, the time averaged average heated surface heat transfer coefficient is defined as:

\[ \overline{\dot{T}}_{\text{wQ}} = \frac{1}{2\pi} \int_{0}^{2\pi} \overline{T}_{\text{wQ}} d\theta \]  

(4.21b)

In this investigation, the time averaged average heated surface heat transfer coefficient is defined as:

\[ \bar{h} = \frac{q_{o}}{(\bar{T}_{t} - T_{o})} \]  

(4.22)

for a constant heat flux, \( q_{o} \), applied to the heated surface at \( y = 0 \).

A time averaged average heated surface Nusselt number, \( \bar{N}u_{s} \), a dimensionless heat transfer coefficient, is defined as:

\[ \bar{N}u_{s} = \left( \frac{\bar{h}b_{o}}{k} \right) \equiv \frac{1}{\overline{T}_{\text{wQ}}} \]  

(4.23)

For convenience in comparing cases with different mean channel widths, a modified Nusselt number and dimensionless temperature denoted by the subscript \( M \) will be defined where \( b_{nc} \) denotes a typical natural convection flow channel width used in the natural convection reference case.

\[ \bar{N}u_{M} = \left( \frac{\bar{h}b_{nc}}{k} \right) \]  

(4.24)

\[ \bar{T}_{M} = \left( \frac{q_{o}b_{nc}}{k} \right) \]  

(4.25)

For ease of reading, in the remainder of this work \( \bar{N}u_{s} \) will be denoted by \( Nu \) and \( \bar{N}u_{M} \) will be denoted by \( Nu_{M} \).
4.4 Squeeze Film Results of Heat Transfer Performance

Important information about the potential effects of transverse oscillations on the thermal conditions in the channel in general and at the heated surface in particular is revealed through the parametric studies performed in this investigation. Parametric studies were completed for $L/b_o$ values of 10, 20, 50, and 100 and $a_o/b_o$ values of 0.25, 0.50, 0.75, and 0.85 with varying oscillation frequencies and a fixed, constant and uniform heat flux and a fixed channel length. Table 4.1 lists the general parameters used in this investigation. Although the solutions for complete time history were determined, results presented are at the “transient steady state.”

To establish a basis from which to measure the cooling effect, the results of the parametric study are compared to the results for a natural convection reference case. This reference case is a fixed wall channel system cooled by pure natural convection with the same thermal boundary conditions and channel length as the present study and a typical natural convection length to width ratio ($L/b_{nc}$) of 10. The natural convection reference results were obtained through a finite element simulation.

In this section, time averaged and transient results of the “squeeze film” investigation are discussed.

<table>
<thead>
<tr>
<th>Table 4.1 Squeeze Film Model Input Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material Property</td>
</tr>
<tr>
<td>$k$ (thermal conductivity)</td>
</tr>
<tr>
<td>$\rho$ (density)</td>
</tr>
<tr>
<td>$c_p$ (specific heat)</td>
</tr>
<tr>
<td>$\nu$ (kinematic viscosity)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0$ (ambient temperature)</td>
<td>25°C</td>
</tr>
<tr>
<td>$q_a$ (applied heat flux)</td>
<td>150 W/m$^2$</td>
</tr>
<tr>
<td>$L$ (half channel length)</td>
<td>0.1 m</td>
</tr>
</tbody>
</table>
4.4.1 Time Averaged Transient Steady State Results

The most significant measure of the potential cooling capabilities of the transverse oscillations is the resulting time-averaged average heated surface Nusselt numbers and time-averaged average heated surface temperatures. The time-averaged results are presented in both tabular and graphical form and clearly indicate the potential for significant cooling for practical system parameter values. Table 4.2 summarizes the maximum improvement in the \( Nu_M \) over pure natural convection that was achieved for each of the parameter cases investigated. From the table, estimations of the potential oscillation induced cooling effect can be made. The potential for an increase in the time averaged heated surface modified Nusselt number, \( Nu_M \), of as much as a 500% relative to the reference natural convection case is possible for the parameter values studied for which the pressure change in the channel is within the allowable limits. (In this investigation when the change in pressure experienced in the channel is over 5% of an atmosphere, the constant property assumption is deemed questionable. A typical pressure distribution at the flow symmetry line \((x=0)\) is given in Figure 4.2.)

While the potential cooling enhancement can be read from the tables, further discussion of the effects of the parameter values \( \omega, L/b_0 \) and \( a_0/b_0 \) is warranted. The influence of the system parameter values on the cooling can be more easily identified through Figure 4.3 through Figure 4.6. These figures show \( Nu_M \) (Eq. (4.24)) as a function of the oscillation frequency, \( \omega \), and \( a_0/b_0 \) for the four \( L/b_0 \) cases investigated. The heated surface time-averaged dimensionless temperature, \( T_M \), data is also plotted in the figures along with the reference natural convection results. Though the discussion of results is
focused on the effect of the oscillations on the Nusselt number, the results for the temperature field are inverse to those of the Nusselt number.

Table 4.2  Maximum Percent Increase in Modified Nusselt Number Over Natural Convection Reference

<table>
<thead>
<tr>
<th>$L/b_0$</th>
<th>$a_0/b_0$</th>
<th>$\omega$ (rad/sec)</th>
<th>$Nu_M$ max</th>
<th>% Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.25</td>
<td>800.00</td>
<td>2.74</td>
<td>10.67</td>
</tr>
<tr>
<td>10</td>
<td>0.50</td>
<td>250.00</td>
<td>3.48</td>
<td>40.67</td>
</tr>
<tr>
<td>10</td>
<td>0.75</td>
<td>266.67</td>
<td>6.51</td>
<td>163.20</td>
</tr>
<tr>
<td>10</td>
<td>0.85</td>
<td>235.29</td>
<td>8.42</td>
<td>240.13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L/b_0$</th>
<th>$a_0/b_0$</th>
<th>$\omega$ (rad/sec)</th>
<th>$Nu_M$ max</th>
<th>% Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.25</td>
<td>1200.00</td>
<td>3.49</td>
<td>41.16</td>
</tr>
<tr>
<td>20</td>
<td>0.50</td>
<td>600.00</td>
<td>5.68</td>
<td>129.52</td>
</tr>
<tr>
<td>20</td>
<td>0.75</td>
<td>500.00</td>
<td>9.97</td>
<td>302.82</td>
</tr>
<tr>
<td>20</td>
<td>0.85</td>
<td>470.59</td>
<td>13.04</td>
<td>426.81</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L/b_0$</th>
<th>$a_0/b_0$</th>
<th>$\omega$ (rad/sec)</th>
<th>$Nu_M$ max</th>
<th>% Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.25</td>
<td>2500.00</td>
<td>4.61</td>
<td>86.32</td>
</tr>
<tr>
<td>50</td>
<td>0.50</td>
<td>1500.00</td>
<td>9.61</td>
<td>288.26</td>
</tr>
<tr>
<td>50</td>
<td>0.75</td>
<td>1000.00</td>
<td>15.52</td>
<td>527.10</td>
</tr>
<tr>
<td>50**</td>
<td>0.85</td>
<td>1176.47</td>
<td>21.85</td>
<td>782.84</td>
</tr>
<tr>
<td>50**</td>
<td>0.85</td>
<td>441.18</td>
<td>12.34</td>
<td>398.78</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L/b_0$</th>
<th>$a_0/b_0$</th>
<th>$\omega$ (rad/sec)</th>
<th>$Nu_M$ max</th>
<th>% Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.25</td>
<td>3000.00</td>
<td>4.66</td>
<td>88.15</td>
</tr>
<tr>
<td>100</td>
<td>0.50</td>
<td>2500.00</td>
<td>12.44</td>
<td>402.73</td>
</tr>
<tr>
<td>100*</td>
<td>0.75</td>
<td>2000.00</td>
<td>21.12</td>
<td>753.46</td>
</tr>
<tr>
<td>100**</td>
<td>0.75</td>
<td>500.00</td>
<td>6.59</td>
<td>166.25</td>
</tr>
<tr>
<td>100*</td>
<td>0.85</td>
<td>1764.71</td>
<td>24.61</td>
<td>894.21</td>
</tr>
<tr>
<td>100**</td>
<td>0.85</td>
<td>294.12</td>
<td>5.14</td>
<td>107.77</td>
</tr>
</tbody>
</table>

*exceeds allowable pressure  
**near lowest allowable pressure  
Note $Nu_M$ is defined in Eq. (4.24)

Increasing the oscillation frequency for fixed $a_0/b_0$ and fixed $L/b_0$ values results in higher average modified Nusselt numbers as can be clearly seen in Figure 4.3 through Figure 4.6. Increasing the oscillation frequency while holding all other parameters constant increases the rate of exchange of the channel air with the surrounding fluid as well as the oscillation velocity amplitude($a_0\omega$) and hence the velocity magnitudes in the
channel. The higher velocities and the faster rate of fluid exchange that occur with the higher oscillation frequency lead to improved fluid and thermal mixing and thus result in the increased cooling effect. Figure 4.3 through Figure 4.6 show that the rate of increase in the cooling effect decreases with increases in the frequency for fixed $a_0/b_0$ and fixed $L/b_0$ values. Despite the higher fluid velocities, further increases in the frequency seem to prohibit the cooler fluid from traveling further along the channel and from reaching and cooling the high temperature region closer to the flow symmetry line. (See Section 4.4.2.1.) Thus, the cooling effect has some limit.

The results also show that higher $Nu_M$ values can be obtained by increasing the value of $L/b_0$ for fixed $a_0/b_0$ and $\omega$ parameter values. (See Figure 4.3 through Figure 4.6.) The channel velocities and hence the cooling effect increase with $L/b_0$. (The magnitude of the average velocity at the channel inlet/outlet is equal to $(a_0 \omega)(L/b_0)$, where $a_0 \omega$ is the oscillation velocity amplitude.) Keeping $L$ fixed and increasing $L/b_0$ produces thinner channels having less fluid mass trapped in the high temperature region of the flow symmetry line. (This is discussed in more detail in Section 4.4.2.1.) While the local maximum temperatures in the symmetry zone ($x=0$) may be higher in this case, the boundary layers are thinner so an increase in $Nu_M$ results. Though the cooling effect is a strong function of the $L/b_0$ value, as $L/b_0$ values continue to increase, a limiting $Nu_M$ appears to be approached. (In this study, an upper limit of 2.0 m/s on the velocity of the plate was imposed.)

The average modified Nusselt number was also found to increase with $a_0/b_0$ for fixed $L/b_0$ and fixed $\omega$ values as seen in Figure 4.3 through Figure 4.6. The parameter $a_0/b_0$ can be interpreted as the ratio of the volume of heated fluid expelled from the
channel or the volume of the ambient fluid drawn into the channel during each quarter of the oscillation cycle to the mean volume of fluid in the channel. Thus, for fixed $\omega$ and $L/b_o$ values, the larger the $a_o/b_o$ value, the larger the percentage of the mean channel volume exchanged with the surroundings and the further the cooler fluid drawn into the channel advances towards the high temperature flow symmetry line region. The exchange of fluid with the surroundings is the driving force behind the effectiveness of the oscillation induced cooling. If the amount of fluid exchanged with the ambient is insufficient to overcome the effects of the thermally "dead zone" that develops at the flow symmetry line (see discussion in Section 4.4.2.1), the enhancement is minimal, or impeded cooling occurs relative to the natural convection reference. Inadequate oscillation source displacements may be the reason for the results for the parameter case $a_o/b_o=0.25$ shown in Figure 4.3 through Figure 4.6. For high $L/b_o$ and $\omega$ values, the cooling conditions for $a_o/b_o=0.25$ are similar to those found at the pure natural convection reference case conditions. However, at lower $L/b_o$ ratios and $\omega$ values, the $Nu_M$ values are lower than those for pure natural convection, and local flow symmetry line maximum temperatures may be as much as three times higher than those for the natural convection reference. (The contour plots in Figure 4.8 clearly demonstrate these effects of the $a_o/b_o$ value on the overall temperature field.) In addition to these findings about the importance of sufficient oscillation displacement, the results also show there is a limit to the cooling potential as the rate of increase in the improvement of the thermal conditions decreases with increasing $a_o/b_o$. For a given $L/b_o$ over a similar range of oscillation frequencies, the time averaged heated surface modified Nusselt number appears to be more sensitive to changes in the $a_o/b_o$ value than to changes in the frequency.
Correlation equations for all Nusselt number data are presented in Table 4.3. In the correlations, $Nu$ (defined in Eq. (4.23)) is expressed as a function of the oscillation frequency in the form $Nu = c + d\omega^f$, with the coefficients being dependent upon the $L/b_o$ and $a_o/b_o$ parameters.

**Table 4.3  Nusselt Number Correlation Equations For Current Data $Nu = c + d\omega^f$**

<table>
<thead>
<tr>
<th>$L/b_o$</th>
<th>$a_o/b_o$</th>
<th>$c$</th>
<th>$D$</th>
<th>$f$</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.25</td>
<td>0.01929</td>
<td>0.10726</td>
<td>0.48577</td>
<td>0.0348</td>
</tr>
<tr>
<td>10</td>
<td>0.50</td>
<td>0.03912</td>
<td>0.37105</td>
<td>0.40476</td>
<td>0.0186</td>
</tr>
<tr>
<td>10</td>
<td>0.75</td>
<td>0.01879</td>
<td>0.83538</td>
<td>0.3688</td>
<td>0.0759</td>
</tr>
<tr>
<td>10</td>
<td>0.85</td>
<td>0.009288</td>
<td>1.00587</td>
<td>0.3911</td>
<td>0.12667</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L/b_o$</th>
<th>$a_o/b_o$</th>
<th>$c$</th>
<th>$D$</th>
<th>$f$</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.25</td>
<td>0.005122</td>
<td>0.033696</td>
<td>0.55746</td>
<td>0.01078</td>
</tr>
<tr>
<td>20</td>
<td>0.50</td>
<td>0.003483</td>
<td>0.16729</td>
<td>0.44383</td>
<td>0.0305</td>
</tr>
<tr>
<td>20</td>
<td>0.75</td>
<td>-0.00839</td>
<td>0.32104</td>
<td>0.44345</td>
<td>0.09815</td>
</tr>
<tr>
<td>20</td>
<td>0.85</td>
<td>0.026387</td>
<td>0.361777</td>
<td>0.4735</td>
<td>0.146</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L/b_o$</th>
<th>$a_o/b_o$</th>
<th>$c$</th>
<th>$D$</th>
<th>$f$</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.25</td>
<td>-0.00717</td>
<td>0.00870779</td>
<td>0.5996</td>
<td>0.0206</td>
</tr>
<tr>
<td>50</td>
<td>0.50</td>
<td>-0.01044</td>
<td>0.03403</td>
<td>0.5551</td>
<td>0.0581</td>
</tr>
<tr>
<td>50</td>
<td>0.75</td>
<td>-0.02421</td>
<td>0.03999</td>
<td>0.63444</td>
<td>0.0992</td>
</tr>
<tr>
<td>50</td>
<td>0.85</td>
<td>-0.04465</td>
<td>0.05516</td>
<td>0.62331</td>
<td>0.104</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$L/b_o$</th>
<th>$a_o/b_o$</th>
<th>$c$</th>
<th>$D$</th>
<th>$f$</th>
<th>Residual</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>0.25</td>
<td>-0.01133</td>
<td>0.00075972</td>
<td>0.807794</td>
<td>0.0196</td>
</tr>
<tr>
<td>100</td>
<td>0.50</td>
<td>-0.01255</td>
<td>0.0032708</td>
<td>0.76393</td>
<td>0.0448</td>
</tr>
<tr>
<td>100</td>
<td>0.75</td>
<td>-0.00942</td>
<td>0.00523</td>
<td>0.791997</td>
<td>0.0433</td>
</tr>
<tr>
<td>100</td>
<td>0.85</td>
<td>-0.02159</td>
<td>0.005555</td>
<td>0.81862</td>
<td>0.0432</td>
</tr>
</tbody>
</table>

Note $Nu$ is defined in Eq. (4.23).
Reference is Natural Convection with $L/b_o = 10$.

**Figure 4.3** Results for $L/b_o = 10$: (a) modified time averaged average heated surface Nusselt number vs. oscillation frequency, (b) dimensionless modified time averaged average heated surface temperatures vs. oscillation frequency.
Reference is Natural Convection with $L/b_0=10$.

**Figure 4.4** Results for $L/b_0 = 20$: (a) modified time averaged average heated surface Nusselt number vs. oscillation frequency, (b) dimensionless modified time averaged average heated surface temperatures vs. oscillation frequency.
Reference is Natural Convection with $L/b_0=10$. Dashed line indicates maximum change in channel pressure over 5% of one atmosphere.

**Figure 4.5** Results for $L/b_0 = 50$: (a) modified time averaged average heated surface Nusselt number vs. oscillation frequency, (b) dimensionless modified time averaged average heated surface temperatures vs. oscillation frequency.
Reference is Natural Convection with \( L/b_0 = 10 \). Dashed line indicates maximum change in channel pressure over 5% of one atmosphere.

**Figure 4.6** Results for \( L/b_0 = 100 \): (a) modified time averaged average heated surface Nusselt number vs. oscillation frequency, (b) dimensionless modified time averaged average heated surface temperatures vs. oscillation frequency.
4.4.2 Transient Results at Transient Steady State

By examining the transient effects of the oscillations, a better understanding of how the oscillations can most effectively be used to enhance the cooling of a system can be gained.

4.4.2.1 General Effects of Oscillations on Velocity and Temperature Fields.

Because oscillation induced cooling relies on the oscillation induced velocity field to improve the thermal conditions in the channel, it is important to be cognizant of the fundamental characteristics of the oscillation flow field and how these characteristics influence the development of the temperature distribution. As seen in Eq. (4.12), the $x$ component of velocity is zero at the flow symmetry line and its magnitude increases linearly towards the channel inlet/outlet. A typical squeeze film stream function distribution indicating these characteristics is given in Figure 4.7. Because of the low velocities, the convective cooling is not significant near the flow symmetry line. Hence, high temperatures develop in this region along with the accompanying low heat transfer coefficients. Near the channel inlet/outlet, however, the higher fluid velocities and the close proximity to the cooler ambient air allow for better fluid exchange and thermal and fluid mixing. This results in higher temperature gradients near the heat source surface and lower temperatures and higher heat transfer coefficients at the heated surface than those occurring near the flow symmetry line. Hence, the thermal and fluid mixing effect caused by the oscillations provides for the cooling. Based on these characteristics, it can also be concluded that for the oscillations to improve the overall thermal conditions relative to those of pure through flow natural convection, the oscillation cooling effect must be strong enough to overcome the “lost heat removal” from the essentially “dead
zone" that develops near the flow symmetry line. Typical temperature contour plots for the parameter values indicated at the same position in the oscillation are shown in Figure 4.8 and illustrate the temperature field characteristics discussed above. Typical dimensionless temperature and local dimensionless heat transfer coefficient distributions at the heated surface for the same parameters as in Figure 4.8 are presented in Figure 4.9 and further exemplify the conditions described above.

![Figure 4.7 Typical squeeze film velocity field stream functions.](image)

The cooling effect was shown to be dependent on the oscillation and geometric parameters as seen in Section 4.4.1. The values of $L/b_o$, $a_0/b_o$, and $\omega$ influence such quantities as the amount of fluid in the channel, the amount of heated fluid expelled from the channel or cool fluid drawn into the channel, the rate of fluid exchange, the distance the cooling fluid travels into the channel, and the fluid velocity magnitudes. A clearer understanding of the relationship between these parameters and the cooling effect can be gained upon examination of the effect of the parameters values on the oscillation induced velocity and temperature fields. For example, Figure 4.8 reveals that as $a_0/b_o$ increases, the cooling fluid travels further into the channel corresponding to the increases in the Nusselt number previously discussed.
4.4.2.2 Effect of Parameters on the Average Heated Surface, Volume Averaged, and Maximum Temperatures. The effects of the system parameters on the transient steady state average heated surface, fluid volume averaged, and maximum temperatures can also yield important information that can lead to a better understanding of the mechanisms behind transverse oscillation cooling. A sample of the transient temperature results for constant $\omega$ and $L/b_0$ values are shown in Figure 4.10a and Figure 4.10b.

Increasing the oscillation frequency for fixed $a_0/b_0$ and $L/b_0$ values was found to decrease the amplitudes of the oscillations and the mean values of these three temperatures. This trend may be an indication of improved cooling. Not only are the mean values of the given temperatures lowered by higher frequencies, but also the maximum value each temperature attains during a plate oscillation is lowered. Conversely, as the value of $\omega$ decreases, the maximum value in the oscillation of the volume averaged temperature approaches the maximum value in the oscillation of the average heated surface temperature. This corresponds to a decreased cooling effect for lower $\omega$ values as the overall fluid in the channel is not being effectively cooled.

Though the mean values of all three characteristic temperatures decrease as the value of $a_0/b_0$ increases for fixed $L/b_0$ and $\omega$ values, the amplitudes of the oscillations in the volume averaged and average heated surface temperatures decrease while the amplitude of the maximum temperature increases. (See Figure 4.10a and Figure 4.10b) The decrease in the amplitudes of the volume averaged temperature and the amplitudes of the average heated surface temperature is likely a signal of the improved overall cooling previously discussed. The increase in the oscillation amplitudes of the maximum
temperature is the result of the advancement of the region of the fluid influenced by the oscillations further into the channel towards the high temperature region.

Finally, higher $L/b_0$ values were found to produce higher maximum temperature levels, but lower fluid volume averaged and average heated surface temperatures. The thinner the fluid film, the higher the maximum temperatures due to the lower fluid volume near the channel symmetry line. However, the higher flow velocities that develop with higher $L/b_0$ values more than compensate for these high temperatures as the surface and volume averaged temperatures decrease.

The oscillations in the volume averaged, average heated surface, and maximum temperatures are not necessarily in phase with the plate oscillations. The maximum temperature is nearly $180^\circ$ out of phase with the plate oscillations. Hence, the maximum of the maximum temperatures occurs when the channel width is near its minimum value and the fluid film is thinnest. The oscillations in the volume averaged and the average heated surface temperatures can slightly lead those of the plate oscillations under certain parameter conditions with the phase difference decreasing with increasing $a_0/b_0$. (See Figure 4.10a and Figure 4.10b.)
Figure 4.8  Typical dimensionless temperature contour plots at same position in oscillation for $L/b_o=100 \ \omega = 2000$ rad/sec: (a) $a_o/b_o=0.25$, (b) $a_o/b_o=0.50$, (c) $a_o/b_o=0.75$. 
Figure 4.9 Typical local heated surface results for $L/b_0 = 100$  $\omega = 2000$ rad/sec:
(a) typical dimensionless temperature along heated surface, (b) typical dimensionless heat transfer coefficient along heated surface.
Figure 4.10 Dimensionless temperatures at transient steady state vs. time for $L/b_o = 100$ $\omega = 2000$ rad/sec: (a) $a_o/b_o = 0.25$, (b) $a_o/b_o = 0.75$. 
4.5 Conclusions

The effects of transverse oscillations of an insulated parallel channel wall on the thermal conditions in the channel in general and at the constant heat flux fixed channel wall in particular were investigated in a two dimensional analytical and numerical study under a squeeze film velocity distribution assumption. The major results of the parametric studies of the investigation reveal that the transverse oscillations have the potential for causing significant localized enhancement in the time averaged Nusselt number over that of pure natural convection for sufficient oscillation amplitudes \((a_o/b_o > 0.50)\). Nusselt numbers as much as five times that of a reference natural convection case could be attained. The cooling effect was found to be highly dependent on the oscillation displacement amplitude to mean channel width ratio and the channel length to width ratio. The cooling effect also increased with the oscillation frequency, but the effect was less pronounced. The developed correlation equations can be used to evaluate and compare the resulting heat transfer coefficients.

In Krussing’s experimental investigation of the use of piezoelectric devices [16], an equivalent heat transfer coefficient on the order of 100 W/m\(^2\)K was obtained. This is of the same order as those obtained in the current work for the smaller clearance, and higher displacement and frequency cases.

Based on the results of this study, the use of transverse oscillations has potential as a practical and innovative means of locally augmenting natural convection cooling of electronics. In light of the limitations of the squeeze film model, further study of the use of the local transverse oscillations is warranted. This leads to the study of the oscillations under more general conditions which will be discussed in Chapter 5.
CHAPTER 5

FINITE VOLUME NUMERICAL INVESTIGATION OF CHANNEL WITH TRANSVERSELY OSCILLATING WALL
MORE GENERAL MODEL WITH NATURAL CONVECTION AND INERTIA EFFECTS

5.1 Introduction

Under the squeeze film velocity field, the placement of a transversely oscillating plate in close proximity to a heat source has been shown to produce a substantial cooling effect at the heat source for sufficient oscillation source displacement amplitudes. The squeeze film velocity used in establishing these results is based on the assumption that the magnitudes of the fluid acceleration or inertia effects as well as the natural convection effects are small relative to the pressure and shear forces. However, the level of cooling achieved through the use of the transverse oscillations is dependent on the strength of the dynamic effects of the oscillations on the fluid flow in the immediate vicinity of the heat source. As was seen in the squeeze film study results, low displacement, low velocity, and low frequency plate oscillations may not provide a cooling effect that is sufficient to surpass that of pure natural convection, yet the higher oscillation parameters may invalidate the squeeze film assumptions. In addition, the oscillation source and, thus, the fluid, continually change direction, and, hence, the inertia forces may not be negligible. Finally, for certain parameters, natural convection effects may provide a means by which the high temperature fluid trapped in the stagnant region near the mid-channel length can be carried away, alleviating the thermal “problem area” identified in Section 4.4.2.1. Due to the nature of the squeeze film velocity field, the natural convection effects may be an important mechanism through which the thermal conditions near the heat source can be
improved. However, any such effect can not be investigated with a model excluding the effects of natural convection. Therefore, the squeeze film model may not be able to reveal the true potential of the oscillation enhancement.

A more general model that takes into account the fluid inertia effects as well as the natural convection effects is needed to investigate the use of the oscillations under more realistic conditions. At present little published information exists concerning the use of transverse oscillations to enhance laminar natural convection particularly under conditions where the effects of both the natural convection and the oscillations are important (mixed convection flow). Hence, the study of the use of the transverse oscillations under less stringent assumptions than those of the squeeze film investigation was undertaken. The objective of this investigation was to arrive at a better understanding of the potential of the transverse oscillations. In addition, this study was initiated to gain knowledge about the relative importance of the natural convection and the oscillations in producing the cooling effect and the conditions under which both of these mechanisms can best work together to contribute to the cooling of a system. In this chapter, the findings of this investigation are discussed.

5.2 Problem Statement for More General Model to Investigate the Use of Transverse Oscillations to Enhance Natural Convection

In order to more fully investigate the use of transverse oscillations as a means of augmenting laminar natural convection cooling in a vertical channel, a more general model was developed to study the effects of the oscillations in a simple parallel plate geometry as in the previous chapter. The acceleration or inertia terms as well as the buoyancy force that were all neglected in the squeeze film investigation are included in
this numerical model. As in the study under the squeeze film velocity assumptions, the system investigated consists of a parallel plate channel where the plate at \( y = 0 \), which simulates the heat source, is fixed and supplies a constant heat flux to the channel. A second parallel plate at \( y = b(t) \) is insulated and oscillates transversely to simulate the oscillation source.

Figure 5.1 Model used to investigate use of transverse oscillations to supplement natural convection cooling.

The location and motion of the plate at \( y = b(t) \) are specified in Eq. (5.1) through Eq. (5.3).

The channel width at any time, \( b(t) \), is given by:

\[
b(t) = b_o + a_o \sin(\omega t)
\]  

(5.1)

The velocity of the moving wall, \( V_{wall}(t) \), is given by:

\[
V_{wall}(t) = V_o \cos(\omega t) \text{ where } V_o = a_o \omega
\]  

(5.2)

The acceleration of the moving wall, \( a_{wall}(t) \), is given by:

\[
a_{wall}(t) = -a_o \omega^2 \cos(\omega t)
\]  

(5.3)
Note that $L$ now represents the entire channel length as the problem is no longer symmetric when natural convection effects are taken into account.

With the inclusion of the inertia and buoyancy forces, the set of coupled governing equations needed to determine the velocity, pressure, and temperature field in the channel is more complex than the simplified set of equations that results from the squeeze film assumptions. Thus, a much more extensive finite volume solution procedure is required. Before undertaking the development of a finite volume program to solve for the flow and temperature field in the channel, a brief finite element investigation was carried out using FIDAP© to examine the extent to which the effects of inertia and buoyancy may alter the results and, thus, to determine if further study was warranted. Under the assumptions of Section 2.2 and for the plate motion described in Eq. (5.1) to Eq. (5.3), the velocity, pressure, and temperature fields within the channel were first solved for numerically under conditions where only the inertia effects are included, but the buoyancy force is still neglected. The results of this study confirmed that the inertia effects are important for certain system parameters. A finite element model that takes into account both the inertia effects and the buoyancy force was developed next. The results of a brief finite element study with this more general model verified that for certain parameter values, the natural convection effects also play a significant role in the development of the velocity and temperature fields and thus the cooling effect that the oscillations are capable of producing. Consequently, these finite element study results justify the need to more fully investigate the effects of oscillation-enhanced natural convection.
In the remainder of this investigation, the velocity, pressure, and temperature fields within the channel for the plate motion described above were solved for numerically through finite volume techniques under the general assumptions of Section 2.2. Specific details about the more extensive finite volume solution method needed to numerically solve for the velocity, pressure, and temperature are provided in Section 5.3.3 and in Appendix E. Because of the complex nature of the finite volume solution method needed for this study relative to that of the squeeze film and because of the subsequent long run times to reach even a few plate oscillations, a limited number of parametric studies varying the oscillation and heat source parameters as well as the channel spacing were run for a fixed channel length. These parametric studies revealed key attributes of the flow and temperature fields resulting from the oscillation/natural convection interaction. Measures of the cooling effect including the time averaged and space averaged heated surface temperatures and heat transfer coefficients were also obtained from the study results. Quantitative and qualitative comparisons of these results for different parameter cases, for the natural convection reference case, and, in some cases, for the squeeze film results were then made. Based on the information gathered and the comparison of results, conclusions were made as to the potential level of cooling that may be provided by the oscillation-enhanced natural convection along with an assessment of the conditions under which the method best functions.
5.3 Non-Dimensionalization

For ease of computation, the following set of dimensionless variables was used in this investigation where the $R$ subscript refers to the dimensionless variables used in this more general model.

$$
\tilde{x}_R = \frac{x}{L} \quad \tilde{y}_R = \frac{y}{b(t)} \quad \tilde{u}_R = \frac{u}{\left(\frac{\omega}{2\pi}\right)L} \quad \tilde{v}_R = \frac{v}{\left(\frac{\omega}{2\pi}\right)b_o}
$$

$$
\tilde{p}_R = \frac{P_D}{\rho \left(\frac{\omega}{2\pi}\right)^2} \quad \tilde{t}_R = \frac{\omega}{2\pi} t \quad \tilde{T}_R = \frac{T - T_o}{\Delta T_{ref}}
$$

The reference temperature difference is selected based on the type of thermal boundary condition at $\tilde{y}_R = 0$.

$$
\Delta T_{ref} = T1 - T_o, \text{ constant surface temperature } T1 \text{ applied at } \tilde{y}_R = 0,
$$

$$
\Delta T_{ref} = \frac{q_o b_o}{k}, \text{ constant heat flux, } q_o, \text{ applied at } \tilde{y}_R = 1
$$

(5.4a)

In this study, as in the study of Chapter 4, the $y$-coordinate is scaled by $b(t)$, the instantaneous channel width. This transformation “fixes” the moving surface in the transformed coordinate system. Thus, in the transformed coordinates the system domain does not change size or shape with time, simplifying the numerical procedure, but not altering the governing equations. The moving wall location, displacement, and acceleration must be written in the form of these dimensionless variables. The non-dimensional channel width at any time, $\tilde{b}_R(t_R)$, can be expressed as:

$$
\tilde{b}_R(t_R) = \frac{b(t_R)}{b_o}
$$

(5.5)

The non-dimensional moving wall velocity, $\tilde{V}_{wall,R}$, can be expressed as:
The non-dimensional moving wall acceleration, $\tilde{a}_{\text{wall}R}$, can be expressed as:

$$
\tilde{V}_{\text{wall}R}(\tilde{t}_R) = \frac{V_{\text{wall}}(\tilde{t}_R)}{\left(\frac{\omega}{2\pi}\right)b_o}
$$

(5.6)

As a result of the transformation of variables, it was found to be convenient to define a new variable $\tilde{v}_{\text{new}R}$ that acts as a modified $\tilde{v}_R$:

$$
\tilde{v}_{\text{new}R} = \tilde{v}_R - \frac{\tilde{y}_R}{\tilde{t}_R} \tilde{V}_{\text{wall}R}
$$

(5.8)

Some additional parameters are also defined to make the equations more compact. These parameters are listed below.

The Grashof number, a measure of the buoyancy force to shear force, is:

$$
Gr_{b_o} = \frac{\beta g \Delta T_{ef} b_o^3}{\nu^2}
$$

(5.9)

Reynolds numbers based on both the channel length and mean channel width, measuring the inertia to shear force, are also defined.

$$
Re_L = \frac{\omega L^2}{\nu} ; \quad Re_{b_o} = \frac{\omega b_o^2}{\nu}
$$

(5.10)

The Prandtl number, a measure of the viscous to thermal diffusion effects, is:

$$
Pr = \left(\frac{k}{\rho c_p} \frac{\nu}{\mu}\right)
$$

where $\nu = \frac{\mu}{\rho}$

(5.11)
5.3.1 Dimensionless Form of Governing Equations

This set of dimensionless variables and dimensionless parameters can then be applied to the continuity, momentum, and energy equations, Eqs. (2.1), (2.12), (2.13), and (2.7) respectively. The non-dimensionalization procedure is complicated by the fact that the non-dimensional variable \( \tilde{y} \) is a function of both dimensional \( y \) and \( t \), and attention must be given to properly determine the necessary derivatives.

Applying the dimensionless variables in Eq. (5.4) to the continuity equation in Eq. (2.1) and simplifying yields:

\[
\frac{\partial \tilde{u}_R}{\partial \tilde{x}_R} + \frac{1}{\tilde{b}_R(\tilde{t}_R)} \frac{\partial \tilde{v}_{newR}}{\partial \tilde{y}_R} = \tilde{V}_{wallR}(\tilde{t}_R) 
\]  

(5.12)

The "weak forms" of the transformed momentum and energy equations are obtained through the use of the transformed continuity equation in Eq. (5.12) and result in Eq. (5.13) through Eq. (5.15) below.

After some manipulation, the application of the dimensionless variables in Eq. (5.4) to the \( x \) component of the momentum equation in Eq. (2.12) and the utilization of the appropriate parameters, this equation can be expressed as:

\[
\frac{\partial \tilde{u}_R}{\partial \tilde{t}_R} + \frac{\partial}{\partial \tilde{x}_R} \left( \tilde{u}_R \tilde{u}_R - \frac{1}{Re_L} \frac{\partial \tilde{u}_R}{\partial \tilde{x}_R} \right) + \frac{\partial}{\partial \tilde{y}_R} \left( \frac{1}{\tilde{b}_R(\tilde{t}_R)} \tilde{u}_R \tilde{v}_{newR} - \frac{1}{\tilde{b}_R(\tilde{t}_R)} \frac{1}{Re_o} \frac{\partial \tilde{u}_R}{\partial \tilde{y}_R} \right) 
\]  

(5.13)

\[
= - \frac{\partial \tilde{p}_R}{\partial \tilde{x}_R} + \frac{Gr_o}{Re_L Re_b} \frac{L}{R} \tilde{V}_{wallR}(\tilde{t}_R) \tilde{u}_R - \frac{V_{wallR}(\tilde{t}_R) \tilde{u}_R}{\tilde{b}_R(\tilde{t}_R)} 
\]

Similarly, using the above parameters and dimensionless variables, it can be shown that the transformed form of the \( y \) component of the momentum equation in Eq. (2.13) can be expressed as:
Finally, after some simplification, substitution of the appropriate dimensionless variables and dimensionless groups into the energy equation in Eq. (2.7) yields:

\[
\frac{\partial \tilde{v}_{\text{newR}}}{\partial t_R} + \frac{\partial}{\partial x_R} \left( u_R \tilde{v}_{\text{newR}} - \frac{1}{Re_L} \frac{\partial \tilde{v}_{\text{newR}}}{\partial x_R} \right) + \frac{\partial}{\partial y_R} \left( \frac{1}{\tilde{b}_R(\tilde{t}_R)} \tilde{v}_{\text{newR}} \tilde{v}_{\text{newR}} - \frac{1}{\tilde{b}_R(\tilde{t}_R)} \left( \frac{\tilde{v}_{\text{newR}}}{\tilde{b}_R(\tilde{t}_R)} \right) \frac{\partial \tilde{v}_{\text{newR}}}{\partial y_R} \right) (5.14)
\]

\[
= - \left( \frac{L}{b_o} \right)^2 \frac{1}{\tilde{b}_R(\tilde{t}_R)} \frac{\partial p_R}{\partial x_R} - 2 \frac{V_{\text{wallR}}(\tilde{t}_R)}{\tilde{b}_R(\tilde{t}_R)} \tilde{v}_{\text{newR}} - \tilde{y}_R a_{\text{wallR}}(\tilde{t}_R)
\]

These "weak forms" of the equations are then solved numerically for the velocity, pressure, and temperature fields. More details on the non-dimensionalization procedure are provided in Appendix E.

### 5.3.2 Initial Conditions and Boundary Conditions

In early runs, zero velocity, zero temperature initial conditions were imposed. Then, in order to speed convergence to a transient "steady state," the fluid was given an initial velocity, pressure, and temperature. A squeeze film velocity distribution and pressure field were assigned as initial conditions for the velocity and pressure. Hence:

\[
\tilde{u}_R(\tilde{x}_R, \tilde{y}_R, \theta) = \frac{3V_{\text{wallR}}(\tilde{t}_R)}{\tilde{b}_R(\tilde{t}_R)} \left( 2\tilde{x}_R - 1 \right) \left( \tilde{y}_R - 1 \right) \left( \tilde{y}_R - 1 \right) (5.18a)
\]
Conversion between the dimensionless variables and geometric parameters used in the squeeze film and those used in this more general model was required to arrive at the above form. By the formulation of a curve fit of the solution data of a finite element model, the temperature field is initially set to the temperature field produced by steady state pure natural convection conditions.

The dimensionless form of the boundary conditions is listed below:

From the no slip boundary conditions at the walls:

\[ \tilde{y}_R = 0, \quad \tilde{u}_R = 0, \quad \tilde{v}_R = 0 \]  
\[ \tilde{y}_R = 1, \quad \tilde{u}_R = 0, \quad \tilde{v}_R = \tilde{V}_{wallR} \]  

(5.19a)  
(5.19b)

In addition, the pressure gradients at the channel walls were approximated as zero, or:

\[ \tilde{y}_R = 0, \quad \frac{\partial \tilde{p}_R}{\partial \tilde{y}_R} = 0 \quad \tilde{y}_R = 1, \quad \frac{\partial \tilde{p}_R}{\partial \tilde{y}_R} = 0 \]  

(5.19c)

As in the squeeze film investigation, to simulate the electronic device, a constant heat flux is applied to the surface at \( y = 0 \).

\[ \tilde{y}_R = 0, \quad \frac{\partial \tilde{T}_R}{\partial \tilde{y}_R} = -\tilde{b}_R \left( \tilde{T}_R \right) \]  

(5.19d)

As in the squeeze film study, a thermally insulated boundary condition is used for the moving surface to provide a "conservative estimate of the maximum temperature"[82].

\[ \tilde{y}_R = 1, \quad \frac{\partial \tilde{T}_R}{\partial \tilde{y}_R} = 0 \]  

(5.19e)

At the channel inlet and outlet, the pressure is assumed to be equal to the local hydrostatic pressure, so the dynamic pressure is set equal to zero.
In addition, when fluid flows into the channel at the inlet or the outlet, the incoming fluid is assumed to be well mixed with the fluid in the surroundings and, thus, takes on the ambient temperature. It is also assumed to have one component of velocity. (See discussion in Chapter 2.) When fluid flows out of the channel, the temperature gradient and the gradient of the y component of velocity are assumed to be zero as per discussion in Chapter 2. Hence along the surfaces indicated, the boundary conditions applied are:

\[ \tilde{x}_R = 0, \quad \tilde{p}_R = 0 \]  \hspace{1cm} (5.19f)

\[ \tilde{x}_R = 1, \quad \tilde{p}_R = 0 \]  \hspace{1cm} (5.19g)

\[ \tilde{u}_R \geq 0 \quad \tilde{T}_R = 0 \quad \tilde{v}_R = 0 \]

\[ \tilde{x}_R = 0, \quad \tilde{u}_R < 0 \quad \frac{\partial \tilde{T}}{\partial \tilde{x}_R} = 0 \]

\[ \tilde{x}_R = 1, \quad \tilde{u}_R > 0 \quad \frac{\partial \tilde{T}}{\partial \tilde{x}_R} = 0 \]

\[ \frac{\partial \tilde{v}}{\partial \tilde{x}_R} = 0 \]

\[ \frac{\partial \tilde{v}}{\partial \tilde{x}_R} = 0 \]  \hspace{1cm} (E.19h)

The x component of velocity at the channel inlet/outlet is then determined so as to satisfy both the momentum and continuity equations. More information on the numerical boundary condition implementation, particularly for the \( u \) component of velocity at the channel inlet and outlet can be found in Appendix E.

5.3.3 Numerical Solution Procedure

For the solution of the velocity, pressure, and temperature fields, the SIMPLER finite volume technique developed by Patankar [83, 84] was implemented. More information regarding this method can be found in Appendices D and E. The program listing is given in Appendix F. In the current model, a 195 x 201 graded grid is used as grid sensitivity studies showed that further increases in the number of grid lines produced less than a
0.2% change in the results. (See Figure E.7.) The grid layout was influenced by the expected flow and temperature field patterns. Due to the nature of the flow that develops in the channel for the parameters investigated, a significant number of grid points in the channel width direction are needed to capture the effect of the plate oscillations on the velocity field. Also, to improve the accuracy of the results, graded grid spacing was utilized in this model. Finer grid spacing was placed where high velocity or temperature gradients were expected to occur: near the channel walls, at the channel inlet and outlet, and near the middle of the channel. The stability of the solution was found to be highly dependent on the grid spacing and time step selection. Decreasing the size of the grid required smaller time steps to maintain convergence. Hence, a flexible time stepping scheme was employed. The program developed allows for the specification of different time steps to be used over the course of one oscillation so that smaller time steps can be used over certain portions of an oscillation. Also, the time step scheme used can differ from one plate oscillation to another since convergence problems tend to diminish as the number of cycles increase. In addition, if the solution does not converge within a specified number of iterations, the time step is reduced by a factor of two. If the solution at this half-time step converges, then the time step is increased to the original time step. In the present work, time steps between 5-e04 and 1e-03 were used.

In the implementation of the finite volume SIMPLER method, a backward difference implicit time discretization scheme is used. In the solution of the discretized equations, the unknowns are solved for one y grid line at a time sweeping first in the \( x_R = 0 \) to \( x_R = 1 \) direction and then in the reverse \( x \) direction. For each governing equation for \( \phi (\phi \text{ represents } \bar{u}_R, \bar{v}_R, \bar{T}_R, \bar{P}_R) \), the discretized equations for all unknown values of \( \phi \)
over an entire $y$ grid line are placed into a tridiagonal matrix and Thomas’ Algorithm is used to solve the set of linear equations. The solution at each time step is defined to have converged when the root mean square average residuals of each of the governing equations as well as the local continuity and overall flow rate balance are less than specified epsilon values. More specific details on the limits used are provided in the Appendix E. The solution proceeds in time until a periodically repeating solution or “transient steady state” is attained. The periodically repeating solution condition is met when the time averaged average heat transfer coefficient over one plate oscillation at the heated wall at $y = 0$ is within 0.01% of its value from the previous plate oscillation. The program was validated by comparing the results to known solutions. More information about this validation can be found in Appendix E along with a brief flow chart of the program sequence.

5.3.4 Calculated Parameters

Once the velocity and temperature results were obtained, the following design parameters were calculated to assist in the analysis of the results. These design parameters include a dimensionless local heat transfer coefficient at the heated surface at any time based on the local temperature and heat flux:

$$\tilde{h}_R = \frac{\tilde{q}_R}{\tilde{T}_{s_R}}$$  \hspace{1cm} (5.20)

The dimensionless heat flux in Eq. (5.20) is defined as follows with a reference heat flux equal to the applied heat flux:

$$\tilde{q}_R = \frac{q}{q_a}$$ \hspace{1cm} (5.21)

An average heat transfer coefficient at the heated surface at any time, $\tilde{h}_R$, is defined as:
The time average of this average heated surface temperature over one period of oscillation, \( \bar{h}_{sR} \), is given by:

\[
\bar{h}_{sR} = \frac{1}{T} \int_0^T h_{sR} \, d\tilde{x}_R
\]  
(5.22)

An average temperature of the heated surface at any time, \( \bar{T}_{sR} \), is defined as:

\[
\bar{T}_{sR} = \frac{1}{T} \int_0^T T_{sR} \, d\tilde{x}_R
\]  
(5.24)

The time average of this average heated surface temperature over one period of oscillation, \( \bar{T}_{sR} \), is given by:

\[
\bar{T}_{sR} = \frac{1}{T} \int_0^T T_{sR} \, d\tilde{x}_R
\]  
(5.25)

For convenience in comparing results for cases with different parameter values, a modified dimensionless heat transfer coefficient and dimensionless temperature denoted by the subscript \( M \) are defined where \( b_{nc} \) denotes the typical natural convection flow channel width used in the natural convection reference case. The dimensionless temperature and heat rate are based on the parameters for Case 3A in Table 5.1a.

\[
\bar{h}_{MR} \equiv \left( \frac{h_{sR}}{k} \right)_{\text{Case 3A}}
\]  
(5.26)

\[
\bar{T}_{MR} = \frac{(T - T_o)}{q_o b_{nc} / k}_{\text{Case 3A}}
\]  
(5.27)
For ease of reading, in the remainder of this work \( \bar{h} \) will be denoted by \( Nu \) and \( \bar{h}_M \) will be denoted by \( Nu_M \). Similarly, for ease of comparison, a dimensionless velocity based on the parameters of Case 3A in Table 5.1a is defined as:

\[
\tilde{u}_{MR} = \frac{u}{\omega L} \left( \frac{2\pi}{\omega L} \right)_{\text{Case3A}}
\]  

(5.27a)

5.4 Analysis of Transverse Oscillation Enhanced Natural Convection

The results of this numerical investigation with the more realistic modeling conditions provide important information from which the differences in the flow and temperature fields caused by the inclusion of the inertia and buoyancy effects from those under squeeze film velocity field can be explored. Hence, the contribution of the inertia and buoyancy forces towards the cooling effect can be investigated as well as the relative strength of these two effects under various conditions. The general parameters used in this investigation are provided in Table 4.1 unless otherwise noted.

5.4.1 General Conclusions and Observations

This section contains some general conclusions drawn from the results of this investigation about the importance of the inertia and natural convection effects for systems involving the oscillating channel wall.

5.4.1.1 Studies Including Inertia and Neglecting Natural Convection. First, some general conclusions and observations regarding the flow and temperature fields that result from the oscillation enhanced natural convection are discussed beginning with the effects of the inertia forces. The effects of inertia were isolated in the initial FIDAP© finite
element investigation as the acceleration terms were included in the governing equations, but not the buoyancy force. Comparisons of the results of these studies to results of the squeeze film assumptions for the same system conditions reveal the extent to which the inertia influences the development of the velocity field. Figure 5.2 depicts the inlet velocity ($\tilde{x}_R = 0$) produced under the squeeze film assumptions and that predicted by FIDAP© model including the inertia terms for the same channel geometry, oscillation parameters, and oscillating plate position. The figure illustrates the significant effect of the inclusion of inertia for the parameter values studied. While the squeeze film velocity distribution is parabolic and symmetric about the mid-channel width ($\tilde{y}_R = 1/2$), the velocity distribution under the current modeling assumptions is non-symmetric. The velocity gradients near both the heated and moving surfaces are greater than those of the squeeze film, though the velocities near the middle channel width are lower with the inertia effects. The higher velocity gradients that occur near the heated surfaces with the inclusion of the acceleration effects indicate higher velocities at the same distance perpendicular to the heated surface and hence a thinner velocity boundary layer than that predicted under the squeeze film model. Because of the thinner velocity boundary layer and the higher velocities closer to the heat source, there is potential for thinner thermal boundary layers and for greater heat removal through convection than those occurring under the squeeze film model. The velocity distributions in the remainder of the channel hold similar profiles to those in Figure 5.2, but the velocity magnitudes decrease moving towards the mid-channel length. Therefore, for the parameters studied, the inclusion of the inertia effects has a considerable influence on the velocity field and, consequently, on
both the temperature field in the channel and the potential cooling enhancement provided by the oscillations.

**Figure 5.2** Comparison of dimensionless velocity distribution under squeeze film assumptions and that with the inclusion of the inertia terms at $\tilde{x}_R=0$ with $b_o=0.010m$, $a_0/b_0=0.10$, $L/b_0=20$, $\omega=82.2$ rad/s.

The differences in the “shape” of the velocity distributions in Figure 5.2 can be directly attributed to the inertia effects. To assist in illustrating this point, the typical velocity distributions at the channel inlet over the course of one plate oscillation are depicted in Figure 5.3. Due to the fluid mass, time is required for the effects of the oscillations to diffuse from the moving wall to a given location within the channel, with the greatest effect of the oscillations restricted to a distance of order $\sqrt{\frac{V}{\omega}}$ from the oscillation source. Thus, phase lags develop between the oscillations of the plate and the oscillations of the fluid at different positions in the channel. Because of these phase shifts, the direction as well as the magnitude of the velocity induced by the oscillations
can differ with the location in the channel. This effect gives rise to the observed altered velocity distribution shape as well as the possibility that the flow across any channel cross-section is not uniform in direction as seen in Figure 5.3. As a result of the existence of these phase shifts in the oscillations, or, harmonics, the oscillation induced velocity field can be thought of as the summation of a number of harmonics. These harmonics result in the laminar flow mixing that is responsible for the thinner thermal and velocity boundary layers which lead to the higher near-wall velocity gradients and the improved cooling potential [55, 57, 75]. Because the inertia effects are neglected and the acceleration terms are not included in the squeeze film model, the squeeze assumptions do not allow for the modeling of such phase shifts. Hence, the squeeze film velocities are in phase with the velocities of the oscillating plate throughout the channel. (i.e. Whenever the plate changes direction, all fluid in the channel changes direction.) For smaller channel widths and lower oscillation frequencies and velocity amplitudes, the fluid inertia is reduced and the differences in the velocities predicted by the current model and those under the squeeze film assumptions should decrease.
Figure 5.3  Typical dimensionless velocity distributions with the inclusion of the inertia terms over the course of a plate oscillation at $\tilde{x}_R=1$ with $b_0=0.010\text{m}$ $a_0/b_0=0.10$ $L/b_0=20$ $\omega=82.2\text{ rad/s}$: (a) portion of cycle $\tilde{V}_{wall}=0$, (b) portion of cycle $\tilde{V}_{wall}<0$. 

(a) 

(b)
While the inclusion of the inertia terms has been shown to have a strong effect on the development of the velocity field in general, the intensity of the effect differs along the channel length since, as noted, the velocity magnitudes and, thus, the inertia effects decrease moving toward the mid-length channel position. In the absence of natural convection, the inertia effects near the mid-channel-length region are minimal due to the low velocities and small changes in the velocity as the flow is symmetric, about $\tilde{x}_R = \frac{1}{2}$.

As a result, a low velocity region develops in the same area as did under the squeeze film assumptions. The potential cooling effect of the plate oscillations even with the inertia effects is limited by this low velocity region just as in the squeeze film study. The low velocities inhibit the flow of heat causing high temperatures to develop near the mid-channel length regardless of the higher inertia forces in other portions of the channel. The high temperature area can be seen in Figure 5.4, which shows a typical temperature distribution in the channel where natural convection effects are excluded. Figure 5.5 plots the average heated surface temperature as a function of time for a model including the inertia effects, but neglecting the natural convection effects as well as for a model with natural convection effects. It illustrates not only the high temperatures that develop but also the long time required for a system to reach thermal equilibrium without natural convection. (It should be noted that while the plate has completed almost 68 oscillations for the case neglecting natural convection, a “steady state” has not yet been reached. For comparison, the squeeze film model for the same conditions predicted an average dimensionless temperature of 16 at steady state.) Recall that the squeeze film results indicate that high displacement/high frequency oscillations are needed to overcome the reduced cooling near the mid-channel length. Therefore, it is important to investigate if
and how natural convection effects, which promote through flow along the entire channel length, can be used to remove the heated air from this stagnant region and move it towards the surroundings. The next set of models, therefore, includes the above mentioned natural convection effects first in a limited finite element study and then for all of the subsequent finite volume investigations. The results of these investigations are discussed in the remainder of this chapter.

Figure 5.4 Typical temperature distribution for model with inertia effects and natural convection included for \( b_o=0.010m \) \( a_o/b_o=0.10 \) \( L/b_o=20 \) \( \omega=82.2 \text{ rad/s} \) \( q_o=150 \text{ W/m}^2 \).
Figure 5.5 Average heated surface temperature as a function of time with inertia effects with and without natural convection for $b_0=0.010m$ $a_0/b_0=0.10$ $L/b_0=20$ $\omega=82.2$ rad/s $q_a=150$ W/m$^2$.

5.4.1.2 Studies Including Inertia and Including Natural Convection. For the parameters investigated, natural convection was found to play an important role in the development of the channel velocity and temperature field and thus the cooling effect. The high mid-channel length temperature characteristic can be altered by natural convection effects. As seen in Figure 5.5, when natural convection effects are included the transient steady state can be reached much faster and for a much lower temperature than when it was neglected. The natural convection induced flow is the cause for this difference. The natural convection effects induce flow mainly in the positive $x$ direction, while the direction of the flow induced by the oscillations changes with time. Because of this, the velocity distribution in the channel is not symmetric about $\bar{x}_R=1/2$. At certain times and locations, the natural convection aids the oscillation induced flow resulting in higher velocities. At other times and locations, the natural convection induced flow
opposes the oscillation induced flow, resulting in velocities lower than those obtained neglecting natural convection effects. While the total amount of fluid entering and exiting the channel must match the fluid volume displaced by the plate motion, the flow through the surface at $\tilde{x}_R=0$ need not equal the flow through the surface at $\tilde{x}_R=1$. As a result of the natural convection induced flows, the location of the low velocity region shifts with time and does not necessarily remain at the mid-channel length. At the mid-channel length, the velocity resembles that of pure natural convection in the positive $x$ direction. (See Figure 5.6.) The direction of this flow does not change with the plate oscillations.

The different flow characteristics caused by the oscillations affects the heat flow in the channel. As previously discussed, due to the low velocity region at the mid-channel length when natural convection is neglected, all of the heat on the “left” half of the channel remains on the “left” half of the channel or exits at $\tilde{x}_R=0$ while all of the heat on the “right” half of the channel remains on the “right” half of the channel or exits at $\tilde{x}_R=1$. Therefore, with no natural convection and the low mid-length velocities, the heat near the middle length of the channel becomes essentially “trapped”. With the inclusion of the natural convection effects, the natural convection induced flow continuously acts to move heat through the mid-channel length towards the $\tilde{x}_R=1$ end of the channel regardless of the direction of the oscillations. This brings the higher temperature fluid closer to the higher temperature gradient – higher oscillation induced velocity regions at the channel ends where it can be more effectively carried to the surroundings. Thus, the natural convection effects can be used to assist the oscillations in improving the thermal conditions in the channel.
Figure 5.6 Velocity distributions at $x_R=1/2$ for model with inertia effects and natural convection included for $b_0=0.010\text{m}$ $a_0/b_0=0.10$ $L/b_0=20$ $\omega=82.2$ rad/s $q_\sigma=150$ W/m$^2$.

The ability of the natural convection induced flows to alter the flow of heat can be most clearly seen by observing the changes in the temperature distribution in the channel as time increases. The results showed that time is required for the temperature field and, hence, the natural convection induced flow to develop. (Note Figure 5.6.) The development of the surface temperature along the heated surface is shown in Figure 5.7 and is compared to the surface temperature with the natural convection effects neglected for a case with a 10% mean channel width displacement amplitude. Figure 5.8 depicts the development of the temperature field in the channel through contour plots for the same case with the natural convection effects.

As the natural convection effects begin to become noticeable, the temperatures along the heated surface near $\tilde{x}_R=0$ are lower than they are without the inclusion of the natural convection effects while those near $\tilde{x}_R=1$ are consistently higher. There is little
difference in the middle length of the channel. (See Figure 5.7b.) Regardless of the
direction of the plate motion, natural convection induced flow brings in more cool air at
\( \tilde{x}_R = 0 \) resulting in more heat being convected towards the \( \tilde{x}_R = 1 \) end of the channel,
resulting in a higher temperature region in the downstream part of the channel than that
for a flow without natural convection. As the temperature field continues to develop and
the temperature difference increases, the natural convection induced flow rate also
increases. Plots of the velocity at \( \tilde{x}_R = 1/2 \) in Figure 5.6 clearly show the growth of the
natural convection induced flow. As the flow increases, more cool fluid is drawn into the
channel at \( \tilde{x}_R = 0 \) and more heat is convected towards \( \tilde{x}_R = 1 \). (See Figure 5.8.) It is
observed that with this flow increase, there is an increase in the portion of the heated
surface near the inlet over which the temperatures are less than those in models where
natural convection effects are neglected. There is also a decrease in the portion of the
heated surface near the outlet over which the temperatures are higher than in models
without natural convection. (See Figure 5.7b.) Eventually, the natural convection
induced flow is strong enough to create through flow along the channel length. (See
Figure 5.8.) As a consequence, the average heated surface temperature with natural
convection is significantly lower than that where natural convection is neglected. (See
Figure 5.7c.) These results clearly illustrate that, for certain parameters, the natural
convection effects working in combination with the transverse oscillations do improve
the thermal conditions in a system. Of course, the specific distributions depend on the
system parameters. Overall, the cooling effect of the transverse oscillations may best be
utilized in the operational regime for which the natural convection effects are comparable
to those of the oscillations.
Figure 5.7 Heated surface temperature development with and without natural convection 
\( b_o = 0.010 \text{m} \quad a_o / b_o = 0.10 \quad L / b_o = 20 \quad \omega = 82.2 \text{ rad/s} \quad q_a = 150 \text{ W/m}^2 \): 
(a) \( t_F = 6 \), (b) \( t_F = 12 \), 
(c) \( t_F = 24 \).
Figure 5.8  Contour plots showing development of temperature field with the inclusion of natural convection for $b_0=0.010\text{m}$ $a_0/b_0=0.10$ $L/b_0=20$ $\omega=82.2$ rad/s $q_a=150$ W/m$^2$: (a) $t_F=11.5$, (b) $t_F=16.5$. (cont.).
Figure 5.8 Contour plots showing development of temperature field with the inclusion of natural convection for $b_0=0.010\text{m}$ $a_0/b_0=0.10$ $L/b_0=20$ $\omega=82.2\text{ rad/s}$ $q_a=150\text{ W/m}^2$: (c) $t_F=19.5$, (d) $t_F=35.5$. 
Thus, the results indicate that the transverse oscillation natural convection enhancement method may best be implemented under “mixed” convection conditions where both the natural convection and oscillation effects play important roles. In general, the ratio of \( \frac{Gr}{Re_{bo} Re_{L}} \) measures the importance of the natural convection effects to those of the forced convection oscillation effects. Typically, the mixed convection regime is for \( \frac{Gr}{Re_{bo} Re_{L}} \) type values between 0.10 and 10 [85]. Therefore, the proper parameter values must be used in order for the beneficial oscillation aided natural convection characteristics described above to persist. While the ratio may indicate that either the natural convection or oscillation effects dominate, the study has also shown that the cooling effect may not necessarily dominate in all regions in the system. Though the effects of the oscillations are greater near the ends of the channel, at the mid channel length natural convection cooling is more significant.

5.4.2 Effect of Parameter Values

Using the available numerical tools, the combined effects of the oscillations and natural convection were investigated for systems with a number of different parameters. Due to the long run times, the quantity of these studies was limited. However, useful information regarding the basic trends in the relationship between the oscillation, geometric and heat source parameters can be gained from these limited studies. Table 5.1 gives a listing of the time-averaged average heat transfer coefficient and temperature values at the heated surface.
Table 5.1a Summary of Results for More General Model Investigation $q_o=150$ W/m$^2$

<table>
<thead>
<tr>
<th>Case</th>
<th>$b_0$ (m)</th>
<th>$L$ (m)</th>
<th>$L/b_0$</th>
<th>$a_0/b_0$ (m)</th>
<th>$\omega$ (rad/sec)</th>
<th>$q_o$ (W/m$^2$)</th>
<th>$\equiv h_R$</th>
<th>$T_M$</th>
<th>%Diff Pure Nat. Conv. $L/b_0=20$</th>
<th>$h_M$</th>
<th>%Diff Pure Nat. Conv. $L/b_0=20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady State*</td>
<td>0.01</td>
<td>0.2</td>
<td>20</td>
<td>n/a</td>
<td>n/a</td>
<td>150</td>
<td>0.3553</td>
<td>2.4752</td>
<td>0.4770</td>
<td>n/a</td>
<td>2.4752</td>
</tr>
<tr>
<td>1A</td>
<td>0.01</td>
<td>0.2</td>
<td>20</td>
<td>0.1</td>
<td>82.252323</td>
<td>150</td>
<td>0.4517</td>
<td>2.2139</td>
<td>0.4517</td>
<td>-5.30%</td>
<td>2.2139</td>
</tr>
<tr>
<td>2A**</td>
<td>0.01</td>
<td>0.2</td>
<td>20</td>
<td>0.1</td>
<td>82.252322</td>
<td>150</td>
<td>0.5606</td>
<td>2.3429</td>
<td>0.5606</td>
<td>17.52%</td>
<td>2.3429</td>
</tr>
<tr>
<td>3A</td>
<td>0.01</td>
<td>0.2</td>
<td>20</td>
<td>0.25</td>
<td>25</td>
<td>150</td>
<td>0.4609</td>
<td>2.7497</td>
<td>0.4609</td>
<td>-3.36%</td>
<td>2.7497</td>
</tr>
<tr>
<td>4A</td>
<td>0.01</td>
<td>0.2</td>
<td>20</td>
<td>0.25</td>
<td>40</td>
<td>150</td>
<td>0.4121</td>
<td>3.0461</td>
<td>0.4121</td>
<td>-13.61%</td>
<td>3.0461</td>
</tr>
</tbody>
</table>

* Note pure oscillation for $L/b_0=20$ produces $T_M>1.2$

** Note different times

Table 5.1b Summary of Results for More General Model Investigation $q_o=600$ W/m$^2$

<table>
<thead>
<tr>
<th>Case</th>
<th>$b_0$ (m)</th>
<th>$L$ (m)</th>
<th>$L/b_0$</th>
<th>$a_0/b_0$ (m)</th>
<th>$\omega$ (rad/sec)</th>
<th>$q_o$ (W/m$^2$)</th>
<th>$\equiv h_R$</th>
<th>$T_M$</th>
<th>%Diff Pure Nat. Conv. $L/b_0=20$</th>
<th>$h_M$</th>
<th>%Diff Pure Nat. Conv. $L/b_0=20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steady State*</td>
<td>0.01</td>
<td>0.2</td>
<td>20</td>
<td>n/a</td>
<td>n/a</td>
<td>600</td>
<td>0.3553</td>
<td>3.2255</td>
<td>1.4211</td>
<td>n/a</td>
<td>3.2255</td>
</tr>
<tr>
<td>1B</td>
<td>0.01</td>
<td>0.2</td>
<td>20</td>
<td>0.1</td>
<td>164.50465</td>
<td>600</td>
<td>0.3741</td>
<td>2.6731</td>
<td>1.4964</td>
<td>5.30%</td>
<td>2.6731</td>
</tr>
<tr>
<td>2B</td>
<td>0.01</td>
<td>0.2</td>
<td>20</td>
<td>0.25</td>
<td>164.50456</td>
<td>600</td>
<td>0.2059</td>
<td>4.8557</td>
<td>0.8238</td>
<td>-42.03%</td>
<td>4.8557</td>
</tr>
<tr>
<td>3B</td>
<td>0.01</td>
<td>0.2</td>
<td>20</td>
<td>0.25</td>
<td>40</td>
<td>600</td>
<td>0.3176</td>
<td>3.6564</td>
<td>1.2705</td>
<td>-10.60%</td>
<td>3.6564</td>
</tr>
<tr>
<td>4B</td>
<td>0.005</td>
<td>0.2</td>
<td>40</td>
<td>0.25</td>
<td>40</td>
<td>600</td>
<td>0.5626</td>
<td>2.2815</td>
<td>1.1251</td>
<td>-20.83%</td>
<td>4.5629</td>
</tr>
</tbody>
</table>

* Note pure oscillation for $L/b_0=20$ produces $T_M>1.72$
In general, the results show that decreasing the mean channel width results in slightly higher heat transfer coefficients and slightly lower temperatures holding all other parameters fixed (Case 3B and Case 4B from Table 5.1b). Decreasing the mean channel width by half increased the maximum velocity by a factor of two. The increase in the cooling effect can be attributed to these higher velocities. In addition to the higher velocity magnitude, the velocity profile is also altered by the changed channel width. For the smaller channel width, just after the plate changes direction, the distribution is more complex and asymmetric than that for the greater spacing likely due to the greater fluid acceleration. (See Figure 5.9.) Yet, as the plate reaches its maximum and minimum velocities, it appears to be more symmetric when the fluid acceleration is lower possibly due to the lower fluid mass. (See Figure 5.10.) In addition, the higher velocities with the decreased channel width result in a more noticeable inertia effect near the mid-length as can be seen from the velocity profiles which have become “distorted” from the typical natural convection distribution that occurs with the larger spacing. (See Figure 5.11.) As a result of the stronger oscillation effects and the weaker natural convection effects for the case with the smaller plate spacing, the region of maximum temperature is further upstream of its location for the higher channel spacing. (See Figure 5.12.)

Though in both these parameter cases, the temperatures achieved are not substantially lower than those for pure natural convection, they are significantly lower than those resulting from pure oscillations as indicated in Table 5.1b.
Figure 5.9 Comparison of inlet velocity distributions (modified dimensionless) at 30% of cycle for $a_o/b_o=0.25$, $L/b_o=10$, $q_a=600 \text{ W/m}^2$, $\omega=40 \text{ rad/sec}$ for $b_o=0.010\text{m}$ and $b_o=0.005\text{m}$. 
Figure 5.10 Comparison of inlet velocity distributions (modified dimensionless) over a cycle for $a_o/b_o=0.25$, $L/b_o=10$, $q_o=600$ W/m$^2$: (a) $b_o=0.010$m, (b) $b_o=0.005$m.
Figure 5.11 Comparison of mid-length velocity distributions (modified dimensionless) over a cycle for \( \frac{a_o}{b_o} = 0.25 \) \( L/b_o = 10 \), \( \omega = 40 \) rad/sec, \( q_o = 600 \) W/m²: (a) \( b_o = 0.010\)m, (b) \( b_o = 0.005\)m.
Figure 5.12 Variation of heated surface temperature distributions with time (modified dimensionless) for \( a_o/b_o = 0.25 \) \( L/b_o = 10 \), \( \omega = 40 \) rad/sec, \( q_o = 600 \) W/m\(^2\): (a) \( b_o = 0.010m \), (b) \( b_o = 0.005m \).
The effect of an increase in the applied heat rate was also investigated, where holding all other parameters fixed, the heat rate was increased by a factor of four (Case 4A and Case 3B). The results of this investigation show that while higher temperatures result for the higher heat rate case because it receives four times the amount of heat, its heat transfer coefficient is slightly higher than that at a lower heat rate. Therefore, for the given parameter condition, the cooling method provides a more effective means of heat removal at higher heat rate. The basic shapes of the velocity distribution curves for the two different heat rates are very similar. (See Figure 5.10a and 5.13.) However, they differ in magnitude with the velocities for the higher $q_a$ case approximately double those with a lower heat rate. In addition, the maximum flow at the mid-length of the channel, mainly induced by natural convection, is about four times greater than that with a lower heat rate. While the velocity distribution at the inlet appears to be in phase between these two cases (the applied oscillations and geometry are the same), there is a greater amount of fluid flowing into the inlet and through the channel for the higher heat flux case than that for the lower heat flux case conditions due to the higher buoyancy effects. (See Figure 5.14.) This difference in the flow field in the channel leads to differences in the temperature distribution in the channel. Though the higher heat rate case attains higher temperature values, the stronger natural convection effect moves the heated fluid towards the downstream end of the channel much more rapidly than in the lower applied heat flux case. (See Figure 5.12a and 5.15.) This action helps to cool the upstream portion of the heated surface. Thus, the greater flow rate and the ensuing higher velocities due to the natural convection effects are responsible for the more effective cooling with the higher applied heat flux.
Note: Compare to Figure 3.10a.

Figure 5.13 Variation of inlet velocity distribution over a cycle (modified dimensionless): $a_o/b_o=0.25$, $L/b_o=20$, $b_o=0.010\text{m}$, $\omega=40\ \text{rad/sec}$, $q_{a}=150\ \text{W/m}^2$.

Note: Compare to Figure 5.9

Figure 5.14 Comparison of inlet velocity distributions 30% of cycle (modified dimensionless) for $a_o/b_o=0.25$, $L/b_o=20$, $b_o=0.010\text{m}$, $\omega=40\ \text{rad/sec}$ for $q_{a}=150\ \text{W/m}^2$ and $q_{a}=600\ \text{W/m}^2$. 
Comparison of the results for two parameters cases where only the frequency differed demonstrated that the higher frequency provided a greater cooling effect (Case 3A and Case 4A). (A 23% increase in the heat transfer coefficient relative to pure natural convection was achieved versus 11% with a lower frequency.) At the inlet and outlet of the channel, the shape of the velocity distribution for the higher frequency case is slightly more complex resulting from the greater inertia effects and can be described by a sum of higher order harmonics. Because of the more complex nature of these flows and the higher inertia effects, the higher frequency flows are more apt to produce flow reversals across the channel width and hence to cause greater fluid mixing. The higher frequency also produces greater velocity magnitudes. These factors contribute to the greater cooling effect. (See Figure 5.13 compared to Figure 5.16. Note difference in the scales.) The velocity distributions for these cases at 30% of an oscillation just after the moving plate
changes direction are depicted in Figure 5.17. The higher velocity magnitudes produced by the higher frequency oscillations case can be easily observed. Because of the lower magnitudes of the velocities for the lower frequency case, the natural convection cooling plays a more significant role in the velocity distribution development and the heat flow for this case. Near the mid-channel length, the flow which is mainly induced by natural convection is slightly higher for the lower frequency case. (See Figure 5.18.) Comparing the surface temperature results over time (Figure 5.19 and Figure 5.15), it is noted that while the lower frequency case results in higher temperatures, the natural convection induced flows, which are relatively larger than those for the higher frequency case, more effectively and more rapidly force the heated air towards the downstream end of the channel as the natural convection flow faces less opposition from the oscillation induced flow due to the smaller oscillation induced flow velocities. The slope of the heated surface temperature distribution reached after eight plate oscillations for the higher frequency case is attained in only five cycles for the lower frequency case. While the natural convection effects may be better at moving the heat from the mid-channel length with the lower oscillations frequency, they are not strong enough to move the heat out of the channel and thus can not cool as effectively as the higher frequency flows. (See Figure 5.19 compared to Figure 5.15.) The velocity field characteristics and their relationship to the temperature distribution illustrate why the higher frequency results in lower temperatures and higher heat transfer coefficients than the lower frequency oscillations.
Note: Compare to Figure 5.13

Figure 5.16 Variation inlet velocity distribution over cycle (modified dimensionless) for \( a_o/b_o=0.25\), \( L/b_o=20\), \( b_o=0.010\)m, \( \omega=25\) rad/sec, \( q_a=150\) W/m\(^2\).

Figure 5.17 Comparison of inlet velocity distributions 30% of cycle (modified dimensionless) for \( a_o/b_o=0.25\), \( L/b_o=20\), \( b_o=0.010\)m, \( q_a=150\) W/m\(^2\) for \( \omega=40\) rad/sec and \( \omega=25\) rad/sec.
Figure 5.18 Comparison of mid-channel length velocity distributions (modified dimensionless) for $a_o/b_o=0.25$, $L/b_o=20$, $b_o=0.010$m, $q_o=150$ W/m$^2$: (a) $\omega=25$ rad/sec., (b) $\omega=40$ rad/sec.
Finally, increasing the displacement amplitude generally was found to result in lower temperatures and higher heat transfer coefficients. Higher amplitudes call for a greater amount of fluid to be moved into and out of the channel at higher velocities during the oscillations (Case 1B and Case 2B). Up to a 50% increase in the time averaged surface averaged heat transfer coefficient was obtained in this study for the higher displacement cases investigated. The plots of the velocities for systems under the same conditions but for 10% and 25% mean channel width oscillation displacement amplitudes at 30% of a plate oscillation in Figure 5.20 show not only the greater velocity magnitudes and flow rates involved but also the more complicated flow patterns that occur for the higher displacement amplitudes. The effects of the oscillations are significant enough to cause some flow reversal even at the mid channel length as shown in Figure 5.21.
The difference in velocity fields caused by the different amplitudes affects the temperature and thus the heat flow and the cooling potential. Comparing the temperature contours and the heated surface temperature distributions among cases with different amplitudes can lead to a clearer understanding of the reason that the higher displacement results in a greater cooler effect. While the general development of the temperature field for the 10% case follows that discussed in Section 5.4.1.2, the temperature field pattern for the higher displacement amplitude differs. Due to the higher amplitude, more ambient air is drawn into the channel which advances further into the channel. This can be seen in the larger region of lower temperatures near the channel ends in both the surface temperature and the temperature contour plots in Figures 5.22 and 5.23 respectively. While the mid-length temperatures may be higher, the overall or average temperatures are lower. As time increases, and the temperature field develops, the natural convection induced flow grows and its effect on the temperature field increases. The surface temperature distribution becomes less symmetric, and the high temperature region begins to move downstream where the heat can be more readily carried out of the channel. The average surface temperature that results is significantly lower than that for the 10% displacement case as can be seen in Figure 5.24. (Both temperatures are significantly lower than that for pure natural convection.) (See Table 5.1b.) The study also indicates that for the same oscillation parameters, heat rates, and channel geometries, the cooling effect under conditions when both the natural convection and oscillations are able to act together is typically greater than that achieved under the squeeze film conditions.
Figure 5.20 Comparison of inlet velocity distributions at 30% of oscillation (modified dimensionless) for $L/b_0=20$, $b_0=0.010m$, $q_a=600 \text{ W/m}^2$, $\omega=164\text{ rad/sec}$ for $a_0/b_0=0.10$ and $a_0/b_0=0.25$.

Figure 5.21 Close-up of mid-channel length region showing flow reversal near the heated wall indicating strong oscillation effects for: $L/b_0=20$, $a_0/b_0=0.25$, $b_0=0.010m$, $\omega=164\text{ rad/sec}$, $q_a=600 \text{ W/m}^2$. 
Figure 5.22 Development of temperature field in channel for $a_0/b_0=0.25$ $L/b_0=20$, $b_0=0.010$m, $\omega=164$rad/sec, $q_a=600$ W/m$^2$: (a) $t_F=4.70$, (b) $t_F=6.20$, (c) $t_F=10.70$ (cont.).
Figure 5.22  Development of temperature field in channel for $a_o/b_o=0.25$ $L/b_o=20$, $b_o=0.010m$, $\omega=164$rad/sec, $q_a=600$ W/m$^2$: (d) $\tilde{t}_F=11.70$, (e) $\tilde{t}_F=12.70$. 
Figure 5.23 Development of surface temperature distributions (modified dimensionless) for: $L/b_0=20$, $a_0/b_0=0.25$, $b_0=0.010\text{m}$, $\omega=164\text{rad/sec}$, $q_a=600\ \text{W/m}^2$.

Figure 5.24 Comparison of temperature distributions along heated surface (modified dimensionless) for different oscillation amplitudes: $L/b_0=20$, $b_0=0.010\text{m}$, $\omega=164\text{rad/sec}$, $q_a=600\ \text{W/m}^2$. 
5.5 Conclusions

From the results of this study using more general modeling assumptions to account for both inertia and natural convection effects, some important conclusions can be drawn. Transverse oscillations may be used to supplement the natural convection cooling in a manner such that the cooling effects produced by both of these methods can contribute to the improvement of the thermal conditions of a system. With the natural convection, cooling comparable to that achieved in the squeeze film study can be reached at lower displacements and lower frequencies. This means that less energy can be expended to produce a similar cooling effect. In addition, the oscillations of the plate should be inaudible, so the frequency of the oscillations must be kept below the audible range (less than 100Hz or 628 rad/sec for dimensions expected to be used in such an application) [7]. Operating in the "mixed" convective flow regime would help to meet this requirement while still allowing for a substantial cooling effect.

Basic information about the relationships between the cooling effect and the system parameters was also gained in this study. The cooling effect, as measured by the time averaged heat surface temperatures and heat transfer coefficients, was found to increase for higher oscillation frequencies, smaller mean channel width values, and most importantly, higher oscillation amplitude values. (At least a 25% displacement was found to be needed for significant cooling relative to the natural convection reference.) The amount of heat supplied to the system, or, the Grashof number, is also an important factor, and the effectiveness of the method seems to increase with increases in the heat rate for the parameters investigated. While direct comparisons cannot be made due to differences in the system setups, Garimella's limited experimental investigation of the
use of a piezoelectric fan in [5] found that between a 24 and 100% increase in the heat transfer coefficients over natural convection could be obtained. These numbers are within the order of magnitude of the enhancement found in the results of this study.

From the results of this study, it can be concluded that the use of transverse oscillations to aid natural convection cooling is a viable method of significantly improving the thermal conditions of a system for a specific range of realistic parameter values. Further investigation of the implementation of this method for a variety of heat source and oscillation source geometries and arrangements would be beneficial in the assessment of the practical application of this method.
CHAPTER 6

FINITE ELEMENT NUMERICAL INVESTIGATIONS OF THE USE OF DISCRETE TRANSVERSE OSCILLATION SOURCES FOR NATURAL CONVECTION ENHANCEMENT WITH VARIOUS GEOMETRIES

6.1 Introduction

The numerical investigations of Chapters 4 and 5 suggest that under the proper system configurations the imposition of transverse oscillations of a body placed near a heated surface can considerably enhance pure natural convection cooling in a vertically oriented channel by producing conditions that facilitate increased heat removal from the heat source vicinity. The results of these studies were used to establish the general range of the potential cooling effect and the parameter values needed to effectively cool. Also, the results were used to ascertain basic trends in the resulting flow and temperatures, and to uncover information about the relationship between the system parameters and the measures of the cooling effect that develops. However, these studies made use of a simplified and restrictive heat source and oscillation source geometry resulting in a system formed by two parallel plates. It is recognized that in order to judge the feasibility of this natural convection enhancement method, studies with more realistic heat source and oscillation source geometries need to be undertaken.

In the next portion of this work, the effects of more complex and realistic system component geometries on the potential cooling enhancement were investigated by employing two dimensional heat source or oscillation source geometries. There are a number of implications behind the use of these more complicated two-dimensional geometries. The oscillation source and heat source become finite discrete entities placed
within a vertically oriented channel and so the space between the oscillation source and the heat source comprises only part of the system. The flow as well as the temperature and pressure through the "entrance and exit" areas to the region between the oscillation source and the heat source must be solved for as part of the solution procedure, as these surfaces no longer coincide with the boundaries to the open surroundings. Also, due to the thin, short plate oscillation source geometry, fluid can freely flow over the oscillation source and around the ends of the oscillation source. The interaction of the effects of natural convection, which act mainly in the downstream positive x direction, the plate vibrations, and the plate and heat source geometry, particularly at the entrance and exit to the region between the moving surface and fixed heat source surface, leads to more complex flow patterns. In addition to these effects, conduction through the oscillation source also occurs. All of these issues cannot be easily modeled with the simple parallel plate geometry. The new flow attributes that result with these additional factors, alter the temperature fields and hence the cooling effect of the oscillations. The impact that the use of the two-dimensional oscillation source and heat source geometry has on the ability of the oscillations to cool was investigated for various heat source geometries and oscillation source arrangements within the channel. This chapter describes the studies that were performed and explains their important findings.

6.2 General Problem Statement

The application of transverse oscillations to discrete oscillation sources strategically positioned near heat sources in a vertically oriented channel was investigated through finite element methods for three basic sets of geometries including an oscillation source
placed in a plain channel, an oscillation source placed just above a rectangular heat
source, and modified rectangular heat source/oscillation source arrangements. For each of
these geometries, the vibration source is modeled as a thin short transversely oscillating
plate with a mean position or clearance, C, from the heat source surface as shown in
Figure 6.1. The specific geometries used in each of these sets are discussed later.

Using the dimensionless variables discussed in Appendix B and the oscillation
specifications in Eqs. (B.52-53), the displacement, not position, of the oscillation source
is given by:

\[ \ddot{d}_{os} (t_F) = \frac{A}{b} \sin \left( \ddot{\omega}_F \tau_F \right) \]  \hspace{1cm} (6.1)

The velocity of the plate in the positive y direction is given by:

\[ \ddot{V}_{os} (\tau_F) = \frac{A\ddot{\omega}_F}{b} \cos \left( \ddot{\omega}_F \tau_F \right) \]  \hspace{1cm} (6.2)

where \( A \) is the displacement of the oscillation source and \( b \) is the reference length \( L_{ref} \).

For ease of reading, \( \frac{A}{b} \) will be denoted by \( d \), \( \frac{A\ddot{\omega}_F}{b} \) by \( V \), and \( \ddot{\omega}_F \) by \( \omega \).

Parametric studies varying the oscillation parameters and, in some cases, the
oscillation source location, \( C \), were performed using finite element methods to determine
the flow and temperature fields that result from the oscillation source motion described above for fixed applied heat rates. Simulations were continued until a transient steady state was reached, at which point the potential cooling effect was examined. This includes studying the temperature and flow fields that result for the cases with the various geometries for information about the effectiveness of the oscillation augmented natural convection method. Also, the space averaged heat transfer coefficients at the appropriate heat source surfaces were determined at each time step, and the local heat transfer coefficients at four distinct heat source surface locations were tracked over time to determine the local impact of the oscillations at various positions. These heat transfer coefficients were then averaged over time so that the effect of the oscillations could be better evaluated. In order to assess the performance of the oscillation sources in supplementing the natural convection, comparisons were made among the results for different oscillation parameters and against the results for steady state pure natural convection with and without the oscillation source.

Because FIDAP© allows for the modeling of moving entities, it was employed to create and solve finite element models for these dynamic problems. Due to the complex nature of the geometric models, the coupling of the governing equations and the need to determine the location of the finite element node points at every time step, the number of parametric studies run was limited. Computations alone were time and memory intensive. In addition, the treatment of the moving body problem by FIDAP© caused some challenging issues. While the displacements and velocities of a moving entity in the finite element model are easily specified, the nodes in the fluid entity must be repositioned at each time step in response to the motion of the body. To find the new
node positions at each time step, the program uses an “elasticity-based meshing algorithm”, treating the mesh like a deforming solid where an artificial Young’s Modulus and Poison ratio are supplied for different sections of the mesh. These parameters were altered during the motion in order to prevent severe mesh distortion. However, the displacements that can be achieved with this method are limited due to the potential for non-convergence and mesh distortion problems. (More information about the FIDAP© solution methods can be found in [86].)

While the results may not be sufficient for a complete quantitative analysis, they are sufficient to provide for a good qualitative understanding of the potential for this heat transfer enhancement method.

6.3 Discrete Transverse Oscillation Sources to Supplement Natural Convection in Plain Channel

In this portion of the study, the first geometry set investigated was the plain channel with a discrete oscillation source. This section describes the important results obtained for this geometry.

6.3.1 Problem Statement – Plain Channel Geometry

The model geometry consists of a vertically oriented channel containing a small, short thin oscillating plate as shown in Figure 6.2. The general assumptions from Section 2.2 are applied to this system. In addition, conduction in the oscillation source is modeled with the application of the appropriate thermal conductivity parameter. At the surface \( y = 0 \), a constant heat flux is supplied. The standard inlet, outlet, and surface boundary conditions are specified as in the finite element investigations including conduction in the
solids in Section 3.3 or as discussed in Section 2.4. The specific parameter values used are listed in Table 6.1. After testing a number of oscillation source clearance values, a mean clearance value, $C$, of 0.15 was selected. For this clearance distance, the plate was sufficiently far from the heat source to allow for natural convection flow between the heat source and oscillation source, but close enough for the oscillations to influence the flow near the heated surface. Different oscillation source displacement amplitudes and frequencies were then tested for a fixed applied heat flux so as to facilitate the examination of the effects of these parameters. The parameter cases used are listed in Table 6.2.

After a grid sensitivity study weighing both the solution accuracy, run time, and the possible mesh distortion concerns, a $261 \times 75$, 9 node quadratic quadrilateral element graded mesh was generated with FIDAP©. Solutions proceeded in time until a transient steady state was reached. In addition to the flow and temperature distributions, measures of the cooling effect were calculated. These include the time-averaged average heat transfer coefficients at the heated surface and the time-averaged local heat transfer coefficients for the four points shown in red in Figure 6.3. These four locations are: just upstream of the oscillation source, two positions under the oscillation source, and one position just downstream of the oscillation source. The resulting temperature and velocity fields and the subsequent calculated parameters were then analyzed.

![Figure 6.2 Plain channel model geometry.](image)
Table 6.1 Data Used in Investigation of Use of Discrete Oscillation Sources To Supplement Natural Convection – Plain Channel Geometry

<table>
<thead>
<tr>
<th>Material Property*</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k$ (thermal conductivity of air)</td>
<td>0.027 W/mK</td>
</tr>
<tr>
<td>$\nu$ (kinematic viscosity of air)</td>
<td>1.717e-5 m/s²</td>
</tr>
<tr>
<td>$\rho$ (density of air)</td>
<td>1.12492 kg/m³</td>
</tr>
<tr>
<td>$c_p$ (specific heat of air)</td>
<td>1005.93 J/kg K</td>
</tr>
<tr>
<td>$\beta$ (volumetric expansion)</td>
<td>1/315.5K</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$g$ (gravitational acceleration)</td>
<td>9.81 m/s²</td>
</tr>
<tr>
<td>$T_o$ (ambient temperature)</td>
<td>25°C</td>
</tr>
<tr>
<td>$q_a$ (applied heat flux)</td>
<td>150 W/m²</td>
</tr>
<tr>
<td>Grashof number based on b</td>
<td>15122.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimensional Parameter**</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b$ (channel height)</td>
<td>0.50 in = 0.0125 m</td>
</tr>
<tr>
<td>$TL$ (channel length)</td>
<td>7b</td>
</tr>
<tr>
<td>$PT$ (plate thickness)</td>
<td>0.1b</td>
</tr>
<tr>
<td>$PL$ (plate length)</td>
<td>1.0b</td>
</tr>
</tbody>
</table>

*Properties at $T_{avg}$ = 42.5°C = 315.5 K
**Dimensions defined in Figure 6.3

Figure 6.3 Dimensioning of plain channel geometry model.

6.3.2 Analysis of Results – Plain Channel Geometry

From the results of the investigation, it is clear that the movement of the plate affects the velocity distribution in the channel, which, in turn, alters the temperature and heat flow patterns to which the velocity is coupled. The greatest impact of the oscillations occurs in close proximity to the oscillation source. The extent of the region of influence varies
in size and intensity with the oscillation displacement amplitude and oscillation frequency. For each of the parameter cases studied the time-averaged average heat transfer coefficients at the heated surface and the time-averaged local heat transfer coefficients for the four points shown in red in Figure 6.3 are listed in Table 6.2. The remainder of this section compares and contrasts the velocity and temperature fields that are produced by the oscillations for different parameters and for a steady state pure natural convection reference case. In addition, the association between the velocity and temperature field characteristics and the resulting heat transfer coefficients that measure the cooling effect are discussed. Further transient and time-averaged results are provided in Appendix G. Note that all dimensionless variables used are defined in Appendix B, and that all time-averaged results are for the “transient steady state” after the completion of a number of plate oscillations.

6.3.2.1 Steady State Velocity and Temperature Fields and Heat Transfer Coefficient. Because the performance of the oscillation enhanced natural convection is measured relative to natural convection flow, the typical temperature and flow field characteristics for the system at steady state cooled by pure natural convection with and without the stationary oscillation source plate in the channel must be analyzed as well. For steady state pure natural convection in a plain vertical channel, without any plate, the velocity in the channel takes on a parabolic-like distribution offset towards the heated surface at \( y = 0 \). The highest velocities occur near the channel outlet for a plain channel. (See Figure 6.4a.) The temperature along the heated surface increases moving downstream while the local heat transfer coefficients decrease. (See Figure 6.5a.) Adding a stationary plate to the channel constricts the flow and causes a significant portion of channel flow to
be diverted over the top surface of the fixed oscillation source instead of flowing between the heat source and oscillation source (See Figure 6.4b). As a result of the lower flow velocities and the resulting thicker momentum boundary layer, the temperatures at the heated surface just under and downstream of the plate are elevated and the heat transfer coefficients are lower than those without the plate. (See Figure 6.5b). Therefore, the effect of the oscillations must be sufficient to overcome this diminished cooling caused by the presence of the plate.

**Figure 6.4** Steady state velocity distributions – plain channel: (a) no plate, (b) plate fixed in channel.
6.3.2.2 Results with Oscillating Plate.

Velocity and Temperature Field Characteristics

While some basic flow and temperature field features that develop due to the oscillations are common among all parameter values, for a fixed geometry and heat parameter set (i.e. Grashof number), the specific velocity field attributes, as expected, are highly dependent on the values of oscillation parameters with the larger parameter values typically producing more complex flow patterns and better cooling. Because a few words or images cannot adequately describe all the specific flow features and their corresponding temperature fields, animations of the velocity field and temperature field were made.

Figure 6.5 Steady state temperature distributions – plain channel: (a) no plate, (b) plate fixed in channel.
These are included on the accompanying CD in the folder labeled "oscillating plate in channel."

Some general observations about the common characteristics of the oscillation-natural convection velocity field are described below. The major impact of the oscillations on the velocity field was found to remain local to the oscillation source. However, much of the flow in the channel downstream of the oscillation source is influenced by the effects of the oscillations. Attention is first given to the region between the oscillating plate and the fixed heated surface. The velocities in this region are not necessarily symmetric with respect to the mid-plate length and the location of the low velocity region under the plate may change with time, indicating that both the natural convection and oscillation forces are at work. When the plate moves upward, natural convection and oscillation effects reinforce one another on the upstream side of the oscillation source and oppose one other on the downstream side. Thus, velocities on the upstream side of this region tend to be higher than those downstream, with the low velocity region shifting downstream. When the plate moves downward, the velocities on the downstream end tend to be higher than those upstream, and the low velocity region shifts upstream. Inertia effects can also be significant for these types of flows. As the flow in this region periodically changes, the velocities and velocity gradients near the heated surface periodically increase and decrease with the regions near the ends of the plate experiencing the greatest change. If the time average of these gradients is higher than that for pure natural convection, indicating a thinner time averaged momentum boundary layer, an improvement in the cooling has been achieved.
The flow into and out of the region between the moving plate and fixed channel wall is much more complex than in previous investigations. It is dependent on the flow outside of the region between the plates which is the resultant of the interaction of many flow effects including the fluid inertia, a phenomena which is significant due to the nature of these oscillations, as well as the natural convection induced flow and the flow over and around the ends of the oscillating plate and the shear and pressure forces. Natural convection forces always act in the positive $x$ direction and are responsible for the main channel flow, some of which is diverted over the top of the plate due to its finite length. The amount of fluid diverted over the top of the oscillation source increases as the spacing between the plate and the heat source decreases. At times some of this diverted flow may separate, reattaching at the heated surface and therefore altering the flow patterns near the heated surface. Much of this flow eventually turns around the ends of the plate due to the finite plate length. Under most conditions, counterclockwise flow around the upstream end and clockwise flow around the downstream end accompany the upward plate motion with circulations reversing direction with the downward motion of the plate. Much of the flow over the top of the oscillation source feeds the flow into the region between the heat source and oscillation source during upward plate motion. During the downward plate motion, much of the fluid expelled from under the plate is carried over the top of the oscillation source. Therefore, flow streams turning around the oscillation source play a significant role in establishing the cooling ability of the method.

The relative influence of the various flow streams establishes the flow pattern and, thus, the cooling effect. Because the flow is unimpeded by the plate, natural convection has a greater influence on the flow near the upstream end of the region just
out from under the moving plate than at the downstream end and tends to counteract the
effects of the oscillations regardless of the direction of the plate motion. At the upstream
side, the natural convection flow opposes both the flow around the top of the plate when
fluid is drawn into the channel as well as the flow of the air exiting from the region
between the heat source and oscillation source. Behind or just downstream of the plate, the
natural convection induced flow is much less significant. In this region there is
greater interaction between the flow expelled or drawn into the channel-like space, the
flow over the top of the oscillation source and the flow turning around the ends of the
plate. Even for low frequencies, as the plate moves in the positive y direction, noticeable
clockwise circulation cells tend to develop at the downstream end of the oscillation
source as a result of such interactions. (In most cases when the plate changes direction,
the circulation tends to move downstream and dissipate, carried by the exiting fluid as
well as the natural convection induced flows.) Because of this type of flow interaction,
there is greater potential for change in the velocity field and, therefore, for change in the
heat transfer coefficients at the heated surface near the downstream end of the oscillation
source. For higher oscillation parameters the greater strength of the oscillations induced
flow is higher. This promotes a more rigorous interaction of the flow streams and leads to
more complex flow patterns indicated by the presence of a number of circulation regions
both within and outside of the region between the oscillation source and the heat source.
Under these conditions the greatest heat transfer enhancement was observed. It is also
important to note that because of the plate geometry, fluids drawn into the channel have
elevated temperatures relative to the ambient, although these temperatures are likely to be
lower than the temperatures right under the moving plate. Also, the natural convection
induced flow can carry the heated fluid expelled from the area between the plate and heat source into the main channel flow at both ends. The higher the velocity of the outgoing fluid, the farther away the heat can travel before the plate changes direction. These additional factors can help to remove heat from the heated surface.

**Overall Results for the Heat Transfer Coefficient**

The velocity field characteristics described above are coupled to the temperature field attributes and thus the heat transfer coefficients that result. As described earlier, when the time averaged velocity gradients are higher than those occurring for pure natural convection, the time averaged momentum boundary layer is thinner. This results in a thinner time averaged thermal boundary layer and higher time averaged heat transfer coefficients indicating cooling improvement. A brief summary of the impact of the oscillations on the local and surface averaged heat transfer coefficients is given along with some general trends. The results of this investigation corroborate the trends in the results of the finite volume parametric investigations of Chapter 4 and Chapter 5 as it was found that the enhancement effect increases with the oscillation frequency and amplitude. As noted previously, the cooling effect is more sensitive to the oscillation displacement amplitude. The higher displacement amplitudes cause a larger portion of the fluid between the heat source and the oscillation source to be expelled into the surrounding fluid and a greater amount of cooler fluid to be drawn into the channel as previously explained. Generally, the greater the amount of fluid exchange, the greater the overall fluid and thermal mixing in the plate/heat source vicinity, the greater the cooling effect.

Because the main impact of the oscillations is felt in the immediate vicinity of the oscillation source, the changes in the local heat transfer coefficients at the four locations
along the heated surface shown in Figure 6.2 that result from the oscillations are a good
indication of the potential cooling effect. For reference these points are called Point 1
through Point 4 going from the upstream to the downstream positions (left to right in the
figure). Examples of the way in which the local heat transfer coefficients vary with time
are shown in Figure 6.13. In another significant finding that supports the importance of
the inertia effects and concept of the flow harmonics discussed in Chapter 5, the
oscillations in the heat transfer coefficients are not necessarily in phase with the plate
oscillations or in phase with the oscillations at other locations along the heated surface. In
general, the results showed that little beneficial effect of the oscillations was felt
upstream of the oscillation source (Point 1) where the natural convection induced flow
acts in the downstream direction and therefore tends to suppress any oscillation-produced
alteration in the local velocity from advancing upstream. In fact, for most parameter
cases, a low velocity region is observed in this area for a large portion of a plate
oscillation since the natural convection and some component of the oscillation-induced
flow continually counteract each other. In contrast, for all of the parameters investigated,
the two locations beneath the oscillation source (Points 2 and 3) did experience
improvements to varying extents relative to steady state conditions with the fixed plate.
However, significant improvement relative to pure natural convection with no plate was
reserved only for the higher oscillation parameters. The velocities induced by the
oscillations with lower parameters are not large enough to overcome the effect of the
reduced velocities near the heated surface resulting from the presence of the plate. The
greatest potential for cooling along the heated surface occurs just downstream of the
plate. In this area, the main channel natural convection induced flow behind the
oscillating plate is low and is not strong enough to damp the effects of the oscillations, thus allowing for the higher velocity fluid exiting and entering the region beneath the plates to affect the velocity near the heated surface. The interaction of the various induced flows in this area of the channel promotes strong fluid mixing (i.e. reduction of stagnant regions, coalescence of fluid streams from various channel locations etc., leading to thinner momentum). The resulting commingling of the heated and cooler fluid improves the temperature gradients and hence the heat flow and heat transfer coefficients. However, while improvements over the steady state conditions with the fixed plate occur for all parameter cases at Point 4, once again, the oscillation parameters must be large enough for cooling improvement to occur over standard natural convection. For the parameters investigated the highest frequency, highest displacement case with \( d = 0.10 \), \( V = 0.4\pi \), \( \omega = 4\pi \), Point 4 experienced a 40% increase in the time-averaged local heat transfer coefficient relative to pure natural convection. For all of the parameters involved, the time-averaged local heat transfer coefficients at Point 4, just downstream of the oscillating plate, are presented in bar graphs in Figure 6.12. The effect of the oscillations on the overall channel length is not as dramatic, as the cooling effect is quite localized. (See Table 6.2 and Appendix G.)
Table 6.2a Summary of Average Dimensionless Heat Transfer Coefficients – Plain Channel Geometry C=0.15

<table>
<thead>
<tr>
<th>Case</th>
<th>Max Time Avg Dimensionless Temperature</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimensionless Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.417051</td>
<td>-</td>
<td>-8.88%</td>
<td>0.042315</td>
<td>-</td>
<td>8.381</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.457684</td>
<td>9.74%</td>
<td>-</td>
<td>0.039050</td>
<td>-1.72%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.05 V=0.10</td>
<td>0.452023</td>
<td>8.38%</td>
<td>-1.24%</td>
<td>0.039554</td>
<td>-6.53%</td>
<td>1.291%</td>
</tr>
<tr>
<td>d=0.05 V=0.10π</td>
<td>0.451578</td>
<td>8.28%</td>
<td>-1.33%</td>
<td>0.039359</td>
<td>-6.99%</td>
<td>0.791%</td>
</tr>
<tr>
<td>d=0.05 V=0.20π</td>
<td>0.445751</td>
<td>6.88%</td>
<td>-2.61%</td>
<td>0.039060</td>
<td>-7.69%</td>
<td>0.025%</td>
</tr>
<tr>
<td>d=0.10 V=0.20</td>
<td>0.448294</td>
<td>7.49%</td>
<td>-2.05%</td>
<td>0.040914</td>
<td>-3.31%</td>
<td>4.77%</td>
</tr>
<tr>
<td>d=0.10 V=0.20π</td>
<td>0.432781</td>
<td>3.77%</td>
<td>-5.44%</td>
<td>0.042363</td>
<td>0.112%</td>
<td>8.48%</td>
</tr>
<tr>
<td>d=0.10 V=0.40π</td>
<td>0.464417</td>
<td>11.35%</td>
<td>1.47%</td>
<td>0.041751</td>
<td>-1.33%</td>
<td>6.92%</td>
</tr>
</tbody>
</table>

Table 6.2b Summary of Average Local Dimensionless Heat Transfer Coefficients – Plain Channel Geometry C=0.15

<table>
<thead>
<tr>
<th>Case</th>
<th>Time Avg Dimm-less Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimm-less Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.037503</td>
<td>-14.68%</td>
<td>-</td>
<td>0.035535</td>
<td>27.04%</td>
<td>24.06%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.032702</td>
<td>-12.80%</td>
<td>-</td>
<td>0.027971</td>
<td>21.29%</td>
<td>0.032563</td>
</tr>
<tr>
<td>d=0.05 V=0.10</td>
<td>0.031686</td>
<td>-15.51%</td>
<td>-1.01%</td>
<td>0.028170</td>
<td>7.13%</td>
<td>0.026652</td>
</tr>
<tr>
<td>d=0.05 V=0.10π</td>
<td>0.032536</td>
<td>-13.24%</td>
<td>-0.505%</td>
<td>0.028615</td>
<td>2.31%</td>
<td>0.029777</td>
</tr>
<tr>
<td>d=0.05 V=0.20π</td>
<td>0.031616</td>
<td>-15.67%</td>
<td>-3.312%</td>
<td>0.028429</td>
<td>1.64%</td>
<td>0.029919</td>
</tr>
<tr>
<td>d=0.10 V=0.20</td>
<td>0.033429</td>
<td>-10.86%</td>
<td>2.22%</td>
<td>0.032525</td>
<td>16.28%</td>
<td>0.031894</td>
</tr>
<tr>
<td>d=0.10 V=0.20π</td>
<td>0.032450</td>
<td>-13.47%</td>
<td>-0.770%</td>
<td>0.031086</td>
<td>-12.52%</td>
<td>0.045995</td>
</tr>
<tr>
<td>d=0.10 V=0.40π</td>
<td>0.030106</td>
<td>-19.72%</td>
<td>-7.93%</td>
<td>0.032979</td>
<td>17.91%</td>
<td>0.045995</td>
</tr>
</tbody>
</table>
In order to better understand the flow patterns and their relationship to the cooling effect, three sample velocity fields and their corresponding temperature fields are described. Because the alteration of the velocity field caused by the oscillations is lessened for lower displacement values, three cases with the higher displacement values but increasing frequency values are discussed here to best illustrate the flow patterns. The first parameter conditions are for \( d = 0.10, \ V = 0.20, \ \omega = 2 \), a relatively low frequency. A sample of the velocity and temperature fields is shown in Figure 6.6 and Figure 6.7. A brief description of the flow follows. As the plate moves upward from its lowest position, the flow over the upstream half of the top of the moving plate has lower velocities than downstream because the natural convection and the flow turning counter clockwise around the upstream end of the plate oppose one another. While a low velocity region occurs at the upstream plate end, no evidence of a circulation region is observed, and flow enters the region between the heat source and oscillation source as the space between these components increases. As some of the fluid that turns around the downstream end of the plate enters the channel, a clockwise rotation cell develops at the downstream edge of the plate (Figure 6.6a). A portion of the higher velocity fluid moving over the top of the downstream end of the plate follows a path such that it impinges on the board further downstream as it must move around this circulation region. Under the plate, a region of low velocity moves from the middle length of the plate downstream to the outlet. As the plate changes direction and starts to move back towards the heated surface, higher velocities occur over the upstream side of the moving plate while slower regions occur at the downstream side where the natural convection effects oppose the counter clockwise flow around that side of the plate. The low velocity region under the
plate begins at the upstream end and moves to the center of the plate. As fluid begins to exit the region between the heat source and the oscillation source with the downward stroke of the plate, the fluid motion carries the downstream circulation region away from the end of the plate and towards the lower heated wall. The flow exiting the region between the oscillation source and heat source is then deflected jet-like between the oscillation source and this low velocity circulation region. (Figure 6.6b). The circulation region then dissipates moving downstream. Once the low velocity region moves far enough away, the high velocity fluid exiting plate region sweeps over the heated surface region just downstream of the oscillation source (Figure 6.6c and Figure 6.6d). Upstream of the plate, a small low velocity circulating region briefly develops due to the opposition of the main channel flow and the flow exiting from beneath the oscillation source. The plate then changes direction and moves upward once more.

The velocities described above directly affect the temperature field and thus the heat transfer coefficients at the heated surface. As illustrated in Figure 6.7, upstream of the oscillation source at the Point 1 location, the temperatures at the heated surface are higher than those without the plate. Due to the low velocities that occur in this region, the momentum boundary layer and thus the thermal boundary are thicker causing the heat transfer coefficients to be lower. At Points 2 and 3, the heat transfer coefficients do change with time as a result of the plate oscillations as evidenced by the increasing and decreasing temperature gradients near the surface with time and the oscillations in the local heat transfer coefficients in Figure 6.13a. (Note these oscillations may be out of phase with one another and the oscillations of the plate.) While improvement over steady state conditions with the plate fixed in the channel does occur at these points, the time
averaged effects do not show any significant improvement over the conditions with no plate in the channel. Among the four points, the Point 4 location experiences the greatest alteration in the temperature and heat transfer coefficient due to the higher velocities and greater flow mixing that occurs at this location. However, though the local coefficient increases relative to the steady state with the plate in the channel, no significant improvement in the average heat transfer coefficient over standard natural convection occurs at this position either. The circulation regions and lower velocities that develop downstream of the plate for certain portions of the cycle prohibit the full extent of the higher velocity flows from reaching the heated surface at this location, and velocities that do pass near this location are not high enough to produce improvements in cooling. The oscillations in the heat transfer coefficients for the four points are very regular, owing to the lower velocities and lower frequencies, and, thus, the lower levels of fluid mixing involved (Figure 6.13a). The oscillation parameters in this case produce fluid motion that is not significant enough to cause heat transfer enhancement relative to pure natural convection.

A greater cooling potential can be reached for higher frequency flows. In the next case, the velocity field is explored for parameter values of $d = 0.10$, $V = 0.20\pi$, $\omega = 2\pi$. As in the previous case, a brief description of the flow field is as follows. As the oscillation source moves upward from its lowest position and flow fluid turns around the top of the oscillation source to enter the space between the plate and the heat source, a clockwise circulation region develops downstream of the oscillation source. This region appears to be larger than that occurring at the smaller frequency and it is positioned closer to the area under the moving plate. As the plate continues to move up, a
counter-clockwise circulation region develops just under the upstream end of the plate as a result of the opposing natural convection flow and the stronger flow turning around the plate end than that at the lower oscillation frequency. The development of this was suppressed in the lower frequency case due to the relative strength of the natural convection effects. Though the stronger oscillation effects facilitate better flow stream interactions, they also result in reduced direct natural convection based through flow between the moving and fixed surfaces. (Less noticeable movement of a distinct low velocity region under the plate as compared to the lower frequency case studied is observed.) As the plate continues to move upward, the circulation region at the downstream end of the plate is drawn slightly under the plate (Figure 6.8a). As the plate begins to move back towards the heated surface and fluid is expelled from the oscillation source region, this circulation region moves down towards the lower heated wall. The flow exits in a jet-like manner between the plate and the recirculation region until this low velocity region moves farther away from the plate and dissipates (Figure 6.8b). Then, a higher velocity sweeps over the heated surface (Figure 6.8c). At the upstream side of the plate, at a time just briefly after the change in direction, a clockwise circulation region is formed near the heated surface from the opposition of the natural convection induced flow, the exiting flow, and the flow moving around the oscillation source. At this time, some fluid is still entering the channel at this upstream end due to inertia or natural convection effects. Note these directions are opposite those velocities induced by the upward motion of the plate and result in through flow along the plate length. As the plate continues to move downwards, a clockwise circulation region develops upstream while a counter-clockwise circulation cell develops downstream.
(Figure 6.8d). Eventually, the flow from over the top of the oscillation source is strong enough to cause the downstream circulating region to move further downstream and away from the plate. The upstream circulation region dissipates when the flow changes direction and the plate begins to move upward.

The velocity field in close proximity to the oscillation plate has significantly more complex flow features with this higher frequency than that of the lower frequency case. Thus, the changes this velocity field instigates in the temperature field are more pronounced as are the changes in the resulting heat transfer coefficients. At the upstream location of Point 1, slight decreases in the heat transfer coefficient as compared to the lower frequency case are found, corresponding to the lower velocities and thicker velocity boundary layers which tend to occur at this location for most of a plate oscillation cycle (Figure 6.9). The velocity results show that in the area just under the plate, greater fluid mixing and higher velocity gradients occur as a result of the oscillations. (This is due in part to the through flow despite opposition to plate motion noted in the previous paragraph.) Therefore, Points 2 and 3 experience a greater cooling effect than that in the previous lower frequency case. The improved thermal conditions are evidenced in the temperature contours just inside the plate ends and indicate higher temperature gradients. The oscillations in the heat transfer coefficients at Points 2 and 3 are of a more complex nature than those for the lower frequency due to the more complex flows under the moving plate as in Figure 6.13b. However, there is still not significant improvement over pure natural convection at these positions. (Point 3 has just a 0.8% improvement over the standard natural convection case.) The greatest improvement in the local heat transfer coefficient remains at Point 4, just downstream of the oscillation
source where a 34% increase relative to standard natural convection was calculated. The velocities of the fluid expelled from beneath the moving plate are higher and thus the fluid velocities near this point are higher than for the previous parameter set. This helps to move the low velocity circulating regions away from plate region more rapidly than for the lower frequency case and also carries the heated fluid farther away from the area so it can be better removed by the main channel flow. In addition, velocity gradients and thus temperature gradients near the heated surface at this position are higher as well. Hence, the flow near Point 4 promotes thinner time-averaged boundary layers and higher heat transfer coefficients. Because of the larger velocities involved, the fluctuations in velocity magnitudes are greater than in the previous two cases, causing larger amplitudes in the oscillations of the heat transfer coefficient at this location (Figure 6.13b). These results illustrate that the more vigorous interaction of the many flow streams involved leads to a more significant cooling of the heated surface.

An even higher oscillation frequency produced an even greater flow complexity and an even greater capacity for cooling enhancement. The set of oscillation parameters for this third case is $d = 0.10$, $V = 0.4\pi$, $\omega = 4\pi$. As the plate moves upward from its lowest clearance positions, two sets of counter-rotating cells develop just under the oscillating plate. The fluid entering the “channel-like” region appears like two symmetrical jets first shooting between the rotational cells and then turning upward (Figure 6.10a). The responsibility of this flow pattern lies with the greater fluid velocities which lead to greater fluid inertia and greater opposition between the natural convection and oscillation induced flows. As the plate slows and reaches the maximum spacing, the outer circulation region moves inward while the inner circulation regions become more
intense and move upward, closer to the moving plate. Despite the circulation regions, some through flow still occurs even though the upward plate motion opposes such flow at the outlet. The flow then takes a serpentine path around these circulation regions and exits in a jet-like fashion just under the downstream end of the moving plate (Figure 6.10b). As the plate changes direction, the clockwise rotating region nearest to the downstream end of the oscillation plate is forced from under the plate and moves downstream. As the plate continues downwards, a small circulation region develops near the heated surface upstream of the moving plate. Upstream clockwise and downstream counterclockwise circulations are formed around the plate ends (Figures 6.10c and 6.10d). The higher frequency produces more intricate flow patterns that can assist in increasing the velocity gradients near the heated surfaces and in moving the heat from under the vibrating plate.

The flow conditions described above correspond directly to the changes in the heat transfer coefficient and heat source temperature characteristics brought about by the oscillations. Due to the greater opposition of the natural convection and oscillation effects near the upstream side of the plate and the lower velocities that result, the heat transfer coefficients at this location (Point 1) are even lower than those with the plate fixed in the channel. For this set of parameters, at the heated surface just under and downstream of the oscillating plate, higher velocity gradients, thinner boundary layers and the subsequent thinner thermal boundary layers, lower temperatures, and higher heat transfer coefficients are periodically generated (Figure 6.11). The more complex oscillations in the heat transfer coefficient at Point 2 and 3 (shown in Figure 6.13c) correspond to the greater inertia effects and more complex flow patterns. The time
averaged heat transfer coefficient at Point 2 is 7% lower than that for pure natural convection, but over 17% higher than that for steady state condition with the fixed plate. The enhancement over a pure natural convection at Point 3 is 7% (33% greater than with the plate). The enhancement in the heat transfer coefficient is most significant at Point 4 (41% and 83% for pure natural convection and that with a plate, respectively), just downstream of the oscillating plate in the region of high fluid mixing and high exit and entrance velocities.
Figure 6.6 Velocity distributions near moving plate for $d=0.10$ $V=0.20$ $\omega = 2$:
(a) $t_F = 9.70$, (b) $t_F = 10.2$ (cont).
Figure 6.6 Velocity distributions near moving plate for $d=0.10$ $V=0.20$ $\alpha = 2$:
(c) $t_F = 10.8$, (d) $t_F = 11.4$. 
**Figure 6.7** Temperature distributions near moving plate for \(d=0.10\) \(V=0.20\) \(\omega = 2\):

(a) \(\bar{t}_F = 9.70\), (b) \(\bar{t}_F = 10.2\) (cont).
Figure 6.7 Temperature distributions near moving plate for $d=0.10, V=0.20$, $\omega = 2$: (c) $\tilde{t}_F=10.8$, (d) $\tilde{t}_F=11.4$. 
Figure 6.8 Velocity distributions near moving plate for $d=0.10$ $V=0.20\pi$ $\omega = 2\pi$:
(a) $\tilde{t}_F = 5.225$, (b) $\tilde{t}_F = 5.300$(cont).
Figure 6.8  Velocity distributions near moving plate for \(d=0.10\) \(V=0.20\pi\) \(\omega = 2\pi\):
(e) \(t_F =5.55\), (d) \(t_F =5.725\).
Figure 6.9 Temperature distributions near moving plate for \( d=0.10 \), \( V=0.20\pi \) \( \omega = 2\pi \):
(a) \( \tau_F = 10.225 \), (b) \( \tau_F = 10.30 \) (cont).
**Figure 6.9** Temperature distributions near moving plate for $d=0.10$, $V=0.20\pi$, $\omega = 2\pi$:

- (c) $t_F = 5.50$, (d) $t_F = 5.725$. 
Figure 6.10  Velocity distributions near moving plate for $d=0.10$ $V=0.40\pi$ $\omega = 4\pi$:

(a) $t_F=3.075$, (b) $t_F=3.15$(cont).
Figure 6.10  Velocity distributions near moving plate for $d=0.10$ $V=0.40\pi$ $\omega = 4\pi$:

(c) $t_F = 3.325$, (d) $t_F = 3.45$. 
**Figure 6.11** Temperature distributions near moving plate for $d=0.10$ $V=0.40\pi$ $\omega = 4\pi$:

(a) $t_F = 3.075$, (b) $t_F = 3.15$ (cont).
Figure 6.11 Temperature distributions near moving plate for $d=0.10$ $V=0.40\pi$ $\omega = 4\pi$:

(c) $t_F = 3.325$, (d) $t_F = 3.45$. 
Figure 6.12 Time averaged dimensionless local heat transfer coefficient at Point 4 along the heated surface for plain channel geometry: (a) $d=0.05$, (b) $d=0.10$. 
Figure 6.13 Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) $d=0.10 \ V=0.20 \ \omega=2$, (b) $d=0.10 \ V=0.20\pi \ \omega=2\pi$, (c) $d=0.10 \ V=0.40\pi \ \omega=4\pi$
6.3.3 Conclusions

Based on the results of this study, some important conclusions can be drawn as to the effectiveness of the use of discrete oscillation sources to locally enhance natural convection cooling in a vertically oriented channel. The use of transverse oscillations of a discrete oscillation source has been shown to supplement pure natural convection under the proper oscillation, heat, and geometric conditions. Though the cooling effect increases with the oscillation frequency, increases in the oscillation displacement amplitude were found to be more effective at increasing heat transfer enhancement for a fixed geometry and applied heat flux. A maximum increase in the local heat transfer coefficient of 40% compared with standard pure natural convection was achieved for the parameters investigated.

The extent of the cooling effect was found to be dependent on the interaction and therefore relative influence of the different flow streams involved including the natural convection flow, the flow over the top of and turning around the sides of the vibrating plate, and the flow entering and exiting the region beneath the plate as well as the shear, inertia, and pressure forces. If the different flow effects are of the same order, the flow that develops is more complex and the potential for improved fluid mixing is greater; thus, there is a greater cooling potential. Because of the varying “strengths” of the different flow streams at different positions, the cooling effect differs significantly with the location along the heated surface. The main channel natural convection induced flow exerts a stronger influence at the upstream end of the oscillation source, acting to suppress the effects of the oscillations. The effects of the main channel flow, just behind and slightly downstream of the oscillation source, are relatively minimal and permit
better mixing of the different flow streams. Higher velocity flows and the accompanying development of multiple circulation regions that move with time, particularly under the oscillating plate, were found to be indicators of improved thermal conditions of the system. From the results, it can be concluded that for this cooling method to produce the velocity fields and flow mixing adequate for improved cooling, the oscillation induced flows and the natural convection should complement one another so that one cooling force does not dominate. The oscillation effects should be strong enough to affect the flow upstream and downstream of the oscillation source, but should still allow for the natural convection to transport some fluid through the region between the heat source and oscillating surface and to carry some fluid expelled from under the moving plate into the main natural convection induced channel flow.

The velocity field characteristics were shown to be directly related to the temperature field and to the heat transfer coefficients. The oscillations in the velocity field produce out of phase oscillations in the temperatures and heat transfer coefficients at the heated surface. The resulting higher velocity gradients near the heat source surfaces cause the thinner momentum boundary layers which produce thinner thermal boundary layers, higher temperature gradients, and consequently higher heat transfer coefficients and lower heat source temperatures. For the proper system geometries and oscillation and heat source parameters, the time averages of the oscillations in the heat transfer coefficients can reach values greater than those for standard natural convection. Under such conditions, the discrete transversely oscillating plate can be used as means of providing effective, focused and localized cooling enhancement to those portions of the heated surface in a vertical channel in the immediate vicinity of the oscillation source.
6.4 Discrete Transverse Oscillation Sources to Supplement Natural Convection in Vertical Channel Containing a Rectangular Heat Source

The short thin moving plate geometry analyzed in the previous section allows for the more realistic modeling of the oscillation sources that can be strategically placed for local heat transfer enhancement. More realistic modeling of the heat source can next be achieved through the use of a two-dimensional rectangular heat-conducting solid with a constant volumetric heat generation rate. This section describes the studies undertaken for this rectangular heat source geometry and discusses the important study results.

6.4.1 Problem Statement

The effectiveness of the use of the transverse oscillations near a two dimensional heat source for enhancing natural convection in a vertically oriented channel was investigated through the model shown in Figure 6.14 in which a rectangular heat source is located at the mid-channel length and a transversely oscillating plate is positioned symmetrically over the top of the heat source. The oscillation source, board, and heat source are modeled as solid entities. The conduction in these solids is accounted for through the assignment of the proper thermal conductivity parameters. Because conduction through the solid board is taken into account, a “second” channel heated only by conduction through the board is also included to better model effects of the heat flow through the bottom surface of the heat source. A constant and uniform volumetric heat generation rate is applied to the rectangular solid. The specific parameter values used are listed in Table 6.3. The general assumptions from Section 2.2 are applied to this system. In addition, the standard inlet, outlet, and surface boundary conditions used in the finite element investigations including conduction in the solids in Section 3.3 or as discussed in Section
Parametric studies for a number of oscillation source displacement amplitudes and frequencies as well as two clearance values, \( C \), were then performed for a fixed applied heat generation rate (Grashof number) so that the effects of these parameters on the potential cooling effect could be investigated. The parameter cases investigated are listed in Table 6.4. Animations of the velocity and temperature fields are provided in the CD in the folder “rectangular heat source.”

A 275 x 167, 9 node quadratic quadrilateral element graded mesh for this geometry was generated for this problem. The velocity and temperature fields were solved for by using FIDAP© until a transient steady state was attained. The local heat transfer coefficients at the four points marked in red in Figure 6.14 and the surface averaged heat transfer coefficients were calculated at each time step and then averaged over time to provide the information from which conclusions can be drawn as to the effectiveness of the use of the oscillations with a two-dimensional heat source and a two-dimensional oscillation source.

*Figure 6.14* Model geometry for rectangular heat source investigation.
Table 6.3 Data Used in Investigation of Use of Discrete Oscillation Sources To Supplement Natural Convection – Rectangular Heat Source Geometry

<table>
<thead>
<tr>
<th>Material Property*</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>k (Thermal conductivity of air)</td>
<td>0.027 W/mK</td>
</tr>
<tr>
<td>ν (kinematic viscosity of air)</td>
<td>1.717e-5 m/s²</td>
</tr>
<tr>
<td>ρ (density of air)</td>
<td>1.12492 kg/m³</td>
</tr>
<tr>
<td>cₚ (specific heat of air)</td>
<td>1005.93 J/kg K</td>
</tr>
<tr>
<td>β (volumetric expansion)</td>
<td>1/315.5K</td>
</tr>
<tr>
<td>k₁ (thermal conductivity of heating elements)</td>
<td>177 W/mK</td>
</tr>
<tr>
<td>c₁ (specific heat of heating elements)</td>
<td>875 J/kgK</td>
</tr>
<tr>
<td>ρ₁ (density of heating elements)</td>
<td>2770 m³/kg</td>
</tr>
<tr>
<td>k₂ (thermal conductivity of board)</td>
<td>0.26 W/mK</td>
</tr>
<tr>
<td>c₂ (specific heat of board)</td>
<td>1173 J/kgK</td>
</tr>
<tr>
<td>ρ₂ (density of board)</td>
<td>1207 kg/m³</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>g (gravitational acceleration)</td>
<td>9.81 m/s²</td>
</tr>
<tr>
<td>T₀ (ambient temperature)</td>
<td>25°C</td>
</tr>
<tr>
<td>Q''ₐ (applied volumetric heat generation rate)</td>
<td>702.99e3 W/m³</td>
</tr>
<tr>
<td>Grashof Number based on b</td>
<td>1157.8</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimensional Parameter**</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>b (block height and length)=Lₜₑₙ (length basis)</td>
<td>0.25 in=0.0635m</td>
</tr>
<tr>
<td>BH=BL (block height=block length)</td>
<td>b</td>
</tr>
<tr>
<td>CH (channel height)</td>
<td>2b</td>
</tr>
<tr>
<td>BT (board thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>TL (channel length)</td>
<td>14b</td>
</tr>
<tr>
<td>PT (plate thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>PL (plate length-plate over heating elements)</td>
<td>0.75b</td>
</tr>
</tbody>
</table>

*Properties at Tₐᵥₑₐ=42.5°C=315.5 K
**Dimensions defined in Figure 6.15.
6.4.2 Analysis of Results — Rectangular Heat Source Geometry

The results of this study show that a small transversely oscillating plate placed near the top surface of a rectangular heat generating solid does alter the velocity field near the heat source and therefore changes the flow of heat from the heat generating element to the surrounding fluid and solids. Again, the oscillations were found to produce a localized cooling effect with the top surface of the heat source receiving the most benefit. While overall the general velocity field characteristics are similar to those of the plate geometry results, due to the different geometry investigated here, certain unique flow features and temperature characteristics develop. For a fixed channel geometry and heat rate, the level of the cooling enhancement attained was found to be dependent on the oscillation amplitude and frequency as well as the clearance spacing. Judgment on the cooling potential of this method is made by comparing the time-averaged average heat transfer coefficients at the heated surfaces and the time-averaged local heat transfer coefficients for the four points shown in red in Figure 6.14 to their values at steady state for standard natural convection and for pure natural convection with the plate fixed over...
the heat source. (The points are named Point 1 through Point 4 going upstream to
downstream, left to right in the figure.) The time averaged results are listed in Table 6.4.
Further transient and time-averaged results are provided in Appendix G. Note that all
dimensionless variables used are defined in Appendix B and that all time-averaged results
are for the "transient steady state".

6.4.2.1 Steady State Natural Convection Induced Flow over a Rectangular Heat
Source in a Vertical Channel. Before exploring the effects of the oscillations on
the characteristics of the flow and heat transfer from a rectangular heat source, the unique
attributes of flow over the rectangular heat source in a vertically oriented channel are
summarized because they are important in understanding how the oscillations can alter
the typical flow patterns to cause enhancement. As expected, the maximum channel flow
occurs over the top of the heat source, due, in part, to the constriction of the flow area.
This flow continues to the channel outlet. (See Figure 6.16.)

As the fluid flows over the heat source, a low velocity area of triangular shape
develops just upstream of the left (upstream) side of the heat source. This is caused by
the sudden redirection of the flow around the block. A small circulation region is formed
near the lower left corner of the block's upstream surface.

In the current study, no flow separation was observed at the upper corner of the
left (upstream) side of the heat source. Similar conclusions were reached by Desrayaud
and Fichera [80] who note that the low natural convection induced velocities may be
responsible.

The flow following the upper surface of the heat source, however, does separate
at the right (downstream) upper corner. The flow reattaches further downstream at the
board surface. A large and easily observable clockwise circulating region develops in the low velocity area behind the heat source as this region is cut off from the rest of the channel flow by this separated flow. (See Figure 6.17.)

Another important flow feature occurs near the upper channel wall downstream of the heat source block. As the flow expands downstream of the block, the higher velocity area remains more or less intact but is redirected downward toward the board surface. The presence of the lower wall changes the direction of the flow resulting in the development of higher near wall velocities. The combination of this flow and the interaction of the pressure, inertia, and shear forces near the upper wall cause the formation of a large counterclockwise circulation region that extends to the channel outlet. This circulation region helps to direct the main channel flow towards the lower wall and restricts the flow over a significant portion of the outlet. This slow recirculating region can also bring cooler ambient air into the channel. Hence, the presence of the two-dimensional heat source alone results in more complex flow patterns than a heated channel wall.

For this geometry, the highest heat transfer coefficients occur at the top heat source surface. The temperatures immediately downstream of the heat source are elevated by the effects of the heat source. (See Figure 6.18.)

When a plate is placed over the top of the heat source, the velocity and coupled temperature distribution are altered. The general path the flow takes and the regions of low velocity and circulations to the sides of the heat source and near the upper wall remain. However, the presence of the plate causes a flow constriction. (See Figure 6.17.) There is less natural convection induced flow moving over the top heat source surface,
and so, the heat source temperature rises. For the parameters studied, this increase was 6%. The average heat transfer coefficients on the three exposed heat source surfaces decrease by as much as 12%. However, at the upstream end of the heat source top surface (Point 2), the velocities near the top heat source surface are slightly larger than then they were without the plate because the flow that turns around the left (upstream) corner of the heat source and enters the space between the heat source and oscillation source is confined to move near the heat source by the presence of the oscillation source. This results in an increase in the local heat transfer coefficient. (For the current investigation a 10% increase occurred.) Also, due to the presence of the plate, the flow that moves over the top of the plate is directed in a more downward fashion towards the right side heat source surface as it moves into the region of expanded flow area. This results in a slight reduction of the size of the higher temperature region behind the rectangular heat source. Overall, however, the presence of the plate causes the thermal conditions of the heat source to worsen. The oscillations must act to overcome these conditions to surpass the standard natural convection cooling.
Figure 6.16 Typical natural convection induced flow over rectangular heat source: (a) velocity field, (b) stream lines.
Figure 6.17 Typical flow over rectangular heat source: (a) without plate, (b) with plate.
Figure 6.18 Typical temperature distribution for natural convection cooling of rectangular heat source: (a) without plate, (b) with plate.
6.4.2.2 Results with Oscillating Plate—Rectangular Heat Source Geometry. Under the proper conditions, the alterations in the velocity field caused by the oscillations of the plate were found to assist in the cooling of the rectangular heat source. The two-dimensional heat source geometry proved to be an important factor in the flow field development and thus the cooling effect. The characteristics of the flow and temperature field and heat transfer coefficients specific to this rectangular heat source geometry and their relationship to the system parameters are explained in this section. Again, animations of velocity and temperature fields for all cases studied were made to help describe all the specific flow features and their corresponding temperature fields. These are included on the accompanying CD in the folder labeled “oscillating plate over block.”

Overall Results

A few general observations about the effect of the oscillations on the flow, temperature field and resulting heat transfer coefficients are discussed before examining specific cases. The basic flow characteristics generated by the interaction of the plate motion, the natural convection effects and the flow over and around the rectangular heat source are similar to those with the plain channel geometry. Due to the motion of the plate, fluid periodically moves in and out of the region between the heat source and the oscillation source and fluid continues to move over and around the oscillating plate. Circulation zones tend to develop near the ends of the plate with the downstream circulation regions being larger and stronger.

However, the rectangular block geometry does result in some different flow features. While the flow over the top of the oscillation source is more restricted than for the plain channel geometry due to the close proximity to the upper channel wall, the heat
source is a finite entity and so the flow upstream and downstream of the oscillation source entering and exiting the region beneath the oscillation source is less confined. This has a number of consequences. First, some of the flow from the lower portion of the channel near the unheated board surface must move over the upper left corner of the heat source. This flow is directed into the region under the oscillation source and thus is a source of cooler fluid. Second, the flow over the top of the oscillation source, which was shown to play an important role in the flow field development, is altered by the finite heat source nature. For the plain channel geometry, the flow over the top of the oscillation source did, at certain times, impinge on the heated surface downstream of the oscillation source keeping the circulation regions that develop near the downstream end of the oscillating plate from moving away from the region just near the oscillation source until the motion of the plate redirected the flow. With this rectangular heat source geometry, the angle at which this flow exits over the top of the oscillation source varies with time, but there is no nearby downstream solid surface to which this flow can reattach and confine the circulation regions. Hence, the circulating regions dissipate much faster with this two-dimensional heat source geometry. During the upward oscillating plate motion, this flow is angled more downstream due to the circulation regions at the end of the plate as was the case for the plain channel geometry. However, in the current investigation, this flow sets up a small circulation region at the upper downstream (right) corner of the heat source, not the oscillation source. Because the velocities near the side surfaces of the heat source are already small relative to those moving over the top heat source surface, this small circulation region does not impede heat removal from the region. In addition, this keeps the fluid directed towards the top heat source surface where it may
better improve the cooling. Also, the more "open" flow to the sides of the heat source better facilitates the removal of heat that is expelled from the "channel like region" between the heat and oscillation source away from the heat source vicinity through transport to the main channel flow and to the outlet flow at the downstream channel end.

The changes in the velocity field around the heat source produced by the presence of the plate and its oscillations alter the temperature field in the channel and, thus, the heat transfer coefficients along the surfaces of the heat source. The velocities near the side heat sources surfaces are already low due to the geometrical arrangement, and the oscillations do not significantly affect the flow near the side surfaces of the heat source since the oscillations are localized in nature. Therefore, the impact of these oscillations is felt most at the positions along the top heat source surface. However, some effect of the motion of the plate does occur at all of the heat source surfaces. On the upstream and downstream side surfaces of the heat source, the motion of the oscillation source has little beneficial influence on the local heat transfer coefficients (at Point 1 or Point 4 respectively) or the space averaged heat transfer coefficients. In fact, up to a 10% decrease in the local and space averaged heat transfer coefficients relative to pure natural convection can occur as a result of the oscillations. This is the result of the lower time averaged flow velocities that occur near the sides of the heat source due to the higher velocity flows entering and exiting the region between the moving plate and the top heat source surface. However, since the heat transfer coefficients along the sides are on average about 5 times lower than those from the top heat source surface, this effect is not significantly detrimental to the overall thermal conditions of the system. Under the moving plate (Points 2 and 3), the changes the oscillations cause in the velocity field and
the velocity gradients are more significant as the effects of the oscillations are more concentrated near the top surface of the heat source. The heated surface experiences improvements in the heat transfer coefficients corresponding to the thinner thermal and fluid boundary layers that result from these higher velocities. Due to the geometry alone, the velocities near the upstream side of the heat source top surface are slightly higher than those without the plate. Consequently, for the lower oscillation parameters, Point 2 experiences the larger improvement in the time averaged local heat transfer coefficient. However, as the oscillation parameters increase, the improvement in the heat transfer coefficient further downstream at Point 3 is more significant with as much as a 52% increase over that of standard natural convection.

In general, the cooling effect was found to increase with the oscillation frequency, but more so for the oscillation amplitude. The smaller clearance space did improve the thermal conditions near the top of the heat source, but not near the sides of the heat source. The higher velocities near the inlet and outlet to the region under the plates are likely responsible for this effect. While the magnitudes may be higher, the higher velocity flow, essentially in the positive x direction, exiting from the tops of the heat sources cuts off the flow to the side surfaces. This, combined with the reduced flow rate between the top surface and the oscillation source, results in higher overall temperatures. The smaller clearance space also restricts the allowable oscillation source displacement and thus the amount of fluid entering and exiting the space between the oscillation source and the heat source, as well as the natural convection induced flows. Therefore, the extent to which the smaller clearance spacing is beneficial is dependent on the specific system parameters and may not be beneficial for all conditions.
Velocity Fields, Temperature Fields, and Heat Transfer Coefficients for Three Cases Investigated

As the specific details of the flow distribution and the related temperature fields are highly dependent on the parameters of a given system, the velocity field and resulting temperature and heat transfer coefficient characteristics are explained for three of the cases investigated.

The first parameter case is for \( d = 0.10, \ \nu = 0.40\pi, \ \omega = 4\pi, \ C = 0.30 \). Velocity fields at four points along a cycle are shown in Figure 6.19. When the plate begins at its lowest position, several circulation regions exist. A counterclockwise motion is observed at the upstream end of the plate where fluid from both above the oscillation source as well as fluid originally from the upstream lower portion of the channel near the board that has moved over the heat source is drawn into the space between the heat source and oscillation source. At the opposite end of the plate a clockwise recirculation region forms just downstream of the oscillation source. In addition, just downstream of the right upper corner of the heat source, a small clockwise circulation region is seen, caused by the downstream deflection of the flow over the top of the oscillation source due to the circulation region near the end of the plate. As the plate moves upward and there is significant through flow under the oscillation source despite opposition to the flow induced by the plate motion. The slow rotating region begins to dissipate, staying briefly near the downstream end of the top heat source surface. The exiting flow is then diverted between the oscillation source and this low velocity region. This low velocity region then begins to shift downstream. The through flow becomes more significant, pushing the low velocity region off of the edge of the heat source and causing it to dissipate. Once this circulation region is out of the way, the flow over the top of the heat source is free to
more closely follow the side heat source surface and exits at a steeper angle. With this, the corner circulation region disappears. As the plate changes direction, a clockwise rotation develops both over and to the side of the oscillation source at its upstream end while a counterclockwise circulation region develops over its downstream end. A small circulation region develops briefly upstream from the oscillation source. As the plate changes direction again, the flow over the plate causes the corner circulation to reform and the cycle begins again.

The velocity field characteristics described above can be correlated to the temperature field and heat transfer coefficients at the heated surface. Illustrative temperature fields are shown in Figure 6.20. Because there is little change in the velocity field near the upstream side surface or the downstream side surface of the heat source, there is little change in the temperature distribution and in the heat transfer coefficients at these side surfaces. This is evidenced by the plot of the local heat transfer coefficient as a function of time in Figure 6.25, which shows little variation at both Points 1 and Point 4. In contrast, the velocity gradients under the oscillation source are generally higher resulting in higher temperature gradients and higher heat transfer coefficients than those for natural convection with the plate fixed over the heat source. (See Figure 6.25 for variation in heat transfer coefficients at Points 2 and 3.) However, compared to standard natural convection with no plate, the improvement is not significant. The average heat transfer coefficient on the top surface increases by 2% relative to standard natural convection. With the through flow occurring in the space beneath the oscillation source, Point 2 experiences an 18% increase in the local heat transfer coefficient relative to the natural convection, and, of the points tracked, this location has the most variation
in the heat transfer coefficient with time. However, at Point 3 the reported 7% reduction in the heat transfer coefficient relative to pure natural convection is due to the low velocity region near the downstream end of the plate that directs the higher velocity flow away from the downstream portion of the top heat source surface. Hence, the oscillation parameters for this case produce a velocity field that results in limited local enhancement of natural convection, but does not enhance the overall thermal conditions of the heat source.

In the second parameter case, the displacement amplitude is doubled, but the oscillation frequency is the same, or, \( d = 0.20, \ V = 0.80\pi, \ \omega = 4\pi \). For this set of oscillation parameters, a more complex flow field results. (See Figure 6.21.) When the oscillation source is at its lowest position, the flow exiting from under the upstream end of the plate is carried up and over the oscillation source by the main channel natural convection flow and the flow around the oscillation source. This leads to the development of a large clockwise circulation region that extends from the upstream edge of the moving plate over the top of the moving plate. Another circulation region caused by the jet-like flow exiting from under the heat source as well as the flow around the oscillation source is formed above the downstream end of the oscillating plate. In addition, this jet-like expelled flow promotes the formation of an additional circulation region at the upper downstream corner of the oscillation source. The flow over the top of the oscillation that enters the space under the oscillation source is constricted to flow around the upstream circulation region and between the downstream circulation region and the oscillation source. As the plate begins to move upward, all circulations regions dissipate except the one at the upper block corner. Circulation regions counterclockwise in nature at the
upstream oscillation source end and clockwise at the downstream end then develop. Due to the upstream circulation region, some of the fluid entering the channel-like region is deflected towards and impinges on the upstream side of the top heat source surface. As the plate continues to move upward, the downstream circulation region is drawn slightly under the plate. The circulation region at the heat source corner dissipates. Despite the upward motion of the plate, a jet-like flow exits between the oscillation source and the circulation region. When the plate changes direction, the fluid under the plate still moves upward. Because of the larger displacement, more fluid is enclosed under the oscillation source and so the inertia effects are more pronounced. This leads to the development of a pair of counter-rotational cells beneath the oscillation source (upstream counterclockwise, downstream clockwise). The fluid entering the "channel" on the upstream side (opposing the downward motion of the plate) moves under the upstream cell and over the downstream cell to the exit. Hence, there is fluid entering the region beneath the oscillation source at the upstream end of the moving plate that exits downstream despite the plate motion. Also, some fluid is drawn into the region near the downstream top surface of the heat source. As the plate continues to move down, the upstream oscillation source dissipates due to the strengthening opposition the oscillations pose to the diminished natural convection flow as the spacing decreases. The downstream circulation region is eventually pushed out from under the moving plate. Then circulation regions form to the sides of the oscillation source and the cycle begins again. The velocities that occur during a cycle for this case are much more directed towards the top heat source surface than in the previous case.
An important finding from the examination of this velocity field is that the larger amplitude of the displacement of the oscillation source allows for larger inertia effects which lead to better fluid mixing, thinner thermal and fluid boundary layers and increased heat transfer coefficients. The plots in Figure 6.22 show the temperature field at a few instances during the oscillations and show that the temperature gradients along the top heat source surface, though varying with time, are significantly higher than those occurring for standard natural convection. The time averaged surface averaged heat transfer coefficient at the top heat source surface is 40% greater than that with no plate. The variation in the local heat transfer coefficients with time is given in Figure 6.25. The most significant variation in the heat transfer with time occurs at Points 2 and 3. The oscillations in these values are complex as a result of the complex nature of the flow. A 43% increase in the heat transfer coefficient at Point 2 occurs while a 52% increase relative to standard natural convection exists at Point 3. The increased amplitude substantially improves the cooling effect provided by the oscillations for this two-dimensional heat source geometry.

The final parameter set discussed involves a reduced clearance space between the oscillation source and the top heat source surface with the same oscillation parameters as in the first case discussed; \( d = 0.10 \), \( V = 0.40\pi \), \( \omega = 4\pi \), \( C = 0.15 \). Figure 6.23 shows the velocity field for this case at specific times along the cycle. At its lowest position, clockwise circulation regions are observed over the top of the oscillation source due to the fluid flowing over the top of this solid as well as at the upper downstream corner of the heat source. A clockwise rotation zone is seen just upstream of the oscillating plate, a likely result of the flow exiting from under the oscillation source. As the plate moves in
the positive $y$ direction, the low velocity upstream region dissipates, and a clockwise circulation region develops at the downstream end of the plate due to the inflow into the region under the plate. The development of this circulation region causes the "corner" circulation region to dissipate as it drags the air closer to the heat source side surface. At the upstream end, fluid moves over the top of the oscillation source and into the spacing between the moving body and the fixed heat source. Flow out of this region on the downstream end of the plate is deflected over the downstream circulation region and under the bottom surface of the oscillation source. When the plate changes direction, the fluid still enters the upstream end of the region for some time. There, it flows over a clockwise circulation region which deflects the flow towards the top surface of the heat source and then over the low velocity region that has been formed by the dissipation of the downstream circulation region. When the fluid motion "catches up" to the motion of the plate, two rotation regions are noticeable at the sides of the oscillation source. As time progresses, the upstream region becomes more intense due to the opposition of the natural convection flows and the exiting oscillation induced flows, while the downstream circulation region dissipates. When this downstream region dissipates, the flow from the top of the oscillation source causes the development of the corner circulation region. The plate then moves upward again.

For the same oscillation parameters, the lower clearance spacing produces a velocity field near the top surface of the heat source that is more conducive to improving the thermal conditions of a system. Figure 6.24 depicts key snapshots temperature field as it changes with time. The closer spacing of the oscillation source results in the formation of higher velocities due to the oscillations. The higher velocities appear to
inhibit the development of the circulation regions particularly downstream and work to move them away from the heat source surfaces. Thus, the flow is not diverted away from the heat source top surface as often, and the higher velocity gradients and temperature gradients can be maintained for longer portions of the oscillations. As a result, both the surface averaged and local heat transfer coefficients on the top surface are higher with this lower clearance than those for the first case discussed which was for the same conditions but with double the clearance spacing. However, due to the lower overall flow rates over the top of the heat source, the temperatures are higher. The average heat transfer coefficient on the top heat source surface is 16% higher than standard natural convection compared to 2% for the previous case. Both Points 2 and 3 experience a 26% increase in the time averaged local heat transfer coefficient. There is only a slight effect of the oscillations on the side surfaces. The temperature contours at four times during an oscillation in Figure 6.24 illustrate the higher temperature gradients above the heat source surface. The variation in the local heat transfer coefficient with time at the four points is plotted in Figure 6.25.

The time-averaged local heat transfer coefficient at Point 2 for each of the cases studied is shown in Figure 6.27. This parameter set study shows that the smaller clearance values as well as the inertia are also important factors in the formulation of the velocity distribution and ultimately the cooling effect. While the greatest increase in heat transfer coefficient relative to steady state occurs with the reduced clearance, the highest heat transfer coefficient values are obtained for the highest displacement amplitude.
### Table 6.4a Summary of Average Dimensionless Heat Transfer Coefficients – Rectangular Heat Source Geometry C=0.30

<table>
<thead>
<tr>
<th>Case C=0.30</th>
<th>Max Time Avg Dimless Temperature</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimm-less Avg Heat Transfer Coeff. Left</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimm-less Avg Heat Transfer Coeff. Top</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimm-less Avg Heat Transfer Coeff. Right</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>1.755475</td>
<td>-5.296%</td>
<td>0.125359</td>
<td>-13.49%</td>
<td>0.146033</td>
<td>-5.60%</td>
<td>0.042630</td>
<td>-14.20%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>1.853636</td>
<td>5.59%</td>
<td>-0.110455</td>
<td>-11.88%</td>
<td>0.138287</td>
<td>-5.30%</td>
<td>0.037329</td>
<td>-12.43%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.05 v=0.10 ω=2 f=1/π</td>
<td>1.853611</td>
<td>5.59%</td>
<td>-0.110654</td>
<td>-11.73%</td>
<td>0.140153</td>
<td>-4.02%</td>
<td>1.34%</td>
<td>0.038812</td>
<td>-8.95%</td>
<td>3.97%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.05 v=0.1π ω=2π f=1</td>
<td>1.853625</td>
<td>5.59%</td>
<td>-0.110064</td>
<td>-12.20%</td>
<td>0.140025</td>
<td>-4.11%</td>
<td>1.25%</td>
<td>0.038990</td>
<td>-8.54%</td>
<td>4.44%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.10 v=0.20 ω=2 f=1/π</td>
<td>1.853547</td>
<td>5.58%</td>
<td>-0.111082</td>
<td>-11.38%</td>
<td>0.145820</td>
<td>-0.14%</td>
<td>5.44%</td>
<td>0.042242</td>
<td>-0.910%</td>
<td>13.16%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.10 v=0.2π ω=2π f=1</td>
<td>1.853591</td>
<td>5.58%</td>
<td>-0.109118</td>
<td>-12.95%</td>
<td>0.146217</td>
<td>0.126%</td>
<td>5.73%</td>
<td>0.045860</td>
<td>7.57%</td>
<td>22.85%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.10 v=0.4π ω=4π f=2</td>
<td>1.853610</td>
<td>5.59%</td>
<td>-0.108623</td>
<td>-13.35%</td>
<td>0.149061</td>
<td>2.07%</td>
<td>7.79%</td>
<td>0.041308</td>
<td>-3.10%</td>
<td>10.65%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.15 v=0.6π ω=4π f=2</td>
<td>1.853585</td>
<td>5.58%</td>
<td>-0.107477</td>
<td>-14.26%</td>
<td>0.169217</td>
<td>15.87%</td>
<td>22.36%</td>
<td>0.045882</td>
<td>7.62%</td>
<td>22.91%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20 v=0.4π ω=2π f=1</td>
<td>1.853517</td>
<td>5.58%</td>
<td>-0.108711</td>
<td>-13.28%</td>
<td>0.190186</td>
<td>30.23%</td>
<td>37.53%</td>
<td>0.057974</td>
<td>35.99%</td>
<td>55.30%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20 v=0.8π ω=4π f=2</td>
<td>1.853551</td>
<td>5.58%</td>
<td>-0.107454</td>
<td>-14.28%</td>
<td>0.204486</td>
<td>40.02%</td>
<td>47.87%</td>
<td>0.052804</td>
<td>23.86%</td>
<td>41.45%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 6.4b Summary of Average Dimensionless Heat Transfer Coefficients – Rectangular Heat Source Geometry C=0.15

<table>
<thead>
<tr>
<th>Case C=0.15</th>
<th>Max Time Avg Dimless Temperature</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimm-less Avg Heat Transfer Coeff. Left</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimm-less Avg Heat Transfer Coeff. Top</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>1.755475</td>
<td>-8.82%</td>
<td>0.125359</td>
<td>-15.90%</td>
<td>0.146033</td>
<td>-28.23%</td>
<td>0.042630</td>
<td>-6.108%</td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>1.925459</td>
<td>9.66%</td>
<td>-0.108158</td>
<td>-13.72%</td>
<td>0.113882</td>
<td>-22.01%</td>
<td>0.040177</td>
<td>-5.756%</td>
<td></td>
</tr>
<tr>
<td>d=0.10 v=0.2π ω=2π f=1</td>
<td>1.924787</td>
<td>9.64%</td>
<td>-0.108144</td>
<td>-13.73%</td>
<td>0.160844</td>
<td>10.14%</td>
<td>41.23%</td>
<td>0.042789</td>
<td>0.326%</td>
</tr>
<tr>
<td>d=0.10 v=0.4π ω=4π f=2</td>
<td>1.925395</td>
<td>9.67%</td>
<td>-0.108054</td>
<td>-13.80%</td>
<td>0.169384</td>
<td>15.99%</td>
<td>48.73%</td>
<td>0.041293</td>
<td>-3.138%</td>
</tr>
</tbody>
</table>
### Table 6.5a Summary of Average Local Dimensionless Heat Transfer Coefficients — Rectangular Heat Source Geometry C=0.30

<table>
<thead>
<tr>
<th>Case C=0.30</th>
<th>Time Avg Dimm-less Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.100253</td>
<td>-11.26%</td>
<td>11.38765</td>
<td>-12.07%</td>
<td>13.73%</td>
<td>0.029077</td>
<td>-15.16%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.090110</td>
<td>-10.12%</td>
<td>9.80%</td>
<td>-10.8275</td>
<td>12.07%</td>
<td>0.025260</td>
<td>-13.16%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.05 v=0.10 ω=2 f=1/π</td>
<td>0.090311</td>
<td>-9.92%</td>
<td>0.223%</td>
<td>0.152664</td>
<td>10.01%</td>
<td>0.111590</td>
<td>-9.38%</td>
<td>3.06%</td>
<td>0.026851</td>
<td>-7.65%</td>
</tr>
<tr>
<td>d=0.10 v=0.20 ω=2 f=1/π</td>
<td>0.085598</td>
<td>-14.62%</td>
<td>-5.01%</td>
<td>0.152571</td>
<td>9.94%</td>
<td>0.104968</td>
<td>-14.76%</td>
<td>-3.05%</td>
<td>0.025149</td>
<td>-13.50%</td>
</tr>
<tr>
<td>d=0.10 v=0.20 ω=2 f=1/π</td>
<td>0.090743</td>
<td>-9.49%</td>
<td>0.702%</td>
<td>0.154752</td>
<td>11.52%</td>
<td>0.120682</td>
<td>-2.00%</td>
<td>11.45%</td>
<td>0.030262</td>
<td>4.07%</td>
</tr>
<tr>
<td>d=0.10 v=0.20 ω=2 f=1/π</td>
<td>0.089299</td>
<td>-10.93%</td>
<td>-0.900%</td>
<td>0.155661</td>
<td>12.17%</td>
<td>0.116569</td>
<td>-5.34%</td>
<td>7.66%</td>
<td>0.029684</td>
<td>2.08%</td>
</tr>
<tr>
<td>d=0.20 v=0.20 ω=2 f=1/π</td>
<td>0.088790</td>
<td>-11.43%</td>
<td>-1.46%</td>
<td>0.164254</td>
<td>18.36%</td>
<td>0.113785</td>
<td>-7.60%</td>
<td>5.08%</td>
<td>0.026379</td>
<td>-9.28%</td>
</tr>
<tr>
<td>d=0.20 v=0.20 ω=4 f=2</td>
<td>0.087763</td>
<td>-12.46%</td>
<td>-2.60%</td>
<td>0.177132</td>
<td>27.64%</td>
<td>0.138344</td>
<td>12.33%</td>
<td>27.77%</td>
<td>0.026379</td>
<td>9.28%</td>
</tr>
<tr>
<td>d=0.20 v=0.80 ω=4 f=2</td>
<td>0.087215</td>
<td>-13.01%</td>
<td>-3.21%</td>
<td>0.198185</td>
<td>42.82%</td>
<td>0.187316</td>
<td>52.10%</td>
<td>73.00%</td>
<td>0.025411</td>
<td>-12.61%</td>
</tr>
</tbody>
</table>

### 6.5b Summary of Average Local Dimensionless Heat Transfer Coefficients — Rectangular Heat Source Geometry C=0.15

<table>
<thead>
<tr>
<th>Case C=0.15</th>
<th>Time Avg Dimm-less Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.100253</td>
<td>-10.79%</td>
<td>0.138765</td>
<td>-27.66%</td>
<td>0.123151</td>
<td>-17.58%</td>
<td>0.029077</td>
<td>-0.68%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.090485</td>
<td>-9.74%</td>
<td>0.108698</td>
<td>-21.668%</td>
<td>0.104733</td>
<td>-14.95%</td>
<td>0.028881</td>
<td>-0.676%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.10 v=0.20 ω=2 f=1</td>
<td>0.090107</td>
<td>-10.12%</td>
<td>-0.418%</td>
<td>0.170732</td>
<td>23.037%</td>
<td>0.144909</td>
<td>17.66%</td>
<td>38.36%</td>
<td>0.028319</td>
<td>-2.60%</td>
</tr>
<tr>
<td>d=0.10 v=0.40 ω=4 f=2</td>
<td>0.089731</td>
<td>-10.49%</td>
<td>-0.833%</td>
<td>0.174875</td>
<td>26.022%</td>
<td>0.156083</td>
<td>26.74%</td>
<td>49.02%</td>
<td>0.028394</td>
<td>-2.35%</td>
</tr>
</tbody>
</table>
Figure 6.19 Velocity field for $d=0.1$ $V=0.4\pi$ $\omega=4\pi$ $C=0.30$ at indicated dimensionless times: (a) $t_F=1.3375$ (b) $t_F=1.00$ (cont.).
Figure 6.19 Velocity field for $d=0.1$, $V=0.4\pi$, $\omega=4\pi$, $C=0.30$ at indicated dimensionless times: (c) $\tilde{t}_F = 1.1125$, (d) $\tilde{t}_F = 1.1875$. 
Figure 6.20  Temperature field for \( d=0.1 \) \( V=0.4\pi \) \( \omega=4\pi \) \( C=0.30 \) at indicated dimensionless times: (a) \( \hat{t}_F=1.3375 \) (b) \( \hat{t}_F=1.000 \) (cont.).
Figure 6.20  Temperature field for $d=0.1$ $V=0.4\pi$ $\omega=4\pi$ $C=0.30$ at indicated dimensionless times: (c) $\tilde{t}_F=1.1125$, (d) $\tilde{t}_F=1.1875$. 
Figure 6.21 Velocity field for $d=0.2$ $V=0.8\pi$ $\omega=4\pi$ $C=0.30$ at indicated dimensionless times: (a) $\tilde{t}_F=1.350$, (b) $\tilde{t}_F=1.0625$, (cont.).
Figure 6.21 Velocity field for \(d=0.2\) \(V=0.8\pi\) \(\omega=4\pi\) \(C=0.30\) at indicated dimensionless times: (c) \(t_F=1.1250\), (d) \(t_F=1.250\).
Figure 6.22 Temperature field for $d=0.2$, $V=0.8\pi$, $\omega=4\pi$, $C=0.30$ at indicated dimensionless times: (a) $\tilde{t}_F=1.3375$, (b) $\tilde{t}_F=1.0625$ (cont.).
Figure 6.22 Temperature field for $d=0.2$ $V=0.8\pi$ $\omega=4\pi$ $C=0.30$ at indicated dimensionless times: (c) $\tilde{t}_F = 1.1250$, (d) $\tilde{t}_F = 1.250$. 
Figure 6.23 Velocity field for $d=0.1$ $V=0.4\pi$ $\omega=4\pi$ $C=0.15$ at indicated dimensionless times: (a) $\tilde{t}_F=1.375$, (b) $\tilde{t}_F=1.0375$ (cont.).
Figure 6.23   Velocity field for $d=0.1$, $V=0.4\pi$, $\phi=4\pi$, $C=0.15$ at indicated dimensionless times: (c) $\tilde{t}_F=1.1125$, (d) $\tilde{t}_F=1.1750$. 
Figure 6.24 Temperature field for $d=0.1$ $V=0.4\pi$ $\omega=4\pi$ $C=0.15$ at indicated dimensionless times: (a) $t_F = 1.375$, (b) $t_F = 1.0375$ (cont.).
(d) Figure 6.24 Temperature field for $d=0.1 \ V=0.4\pi \ \omega=4\pi \ C=0.15$ at indicated dimensionless times: (c) $t_F=1.1125$, (d) $t_F=1.1750$. 
Figure 6.25 Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) $d=0.10$ $V=0.40\pi$ $\omega=4\pi$ $C=0.30$, (b) $d=0.20$ $V=0.80\pi$ $\omega=4\pi$ $C=0.30$, (c) $d=0.10$ $V=0.40\pi$ $\omega=4\pi$ $C=0.15$. 
Figure 6.26 Time averaged local dimensionless heat transfer coefficient at Point 2:
(a) $d = 0.05 \ C = 0.30$, (b) $d = 0.10 \ C = 0.30$. 
Figure 6.27 Time averaged local dimensionless heat transfer coefficient at Point 2:
(a) $d=0.15$ $d=0.20$ $d=0.20$, respectively $C=0.30$, (b) $d=0.10$ $C=0.15$. 
6.4.3 Conclusions

This portion of the current work investigated the use of two dimensional oscillation and heat source geometries. The study results revealed a number of significant findings. First, the discrete nature of both the oscillation source and the heat source opens up the possibility of different flow angles and flow paths that at times can better direct the flow toward the top heat source surface, can help to destroy the downstream circulation regions, and can allow for flow from different channel areas to be brought to the area beneath the oscillation source. Therefore, the finite oscillation source and heat source geometries are more conducive to facilitating the improvement of the heat source cooling.

In addition to these geometric effects, some important trends and relationships between the system parameters and the cooling effect were also revealed. First, the cooling effect increases with the oscillation frequency and displacement, with the displacement holding more influence. Decreasing the clearance between the oscillation source and the heat source improved the heat transfer coefficients from the top heat source surface, but resulted in higher overall heat source temperatures. The decreased clearance as well as the increased displacement amplitude cases result in larger velocities, and the higher amplitude case involves more fluid mass within the channel-like region between the heat source and oscillation source. Therefore, the inertia forces are more significant. While the inertia may contribute to the development of the low velocity circulation regions, the inertia effects also promote some through-flow along the oscillation source length in opposition to the fluid motion caused by the moving plate and to help bring fresher fluid closer to the heat source. Hence, the inertia forces tend to
cause better fluid mixing and, hence, help to improve the cooling effect. As much as a 40% increase in the average heat transfer coefficient from the top heat source and 50% increase locally were achieved through the local application of the oscillation source.

The results of this study showed that the transverse oscillations of a discrete oscillation source placed near the top surface of a heat source placed in a vertically oriented channel can substantially improve the thermal conditions of the heat source over that of pure natural convection for a range of viable oscillation, heat source, and geometric parameters.

6.5 Modified Rectangular Heat Source Geometry

In an attempt to take advantage of the high velocity “jet-like” expulsion of fluid from beneath the oscillation source and further improve the cooling of a rectangular heat source, different arrangements of the oscillation source relative to the rectangular heat source were also investigated. These modified arrangements and the resulting velocity, temperature, and heat transfer coefficients are briefly described below. As before, animations of velocity and temperature fields for all cases studied were made to help describe all the specific flow features and their corresponding temperature fields. These are included on the accompanying CD in the folder labeled “oscillating plate modified arrangement - one block.”

6.5.1 Dummy Block Modified Arrangement

In the first modified arrangement, the oscillation source is placed over a “dummy” conducting unheated rectangular block placed just upstream of the heated rectangular block positioned in an attempt to force the high velocity fluid that exits the space under
the oscillation source to pass near the top heat source surface. The arrangement used is shown in Figure 6.28 where an oscillation source that the top heat source surface better experiences the effects of the oscillations. The material property and system parameters are the same as those in Table 6.3. The geometric parameters for this case are shown in Table 6.6.

![Figure 6.28](image)

**Figure 6.28** Model geometry for dummy block investigation.

**Table 6.6** Dimensional Parameter Values for Dummy Block Geometry

<table>
<thead>
<tr>
<th>Dimensional Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>b = L_{ref} (length basis)</td>
<td>0.25 in = 0.0635 m</td>
</tr>
<tr>
<td>BL = BH (block length = block height)</td>
<td>b</td>
</tr>
<tr>
<td>BT (board thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>CHL, CHU (channel heights)</td>
<td>2b</td>
</tr>
<tr>
<td>CLB (plate clearance)</td>
<td>0.15b</td>
</tr>
<tr>
<td>SPACE (spacing between blocks)</td>
<td>0.50b</td>
</tr>
<tr>
<td>PLB (plate length)</td>
<td>0.75b</td>
</tr>
<tr>
<td>TL (channel length)</td>
<td>14b</td>
</tr>
<tr>
<td>PLH (plate thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>SL (starting length)</td>
<td>5.75b</td>
</tr>
</tbody>
</table>

*Dimensions defined in Figure 6.29 (313x205 graded mesh generated)*

![Figure 6.29](image)

**Figure 6.29** Dimensioning for dummy block investigation.
<table>
<thead>
<tr>
<th>Case C=0.15</th>
<th>Max Time Avg Dim-ness Temperature</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS Without Plate and Block</th>
<th>Time Avg Dim-ness Avg Heat Transfer Coeff. Left</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS Without Plate and Block</th>
<th>Time Avg Dim-ness Avg Heat Transfer Coeff. Top</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS Without Plate and Block</th>
<th>Time Avg Dim-ness Avg Heat Transfer Coeff. Right</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS Without Plate and Block</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>1.79739</td>
<td>-</td>
<td>-3.24%</td>
<td>0.753%</td>
<td>0.0936226</td>
<td>-</td>
<td>12.905%</td>
<td>-24.31%</td>
<td>0.1621125</td>
<td>-7.52%</td>
<td>12.53%</td>
<td>0.0444780</td>
<td>-5.76%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>1.85771</td>
<td>3.35%</td>
<td>4.13%</td>
<td>0.0829217</td>
<td>-11.43%</td>
<td>-</td>
<td>-32.96%</td>
<td>-7.00%</td>
<td>0.1507618</td>
<td>-4.65%</td>
<td>0.0471978</td>
<td>6.11%</td>
<td>-</td>
</tr>
<tr>
<td>SS without plate and block</td>
<td>1.78395</td>
<td>-0.748%</td>
<td>-3.97%</td>
<td>0.1237041</td>
<td>32.13%</td>
<td>49.182%</td>
<td>-</td>
<td>0.1440511</td>
<td>-11.14%</td>
<td>-4.45%</td>
<td>0.043114</td>
<td>-3.06%</td>
<td>-8.65%</td>
</tr>
<tr>
<td>d=0.10 v=0.2 (\pi ) w = 2 (\pi ) f =1</td>
<td>1.85359</td>
<td>3.12%</td>
<td>-0.22%</td>
<td>3.90%</td>
<td>0.086697</td>
<td>-7.39%</td>
<td>4.553%</td>
<td>-29.91%</td>
<td>0.148467</td>
<td>-8.41%</td>
<td>-1.52%</td>
<td>3.06%</td>
<td>0.046418</td>
</tr>
<tr>
<td>d=0.10 v=0.4 (\pi ) w = 4 (\pi ) f =2</td>
<td>1.85361</td>
<td>3.12%</td>
<td>-0.221%</td>
<td>3.90%</td>
<td>0.084795</td>
<td>-9.42%</td>
<td>2.259%</td>
<td>-31.45%</td>
<td>0.145358</td>
<td>-10.33%</td>
<td>-3.58%</td>
<td>0.907%</td>
<td>0.047239</td>
</tr>
</tbody>
</table>
Table 6.7b Summary of Average Local Dimensionless Heat Transfer Coefficients – Dummy Block Geometry C=0.15

<table>
<thead>
<tr>
<th>Case</th>
<th>C=0.15</th>
<th>Time Avg Dimensionless Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate and Block</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS Without Plate and Block</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS Without Plate and Block</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS Without Plate and Block</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.064087</td>
<td>-</td>
<td>4.24%</td>
<td>-35.51%</td>
<td>0.160191</td>
<td>-</td>
<td>10.16%</td>
<td>18.20%</td>
<td>0.132655</td>
<td>-</td>
<td>0.635%</td>
<td>8.68%</td>
<td>0.029776</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.061479</td>
<td>-4.06%</td>
<td>-</td>
<td>-38.13%</td>
<td>0.145407</td>
<td>-9.23%</td>
<td>-</td>
<td>0.941%</td>
<td>0.131818</td>
<td>-0.631%</td>
<td>-</td>
<td>8.00%</td>
<td>0.031029</td>
</tr>
<tr>
<td>SS without plate and block</td>
<td>0.099378</td>
<td>55.06%</td>
<td>61.64%</td>
<td>-</td>
<td>0.135519</td>
<td>-15.40%</td>
<td>-6.80%</td>
<td>-</td>
<td>0.122052</td>
<td>-7.99%</td>
<td>-7.40%</td>
<td>-</td>
<td>0.029238</td>
</tr>
<tr>
<td>d=0.10 v=0.2π (\omega = 2\pi ) f=1</td>
<td>0.061527</td>
<td>-3.99%</td>
<td>0.078%</td>
<td>-38.07%</td>
<td>0.144188</td>
<td>-9.99%</td>
<td>-0.838%</td>
<td>-</td>
<td>0.125984</td>
<td>-5.02%</td>
<td>-4.42%</td>
<td>3.22%</td>
<td>0.031010</td>
</tr>
<tr>
<td>d=0.10 v=0.4π (\omega = 4\pi ) f=2</td>
<td>0.061128</td>
<td>-4.61%</td>
<td>-0.572%</td>
<td>-38.49%</td>
<td>0.132703</td>
<td>-17.16%</td>
<td>-8.73%</td>
<td>-7.87%</td>
<td>0.131388</td>
<td>-0.955%</td>
<td>-0.326%</td>
<td>7.65%</td>
<td>0.031026</td>
</tr>
</tbody>
</table>
The expected cooling effect for this oscillation source arrangement did not materialize. The dummy block, while unheated, obstructs the flow of unheated fluid to the surfaces of the heat source further downstream. The placement of the plate over the top of the dummy block further blocks the flow to the heat source. Hence, the velocities near the top as well as the left (upstream) heat source surfaces are significantly lower than those for just a single rectangular heat source placed in a vertically oriented channel under steady state conditions. The only benefit is a slight improvement in the heat removal from the upstream side of the heat source as there is significant conduction through the fluid from the active heat source into the dummy block.

The effects of the oscillations do not act to improve these thermal conditions in the channel for the two parameter cases investigated. The oscillations do cause slight amounts of fluid to flow into and out of the space between the dummy block and heat source. This can result in a penetration and deflection of the flow in this region as well as the development of circulation zones. The depth of the penetration varies with time. For most of an oscillation cycle, the circulation regions or low velocity regions persist downstream of the oscillation source. They are slower to dissipate than for the single block geometry due to the modified arrangement and tend to further obstruct the flow. This prevents the higher velocity flow from reaching the heat source top surface, and, thus, the expected cooling effect does not occur. Heat transfer coefficients on the top heat source surface were either lower than those for the single heat source geometry or experienced little change. While a smaller clearance value was used in the current investigation, restricting the possible displacements, higher displacements may be required to change the flow patterns to be more beneficial. However, given the general
flow characteristics, this "dummy block" arrangement does not appear to have promise for facilitating significant heat transfer enhancement.

Figure 6.30 Typical natural convection induced flow over dummy block geometry: (a) velocity field, (b) temperature field. $d=0.10$ $V=0.4\pi$ $\omega=4\pi$ $C=0.15$. 
6.5.2 Plate Extension Modified Arrangement

To reduce the flow obstruction while still attempting to position the oscillation source and heat source so as to exploit the cooling potential of the flow expelled from under the oscillation source, the geometry shown in Figure 6.32 was then investigated. A short thin conduction plate extension was attached to the upstream top surface of the heat source, and the oscillation source was placed above this extension. Because of its thinner profile, the plate extension geometry does not obstruct the flow as significantly as the dummy block. The geometric parameters used are listed in Table 6.8 while the parameters investigated are listed in Table 6.9.
Figure 6.32 Model geometry for extension plate investigation.

Table 6.8 Dimensional Parameter Values for Extension Geometry

<table>
<thead>
<tr>
<th>Dimensional Parameter**</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b=L_{ref}$ (length basis)</td>
<td>0.25 in=0.0635m</td>
</tr>
<tr>
<td>BH=BL (block height = block length)</td>
<td>$b$</td>
</tr>
<tr>
<td>BT (board thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>CHL, CHU (channel heights)</td>
<td>2b</td>
</tr>
<tr>
<td>CLB (plate clearance)</td>
<td>0.15b, 0.30b</td>
</tr>
<tr>
<td>PLB (plate length)</td>
<td>0.75b</td>
</tr>
<tr>
<td>PLTH (plate thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>PLL (extension length)</td>
<td>0.915b</td>
</tr>
<tr>
<td>PLT (extension thickness)</td>
<td>0.02b</td>
</tr>
<tr>
<td>TL (channel length)</td>
<td>14b</td>
</tr>
<tr>
<td>SL (location of plate end)</td>
<td>6.585b</td>
</tr>
</tbody>
</table>

**Dimensions defined in Figure 6.33 (335x205 non-uniform mesh generated)

Figure 6.33 Dimensioning for extension plate investigation.
<table>
<thead>
<tr>
<th>Case C = 0.15</th>
<th>Max Time Avg Dimn-less Temp.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimn-less Avg Heat Transfer Coeff. Left</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimn-less Avg Heat Transfer Coeff. Top</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimn-less Avg Heat Transfer Coeff. Right</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Extension</th>
<th>% Diff from SS with Plate and Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS w/out plate</td>
<td>1.597618</td>
<td>-</td>
<td>-6.33%</td>
<td>0.028309</td>
<td>-</td>
<td>3.58%</td>
<td>-75.75%</td>
<td>0.114271</td>
<td>-</td>
<td>9.48%</td>
<td>-19.94%</td>
<td>0.039886</td>
<td>-</td>
<td>-8.16%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>1.705680</td>
<td>6.76%</td>
<td>-</td>
<td>0.027331</td>
<td>-3.45%</td>
<td>-</td>
<td>-76.58%</td>
<td>0.104371</td>
<td>-</td>
<td>-8.66%</td>
<td>-26.88%</td>
<td>0.043431</td>
<td>8.89%</td>
<td>-</td>
</tr>
<tr>
<td>SS w/out plate and extension</td>
<td>1.765670</td>
<td>10.51%</td>
<td>3.51%</td>
<td>0.116740</td>
<td>312.37%</td>
<td>327.14%</td>
<td>-</td>
<td>0.142744</td>
<td>24.91%</td>
<td>36.76%</td>
<td>-</td>
<td>0.043112</td>
<td>8.08%</td>
<td>-0.736%</td>
</tr>
<tr>
<td>d=0.10 v=0.2π ω = 2π f = 1</td>
<td>1.705653</td>
<td>6.76%</td>
<td>-0.002%</td>
<td>0.027375</td>
<td>-3.30%</td>
<td>0.163%</td>
<td>-76.55%</td>
<td>0.107508</td>
<td>-5.91%</td>
<td>3.00%</td>
<td>-24.68%</td>
<td>0.041777</td>
<td>4.74%</td>
<td>-3.80%</td>
</tr>
<tr>
<td>d=0.10 v=0.4π ω = 4π f = 2</td>
<td>1.705649</td>
<td>6.76%</td>
<td>-0.002%</td>
<td>0.027346</td>
<td>-3.40%</td>
<td>0.056%</td>
<td>-76.57%</td>
<td>0.094745</td>
<td>-17.08%</td>
<td>-9.22%</td>
<td>-18.84%</td>
<td>0.042153</td>
<td>5.68%</td>
<td>-2.94%</td>
</tr>
<tr>
<td>d=0.10 v=0.8π ω = 8π f = 4</td>
<td>1.705644</td>
<td>6.76%</td>
<td>-0.002%</td>
<td>0.027331</td>
<td>-3.45%</td>
<td>0.003%</td>
<td>-76.58%</td>
<td>0.093296</td>
<td>-18.35%</td>
<td>-10.61%</td>
<td>-20.08%</td>
<td>0.043535</td>
<td>9.15%</td>
<td>0.239%</td>
</tr>
</tbody>
</table>
Table 6.9b Summary of Average Local Dimensionless Heat Transfer Coefficients – Extension Plate Geometry  C=0.15

<table>
<thead>
<tr>
<th>Case C=0.15</th>
<th>Time Avg Dimnless Heat Transfer Coeff. Point 1</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Extension</th>
<th>Time Avg Dimnless Heat Transfer Coeff. Point 2</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Extension</th>
<th>Time Avg Dimnless Heat Transfer Coeff. Point 3</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Extension</th>
<th>Time Avg Dimnless Heat Transfer Coeff. Point 4</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS w/out plate</td>
<td>0.022578</td>
<td>-</td>
<td>5.67%</td>
<td>5.67%</td>
<td>-</td>
<td>-</td>
<td>5.67%</td>
<td>5.67%</td>
<td>-</td>
<td>5.67%</td>
<td>5.67%</td>
<td>-</td>
<td>5.67%</td>
<td>5.67%</td>
<td>-</td>
<td>5.67%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.021365</td>
<td>-</td>
<td>5.37%</td>
<td>-</td>
<td>5.37%</td>
<td>-</td>
<td>-</td>
<td>5.37%</td>
<td>-</td>
<td>5.37%</td>
<td>-</td>
<td>5.37%</td>
<td>-</td>
<td>5.37%</td>
<td>-</td>
<td>5.37%</td>
</tr>
<tr>
<td>SS w/out plate and extension</td>
<td>0.098641</td>
<td>336.88%</td>
<td>361.69%</td>
<td>361.69%</td>
<td>336.88%</td>
<td>-</td>
<td>361.69%</td>
<td>336.88%</td>
<td>-</td>
<td>361.69%</td>
<td>336.88%</td>
<td>-</td>
<td>361.69%</td>
<td>336.88%</td>
<td>-</td>
<td>361.69%</td>
</tr>
<tr>
<td>d=0.10 v=0.2π ω=2π f=1</td>
<td>0.021367</td>
<td>-5.36%</td>
<td>-0.010%</td>
<td>-5.36%</td>
<td>0.021367</td>
<td>-</td>
<td>-0.010%</td>
<td>-5.36%</td>
<td>0.021367</td>
<td>-</td>
<td>-0.010%</td>
<td>-5.36%</td>
<td>0.021367</td>
<td>-</td>
<td>-0.010%</td>
<td>-5.36%</td>
</tr>
<tr>
<td>d=0.10 v=0.4π ω=4π f=2</td>
<td>0.021350</td>
<td>-5.43%</td>
<td>-0.070%</td>
<td>-5.43%</td>
<td>0.021350</td>
<td>-</td>
<td>-0.070%</td>
<td>-5.43%</td>
<td>0.021350</td>
<td>-</td>
<td>-0.070%</td>
<td>-5.43%</td>
<td>0.021350</td>
<td>-</td>
<td>-0.070%</td>
<td>-5.43%</td>
</tr>
<tr>
<td>d=0.10 v=0.8π ω=8π f=4</td>
<td>0.021345</td>
<td>-5.46%</td>
<td>-0.096%</td>
<td>-5.46%</td>
<td>0.021345</td>
<td>-</td>
<td>-0.096%</td>
<td>-5.46%</td>
<td>0.021345</td>
<td>-</td>
<td>-0.096%</td>
<td>-5.46%</td>
<td>0.021345</td>
<td>-</td>
<td>-0.096%</td>
<td>-5.46%</td>
</tr>
</tbody>
</table>
### Table 6.10a Summary of Average Dimensionless Heat Transfer Coefficients – Extension Plate Geometry  C=0.30

<table>
<thead>
<tr>
<th>Case C=0.30</th>
<th>Max Time Avg Dimm-less Temp.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Extension</th>
<th>Time Avg Dimm-less Avg Heat Transfer Coeff. Left</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Extension</th>
<th>Time Avg Dimm-less Avg Heat Transfer Coeff. Top</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Extension</th>
<th>Time Avg Dimm-less Avg Heat Transfer Coeff. Right</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Extension</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>1.594618</td>
<td>-</td>
<td>-5.71%</td>
<td>-9.68%</td>
<td>0.028309</td>
<td>-</td>
<td>2.72%</td>
<td>-75.75%</td>
<td>0.114271</td>
<td>-</td>
<td>31.573%</td>
<td>-19.94%</td>
<td>0.043429</td>
<td>-</td>
<td>10.052%</td>
<td>-</td>
</tr>
<tr>
<td>SS with plate</td>
<td>1.691295</td>
<td>6.063%</td>
<td>-</td>
<td>-4.20%</td>
<td>0.027569</td>
<td>-2.65%</td>
<td>-</td>
<td>-76.39%</td>
<td>0.086850</td>
<td>-23.99%</td>
<td>-</td>
<td>-39.15%</td>
<td>0.039462</td>
<td>-9.134%</td>
<td>-</td>
<td>-8.46%</td>
</tr>
<tr>
<td>SS without plate and extension</td>
<td>1.765607</td>
<td>10.723%</td>
<td>4.39%</td>
<td>-</td>
<td>0.116740</td>
<td>312.37%</td>
<td>323.5%</td>
<td>-</td>
<td>0.142744</td>
<td>24.91%</td>
<td>64.357%</td>
<td>-</td>
<td>0.043112</td>
<td>-0.730%</td>
<td>9.248%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.20</td>
<td>1.691290</td>
<td>6.062%</td>
<td>0.000%</td>
<td>-4.20%</td>
<td>0.027586</td>
<td>-2.55%</td>
<td>0.097%</td>
<td>-78.37%</td>
<td>0.105411</td>
<td>-7.75%</td>
<td>21.372%</td>
<td>-14.788%</td>
<td>0.038125</td>
<td>-12.214%</td>
<td>-3.390%</td>
<td>-11.56%</td>
</tr>
<tr>
<td>Case</td>
<td>Time Avg Dimnless Heat Transfer Coeff. Point 1</td>
<td>% Diff from SS without Plate</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Extension</td>
<td>Time Avg Dimnless Heat Transfer Coeff. Point 2</td>
<td>% Diff from SS without Plate</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Extension</td>
<td>Time Avg Dimnless Heat Transfer Coeff. Point 3</td>
<td>% Diff from SS without Plate</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Extension</td>
<td>Time Avg Dimnless Heat Transfer Coeff. Point 4</td>
<td>% Diff from SS without Plate</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Extension</td>
</tr>
<tr>
<td>-------</td>
<td>---------------------------------------------</td>
<td>-----------------------------</td>
<td>---------------------------</td>
<td>-------------------------------------------</td>
<td>---------------------------------------------</td>
<td>-----------------------------</td>
<td>---------------------------</td>
<td>-------------------------------------------</td>
<td>---------------------------------------------</td>
<td>-----------------------------</td>
<td>---------------------------</td>
<td>-------------------------------------------</td>
<td>---------------------------------------------</td>
<td>-----------------------------</td>
<td>---------------------------</td>
<td>-------------------------------------------</td>
</tr>
<tr>
<td>SS w/out plate</td>
<td>0.022578</td>
<td>-</td>
<td>4.54%</td>
<td>-77.11%</td>
<td>0.110923</td>
<td>-</td>
<td>44.80%</td>
<td>-17.38%</td>
<td>0.109959</td>
<td>-</td>
<td>25.83%</td>
<td>-9.21%</td>
<td>0.027678</td>
<td>-</td>
<td>5.88%</td>
<td>-9.42%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.021597</td>
<td>-4.34%</td>
<td>-</td>
<td>-78.10%</td>
<td>0.076604</td>
<td>-30.94%</td>
<td>-</td>
<td>-42.94%</td>
<td>0.087383</td>
<td>-20.53%</td>
<td>-</td>
<td>-27.85%</td>
<td>0.026141</td>
<td>-5.55%</td>
<td>-</td>
<td>-10.22%</td>
</tr>
<tr>
<td>SS w/out plate and extension</td>
<td>0.098641</td>
<td>336.88%</td>
<td>356.74%</td>
<td>-</td>
<td>0.134267</td>
<td>21.04%</td>
<td>75.27%</td>
<td>-</td>
<td>0.121116</td>
<td>10.14%</td>
<td>38.60%</td>
<td>-</td>
<td>0.029116</td>
<td>5.19%</td>
<td>11.38%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.20 v=0.8 r(0)=4π f=2</td>
<td>0.021517</td>
<td>-4.69%</td>
<td>-0.369%</td>
<td>-78.18%</td>
<td>0.106113</td>
<td>-4.33%</td>
<td>38.52%</td>
<td>-20.96%</td>
<td>0.086120</td>
<td>-21.68%</td>
<td>-1.44%</td>
<td>-28.89%</td>
<td>0.026152</td>
<td>-5.51%</td>
<td>0.045%</td>
<td>-10.18%</td>
</tr>
</tbody>
</table>
Despite the thin profile of the extension to the heat source, the extension continues to obstruct the flow near the heat source and causes the development of a large circulation region that encompasses the entire area beneath the extension. Hence, the heat removal from the upstream side surface of the heat source is negligible. The only advantage of this arrangement is that a significant amount heat is conducted to the extension from the heat source. This results in heat source temperatures that are significantly lower than those for a single rectangular heat source placed in the channel.

A number of lower oscillation displacement cases were first tested for a smaller clearance spacing. The motion of the plate did not produce any significant cooling effect at the top surface of the heated surface. Instead, the presence of the plate obstructs the flow. The lower velocities together with the subsequent circulation regions that develop downstream of the oscillation source result in the development of a much thicker momentum and thermal boundary layers on the top heat source surface. After increasing the oscillation frequency by a factor of 4 with no beneficial effects, the clearance spacing was increased by a factor of 2 allowing for twice the displacement of amplitude for the vibrating plate. With the higher displacement and clearance, the circulation regions dissipated much more rapidly due to the higher velocities and greater natural convection effects, thus allowing for some of the higher velocities caused by the plate motion to wash over the top heat source surface. However, the improvement in the heat transfer coefficient at the top heat source surface relative to standard natural convection was minimal, though there was some improvement relative to steady state with the plate fixed over the channel. Overall, the heat transfer coefficients are lower than those for a rectangular heat source cooled by standard natural convection.
Figure 6.34 Typical natural convection induced flow over extension plate geometry: (a) velocity field, (b) temperature field. $d=0.20 \ V=0.8\pi \ \omega=4\pi \ C=0.30$. 
6.5.3 Upstream Oscillation Source Modified Arrangement

Based on the velocity fields for the first two modified arrangements, the obstruction of the flow by the positioning of a fixed solid entity upstream of the heat source can not be easily overcome by the effects of the oscillations. This led to a third modified geometry where an oscillation source alone is placed upstream of the heated surface at the level of the heat source top surface. This arrangement is shown in Figure 6.36 while the geometric parameters used are given in Table 6.11.
Table 6.11 Dimensional Parameter Values for Upstream Oscillation Source

<table>
<thead>
<tr>
<th>Dimensional Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b=\text{L}_{\text{ref}} ) (length basis) = ( b )</td>
<td>0.25 in = 0.0635 m</td>
</tr>
<tr>
<td>BH = BL (block height = block length)</td>
<td>( b )</td>
</tr>
<tr>
<td>BT (board thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>CHL, CHU (channel heights)</td>
<td>2b</td>
</tr>
<tr>
<td>CLB (plate clearance)</td>
<td>0.30b</td>
</tr>
<tr>
<td>SPACE (spacing between block and plate)</td>
<td>0.20b</td>
</tr>
<tr>
<td>PLB (plate length)</td>
<td>0.50b</td>
</tr>
<tr>
<td>PLH (plate thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>TL (channel length)</td>
<td>14b</td>
</tr>
<tr>
<td>SL (starting length)</td>
<td>7.5b</td>
</tr>
</tbody>
</table>

**Dimensions Defined in Figure 6.37. (337x217 non uniform mesh generated)**

Figure 6.37 Dimensioning for Upstream Oscillation Source Investigation.
### Table 6.12a Summary of Average Dimensionless Heat Transfer Coefficients – Upstream Oscillation Source C=0.30

<table>
<thead>
<tr>
<th>Case C=0.30</th>
<th>Max Time Avg Dimn-less Temperature</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimn-less Avg Heat Transfer Coeff. Left</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimn-less Avg Heat Transfer Coeff. Top</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimn-less Avg Heat Transfer Coeff. Right</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>1.778400</td>
<td>-</td>
<td>-0.262%</td>
<td>0.122582</td>
<td>-</td>
<td>-3.22%</td>
<td>0.142945</td>
<td>-</td>
<td>3.637%</td>
<td>0.043147</td>
<td>-</td>
<td>0.776%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>1.783073</td>
<td>0.263%</td>
<td>-</td>
<td>0.126664</td>
<td>3.33%</td>
<td>0.000%</td>
<td>0.137928</td>
<td>-3.51%</td>
<td>-</td>
<td>0.042815</td>
<td>-0.770%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.10 v=0.2π ω=2π f =1</td>
<td>1.783312</td>
<td>0.276%</td>
<td>0.013%</td>
<td>0.126485</td>
<td>3.18%</td>
<td>-0.141%</td>
<td>0.140256</td>
<td>-1.88%</td>
<td>1.68%</td>
<td>0.043679</td>
<td>1.23%</td>
<td>2.02%</td>
</tr>
<tr>
<td>d=0.20 v=0.4π ω=2π f =1</td>
<td>1.783298</td>
<td>0.275%</td>
<td>0.013%</td>
<td>0.1250228</td>
<td>1.99%</td>
<td>-1.29%</td>
<td>0.153981</td>
<td>7.72%</td>
<td>11.63%</td>
<td>0.045871</td>
<td>6.31%</td>
<td>7.13%</td>
</tr>
</tbody>
</table>

### Table 6.12b Summary of Average Local Dimensionless Heat Transfer Coefficients – Upstream Oscillation Source C=0.30

<table>
<thead>
<tr>
<th>Case C=0.30</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 1</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 2</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 3</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 4</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.098725</td>
<td>-</td>
<td>-2.56%</td>
<td>0.134403</td>
<td>-</td>
<td>4.42%</td>
<td>0.121302</td>
<td>-</td>
<td>5.31%</td>
<td>0.029115</td>
<td>-</td>
<td>2.69%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.101323</td>
<td>2.63%</td>
<td>-</td>
<td>0.128711</td>
<td>-4.23%</td>
<td>-</td>
<td>0.115181</td>
<td>-5.04%</td>
<td>-</td>
<td>0.028350</td>
<td>-2.62%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.10 v=0.2π ω=2π f =1</td>
<td>0.101358</td>
<td>2.66%</td>
<td>0.035%</td>
<td>0.130743</td>
<td>-2.72%</td>
<td>1.57%</td>
<td>0.118241</td>
<td>-2.52%</td>
<td>2.65%</td>
<td>0.028583</td>
<td>-1.83%</td>
<td>0.623%</td>
</tr>
<tr>
<td>d=0.20 v=0.4π ω=2π f =1</td>
<td>0.100887</td>
<td>2.19%</td>
<td>-0.430%</td>
<td>0.146742</td>
<td>9.18%</td>
<td>14.00%</td>
<td>0.131678</td>
<td>8.55%</td>
<td>14.32%</td>
<td>0.023321</td>
<td>0.706%</td>
<td>3.42%</td>
</tr>
</tbody>
</table>
In order to be beneficial, the oscillation source must be positioned close enough to the heat source to have an impact on the velocity but not so close that its presence reduces the flow over the heat source. Hence a number of clearance spacings were investigated.

For the final clearance selected, two cases were run where two different oscillation displacement amplitudes were applied to the oscillation source for the same oscillation frequency, and the resulting temperature and velocity fields were determined for this upstream oscillation source arrangement. In general, as the oscillation source moves upward, angled high velocity flow between the lower right corner of the oscillation source and the upper left corner of the heat source turns towards the upper heat source surface. The flow over the top of the oscillation source turns down and sweeps across the top heat source surface. The flow squeezed between the heat and oscillation sources becomes nearly parallel to top the heat source surface. As the plate changes direction, a high velocity region develops between the oscillation source and the heat source and the interaction of this flow and the flow over the top of the heat source results in a circulation region. However, contrary to the other cases, this circulation region is far from the heat source. The influence of the oscillations is restricted to the top surfaces of the heat source. The magnitude of the velocity this flow produces near the heated surface relative to that produced by pure natural convection dictates whether significant cooling enhancement is achievable for a particular parameter set. For the lower amplitude case, the results show that the oscillations of the plate do cause the heat transfer coefficients at the top heat source surfaces to oscillate, but any cooling enhancement is negligible because the flow from the top of the oscillation source is not
strong enough to produce the velocity and velocities gradients near the top heat source surface that are required for heat transfer enhancement. Increasing the oscillation displacement amplitude resulted in flow over the top of the oscillation source that almost directly impinges on the top heat source surface. As a consequence, increased velocity gradients, thinner fluid and thermal boundary layers and higher heat transfer coefficients develop. For the parameter set \( d=0.20, V=0.40\pi, \omega=2\pi, C=0.30 \), a maximum 7% improvement in the average heat transfer coefficient was achieved on the top side of the heat source while a maximum of 10% improvement in the local heat transfer coefficient relative to standard natural convection was reached. This range of increase is comparable to that achieved with the positioning of the oscillation source over the heat source. However, the maximum temperature of the heat source is much lower due to the less constricting flow geometry.

(a) Figure 6.38 Typical natural convection induced flow over upstream oscillating plate geometry: (a) velocity field, \( d=0.20, V=0.4\pi, \omega=2\pi, C=0.30 \) (cont.).
**Figure 6.38** Typical natural convection induced flow over upstream oscillating plate geometry: (b) temperature field, $d=0.20 \ V=0.4\pi \ \omega=2\pi \ C=0.30$.

<table>
<thead>
<tr>
<th>Temperature Contour Plot</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>LEGEND</strong></td>
</tr>
<tr>
<td>---</td>
</tr>
<tr>
<td>0.9000E-01</td>
</tr>
<tr>
<td>0.7000E-01</td>
</tr>
<tr>
<td>0.5000E-01</td>
</tr>
<tr>
<td>0.3000E-01</td>
</tr>
<tr>
<td>0.1000E+00</td>
</tr>
<tr>
<td>0.0000E+00</td>
</tr>
<tr>
<td>0.1170E+01</td>
</tr>
<tr>
<td>0.1300E+01</td>
</tr>
<tr>
<td>0.1700E+01</td>
</tr>
</tbody>
</table>

**MINIMUM**
0.000000E+00

**MAXIMUM**
0.170003E+01

**TIME** 0.294E+01
**SCREEN LIMITS**
XMIN 0.555E+01
XMAX 0.105E+02
YMIN 0.260E-01
YMAX 0.419E+01
**FIDAP 8.7.2**
25 Oct 84
11:42:09

**Figure 6.39** Local dimensionless heat transfer coefficient as a function of dimensionless time: $d=0.20 \ V=0.4\pi \ \omega=2\pi \ C=0.30$. 

- left $y^*=2.68$
- top $x^*=7.75$
- top $x^*=8.25$
- right $y^*=2.68$
Based on the modified arrangements of the oscillation sources, improvement in the cooling effect can be achieved by positioning the oscillation source upstream of the heat source in a manner such that the obstruction of the flow over the heat source is minimized.

6.6 Conclusions

The results of this investigation clearly indicate that the strategic positioning of the oscillation sources in the vicinity of the heat source has practical potential as a means of enhancing laminar natural convection in a vertically oriented channel. The results demonstrate that the relative influence of the natural convection effects and the oscillation effects dictates the level of improvement that can be achieved. Important flow streams were found to be the natural convection induced flows, the oscillation induced flows, including flow entering and exiting from beneath the oscillation source as well as flow around the plate ends and top surfaces. When each of these effects as well as the inertia and the shear and pressure force flow mechanisms are able to exert influence, a “well mixed” flow field results. Under these conditions, any low velocity regions move with time and fluid streams from different regions of the channel and are forced to interact (i.e. high and low temperatures). This type of flow field leads to higher time averaged velocity gradients at the heated surfaces, thinner momentum boundary layers, thinner thermal boundary layers, improved heat transfer coefficients and lower heat source temperatures.

The results of the study also imply that the two-dimensional heat source geometry may better exploit the potential of the oscillation-induced cooling effect. Because of the
freer nature of the flow to the sides of the heat source top surface, the incoming and outgoing flow can take any number of paths to or from the top heated surface. Since the interaction of different flow streams is the basis of this cooling method, this characteristic is significant.

For the parameter cases studied significant improvement in the thermal conditions of the system, as measured by the time averaged heat transfer coefficients relative to standard natural convection, were produced by the oscillation-enhanced method. For the parameter range investigated, the cooling effect increased with oscillation displacement amplitude and frequency and decreased clearance spacing. A maximum enhancement in the local heat transfer coefficient of 50% was attained. The forced convection studies of Yang [62] and Fu [58-61] for an oscillation source placed upstream of the heat sources report increases in the heat transfer coefficient measured relative to unenhanced forced convection of up to 116%. Thus, the level of improvement in the heat transfer coefficient for the current investigation is reasonable.

Therefore, based on the information gathered in the current investigation, the use of discrete oscillation sources is a feasible method of enhancing pure natural convection cooling of discrete heat sources in a vertically oriented channel.
CHAPTER 7

NUMERICAL INVESTIGATION OF COMBINED METHOD

7.1 Introduction

Thus far this work has explored two methods of enhancing natural convection in a vertically oriented channel — the static method of alternate cross flow passages as described in Chapter 3 and the dynamic method of the strategic placement of the transverse oscillation sources discussed in Chapters 4, 5, and 6. The study results have shown that under the proper conditions, individually each of these methods has practical potential to improve the thermal conditions of a given system. In this chapter the feasibility of the use of the combination of these two methods is investigated.

Both the alternate cross flow passages and the transverse oscillations act to improve the cooling of a heat source by adjusting the flow near the heating elements. The alternate cross-flow passages open new flow paths to redirect the flow of cooler fluid to specific areas of need by solely altering the system geometry. The oscillations locally alter the flow distribution by actively creating new flow streams, forcing different flow streams to interact, and by increasing the velocities nearby the heat sources. If these two methods are used in tandem, the oscillations may help to promote the mixing of the cooler fluid streams created by the alternate cross-flow paths with fluid flow near the oscillation source and therefore the heat source. In this manner, the beneficial effects behind each of these individual alternative cooling techniques can be enhanced.

A numerical investigation of the combination of the static and dynamic enhancement methods is undertaken in the final portion of the current work. The use of
the combined cooling is tested for a number of different heat source and oscillation source geometries and arrangements. The simplest involves a plain channel containing a board with a constant heat flux surface and an opening. Later, the method is applied to systems consisting of two rectangular heat sources separated by a board opening with various positions of the oscillation source tested. Through such investigations, the cooling effects capable of being produced by this combined method are better understood.

7.2 Combined Method For Plain Channel Geometry

7.2.1 Problem Statement

The first investigation of this combined method involves a plain channel containing a heated board surface with an alternate cross flow passage formed by a board opening as shown in Figure 7.1. A constant and uniform heat flux is applied at \( y = 0 \). A two dimensional oscillating plate is then placed just over the opening at a distance \( C \) as defined in Figure 6.1. The motion of the plate is governed by Eqs. (6.1) and (6.2) as in all previous studies in Chapter 6. The standard boundary conditions with a constant heat flux at \( y = 0 \) as described in Section 6.3 and Section 2.4 are specified, and assumptions applied in the previous finite element investigations and discussed in Section 2.2 are again applied here. For this initial investigation, conduction in the board or heat source is neglected while that in the oscillation source is taken into account. The system and material property parameters used appear in Table 6.1 with the dimension parameters listed in Table 7.1. The length of the oscillating plate was purposely made longer than the opening to ensure that some of the flow through the opening is directed towards the
heated surface and is not just carried away with the main channel flow. For a fixed applied heat flux, the oscillation source displacement amplitude and frequency are varied. Tables 7.2 list the system parameters that were utilized.

As in the previous studies, the velocities and temperature distributions that result from this cooling method were determined through finite element methods using FIDAP©. In this case a graded 251x75 mesh was used for the model. The measures of the cooling effect, including the time-averaged average heat transfer coefficients on the upstream and downstream portions of the heated surface as well as the time averaged local heat transfer coefficients at the four positions indicated in red in Figure 7.1, are determined for each case. As with the other FIDAP studies these four positions are named Point 1 through Point 4 upstream to downstream (left to right in the figure). A comparison of results yields information as to the effectiveness of the use of the opening and the oscillations to bring cooler fluid to the closer heated surface.

Figure 7.1 Combined method – plain channel geometry layout.
Table 7.1 Dimensional Parameters for Combined Method-Plain Channel Geometry

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b(\text{channel height}) = L_{\text{ref}} )</td>
<td>0.50 in = 0.0125 m</td>
</tr>
<tr>
<td>( b(\text{channel height}) )</td>
<td>1b</td>
</tr>
<tr>
<td>( TL(\text{channel length}) )</td>
<td>7b</td>
</tr>
<tr>
<td>( BT(\text{board thickness}) )</td>
<td>0.1b</td>
</tr>
<tr>
<td>( PT(\text{plate thickness}) )</td>
<td>0.1b</td>
</tr>
<tr>
<td>( PL(\text{plate length}) )</td>
<td>1.0b</td>
</tr>
<tr>
<td>( HL(\text{opening width}) )</td>
<td>0.50b</td>
</tr>
</tbody>
</table>

*Dimensions defined in Figure 7.2

7.2.2 Analysis of Results – Combined Method Plain Channel Geometry

The presence of the opening provides an additional fluid stream that must interact with the natural convection induced flow, the flows caused by the oscillating plate, and the other flow effects described earlier. Thus, with this geometry some distinctive velocity fields and, hence, heat transfer attributes arise, compared with those of the previous investigations. In general, based on the results of this investigation, the application of the oscillations to the system described above has the potential to cause heat transfer enhancement. The manner in which these oscillations contribute to this enhancement is...
discussed in this section. A summary of the average heat transfer coefficient results for the cases investigated is provided in Table 7.2 with additional data given in Appendix G.

7.2.2.1 Steady State Solutions. Before describing the flow and heat transfer coefficients resulting from the oscillations, the steady state results can provide insight into the basic flow and temperature field characteristics and how the cooling methods can best be used to alter these flow and temperature fields. The steady state results have shown that without the plate fixed over the opening there is little flow through the opening as there is little pressure difference between the lower channel and the upper channel at this location. Though the placement of the plate in the channel does obstruct the flow causing much of the natural convection induced flow to be diverted over the top of the oscillation source leading to higher maximum temperatures, it also causes a larger pressure difference in the opening region and therefore some flow through the opening occurs. Due to this flow, the region of higher temperature shifts downstream. This may be an important effect if a specific area needs to be targeted for cooling. Comparisons of the steady state velocity and temperature distributions with and without the plate are shown in Figures 7.3 and 7.4, respectively.
Figure 7.3  Typical steady state velocity distributions for plain channel geometry: (a) without plate, (b) with plate.
Figure 7.4 Typical steady state temperature distributions for plain channel geometry: (a) without plate, (b) with plate.

7.2.2.2 Results For Oscillating Plate. The ability of the oscillations of an extended plate placed just above an opening to promote the development of velocity fields which more effectively remove heat from the vicinity of the heat source can be analyzed from results of this investigation. The regions immediately under and downstream as well as
those immediately upstream of the oscillation source were found to receive the most thermal benefit. Before discussing the results for specific parametric cases, some general observations on velocity, temperature, and corresponding heat transfer coefficients that occur in the channel as a result of the oscillations are reported. Again, because of the complex and parameter-dependent flow distributions and corresponding temperature fields, the accompanying CD contains animations of the velocity and temperature fields in the directory “oscillating plate over an opening in channel.”

**General Observations: Velocity, Temperature, and Heat Transfer Coefficients**

The most obvious difference in the flow caused by the opening/oscillation source arrangement is the flow through the opening. This flow periodically changes direction with the quantity and direction of this flow, not simply dependent on the direction of the motion of the plate, but on a combination of the natural convection induced flows, the effects of the oscillations, inertia, and the pressure and shear forces. The opening flow is capable of bringing cooler fluid from the lower channel right to the heat source and of expelling heated fluid into the lower channel. As a consequence of this flow, there is no longer a low velocity region that remains near the mid-length of the plate regardless of how high the oscillation parameters are as occurs for the other geometries investigated. The path the opening flow takes upon moving from the lower channel and exiting in the upper channel is dependent on the relative strengths of the natural convection effects, the oscillation induced flow around the ends of the plate and the opening flow. For small oscillation parameters, most of the opening inflow proceeds downstream to the downstream end of the plate due to the natural convection effects. For larger oscillation parameters fluid enters into the upper channel through both the upstream and downstream
ends of the oscillating plate. Similarly, when there is flow from the upper channel to the lower channel, for many of the lower oscillation parameter cases, most of the fluid downstream of the opening exits to the downstream end of the plate. However, most of the fluid above the opening and upstream of the opening exits the space through the opening. Only when the plate begins to come near the heated surface is some of the flow seen exiting through the upstream side of the moving plate. When the oscillation induced flows are higher in magnitude, fluid exits the region under the moving plate through the opening as well as through both the upstream and downstream ends of the plate. As the plate reaches its minimum position, flow exits both through the opening and into the main channel flow at both the upstream and downstream ends of the plate, leaving isolated low velocity regions in the spaces between the moving plate and upstream and downstream portions of the heated surface no matter the oscillation parameters. However, prior to reaching this condition, the velocities near this portion of the heated surface are quite high. The time period for which the velocities under the plate are low is much reduced from that with the solid board geometries.

The additional flow through the opening helps to improve the removal of heat from under the oscillating plate and near the heated surface to the sides of the opening, but also alters the flow patterns in both the upper and lower channels. When the opening flow moves from the lower channel to the upper channel, the flow rates and velocities are comparable to those caused by the natural convection and by other oscillation-induced flows. Because the opening flow generally induces the greatest fluid velocities under the moving plate, the opening flow better removes the heated fluid from under the heat source. However, because this inflow must then enter either the main channel or the
lower channel, the flow can also affects the downstream portions of the upper lower channels. When the heated fluid is expelled through the opening and into the lower channel, in addition to increasing the velocities in the vicinity of the opening and oscillation source, the opening flow reduces the amount of heat that travels downstream in the upper channel by carrying this heat into the lower channel. Heated fluid from under the plate that does exit to the upper channel is of a lower temperature than that without the opening since this fluid has been mixed with the cooler lower channel fluid. This is an additional means by which the opening/oscillation method improves the cooling.

In general, the downstream portion of the heated surface experiences a greater cooling effect due to the openings. The opening flow has a greater tendency to move downstream upon entering the main channel and thus to cool the downstream side of the heated surface due to opposing natural convection flow at the upstream end of the region under the plate. Hence, the velocities near the heated surface upstream of the plate are typically lower than those under and just downstream of the plate. Overall, there is little difference in the surface-averaged heat transfer coefficients along the upstream half of the heated surface with the oscillations though the local heat transfer coefficient at Point 1 drops by as much as 18% due to the decreased velocity at this location resulting from the opposing natural convection and opening flows. The beneficial effects of the oscillations are localized to the region between the oscillation source and the heated surface. An increase in the local heat transfer coefficient relative to the standard natural convection conditions as high as 51% can occur at Point 2, located just under the plate, while as much as a 29% increase can occur at Point 3. (Note that Point 3 has higher heat transfer coefficients.) The higher local velocities that can occur during portions of the cycle even
as the plate is nearing its minimum clearance position cause this effect. The overall average heat transfer coefficient of the downstream half of the channel was found to increase by as much as 20% for the parameters of this study. However, far downstream of the oscillation source, regions of increasingly high temperatures develop since, for a portion of an oscillation, the flow downstream of the opening in the upper channel is restricted due to the movement of a significant amount of fluid into the lower channel. (This effect can become significant for higher oscillation parameters.) In the vicinity of the oscillation source and opening, however, the supplementary flow though the opening improves the ability of the fluid to transport the heat from the heat source.

**Velocity Fields, Temperature Fields, and Heat Transfer Coefficients for Three Cases Investigated**

A few specific parameter case results are examined next to obtain a more detailed understanding of the flow and temperature fields that result. A lower frequency case is discussed first. This case has the parameters $d = 0.10$, $V = 0.2$, $\omega = 2$, $C = 0.15$. When the plate is at its minimum position, most of the flow is over the top of the oscillating plate and there is little flow through the opening. As the plate begins to more upward, some fluid enters the opening and is carried in a downstream direction due to the natural convection effects. Flow also enters the region through both plate ends. These opposing flows are clearly seen in Figure 7.5a. Because the opening flow turns downstream, the oscillation induced/natural convection inflow at the upstream end of the plate eventually dominates. Downstream, the opening flow dominates and a clockwise circulation region develops due to the opposing upward plate motion and the flow over the top of the moving plate. (See Figure 7.5b.) As the plate continues to move upward and the amount of opening flow increases, the circulation region detaches from the area near the end of
the plate. As the flow changes direction, most of the fluid under the plate flows to the lower channel via the opening, but most of the fluid downstream of the opening enters the main channel flow through the downstream end of the plate. (See Figure 7.5c.) Only when the plate moves closer to the heated surface and the flow is redirected by this surface is there flow out of the upstream end of the plate to the main channel. As the plate continues down, a circulation region develops upstream and then dissipates as the plate slows. As the plate nears its lowest position, outflow into both the main channel and into the opening occur both upstream and downstream. (See Figure 7.5d.)

The velocity field described above results in higher velocities and velocity gradients at some locations along the heated surface and lower values at others. The resulting temperature fields and heat transfer coefficients are described below. Due to the lower velocities upstream of the plate, the time averaged local heat transfer coefficient at Point 1 drops by 13% from its steady state value with no plate in the channel. While the local heat transfer coefficient at Point 2 increases by 13% over its steady state value with the plate, the oscillation and opening flows are not sufficient to cause improvement over the no plate steady state conditions. Just downstream of the opening, however, the flow rates and flow velocities due to the flow through the openings are much higher, resulting in higher temperature gradients downstream of the opening as seen in Figure 7.6. While the surface average heat transfer coefficient shows little change, the heat transfer coefficients at Points 3 and 4 increased by 23% and 28%, respectively, from steady state with the plate conditions, but they showed negligible change from the no plate steady state conditions. The variation in the heat transfer coefficient with time is plotted in Figure 7.11 and a bar graph of the time averaged local heat transfer coefficient at Point 3
is shown in Figure 7.12. Hence, for this parameter set, the cooling effect is not sufficient to surpass the pure natural convection cooling effects. The results of this parameter set also show that though the major effects of the oscillations occur in the immediate vicinity of the opening because of the significant flow through the opening, flow in both the upper and lower channels is affected by the oscillations.

Increasing the frequency of the oscillations resulted in increased flow complexity and stronger flow through the openings as is seen in the results for the case of $d = 0.10$, $V = 0.2\pi$, $\omega = 2\pi$, $C = 0.15$. For the minimum spacing position of the plate, a clockwise circulation region develops both to the side and over the top upstream corner of the oscillation source as well as just under the lower left corner of the board opening. (See Figure 7.7a.) As the motion of the plate turns upward, there is more significant flow through the opening to the upper channel. The upstream circulation region dissipates. A counter clockwise circulation region develops upstream, while a clockwise circulation region develops downstream of the oscillating plate. Because of the stronger opening flow, the direction of this flow is not angled downstream but is perpendicular to the moving plate, and the flow is cut into three main sections as seen in Figure 7.7b. As the plate slows down, the upstream circulation region moves slightly under the plate due to the stronger influence of the natural convection as the oscillation induced velocities decrease. The circulation region, the natural convection effects, and the opposing motion of the plate after it changes direction cause the opening flow to be angled downstream. (See Figure 7.7c.) The downstream circulation is carried out by this “opening flow,” and the exiting flow forms a high velocity stream to escape from under the plate. As the plate lowers and the presence of the heated surface begins to influence the flow, some of the
opening inflow leaves through the upstream end of the oscillating plate. A new clockwise upstream recirculation zone develops, and a new downstream counterclockwise circulation zone is formed around the ends of the plate. (See Figure 7.7d.) Also, a circulation briefly develops at the upstream side of the opening as the direction of the flow through the opening begins to change. The most significant difference in this velocity distribution compared to that for the lower frequency is the significant flow exiting from under the plate to the main channel at the upstream plate end during certain stages of the cycle. This flow develops because with the higher oscillation parameters, the oscillation induced opening flow is sufficient to overcome the natural convection effects at some times. This flow carries the lower temperature opening fluid through this region and, therefore, helps to cool the portion of the heated surface just upstream of the oscillation source.

As a result of the greater flow through the openings and the higher velocities involved, this set of system parameters produced a more significant improvement in thermal conditions in close proximity to the oscillation source and the opening. The higher flow rates bring cooler fluid to the heat source. Since the inflow is angled slightly downstream for a good part of a plate cycle, the greatest cooling effect is expected to occur downstream and does. (See Figure 7.8.) For these parameters, Points 3 and 4 show a 20% and 21% increase in the time averaged local heat transfer coefficient, respectively. However, due to the more significant upstream flow even Point 2 experiences a 19% increase in the heat transfer coefficient above the steady state no plate results. (The heat transfer coefficient is about 30% lower than at Point 3.) The time variation of the local
heat transfer coefficient is depicted in Figure 7.11 and bar graphs comparing the heat transfer coefficient at Point 3 are given in Figure 7.12.

Finally, the results for a case with double the frequency of the previous case are described where \(d=0.10, \ V=0.4\pi, \ \omega=4\pi, \ C=0.15\). The flow through the opening in this case is so great that the flow in the downstream portion of the upper channel is severely constricted which is clearly seen in the “velocity” animation. At the minimum spacing position of the plate, a clockwise circulation region develops both to the side and over the top corner of the upstream end of the vibrating plate and just under the lower left corner of the board opening unlike the previous case. A counter clockwise circulation region develops both to the side and over the top corner of the downstream end of the oscillation source and just under the lower right corner of the board opening. (See Figure 7.9a.) As the plate moves up, there is significant flow through the opening to the upper channel and the circulation regions dissipate. A counter clockwise circulation region develops upstream while a clockwise circulation region develops downstream of the oscillating plate. The flow enters the space between the heat source and oscillation source in a sweeping curve. The division between these flows and the flow through the opening is marked by a region of low velocity. In response to these high velocities, a large downstream counter clockwise region forms. (See Figure 7.9b.) As the plate slows down, the circulation regions nearby the oscillation source move slightly under the plate since the influence of natural convection becomes stronger as the plate velocity decreases. As the plate changes direction, the opening flow becomes angled downstream due to the opposing motion of the plate and the presence of the circulation region upstream and the natural convection effects. The downstream circulation is carried out by this “opening
flow,” and the exiting flow forms a high velocity stream to escape from under the plate. (See Figure 7.9c.) As the plate lowers and the presence of the heated surface begins to influence the flow, some of the opening inflow leaves through the upstream end of the oscillating plate. A new clockwise upstream recirculation zone develops and a new downstream counterclockwise circulation zone is formed around the ends of the plate. (See Figure 7.9d.) Also, a circulation zone briefly develops at the upstream side of the opening as the direction of the flow through the opening begins to change.

For this set of parameters, the velocities and the flow rates through the opening are much more significant than in any of the other cases investigated. This results in a more beneficial flow pattern that significantly alters the temperature field as seen in the Figure 7.10. The effect of this flow pattern is evidenced by the heat transfer coefficient results. (See Figures 7.11 and 7.12.) Under these conditions, Point 2 experiences a 51% rise in the time averaged local heat transfer coefficient relative to pure natural convection without the plate. The downstream portion of the channel experiences a 16% increase in the time averaged heat transfer coefficient, and Points 3 and 4 have local heat transfer coefficients of about 29% higher than that at steady state with no plate over the opening. Therefore, despite the higher maximum temperatures and the restricted flow that occurs far downstream of the opening, in the vicinity of the opening, the cooling effect is significant.
Figure 7.5  Velocity distribution for combined method plain channel with opening 
d=0.10 V=0.20 ω=2: (a) $\bar{I}_F=11.5$, (b) $\bar{I}_F=12.6$(cont.).
Figure 7.5 Velocity distribution for combined method plain channel with opening
d=0.10 V=0.20 \omega=2: (c) \hat{t}_f=10.2, (d) \hat{t}_f=10.9.
Figure 7.6 Temperature distribution for combined method plain channel with opening \(d=0.10\) \(\nu=0.20\) \(\omega=2\): (a) \(T_F=11.5\), (b) \(T_F=12.6\) (cont.).
Figure 7.6 Temperature distribution for combined method plain channel with opening $d=0.10$, $V=0.20$, $\omega=2$: (c) $\tilde{t}_F=10.2$, (d) $\tilde{t}_F=10.9$. 
Figure 7.7  Velocity distribution for combined method plain channel with opening $d=0.10$ $d=0.10$ $V=0.20\pi$ $\omega=2\pi$: (a) $t_f=6.75$, (b) $t_f=6.90$ (cont.).
Figure 7.7  Velocity distribution for combined method plain channel with opening 
d=0.10 d=0.10 V=0.20π ω=2π: (c) \( t_p = 6.1875 \), (d) \( t_p = 6.6125 \).
Figure 7.8 Temperature distribution for combined method plain channel with opening $d=0.10$ $V=0.20\pi$ $\omega=2\pi$: (a) $t_F=6.75$, (b) $t_F=6.90$ (cont.).
Figure 7.8 Temperature distribution for combined method plain channel with opening $d=0.10$, $V=0.20\pi$, $\omega=2\pi$: (c) $t_F = 6.1875$, (d) $t_F = 6.6125$. 
Figure 7.9 Velocity distribution for combined method plain channel with opening $d=0.10$, $V=0.40\pi$, $\omega=4\pi$: (a) $\tilde{t}_p=3.375$, (b) $\tilde{t}_p=3.50$ (cont.).
**Figure 7.9** Velocity distribution for combined method plain channel with opening $d=0.10$ $V=0.40\pi$ $\omega=4\pi$: (c) $\tilde{t}_f=3.125$, (d) $\tilde{t}_f=3.2875$. 
Figure 7.10 Temperature distribution for combined method plain channel with opening $d=0.10$ $V=0.40\pi$ $\omega=4\pi$: (a) $\bar{t}_F=3.375$, (b) $\bar{t}_F=3.50$ (cont.)
Figure 7.10 Temperature distribution for combined method plain channel with opening $d=0.10$ $V=0.40\pi \phi=4\pi$: (c) $\tilde{t}_f=3.125$, (d) $\tilde{t}_f=3.2875$. 
Figure 7.11 Variation in the heat transfer coefficient as a function of time plain channel with opening: (a) $d=0.10 \ V=0.20 \ \omega=2$, (b) $d=0.10 \ V=0.20\pi \ \omega=2\pi$ (cont.).
Figure 7.11 Variation in the heat transfer coefficient as a function of time plain channel with opening: (c) \( d=0.10 \ V=0.40\pi \ \phi=4\pi \).
Figure 7.12 Time-averaged local heat transfer coefficients at Point 3: (a) d=0.05, (b) d=0.10.
Table 7.2a Summary of Average Heat Transfer Coefficient Results – Combined Method - Plain Channel Geometry C=0.15

<table>
<thead>
<tr>
<th>Case</th>
<th>Time Avg Max Dimensionless Temperature</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Average Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimensionless Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.401183</td>
<td>-</td>
<td>-3.69%</td>
<td>0.055342</td>
<td>-</td>
<td>60.38%</td>
<td>0.030738</td>
<td>-</td>
<td>-10.91%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.416659</td>
<td>3.83%</td>
<td>-</td>
<td>0.053136</td>
<td>-3.98%</td>
<td>-</td>
<td>0.034506</td>
<td>12.25%</td>
<td>-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.05 v=0.10, ( \omega = 2 ), ( f = \frac{1}{\pi} )</td>
<td>0.416670</td>
<td>3.86%</td>
<td>0.024%</td>
<td>0.053103</td>
<td>-4.04%</td>
<td>50.89%</td>
<td>0.031780</td>
<td>3.38%</td>
<td>-7.90%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.05 v=0.314, ( \omega = 2\pi ), ( f = 1 )</td>
<td>0.415994</td>
<td>3.69%</td>
<td>-0.138%</td>
<td>0.053056</td>
<td>-4.130%</td>
<td>53.76%</td>
<td>0.031337</td>
<td>1.94%</td>
<td>-9.18%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.10 v=0.20, ( \omega = 2 ), ( f = \frac{1}{\pi} )</td>
<td>0.417860</td>
<td>4.15%</td>
<td>0.310%</td>
<td>0.052728</td>
<td>-4.724%</td>
<td>52.80%</td>
<td>0.031170</td>
<td>1.40%</td>
<td>-9.66%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.10 v=0.628, ( \omega = 2 \pi ), ( f = 1 )</td>
<td>0.437071</td>
<td>8.94%</td>
<td>4.92%</td>
<td>0.052453</td>
<td>-5.22%</td>
<td>52.01%</td>
<td>0.036424</td>
<td>18.49%</td>
<td>5.56%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.10 v=1.256, ( \omega = 4 \pi ), ( f = 2 )</td>
<td>0.426320</td>
<td>6.26%</td>
<td>2.34%</td>
<td>0.053510</td>
<td>-3.31%</td>
<td>55.04%</td>
<td>0.035831</td>
<td>16.56%</td>
<td>3.84%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 7.2b Summary of Local Heat Transfer Coefficient Results – Combined Method - Plain Channel Geometry C=0.15

<table>
<thead>
<tr>
<th>Case</th>
<th>Time Avg Dimm-less Heat Transfer Coeff. Point 1</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimm-less Heat Transfer Coeff. Point 2</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimm-less Heat Transfer Coeff. Point 3</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Time Avg Dimm-less Heat Transfer Coeff. Point 4</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.036916</td>
<td>-</td>
<td>14.30%</td>
<td>0.035880</td>
<td>-</td>
<td>8.58%</td>
<td>0.049827</td>
<td>-</td>
<td>20.79%</td>
<td>0.039633</td>
<td>-</td>
<td>31.12%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.032297</td>
<td>-12.51%</td>
<td>-</td>
<td>0.033043</td>
<td>-7.90%</td>
<td>-</td>
<td>0.041249</td>
<td>-17.216%</td>
<td>-</td>
<td>0.030225</td>
<td>-23.73%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.05 v=0.10, ( \omega = 2 ), ( f = \frac{1}{\pi} )</td>
<td>0.032170</td>
<td>-12.85%</td>
<td>-0.393%</td>
<td>0.034192</td>
<td>-4.70%</td>
<td>3.47%</td>
<td>0.043295</td>
<td>-13.108%</td>
<td>4.96%</td>
<td>0.033175</td>
<td>-16.29%</td>
<td>9.75%</td>
</tr>
<tr>
<td>d=0.05 v=0.314, ( \omega = 2\pi ), ( f = 1 )</td>
<td>0.032163</td>
<td>-12.87%</td>
<td>-0.414%</td>
<td>0.034482</td>
<td>-3.89%</td>
<td>4.35%</td>
<td>0.042845</td>
<td>-14.012%</td>
<td>3.870%</td>
<td>0.034121</td>
<td>-13.90%</td>
<td>12.89%</td>
</tr>
<tr>
<td>d=0.10 v=0.20, ( \omega = 2 ), ( f = \frac{1}{\pi} )</td>
<td>0.031857</td>
<td>-13.70%</td>
<td>-1.36%</td>
<td>0.037409</td>
<td>4.26%</td>
<td>13.21%</td>
<td>0.050777</td>
<td>1.908%</td>
<td>23.10%</td>
<td>0.038946</td>
<td>-1.73%</td>
<td>28.85%</td>
</tr>
<tr>
<td>d=0.10 v=0.628, ( \omega = 2 \pi ), ( f = 1 )</td>
<td>0.029940</td>
<td>-18.89%</td>
<td>-7.29%</td>
<td>0.042905</td>
<td>19.58%</td>
<td>29.84%</td>
<td>0.059981</td>
<td>20.379%</td>
<td>45.41%</td>
<td>0.047993</td>
<td>21.09%</td>
<td>58.78%</td>
</tr>
<tr>
<td>d=0.10 v=1.256, ( \omega = 4 \pi ), ( f = 2 )</td>
<td>0.030833</td>
<td>-16.47%</td>
<td>-4.53%</td>
<td>0.054263</td>
<td>51.23%</td>
<td>64.22%</td>
<td>0.064748</td>
<td>29.946%</td>
<td>56.97%</td>
<td>0.051199</td>
<td>29.18%</td>
<td>69.39%</td>
</tr>
</tbody>
</table>
7.2.3 Conclusions

The results of this investigation of the use of the oscillating plate above an opening in the heat source indicate a strong potential for the combined alternate cross-flow passage-transverse oscillation source method. The oscillations of a plate just above an opening can lead to significant flow through the opening. When the opening flow is from the lower channel to the upper channel, the path the fluid takes to the main channel depends on the relative strengths of the natural convection and the oscillations. The opening flow tends to contribute more to the cooling of the downstream end of the heat source due to the opposition of the natural convection flow. When the opening flow is from the upper channel to the lower channel, most of the fluid upstream of the downstream edge of the opening in the lower oscillation parameters exits through the opening and exits both through the opening as well as through the upstream and downstream regions under the plate for higher oscillation parameters. When the plate becomes sufficiently close to the heated surface for any oscillation parameters, there is flow into the opening and into the upper channel at both the upstream and downstream sides of the plate. The thermal benefits that result from these flows caused by the opening of a cross-flow passage are clear for the immediate vicinity of the oscillation source and opening. However, the flow through the opening has consequences outside of the opening/plate region, and these consequences may not always be beneficial. Downstream of the opening there may be significant flow constriction due to the flow rate oscillations as fluid periodically enters and exits the upper channel. Areas of high temperature can develop far downstream of the opening. This is not of concern if the region that has been targeted for cooling is
nearby the opening and oscillation source for it is in this region that the study results show this method has strong capabilities.

7.3 Combined Method with Rectangular Heat Sources

With the conclusions drawn from the previous study, further exploration of the use of the oscillations together with the alternate cross flow passages is certainly warranted. In this section the combined cooling method approach is applied to a two-dimensional rectangular heat source geometry. The system investigated consists of a two-heat source array in a vertical channel with a board opening as shown in Figure 7.13. The oscillation source is positioned at various locations near the heat sources. The positions chosen were those thought to have the most potential to promote the mixing of the fluid in the opening region, to draw fluid into the opening, or to assist in the transport of the opening fluid elsewhere in the channel while causing minimal flow blockages. Brief parametric studies were performed to identify any cooling effect.

![Diagram of heat sources and oscillation source](image)

**Figure 7.13** General configuration for the two block studies.
7.3.1 Problem Statement

A number of oscillation source arrangements were tested in this investigation to uncover which arrangement best takes advantage of the opening and oscillation flow characteristics for the two rectangular heat source array. The specific geometries used are discussed later in this section. In these studies, a constant volumetric heat generation rate was applied to each of the two heat sources. The oscillation source, board, and heat source are modeled as solid entities. The conduction in these solids is accounted for through the assignment of the proper thermal conductivity parameters. As in the studies with rectangular heat sources in Sections 6.4 and 6.5, a “second” channel that does not contain any heat sources is also included to better model the effects of the heat flow through the bottom surfaces of the heat sources. The specific parameter values used are listed in Table 6.3. The general assumptions from Section 2.2 are applied to this system. In addition, the standard inlet, outlet, and surface boundary conditions used in the finite element investigations including conduction in the solids in Section 3.4 or as discussed in Section 2.4 are specified. Due to the complexities of the models involved, parametric studies for a limited number of oscillation source displacement amplitudes and frequencies and in a few cases the clearance, C, were then performed for a fixed applied heat generation rate (Grashof number) so that the effects of these parameters on the potential cooling effect could be investigated. Animations of the velocity and temperature fields are provided in the CD in the folder “oscillating plate with two blocks.”

After generating, finite element models for each of the arrangements to be described later, the velocity and temperature fields in each case were solved for using FIDAP© until a transient steady state was attained The local heat transfer coefficients at
the four points marked in red in the figures of each arrangement and the surface averaged heat transfer coefficients along the heat sources were calculated at each time step and then averaged over time, providing the information necessary to investigate the consequences of the two-dimensional heat source and oscillation source geometries. Comparisons were then made to steady state conditions with the oscillation source fixed, with no oscillation source, and with no oscillation source and opening.

7.3.2 Study Results — Combined Method with Rectangular Heat Sources

Before discussing the effects of the oscillations, the different geometric cases are introduced and a comparison of the steady state average temperatures and heat transfer coefficients are provided. A brief description of the oscillation source arrangements as well as a summary of the steady state results are provided in Table 7.3. As the aim of this study is to determine how to improve the thermal conditions over steady state conditions, the temperature fields at steady state with a board opening and steady state without the board are shown in Figure 7.14 for a qualitative evaluation so that the regions in the channel that most need improvement can be identified.

Based on the steady state results, it was decided that certain arrangements would not be effective at cooling in the manner described above because of their elevated temperatures or low heat transfer coefficients. One such arrangement was Case A - the oscillation source over Block 2. Besides the elevated temperatures of Block 2, the arrangement obstructs the opening flow from impinging on the top surface of the second heat source. The other was Case B - the oscillation sources over both heat sources, where the oscillation sources obstruct the flow over both the heat sources and divert the opening flow away from the top of the heat sources. Of the remaining cases examined, the
upstream oscillation source case produced the lowest heat source temperatures due to the lack of flow constriction because of the upstream plate position. The plate over the first or upstream heating element had the highest side surface heat transfer coefficient. For this geometry at steady state conditions, the flow that was diverted over the top of the fixed oscillation source and the flow patterns that developed near the downstream end of the plate helped to channel the "opening" flow from the cavity between the heat sources towards the downstream heat source. Hence, the average temperature of the downstream heat source was lower than that of the upstream heat source, and the heat transfer coefficient at the upstream side of the second heat source was 34% greater than it was without the plate over the first heat source. The fixed plate located above the opening at the level of the top of the heat sources appeared to direct the opening flow towards the downstream heat source as well. While the geometry for the heat source located in the cavity between the openings resulted in higher heat source temperatures, there appears to be potential for the oscillations of this plate to facilitate mixing the flow through the opening with the higher temperature fluid near the heat source surfaces. Because of this, further study of the geometry was also performed.
<table>
<thead>
<tr>
<th>Case</th>
<th>Vel Max</th>
<th>TBlock1</th>
<th>TBlock2</th>
<th>HB1L</th>
<th>HB1T</th>
<th>HB1R</th>
<th>HB2L</th>
<th>HB2T</th>
<th>HB2R</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard – No Plate</td>
<td>2.862</td>
<td>1.758109</td>
<td>1.957616</td>
<td>0.124566</td>
<td>0.135164</td>
<td>0.048191</td>
<td>0.120075</td>
<td>0.083777</td>
<td>0.035095</td>
</tr>
<tr>
<td>No plate or Hole</td>
<td>2.497</td>
<td>1.880584</td>
<td>1.968683</td>
<td>0.130303</td>
<td>0.145678</td>
<td>0.023270</td>
<td>0.055867</td>
<td>0.136203</td>
<td>0.039005</td>
</tr>
<tr>
<td>A - Plate Over Block 2</td>
<td>2.590</td>
<td>1.886680</td>
<td>2.380601</td>
<td>0.117432</td>
<td>0.128518</td>
<td>0.036093</td>
<td>0.068196</td>
<td>0.096565</td>
<td>0.029101</td>
</tr>
<tr>
<td>B - Plates Over Blocks 1 and 2</td>
<td>2.871</td>
<td>1.880584</td>
<td>1.968683</td>
<td>0.105112</td>
<td>0.116545</td>
<td>0.046737</td>
<td>0.121497</td>
<td>0.081844</td>
<td>0.027813</td>
</tr>
<tr>
<td>C - Plate Upstream of Blocks</td>
<td>2.841</td>
<td>1.731758</td>
<td>1.883230</td>
<td>0.134171</td>
<td>0.126314</td>
<td>0.049318</td>
<td>0.130771</td>
<td>0.083189</td>
<td>0.034831</td>
</tr>
<tr>
<td>D - Plate Over Hole</td>
<td>2.665</td>
<td>1.792665</td>
<td>2.283827</td>
<td>0.128430</td>
<td>0.141083</td>
<td>0.039604</td>
<td>0.073780</td>
<td>0.098732</td>
<td>0.037148</td>
</tr>
<tr>
<td>E - Plate Over Hole Near Top Of Heat Source</td>
<td>2.898</td>
<td>1.816176</td>
<td>2.115428</td>
<td>0.119396</td>
<td>0.130020</td>
<td>0.045639</td>
<td>0.103948</td>
<td>0.079454</td>
<td>0.033440</td>
</tr>
<tr>
<td>F - Plate Over Block 1</td>
<td>3.015</td>
<td>1.760370</td>
<td>1.711338</td>
<td>0.111123</td>
<td>0.128878</td>
<td>0.049605</td>
<td>0.161337</td>
<td>0.077895</td>
<td>0.034767</td>
</tr>
</tbody>
</table>

Geometry in Figure 7.13. L, R and T refer to the left, right and top surfaces of block 1 (B1) or block 2 (B2).

**Figure 7.14** Temperature distributions at steady state: (a) without plate, (b) without plate or hole.
7.3.2.1 Oscillating Plate Upstream of Two Blocks. The upstream oscillation source geometry shown in Figure 7.15 is much like the arrangement used for the modified heat source geometry in the single block study. Because of the lack of flow constriction, studies of this arrangement were continued to determine if the oscillations could be used to provide any further heat transfer enhancement. The specific parameter cases investigated are listed in Tables 7.5 and 7.6 with the geometric conditions in Table 7.4. Lower dimensionless displacements of 0.05 and 0.10 were tested but showed little effect on the velocity and temperature distribution at the heat source. Even with frequencies increasing from $2\pi$ and $8\pi$ for cases with displacement of 0.20, minimal changes in the velocity were noted only at the upstream upper corner of the first heat source. (See Figure 7.19.) The nearly constant heat transfer coefficients as a function of time plotted in Figure 7.17 and Figure 7.18 demonstrate the negligible effect that the oscillation source has on the heat transfer at the heat sources. The higher oscillation induced velocities were confined to the vicinity of the oscillation source particularly for the higher frequency cases. Either larger displacements or closer positioning to the heat source may be required for this arrangement to produce a beneficial cooling effect.

![Figure 7.15 Oscillating plate upstream of two blocks - geometry layout.](image-url)
Table 7.4 Dimensional Parameter Values for Upstream Oscillation Source

<table>
<thead>
<tr>
<th>Dimensional Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>b = L_{ref} (length basis)</td>
<td>0.25 in = 0.0635 m</td>
</tr>
<tr>
<td>BL = BH (block length and height)</td>
<td>b</td>
</tr>
<tr>
<td>BT (board thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>CHL, CHU (channel heights)</td>
<td>2b</td>
</tr>
<tr>
<td>CLB (plate clearance)</td>
<td>0.15b</td>
</tr>
<tr>
<td>SPACE (spacing between blocks)</td>
<td>0.50b</td>
</tr>
<tr>
<td>S2 (spacing between plate and first block)</td>
<td>1.00b</td>
</tr>
<tr>
<td>HW (width of the hole)</td>
<td>0.80b</td>
</tr>
<tr>
<td>PLB (plate length)</td>
<td>0.75b</td>
</tr>
<tr>
<td>PLH (plate thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>TL (channel length)</td>
<td>14b</td>
</tr>
<tr>
<td>SL (location of the first block)</td>
<td>5.5b</td>
</tr>
</tbody>
</table>

** Dimensions Defined in Figure 7.16 (337x217 mesh generated)**

Figure 7.16 Oscillating plate upstream of two blocks - geometry definitions.
**Figure 7.17** Local dimensionless heat transfer coefficient as a function of time $d=0.20$, $V=1.6\pi$, $\omega=8\pi$: (a) heat source 1, (b) heat source 2.

**Figure 7.18** Time-averaged local heat transfer coefficients at heat source 1 Point 1.
Figure 7.19 Typical velocity distribution and temperature field for upstream oscillation source: $d=0.20, V=0.8\pi, \omega=4\pi$: (a) velocity, (b) temperature.
### Table 7.5a Summary of Average Heat Transfer Coefficient Results – Upstream Oscillating Source – Two Block Geometry – Block 1

<table>
<thead>
<tr>
<th>C = 0.30 Case</th>
<th>Max Time</th>
<th>Dimn-less Temp.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimn-less Avg Heat Transfer Coeff. Left</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimn-less Avg Heat Transfer Coeff. Top</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimn-less Avg Heat Transfer Coeff. Right</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>1.758109</td>
<td>-</td>
<td>1.52%</td>
<td>4.82%</td>
<td>-</td>
<td>-7.15%</td>
<td>4.40%</td>
<td>-</td>
<td>7.00%</td>
<td>-7.21%</td>
<td>0.048191</td>
<td>-</td>
<td>2.78%</td>
<td>-7.21%</td>
<td>0.048191</td>
<td>-</td>
<td>2.78%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>1.731758</td>
<td>-1.49%</td>
<td>6.25%</td>
<td>7.71%</td>
<td>-</td>
<td>2.96%</td>
<td>-6.54%</td>
<td>-</td>
<td>-13.29%</td>
<td>0.049318</td>
<td>2.33%</td>
<td>-</td>
<td>111.94%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS without Plate and Hole</td>
<td>1.847226</td>
<td>5.06%</td>
<td>6.68%</td>
<td>-</td>
<td>4.60%</td>
<td>-2.88%</td>
<td>7.77%</td>
<td>15.33%</td>
<td>-</td>
<td>0.023270</td>
<td>-51.71%</td>
<td>-52.81%</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20, v=0.8 π</td>
<td>1.73176</td>
<td>-1.49%</td>
<td>0.000%</td>
<td>-6.25%</td>
<td>9.98%</td>
<td>-1.60%</td>
<td>1.31%</td>
<td>-10.62%</td>
<td>-4.35%</td>
<td>-17.07%</td>
<td>0.049460</td>
<td>2.63%</td>
<td>0.288%</td>
<td>112.55%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20, v=1.6 π</td>
<td>1.73176</td>
<td>-1.49%</td>
<td>0.000%</td>
<td>-6.25%</td>
<td>8.70%</td>
<td>0.92%</td>
<td>3.91%</td>
<td>9.17%</td>
<td>-2.80%</td>
<td>-5.78%</td>
<td>0.049397</td>
<td>2.50%</td>
<td>0.161%</td>
<td>112.28%</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7.5b Summary of Local Heat Transfer Coefficient Results – Upstream Oscillating Source – Two Block Geometry – Block 1

<table>
<thead>
<tr>
<th>C = 0.3 Case</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 1</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 2</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.099936</td>
<td>-</td>
<td>-9.88%</td>
<td>-3.90%</td>
<td>0.133256</td>
<td>-</td>
<td>8.85%</td>
<td>-5.49%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.110903</td>
<td>10.97%</td>
<td>-</td>
<td>6.63%</td>
<td>0.122413</td>
<td>-8.13%</td>
<td>-</td>
<td>-13.18%</td>
</tr>
<tr>
<td>SS without Plate and Hole</td>
<td>0.104000</td>
<td>4.06%</td>
<td>-6.22%</td>
<td>-</td>
<td>0.141000</td>
<td>5.81%</td>
<td>15.18%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.20 v=0.8π ω = 4π f = 2</td>
<td>0.109636</td>
<td>9.70%</td>
<td>-1.14%</td>
<td>5.42%</td>
<td>0.113879</td>
<td>-14.54%</td>
<td>-6.97%</td>
<td>-19.23%</td>
</tr>
<tr>
<td>d=0.20 v=1.6π ω = 8π f = 4</td>
<td>0.115611</td>
<td>15.68%</td>
<td>4.24%</td>
<td>11.16%</td>
<td>0.111008</td>
<td>-16.69%</td>
<td>-9.31%</td>
<td>-21.271%</td>
</tr>
<tr>
<td>C = 0.3 Case</td>
<td>Time Avg Dimn-less Heat Transfer Coeff. Point 3</td>
<td>% Diff from SS without Plate</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Hole</td>
<td>Time Avg Dimn-less Heat Transfer Coeff. Point 4</td>
<td>% Diff from SS without Plate</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Hole</td>
</tr>
<tr>
<td>SS without plate</td>
<td>0.109500</td>
<td>-</td>
<td>7.31%</td>
<td>-8.70%</td>
<td>0.043456</td>
<td>-3.251%</td>
<td>378.56%</td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.102041</td>
<td>-6.81%</td>
<td>-</td>
<td>-14.96%</td>
<td>0.044916</td>
<td>3.36%</td>
<td>-</td>
<td>394.66%</td>
</tr>
<tr>
<td>SS without Plate and Hole</td>
<td>0.120000</td>
<td>9.58%</td>
<td>17.60%</td>
<td>-</td>
<td>0.009080</td>
<td>-79.10%</td>
<td>-79.784%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.20 v=0.8π ω = 4π f = 2</td>
<td>0.100309</td>
<td>-8.39%</td>
<td>-1.69%</td>
<td>-16.40%</td>
<td>0.044587</td>
<td>2.60%</td>
<td>-0.733%</td>
<td>391.03%</td>
</tr>
<tr>
<td>d=0.20 v=1.6π ω = 8π f = 4</td>
<td>0.102381</td>
<td>-6.52%</td>
<td>0.33%</td>
<td>-14.68%</td>
<td>0.045178</td>
<td>3.96%</td>
<td>0.584%</td>
<td>397.56%</td>
</tr>
<tr>
<td>Case</td>
<td>Time Avg Dimn-less Temp.</td>
<td>% Diff from SS without Plate</td>
<td>% Diff from SS with Plate</td>
<td>Time Avg Dimn-less Avg Heat Transfer Coeff. Left</td>
<td>% Diff from SS without Plate and Hole</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Hole</td>
<td>Time Avg Dimn-less Avg Heat Transfer Coeff. Top</td>
</tr>
<tr>
<td>------</td>
<td>--------------------------</td>
<td>------------------------------</td>
<td>--------------------------</td>
<td>---------------------------------</td>
<td>-------------------------------------------</td>
<td>-------------------------------</td>
<td>-------------------------------</td>
<td>---------------------------------</td>
</tr>
<tr>
<td>SS without plate</td>
<td>1.957616</td>
<td>-</td>
<td>3.95%</td>
<td>-16.85%</td>
<td>0.120075</td>
<td>-</td>
<td>-8.17%</td>
<td>114.93%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>1.883230</td>
<td>-3.80%</td>
<td>-</td>
<td>-20.01%</td>
<td>0.130771</td>
<td>8.90%</td>
<td>0.000%</td>
<td>134.07%</td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>2.354531</td>
<td>20.27%</td>
<td>25.02%</td>
<td>-</td>
<td>0.055867</td>
<td>-53.43%</td>
<td>-57.279%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.20 v=0.8 ( \nu = 4 \Rightarrow f = 2 )</td>
<td>1.883565</td>
<td>-3.78%</td>
<td>0.018%</td>
<td>-20.00%</td>
<td>0.131306</td>
<td>9.35%</td>
<td>0.409%</td>
<td>135.03%</td>
</tr>
<tr>
<td>d=0.20 v=1.6 ( \nu = 8 \Rightarrow f = 4 )</td>
<td>1.883565</td>
<td>-3.78%</td>
<td>0.018%</td>
<td>-20.00%</td>
<td>0.131542</td>
<td>9.55%</td>
<td>0.589%</td>
<td>135.45%</td>
</tr>
</tbody>
</table>
Table 7.6b Summary of Local Heat Transfer Coefficient Results – Upstream Oscillating Source – Two Block Geometry – Block 2

<table>
<thead>
<tr>
<th>C = 0.3 Case</th>
<th>Time Avg Dimn-less Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimn-less Heat Transfer Coeff.</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.109042</td>
<td>-</td>
<td>-10.12%</td>
<td>306.68%</td>
<td>0.076637</td>
<td>-</td>
<td>0.193%</td>
<td>-41.72%</td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.121331</td>
<td>11.27%</td>
<td>-</td>
<td>352.52%</td>
<td>0.076489</td>
<td>-0.193%</td>
<td>-</td>
<td>-41.83%</td>
<td></td>
</tr>
<tr>
<td>SS without Plate and Hole</td>
<td>0.026812</td>
<td>-75.41%</td>
<td>-77.90%</td>
<td>-</td>
<td>0.131502</td>
<td>71.59%</td>
<td>71.922%</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>d=0.20 (v=0.8 \pi ) (w=4 \pi ) (f=2)</td>
<td>0.120547</td>
<td>10.55%</td>
<td>-0.646%</td>
<td>349.59%</td>
<td>0.077314</td>
<td>0.883%</td>
<td>1.078%</td>
<td>-41.20%</td>
<td></td>
</tr>
<tr>
<td>d=0.20 (v=1.6 \pi ) (\omega = 8 \pi ) (f=4)</td>
<td>0.122970</td>
<td>12.77%</td>
<td>1.35%</td>
<td>358.63%</td>
<td>0.075155</td>
<td>-1.934%</td>
<td>-1.745%</td>
<td>-42.84%</td>
<td></td>
</tr>
<tr>
<td>C = 0.3 Case</td>
<td>Time Avg Dimn-less Heat Transfer Coeff.</td>
<td>% Diff from SS without Plate</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Hole</td>
<td>Time Avg Dimn-less Heat Transfer Coeff.</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Hole</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Hole</td>
</tr>
<tr>
<td>SS without plate</td>
<td>0.073309</td>
<td>-</td>
<td>2.32%</td>
<td>-37.19%</td>
<td>0.028310</td>
<td>-</td>
<td>0.810%</td>
<td>0.631%</td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.071643</td>
<td>-2.27%</td>
<td>-</td>
<td>-38.62%</td>
<td>0.028082</td>
<td>-0.803%</td>
<td>-</td>
<td>-0.177%</td>
<td></td>
</tr>
<tr>
<td>SS without Plate and Hole</td>
<td>0.116730</td>
<td>59.23%</td>
<td>62.93%</td>
<td>-</td>
<td>0.028132</td>
<td>-0.627%</td>
<td>0.178%</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>d=0.20 (v=0.8 \pi ) (\omega = 4 \pi ) (f=2)</td>
<td>0.073151</td>
<td>-0.216%</td>
<td>2.10%</td>
<td>-37.33%</td>
<td>0.028259</td>
<td>-0.177%</td>
<td>0.631%</td>
<td>0.453%</td>
<td></td>
</tr>
<tr>
<td>d=0.20 (v=1.6 \pi ) (\omega = 8 \pi ) (f=4)</td>
<td>0.071284</td>
<td>-2.76%</td>
<td>-0.501%</td>
<td>-38.93%</td>
<td>0.028128</td>
<td>-0.640%</td>
<td>0.165%</td>
<td>-0.013%</td>
<td></td>
</tr>
</tbody>
</table>
7.3.2.2 Oscillating Source Over Board Opening. The arrangement with the plate over the opening in the cavity between the heat sources was thought to have potential to better mix the opening flow with the high temperature fluid in the cavity and, thus, to take advantage of the benefits of the opening and the oscillations. This arrangement is shown in Figure 7.20, and the parameter values for the geometry are provided in Table 7.7. A summary of the heat transfer coefficient results is provided in Table 7.8 and Table 7.9. Lower displacement cases were tested but the oscillations did not significantly alter the velocity field. For the 0.20 displacement amplitude cases, the oscillations did change the flow pattern in the channel, particularly within and just above the cavity. In general, the kinds of flow patterns that develop are discussed below with the higher frequency cases resulting in a slightly more complex flow. Beginning with the plate at its lowest position, a pair of counter-rotating zones is observed just above the oscillation source. These zones are non-symmetric, owing to the effects of the natural convection. As the plate moves upward, circulation regions form to the sides of the plate. Eventually, these regions block the flow through the opening that moves towards the heat sources. These circulations eventually move under the oscillation source as the plate motion slows. This frees the fluid to move through the cavity and promotes inflow through the opening. The circulations dissipate as the plate changes direction and moves downward. Even as the plate moves downward, the circulations zones that develop facilitate the motion of fluid along the downstream side of the cavity to the main channel flow. As the plate nears its lowest position, the circulation regions move over the top of the oscillation source. (See Figure 7.24.) While there is significant flow through the opening from the lower channel to the upper channel, there is slight outflow into the
lower channel from the downstream side of the opening. Based on this description of the velocity field, it is clear that the greatest effect of the oscillations occurs at the downstream side of the cavity and for the flow over the top of the downstream heat source. As much as a 28% increase in the local heat transfer coefficient relative to natural convection without the plate occurs at Point 3 (Block 2) as a result of the altered flow pattern over the top surface of the second heat source. While the side surfaces experience increases over their steady state values with the oscillations source fixed in position, there is no improvement over conditions without any oscillation source. These characteristics can be seen in Figure 7.22, which shows the local heat transfer coefficients as a function of time as well as in the bar graphs in Figure 7.23. The flow constriction caused by the location of the oscillation source in the cavity hinders potential benefits of the opening flow, for the oscillations do not cause a substantial improvement in the thermal conditions of the system. Though the flow patterns that develop are quite complex and interesting, it can be concluded that the system geometry is not conducive to heat transfer enhancement.

![Figure 7.20 Oscillating source over board opening - geometry layout.](image-url)
Table 7.7 Dimensional Parameter Values for Oscillation Source Over Board Opening

<table>
<thead>
<tr>
<th>Dimensional Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>b(block height)=L_{ref}(length basis)</td>
<td>0.25 in=0.0635m</td>
</tr>
<tr>
<td>BL=BH(block length = block height)</td>
<td>b</td>
</tr>
<tr>
<td>BT(board thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>CHL, CHU(channel heights)</td>
<td>2b</td>
</tr>
<tr>
<td>CLH(plate clearance)</td>
<td>0.50b</td>
</tr>
<tr>
<td>SPACE(spacing between blocks)</td>
<td>1.00b</td>
</tr>
<tr>
<td>PLB(plate length)</td>
<td>0.50b</td>
</tr>
<tr>
<td>PLH(plate thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>HW (hole width)</td>
<td>0.80b</td>
</tr>
<tr>
<td>TL(channel length)</td>
<td>14b</td>
</tr>
<tr>
<td>SL(location of first block)</td>
<td>5.5b</td>
</tr>
</tbody>
</table>

** Dimensions defined in Figure 7.21 (335x217 mesh generated)

Figure 7.21 Oscillating source over board opening - geometry definitions.
Figure 7.22 Local dimensionless heat transfer coefficient as a function of time $d=0.20$, $V=0.8\pi$, $\omega=4\pi$: (a) heat source 1, (b) heat source 2.

Figure 7.23 Time-averaged local dimensionless heat transfer coefficient Heat Source 2 Point 1.
Figure 7.24 Typical velocity distribution and temperature field oscillating source over board opening: $d=0.20$, $V=0.8\pi$, $\omega=4\pi$: (a) velocity, (b) temperature.
### Table 7.8a Summary of Average Heat Transfer Coefficient Results – Oscillating Source Over Board Opening – Two Block Geometry – Block1 C=0.50

<table>
<thead>
<tr>
<th>Case</th>
<th>Max Time Avg Dimn-less Temp</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>1.758109</td>
<td>-</td>
<td>-1.92%</td>
<td>0.124566</td>
<td>-3.00%</td>
<td>0.135164</td>
<td>-4.19%</td>
<td>0.048191</td>
<td>-21.68%</td>
<td>107.09%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>1.792665</td>
<td>1.96%</td>
<td>-2.95%</td>
<td>0.128430</td>
<td>3.10%</td>
<td>0.141083</td>
<td>4.37%</td>
<td>0.039604</td>
<td>-17.82%</td>
<td>-70.19%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>1.847226</td>
<td>5.06%</td>
<td>3.04%</td>
<td>0.130303</td>
<td>4.60%</td>
<td>0.145678</td>
<td>7.77%</td>
<td>0.023270</td>
<td>-51.71%</td>
<td>-41.24%</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20 v=0.4 π ω = 2 π f = 1</td>
<td>1.792665</td>
<td>1.96%</td>
<td>0.00%</td>
<td>0.128763</td>
<td>3.36%</td>
<td>0.140867</td>
<td>4.21%</td>
<td>0.037323</td>
<td>-22.55%</td>
<td>-5.75%</td>
<td>60.39%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20 v=0.8 π ω = 4 π f = 2</td>
<td>1.792662</td>
<td>1.96%</td>
<td>0.00%</td>
<td>0.12921644</td>
<td>3.73%</td>
<td>0.141722</td>
<td>4.85%</td>
<td>0.038601</td>
<td>-19.90%</td>
<td>-2.53%</td>
<td>65.88%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7.8b  Summary of Local Heat Transfer Coefficient Results – Oscillating Source Over Board Opening – Two Block Geometry – Block1 C=0.50

<table>
<thead>
<tr>
<th>Case</th>
<th>Time Avg</th>
<th>Dimn-less Heat Transfer Coeff. Point 1</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 2</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.099936</td>
<td>-</td>
<td>-2.64%</td>
<td>-3.94%</td>
<td>0.133256</td>
<td>-3.64%</td>
<td>-5.53%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.102650</td>
<td>2.71%</td>
<td>-</td>
<td>-1.33%</td>
<td>0.138299</td>
<td>3.78%</td>
<td>-1.96%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>0.104037</td>
<td>4.10%</td>
<td>1.351%</td>
<td>-</td>
<td>0.141069</td>
<td>5.66%</td>
<td>2.00%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20 v=0.4 π ω = 2 π f = 1</td>
<td>0.102970</td>
<td>3.03%</td>
<td>0.312%</td>
<td>-1.02%</td>
<td>0.138353</td>
<td>3.62%</td>
<td>0.039%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20 v=0.8 π ω = 4 π f = 2</td>
<td>0.103348</td>
<td>3.41%</td>
<td>0.680%</td>
<td>-0.66%</td>
<td>0.138796</td>
<td>4.15%</td>
<td>0.359%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS without plate</td>
<td>0.109500</td>
<td>-</td>
<td>-5.11%</td>
<td>-8.94%</td>
<td>0.043456</td>
<td>-12.94%</td>
<td>378.38%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.115397</td>
<td>5.38%</td>
<td>-</td>
<td>-4.04%</td>
<td>0.038475</td>
<td>-11.46%</td>
<td>-232.55%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>0.120258</td>
<td>9.82%</td>
<td>4.21%</td>
<td>-</td>
<td>0.009084</td>
<td>-79.09%</td>
<td>-76.39%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20 v=0.4 π ω = 2 π f = 1</td>
<td>0.115241</td>
<td>5.24%</td>
<td>-0.135%</td>
<td>-4.17%</td>
<td>0.033252</td>
<td>-23.48%</td>
<td>-13.57%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.20 v=0.8 π ω = 4 π f = 2</td>
<td>0.116223</td>
<td>6.13%</td>
<td>0.716%</td>
<td>-3.35%</td>
<td>0.036983</td>
<td>-14.89%</td>
<td>-3.87%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 7.9a Summary of Average Heat Transfer Coefficient Results – Oscillating Source Over Board Opening – Two Block Geometry– Block 2 C=0.50

<table>
<thead>
<tr>
<th>Case</th>
<th>Max Time Avg Dimm less Temp.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimm less Avg Heat Transfer Coeff. Left</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimm less Avg Heat Transfer Coeff. Top</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimm less Avg Heat Transfer Coeff. Right</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>1.957616</td>
<td>-</td>
<td>-14.28%</td>
<td>0.120075</td>
<td>-</td>
<td>62.747%</td>
<td>114.93%</td>
<td>0.083777</td>
<td>-</td>
<td>-15.14%</td>
<td>-38.49%</td>
<td>0.035095</td>
</tr>
<tr>
<td>SS with plate</td>
<td>2.283827</td>
<td>16.66%</td>
<td>-3.00%</td>
<td>0.073780</td>
<td>-38.55%</td>
<td>32.06%</td>
<td>0.098732</td>
<td>17.85%</td>
<td>-</td>
<td>-27.51%</td>
<td>0.037148</td>
<td>5.849%</td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>2.354531</td>
<td>20.27%</td>
<td>3.096%</td>
<td>0.055867</td>
<td>-53.47%</td>
<td>-24.27%</td>
<td>0.136203</td>
<td>62.57%</td>
<td>37.95%</td>
<td>-</td>
<td>0.039005</td>
<td>11.140%</td>
</tr>
<tr>
<td>d=0.20 v=0.4 π ω = 2 π f = 1</td>
<td>2.284060</td>
<td>16.67%</td>
<td>0.010%</td>
<td>0.074881</td>
<td>-37.63%</td>
<td>1.49%</td>
<td>34.03%</td>
<td>0.100987</td>
<td>20.54%</td>
<td>2.28%</td>
<td>-25.85%</td>
<td>0.037211</td>
</tr>
<tr>
<td>d=0.20 v=0.8 π ω = 4 π f = 2</td>
<td>2.284058</td>
<td>16.67%</td>
<td>0.010%</td>
<td>0.080229</td>
<td>-33.18%</td>
<td>8.74%</td>
<td>43.60%</td>
<td>0.098428</td>
<td>17.48%</td>
<td>-0.308%</td>
<td>76.183%</td>
<td>0.036944</td>
</tr>
</tbody>
</table>
Table 7.9b Summary of Local Heat Transfer Coefficient Results – Oscillating Source Over Board Opening – Two Block Geometry–Block 2 \( C = 0.50 \)

<table>
<thead>
<tr>
<th>Case</th>
<th>Time Avgr Dimn-less Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate and Hole</th>
<th>Time Avgr Dimn-less Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.109042</td>
<td>-</td>
<td>55.18%</td>
<td>306.68%</td>
<td>0.076637</td>
<td>-</td>
<td>-15.20%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.070267</td>
<td>-35.56%</td>
<td>-</td>
<td>162.07%</td>
<td>0.090381</td>
<td>17.934%</td>
<td>-</td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>0.026812</td>
<td>-75.41%</td>
<td>-61.84%</td>
<td>-</td>
<td>0.131502</td>
<td>71.59%</td>
<td>45.49%</td>
</tr>
<tr>
<td>( d=0.20 ) ( v=0.4\pi \omega ) ( =2\pi ) ( f=1 )</td>
<td>0.066305</td>
<td>-39.19%</td>
<td>-5.639%</td>
<td>147.29%</td>
<td>0.092565</td>
<td>20.78%</td>
<td>2.41%</td>
</tr>
<tr>
<td>( d=0.20 ) ( v=0.8\pi \omega ) ( =4\pi ) ( f=2 )</td>
<td>0.079629</td>
<td>-26.97%</td>
<td>13.323%</td>
<td>196.98%</td>
<td>0.090657</td>
<td>18.29%</td>
<td>0.305%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case</th>
<th>Time Avgr Dimn-less Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate and Hole</th>
<th>Time Avgr Dimn-less Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.073309</td>
<td>-</td>
<td>-20.70%</td>
<td>-37.19%</td>
<td>0.028310</td>
<td>-</td>
<td>0.000%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.092449</td>
<td>26.10%</td>
<td>-</td>
<td>-20.80%</td>
<td>0.028310</td>
<td>0.000%</td>
<td>-</td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>0.116730</td>
<td>59.23%</td>
<td>26.26%</td>
<td>-</td>
<td>0.028132</td>
<td>-0.627%</td>
<td>-0.627%</td>
</tr>
<tr>
<td>( d=0.20 ) ( v=0.4\pi \omega ) ( =2\pi ) ( f=1 )</td>
<td>0.094349</td>
<td>28.70%</td>
<td>2.055%</td>
<td>-19.17%</td>
<td>0.028296</td>
<td>-0.047%</td>
<td>-0.047%</td>
</tr>
<tr>
<td>( d=0.20 ) ( v=0.8\pi \omega ) ( =4\pi ) ( f=2 )</td>
<td>0.092189</td>
<td>25.75%</td>
<td>-0.281%</td>
<td>-21.02%</td>
<td>0.028283</td>
<td>-0.094%</td>
<td>-0.093%</td>
</tr>
</tbody>
</table>
7.3.2.3 Oscillating Source Over Board Opening at Level of Top Heat Source Surface. The placement of the oscillation source over the opening but at the level of the top of the heat source surfaces shows more promise for enhancing the cooling of the heat sources as it limits the constriction of the opening flow. The position of the oscillation source is shown in Figure 7.25 with the geometric parameter data given in Table 7.10, and the oscillation parameters given in Tables 7.11 and 7.12. Because of its placement farther away from the heat source surfaces and opening geometry, the oscillation source is less constricting to the flow over the heat sources and through the opening. With the new arrangement, there is more fluid flowing through the opening to the main channel. Not only does the presence of the oscillation source help to direct the opening flow towards the downstream heat source and to prevent the interaction with the main channel flow that reduces the velocities over the top surfaces as found in Chapter 3, but also the velocities induced by the oscillations behave much like the beneficial upstream oscillation source used for one rectangular heat source in Section 6.5.3. As the plate moves upward, flow over the top of the oscillation source moves towards the top of the downstream heat source. As the plate changes direction, fluid is squeezed between the oscillation source and the heat source, creating a higher velocity "jet-like" region angled over the top of the second heat source. (See Figure 7.29.) Circulation regions that develop in the cavity keep the through flow moving towards the downstream heat source. Close to a 43% increase in the local heat transfer coefficient at Point 3 on the downstream heat source was achieved in this study. (See Figure 7.28.) The upstream heat source feels little impact of the oscillations of the plate as seen in Figure 7.27. However, because the fresh inlet fluid flows over this heat source, its temperatures and heat transfer coefficients are
comparable to those of heat source that is further downstream. Hence this arrangement can be used to create a more uniform temperature distribution among the heat sources.

**Figure 7.25** Oscillating source over board opening -level of top heat source surface - geometry layout.

**Table 7.10** Dimensional Parameter Values for Oscillation Source Over Board Opening at Level of Top Heat Source Surface

<table>
<thead>
<tr>
<th>Dimensional Parameter**</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( b()height) (=L_{ref}()length basis)</td>
<td>0.25 in=0.0635m</td>
</tr>
<tr>
<td>BH=BL(block height=block length)</td>
<td>(b)</td>
</tr>
<tr>
<td>BRTH(board thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>CHL, CHU(channel heights)</td>
<td>2b</td>
</tr>
<tr>
<td>CLRB(plate clearance)</td>
<td>0.30b</td>
</tr>
<tr>
<td>SPACE(spacing between blocks)</td>
<td>0.50b</td>
</tr>
<tr>
<td>PLB(plate length)</td>
<td>0.75b</td>
</tr>
<tr>
<td>PLTH(plate thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>TL(channel length)</td>
<td>14b</td>
</tr>
<tr>
<td>SL(location of first block)</td>
<td>5.5b</td>
</tr>
</tbody>
</table>

*Dimensions defined in Figure 7.26 (335x221 graded grid generated).*

**Figure 7.26** Oscillating source over board opening at level of top heat source surface - geometry definitions.
Figure 7.27 Local dimensionless heat transfer coefficient as a function of time $d=0.20$, $V=0.4\pi$, $\omega=2\pi$: (a) heat source 1, (b) heat source 2.

Figure 7.28 Time averaged local dimensionless heat transfer coefficient heat source 2 Point 3.
Figure 7.29 Typical velocity distribution and temperature field oscillating source over opening level of top of block: $d=0.20$, $V=0.4\pi$, $\omega=2\pi$: (a) velocity, (b) temperature.
Table 7.11a Summary of Average Heat Transfer Coefficient Results – Oscillating Source Over Board Opening at Level of Top Heat Source Surface – Two Block Geometry – Block1 C=0.30

| Case                          | Max Time Avg Dimn-less Temp. | % Diff from SS without Plate | % Diff from SS with Plate | % Diff from SS without Plate and Hole | Time Avg Dimn-less Avg Heat Transfer Coeff. Left | % Diff from SS without Plate | % Diff from SS without Plate and Hole | Time Avg Dimn-less Avg Heat Transfer Coeff. Top | % Diff from SS without Plate | % Diff from SS without Plate and Hole | Time Avg Dimn-less Avg Heat Transfer Coeff. Right | % Diff from SS without Plate | % Diff from SS without Plate and Hole | % Diff from SS without Plate |
|-------------------------------|-----------------------------|------------------------------|---------------------------|---------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|-----------------------------------------------|
| SS without plate              | 1.758109                   | -                            | -4.82%                    | 0.124566                              | -4.40%                                        | 0.135164                                     | -3.95%                                        | -7.21%                                        | 0.048191                                     | -5.592%                                     | 107.09%                                      |                                              |                                              |                                              |
| SS with plate                 | 1.816176                   | 3.30%                        | -1.68%                    | 0.119396                              | -8.37%                                        | 0.1300020                                   | -3.80%                                        | -10.74%                                       | 0.045639                                     | -5.29%                                      | 96.13%                                       |                                              |                                              |                                              |
| SS without plate and hole     | 1.847226                   | 5.06%                        | 1.710%                    | 0.130303                              | 9.13%                                         | 0.145678                                    | 12.04%                                        |                                  | 0.023270                                     | -51.71%                                     | -49.014%                                    |                                              |                                              |                                              |
| d=0.10 v=0.2 w=2 Ø=2 f=1      | 1.816176                   | 3.30%                        | 0.000%                    | 0.119070                              | -8.62%                                        | 0.129781                                    | -3.98%                                        | -10.91%                                       | 0.045774                                     | -5.01%                                      | 96.71%                                      |                                              |                                              |                                              |
| d=0.20 v=0.4 w=2 Ø=2 f=1      | 1.816176                   | 3.30%                        | 0.000%                    | 0.1190840                             | -8.61%                                        | 0.1294129                                   | -4.25%                                        | -0.683%                                       | 0.046335                                     | -3.85%                                      | 99.12%                                      |                                              |                                              |                                              |
Table 7.11b Summary of Local Heat Transfer Coefficient Results – Oscillating Source Over Board Opening at Level of Top Heat Source Surface – Two Block Geometry – Block1

<table>
<thead>
<tr>
<th>C = 0.3 Case</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 1</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 2</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.099936</td>
<td>-</td>
<td>3.38%</td>
<td>-3.94%</td>
<td>0.133256</td>
<td>-</td>
<td>5.33%</td>
<td>-5.53%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.096661</td>
<td>-3.27%</td>
<td>-</td>
<td>-7.08%</td>
<td>0.126504</td>
<td>-5.06%</td>
<td>-</td>
<td>-10.32%</td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>0.104037</td>
<td>4.10%</td>
<td>7.63%</td>
<td>-</td>
<td>0.141069</td>
<td>5.86%</td>
<td>11.51%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.10 v=0.2 π ( \omega = 2 \pi \ f = 1 )</td>
<td>0.096549</td>
<td>-3.39%</td>
<td>-0.117%</td>
<td>-7.19%</td>
<td>0.126432</td>
<td>-5.12%</td>
<td>-0.057%</td>
<td>-10.37%</td>
</tr>
<tr>
<td>d=0.20 v=0.4 π ( \omega = 2 \pi \ f = 1 )</td>
<td>0.096432</td>
<td>-3.50%</td>
<td>-0.237%</td>
<td>-7.31%</td>
<td>0.1263505</td>
<td>-5.18%</td>
<td>-0.122%</td>
<td>-10.43%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C = 0.3 Case</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 3</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 4</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.109500</td>
<td>-</td>
<td>4.91%</td>
<td>-8.94%</td>
<td>0.043456</td>
<td>-</td>
<td>15.21%</td>
<td>378.38%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.104369</td>
<td>-4.68%</td>
<td>-</td>
<td>-13.21%</td>
<td>0.037716</td>
<td>-13.20%</td>
<td>-</td>
<td>315.20%</td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>0.120258</td>
<td>9.82%</td>
<td>15.22%</td>
<td>-</td>
<td>0.009084</td>
<td>-79.09%</td>
<td>-75.91%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.10 v=0.2 π ( \omega = 2 \pi \ f = 1 )</td>
<td>0.104138</td>
<td>-4.89%</td>
<td>-0.222%</td>
<td>-13.40%</td>
<td>0.037793</td>
<td>-13.03%</td>
<td>0.203%</td>
<td>316.05%</td>
</tr>
<tr>
<td>d=0.20 v=0.4 π ( \omega = 2 \pi \ f = 1 )</td>
<td>0.1039324</td>
<td>-5.08%</td>
<td>-0.419%</td>
<td>-13.57%</td>
<td>0.0384749</td>
<td>-11.46%</td>
<td>2.01%</td>
<td>323.55%</td>
</tr>
</tbody>
</table>
Table 7.12a Summary of Average Heat Transfer Coefficient Results – Oscillating Source Over Board Opening at Level of Top Heat Source Surface – Two Block Geometry – Block 2 C=0.30

<table>
<thead>
<tr>
<th>Case</th>
<th>Max Time</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate and Hole</th>
<th>Time Avg Dimnless Avg Heat Transfer Coeff. Left</th>
<th>% Diff from SS without Plate and Hole</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimnless Avg Heat Transfer Coeff. Top</th>
<th>% Diff from SS without Plate and Hole</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimnless Avg Heat Transfer Coeff. Right</th>
<th>% Diff from SS without Plate and Hole</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>1.957616</td>
<td>-</td>
<td>-7.46%</td>
<td>-16.85%</td>
<td>0.120075</td>
<td>-</td>
<td>15.51%</td>
<td>114.93%</td>
<td>0.083777</td>
<td>-</td>
<td>5.44%</td>
<td>-38.49%</td>
<td>0.035095</td>
<td>-</td>
<td>4.95%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>2.115428</td>
<td>8.06%</td>
<td>-</td>
<td>-10.15%</td>
<td>0.103948</td>
<td>-13.43%</td>
<td>-</td>
<td>86.06%</td>
<td>0.079454</td>
<td>-5.16%</td>
<td>-</td>
<td>-41.66%</td>
<td>0.033440</td>
<td>-4.71%</td>
<td>-</td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>2.354531</td>
<td>20.27%</td>
<td>11.30%</td>
<td>-</td>
<td>0.055867</td>
<td>-53.47%</td>
<td>-46.25%</td>
<td>-</td>
<td>0.136203</td>
<td>62.57%</td>
<td>71.42%</td>
<td>-</td>
<td>0.039005</td>
<td>11.14%</td>
<td>16.64%</td>
</tr>
<tr>
<td>d=0.10 v=0.2 ( \omega = 2 \pi ) f=1</td>
<td>2.115419</td>
<td>8.06%</td>
<td>0.000%</td>
<td>-10.15%</td>
<td>0.103579</td>
<td>-13.73%</td>
<td>-0.355%</td>
<td>85.40%</td>
<td>0.085973</td>
<td>2.62%</td>
<td>8.20%</td>
<td>-36.87%</td>
<td>0.035158</td>
<td>0.179%</td>
<td>5.13%</td>
</tr>
<tr>
<td>d=0.20 v=0.4 ( \omega = 2 \pi ) f=1</td>
<td>2.115648</td>
<td>8.07%</td>
<td>0.010%</td>
<td>-10.14%</td>
<td>0.102377</td>
<td>-14.73%</td>
<td>-1.51%</td>
<td>83.25%</td>
<td>0.107575</td>
<td>28.40%</td>
<td>35.39%</td>
<td>92.55%</td>
<td>0.038843</td>
<td>10.68%</td>
<td>16.18%</td>
</tr>
</tbody>
</table>
### Table 7.12b Summary of Local Heat Transfer Coefficient Results – Oscillating Source Over Board Opening at Level of Top Heat Source Surface – Two Block Geometry – Block 2 C=0.30

<table>
<thead>
<tr>
<th>C = 0.3</th>
<th>Case</th>
<th>Time Avg Dimm-less Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimm-less Heat Transfer Coeff.</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.109042</td>
<td>-</td>
<td>20.95%</td>
<td>306.68%</td>
<td>0.076637</td>
<td>-</td>
<td>6.76%</td>
<td>-41.72%</td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.090150</td>
<td>-17.32%</td>
<td>-</td>
<td>236.22%</td>
<td>0.071784</td>
<td>-6.332%</td>
<td>-</td>
<td>-45.41%</td>
<td></td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>0.026812</td>
<td>-75.41%</td>
<td>-70.25%</td>
<td>-</td>
<td>0.131502</td>
<td>71.59%</td>
<td>83.19%</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>d=0.10 v=0.2π</td>
<td>ω = 2π f = 1</td>
<td>0.089724</td>
<td>-17.71%</td>
<td>-0.473%</td>
<td>234.63%</td>
<td>0.073858</td>
<td>-3.62%</td>
<td>2.88%</td>
<td>-43.83%</td>
</tr>
<tr>
<td>d=0.20 v=0.4π</td>
<td>ω = 2π f = 1</td>
<td>0.088375</td>
<td>-18.95%</td>
<td>-1.97%</td>
<td>229.60%</td>
<td>0.092418</td>
<td>20.59%</td>
<td>28.74%</td>
<td>-29.72%</td>
</tr>
<tr>
<td>C = 0.3</td>
<td>Case</td>
<td>Time Avg Dimm-less Heat Transfer Coeff.</td>
<td>% Diff from SS without Plate</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Hole</td>
<td>Time Avg Dimm-less Heat Transfer Coeff.</td>
<td>% Diff from SS without Plate</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Hole</td>
</tr>
<tr>
<td>SS without plate</td>
<td>0.073309</td>
<td>-</td>
<td>2.58%</td>
<td>-37.19%</td>
<td>0.028310</td>
<td>-</td>
<td>12.08%</td>
<td>0.631%</td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.071460</td>
<td>-2.52%</td>
<td>-</td>
<td>-38.78%</td>
<td>0.025258</td>
<td>-10.77%</td>
<td>-</td>
<td>-10.21%</td>
<td></td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>0.116730</td>
<td>59.23%</td>
<td>63.35%</td>
<td>-</td>
<td>0.028132</td>
<td>-0.627%</td>
<td>11.37%</td>
<td>-</td>
<td></td>
</tr>
<tr>
<td>d=0.10 v=0.2π</td>
<td>ω = 2π f = 1</td>
<td>0.080032</td>
<td>9.17%</td>
<td>11.99%</td>
<td>-31.43%</td>
<td>0.025648</td>
<td>-7.79%</td>
<td>1.542</td>
<td>-8.83%</td>
</tr>
<tr>
<td>d=0.20 v=0.4π</td>
<td>ω = 2π f = 1</td>
<td>0.104597</td>
<td>42.68%</td>
<td>46.37%</td>
<td>-10.39%</td>
<td>0.026103</td>
<td>-7.79%</td>
<td>3.34%</td>
<td>-7.21%</td>
</tr>
</tbody>
</table>
7.3.2.4 Oscillating Source Over First Heat Source. The final arrangement investigated is for the oscillation source placed over the first heating element as depicted in Figure 7.30. The dimensional parameters are listed in Table 7.13, and summaries of the heat transfer coefficients were given in Table 7.14 and Table 7.15. This arrangement removes any flow obstruction from the vicinity of the opening flow. Since the oscillation source is upstream of the second heat source, it was thought that some cooling benefit could be produced by the oscillations at the second heat source as well. The flow near the top surface of the first heat source due to the oscillating plate is highly independent of the downstream flow and is very similar to that for a single rectangular heat source. The only difference is that the flow over the plate and exiting from beneath the oscillation source influences the flow near the second heat source. (See Figure 7.34.) When the plate motion is upward, the flow moving over the top of the heat source flows into the cavity region and sets-up circulation regions, thus keeping the opening flow mainly directed downstream. During downward plate motion, the flow ejected from beneath the oscillation source tends to keep the inflow from the opening moving downstream. Hence cooler fluid is again brought closer to the downstream heat source. Despite this, Figure 7.32 shows that while the oscillations have a significant impact on the heat transfer coefficients at the top surface of the upstream heat source, the oscillations have a minimal impact on the heat transfer coefficients on the downstream heat source. The maximum improvement in the local heat transfer coefficient for this arrangement was 70% at Point 3 on the upstream heat source caused by the motion of the plate. (See Figure 7.32 and Figure 7.33.) The downstream heat source experienced a maximum of a 44% increase in the local heat transfer coefficient at Point 1 relative to standard natural convection with
the opening, due mainly to the opening flow and the diversion of the main channel flow caused by the presence of the plate. In addition, the temperature of the downstream heat source is lower than that of the upstream heat source as a result of this opening flow. It can be concluded that for this arrangement, the upstream heat source is cooled largely by the oscillations while the downstream heat source is cooled largely by the opening flow.

Figure 7.30  Oscillating source over first heat source - geometry layout.
Table 7.13 Dimensional Parameter Values for Oscillation Source Over First Heat Source

<table>
<thead>
<tr>
<th>Dimensional Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>b=L_ref(length basis)</td>
<td>0.25 in=0.0635m</td>
</tr>
<tr>
<td>BH=BL(board height = board width)</td>
<td>b</td>
</tr>
<tr>
<td>BT(board thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>CHL, CHU(channel heights)</td>
<td>2b</td>
</tr>
<tr>
<td>CLB(plate clearance)</td>
<td>0.30b</td>
</tr>
<tr>
<td>SPACE(spacing between blocks)</td>
<td>0.50b</td>
</tr>
<tr>
<td>PLB(plate length)</td>
<td>0.75b</td>
</tr>
<tr>
<td>PLH(plate thickness)</td>
<td>0.2b</td>
</tr>
<tr>
<td>HW(hole width)</td>
<td>0.80b</td>
</tr>
<tr>
<td>TL(channel length)</td>
<td>14b</td>
</tr>
<tr>
<td>SL(location of first block)</td>
<td>5.5b</td>
</tr>
</tbody>
</table>

**Dimensioned defined in Figure 7.31 (335x217 graded grid generated)

Figure 7.31 Oscillating source over first heat source - geometry definitions.
Figure 7.32  Local dimensionless heat transfer coefficient as a function of time $d=0.20$ $V=0.8\pi$ $\omega=4\pi$: (a) heat source 1, (b) heat source 2.

Figure 7.33  Time averaged local dimensionless heat transfer coefficient heat source 1 Point 3.
Figure 7.34 Typical velocity distribution and temperature field oscillating source over first heat source: $d=0.20 \ V=0.8\pi \ \omega=4\pi$: (a) velocity, (b) temperature.
Table 7.14a Summary of Average Heat Transfer Coefficient Results – Oscillating Source Over First Heat Source – Two Block Geometry – Block1 C=0.30

<table>
<thead>
<tr>
<th>C = 0.3</th>
<th>Max Time</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>Avg Dimn-less Temp.</th>
<th>Avg Dimn-less Avg Heat Transfer Coeff. Left</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimn-less Avg Heat Transfer Coeff. Top</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimn-less Avg Heat Transfer Coeff. Right</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>1.758109</td>
<td>-0.128%</td>
<td>-4.82%</td>
<td>0.124566</td>
<td>-12.09%</td>
<td>-4.40</td>
<td>0.135164</td>
<td>-4.87%</td>
<td>-7.21%</td>
<td>0.048191</td>
<td>-2.85%</td>
<td>107.09%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS with plate</td>
<td>1.760370</td>
<td>0.129%</td>
<td>-4.70%</td>
<td>0.111123</td>
<td>-10.79%</td>
<td>-14.71%</td>
<td>0.128878</td>
<td>-4.65%</td>
<td>-11.53%</td>
<td>0.049605</td>
<td>2.93%</td>
<td>113.17%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>1.847226</td>
<td>5.069%</td>
<td>4.93%</td>
<td>-0.130303</td>
<td>4.60%</td>
<td>17.26%</td>
<td>-0.145678</td>
<td>7.77%</td>
<td>13.03%</td>
<td>0.023270</td>
<td>-51.71%</td>
<td>-53.09%</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>d=0.2</td>
<td>v=0.4 \pi \omega = 2 \pi / f = 1</td>
<td>1.760561</td>
<td>0.139%</td>
<td>0.011%</td>
<td>-4.69%</td>
<td>0.109041</td>
<td>-12.46%</td>
<td>-1.87%</td>
<td>-16.31%</td>
<td>0.171058</td>
<td>26.55%</td>
<td>32.72%</td>
<td>17.42%</td>
<td>0.058107</td>
<td>20.57%</td>
<td>17.13%</td>
</tr>
<tr>
<td>d=0.2</td>
<td>v=0.8 \pi \omega = 4 \pi / f = 2</td>
<td>1.7605734</td>
<td>0.140%</td>
<td>0.012%</td>
<td>-4.69%</td>
<td>0.109926624</td>
<td>-11.75%</td>
<td>-1.07%</td>
<td>-15.63%</td>
<td>0.199325037</td>
<td>47.46%</td>
<td>52.97%</td>
<td>52.97%</td>
<td>0.05877901</td>
<td>21.97%</td>
<td>18.49%</td>
</tr>
</tbody>
</table>
Table 7.14b  Summary of Local Heat Transfer Coefficient Results – Oscillating Source Over First Heat Source – Two Block Geometry – Block1 C=0.30

<table>
<thead>
<tr>
<th>Case</th>
<th>Time Avg Dimm-less Heat Transfer Coeff. Point 1</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimm-less Heat Transfer Coeff. Point 2</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.099936</td>
<td>-</td>
<td>10.72%</td>
<td>-3.94%</td>
<td>0.133256</td>
<td>-</td>
<td>-5.94%</td>
<td>-5.53%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.090260</td>
<td>-9.68%</td>
<td></td>
<td>-13.24%</td>
<td>0.141675</td>
<td>6.31%</td>
<td>-</td>
<td>0.430%</td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>0.104037</td>
<td>4.10%</td>
<td>15.26%</td>
<td>-</td>
<td>0.141069</td>
<td>5.86%</td>
<td>-0.428%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.20 v=0.4π ω = 2π f = 1</td>
<td>0.089165</td>
<td>-10.77%</td>
<td>-1.21%</td>
<td>-14.29%</td>
<td>0.184652</td>
<td>38.57%</td>
<td>30.33%</td>
<td>30.89%</td>
</tr>
<tr>
<td>d=0.20 v=0.8π ω = 4π f = 2</td>
<td>0.088992</td>
<td>-10.95%</td>
<td>-1.40%</td>
<td>-14.46%</td>
<td>0.198253</td>
<td>48.77%</td>
<td>39.93%</td>
<td>40.53%</td>
</tr>
</tbody>
</table>

For C = 0.3

<table>
<thead>
<tr>
<th>Case</th>
<th>Time Avg Dimm-less Heat Transfer Coeff. Point 3</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimm-less Heat Transfer Coeff. Point 4</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td>0.109500</td>
<td>-</td>
<td>8.24%</td>
<td>-8.94%</td>
<td>0.043456</td>
<td>-</td>
<td>-7.12%</td>
<td>378.38%</td>
</tr>
<tr>
<td>SS with plate</td>
<td>0.101164</td>
<td>-7.61%</td>
<td></td>
<td>-15.87%</td>
<td>0.046790</td>
<td>7.67%</td>
<td>-</td>
<td>415.09%</td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td>0.120258</td>
<td>9.82%</td>
<td>18.87%</td>
<td>-</td>
<td>0.009084</td>
<td>-79.09%</td>
<td>-80.58%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.20 v=0.4π ω = 2π f = 1</td>
<td>0.1553102</td>
<td>41.83%</td>
<td>53.52%</td>
<td>29.14%</td>
<td>0.047822</td>
<td>10.04%</td>
<td>2.20%</td>
<td>426.46%</td>
</tr>
<tr>
<td>d=0.20 v=0.8π ω = 4π f = 2</td>
<td>0.186151</td>
<td>70.00%</td>
<td>84.01%</td>
<td>54.79%</td>
<td>0.047369</td>
<td>9.00%</td>
<td>1.23%</td>
<td>421.47%</td>
</tr>
</tbody>
</table>
Table 7.15a Summary of Average Heat Transfer Coefficient Results – Oscillating Source Over First Heat Source – Two Block Geometry– Block 2

<p>| Case   | C = 0.3 | Max Time Avg Dimensionless Temperature | % Diff from SS without Plate | % Diff from SS with Plate | % Diff from SS without Plate and Hole | % Diff from SS with Plate and Hole | % Diff from SS without Plate and Hole | % Diff from SS with Plate and Hole | % Diff from SS without Plate and Hole | % Diff from SS with Plate and Hole | % Diff from SS without Plate and Hole | % Diff from SS with Plate and Hole | % Diff from SS without Plate and Hole |
|--------|---------|----------------------------------------|-----------------------------|--------------------------|--------------------------------------|-----------------------------------|-------------------------------------|-----------------------------------|-------------------------------------|------------------------------------|-------------------------------------|-----------------------------------|-------------------------------------|-------------------------------------|
| SS     | -0.20   | 1.957616                               | -                           | -14.391%                 | 14.996%                             | 0.083777                          | -                                   | 7.551%                            | -38.49%                             | 0.035095                           | -                                   | 0.943%                            | -10.02%                            |
| without plate       |                      |                                       |                            |                          |                                     |                                   |                                     |                                   |                                     |                                    |                                     |                                   |                                     |
| SS     | -0.20   | 1.711338                               | -12.58%                     | -                        | -27.31%                             | 0.161337                          | -                                   | 188.78%                           | -42.81%                             | 0.034767                           | -                                   | -0.934%                           | -10.86%                            |
| with plate       |                      |                                       |                            |                          |                                     |                                   |                                     |                                   |                                     |                                    |                                     |                                   |                                     |
| SS     | -0.20   | 2.354531                               | 20.27%                      | 37.584%                  | -                                   | 0.056867                          | -                                   | 0.0136203                         | 74.855%                             | 0.039005                           | 11.14%                             | 12.188%                           | -                                   |
| without plate and hole |                  |                                       |                            |                          |                                     |                                   |                                     |                                   |                                     |                                    |                                     |                                   |                                     |
| d=0.20  | v=0.4 π  | 1.711336                               | -12.58%                     | 0.000%                   | -27.31%                             | 0.162173                          | 35.06%                             | 0.076142                          | -6.72%                             | 0.034899                           | -0.559%                            | 0.378%                             | -10.52%                            |
| f=1    |                      |                                       |                            |                          |                                     |                                   |                                     |                                   |                                     |                                    |                                     |                                   |                                     |
| d=0.20  | v=0.8 π  | 1.711336                               | -12.58%                     | 0.000%                   | -27.31%                             | 0.163968                          | 36.55%                             | 0.076212                          | -6.64%                             | 0.0349737                          | -0.346%                            | 0.593%                             | -10.33%                            |
| f=2    |                      |                                       |                            |                          |                                     |                                   |                                     |                                   |                                     |                                    |                                     |                                   |                                     |</p>
<table>
<thead>
<tr>
<th>C = 0.3</th>
<th>Case</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 1</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
<th>Time Avg Dimn-less Heat Transfer Coeff. Point 2</th>
<th>% Diff from SS without Plate</th>
<th>% Diff from SS with Plate</th>
<th>% Diff from SS without Plate and Hole</th>
</tr>
</thead>
<tbody>
<tr>
<td>SS without plate</td>
<td></td>
<td>0.109042</td>
<td>-</td>
<td>-28.67%</td>
<td>275.92%</td>
<td>0.076637</td>
<td>-</td>
<td>8.79%</td>
<td>-41.72%</td>
</tr>
<tr>
<td>SS with plate</td>
<td></td>
<td>0.152870</td>
<td>40.19%</td>
<td>-</td>
<td>427.02%</td>
<td>0.070441</td>
<td>-8.08%</td>
<td>-</td>
<td>-46.43%</td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td></td>
<td>0.029006</td>
<td>-73.39%</td>
<td>-81.02%</td>
<td>-</td>
<td>0.131502</td>
<td>71.59%</td>
<td>86.68%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.20 v=0.4 π ω = 2π f = 1</td>
<td></td>
<td>0.1545925</td>
<td>41.77%</td>
<td>1.12%</td>
<td>432.96%</td>
<td>0.0706431</td>
<td>-7.82%</td>
<td>0.286%</td>
<td>-46.28%</td>
</tr>
<tr>
<td>d=0.20 v=0.8 π ω = 4π f = 2</td>
<td></td>
<td>0.1571575</td>
<td>44.12%</td>
<td>2.80%</td>
<td>441.80%</td>
<td>0.0686119</td>
<td>-10.47%</td>
<td>2.59%</td>
<td>-47.82%</td>
</tr>
<tr>
<td>C = 0.3</td>
<td>Case</td>
<td>Time Avg Dimn-less Heat Transfer Coeff. Point 3</td>
<td>% Diff from SS without Plate</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Hole</td>
<td>Time Avg Dimn-less Heat Transfer Coeff. Point 4</td>
<td>% Diff from SS without Plate</td>
<td>% Diff from SS with Plate</td>
<td>% Diff from SS without Plate and Hole</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
<td>---------------------------------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>---------------------------------</td>
<td>-----------------</td>
<td>-----------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>SS without plate</td>
<td></td>
<td>0.073309</td>
<td>-</td>
<td>9.47%</td>
<td>-37.19%</td>
<td>0.028310</td>
<td>-</td>
<td>1.53%</td>
<td>0.631%</td>
</tr>
<tr>
<td>SS with plate</td>
<td></td>
<td>0.066987</td>
<td>-8.65%</td>
<td>-</td>
<td>-42.63%</td>
<td>0.027881</td>
<td>-1.51%</td>
<td>-</td>
<td>-0.893%</td>
</tr>
<tr>
<td>SS without plate and hole</td>
<td></td>
<td>0.116730</td>
<td>59.23%</td>
<td>74.31%</td>
<td>-</td>
<td>0.028132</td>
<td>-0.627%</td>
<td>0.901%</td>
<td>-</td>
</tr>
<tr>
<td>d=0.20 v=0.4 π ω = 2π f = 1</td>
<td></td>
<td>0.0655820</td>
<td>-10.54%</td>
<td>-2.06%</td>
<td>-43.81%</td>
<td>0.0279293</td>
<td>-1.34%</td>
<td>0.174%</td>
<td>-0.721%</td>
</tr>
<tr>
<td>d=0.20 v=0.8 π ω = 4π f = 2</td>
<td></td>
<td>0.0686119</td>
<td>-6.40%</td>
<td>2.45%</td>
<td>-41.22%</td>
<td>0.0280407</td>
<td>-0.949%</td>
<td>0.573%</td>
<td>-0.324%</td>
</tr>
</tbody>
</table>
7.3.3 Conclusions

The results of the studies showed that with the two-dimensional heat source geometries, the placement of an oscillation source immediately over the opening constricted the flow through the opening and did not provide significant cooling benefits. In keeping with the results from the modified geometries for the rectangular heat sources, the geometries that produced the least flow constriction while still producing increased velocities and velocity gradients near the heat sources provided the greatest thermal benefits. This was exemplified in the case for the plate over the upstream heat source and the plate over the opening at the top heat source level. These two cases showed that both the oscillation source geometry and the flow caused by the oscillations could be used to divert fluid including the opening flow to cool a specific area. It is important to note that while the placement of the oscillation source upstream of the heat sources did not provide much improvement in the cooling of the cases examined in this study, its effects are likely to increase when it is positioned closer to the upstream heat source.

7.4 Conclusions

The results of the various studies of the combined method showed that the oscillations can be utilized to promote flow through the openings and that the additional opening flow stream can be used to increase the flow velocities passing over the heat sources and nearby the oscillation sources, thereby enhancing the cooling effect of the oscillations. For the plain channel geometry, the direction of the flow through the opening oscillated, periodically delivering cooler fluid to upper channel and carrying the heated fluid away. Used in a similar manner, the placement of an opening between two rectangular heat
sources, did not result in any periodic flow reversal for the parameters studied due to the
differences in the geometry and the larger pressure differences across the opening that
result. While flow through the opening did occur, the presence of the oscillating plate
was largely detrimental as it obstructed the opening flow. Beneficial cooling effects were
found to occur for those arrangements in which the opening flow is largely unobstructed
by the oscillation sources, but the oscillation sources themselves and the flow they
generate have a strong influence in facilitating the movement of the cooler opening flow
to where it is needed to reduce the temperatures of the heat sources. When applied in
such a manner, the balanced combination of transverse-oscillations and the alternate
cross-flow passage is a viable alternate cooling method for enhancing natural convection.
CHAPTER 8
SUMMARY, CONCLUSIONS AND FUTURE WORK

8.1 Summary and Conclusions
In this work a number of alternative static and dynamic techniques for enhancing natural convection in a vertically oriented channel were investigated through two-dimensional laminar flow numerical parametric studies. A finite volume program was developed for studies using a simplified geometry. More complex geometries and boundary conditions necessitated the use of a finite element package. The results of the investigation showed that for the proper system parameters significant cooling enhancement could be achieved through the application of the alternate cross-flow passages, the strategic placement of transverse oscillation sources, as well as the combination of these methods. In general, these methods improve the cooling of the heat source by altering the velocity field nearby the heat sources, leading to increased velocity magnitudes and velocity gradients near the heated surfaces. This, in turn, causes thinner momentum and thermal boundary layers as well as increased convective effects. The resulting increase in the temperature gradients leads to higher rates of heat convected by the fluid away from the heat source. Lower heat source temperatures and higher heat transfer coefficients result.

In this work, the capability of the new flow paths created through the application of the alternate cross-flow path method to redirect the flow of cooler fluid to specific areas through changes in geometry was illustrated. The additional flow streams created by openings in the board surface were found to improve the thermal conditions of a system. Not only does the cross-flow increase the velocities and velocity gradients
nearby the heated surfaces, but it also acts to carry the heat away from the heat source. Under the proper conditions, the temperatures at downstream heat sources may even be lower than those upstream due to the action of the opening flow. The cooling effect is dependent on the amount of flow through the openings, which is related to the temperature differences near the opening and thus the heat source and geometric parameters. The results showed that sufficient heating element heights are necessary to produce the flow rates needed for cooling. In addition, the cooling effect was found to increase with decreases in the heating element and opening width. The results also indicate that there may be an optimal heat rate for which the method is most beneficial. The opening arrangement also affects the cooling potential. For the parameters investigated, using this static enhancement method, a maximum decrease in the heat source temperature of 40% and an increase in a side surface heat transfer coefficient of 180% relative to standard natural convection were attained. This method is expected to be used for enhancement in the laminar natural convection regime for which \( Gr \) is less than approximately \( 10^7 \).

For conditions where static cooling enhancement is insufficient, an alternative dynamic enhancement method, the use of transverse oscillations to supplement the natural convection, was investigated in a number of studies. These oscillations locally alter the flow by actively creating new flow streams, causing different flow streams to interact, and increasing the velocities near the heat source surfaces. First, a simplified model geometry with a simplified set of governing equations was employed to investigate this method. A finite volume program was developed to solve for the temperature field under a squeeze film velocity field where inertia and buoyancy forces were neglected.
The low velocity region near the flow symmetry line that develops under the squeeze film type velocity field may restrict the cooling effect that can be achieved by the oscillations. The results of this study showed that the cooling effect provided by the oscillations increases with the channel width to length ratio and the oscillation displacement amplitude and frequency. However, the most important parameter was found to be the oscillation displacement amplitude. Significant cooling relative to a pure natural convection reference case did not occur until the displacement amplitude values reached 50% of the mean channel width. For $L/b_0$ ratios of 10, 20, 50, and 100, cooling was found to take place for oscillation frequencies greater than 150, 200, 300, and 500 rad/s respectively. Because a wide range of parameters was investigated, correlation equations of the time averaged Nusselt numbers were derived. For the parameters investigated, a maximum improvement of 500% relative to a pure natural convection reference case was attained. However, particularly for high $L/b_0$ ratios, the constant property assumption limits the oscillation parameters. The squeeze film model assumptions are valid for $b_0/L << 1$ and $Re = a_0 b_0 \omega / \nu << 1$.

Under more general conditions, a more extensive finite volume program was developed to investigate the effects of the transverse oscillations where inertia and buoyancy are taken into account. Due to the long run times involved, the number of parameter cases that were tested was limited. However, the results showed that inertia forces can play a significant role in the formation of the velocity field and thus the cooling effect. The inertia forces not only result in a velocity distribution with higher velocities and velocity gradients near both the moving and the stationary walls, but also allow for the possibility of a non-uniform flow direction across the channel width. As a
result, the inertia effects were found to improve the heat removal from the region near the heated surface by increasing the temperature gradients. The complexity of the flow was found to increase with the oscillation amplitude and frequency (higher inertia) and, as a result, so did the cooling effect. Due to the symmetric nature of the problem, when natural convection effects are neglected, the mid-channel length is an area of high temperatures and low velocities and thus low inertia effects. It was concluded that better overall thermal conditions could be achieved with the inclusion of the natural convection effects so that one cooling method does not dominate but the beneficial effects of both methods can be utilized. Operation in this regime ($Gr_{bo}/Re_{bo} Re_{L} \sim 0.1$ to $10$) produces lower heated surface temperatures due to the transport of heat from the mid-channel length carried by the natural convection induced velocities. Regardless of the ratio, the study also showed that one cooling method does not dominate the flow in all portions of the channel with the natural convection effects most important near the mid-channel length and the oscillation effects more important near the ends of the channel. In addition, when natural convection and inertia are taken into account, the same cooling as that obtained under the squeeze film conditions can be attained at lower oscillation displacement amplitudes and frequencies. Similar trends in the relationship between the cooling enhancement and the system parameters as in the squeeze film investigation were found with the benefits of the oscillations increasing with the increasing frequency, heat rate, and amplitude and decreasing channel spacing. This method is expected to be most beneficial when used over the domain where $Gr_{bo}/Re_{bo} Re_{L} \sim 0.1$ to $10$. 
The use of the transverse oscillations was then investigated for more complex two-dimensional oscillation source and heat source geometries through finite element studies. In general, the findings showed that the two-dimensional heat source and oscillation source geometry is a significant factor in the determination of the cooling potential. Flow over the top as well as the flow turning around the sides of the oscillation source and the unique flow patterns over a two-dimensional rectangular heat source contribute to the development of the velocity and temperature fields. Near the heated surface and the oscillation source, the flow is a result of the interaction of many flow streams including the natural convection and oscillation induced flows, as well as the inertia effects and the shear and pressure forces. Typical flow characteristics include the development of multiple circulation regions. While natural convection effects assist in the heat removal enhancement by carrying warmer fluids away from the heated surface downstream of the oscillation source (including the fluid expelled from under the moving plate as well as the flow over the oscillation source), natural convection can oppose the effects of the oscillations particularly at the upstream end of the moving plate. As a result, the cooling is minimal at locations upstream of the oscillation source. However, without the influence of natural convection, the important flow interactions necessary for the cooling effect could not be generated at other portions of the channel.

The transverse oscillation sources were applied to a number of geometries. For the plain channel geometry, the cooling effect was found to increase with the oscillation displacement and frequency. Improvement relative to standard natural convection occurs at locations just under the downstream end of the oscillating plate and just downstream of the plate only for oscillation displacement amplitudes of at least 10% of the channel.
height with a dimensionless frequency of at least $2\pi$ (82 rad/sec). The maximum improvement in the local heat transfer coefficient just downstream of the oscillation source was 40%.

The two-dimensional heat source geometry allows for freer flow to the sides of the heat source making circulation regions quicker to dissipate. The cooling effect was found to increase with the oscillation displacement amplitude and frequency as well as with the reduction in the clearance between the oscillation source and the heat source. Significant cooling did not occur until a 10% heat source height oscillation displacement amplitude was applied for an oscillation dimensionless frequency of at least $2\pi$ (91 rad/sec). This cooling effect is mainly restricted to the top heat source surface where as high as a 52% increase in the local heat transfer coefficient relative to pure natural convection was achieved.

The position of the oscillation source was also modified to try to take advantage of the higher velocity flows entering and exiting the region under the plate. The use of an oscillation source over a dummy unheated block only created a flow obstruction, and the oscillations did not improve these conditions despite the use of a smaller clearance and higher oscillation frequency. A thin upstream plate extension of the heat source over which the oscillation device was placed improved the temperature of the heat source, but only through conduction through this plate. The use of an upstream oscillation source placed close enough to the heat source showed more promise as the flow over the top of the oscillation source and the flow squeezed from between the oscillation source and the heat source pass near the top heat source surface. For a 20% heat source height displacement, up to a 10% increase in the local heat transfer coefficient was obtained, and
due to the reduced flow obstruction, the heat source temperatures were lower than they were with the plate positioned over the top heat source surface.

Based on these investigations, not only are the amplitude and frequency of the oscillations and clearance spacing and hence the relative strength of the natural convection and oscillation induced flows important factors in the determination of the cooling effect, but the positioning of the oscillation sources also plays a significant role.

Finally, two dimensional finite element investigations of the combined method were performed. The studies for the plain channel geometry showed that significant flow through the openings provides for an additional flow stream through which cooler fluid can be brought to the heat source and heated fluid can be moved away. For lower oscillation parameters, most of the flow entering the space under the plate is carried downstream by natural convection. For higher oscillation parameters, some opening flow exits to the upper channel under both the upstream and downstream ends of the moving plate. In addition for lower oscillation parameters, when flow is from the upper channel to the lower, most of the flow upstream and over the opening exits through the opening. For higher oscillation parameters, flow through the opening as well as through the areas to the sides of the moving plate occurs when the opening flow is from the upper channel to the lower channel. When the plate comes close to the heat source flow is directed both into the opening and the upper channel. The altered velocity distribution therefore best cools just upstream of the opening but still under the moving plate as well as just downstream of the opening. The cooling effect logically increases with the oscillation amplitude and frequency with displacement of 10% of the channel width and oscillation frequency of at least $2\pi$ (82 rad/sec) being required for significant enhancement. Up to a
50% enhancement over the natural convection results with just the opening was achieved. However, there are some drawbacks to this method. While the higher oscillations parameters produce better thermal conditions near the heated surface close to the opening, the higher flow rates through the opening significantly change the flow patterns in both the upper and lower channels. In the upper channel, the flow just after the opening can be significantly restricted due to the outflow to the lower channel. This produces a pulsing type of flow in the upper channel, downstream of the opening, leading to a region of higher temperatures far from the opening.

The use of the combined method was then studied for a system consisting of two rectangular heat sources with an opening. Various oscillating plate positions were tested. It was found that the oscillation source placed upstream of both heat sources had little effect on the flow near the heat sources. However, the effect may be increased with the placement of the oscillation source closer to the heat source. An oscillation source placed just above the opening with a length less than the opening width but in the cavity between the heat sources obstructed the flow through the opening and resulted in higher temperatures than those without the oscillation source. The heat transfer coefficients on the side surfaces were lower than those without the oscillation source. Only a location on the top of the downstream heat source experienced an overall improvement. For the oscillation source placed over the opening, but positioned above the top surfaces of the heat source, little effect was found on the upstream heat source, but the downstream heat source experienced up to a 42% increase in the local heat transfer coefficient for a displacement of 20% of the block height. Finally, with the positioning of the oscillation source over the upstream heat source while the upstream heat source was cooled mainly
due to the effects of the oscillations, the downstream heat source was cooled mainly by the flow through the opening. The presence of the upstream plate prevented the decrease in the velocities in the flow over top of the downstream heat source that usually accompanies the use of the openings. A maximum increase in the local heat transfer coefficient relative to standard natural convection of 70% was achieved at the upstream heat source for the 20% heat source height displacement case while a 44% increase was achieved at the downstream heat source. Based on these results, the oscillation source position either over the upstream heat source or over the opening at the heat source top level shows greater potential for heat transfer enhancement.

The use of the oscillations as well as the combined method with the more complex heat and oscillation source geometries are expected to be most beneficial when used over the mixed convection regime where $\frac{Gr}{Re^2} \sim 0.1$ to $10$.

Comparisons to published results for systems using the flow passages and oscillation sources individually, though under forced convection conditions and for different geometries and systems, showed that the enhancement effects reported in this investigation are reasonable.

A number of significant findings regarding the cooling potential of the three cooling methods investigated were reported in this work and demonstrate that these methods have the potential for significant heat transfer enhancement of laminar natural convection cooling of electronics. The major advantages these methods offer over the convention methods are greater energy and space efficiency and reliability as well as the ability to target specific locations for cooling. This work provides the foundation for further exploration into the practical application of these three cooling methods.
8.2 Future Work

With the complexity of the finite element models and the extended finite volume program, the limited computational resources hindered the studies that could be undertaken due to the slow run times encountered.

For the finite element investigation, more extensive finite element studies of the effects of the oscillations for cases with higher displacements as well as for a range of Grashof numbers should be undertaken to determine the limits such a cooling method can provide under different oscillation and natural convection influences. The FIDAP© meshing algorithm also limited the displacements that could be used. Especially for higher oscillation parameters, larger circulation regions develop, resulting in low velocity, high temperature regions [60]. While this was briefly encountered for the opening geometry, it did not appear that such conditions were reached for other geometries.

The use of different oscillation source and heat source arrangements can also be explored. Perhaps a shorter channel length could be used with the plain channel with board opening geometry to suppress the development of the higher temperature region downstream. The upstream oscillation source geometry also seems to hold potential to be a beneficial arrangement. Not only does the reduced flow obstruction result in lower temperatures, but it also appears that the mean location of the oscillation source can be modified so as to force the flow over the top of the heat source and to cause the flow squeezed between the oscillation source and the heat source to fall just along the top heat source surface. Various double block arrangements with the openings also offer a number of possible future investigations. In addition, different oscillation source lengths
and orientations (direction of oscillations), and the use of multiple oscillation sources can be explored. Studies with non-square heat sources can also be initiated.

The developed finite volume program can be used for a more extensive investigation into the general effects of the oscillations. The effects of the oscillations can be tested for a broader parameter range of \( \frac{L}{b_o} \), \( a_o/b_o \), \( b_o \), frequency and heat source values. In this way a larger range of the relative natural convection/forced convection influences (\( Gr \) and \( Re \)) can be investigated. In addition, the developed program is general enough that it can be easily extended to study two-dimensional heat source geometries at the fixed surface. These studies would help to further investigate the flow over the two-dimensional heat sources and the cooling enhancement that can be achieved.
APPENDIX A

VERIFICATION OF FIDAP© SOLUTIONS FOR BUOYANCY DRIVEN FLOWS AND TRANSIENT MOVING BOUNDARY FLOWS

In order to validate the use of FIDAP© in this study, comparisons of FIDAP© solutions to benchmark theoretical and experimental solutions for both buoyancy driven flows and transient flows involving moving boundaries were made. Comparisons were performed for the following cases:

I. Buoyancy Driven Flows — Comparison With Theoretical and Experimental Results
   A. Buoyancy Driven Flows- Comparison with Theoretical Approximate Solution by Aung [24]
      i. Parallel Vertical Plates-Constant But Unequal Temperatures
      ii. Parallel Vertical Plates-Constant But Unequal Heat Fluxes
   B. Buoyancy Driven Flows – Comparison with Experimental Solutions by Darbe[87]
      i. Constant Temperature Block in Vertical Channel 60°C
      ii. Constant Temperature Block in Vertical Channel 101°C

II. Transient Moving Boundary Flows – Comparison With Theoretical Results
   A. Channel Flow- One Plate Fixed One Plate Moving at Constant Velocity-Fluid From Rest
   B. Channel Flow- One Plate Fixed One Plate Starts From Rest and Velocity Increases Linearly to a Constant Velocity-Fluid From Rest
   C. Channel Flow-One Plate Fixed One Plate Moving with Sinusoidal Velocity
A.1 Buoyancy Driven Flows – Comparison With Theoretical and Experimental Results

A.1.1 Buoyancy Driven Flows–Comparison with Theoretical Approximate Solution by Aung[24]

The first evaluation of the validity of the FIDAP© solutions was made comparing the finite element model solution results to two approximate theoretical solutions developed by Aung[24] for the case of “fully developed” laminar natural convection between two parallel vertical plates held at constant temperatures and then at constant heat fluxes.

Parallel Vertical Plates—Constant But Unequal Temperatures

For the case of the laminar natural convection between vertical flat plates held at constant but unequal temperatures, a comparison of the scaled dimensionless velocity and dimensionless temperature for the FIDAP© solution at the channel outlet and the theoretical solution is presented in Figure A.1 and shows the good agreement. For the scaled dimensionless velocity, the average percent root mean square error between the two solutions is less than 0.334% based on the maximum velocity and for the dimensionless temperature, the difference value based on the maximum temperature is less than 0.026%. The slight deviation in the results may be attributed to the fact that the flow at the channel outlet for the FIDAP© study may not satisfy the fully developed conditions imposed by Aung in his work.

**Figure A.1** Comparison of theoretical calculations for fully developed flow to FIDAP© results at channel outlet for constant temperature plates.
Parallel Vertical Plates-Constant But Unequal Heat Fluxes

The comparison between the "fully developed" laminar flow theoretical approximate solutions developed by Aung and the solutions from FIDAP© for the case of natural convection in a vertical channel with constant yet unequal wall heat fluxes was also made. Plots comparing the scaled dimensionless velocity and dimensionless temperature for the constant heat flux case for FIDAP© solution at the channel outlet and the theoretical solution are shown in Figure A.2 For the scaled dimensionless velocity, the average percent root mean square error between the two solutions is less than 0.118% based on the maximum velocity and for the dimensionless temperature, the difference value based on the maximum temperature is less than 0.309%. Again, this small difference results because the FIDAP© model does not completely match the fully developed conditions studied in the theoretical model. However, overall, there is good agreement between the approximate theoretical and FIDAP© solutions.
Figure A.2 Comparison of theoretical calculations for fully developed flow to FIDAP© results at channel outlet for constant heat flux plates.
A.1.2 Buoyancy Driven Flows - Comparison with Experimental Solutions by Darbe [87]

Experimental results for a natural convection problem were also tested against the FIDAP© solutions. Using an interferometer, Darbe [87] measured the resulting temperatures when a block heated to a constant temperature is placed in a vertical channel and cooled under natural convection. The results for constant block temperatures of 60 and 101°C are given. To arrive at the FIDAP© solution, an insulated boundary condition along the channel surfaces was applied. However, the actual boundary condition in the experiment might not have been fully insulated. The temperature contours are shown for constant block temperatures of 60°C in Figure A.3 and 101°C in Figure A.4. Because the temperature contours were not drawn at the same temperature levels, direct comparison of values is not possible. However, the plots show that the dimensionless temperature magnitudes are comparable. In addition, the basic trends in the contours are similar. Just behind the block, the contours fall vertically until intersecting the lower vertical channel surface perpendicularly. The temperature contours further from the block drop down after the block and continue to the outlet. Thus, it is concluded that the experimental results for natural convection in the vertical channel compare favorably to the FIDAP© results.
Note: Temperature Contours of the Same Levels were not possible, but the basic trends and range of magnitudes are similar.


**Figure A.3** Comparison of temperature contours constant surface temperature of 60.0°C: (a) experimental results Darbhe, (b) FIDAP© results.
Note: Temperature contours of the same levels were not possible, but the basic trends and range of magnitudes are similar.


**Figure A.4** Comparison of temperature contours constant surface temperature of 101.0°C: (a) experimental results Darbhe, (b) FIDAP© results.
A.2 Transient Moving Boundary Flows – Comparison With Theoretical Results

Analytical solutions for the velocity field within a parallel plate channel with one axially moving channel wall were determined for three different moving wall velocities. These solutions were then compared to the results obtained from a corresponding FIDAP© model. In each case the percent difference between the analytical solution and the solution predicted by FIDAP© were within 0.05% except for the first time step where the difference near x=1 approached 1%. Thus, the results showed excellent agreement between the analytical and FIDAP© transient results. The system investigated is sketched in Figure A.5. The channel wall at $x = 0$ is fixed and the channel wall at $x = 1$ is given a velocity $V(t)$ in the $z$ direction. For all cases, in the determination of the analytical solutions, the flow is assumed to be one-dimensional. Temperature variations are not included and all fluid is initially at rest. With these assumptions, the set of equations necessary to determine the $z$ component of velocity $v(x, t)$ for all of these benchmark cases reduce to:

$$\frac{\partial v}{\partial t} = \nu \frac{\partial^2 v}{\partial x^2}$$  \hspace{1cm} (A.1)

where $\nu$ is the kinematic viscosity and $v$ is the velocity in the $z$ direction.

Figure A.5 Sketch of system investigated.
A brief problem description for each of the three cases is given below followed by the comparison of the results.

**A.2.1 Case A-Channel Flow - One Plate Fixed One Plate Moving at Constant Velocity-Fluid Starts from Rest**

For this problem, a fixed plate is located at $x=0$ while a plate at $x = 1$ moves with a constant velocity as in the sketch below in Figure A.6. The fluid is initially at rest.

![Figure A.6 Sketch of system investigated for Case A.](image)

The exact theoretical solution for Case A is given by the following expression:

$$v(x,t) = \frac{x}{l} + \sum_{n=1}^{\infty} \left\{ \left( -\frac{2}{n \pi} \right)^n \sin \left( \frac{n \pi x}{l} \right) \exp \left( -\nu \left( \frac{n \pi}{l} \right)^2 t \right) \right\}$$

(A.2)

where for the system in Figure A.4a, with $l=1$, $Z=10$ and $\nu=1$.

The comparison between the FIDAP and exact theoretical solutions for $v(x,t)$ is given in Figure A.7.
Figure A.7 Comparison of results for Case A - plate moving at a constant velocity-fluid initially at rest- theoretical and FIDAP© solutions.
A.2.2 Case B - Channel Flow -One Plate Fixed One Plate Starting From Rest and Velocity Increases Linearly to a Constant Velocity-Fluid From Rest

For this case, a fixed plate is located at $x=0$ while a plate at $x = 1$ starts from rest and linearly reaches a constant velocity at $t = 1/a$. The fluid is initially at rest.

\[ \begin{align*}
\text{v}(x=0,t) &= 0 \\
\text{v}(x=1,t) &= at \quad t \leq 1/a \\
\text{v}(x=1,t) &= 1 \quad t > 1/a \\
\text{where} \; a &= 10 \\
\text{v}(x,t=0) &= 0 \\
\nu &= 1
\end{align*} \]

**Figure A.8** Sketch of system investigated for Case B.

The exact theoretical solution for Case B is given by the following expression:

For $t \leq \frac{1}{a}$

\[
\text{v}(x,t) = \sum_{n=1}^{\infty} \left\{ \left( \frac{-2}{n\pi} \right)^n \sin \left( \frac{n\pi x}{l} \right) \left[ \left( \frac{-1}{\nu \left( \frac{n\pi}{l} \right)^2} + t \right) - \exp \left( -\nu \left( \frac{n\pi}{l} \right)^2 t \right) \right] \right\}
\]

For $t > \frac{1}{a}$

\[
\text{v}(x,t) = \sum_{n=1}^{\infty} \left\{ \left( \frac{-2}{n\pi} \right)^n \sin \left( \frac{n\pi x}{l} \right) \left[ 1 - \exp \left( -\nu \left( \frac{n\pi}{l} \right)^2 \left( t - \frac{1}{a} \right) \right) \right] \right\} + a \sum_{n=1}^{\infty} \left\{ \left( \frac{-2}{n\pi} \right)^n \left( \nu \left( \frac{n\pi}{l} \right)^2 \right) \sin \left( \frac{n\pi x}{l} \right) \left[ \exp \left( -\nu \left( \frac{n\pi}{l} \right)^2 t \right) \right] \right\}
\]

\[
\left[ \left( \frac{1}{a} - 1 \right) \frac{\exp \left( -\nu \left( \frac{n\pi}{l} \right)^2 \frac{1}{a} \right)}{\nu \left( \frac{n\pi}{l} \right)^2} + 1 \right]
\]

where for the current problem $l=1$, $Z=10$, $a=10$, and $\nu =1$. The comparison between the FIDAP and exact theoretical solutions for $v(x,t)$ is given in Figure A.9.
Figure A.9 Comparison of results for Case B- plate moving from rest increasing linearly to constant velocity-fluid initially at rest- theoretical and FIDAP© solutions.
A.2.3 Case C-Channel Flow- One Plate Fixed One Plate Moving with Sinusoidal Velocity-Fluid From Rest

For this problem, a fixed plate is located at $x = 0$ while a plate at $x = 1$ moves with a sinusoidal velocity. The fluid is initially at rest.

\[
\begin{align*}
v(x=0,t) &= 0 \\
v(x=1,t) &= A \omega \cos(\omega t) \\
\text{where } A &= 1 \\
\omega &= 1 \\
v(x,t=0) &= 0 \\
v &= 1
\end{align*}
\]

**Figure A.10** Sketch of system investigated for Case C.

The exact theoretical solution for Case C is given by the following expression:

\[
v(x,t) = \frac{A \omega x}{l} \cos(\omega t) + \sum_{n=1}^{\infty} \left\{ A \omega \left( \frac{2}{n\pi} \right) (-1)^n \sin \left( \frac{n\pi x}{l} \right) \exp \left[ -\nu \left( \frac{n\pi}{l} \right)^2 t \right] \right\}
\]

\[
+ \sum_{n=1}^{\infty} \left\{ \frac{-A \omega^2}{1 + \frac{\omega^2}{\left( \frac{2}{n\pi} \right)^2}} \left( \frac{2}{n\pi} \right) (-1)^n \sin \left( \frac{n\pi x}{l} \right) \right\}
\]

\[
\left[ \sin(\omega t) - \frac{\omega \cos(\omega t)}{\left( \frac{n\pi}{l} \right)^2} + \frac{\omega}{\left( \frac{n\pi}{l} \right)^2} \exp \left[ -\nu \left( \frac{n\pi}{l} \right)^2 t \right] \right]\]

(A.4)

with $l=1$, $Z=10$, $A=1$, $\omega=1$, and $\nu =1$. The comparison between the FIDAP and exact theoretical solutions for $v(x,t)$ is given in Figure A.11.
Figure A.11 Comparison of results for Case C - plate moving with a sinusoidal velocity - fluid initially at rest - theoretical and FIDAP© solutions.
The development of the dimensionless forms of the governing equations used in the finite element portion of this investigation is discussed in this appendix. The nature of the finite element package used in this work, FIDAP©, restricts the form of the dimensionless variables that may be used. In the scope of the present work, FIDAP© solves the equations governing fluid flow and heat transfer, solid/fluid interactions, and heat flow in the solids of a specific form that cannot be altered. This set form follows the equations presented in Chapter 2. In order for the dimensionless equations be of the proper form, only one reference length can be used for non-dimensionalizing both $x$ and $y$ and one reference velocity must be used to non-dimensionalize both $u$ and $v$. The reference for time and pressure must be chosen so as to be consistent. A reference temperature difference, a constant value used for both the temperatures in the fluid and any solids, must also be specified. Therefore, non-dimensionalizing a model for use with FIDAP© is a process that can be thought of as scaling the physical domain and specifying "equivalent property" values for the system materials and system properties obtained by comparing the coefficients of dimensionless form of the equations to those coefficients in the standard set of governing equations as listed in Section 2.3. Hence, in this investigation, the same basic set of dimensionless variables is used in the steady state investigations of Chapter 3 as in the transient oscillating body studies in Chapters 6 and 7. The specific set of dimensionless primary and calculated variables used is discussed below.
B.1 Dimensionless Variables and FIDAP©

The general set of dimensionless primary variables in (B.1) can be applied to the governing equations in Section 2.3 with the subscript F denoting the dimensionless quantities used with FIDAP©.

\[
\begin{align*}
\tilde{x}_F &= \frac{x}{L_{ref}} \quad \tilde{y}_F &= \frac{y}{L_{ref}} \quad \tilde{g}_{comp} &= \frac{g_{comp}}{g} \quad \tilde{P}_F &= \frac{P_D}{P_{ref}} \\
\tilde{u}_F &= \frac{u}{U_{ref}} \quad \tilde{v}_F &= \frac{v}{U_{ref}} \quad \tilde{t}_F &= \frac{t}{t_{ref}} \quad \tilde{T}_F &= \frac{(T-T_0)}{\Delta T_{ref}}
\end{align*}
\]  

(B.1)

The reference values of the dimensionless variables are either set to significant system parameters or specified to maintain consistency or a certain equation form [86]. In this investigation, the reference length, \(L_{ref}\), is set to an important system dimension and \(g\) is the magnitude of the gravitational acceleration. The remaining reference values, with the exception of the reference temperature difference, are specified during the process of non-dimensionalizing the momentum equation and can be selected in a similar manner regardless of the problem being investigated. The value of the reference temperature difference depends on the problem being investigated because in the FIDAP© investigations in the present work, two types of heat sources are used: a constant heat flux surface and a constant volumetric heat source solid. In the current investigation, for the constant heat flux surface case, the reference temperature difference is selected as

\[
\Delta T_{ref} = \frac{q_a L_{ref}}{k} \quad (q_a \text{ is applied heat flux}),
\]

while for the constant volumetric heat source solid heat source, a reference heat generation rate is selected equal to the heat source heat generation rate, or \(Q_{ref}" = Q_a"\) (\(Q_a"\) is applied volumetric heat rate). The reference temperature difference for this case is then determined by the requirement of consistency.
of the heat flow across solid-solid and solid-fluid boundaries with more specific information discussed later. The equivalent thermal properties of all solids are also determined through the consistent heat flux requirement as well.

In addition to the primary variables, important values are calculated from the velocity, pressure, and temperature field results determined by FIDAP©. Just as the equations that FIDAP© solves are of a preset format, so are the equations that it uses to calculate values from the results. Specifically for the temperature field, the heat flux values given are of the form:

$$q = -k \frac{\partial T}{\partial n}$$  \hspace{1cm} (B.2)

and the heat transfer coefficients are calculated as:

$$h = \frac{q}{T}$$  \hspace{1cm} (B.3)

Consequently, as the $\Delta T_{ref}$ differs for these two heat sources type models so do the reference values for the related calculated variables including the reference heat flux and heat transfer coefficients needed for interpreting the results.

For the two-dimensional study, the flow rate through a given surface per unit depth is calculated as:

$$Q_{flow} = \int_{s} \vec{v} \cdot d\vec{s}$$  \hspace{1cm} (B.4)

where $\vec{v}$ is the velocity vector

$\vec{s}$ is a vector with direction a perpendicular to the surface and a magnitude equal to the length along the surface.

Regardless of these differences, the general process of non-dimensionalization the heat and fluid flow equations is the same.
B.2 Dimensionless Equations Governing Fluid Flow and Heat Transfer

For both the heat flux surface and the solid body heat source cases, the same form of the fluid flow equations must be solved and the same general process of non-dimensionalizing the heat and fluid flow equations regardless of the differences in the reference temperature difference values. Applying the dimensionless variables in (B.1) to the continuity equation in Eq. (2.1) yields:

$$\frac{\partial \tilde{u}_F}{\partial \tilde{x}_F} + \frac{\partial \tilde{v}_F}{\partial \tilde{y}_F} = 0$$  \hspace{1cm} (B.5)

Applying the dimensionless variables to the x component of the momentum equation in Eq. (2.12), it is clear that to be consistent with the form of the equation FIDAP© solves, the reference time must be:

$$t_{ref} = \frac{L_{ref}}{U_{ref}}  \hspace{1cm} (B.6)$$

Making this substitution in Eq. (2.12), and dividing by $g\beta \rho \Delta T_{ref}$ produces:

$$\frac{\rho U_{ref}^2}{L g \beta \rho \Delta T_{ref}} \left( \frac{\partial \tilde{u}_F}{\partial \tilde{x}_F} + \tilde{u}_F \frac{\partial \tilde{u}_F}{\partial \tilde{x}_F} + \tilde{v}_F \frac{\partial \tilde{u}_F}{\partial \tilde{y}_F} \right) =$$

$$- \frac{P_{ref}}{L g \beta \rho \Delta T_{ref}} \frac{\partial \tilde{p}_F}{\partial \tilde{x}_F} + \frac{\mu U_{ref}}{L^2 g \beta \rho \Delta T_{ref}} \left( \frac{\partial^2 \tilde{u}_F}{\partial \tilde{x}_F^2} + \frac{\partial^2 \tilde{u}_F}{\partial \tilde{y}_F^2} \right) - g_{comp} \tilde{T}_F  \hspace{1cm} (B.7)$$

In the development of this general set of dimensionless equations, no term is assumed more important than another for the problems being investigated and as such the non-dimensional expressions in each term may not be of the same order of magnitude. In this investigation, the following reference values are then selected.

$$U_{ref} = \sqrt{L g \beta \Delta T_{ref}}  \hspace{1cm} (B.8)$$

$$P_{ref} = L g \beta \rho \Delta T_{ref}  \hspace{1cm} (B.9)$$
The coefficient of the shear force term is related to a dimensionless group that can be defined called the Grashof number, $Gr$, which is a measure of the buoyancy to shear to inertia forces and buoyancy forces.

$$Gr = \frac{\beta g \Delta T_{ref} L^3}{\nu^2} \quad \text{(B.10)}$$

The dimensionless form of the $x$ component of the momentum equation can be written as:

$$\left( \frac{\partial \bar{u}_x}{\partial \bar{x}_F} + \bar{u}_x \frac{\partial \bar{u}_x}{\partial \bar{x}_F} + \bar{v}_F \frac{\partial \bar{u}_x}{\partial \bar{y}_F} \right) = -\frac{\partial \bar{p}_x}{\partial \bar{x}_F} + \frac{1}{\sqrt{Gr}} \left( \frac{\partial^2 \bar{u}_x}{\partial \bar{x}_F^2} + \frac{\partial^2 \bar{u}_x}{\partial \bar{y}_F^2} \right) - \bar{g}_{comp} \bar{T}_F \quad \text{(B.11)}$$

As mentioned previously, because FIDAP© solves an equation of the set form given in Eq.(2.12), the altered coefficients in the dimensionless form in Eq.(B.11) can be interpreted as equivalent properties, or Eq. (B.11) can be written as:

$$\rho_{equ} \left( \frac{\partial \bar{u}_x}{\partial \bar{x}_F} + \bar{u}_x \frac{\partial \bar{u}_x}{\partial \bar{x}_F} + \bar{v}_F \frac{\partial \bar{u}_x}{\partial \bar{y}_F} \right) = -\frac{\partial \bar{p}_x}{\partial \bar{x}_F} + \rho_{equ} \left( \frac{\partial^2 \bar{u}_x}{\partial \bar{x}_F^2} + \frac{\partial^2 \bar{u}_x}{\partial \bar{y}_F^2} \right) - (\rho \beta g)_{equ} \bar{T}_F \quad \text{(B.12)}$$

with the following equivalent properties defined:

$$\rho_{equ} = 1 \quad \text{(B.12a)}$$

$$\mu_{equ} = \frac{1}{\sqrt{Gr}} \quad \text{(B.12b)}$$

$$\beta g_{equ} = 1 \quad \text{(B.12c)}$$

The $y$ component of the momentum equation in Eq. (2.13) is non-dimensionalized in a similar manner yielding:

$$\left( \frac{\partial \bar{v}_y}{\partial \bar{t}_F} + \bar{u}_x \frac{\partial \bar{v}_y}{\partial \bar{x}_F} + \bar{v}_F \frac{\partial \bar{v}_y}{\partial \bar{y}_F} \right) = -\frac{\partial \bar{p}_y}{\partial \bar{y}_F} + \frac{1}{\sqrt{Gr}} \left( \frac{\partial^2 \bar{v}_y}{\partial \bar{x}_F^2} + \frac{\partial^2 \bar{v}_y}{\partial \bar{y}_F^2} \right) \quad \text{(B.13)}$$

or recalling the equivalent properties defined above:

$$\rho_{equ} \left( \frac{\partial \bar{v}_y}{\partial \bar{t}_F} + \bar{u}_x \frac{\partial \bar{v}_y}{\partial \bar{x}_F} + \bar{v}_F \frac{\partial \bar{v}_y}{\partial \bar{y}_F} \right) = -\frac{\partial \bar{p}_y}{\partial \bar{y}_F} + \rho_{equ} \left( \frac{\partial^2 \bar{v}_y}{\partial \bar{x}_F^2} + \frac{\partial^2 \bar{v}_y}{\partial \bar{y}_F^2} \right) \quad \text{(B.14)}$$
The temperature field in the fluid is found through the energy equation given in Eq. (2.7). Introducing the appropriate dimensionless variables in Eq. (B.1) into this equation, dividing by $U_{ref}$, and making the proper parameter substitutions, the non-dimensional form of the energy equation becomes:

$$
\left( \frac{\partial \tilde{T}_e}{\partial \tilde{x}_e} + \tilde{u}_e \frac{\partial \tilde{T}_e}{\partial \tilde{x}_e} + \tilde{v}_e \frac{\partial \tilde{T}_e}{\partial \tilde{y}_e} \right) = \frac{1}{Pr \sqrt{Gr}} \left( \frac{\partial^2 \tilde{T}_e}{\partial \tilde{x}_e^2} + \frac{\partial^2 \tilde{T}_e}{\partial \tilde{y}_e^2} \right) \quad (B.15)
$$

Again, comparing the coefficients of each term with those in the set form of the energy equation in Eq. (2.7), this equation can be written in the form:

$$
(\rho c_p)_{equ} \left( \frac{\partial \tilde{T}_e}{\partial \tilde{x}_e} + \tilde{u}_e \frac{\partial \tilde{T}_e}{\partial \tilde{x}_e} + \tilde{v}_e \frac{\partial \tilde{T}_e}{\partial \tilde{y}_e} \right) = k_{equ} \left( \frac{\partial^2 \tilde{T}_e}{\partial \tilde{x}_e^2} + \frac{\partial^2 \tilde{T}_e}{\partial \tilde{y}_e^2} \right) \quad (B.16)
$$

by defining the following equivalent properties:

$$
\rho_{equ} = 1 \quad (B.16a)
$$

$$
c_{pequ} = 1 \quad (B.16b)
$$

$$
k_{equ} = \frac{1}{Pr \sqrt{Gr}} \quad (B.16c)
$$

**B.3 Dimensionless Energy Equation in Solid Bodies**

For problems involving solid bodies or constant volumetric heat generation heat sources, the temperature distribution in the solid must also be determined. (Again all solids are assumed rigid and non-deforming for this investigation with any motion strictly specified). The energy equation in the form of Eq. (2.14) governs the temperature in these solid bodies. For a solid body not generating heat, the value of $Q'''$ is zero. Here, the method of non-dimensionalizing is dependent on the problem being investigated.
B.3.1 Constant Heat Flux Heat Source

For the constant heat flux heat source cases where all solids do not generate heat, the reference temperature difference has already been specified. A subscript $s$ will denote the properties associated with the solid. After direct substitution of the non-dimensional temperature into Eq. (2.14) with $Q''$ set equal to zero and multiplying by $\frac{k_s}{U_{ref} L_{ref}}$ the non-dimensional form of the energy equation can be written as:

$$
\rho_s c_p \frac{\partial \tilde{T}_F}{\partial t_F} = \frac{k_s}{L_{ref} U_{ref}} \left( \frac{\partial^2 \tilde{T}_F}{\partial x_F^2} + \frac{\partial^2 \tilde{T}_F}{\partial y_F^2} \right)
$$

(B.17)

However, for use with FIDAP®, Eq. (B.17) must be in the form:

$$(\rho c_p)_e \frac{\partial \tilde{T}_F}{\partial t_F} = k_e \left( \frac{\partial^2 \tilde{T}_F}{\partial x_F^2} + \frac{\partial^2 \tilde{T}_F}{\partial y_F^2} \right)
$$

(B.18)

The values used in the non-dimensional forms of all equations are not arbitrary and must be consistent between the various materials and governing equations as well as the boundary conditions. Since perfect thermal contact is assumed, both the temperatures of the two substances at the boundary must be equal as well as the heat fluxes through each material at the boundary. The non-dimensionalization of these heat flux boundary conditions provides the additional constraints needed to link non-dimensional parameters related to the fluid and the solid regions. Therefore, the continuity of the heat flux is needed to specify the equivalent properties. The consistency of the heat flux yields:

$$
-k_s \frac{\partial T}{\partial n} \bigg|_s = -k \frac{\partial T}{\partial n} \bigg|_F
$$

(B.19)

where $n$ can represent any spatial direction.
Non-dimensionalizing:

\[- \frac{k_s \Delta T_{ref}}{L_{ref}} \frac{\partial \tilde{T}_F}{\partial \tilde{n}_F} \]  
\[- \frac{k \Delta T_{ref}}{L_{ref}} \frac{\partial \tilde{T}_F}{\partial \tilde{n}_F} \]  

This can be written in the form:

\[- k_{equ} \frac{\partial \tilde{T}_F}{\partial \tilde{n}_F} \]  
\[- k \frac{\partial \tilde{T}_F}{\partial \tilde{n}_F} \]  

From the ratio of Eq. (B.20) and Eq. (B.21), it can be shown that:

\[ k_{equ} = k \left( \frac{k_s}{k} \right) \]  

where \( k_{equ} = \frac{1}{Pr} \frac{1}{\sqrt{Gr}} \) Eq. (B.16c) and \( Gr \) is defined in Eq. (B.10)

Consequently, from the ratio of Eq. (B.17) and Eq. (B.18):

\[ \left( \frac{\rho c_p}{\rho s c_p} \right)_{equ} = \frac{\rho c_p}{\rho s c_p} \]  

With these specifications, the dimensionless form of the governing energy equation for the solid under the heat flux heat source case is complete.

The reference values for non-dimensionalizing the calculated values related to the temperature including the heat flux and heat transfer coefficient can be specified at this point. Applying the dimensionless variables in Eq. (B.1) to Eq. (B.2) yields:

\[ q = - \frac{k \Delta T_{ref}}{L_{ref}} \frac{\partial \tilde{T}_F}{\partial \tilde{n}_F} \]  

The heat flux calculated by FIDAP© is of the form of Eq. (B.2) or in this case:
Applying the dimensionless variables in Eq. (B.1) to heat transfer coefficient in Eq. (13.3) yields:

\[ \bar{q}_F = \frac{q}{q_{ref}} = -k_{equ} \frac{\partial \bar{T}_F}{\partial n_F} \]  (B.25)

It can be shown that:

\[ q_{ref} = q_a \Pr \sqrt{Gr} \]  (B.26)

Applying the dimensionless variables in Eq. (B.1) to heat transfer coefficient in Eq. (B.3) yields:

\[ h = \frac{q_{ref} \bar{q}_F}{\Delta T_{ref} \bar{T}_F} \]  (B.27)

The heat transfer coefficient calculated by FIDAP© is of the form of Eq. (B.3) or, for this case:

\[ \bar{h}_F = \frac{h}{h_{ref}} = \frac{\bar{q}_F}{\bar{T}_F} \]  (B.28)

It can be shown that:

\[ h_{ref} = \frac{k}{k_{equ} T_{ref}} \]  (B.29)

With this, the non-dimensionalization process for the case with the constant heat flux boundary condition is complete. In addition to the reference values used for the primary variables given in Table 2.1, Table B.1 lists the equivalent property values while Table B.2 lists the reference values for the heat flux, heat transfer coefficient and flow rate.

**B.3.2 Constant Volumetric Heat Source Solid**

For the constant volumetric heat source solid cases, the energy equation in the heat source, the solids not generating heat, and the consistency of the heat flux across solid-fluid and solid-solid boundaries are used to complete the non-dimensionalization process.
A solid generating heat is denoted by 1, and a solid not generating heat is denoted by 2.

Substituting the non-dimensional variables into Eq. (2.7) and dividing by \( Q_{ref}^* \) yields:

\[
\frac{\rho c_{pl} U_{ref} \Delta T_{ref}}{Q_{ref}^* L_{ref}} \frac{\partial \tilde{T}_F}{\partial \tilde{t}_F} = \frac{k_1 \Delta T_{ref}}{L_{ref}^2 Q_{ref}^*} \left( \frac{\partial^2 \tilde{T}_F}{\partial \tilde{x}_F^2} + \frac{\partial^2 \tilde{T}_F}{\partial \tilde{y}_F^2} \right) + \tilde{Q}_F^* \tag{B.30}
\]

where

\[
\tilde{Q}_F^* = \frac{Q^*}{Q_{ref}^*} \quad Q_{ref}^* = Q_o^*
\]

Again, comparing the coefficients of each term with those in the set form of the energy equation in Eq. (2.7), this equation can be written in the form:

\[
\rho_{equ} c_{equ} \frac{\partial \tilde{T}_F}{\partial \tilde{t}_F} = k_{equ} \left( \frac{\partial^2 \tilde{T}_F}{\partial \tilde{x}_F^2} + \frac{\partial^2 \tilde{T}_F}{\partial \tilde{y}_F^2} \right) + \tilde{Q}_F^* \tag{B.32}
\]

where:

\[
k_{equ} = \frac{k_1 \Delta T_{ref}}{L_{ref}^2 Q_{ref}^*} \tag{B.32a}
\]

\[
\rho_{equ} = 1 \tag{B.32b}
\]

\[
c_{equ} = \frac{\rho c_{pl} U_{ref} \Delta T_{ref}}{Q_{ref}^* L_{ref}} \tag{B.32c}
\]

Though \( Q_{ref}^* \) is a known quantity, the value of the \( \Delta T_{ref} \) is still an unknown that needs to be determined. First, the energy equation for solid 2, not generating heat is non-dimensionalized \((Q^* = 0)\) in a similar manner yielding:

\[
\rho_2 c_{p2} U_{ref} \frac{\partial \tilde{T}_F}{\partial \tilde{t}_F} = \frac{k_2}{L_{ref}} \left( \frac{\partial^2 \tilde{T}_F}{\partial \tilde{x}_F^2} + \frac{\partial^2 \tilde{T}_F}{\partial \tilde{y}_F^2} \right) \tag{B.33}
\]
Again, comparing the coefficients of each term with those in the set form of the energy equation in Eq. (2.7), this equation can be written in the form:

\[
\rho_{2equ} c_{p2equ} \frac{\partial \tilde{T}_F}{\partial t_F} = k_{2equ} \left( \frac{\partial^2 \tilde{T}_F}{\partial x_F^2} + \frac{\partial^2 \tilde{T}_F}{\partial y_F^2} \right)
\] (B.34)

Taking the ratio of Eq. (B.33) and Eq. (B.34), it can be shown that with

\[
\rho_{2equ} = 1
\] (B.34a)

and

\[
c_{p2equ} = \frac{k_{2equ} L_{ref} U_{ref} \rho_2 c_{p2}}{k_2}
\] (B.34c)

can be specified. An additional constraint is required to determine \( k_{2equ} \). As previously described, since perfect thermal contact between the two solids is assumed, both the temperatures of any two substances at the boundary must be equal as well as the heat flux through each material at the boundary. The non-dimensionalization of these heat flux boundary conditions provides the additional constraints needed to link non-dimensional parameters related to the fluid and the two solid regions. At the solid 1 – fluid boundary, the continuity of the heat flux yields:

\[
-k_1 \frac{\partial T}{\partial n} \bigg|_1 = -k \frac{\partial T}{\partial n} \bigg|_\text{ref}
\] (B.35)

or non-dimensionalizing:

\[
- \frac{k_1 \Delta T_{ref}}{L_{ref}} \frac{\partial \tilde{T}_F}{\partial n_F} \bigg|_1 = - \frac{k \Delta T_{ref}}{L_{ref}} \frac{\partial \tilde{T}_F}{\partial n_F}
\] (B.36)
Writing Eq. (B.36) in the form:

$$-k_{leq} \frac{\partial \tilde{T}_F}{\partial n_F} = -k_{leq} \frac{\partial \tilde{T}_F}{\partial n_F}$$  \hspace{1cm} (B.37)

and taking the ratio of Eq. (B.36) and Eq. (B.37), it can be shown that:

$$k_{leq} = k_{leq} \frac{k_i}{k}$$  \hspace{1cm} (B.38)

where

$$k_{leq} = \frac{1}{Pr} \frac{1}{\sqrt{Gr}}$$  \hspace{1cm} (B.16c)

and $Gr$ is defined in Eq. (B.10)

The expression in Eq. (B.38) can then be used to determine the $\Delta T_{ref}$ value. Equating the expression in Eq. (B.38) to the expression in Eq. (B.32a) and making the appropriate substitutions for $Gr$ and $k_{leq}$, it can be shown that:

$$\Delta T_{ref} = \left( \frac{Q^m_{ref}}{\rho c_p} \right)^{\frac{1}{2}} \left( \frac{L_{ref}}{\beta g} \right)^{\frac{1}{2}}$$  \hspace{1cm} (B.39)

Knowing the expression for $\Delta T_{ref}$, the expression for $c_{pleq}$ can be found. Substituting Eq. (B.16c) and Eq. (B.39) into Eq. (B.32c), it can be shown that:

$$c_{pleq} = \frac{c_{pl}}{\rho c_p}$$  \hspace{1cm} (B.40)

At the solid 2-fluid heat flux boundary condition, similar results for the expressions relating the equivalent thermal conductivities of the solid 2, which does not generate heat, and the fluid can be found at the boundary between the solid 2, and the fluid. The boundary condition for the continuity of the heat flux is:
Using a similar procedure as above it can be shown that

\[
-k_2 \frac{\partial T}{\partial n}_2 = -k \frac{\partial T}{\partial n}
\]  

(B.41)

This expression for \(k_{2\text{equ}}\) can then be used to determine the expression for \(c_{p2\text{equ}}\).

Substituting Eq. (B.41) and Eq. (B.16c) into Eq. (8.34c), it can be shown that:

\[
c_{p2\text{equ}} = \frac{\rho_2 c_{p2}}{\rho c_p}
\]  

(B.43)

Finally, the continuity of the heat flux between the two solids yields:

\[
-k_2 \frac{\partial T}{\partial n}_2 = -k_1 \frac{\partial T}{\partial n}_1
\]  

(B.44)

Using the procedure discussed earlier:

\[
k_{2\text{equ}} = \frac{k_2}{k_1} k_{1\text{equ}}
\]  

(B.45)

This is consistent with the equivalent thermal conductivities found in Eqs. (B.38) and (B.42).

The reference values for non-dimensionalizing the calculated values related to the temperature including the heat flux and heat transfer coefficient can be specified at this point A before, applying the dimensionless variables in Eq. (B.1) to Eq. (B.2) yields:

\[
q = -\frac{k \Delta T_{\text{ref}}}{L_{\text{ref}}} \frac{\tilde{T}_F}{\tilde{n}_F}
\]  

(B.24)
The heat flux calculated by FIDAP© is of the form of Eq.(B.2) or in this case:

\[ \tilde{q}_F = \frac{q}{q_{ref}} = -k_{equ} \tilde{T}_{F} \frac{\partial \tilde{T}_{F}}{\partial n_{F}} \]  

(B.25)

It can be shown that for this case:

\[ q_{ref} = Q^{m}_{ref} L_{ref} = Q^{m}_{a} L_{ref} \]  

(B.46)

As before, applying the dimensionless variables in Eq. (B.1) to heat transfer coefficient in Eq. (B.3) yields:

\[ h = \frac{q_{ref} q_F}{\Delta T_{ref} T_{F}} \]  

(B.27)

The heat transfer coefficient calculated by FIDAP© is of the form of Eq. (B.3) or, for this case:

\[ \tilde{h}_F = \frac{h}{h_{ref}} = \frac{\tilde{q}_F}{T_{F}} \]  

(B.40)

It can be shown that:

\[ h_{ref} = \frac{k}{k_{equ}} L_{ref} \frac{Q^{m}_{a} L_{ref}}{\Delta T_{ref}} \]  

(B.47)

With this, the non-dimensionalization process for the case with the solid volumetric heat rate is complete. In addition to the reference values used for the primary variables given in Table 2.1, Table B.1 lists the equivalent property values while Table B.2 lists the reference values for the heat flux, heat transfer coefficient and flow rate.
B.4 Non-Dimensionalization of Flow Rate

In addition to the heat flux and heat transfer coefficient there are important values used in this study that are non-dimensionalized. The volume flow rate is an important value in determining the cooling effect. The flow rate per unit depth as calculated by FIDAP© is of the form given in Eq. (B.4):

\[ \tilde{Q}_{\text{flow}} = \int_{s} \tilde{v} \cdot ds \]  

(B.48)

Non-dimensionalizing Eq. (B.4) and using Eq. (B.1):

\[ Q_{\text{flow}} = Q_{\text{flow,ref}} \tilde{Q}_{\text{flow}} = U_{\text{ref}} I_{\text{ref}} \int_{s} \tilde{v} \cdot ds \]  

(B.49)

Therefore,

\[ Q_{\text{flow,ref}} = U_{\text{ref}} I_{\text{ref}} \]  

(B.50)

---

**Table B.1 Summary of Equivalent Properties For Studies Indicated**

<table>
<thead>
<tr>
<th>Study</th>
<th>( \rho_{\text{equ}} )</th>
<th>( c_{p,\text{equ}} )</th>
<th>( k_{\text{equ}} )</th>
<th>( \mu_{\text{equ}} )</th>
<th>( \beta_{\text{equ}} )</th>
<th>( g_{\text{equ}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIDAP — Fluid</td>
<td>1</td>
<td>1</td>
<td>( \frac{1}{\text{Pr} \sqrt{Gr}} )</td>
<td>( \frac{1}{\sqrt{Gr}} )</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>FIDAP - Constant Heat Flux Heat Source</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Solid No Heat Generation(s)</td>
<td>1</td>
<td>( \frac{\rho_{1} c_{p,1}}{\rho_{c} p} )</td>
<td>( \frac{k_{1} k_{\text{equ}}}{k} )</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>FIDAP Constant Volumetric Rate Solid Heat Source</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>FIDAP Solid No Heat Generation(1)</td>
<td>1</td>
<td>( \frac{\rho_{1} c_{p,1}}{\rho_{c} p} )</td>
<td>( \frac{k_{1} k_{\text{equ}}}{k} )</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
<tr>
<td>Solid Heat Generation(2)</td>
<td>1</td>
<td>( \frac{\rho_{2} c_{p,2}}{\rho_{c} p} )</td>
<td>( \frac{k_{2} k_{\text{equ}}}{k} )</td>
<td>N/A</td>
<td>N/A</td>
<td>N/A</td>
</tr>
</tbody>
</table>

where \( k_{\text{equ}} = \frac{1}{\text{Pr} \sqrt{Gr}} \)

\[ Gr = \frac{\beta g \Delta T_{\text{ref}} L^{3}}{v^{2}} \]

See Table 2.1 p. 65 for \( \Delta T_{\text{ref}} \)
Table B.2 Reference Values for Non-Dimensionalizing Calculated Values for Cases Indicated

<table>
<thead>
<tr>
<th>Study</th>
<th>( q_{\text{ref}} )</th>
<th>( Q''''_{\text{ref}} )</th>
<th>( h_{\text{ref}} )</th>
<th>( Q_{\text{flowref}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FIDAP - Constant Heat Flux Heat Source</td>
<td>( q_a )</td>
<td>( N/A )</td>
<td>( k )</td>
<td>( U_{\text{ref}} L_{\text{ref}} )</td>
</tr>
<tr>
<td>FIDAP Constant Volumetric Rate Solid Heat Source</td>
<td>( Q''<em>{\text{ref}} L</em>{\text{ref}} )</td>
<td>( Q'''_{\text{ref}} a )</td>
<td>( k )</td>
<td>( U_{\text{ref}} L_{\text{ref}} )</td>
</tr>
</tbody>
</table>

where \( k_{\text{equ}} = \frac{1}{Pr} \frac{1}{\sqrt{Gr}} \), \( Gr = \frac{\beta g \Delta T_{\text{ref}} L^3}{v^2} \). See Table 2.1 p.65 for \( \Delta T_{\text{ref}} L_{\text{ref}} U_{\text{ref}} \)

B.5 Additional Non-Dimensionalization For Oscillating Body

Finally, in the investigations of the effects of the oscillating moving body on heat transfer, additional variables and parameters must be defined. The dimensional displacement and velocity of the oscillating body must be specified and are given below:

\[ d_{\text{os}} = A \sin(\omega t) \]  \hspace{1cm} (B.51)

\[ V_{\text{os}} = A \omega \cos(\omega t) \]  \hspace{1cm} (B.52)

where \( A \) is the displacement amplitude and \( \omega \) is the frequency of the oscillation. This displacement and velocity can then be non-dimensionalized. While the oscillation source displacement can be non-dimensionalized by the reference length, \( L_{\text{ref}} \), and the oscillation velocity can be non-dimensionalized by the reference velocity \( U_{\text{ref}} \), a new non-dimensional variable, the dimensionless frequency, needs to be defined to maintain consistency with the time reference already specified.

\[ \tilde{\omega}_{v} = \omega \frac{L_{\text{ref}}}{U_{\text{ref}}} \]  \hspace{1cm} (B.53)

Applying the non-dimensional variables as defined Eq. (B.1) and Eq. (B.53), the dimensionless displacement and velocity of the oscillation source can then be written as:
\[ \tilde{a}_{\text{osf}}(\tilde{t}_F) = \left( \frac{A}{L_{\text{ref}}} \right) \sin(\tilde{\omega}_F \tilde{t}_F) \]  
(B.54)

\[ \tilde{V}_{\text{osf}}(\tilde{t}_F) = \left( \frac{A \tilde{\omega}_F}{L_{\text{ref}}} \right) \cos(\tilde{\omega}_F \tilde{t}_F) \]  
(B.55)

The dimensionless period of oscillation is defined as:

\[ \tilde{T}_{\text{osf}} = \frac{2\pi}{\tilde{\omega}_F} \]  
(B.56)

The dimensionless parameters are summarized in Table B.3.

**Table B.3 Dimensionless Oscillation Source Parameters**

<table>
<thead>
<tr>
<th>Dimensionless Value</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimensionless Oscillation Displacement</td>
<td>[ \tilde{a}_{\text{osf}}(\tilde{t}<em>F) = \left( \frac{A}{L</em>{\text{ref}}} \right) \sin(\tilde{\omega}_F \tilde{t}_F) ]</td>
</tr>
<tr>
<td>Dimensionless Oscillation Velocity</td>
<td>[ \tilde{V}_{\text{osf}}(\tilde{t}_F) = \left( \frac{A \tilde{\omega}<em>F}{L</em>{\text{ref}}} \right) \cos(\tilde{\omega}_F \tilde{t}_F) ]</td>
</tr>
<tr>
<td>Dimensionless Oscillation Frequency</td>
<td>[ \tilde{\omega}<em>F = \omega \frac{L</em>{\text{ref}}}{U_{\text{ref}}} ]</td>
</tr>
<tr>
<td>Dimensionless Period of Oscillation</td>
<td>[ \tilde{T}_{\text{osf}} = \frac{2\pi}{\tilde{\omega}_F} ]</td>
</tr>
</tbody>
</table>

See Table 2.1 p.65 for \( L_{\text{ref}} \), \( U_{\text{ref}} \)
APPENDIX C

FINITE ELEMENT METHOD FOR FLUID FLOW AND HEAT TRANSFER

The finite element method is used to approximate the solutions of complex differential equations particularly with complex geometries and boundary conditions. In this method, the domain under study is discretized into a number of cells or finite elements. A weighted integral formulation of the governing partial differential equations over each of these elements is then used to approximate the solution of the partial differential equations at particular locations in these elements, or nodes. Though more complex, this method has an advantage over finite difference methods in that it can easily handle irregular geometries [82-84]. For the more complex geometries in this investigation, the finite element method is used. In this appendix the general procedure followed when approximating the solution to a governing partial differential equation through the use of the finite element method. In addition, the finite element forms of the governing equations for heat transfer and fluid flow are developed.

C.1 Finite Element Procedure

The basic procedure used in the finite element method is the outlined below. First, the domain over which the solution is desired is discretized into a finite element mesh. The type of elements to be used is specified, including the element shape and number of nodes per element. The nodes are the locations on the element at which the approximate solution to the governing equation will be found. The specific locations of each of these elements as well as the element node locations are determined.
Next, for each of the unknown variables, the form of the interpolation function for the elements is specified. The interpolation functions are functions of an assumed form that are used to approximate how the unknown quantity value at a particular node contributes to the value of the unknown variable over one finite element. Hence, they are used to approximate the variation of the unknown variable over the finite element. One interpolation function is associated with each node on the element and has value of one at the particular associated node and a value of zero at all other nodes. The form of this function depends on the number of nodes on the element as well as the order of the differential equation to be solved. If $u^e$ is the finite element approximate solution for $u$ over a typical interior element $e$, $u_j$ is the finite element approximate solution for $u$ at node $j$ on element $e$, and $\psi_j$ is interpolation function associated with node $j$ in element $e$, then the approximate solution for the $u^e$ for an element of $n$ nodes or $n$ unknown values can be expressed as:

$$u^e = \sum_{j=1}^{n} u_j \psi_j$$  \hspace{1cm} (C.1)

After specifying the interpolation functions, the finite element equations over each element are developed. The finite element method does not solve the partial differential equations, but a weighted integral formulation of the equation. One of the more common methods of formulating this integral form of the equation is the Galerkin method. For a two dimensional domain, a governing partial differential equation for $u$ is given by:

$$Du(x, y, t) + g(x, y, t) = 0$$  \hspace{1cm} (C.2)
where $D$ is a differential operator operating on $u$. $u$ satisfies this differential equation. However, substituting the approximate solution for $u$, $u^e$, into Eq. (C.2) will yield a residual because $u^e$ is not the exact solution.

\[ Du^e(x, y, t) + g(x, y, t) = \varepsilon \]  \hspace{1cm} \text{(C.3)}

where

\[ \varepsilon \neq 0 \]  \hspace{1cm} \text{(C.4)}

In order to develop the approximate solution, a method of weighed residuals is used. The weighted integral form of the residual is set to zero where the values of the $u_j$ are selected to satisfy this condition and $W_k$ represent $n$ linearly independent weighing functions, such that the weighted integral form of the governing equations is zero.

\[ \int W_i(x, y) \varepsilon(x, y, u_j) \, dVol = 0 \]  \hspace{1cm} \text{(C.5)}

where $i=1,2,\ldots,n$ and $j=1,2,\ldots,n$ and $n$ is the number of nodes or unknowns per element. For the Galerkin method the weighing functions $W_i$ are set equal to the interpolation functions, $\psi_i$, used for approximating how the node values, $u_j$, contribute to $u^e$ across the entire element. It should be noted that in the integration, the $u_j$ values are at best functions of time alone, while the $W_i$ or $\psi_j$ are functions of $x$ and $y$ only.

Commonly, the weak form of the integral equation in Eq. (C.5), arrived at by integration by parts, is used to form the finite element equations. In a weak form, the differentiation is distributed between the weighing functions $\psi_i$ and the finite element interpolation functions $\psi_j$. This is important because this form requires less continuity of solution constraints for $u^e$ across the element. The derivative type boundary conditions or natural boundary conditions are automatically incorporated into the weak formulation of the equation.
A set of \( n \) linear equations for the nodal solution values \( u_j \) is generated from Eq. (C.5) that can be expressed in the form:

\[
\begin{bmatrix} A^e \end{bmatrix}\{ \dot{u}^e \} + \begin{bmatrix} K^e \end{bmatrix}\{ u^e \} = \{ G^e \}
\]  \hspace{1cm} (C.6)

where \( \{ \dot{u}^e \} \) represents the time derivative of the \( u_j \) values.

Once the finite element equations are developed for each finite element in the domain, the set of equations over each finite element is assembled together to form a global matrix containing the equations necessary to determine the nodal values at all node points in the domain. After the boundary and initial conditions are applied, this global matrix of linear equations is used to determine the \( u_j \) values. Thus, with the finite element method the approximate solution to the governing equation at the node locations is reduced to solving a set of algebraic linear equations.

**C.2 Development of Finite Element Equations for Heat Transfer and Fluid Flow**

Because the set governing equations for heat transfer and fluid flow are coupled, the finite element formulation of the governing equations of the heat and fluid flow is unique and is described below. The form of the governing equations including the continuity Eq. (2.1), momentum equations Eq. (2.12) and Eq. (2.13) and the energy equation Eq. (2.4) used in this investigation were discussed in Chapter 2. The finite element method must be used to approximate the valued of \( u, v, P, \) and \( T \) over the given domain. For purposes of simplification, the symbol

\[
\hat{T} \quad \text{will be used to represent } T - T_e
\]  \hspace{1cm} (C.7)
C.2.1 Interpolation Functions

In developing the finite element equations for fluid flow and heat transfer the first complication is in the selection of the interpolation functions. Different sets of weighing functions may be used for the velocity, temperature, and pressure. The order of the interpolation functions for the velocity and temperature must be at least one order higher than that for the pressure to make the order of approximation the same for velocity, temperature, and pressure. While not necessary, in this investigation the same interpolation functions are used for velocity and temperature. Following the form of approximation given in Eq. (C.1), the velocity, pressure and temperature are approximated by:

\[ u^e = \sum_{j=1}^{n} \psi_j u_j \]  
\[ v^e = \sum_{j=1}^{n} \psi_j v_j \]  
\[ \tilde{T}^e = \sum_{j=1}^{n} \psi_j \tilde{T}_j \]  
\[ P^e = \sum_{j=1}^{m} \phi_j P_j \]

Here \( \psi_j \) represents the interpolation functions for the velocity and temperature

\( n \) represents the number of element nodes for which \( u_j, v_j, \) and \( T_j \) are unknown

\( \phi_j \) represents the interpolation functions for the pressure

\( m \) represents the number of element nodes for which \( P_i \) are unknown

\( u_j, v_j, \tilde{T}_j, \) and \( P_j \) represent the unknown values at the element nodes
It is important to note again that the interpolation functions $\psi_j$ and $\phi_j$ are functions of $x$ and $y$ alone and are independent of $t$ and that $u_j$, $v_j$, $T_j$, and $P_j$ values are at best functions of $t$ alone and are independent of $x$ and $y$.

C.2.2 Mathematical Theorems

Before applying these approximations to the governing equations, some mathematical theorems and expressions of use in developing the weak forms of the equations are reviewed.

The divergence theorem is given by:

$$\int_V \nabla \cdot F dV = \int_S N \cdot F dS \quad (C.10)$$

where $V$ represents a volume

$S$ represents a surface area

$F$ is a vector

$N$ is the outward directed unit normal vector to the surface

The gradient theorem is given by:

$$\int_A \nabla F dx dy = \int_S \vec{N} F dS \quad (C.11)$$

or equivalently:

$$\int_A \left( \frac{\partial F}{\partial x} \vec{i} + \frac{\partial F}{\partial y} \vec{j} \right) dx dy = \int_S (N_x \vec{i} + N_y \vec{j}) F dS \quad (C.11a)$$

Thus,

$$\int_A \left( \frac{\partial F}{\partial x} \right) dx dy = \int_S N_x F dS \quad \text{and} \quad \int_A \left( \frac{\partial F}{\partial y} \right) dx dy = \int_S N_y F dS \quad (C.11b)$$
where $S$ represents a boundary curve

$F$ is a vector

$N$ is the outward directed normal vector to the surface

### C.2.3 Development of Finite Element Equations

The procedure development of the finite element equations is based on the procedure outlined in [83]. First, the finite element form of the continuity equation will be developed. The continuity equation from Eq. (2.1) applied to the element is given by:

$$
\frac{\partial u^e}{\partial x} + \frac{\partial v^e}{\partial y} = 0
$$

(C.12)

To put this equation in the finite element form, it must be multiplied by one of the weighing function for pressure, $\phi_i$ as defined in Eq. (C.9), and integrated over the element volume $V_{e} = dx \, dy \, l$. This procedure must be repeated $m$ times, once for each of the pressure weighing functions. As a result, a set of $m$ equations will be formed.

$$
\int_{V_{e}} \phi_i \left( \frac{\partial u^e}{\partial x} + \frac{\partial v^e}{\partial y} \right) dx \, dy = 0 \quad i = 1 \text{ to } m
$$

(C.13)

Substituting for $u^e$ and $v^e$ from (C.8a) and (C.8b) yields:

$$
\int_{V_{e}} \phi_i \left( \frac{\partial}{\partial x} \left( \sum_{j=1}^{n} \psi_j u_j \right) + \frac{\partial}{\partial y} \left( \sum_{j=1}^{n} \psi_j v_j \right) \right) dx \, dy = 0 \quad i = 1 \text{ to } m
$$

(C.14)

Remembering that $u_j$ and $v_j$ are not functions of $x$ or $y$ and changing the order of the summation and differentiation Eq. (C.14) can be expressed as:

$$
\int_{V_{e}} \phi_i \left( \sum_{j=1}^{n} \frac{\partial \psi_j}{\partial x} u_j + \sum_{j=1}^{n} \frac{\partial \psi_j}{\partial y} v_j \right) dx \, dy = 0 \quad i = 1 \text{ to } m
$$

(C.15)
Changing the order of the summation and integration:

\[
\sum_{j=1}^{n} \left[ \left( \int_{vol} \phi_j \frac{\partial \psi_j}{\partial x} \, dx \, dy \right) u_j \right] + \left[ \left( \int_{vol} \phi_j \frac{\partial \psi_j}{\partial y} \, dx \, dy \right) v_j \right] = 0 \quad i = 1 \to m \tag{C.16}
\]

This is the set of m equations resulting from the continuity equation.

The finite element form of the x component of the momentum equation will be developed next. Because it is desired to solve a set of linear equations in matrix form iteratively, to remove the nonlinearities in the momentum equation, \( u \) and \( v \) values as the coefficients in the acceleration terms are approximated by \( u_s \) and \( v_s \), the solutions from the previous solution iteration. Applying this, the x component of the momentum equation in Eq. (2.12) on an element yields:

\[
\rho \left( \frac{\partial u^e}{\partial t} + u^e_s \frac{\partial u^e}{\partial x} + v^e_s \frac{\partial u^e}{\partial y} \right) = -\frac{\partial P_D^e}{\partial x} + \mu \left( \frac{\partial^2 u^e}{\partial x^2} + \frac{\partial^2 u^e}{\partial y^2} \right) + g \beta \rho T^e \tag{C.17}
\]

To determine the finite element form of the equation, this equation is multiplied by one of the interpolation functions for the x component of velocity, \( \psi_i \), as defined in Eq. (C.8a) and then integrated over the element volume \( Vol^e = dx \, dy \). This procedure must be repeated \( n \) times, once for each of the x component of velocity weighing functions. As a result, a set of \( n \) equations will be formed.

\[
\int_{vol} \psi_i \left\{ \rho \left( \frac{\partial u^e}{\partial t} + u^e_s \frac{\partial u^e}{\partial x} + v^e_s \frac{\partial u^e}{\partial y} \right) \right\} \, dx \, dy = 0 \quad i = 1 \to n \tag{C.18}
\]
The weak form of this expression can be arrived at in the following way. The term
\[
\int_{\Omega} \left( \mu \psi_i \left( \frac{\partial^2 u^e}{\partial x^2} + \frac{\partial^2 u^e}{\partial y^2} \right) \right) dxdy \text{ or equivalently } \int_{\Omega} \left( \psi_i \left( \nabla \cdot \left( \mu \nabla u^e \right) \right) \right) dxdy
\]

\((i = 1 \text{ to } n)\) can be reduced to its weak form using integration by parts and the divergence theorem given in Eq. (C.10) yielding:

\[
\int_{\Omega} \left( \mu \psi_i \left( \frac{\partial^2 u^e}{\partial x^2} + \frac{\partial^2 u^e}{\partial y^2} \right) \right) dxdy = - \int_{\Omega} \left( \nabla \psi_i \cdot \left( \mu \nabla u^e \right) \right) dxdy + \int_{s} \left( \psi_i \vec{N} \cdot \left( \mu \nabla u^e \right) \right) dS^e
\]

\((C.19)\)

\(i = 1 \text{ to } n\)

The term \(\int_{\Omega} \left( \psi_i \frac{\partial P_D^e}{\partial x} \right) dVol^e \quad i = 1 \text{ to } n\) can be expressed in its weak form using integration by parts and the gradient theorem given in Eq. (C.11b) yielding:

\[
\int_{\Omega} \psi_i \frac{\partial P_D^e}{\partial x} dxdy = - \int_{\Omega} \frac{\partial \psi_i}{\partial x} P_D^e dxdy + \int_{s} \left( \psi_i P_D^e \right) N_x dS^e
\]

\((C.20)\)

\(i = 1 \text{ to } n\)

Substituting Eq. (C.19) and Eq. (C.20) into Eq. (C.18)

\[
\int_{\Omega} \left( \psi_i \left( \rho \left( \frac{\partial u^e}{\partial t} + u^e \frac{\partial u^e}{\partial x} + v^e \frac{\partial u^e}{\partial y} - g \beta p \psi_i \hat{T}^e \right) \right) - \psi_i \frac{\partial P_D^e}{\partial x} \right) dxdy
\]

\[
+ \int_{\Omega} \left( \mu \left( \frac{\partial \psi_i}{\partial x} \frac{\partial u^e}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial u^e}{\partial y} \right) \right) dxdy = \int_{s} \psi_i \left( -P_D^e N_x + \mu \nabla u^e \cdot \vec{N} \right) dS^e
\]

\((C.21)\)

\(i = 1 \text{ to } n\)

Substituting for \(u^e, v^e, \hat{T}^e, \) and \(P^e\) from Eq. (C.8a), Eq. (C.8b), Eq. (C.8c), and Eq. (C.9) yields:
Following a similar procedure, the finite element form of the \( y \) component of the momentum equation can be developed. The finite element method can be applied to the \( y \) component of the momentum equation in Eq. (2.13).

\[
\sum_{j=1}^{n} \left[ \int_{\Omega_{e}} \left[ \rho \psi_{j} \frac{\partial \psi_{j}}{\partial t} \right] dxdy \right] \dot{u}_{j}
\]
\[
+ \sum_{j=1}^{n} \left[ \int_{\Omega_{e}} \left[ \left( \rho u_{j}^{*} \psi_{j} \frac{\partial \psi_{j}}{\partial x} + \rho v_{j}^{*} \psi_{j} \frac{\partial \psi_{j}}{\partial y} \right) \right] dxdy \right] u_{j}
\]
\[
+ \sum_{j=1}^{n} \left[ \int_{\Omega_{e}} \mu \left( \frac{\partial \psi_{j}}{\partial x} \frac{\partial \psi_{j}}{\partial x} + \frac{\partial \psi_{j}}{\partial y} \frac{\partial \psi_{j}}{\partial y} \right) dxdy \right] u_{j}
\]
\[
- \sum_{j=1}^{n} \left[ \int_{\Sigma_{e}} \left( \frac{\partial \psi_{j}}{\partial x} \phi_{j} \right) dxdy \right] P_{Dj}
\]
\[
- \sum_{j=1}^{n} \left[ \int_{\Sigma_{e}} \left( g \beta \rho \psi_{j} \right) dxdy \right] \dot{\bar{\nu}}_{j}
\]
\[
\int_{S_{e}} \left[ \mu \overline{N} \cdot \nabla \psi_{j} - P_{D}^{e} N_{e} \right] dS_{e}
\]

\( i = 1 \) to \( n \)

where

\[
\dot{u}_{j}^{e} = \sum_{j=1}^{m} \psi_{j} \dot{u}_{j}
\]
(C.22a)

\[
u_{j}^{e} = \sum_{j=1}^{n} \psi_{j} u_{j}
\]
(C.22b)

\[
u_{j}^{e} = \sum_{j=1}^{n} \psi_{j} v_{j}
\]
(C.22c)

Following a similar procedure, the finite element form of the \( y \) component of the momentum equation can be developed. The finite element method can be applied to the \( y \) component of the momentum equation in Eq. (2.13).

\[
\rho \left( \frac{\partial v_{j}^{e}}{\partial t} + u_{j}^{e} \frac{\partial v_{j}^{e}}{\partial x} + v_{j}^{e} \frac{\partial v_{j}^{e}}{\partial y} \right) = - \frac{\partial P_{Dj}^{e}}{\partial y} + \mu \left( \frac{\partial^{2} v_{j}^{e}}{\partial x^{2}} + \frac{\partial^{2} v_{j}^{e}}{\partial y^{2}} \right)
\]
(C.23)
Multiplying by the weighing functions for the y component of velocity:

\[
\int \psi_i \left( \rho \left( \frac{\partial u^e}{\partial t} + u^e \frac{\partial v^e}{\partial x} + v^e \frac{\partial v^e}{\partial y} \right) - \mu \left( \frac{\partial^2 v^e}{\partial x^2} + \frac{\partial^2 v^e}{\partial y^2} \right) + \frac{\partial P_{D_i^e}}{\partial y} \right) \, dx \, dy = 0
\]

(C.24)

\[i = 1 \text{ to } n\]

It can be shown that the finite element equations can be expressed as:

\[
\sum_{i=1}^{n} \left( \int_{V} \left[ \rho v_i \psi_i \right] \, dx \, dy \right) \hat{v}_i
\]

\[
+ \sum_{i=1}^{n} \left( \int_{V} \left[ \rho u_i \frac{\partial \psi_i}{\partial x} + \rho v_i \frac{\partial \psi_i}{\partial y} \right] \, dx \, dy \right) \hat{v}_j
\]

\[
+ \sum_{i=1}^{n} \left( \int_{V} \mu \left( \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_i}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_i}{\partial y} \right) \, dx \, dy \right) \hat{v}_j
\]

\[
- \sum_{i=1}^{n} \left( \int_{V} \left( \frac{\partial \psi_i}{\partial y} \phi \right) \, dx \, dy \right) P_{D_i} =
\]

\[
\int_{s} \psi_i \left( \mu \nabla \nabla \cdot v^e - P_{D_i} N_y \right) \, ds
\]

\[i = 1 \text{ to } n\]

where

\[
\hat{v}^e = \sum_{j=1}^{n} \psi_j \hat{v}_j 
\]

(C.25a)

\[
u_s^e = \sum_{j=1}^{n} \psi_j u_s^j 
\]

(C.25b)

\[
v_s^e = \sum_{j=1}^{n} \psi_j v_s^j 
\]

(C.25c)

Finally, the finite element form of the energy equation will be developed. Applying the energy equation in Eq. (2.4) to the element yields:

\[
\rho_o c_p \left( \frac{\partial \hat{T}^e}{\partial t} + u^e \frac{\partial \hat{T}^e}{\partial x} + v^e \frac{\partial \hat{T}^e}{\partial y} \right) = k \left( \frac{\partial^2 \hat{T}^e}{\partial x^2} + \frac{\partial^2 \hat{T}^e}{\partial y^2} \right) + Q^e
\]

(C.26)
The finite element equations can be found by multiplying this equation by one of the weighing function for temperature, \( \psi_i \) as defined in Eq. (C.8c), and integrating over the element volume \( dVol^e = dx \, dy \). This procedure must be repeated \( n \) times, once for each of the temperature weighing functions. As a result, a set of \( n \) equations will be formed.

\[
\int_{Vol^e} \psi_i \left\{ \rho \cdot c_p \left( \frac{\partial \hat{T}^e}{\partial t} + u^e \frac{\partial \hat{T}^e}{\partial x} + v^e \frac{\partial \hat{T}^e}{\partial y} \right) - k \left( \frac{\partial^2 \hat{T}^e}{\partial x^2} + \frac{\partial^2 \hat{T}^e}{\partial y^2} \right) - Q'' \right\} \, dx \, dy = 0
\]

\( i = 1 \) to \( n \)

Again, this equation needs to be placed into its weak form as discussed earlier. The term

\[
\int_{Vol^e} k \psi_i \left( \frac{\partial^2 \hat{T}^e}{\partial x^2} + \frac{\partial^2 \hat{T}^e}{\partial y^2} \right) \, dx \, dy
\]

or, equivalently,

\[
\int_{Vol^e} \psi_i \left( \nabla \cdot \left( k \nabla \hat{T}^e \right) \right) \, dx \, dy
\]

\( i = 1 \) to \( n \) can be reduced to its weak form using integration by parts and the divergence theorem given in Eq. (C.10) yielding:

\[
\int_{Vol^e} k \psi_i \left( \frac{\partial^2 \hat{T}^e}{\partial x^2} + \frac{\partial^2 \hat{T}^e}{\partial y^2} \right) \, dx \, dy = - \int_{Vol^e} \left( \nabla \psi_i \cdot \left( k \nabla \hat{T}^e \right) \right) \, dx \, dy + \int_{\partial Vol^e} \left( \psi_i \mathbf{N} \cdot \left( k \nabla \hat{T}^e \right) \right) \, dS^e
\]

\( i = 1 \) to \( n \)

Substituting Eq. (C.29) and the expression for \( \hat{T}^e \) in Eq. (C.8c) into Eq. (C.28) yields:
The matrix for the set of finite element equations from the continuity, momentum, and energy equations over one finite element can be written as:

\[
\sum_{j=1}^{n} \left( \int_{Vol^e} \left[ \rho \frac{c_p}{c} \psi_i \psi_j \right] dxdy \right) \hat{T}_j \\
+ \sum_{j=1}^{n} \left[ \int_{Vol^e} \left[ \left( \rho c_p u^e_i \frac{\partial \psi_j}{\partial x} + \rho c_p v^e_i \frac{\partial \psi_j}{\partial y} \right) \right] dxdy \right) \hat{T}_j \\
+ \sum_{j=1}^{n} \left[ \int_{Vol^e} \left[ k \left( \frac{\partial \psi_i}{\partial x} \frac{\partial \psi_j}{\partial x} + \frac{\partial \psi_i}{\partial y} \frac{\partial \psi_j}{\partial y} \right) \right] dxdy \right) \hat{T}_j \\
\int_{Vol^e} \psi_i q^m dxdy + \int_{s^e} \left( \psi_i \left( k \nabla \hat{T}^e \cdot \hat{N} \right) \right) dS^e
\]

\[i = 1 \text{ to } n\]

where

\[\hat{T}^e = \sum_{j=1}^{m} \psi_j \hat{T}_j\]  \hspace{1cm} \text{(C.29a)}

\[u^e_s = \sum_{j=1}^{n} \psi_j u_{sj}\]  \hspace{1cm} \text{(C.99b)}

\[v^e_s = \sum_{j=1}^{n} \psi_j v_{sj}\]  \hspace{1cm} \text{(C.29c)}

The matrix for the set of finite element equations from the continuity, momentum, and energy equations over one finite element can be written as:

\[[A]\{\dot{\theta}\} + [B]\{\theta\} = \{F\}\]  \hspace{1cm} \text{(C.30)}

where \(\theta, \dot{\theta}, \text{ and } F\) are \((3n+m) \times 1\) and \(A\) and \(B\) are \((3n+m) \times (3n+m)\)
\[
\{ \dot{\theta} \} = \begin{bmatrix}
\dot{u}_1 \\
\dot{u}_2 \\
\vdots \\
\dot{u}_{n-1} \\
\dot{u}_n \\
\dot{v}_1 \\
\vdots \\
\dot{v}_{n-1} \\
\dot{v}_n \\
0 \\
0 \\
0 \\
0 \\
0 
\end{bmatrix}
\]

where \( \dot{\theta} = \frac{\partial \theta}{\partial t} \)

\[
\{ \theta \} = \begin{bmatrix}
u_1 \\
v_2 \\
\vdots \\
v_{n-1} \\
v_n \\
\Tilde{T}_1 \\
\Tilde{T}_2 \\
\vdots \\
\Tilde{T}_{n-1} \\
\Tilde{T}_n \\
0 \\
0 \\
0 \\
0 \\
0 
\end{bmatrix}
\]

\[
A = \begin{bmatrix}
\rho_o [A1] & [0] & [0] & [0] \\
[0] & \rho_o [A1] & [0] & [0] \\
[0] & [0] & \rho_o c_p [A1] & [0] \\
[0] & [0] & [0] & [0] 
\end{bmatrix}
\]

where \([A1]_{ij} = \int_{vor} \left[ \psi_i \psi_j \right] dxdy \quad i = 1 \text{ to } n, \quad j = 1 \text{ to } m\)

\[
B = \begin{bmatrix}
[0] & [0] & [B2] & [0] \\
\end{bmatrix}
\]
\[ [B_1]_{i,j} = \int_{V_{i}^{r}} \left\{ \rho \left( u_{i}^{r} \frac{\partial \psi_{i}}{\partial x} + v_{i}^{r} \frac{\partial \psi_{i}}{\partial y} \right) + \mu \left( \frac{\partial \psi_{i}}{\partial x} \frac{\partial \psi_{i}}{\partial x} + \frac{\partial \psi_{i}}{\partial y} \frac{\partial \psi_{i}}{\partial y} \right) \right\} \, dx \, dy \quad i = 1 \text{ to } n \quad j = 1 \text{ to } n \]

\[ [B_2]_{i,j} = \int_{V_{i}^{r}} \left\{ \rho c_p u_{i}^{r} \frac{\partial \psi_{i}}{\partial x} + v_{i}^{r} \frac{\partial \psi_{i}}{\partial y} \right\} + \int_{V_{i}^{r}} k \left( \frac{\partial \psi_{i}}{\partial x} \frac{\partial \psi_{i}}{\partial x} + \frac{\partial \psi_{i}}{\partial y} \frac{\partial \psi_{i}}{\partial y} \right) \, dx \, dy \quad i = 1 \text{ to } n \quad j = 1 \text{ to } n \]

\[ [B_3]_{i,j} = \int_{V_{i}^{r}} \left[ -\frac{\partial \psi_{i}}{\partial x} \phi_{j} \right] \, dx \, dy \quad i = 1 \text{ to } n \quad j = 1 \text{ to } m \]

\[ [B_4]_{i,j} = \int_{V_{i}^{r}} \left[ -\frac{\partial \psi_{i}}{\partial y} \phi_{j} \right] \, dx \, dy \quad i = 1 \text{ to } n \quad j = 1 \text{ to } m \]

\[ [B_5]_{i,j} = -\int_{V_{i}^{r}} \left[ g \beta \rho \psi_{i} \psi_{j} \right] \, dx \, dy \quad i = 1 \text{ to } n \quad j = 1 \text{ to } n \]

\[ [B_6]_{i,j} = \int_{V_{i}^{r}} \left[ -\frac{\partial \psi_{i}}{\partial x} \phi_{j} \right] \, dx \, dy \quad i = 1 \text{ to } m \quad j = 1 \text{ to } n \]

\[ [B_7]_{i,j} = \int_{V_{i}^{r}} \left[ -\frac{\partial \psi_{i}}{\partial y} \phi_{j} \right] \, dx \, dy \quad i = 1 \text{ to } m \quad j = 1 \text{ to } n \]

\[ F_r = \int_{S_{i}^{r}} \psi_{i} \left( \mu \vec{N} \cdot \nabla u^{e} - N \cdot P_{D}^{e} \right) \, ds \quad i = 1 \text{ to } n \quad r = 1 \text{ to } n \]

\[ F_{r} = \int_{S_{i}^{r}} \psi_{i} \left( \mu \vec{N} \cdot \nabla v^{e} - N \cdot P_{D}^{e} \right) \, ds \quad i = 1 \text{ to } n \quad r = n+1 \text{ to } 2n \]

\[ F_{r} = \int_{S_{i}^{r}} \psi_{i} \left( k \vec{N} \cdot \nabla \hat{T}^{e} \right) \, ds + \int_{V_{i}^{r}} \psi_{i} Q^{e} \, dx dy \quad i = 1 \text{ to } n \quad r = 2n+1 \text{ to } 3n \]

\[ F_{r} = 0 \quad i = 1 \text{ to } n \quad r = 3n+1 \text{ to } 3n+m \]

\[ u^{e} = \sum_{j=1}^{n} \psi_{j} u_{j} \quad v^{e} = \sum_{j=1}^{n} \psi_{j} v_{j} \quad P_{D}^{e} = \sum_{j=1}^{m} \phi_{j} P_{D_{j}} \quad \hat{T}^{e} = \sum_{j=1}^{n} \psi_{j} \hat{T}_{j} \]

\[ u^{e} = \sum_{j=1}^{n} \psi_{j} u_{j} \quad v^{e} = \sum_{j=1}^{n} \psi_{j} v_{j} \]

\[ \dot{u}^{e} = \sum_{j=1}^{m} \psi_{j} \dot{u}_{j} \quad \dot{v}^{e} = \sum_{j=1}^{m} \psi_{j} \dot{v}_{j} \quad \dot{T}^{e} = \sum_{j=1}^{m} \psi_{j} \dot{T}_{j} \]
APPENDIX D

GOVERNING EQUATIONS AND FINITE VOLUME NUMERICAL METHOD USED IN SQUEEZE FILM MODEL

In this appendix more details are given about the governing equations used in the squeeze film model of Chapter 4. Also included is a brief explanation of the SIMPLER finite volume numerical method used to numerically solve for the temperature field that develops between the heated fixed plate and the oscillating insulated plate in Figure 4.1.

D.1 Squeeze Film Model Governing Equations

In the squeeze film study, the velocity, pressure, and temperature fields in the parallel plate channel described in Chapter 4 were determined. The four governing equations used to solve for $u(x,y,t)$, $v(x,y,t)$, $p(x,y,t)$, and $T(x,y,t)$ in the system are the continuity equation, the momentum equations, and the energy; Eqs.(2.1), (2.12), (2.13), and (2.7), respectively. The squeeze film assumptions are applied to the velocity field in the channel. For the system under investigation, as discussed in Chapter 4, it can be shown that the squeeze film assumptions are:

$$\frac{b_o}{L} \ll 1 \quad (D.1)$$

$$\frac{(\omega \alpha_o)b_o}{\nu} \ll 1 \quad (D.2)$$

Using the dimensionless variables defined in Eq. (4.3) and applying the squeeze film assumptions represented by Eqs. (D.1) and (D.2), the $x$ component of the momentum equation, Eq.(2.12), reduces to:
Similarly, the \( y \) component of the momentum equation, Eq. (2.13), reduces to:

\[
\frac{\partial \tilde{P}_Q}{\partial \tilde{y}_Q} \approx 0 \tag{4.7}
\]

Thus, \( \tilde{P}_Q \), the dynamic pressure, is dependent on \( \tilde{x}_Q \) and \( \tilde{t}_Q \) alone.

Finally, the temperature distribution, \( \tilde{T}_Q(\tilde{x}_Q, \tilde{y}_Q, \tilde{t}_Q) \), is governed by the energy equation, Eq.(2.7). Applying the dimensionless variables to Eq.(2.7) yields:

\[
\left\{ \frac{\partial \tilde{T}_Q}{\partial \tilde{t}_Q} + \frac{\partial}{\partial \tilde{x}_Q}\left( \tilde{u}_Q \tilde{T}_Q \right) - K_x \frac{\partial \tilde{T}_Q}{\partial \tilde{x}_Q} \right\} + \frac{\partial}{\partial \tilde{y}_Q}\left( \tilde{v}_Q \tilde{T}_Q \right) - K_y \frac{\partial \tilde{T}_Q}{\partial \tilde{y}_Q} = - \frac{\tilde{v}_w \tilde{T}_Q}{b \left( \frac{\tilde{t}_Q}{\omega} \right) / b_o} \tag{4.19}
\]

This equation was solved numerically for the temperature.

In all of these governing equations, a constant property assumption was made. The validity of this assumption is related to the pressure change experienced by the fluid in the channel. For this investigation, when the maximum pressure difference experienced by the fluid is less than 5% of one atmosphere, the constant property assumption is defined to be valid.

The initial conditions applied for this investigation are given in Eq. (4.9) and all boundary conditions are listed in Eqs. (4.12-4.18).
D.2 Finite Volume Numerical Method

A modified SIMPLER numerical scheme is adopted to solve Eq.(4.19) numerically. This method is briefly explained in this section. The SIMPLER method, developed by Patankar [83,84], can be used to solve for the fluid flow, pressure field, and temperature fields in a given system, among other quantities. For two dimensional studies, the generic form of the governing equation used in the Patankar's [83,84] finite volume scheme is:

\[
\frac{\partial}{\partial t} (\rho \phi) + \frac{\partial}{\partial x} \left( \rho_x u \phi - \Gamma_x \frac{\partial \phi}{\partial x} \right) + \frac{\partial}{\partial y} \left( \rho_y v \phi - \Gamma_y \frac{\partial \phi}{\partial y} \right) = S_p \tag{D.3}
\]

The governing equation in Eq. (D.3) can be written as:

\[
\frac{\partial}{\partial t} (\rho \phi) + \frac{\partial J_x}{\partial x} + \frac{\partial J_y}{\partial y} = S_p \tag{D.4}
\]

where

\[
J_x = \rho_x u \phi - \Gamma_x \frac{\partial \phi}{\partial x} \tag{D.4a}
\]

\[
J_y = \rho_y v \phi - \Gamma_y \frac{\partial \phi}{\partial y} \tag{D.4b}
\]

The discretized form of Eq. (D.4) is then obtained through a finite volume approximation method. This means that the governing equations are integrated over a small finite control volume such as that depicted by the shaded region in Figure D.1. The integrated forms of these equations are then discretized. The center point is given the name \(P\) and the four surrounding grid points are named \(E\), \(W\), \(N\), and \(S\). The control volume surfaces are given the names \(e\), \(w\), \(n\), and \(s\). The integration of the governing equation in Eq. (D.4) over the discretized control volume indicated by the shaded region in Figure D.1 can be approximated by:
where \( dVol \) represents the volume of the control volume \( \Delta x \Delta y \).

\[
\left( \frac{\phi_p \rho_p - \phi^0_p \rho^0_p}{\Delta t} \right) dVol + \overline{J_e} A_e - \overline{J_w} A_w + \overline{J_n} A_n - \overline{J_s} A_s = \overline{S_s} dVol
\]

(D.5)

\( \overline{J_e} \) represents the average value of the flux \( \left[ \int J_e dy \right]_e \) through the \( e \) surface

\( \overline{J_w} \) represents the average value of the flux \( \left[ \int J_w dy \right]_w \) through the \( w \) surface

\( \overline{J_n} \) represents the average value of the flux \( \left[ \int J_n dy \right]_n \) through the \( n \) surface

\( \overline{J_s} \) represents the average value of the flux \( \left[ \int J_s dy \right]_s \) through the \( s \) surface

\( \overline{S_s} \) represents the average value of \( S_s \) in the control volume

\( ^0 \) superscript represents the previous time step

Figure D.1 Typical control volume used in this study as indicated by shaded area.
The continuity equation plays an important role in this solution method as it is used to alter the form of the equations governing a general unknown quantity $\phi$. The generalized governing equation in Eq. (D.3) takes the form of the continuity equation when $\phi$ is equal to 1 and its discretized form is:

$$\left(\frac{\rho_r - \rho_r^0}{\Delta t}\right) dVol + F_e - F_w + F_n - F_s = \overline{S}_1 dVol$$  \hspace{1cm} (D.6)$$

with

$$F_e = \left((\rho u)_{s}\right) \Delta y \hspace{1cm} (D.6a)$$

$$F_w = \left((\rho u)_{w}\right) \Delta y \hspace{1cm} (D.6b)$$

$$F_n = \left((\rho v)_{n}\right) \Delta x \hspace{1cm} (D.6c)$$

$$F_s = \left((\rho v)_{s}\right) \Delta x \hspace{1cm} (D.6d)$$

$$\overline{S}_1 = \overline{S}_\phi \text{ for } \phi = 1 \hspace{1cm} (D.6e)$$

Note the $u$ and $v$ values above are the mean velocities through the surfaces of the control volume in Figure D.1 as indicated.

For a general $\phi$, the discretized form of Eq. (D.3) used to numerically determine is formulated by subtracting $\phi_p$, the value of $\phi$ at location $P$ in the control volume in Figure (D.3), multiplied by the discretized continuity equation in Eq. (D.6) from Eq. (D.3), or \( (Eq.(D.5) - \phi_p (Eq.(D.6))) \), yielding:

$$\left(\frac{\phi_p - \phi_p^0}{\Delta t}\right) \rho_r^0 dVol + (\overline{J}_{e} A_{e} - F_{e} \phi_p) - (\overline{J}_{w} A_{w} - F_{w} \phi_p)$$

$$+ (\overline{J}_{n} A_{n} - F_{n} \phi_p) - (\overline{J}_{s} A_{s} - F_{s} \phi_p) = \overline{S}_\phi dVol - \phi_p \overline{S}_1 dVol$$  \hspace{1cm} (D.7)$$

It is clear from the definitions of $J_x$ and $J_y$ given in Eqs. (D4.a) and (D4.b), which represent the total $\phi$ transfer per unit area, that a method for approximating the $\phi$ value at
each of the control volume surface locations e, w, n, and s must be developed in order to fully discretize the equations. Some important factors must be kept in mind when formulating such a method of approximation. Diffusion "affect(s) the distribution of \( \phi \) along gradients in all directions." However, convection "spreads its influence only in the flow direction." Therefore, the method of approximating the \( \phi \) values must be able to "recognize the direction of flow and the relative strength of the convection and diffusion." [91] The following parameters may help to determine the strength of the convection and diffusion effects. The \( D \) values are a measure of the diffusion effects while the \( F \) values are a measure of the convective effects. The Peclet number, \( Pe \), can then be defined as the ratio of the convective to diffusive effects.

\[
\begin{align*}
D_e &= \Gamma_e \frac{\Delta y}{\delta x_e} & D_n &= \Gamma_n \frac{\Delta x}{\delta y_n} \\
D_w &= \Gamma_w \frac{\Delta y}{\delta x_w} & D_s &= \Gamma_s \frac{\Delta x}{\delta y_s} \\
F_e &= ((\rho u)_e) \Delta y & F_n &= ((\rho v)_n) \Delta x \\
F_w &= ((\rho u)_w) \Delta y & F_s &= ((\rho v)_s) \Delta x \\
Pe_e &= \frac{F_e}{D_e} & Pe_n &= \frac{F_n}{D_n} & Pe_s &= \frac{F_s}{D_s}
\end{align*}
\] (D.8)

Thus the approximations for the transport of \( \phi \) through each of the control volume surfaces must be a function of the Peclet number and can be expressed in the form below where \( \overline{\phi} \) represents an average value of \( \phi \) crossing through the indicated control volume surfaces and \( A(Pe) \) represents a function of the \( Pe \).

\[
\overline{J}_e A_e = \left( \rho u_e \overline{\phi}_e - \Gamma \frac{\partial \phi}{\partial x} \right) A_e \approx F_e \overline{\phi}_e - D_e A(Pe_e) (\overline{\phi}_e - \overline{\phi}_r)
\] (D.9a)
In this work, the function \( A(|Pe|) \) is assigned the power law scheme to approximate the convection and diffusion effects in the flow through the control volume surfaces. In this scheme, when the Peclet number, \( Pe \), has an absolute value greater than 10, the diffusion effects are neglected. For absolute values less than 10, a combination of the effects of diffusion and convection is used. An upwinding scheme, where the convective effect is determined by the direction of the flow, is included. Accordingly, the function

\[
 A(|Pe|) = \left[ 0, (1-0.1(|Pe|)^5) \right]
\]

is used in this investigation where \( [a,c] \) gives the maximum value in the brackets (either a or c).

Defining the following \( a \) coefficients:

\[
 a_{c}(\phi_{p} - \phi_{c}) = (\overline{J}_{c} A_{c} - F_{c} \phi_{p}) \quad \text{(D.10a)}
\]

\[
 a_{w}(\phi_{w} - \phi_{p}) = (\overline{J}_{w} A_{w} - F_{w} \phi_{p}) \quad \text{(D.10b)}
\]

\[
 a_{n}(\phi_{p} - \phi_{n}) = (\overline{J}_{n} A_{n} - F_{n} \phi_{p}) \quad \text{(D.10c)}
\]

\[
 a_{s}(\phi_{s} - \phi_{p}) = (\overline{J}_{s} A_{s} - F_{s} \phi_{p}) \quad \text{(D.10d)}
\]
Eq. (D.7) can be written in a more compact form:

\[ a_P \Phi_P = a_E \Phi_E + a_W \Phi_W + a_N \Phi_N + a_S \Phi_S + C \]  

(D.11)

where the locations \( P, E, W, N, \) and \( S \) are defined in Figure D.1 and

\[ a_E = D_e A(|Pe_e|) + [-F_e, 0] \]  

(D.11a)
\[ a_W = D_w A(|Pe_w|) + [F_w, 0] \]  

(D.11b)
\[ a_N = D_n A(|Pe_n|) + [-F_n, 0] \]  

(D.11c)
\[ a_S = D_s A(|Pe_s|) + [F_s, 0] \]  

(D.11d)

After \( \overline{S}_\phi \) is linearized in the form

\[ \overline{S}_\phi = S_c + S_p \Phi_p \]  

(D.11e)

the remaining parameters become:

\[ C = S_c \Delta x \Delta y \]  

(D.11f)

\[ a_p = a_E + a_W + a_N + a_S + \rho \frac{\Delta x \Delta y}{\Delta t} - S_p \Delta x \Delta y + \overline{S}_1 \Delta x \Delta y \]  

(D.11g)

For the problem at hand, the transformed energy equation in Eq. (4.19) is of the form of Eq. (D.3). The discretized form of Eq. (4.19) can be placed in the form of Eq. (D.11) using:

\[
\begin{align*}
  x &= \tilde{x}_Q & u &= \tilde{u}_Q & \rho &= \rho_x = \rho_y = \rho^0 = 1 & \overline{S}_1 &= -\frac{\tilde{V}_{w\tilde{Q}}}{b(t_0) / b_o} \\
  y &= \tilde{y}_Q & v &= \tilde{v}_{newQ} & \Gamma_x &= K_x & S_p &= -\frac{\tilde{V}_{w\tilde{Q}}}{b(t_0) / b_o} \\
  t &= \tilde{t}_Q & \phi &= \tilde{T}_Q & \Gamma_y &= K_y & S_c &= 0
\end{align*}
\]

(D.12)

Where \( K_x \) is defined in Eq. (4.19c) and \( K_y \) in Eq. (4.19d).

More information about this method can be found in Patankar [83,84].
APPENDIX E

TRANSFORMED GOVERNING EQUATIONS AND APPLICATION OF SIMPLER FINITE VOLUME METHOD FOR SOLUTION OF VELOCITY, PRESSURE, AND TEMPERATURE

In this appendix, more details are given about the application of the SIMPLER method to the solution of the velocity, pressure and temperature fields within the vertically oriented parallel plate channel with one fixed heated wall and one oscillating insulated wall discussed in Chapter 5 and shown in Figure 5.1. The transformation of the governing equations is discussed in more detail. In addition, the general use of and procedures for the SIMPLER method for numerically determining heat and fluid flow are explained. For the specific form of the transformed governing equations in the system under investigation, the specific parameter values corresponding to those in the general formulation of the SIMPLER method are identified, the finite volume grid set up and boundary condition implementation is discussed and convergence criteria are specified. Additionally, a flow chart of the program developed is included along with the program verification.

E.1 Transformation of Governing Equations

For the system described in Chapter 5 with the oscillating wall location, velocity, and acceleration given by Eqs. (5.1-3), the velocity, pressure, and temperature fields within the channel need to be determined in order to analyze the effects of the transverse oscillations under conditions where both natural convection and oscillation effects are important. The set of dimensionless variables in Eq. (5.4) and dimensionless parameters
in Eqs. (5.5-5.11) can be applied to the continuity, momentum, and energy equations given in Eqs. (2.1), (2.12) and (2.13), and (2.7). The non-dimensionalization procedure is complicated by the fact that \( \tilde{y}_R \) is made dimensionless by, \( b(t) \), a non-constant value. Care must be taken in finding the derivatives of this dimensionless \( \tilde{y}_R \) variable, particularly the time derivative. In formulating the time derivative, it is important to recognize that:

\[
\frac{\partial b(t)}{\partial t} = V_{wall}(t) = \left( \frac{\omega}{2\pi} \right) b_o \tilde{V}_{wallR}(\tilde{t}_R)
\]  

(E.1)

With this, it can be shown that:

\[
\frac{\partial \tilde{y}_R}{\partial t}_{x,y} = -\left( \frac{\omega}{2\pi} \right) \tilde{y}_R \frac{\tilde{V}_{wallR}(\tilde{t}_R)}{b_R(\tilde{t}_R)}
\]  

(E.2)

Using this information, the transformed governing equations can be derived and these equations are given below.

Applying the dimensionless variables in Eq. (5.4) to the continuity equation in Eq. (2.1) and simplifying yields:

\[
\frac{\partial \tilde{u}_R}{\partial \tilde{t}_R} + \frac{1}{b_R(\tilde{t}_R)} \frac{\partial \tilde{v}_R}{\partial \tilde{y}_R} = 0
\]  

(E.3)

After some manipulation, the application of the dimensionless variables in Eq.(5.4) to the x component of the momentum equation in (2.12) and the utilization of the appropriate parameters yields:

\[
\frac{\partial \tilde{u}_R}{\partial \tilde{t}_R} + \frac{1}{b_R(\tilde{t}_R)} \left( \tilde{v}_R - \tilde{y}_R \tilde{V}_{wallR}(\tilde{t}_R) \right) \frac{\partial \tilde{u}_R}{\partial \tilde{y}_R} + \tilde{u}_R \frac{\partial \tilde{u}_R}{\partial \tilde{x}_R} =
\]

\[
- \frac{\partial \tilde{p}_R}{\partial \tilde{x}_R} + \frac{1}{Re_L} \frac{\partial^2 \tilde{u}_R}{\partial \tilde{x}_R^2} + \frac{1}{Re_b} \left( \frac{1}{b_R(\tilde{t}_R)} \right)^2 \frac{\partial^2 \tilde{u}_R}{\partial \tilde{y}_R^2} + \frac{Gr_{bt}}{Re_L Re_b} \frac{L}{b_o} \tilde{T}_R
\]  

(E.4)
Similarly, using the above parameters and dimensionless groups, it can be shown that the transformed form of the y component of the momentum equation in Eq. (2.13) can be expressed as:

\[
\frac{\partial \tilde{v}_R}{\partial t_R} + \frac{1}{b_R(t_R)} \left\{ \tilde{v}_R - \tilde{y}_R \tilde{V}_{wall}(t_R) \right\} \frac{\partial \tilde{v}_R}{\partial y_R} + \tilde{u}_R \frac{\partial \tilde{v}_R}{\partial x_R} = \]
\[
- \frac{1}{L} \left( \frac{L}{b_R(t_R)} \right) ^2 \frac{\partial \tilde{p}_R}{\partial y_R} + \frac{1}{Re_L} \frac{\partial ^2 \tilde{v}_R}{\partial x_R ^2} + \frac{1}{Re_b} \frac{1}{\left( \tilde{b}_R(t_R) \right) ^2} \frac{\partial ^2 \tilde{v}_R}{\partial y_R ^2} \tag{E.5}
\]

Finally, substitution of the appropriate dimensionless variables and dimensionless groups into the energy equation in Eq. (2.7) yields:

\[
\frac{\partial \tilde{T}_R}{\partial t_R} + \frac{1}{b_R(t_R)} \left\{ \tilde{v}_R - \tilde{y}_R \tilde{V}_{wall}(t_R) \right\} \frac{\partial \tilde{T}_R}{\partial y_R} + \tilde{u}_R \frac{\partial \tilde{T}_R}{\partial x_R} = \]
\[
\frac{1}{Pr} \left[ \frac{1}{Re_L} \frac{\partial ^2 \tilde{T}_R}{\partial x_R ^2} + \frac{1}{Re_b} \frac{1}{\left( \tilde{b}_R(t_R) \right) ^2} \frac{\partial ^2 \tilde{T}_R}{\partial y_R ^2} \right] \tag{E.6}
\]

after some simplification.

Eqs. (E.3) through (E.6) are then placed into the “weak form” by adding the transformed continuity equation multiplied by the \( \tilde{u}_R, \tilde{v}_R, \) and \( \tilde{T}_R \), respectively to the appropriate governing equation. This procedure together with the substitution for the expression for the repeated quantity \( \tilde{v}_{newR} \) defined in Eq.(5.9) yields Eqs. (5.13) through (5.15). As explained in Chapter 5, this set of governing equation was solved numerically.


Though the SIMPLER finite volume scheme discussed in Appendix D can be directly applied to the continuity, momentum and energy equations in Eqs. (5.12) through (5.15), this set of governing equations has some unique characteristics. These attributes are due
to the fact that the pressure field must be determined as part of the solution procedure though there is no explicit, independent equation to solve for pressure. This did not pose any concern in the squeeze film investigation because analytical solutions for the velocity and pressure fields were formulated and only the temperature field was solved for numerically. Previous research has shown that when velocity and pressure are calculated at the same grid points, the pressure at the center of the control volume plays no role in the determination of the velocity at this center point of the control volume when a local linear approximation in the pressure is used. This leads to physically unrealistic pressure and velocity fields.

One of the more common numerical remedies for this problem is the use of a staggered grid where the velocities, \( u \) and \( v \), and pressure, \( p \), are stored at different grid locations. (Temperature values, \( T \), are stored at the same location as the pressure.) A typical staggered grid set-up is shown in Figure E.1. With the staggered grid, the control volumes used to solve for \( u \), \( v \), and pressure, \( p \), and temperature, \( T \), must be different locations as illustrated in the figure. The staggered grid allows for the pressure difference between pressure grid points adjacent to a velocity grid point to drive the flow and allows for the discretization of the continuity equation over a \( p \)-centered control volume. Hence, unrealistic solutions no longer results.

In addition to these modifications to the finite volume “grid,” a modified solution procedure is necessary for the determination of the pressure field. A pressure field must be established to solve the momentum equations for the velocity field. For a given pressure field, a velocity field that satisfies the momentum equation can be found.
However, such a velocity field does not necessarily satisfy the continuity equation. Thus, the additional constraint used to set the pressure in the continuity equation.

One solution technique which solves for fluid flow and heat transfer using the staggered grid and utilizes various forms of the continuity equation to link the pressure and velocity fields is the SIMPLER pressure correction method. The SIMPLER method and its variations have been used widely in numerical solutions for fluid flow and heat transfer. The basic procedures for the SIMPLER method are explained in the remainder of this section. Application of this method to the problem of the oscillating channel wall in Chapter 5 is then outlined.

Figure E.1 Typical staggered grid.
E.2.1 SIMPLER Pressure Correction Method for Momentum Equations

The x component of the momentum equation will be addressed first. Applying the discretization procedure described in Section D.2 to a general form of the x component of the momentum equation for the \( u \) control volume shown in Figure E.2, the discretized form of the x component of the momentum equation, can be written in the form:

\[
a_p u_p = a_E u_E + a_W u_W + a_N u_N + a_S u_S + A_e (p_e - p_n) + b_u
\]  

(E.7)

\( A_e \) is the area of the \( e \) control volume surface and the \( a \) coefficients are specified by Patankar’s power law scheme as in Eqs. (D.9 a-d). The discretization of any source terms in addition to the pressure will appear in the \( b_u \) and/or modified \( a_p \) terms.

Similarly, applying the discretization procedure to the y component of the momentum equation for the \( v \) control volume shown in Figure E.3, the discretized form of the y component of the momentum equation can be written as:

\[
a_p v_p = a_E v_E + a_W v_W + a_N v_N + a_S v_S + A_n (p_s - p_n) + b_v
\]  

(E.8)
$A_n$ is the area of the $n$ control volume surface. The $a$ coefficients are specified by Patankar’s power law scheme [83,84] in Eqs.(D.9 a-d). Any source terms in addition to the pressure appear in the $b$, or modified $a_p$ terms.

Eqs. (E.7) and (E.8) can be written in a more compact form:

$$a_p u_p = \sum_{nb} a_{nb} u_{nb} + A_e (p_w - p_e) + b_u \tag{E.9}$$

$$a_p v_p = \sum_{nb} a_{nb} v_{nb} + A_n (p_v - p_n) + b_v \tag{E.10}$$

where nb stands for the neighboring points E, W, N, S.

![Control Volume Diagram](image)

**Figure E.3** v control volume.

The use of relaxation factors may be necessary for convergence. The newest value of the unknown quantity $\phi$ then becomes some combination of the previous or old value, $\phi_{old}$, and newly calculated value, $\phi_{\text{calculated}}$, with the weighing factor $\alpha$

or $\phi = \alpha \phi_{\text{calculated}} + (1 - \alpha) \phi_{\text{old}}$. With the relaxation factor, $\alpha_u$, used with the $x$ momentum equation and $\alpha_v$, used with the $y$ momentum equation, the discretized momentum equations will take the form:
E.2.2 SIMPLER Pressure Correction Method for Pressure Solution

With the equations for determining the velocity field clearly established above, attention can be given to the scheme used to determine the pressure field. The SIMPLER method uses the continuity equation to develop an equation that is used to solve for the pressure field. To make use of the continuity equation, the expressions for velocities through the surfaces of the pressure control volume as shown in Figure E.4 are formulated. For a pressure field, \( p \), the velocities \( u \) and \( v \) at the control volume surfaces can be expressed as:

\[
\frac{a_p}{\alpha_u} u_p = \sum_{nb} a_{nb} u_{nb} + A_e \left( p_e - p_p \right) + b_u + \frac{1 - \alpha_u}{\alpha_u} a_p u_{p,old} \quad (E.11)
\]

\[
\frac{a_p}{\alpha_v} v_p = \sum_{nb} a_{nb} v_{nb} + A_n \left( p_n - p_p \right) + b_v + \frac{1 - \alpha_v}{\alpha_v} a_p v_{p,old} \quad (E.12)
\]

\[
u_x = \frac{\alpha_x}{a_x} \left( \sum_{nb} a_{nb} u_{nb} + A_e \left( p_e - p_p \right) + b_u \right) + (1 - \alpha_x) u_{x,old} \quad (E.13a)
\]

\[
u_w = \frac{\alpha_w}{a_w} \left( \sum_{nb} a_{nb} u_{nb} + A_e \left( p_e - p_p \right) + b_u \right) + (1 - \alpha_w) u_{w,old} \quad (E.13b)
\]

\[
u_n = \frac{\alpha_n}{a_n} \left( \sum_{nb} a_{nb} v_{nb} + A_n \left( p_n - p_p \right) + b_v \right) + (1 - \alpha_n) v_{n,old} \quad (E.13c)
\]

\[
u_s = \frac{\alpha_s}{a_s} \left( \sum_{nb} a_{nb} v_{nb} + A_n \left( p_s - p_p \right) + b_v \right) + (1 - \alpha_s) v_{s,old} \quad (E.13d)
\]

from Eqs. (E.11) and (E.12) where nb stands for the appropriate four neighboring points and the b values must be recalculated for each location.
In order to arrive at a form of the continuity equation expressed in terms of pressures, a new set of “pseudo” velocities was defined by Patankar. These “pseudo” velocities do not include the effects of the pressure difference in the source term and are defined below:

\[
\hat{u}_e = \left( \frac{\alpha_n}{a_e} \right) \left( \sum_{nb} a_{nb} u_{n} + b_u \right) + \left( 1 - \alpha_n \right) u_{old} \quad (E.14a)
\]

\[
\hat{u}_w = \left( \frac{\alpha_n}{a_w} \right) \left( \sum_{nb} a_{nb} u_{n} + b_u \right) + \left( 1 - \alpha_n \right) u_{wold} \quad (E.14b)
\]

\[
\hat{v}_n = \left( \frac{\alpha_n}{a_n} \right) \left( \sum_{nb} a_{nb} v_{n} + b_v \right) + \left( 1 - \alpha_n \right) v_{nold} \quad (E.14c)
\]

\[
\hat{v}_s = \left( \frac{\alpha_n}{a_s} \right) \left( \sum_{nb} a_{nb} v_{n} + b_v \right) + \left( 1 - \alpha_n \right) v_{sold} \quad (E.14d)
\]

Figure E.4 $p, T$ control volume.
Hence, the velocities \( u \) and \( v \) can then be expressed as a sum of the "pseudo" velocities and a pressure difference in the following form:

\[
\begin{align*}
\hat{u}_e &= u_e + d_e (p_p - p_R) \\
\hat{u}_w &= u_w + d_w (p_w - p_R) \\
\hat{v}_n &= v_n + d_n (p_p - p_R) \\
\hat{v}_s &= v_s + d_s (p_s - p_R)
\end{align*}
\]  

(E.15a, b, c, d)

where

\[
\begin{align*}
d_e &= \frac{\alpha_e A_e}{a_e} \\
d_w &= \frac{\alpha_w A_w}{a_w} \\
d_n &= \frac{\alpha_n A_n}{a_n} \\
d_s &= \frac{\alpha_s A_s}{a_s}
\end{align*}
\]  

(E.16)

Substituting the expressions for \( u_e, u_w, v_n, v_s \) in Eqs. (E.15 a-d) into the continuity equation in (D.4) yields:

\[
\left( \frac{\rho_e - \rho_e^0}{\Delta t} \right) \text{dVol} + F_e - F_w + F_n - F_s = \bar{S}_1 \, \text{dVol}
\]  

(D.6)

This yields an expression for the pressure \( p \).

\[
a_p \, p_p = a_e \, p_e + a_w \, p_w + a_n \, p_n + a_s \, p_s + b_p
\]  

(E.17)

Where

\[
\begin{align*}
a_e &= (\rho_e d_e A_e) \\
a_w &= (\rho_w d_w A_w) \\
a_n &= (\rho_n d_n A_n) \\
a_s &= (\rho_s d_s A_s)
\end{align*}
\]  

(E.17a, b, c, d)

\[
b_p = \bar{S}_1 \, \text{dVol} + \left( \frac{\rho_p - \rho_p^0}{\Delta t} \right) \frac{\text{dVol}}{} + \left( \rho_w \hat{u}_w A_w \right) - \left( \rho_e \hat{u}_e A_e \right) + \left( \rho_s \hat{v}_s A_s \right) - \left( \rho_n \hat{v}_n A_n \right)
\]  

(E.17e)
Applying a relaxation factor \( \alpha_p \) to (E.17) yields:

\[
\frac{\alpha_p}{\alpha_p} \frac{p_p}{p_p} = \sum_{nb} a_{nb} p_{nb} + b_p + \frac{(1-\alpha_p)}{\alpha_p} a_p p_{old}
\]  
(E.18)

Hence, Eq.(3.18) is the equation used to determine the pressure field from a given velocity field.

The final component of the SIMPLER method is the update of the velocity field determined from the momentum equation \((u^*, v^*)\) to ensure satisfaction of the continuity equation. The updated velocity field can be expressed as below where the prime(') term indicates the correction to the velocities.

\[
u = v^* + v'
\]  
(E.19a)

\[
u = v^* + v'
\]  
(E.19b)

An updated pressure field, with a pressure correction \( p' \), is also defined as:

\[
u = v^* + p'
\]  
(E.19c)

Applying the discretized form of the continuity equation in (D.4) to the pressure control volume in Figure E.4 an expression for the pressure correction can be found after the velocities at the \( e, w, n, \) and \( s \) control volume surfaces are expressed in terms of the pressure correction. These pressure corrections can then be used to update the velocity values. A summary of the procedure used is included below.

First expressions for the current estimation of the velocities at the \( e, w, n, \) and \( s \) control volume surfaces, denoted with the * superscript, are formulated from the momentum equations based on the pressure field \( p^* \):

\[
u^*_e = \frac{\alpha_u}{\alpha_e} \left( \sum_{nb} a_{nb} u^*_{nb} + A_e (p^*_p - p^*_e) + b_u \right) + (1-\alpha_u) u_{old}^*
\]  
(E.20a)

\[
u^*_w = \frac{\alpha_u}{\alpha_w} \left( \sum_{nb} a_{nb} u^*_{nb} + A_e (p^*_w - p^*_p) + b_u \right) + (1-\alpha_u) u_{old}^*
\]  
(E.20b)
nb stands for the appropriate four neighboring points.

Comparing Eqs. (E.13a-d) and Eqs. (E.20a-d), it is clear that the correction factor for the velocity and pressure can be found by subtracting the equations in (Eqs.(E.20a-d)) from the corresponding equations in Eqs. (E.13a-d). Because an iterative procedure is used in this SIMPLER procedure, the differences \( b - b' \), \( \sum_{nb} a_{nb} u_{nb}' \), \( \sum_{nb} a_{nb} v_{nb}' \) and \( v_{old} \) and \( u_{old} \) are neglected in this method. With these assumptions, the updated velocity fields are:

\[
\begin{align*}
v^*_n &= \frac{\alpha_s}{a_n} \left( \sum_{nb} a_{nb} v^*_{nb} + A_n \left( p^*_n - p^*_n \right) + b_v \right) + (1 - \alpha_s) v_{old}^* \\
v^*_s &= \frac{\alpha_s}{a_s} \left( \sum_{nb} a_{nb} v^*_{nb} + A_s \left( p^*_s - p^*_p \right) + b_v \right) + \frac{(1 - \alpha_s)}{\alpha_v} a_s v_{old}^* 
\end{align*}
\]  
(E.20c, d)

Substitution of these expression for velocity in Eqs. (E.21a-b) into the discretized continuity equation in Eq. (D.6) yield an equation for \( p' \) over the control volume:

\[
a_p p' = a_E p'_E + a_w p'_w + a_N p'_N + a_S p'_S + b_{pp} 
\]  
(E.22)

where

\[
\begin{align*}
a_s &= (\rho_s d_s A_s) \\
a_w &= (\rho_s d_w A_w) \\
a_N &= (\rho_n d_n A_n) \\
a_S &= (\rho_s d_s A_s)
\end{align*}
\]  
(E.22a-d)
Eq. (E.22) is then used to solve for the pressure correction over the system domain. Then, the pressure correction can be applied to update the velocity fields as given in Eqs. (E.21a) and (E.21b). It should be noted that the pressure is only updated through the use of Eq. (E.18). The pressure correction factors are only used to update the velocity field not the pressure.

**E.2.3 SIMPLER Pressure Correction Method for Solution of the Energy Equation**

The energy equation is the last governing equation to which the finite volume procedure must be applied. Applying the discretization procedure described in Section D.2 to a general form of the energy equation for the $T$ control volume shown in Figure E.4, the discretized form energy equation can be written as:

$$a_p T_p = a_s T_s + a_w T_w + a_n T_n + a_s T_s + b_T$$

(E.23)

The $u$ and $v$ values at the temperature control volume surfaces are known values. The $a$ values are be given by Patankar's power law scheme as in Section D.2. Any necessary source terms will be included in the $b_T$ and or modified $a_p$ terms.

Again, using a relaxation factor $\alpha_T$ on the energy equation, gives an expression of the form:

$$\frac{a_p}{\alpha_T} T_p = \sum_{nb} a_{nb} T_{nb} + b + \frac{(1 - \alpha_T)}{\alpha_T} a_p T_{old}$$

(E.24)
E.2.4 Outline of SIMPLER Solution Procedure

The SIMPLER solution procedure is:

1. Guess the velocity field \(u^*, v^*\).

2. Calculate \( \hat{u}, \hat{v} \) based on the guessed velocity field from Eqs.(E.15 a-d).

3. Solve the pressure equation for \(p^*\), the pressure field Eq.(E.18).

4. Solve the x and y components of the momentum equation for \(u^*\) and \(v^*\) for the given pressure field \(p^*\) in Eqs. (E.11) and (E.12).

5. Using the continuity equation, solve for the pressure correction values \(p'\) Eq.(E.22).

6. Update the velocities \( u = u^* + u' \)
   \( v = v^* + v' \)
   using Eqs.(E.21a) and (E.21b)

7. Solve for the temperature, \(T\), using the discretized energy equation in Eq.(E.24).

8. Repeat until convergence is reached using the updated velocities as the new \(u^*, v^*\).
   The SIMPLER procedure must be followed for each time step for a transient problem.
   The initial guess for the solution can be set equal to the solution at the previous time step to speed convergence.

E.2.5 SIMPLER Method Boundary Conditions

Though the discretized forms of the governing equations have been determined, the discretized form of the boundary conditions must also be formulated. The selection of the grid layout is important in determining how the boundary condition will be implemented.

For ease of the implementation of the boundary conditions, the grid used in the current investigation follows that in Figure E.1. The grid lines for \(i = 1\) and \(i = 2\) are both located at \(x = 0\) (\(x_R = 0\)) and the grid lines for \(i = M-2\) and \(i = M-3\) are both located at \(x = L(x_R = 1)\). (See Figure 5.1 for channel geometry). Similarly in the y direction, the grid lines for \(j = 1\) and \(j = 2\) are both located at \(y = 0\) (\(y_R = 0\)) and the grid lines for \(j = N\) and
$j = N-1$ are both located at $y = b(t)(\bar{y}_R = 1)$. With this arrangement, no half control volume are needed near any of the boundaries. There are “regular” control volumes for all unknown values closest to the boundary.

For boundaries where values of $u$, $v$, or $T$ are known, the implementation of the boundary conditions for these quantities is simple. The value is set at the appropriate node.

When $u$ or $v$ is known at a boundary or a zero pressure gradient boundary condition at a surface is specified, some modification must be made to the coefficients and $b$ values in the pressure and pressure correction equations. Because the velocities are known and do not need to be updated, the appropriate “pseudo velocity” in Eq. (E.20a) and Eq. (E.20b), and the appropriate pressure corrected velocity in Eqs.(E.15a-d) are needed and are not used. The $a$ coefficient in the direction for which the velocity is known set to zero and the appropriate velocity appearing in the $b_p$ or $b_{pp}$ term is set to the known velocity value. In addition, $p_{pp}'$ or $p_p$ at the wall grid point is set equal to $p'$ or $p$ at the first interior $p'$ or $p$ grid point in the direction of the known velocity. For example if $v_s$ is known on a boundary:

$$a_s = 0$$  \hspace{1cm} (E.25a)

$$v_{s*} = v_s$$  \hspace{1cm} (E.25b)

$$\hat{v}_s = v_s$$

$$b_{pp} = \bar{S}_1dVol + (\rho^o_p - \rho_p)\frac{dVol}{\Delta t} + (\rho_w u_w * A_w) - (\rho_v u_v * A_v) + (\rho_s v_s A_s) - (\rho_n v_n * A_n)$$  \hspace{1cm} (E.25c)

$$b_p = \bar{S}_1dVol + (\rho^o_p - \rho_p)\frac{dVol}{\Delta t} + (\rho_w \hat{u}_w A_w) - (\rho_c \hat{u}_c A_c) + (\rho_s v_s A_s) - (\rho_n \hat{v}_n A_n)$$  \hspace{1cm} (E.25d)
When the gradient of a velocity or temperature is zero, the $a$ coefficient in the direction of the boundary is set to zero and $\phi_{\text{boundary}} = \phi$ where $\phi$ is either the velocity or temperature.

When the pressure is known at a particular boundary, the value of $p'$, the pressure correction, must be set to zero as per Eq. (E.19c). However, at the inlet and outlet boundary, the implementation of the boundary conditions is complicated by the fact that the inlet and outlet $u$ component of velocity must also be determined. A method developed by Kwak [33] that allows for the specification of a known pressure boundary condition as well as the determination of the velocity at the channel inlet satisfying the continuity and momentum equations was used in the current investigation. In this procedure, the $x$ component of the momentum equations as well as the pressure equations, the pressure correction equations and the equations used to determine the "pseudo" velocity, $\hat{u}$, are modified near the inlet and outlet boundary. The procedure is outlined for the inlet velocity. A similar procedure is followed at the channel outlet. For the $u$ control volume nearest to the inlet (one u grid point from inlet) as sketched below in Figure E.5, the $x$ component of the momentum equation takes on the form:

$$a_e u_e = \sum_{\substack{\text{nb} \not= w}} a_{nb} u_{nb} + a_w u_w + A_e (p_p - p_E) + b_u$$  \hspace{1cm} (E.26)

Subtracting $a_w u_w$ from both side of Eq. (E.26)

$$a_e u_e - a_w u_w = \sum_{\substack{\text{nb} \not= w}} a_{nb} u_{nb} + a_w (u_w - u_e) + A_e (p_p - p_E) + b_u$$  \hspace{1cm} (E.27)

Using a relaxation factor, $r$, taking $(1-r)$ Eq. (E.26) + $r$ Eq. (E.27) yields:
\[ a_e^* u_e = \sum_{\text{not } w} a_{eb} u_{eb} + a_w^* u_w + A_e (p_p - p_E) + b_u^* \]  
(E.28)

where

\[ a_e^* = a_e - ra_w, \quad a_w^* = (1-r)a_w, \quad b_u^* = b_u + ra_w (u_w - u_e) \]  
(E.28a)

Eq (E.28) is used for the solution of the interior \( u \) values one \( u \) grid point from the inlet.

\[ w, W \quad P \quad e \quad E \]

\[ \bullet \quad \bigcirc \quad \bullet \quad \bigcirc \quad \bigcirc \quad = p \]

\[ \Delta x_1 \quad \Delta x_2 \]

\[ \bullet \quad = u \]

**Figure E.5** Sketch of channel inlet region \( w, W \) location coincides with channel inlet \( e \) is location of center first interior \( u \) control volume.

The pressure equation must also be altered near the inlet. In Kwak’s scheme while the original expression for \( u_e \) is used.

\[ u_e = \hat{u}_e + d_e (p_p - p_E) \]  
(E.20)

the velocity at the inlet, \( u_w \), is defined as:

\[ u_w = \hat{u}_w + d_e \frac{\Delta x_2}{\Delta x_1} (p_w - p_p) \]  
(E.29)

Hence in the pressure equation

\[ a_w = \frac{\Delta x_2}{\Delta x_1} a_E \]  
(E.30)

For numerical stability, in \( b_p \), the following expression is used for \( \hat{u}_w \):

\[ \hat{u}_w = u_w - d_e (p_p - p_E) \]  
(E.31)

Similarly for the pressure correction equation:
Eq. (E.32) is used exclusively to solve for the velocity at the channel inlet and its corresponding form is used for the outlet velocity. This ensures that the inlet and outlet velocities satisfy the continuity equation. It should be noted that the modified coefficients of the momentum equation used in Eq. (E.28) and Eq. (E.28a) must be used in the calculation of the $d_e$ and $d_w$ values of the pressure and pressure correction equations to maintain consistency. Therefore, Kwak's method allows for the application of a known pressure boundary condition as well as the determination of the inlet velocity.

E.2.6 Specification of Parameters for the Current Investigation

In this section, the specific parameters for the problem under investigation will be applied to the general form of the SIMPLER procedure. Because of the constant property assumption, the density of the fluid is a constant, except in the buoyancy force. The density is therefore, not a function of time, location, or temperature. The following lists the parameters for each of the governing equations in the form of Eq. (D.3) and discretized in the form of Eq. (D.11).
Continuity Equation
For the continuity equation in Eq. (D.6):

\[ x = \tilde{x}_R \quad u = \tilde{u}_R \quad \rho_x = 1 \quad \rho_y = 1 \]
\[ y = \tilde{y}_R \quad v = \tilde{v}_\text{new}_R \quad \Gamma_x = 0 \quad \Gamma_y = 0 \]
\[ t = \tilde{t}_R \quad \phi = 1 \]
\[ S_p = -\frac{\tilde{V}_\text{wall}R}{\tilde{b}(\tilde{t}_R)} \quad \tilde{S}_1 = -\frac{\tilde{V}_\text{wall}R}{\tilde{b}(\tilde{t}_R)} \]
\[ S_C = 0 \]
\[ a_p = \left( \frac{\Delta \tilde{x}_R \Delta \tilde{y}_R}{\Delta \tilde{t}_R} \right) + a_E + a_w + a_N + a_S - S_p \Delta \tilde{x}_R \Delta \tilde{y}_R + \tilde{S}_1 \Delta \tilde{x}_R \Delta \tilde{y}_R \]

(E.35)

x Component of Momentum Equation
For the x component of momentum Eq. (E.9):

\[ x = \tilde{x}_R \quad u = \tilde{u}_R \quad \rho_x = 1 \quad \Gamma_x = \frac{1}{Re_L} \]
\[ y = \tilde{y}_R \quad v = \tilde{v}_\text{new}_R \quad \rho_y = \frac{1}{\tilde{b}(\tilde{t}_R)} \quad \Gamma_y = \frac{1}{Re_b} \left( \frac{1}{\tilde{b}(\tilde{t}_R)} \right)^2 \]
\[ t = \tilde{t}_R \quad \phi = \tilde{u}_R \]
\[ S_p = -\frac{\tilde{V}_\text{wall}R}{\tilde{b}(\tilde{t}_R)} \quad \tilde{S}_1 = -\frac{\tilde{V}_\text{wall}R}{\tilde{b}(\tilde{t}_R)} \]
\[ S_C = \left( \frac{\tilde{p}_R - \tilde{p}_\text{Re}}{\Delta \tilde{x}_R} \right) + \frac{Gr_b}{Re_L Re_b} \frac{L}{b_o} T_{\text{avg}R} + \frac{\Delta \tilde{x}_R \Delta \tilde{y}_R}{\Delta \tilde{t}_R} \tilde{u}_R \]
\[ a_p = \left( \frac{\Delta \tilde{x}_R \Delta \tilde{y}_R}{\Delta \tilde{t}_R} \right) + a_E + a_w + a_N + a_S - S_p \Delta \tilde{x}_R \Delta \tilde{y}_R + \tilde{S}_1 \Delta \tilde{x}_R \Delta \tilde{y}_R \]
\[ b_u = \frac{Gr_b}{Re_L Re_b} \frac{L}{b_o} T_{\text{avg}R} + \frac{\Delta \tilde{x}_R \Delta \tilde{y}_R}{\Delta \tilde{t}_R} \tilde{u}_R \]
y Component of Momentum Equation
For the y component of momentum Eq. (E.10):

\[
\begin{align*}
x &= \tilde{x}_R \quad u = \tilde{u}_R \quad \rho_x = 1 \quad \Gamma_x = \frac{1}{Re_L} \\
y &= \tilde{y}_R \quad v = \tilde{v}_{\text{new}R} \quad \rho_y = \frac{1}{b(\tilde{t}_R)} \quad \Gamma_y = \frac{1}{Re_{\text{hy}}} \frac{1}{(b(\tilde{t}_R))^2} \\
t &= \tilde{t}_R \quad \phi = \tilde{v}_{\text{new}R} \\
S_p &= -\frac{2\tilde{V}_{\text{wall}R}}{\tilde{b}(\tilde{t}_R)} \quad \tilde{S}_1 = -\frac{\tilde{V}_{\text{wall}R}}{\tilde{b}(\tilde{t}_R)} \\
S_C &= \left(\frac{L}{b_o}\right)^2 \frac{1}{\tilde{b}(\tilde{t}_R)} \left(\tilde{p}_{R_s} - \tilde{p}_{R_h}\right) - \tilde{y}_R \tilde{a}_{\text{wall}R} + \frac{\Delta x_R \Delta \tilde{y}_R}{\Delta t_R} \tilde{v}_{\text{new}R} \quad \Delta t_R \\
a_p &= \left(\frac{\Delta x_R \Delta \tilde{y}_R}{\Delta t_R}\right) + a_E + a_w + a_N + a_s - S_p \Delta \tilde{x}_R \Delta \tilde{y}_R + S_1 \Delta \tilde{x}_R \Delta \tilde{y}_R \\
b_r &= -\tilde{y}_R \tilde{a}_{\text{wall}R} + \frac{\Delta x_R \Delta \tilde{y}_R}{\Delta t_R} \tilde{v}_{\text{new}R} \quad \Delta t_R \tilde{t}_{\text{new}R} \quad \tilde{T}_R
\end{align*}
\]  
(E.37)

Energy Equation
For the energy equation in Eq. (E.24):

\[
\begin{align*}
x &= \tilde{x}_R \quad u = \tilde{u}_R \quad \rho_x = 1 \quad \Gamma_x = \frac{1}{Pr Re_L} \\
y &= \tilde{y}_R \quad v = \tilde{v}_{\text{new}R} \quad \rho_y = \frac{1}{b(\tilde{t}_R)} \quad \Gamma_y = \frac{1}{Pr Re_{\text{hy}}} \frac{1}{(b(\tilde{t}_R))^2} \\
t &= \tilde{t}_R \quad \phi = \tilde{T}_R \\
S_p &= -\frac{\tilde{V}_{\text{wall}R}}{\tilde{b}(\tilde{t}_R)} \quad \tilde{S}_1 = -\frac{\tilde{V}_{\text{wall}R}}{\tilde{b}(\tilde{t}_R)} \quad S_C = \frac{\Delta x_R \Delta \tilde{y}_R}{\Delta t_R} \tilde{T}_R \quad \Delta t_R \\
a_p &= \left(\frac{\Delta x_R \Delta \tilde{y}_R}{\Delta t_R}\right) + a_E + a_w + a_N + a_s - S_p \Delta x_R \Delta \tilde{y}_R + S_1 \Delta x_R \Delta \tilde{y}_R \\
b_r &= \frac{\Delta x_R \Delta \tilde{y}_R}{\Delta t_R} \tilde{T}_{\text{new}R} \quad \Delta t_R \tilde{t}_{\text{new}R} \quad \tilde{T}_R
\end{align*}
\]  
(E.38)
Pressure Equation
For the pressure equation in Eq. (E.17):

\[
\begin{align*}
    x &= \tilde{x}_R \\
    y &= \tilde{y}_R \\
    u &= \tilde{u}_R \\
    v &= \tilde{v}_{newR} \\
    \rho_x &= 1 \\
    \rho_y &= \frac{1}{\tilde{b}(\tilde{t}_R)} \\
    \tilde{S}_1 &= -\frac{\tilde{V}_{wallR}}{\tilde{b}(\tilde{t}_R)} \\
    a_p &= a_E + a_W + a_N + a_S \\
    b_{pp} &= \tilde{S}_1 \Delta \tilde{x}_R \Delta \tilde{y}_R + \left( \rho_u \tilde{u}_{Rw} \ast A_w \right) - \left( \rho_v \tilde{u}_{Re} \ast A_e \right) + \left( \rho_s \tilde{v}_{newRs} \ast A_s \right) - \left( \rho_n \tilde{v}_{newRn} \ast A_n \right)
\end{align*}
\]

(E.39)

Pressure Correction Equation
For the pressure correction equation in Eq. (E.22):

\[
\begin{align*}
    x &= \tilde{x}_R \\
    y &= \tilde{y}_R \\
    u &= \tilde{u}_R \\
    v &= \tilde{v}_{newR} \\
    \rho_x &= 1 \\
    \rho_y &= \frac{1}{\tilde{b}(\tilde{t}_R)} \\
    \tilde{S}_1 &= -\frac{\tilde{V}_{wallR}}{\tilde{b}(\tilde{t}_R)} \\
    a_p &= a_E + a_W + a_N + a_S \\
    b_{pp} &= \tilde{S}_1 \Delta \tilde{x}_R \Delta \tilde{y}_R + \left( \rho_u \tilde{u}_{Rw} \ast A_w \right) - \left( \rho_v \tilde{u}_{Re} \ast A_e \right) + \left( \rho_s \tilde{v}_{newRs} \ast A_s \right) - \left( \rho_n \tilde{v}_{newRn} \ast A_n \right)
\end{align*}
\]

(E.40)

E.2.7 Convergence Criteria
Convergence of the governing equations will be defined through the use of residuals of the governing equations. For each control volume, the residual for each equation solved is defined as:

\[
\text{Res}_{equi} = a_E \phi_E + a_W \phi_W + a_N \phi_N + a_S \phi_S + b - a_p \phi_p
\]

(E.41)

This expression is a measure of how well the approximate solution satisfies the specific governing equation for a single control volume. To determine an overall or average
\[ Residual = \sqrt{\frac{\sum_{neq} Res_{equ}^2}{neq}} \]  
(E.42)

where \( Res_{equ} \) = residual for over each individual control volume as in (E.41)

\[ neq = \text{total number of control volumes to which the governing equation is applied} \]

The convergence of the solution for the governing equation will be based on this residual value. If \( Residual < \varepsilon \) (E.43), then the solution of that governing equation is defined to be converged. The convergence of each of the momentum, pressure and energy equations must be achieved to have a converged solution. [92] Adequate convergence limits were found to be \( 10^{-08} \) for \( \tilde{u}_R \), \( 10^{-08} \) for \( \tilde{v}_R \), \( 10^{-10} \) for \( \tilde{p}_R \), and \( 10^{-09} \) for \( \tilde{T}_R \). In addition, a maximum local residual for the continuity equation of \( 10^{-09} \) was established as well as an overall mass balance of 0.7% of the mass flow during one quarter of the plate oscillation. If all of the conditions described above are satisfied, the solution is defined to be converged.

**E.2.8 Flow Chart**

A FORTRAN program was written to carry out the numerical finite volume SIMPLER solutions for the velocity, pressure field, and temperature field in the channel of Chapter 5. A Flow chart of the program procedure is shown in Figure E.6.

**E.2.9 Program Verification**

In order to verify SIMPLER FORTRAN program is, the program results were compared with the results of a FIDAP© finite element model for an equivalent system. The grid used in this verification and for the ensuing studies is shown in Figure E.7. The plots in Figure E.8 show the velocity distributions at the channel inlet for a case where both the
plate oscillations and the natural convection effects are taken into account. The plot shows the good agreement in the results as the average root mean square average percent error is under 0.5%. In additional the surface average heat transfer coefficients and temperatures were also compared with a maximum 0.1% difference. Thus, there is good agreement between the FIDAP© results and those of the author generated SIMPLER program. The parameters used are listed in Table E.1.

Table E.1 Parameters Used in Verification of SIMPLER Program

<table>
<thead>
<tr>
<th>Material Property*</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>k (thermal conductivity of air)</td>
<td>0.027216 W/mK</td>
</tr>
<tr>
<td>ν (kinematic viscosity of air)</td>
<td>1.717e-5 m/s²</td>
</tr>
<tr>
<td>ρ (density of air)</td>
<td>1.12492 kg/m³</td>
</tr>
<tr>
<td>cₚ (specific heat of air)</td>
<td>1005.93 J/kg K</td>
</tr>
<tr>
<td>β (volumetric expansion of air)</td>
<td>1/315.5K</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>T₀ (ambient temperature)</td>
<td>25°C</td>
</tr>
<tr>
<td>qₐ (applied heat flux)</td>
<td>150W/m²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Oscillation Parameter**</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₀ (oscillation displacement amplitude)</td>
<td>0.08225m/s</td>
</tr>
<tr>
<td>ω (oscillation frequency)</td>
<td>82.25rad/sec</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Dimensional Parameter***</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>b₀ (mean channel width)</td>
<td>0.010m</td>
</tr>
<tr>
<td>L (channel length)</td>
<td>0.200m</td>
</tr>
</tbody>
</table>

* Material properties at 42.5°C
** See Figure 5.1 and Eq.(5.1-5.3)
*** See Figure 5.1
Set necessary parameters including: initial conditions, fluid properties, channel dimensions, graded grid setup, constant temperature/constant heat flux boundary conditions, oscillation parameters, u,v, P boundary conditions, time step set-up, relaxation factors, iteration limit, time limits, residual limits for convergence

Begin Time Iterations: \( t_i = t_{i+\Delta t} \)
Set necessary parameters including: update time set initial guess, fluid properties, oscillation parameters and thermal parameters, u,v, P, T boundary conditions, relaxation factors, iteration limit, residual limits for convergence

Calculate \( \hat{u}, \hat{v} \)
P iterations
Calculate P, Residual<\( \varepsilon_P \), iteration<iterationmax

\( u \) iterations
Calculate u, Residual<\( \varepsilon_u \), iteration<iterationmax

\( v \) iterations
Calculate v, Residual<\( \varepsilon_v \), iteration<iterationmax

\( P' \) iterations
Calculate P', Residual<\( \varepsilon_{PP} \), iteration<iterationmax

Update u, v from P'

T iterations
Calculate T, Residual<\( \varepsilon_T \), iteration<iterationmax

Convergence Check -u,v,T,P, continuity, mass balance converged, iteration<iterationmax

Calculate Heat Transfer Coefficients, Fluxes, Output Data

End Y
If at End of cycle check if reached steady state, time limit N

Figure E.6 SIMPLER program flow chart.
Figure E.7 A representative grid for the finite volume studies.
Figure E.8 Comparison of dimensionless $\tilde{u}_R$ component of velocity from SIMPLER program results to those of FIDAP© for $b_0=0.01\text{m}$ $L/b_0=20$ $a_0/b_0=0.10$ $\omega=82.25\text{rad/s}$ $q=150\text{ W/m}^2$ : (a) $\tilde{t}_R=0.20$, (b) $\tilde{t}_R=1.60$. 
APPENDIX F

AUTHOR - WRITTEN FORTRAN PROGRAMS

F.1 Squeeze Film

The author developed Fortran Code for the squeeze film study in Chapter 4 can be found on the accompanying CD in the file squeezefilmLAF.pdf.

F.2 Simpler

The author developed Fortran Code for the finite volume study in Chapter 5 can be found on the accompanying CD in the file simplerLAF.pdf.
Appendix G

FIDAP© Finite Element Results for Transverse Oscillation Investigations

This appendix includes additional plots of the results of the FIDAP© finite element studies of the use of a transversely oscillating plate for enhancement of pure natural convection.

G.1 Plain Channel Geometry

![Graph showing local dimensionless heat transfer coefficient as a function of dimensionless time.]

Figure G.1  Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) d=0.05 V=0.10 ω=2(cont.).
Figure G.1  Local dimensionless heat transfer coefficient as a function of dimensionless time: (b) $d=0.05 \ V=0.1\pi \ \omega=2\pi$, (c) $d=0.05 \ V=0.2\pi \ \omega=4\pi$ (cont.).
Figure G.1  Local dimensionless heat transfer coefficient as a function of dimensionless time: (d) $d=0.10 \ V=0.20 \ \omega=2$, (e) $d=0.10 \ V=0.20\pi \ \omega=2\pi$ (cont.).
Figure G.1  Local dimensionless heat transfer coefficient as a function of dimensionless time: (f) $d=0.10 \ V=0.40\pi \ \omega=4\pi$. 
Figure G.2 Time averaged local dimensionless heat transfer coefficient at Point 1: (a) $d=0.05$, (b) $d=0.10$. 
Figure G.3  Time averaged local dimensionless heat transfer coefficient at Point 2: (a) \(d=0.05\), (b) \(d = 0.10\).
Figure G.4  Time averaged local dimensionless heat transfer coefficient at Point 3: (a) \(d=0.05\), (b) \(d = 0.10\).
Figure G.5  Time averaged local dimensionless heat transfer coefficient at Point 4: (a) $d=0.05$, (b) $d = 0.10$. 
Figure G.6 Variation of local heat transfer coefficient along heated surface at dimensionless times indicated $d=0.10 \; V=0.4\pi \; \omega=2\pi$.

Figure G.7 Variation of surface average heat transfer coefficient with dimensionless time $d=0.10 \; V=0.4\pi \; \omega=2\pi$. 
Figure G.8  Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) $d=0.05$ $V=0.10$ $\omega=2$ $c=0.30$, (b) $d=0.05$ $V=0.10\pi$ $\omega=2\pi$ $c=0.30$ (cont.).
Figure G.8 Local dimensionless heat transfer coefficient as a function of dimensionless time: (c) $d=0.10 \, V=0.20 \, \omega=2, \, c=0.3$, (d) $d=0.10 \, V=0.4\pi \, \omega=4\pi \, c=0.30$ (cont.).
Figure G.8 Local dimensionless heat transfer coefficient as a function of dimensionless time: (e) $d=0.10 \ V=0.2\pi \ \omega=2\pi \ c=0.30$, (f) $d=0.15 \ V=0.6\pi \ \omega=4\pi \ c=0.30$ (cont.).
Figure G.8 Local dimensionless heat transfer coefficient as a function of dimensionless time: (g) $d=0.20 \ V=0.4\pi \ \omega=2\pi \ c=0.30$, (h) $d=0.20 \ V=0.8\pi \ \omega=4\pi \ c=0.30$ (cont.).
Figure G.8  Local dimensionless heat transfer coefficient as a function of dimensionless time: (i) \( d=0.10 \) \( V=0.4\pi \) \( \omega=4\pi \) \( c=0.15 \), (j) \( d=0.10\pi \) \( V=0.2\pi \) \( \omega=2\pi \) \( c=0.15 \).
Figure G.9 Time averaged local dimensionless heat transfer coefficient at Point 1: (a) $d=0.05$ $c=0.30$, (b) $d=0.10$ $c=0.30$ (cont.).
Figure G.9 Time averaged local dimensionless heat transfer coefficient at Point 1: (c) $d=0.15$ and $d=0.20 \ c=0.30$, (d) $d=0.10 \ c=0.15$. 
Figure G.10 Time averaged local dimensionless heat transfer coefficient at Point 2: (a) \(d=0.05 \; c=0.30\), (b) \(d = 0.10 \; c=0.30\) (cont.).
Figure G.10  Time averaged local dimensionless heat transfer coefficient at Point 2: (c) $d=0.15$ and $d=0.20$ $c=0.30$, (d) $d=0.10$ $c=0.15$. 
Figure G.11  Time averaged local dimensionless heat transfer coefficient at Point 3: (a) \( d=0.05 \) \( c=0.30 \), (b) \( d = 0.10 \) \( c=0.30 \) (cont.).
Figure G.11 Time averaged local dimensionless heat transfer coefficient at Point 3: (c) $d=0.15$ and $d=0.20$, $c=0.30$, (d) $d=0.10$, $c=0.15$. 
Figure G.12 Time averaged local dimensionless heat transfer coefficient at Point 4: (a) $d=0.05 \ c=0.30$, (b) $d=0.10 \ c=0.30$. (cont.)
Figure G.12  Time averaged local dimensionless heat transfer coefficient at Point 4: (c) \( d=0.15 \) and \( d=0.20 \) \( c=0.30 \), (d) \( d=0.10 \) \( c=0.15 \).
Figure G.13 Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: $d=0.20 \ V=0.8\pi \ \omega=4\pi \ c=0.30$.

Figure G.14 Variation of surface average heat transfer coefficient with time: $d=0.20 \ V=0.8\pi \ \omega=4\pi \ c=0.30$: (a) left (cont.).
Figure G.14  Variation of surface average heat transfer coefficient with time: \( d=0.20 \)
\( V=0.8\pi \) \( \omega=4\pi \) \( c=0.30 \): (b) top, (c) right.
G.3 Plain Channel With Opening Geometry

Figure G.15 Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) $d=0.05$ $V=0.10$ $\omega=2$, $c=0.30$, (b) $d=0.05$ $V=0.10\pi$ $\omega=2\pi$, $c=0.15$ (cont.).
Figure G.15 Local dimensionless heat transfer coefficient as a function of dimensionless time: (c) $d=0.10 \ V=0.20 \ \omega=2, \ c=0.15$, (d) $d=0.10 \ V=0.20\pi \ \omega=2\pi \ c=0.15$ (cont.).
Figure G.15  Local dimensionless heat transfer coefficient as a function of dimensionless time: (e) $d=0.10 \ V=0.40\pi \ \omega=4\pi, \ c=0.15$. 
Figure G.16 Time averaged local dimensionless heat transfer coefficient at Point 1: (a) $d=0.05$, (b) $d=0.10$. 
Figure G.17  Time averaged local dimensionless heat transfer coefficient at Point 2: (a) $d=0.05$, (b) $d = 0.10$. 
Figure G.18 Time averaged local dimensionless heat transfer coefficient at Point 3: (a) \(d=0.05\), (b) \(d = 0.10\).
Figure G.19  Time averaged local dimensionless heat transfer coefficient at Point 4: (a) 
d=0.05, (b) d = 0.10.
Figure G.20 Variation of local heat transfer coefficient over the heated surface at dimensionless times indicated: $d=0.10, V=0.4\pi, \omega=4\pi, c=0.15$. 
Figure G.21  Variation of surface average heat transfer coefficient with time: \( d=0.10 \)
\( V=0.4\pi \) \( \omega=4\pi \) \( c=0.15 \): (a) left, (b) right.
G.4 Modified Heat Source Geometry

G.4.1 Extension Plate

Figure G.22 Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) \( d=0.10 \) \( V=0.20\pi \) \( \omega=2\pi \) \( c=0.15 \), (b) \( d=0.10 \) \( V=0.40\pi \) \( \omega=4\pi \) \( c=0.15 \) (cont.).
Figure G.22 Local dimensionless heat transfer coefficient as a function of dimensionless time: (c) $d=0.10 \, V=0.80\pi \, \omega=8\pi, \, c=0.15$, (d) $d=0.20 \, V=0.80\pi \, \omega=4\pi, \, c=0.30$. 
Figure G.23 Time averaged local dimensionless heat transfer coefficient at Point 1: (a) \( c = 0.15 \), (b) \( c = 0.30 \).
Figure G.24 Time averaged local dimensionless heat transfer coefficient at Point 2: (a) $c=0.15$, (b) $c=0.30$. 
Figure G.25 Time averaged local dimensionless heat transfer coefficient at Point 3: (a) $c=0.15$, (b) $c=0.30$. 
Figure G.26  Time averaged local dimensionless heat transfer coefficient at Point 4: (a) $c=0.15$, (b) $c=0.30$. 
Figure G.27 Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: d=0.20 V=0.8\pi \dot{\omega}=4\pi c=0.30.

Figure G.28 Variation of surface average heat transfer coefficient with time: d=0.20 V=0.8\pi \dot{\omega}=4\pi c=0.30: (a) left (cont.).
Figure G.28  Variation of surface average heat transfer coefficient with time: $d=0.20$ $V=0.8\pi$ $\omega=4\pi$ $c=0.30$: (b) top, (c) right.
Figure G.29  Local dimensionless heat transfer coefficient as a function of dimensionless time:  (a) d=0.10 V=0.20\pi \ \omega=2\pi \ c=0.15, (b) d=0.10 V=0.40\pi \ \omega=4\pi \ c=0.15.
Figure G.30 Time averaged local dimensionless heat transfer coefficient at Point 1.

Figure G.31 Time averaged local dimensionless heat transfer coefficient at Point 2.
**Figure G.32** Time averaged local dimensionless heat transfer coefficient at Point 3.

**Figure G.33** Time averaged local dimensionless heat transfer coefficient at Point 4.
Figure G.34 Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: d=0.10 V=0.40π ω=4π c=0.15.

Figure G.35 Variation of surface average heat transfer coefficient with time: d=0.10 V=0.40π ω=4π c=0.15, (a) left. (cont.)
Figure G.35  Variation of surface average heat transfer coefficient with time: d=0.10 V=0.40π ω=4π c=0.15: (b) top, (c) right.
Figure G.36 Local dimensionless heat transfer coefficient as a function of dimensionless time: (a) $d=0.10$ $V=0.20\pi$ $\omega=2\pi$ $c=0.30$, (b) $d=0.20$ $V=0.40\pi$ $\omega=2\pi$ $c=0.30$. 
Figure G.37 Time averaged local dimensionless heat transfer coefficient at Point 1.

Figure G.38 Time averaged local dimensionless heat transfer coefficient at Point 2.
Figure G.39  Time averaged local dimensionless heat transfer coefficient at Point 3.

Figure G.40  Time averaged local dimensionless heat transfer coefficient at Point 4.
Figure G.41 Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: d=0.20 V=0.40π ω=2π c=0.30.

(a)

Figure G.42 Variation of surface average heat transfer coefficient with time: d=0.20 V=0.40π ω=2π c=0.30: (a) left (cont.).
Figure G.42  Variation of surface average heat transfer coefficient with time: 
\[ d = 0.20 \quad V = 0.40 \pi \quad \omega = 2\pi \quad c = 0.30 \]  
(b) top, (c) right.
Figure G.43 Local dimensionless heat transfer coefficient as a function of dimensionless time: \( d=0.20 \) \( V=0.80\pi \) \( \omega=4\pi \) \( c=0.30 \): (a) block 1, (b) block 2.
Figure G.44 Local dimensionless heat transfer coefficient as a function of dimensionless time: $d=0.20 \ V=1.6\pi \ \omega=8\pi \ c=0.30$: (a) block 1, (b) block 2.
Figure G.45  Time averaged local dimensionless heat transfer coefficient at Point 1: (a) block 1, (b) block 2.
Figure G.46 Time averaged local dimensionless heat transfer coefficient at Point 2: (a) block 1, (b) block 2.
Figure G.47 Time averaged local dimensionless heat transfer coefficient at Point 3: (a) block 1, (b) block 2.
Figure G.48 Time averaged local dimensionless heat transfer coefficient at Point 4: (a) block 1, (b) block 2.
Figure G.49 Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: $d=0.20 \, V=1.6\pi \, \omega=8\pi \, c=0.3$: (a) block 1, (b) block 2.
Figure G.50  Variation of surface average heat transfer coefficient with time: \( d=0.20 \)
\( V=1.6\pi \omega=8\pi c=0.30 \): (a) block 1 left, (b) block 1 top (cont.).
Figure G.50  Variation of surface average heat transfer coefficient with time: $d=0.20$ $V=1.6\pi$ $\omega=8\pi$ $c=0.30$: (c) block 1 right, (d) block 2 left (cont.).
Figure G.50 Variation of surface average heat transfer coefficient with time: $d=0.20$ $V=1.6\pi \omega = 8\pi$ $c=0.30$: (e) block 2 top (f) block 2 right.
G.5.2 Plate Between Heating Elements – Top of Heat Source Level

Figure G.51 Local dimensionless heat transfer coefficient as a function of dimensionless time: \( d=0.10 \), \( V=0.20\pi \), \( \omega=2\pi \), \( c=0.30 \): (a) block 1, (b) block 2.
Figure G.52  Local dimensionless heat transfer coefficient as a function of dimensionless time: d=0.20 V=0.40π ω=2π c=0.30: (a) block 1, (b) block 2.
Figure G.53  Time averaged local dimensionless heat transfer coefficient at Point 1: (a) block 1, (b) block 2.
Figure G.54  Time averaged local dimensionless heat transfer coefficient at Point 2: (a) block 1, (b) block 2.
Figure G.55 Time averaged local dimensionless heat transfer coefficient at Point 3: (a) block 1, (b) block 2.
Figure G.56  Time averaged local dimensionless heat transfer coefficient at Point 4: (a) block 1, (b) block 2.
Figure G.57 Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: $d=0.20 \ V=0.4\pi \ \omega=2\pi \ c=0.30$: (a) block 1, (b) block 2.
Figure G.58  Variation of surface average heat transfer coefficient with time: \( d=0.20 \)
\( V=0.4\pi \) \( \omega=2\pi \) \( c=0.30 \): (a) block 1 left, (b) block 1 top (cont.).
Figure G.58  Variation of surface average heat transfer coefficient with time: $d=0.20$, $V=0.4\pi$, $\omega=2\pi$, $c=0.30$: (c) block 1 right, (d) block 2 left (cont.).
Figure G.58  Variation of surface average heat transfer coefficient with time: \( d=0.20 \), \( V=0.4\pi \), \( \omega=2\pi \), \( c=0.30 \): (e) block 2 top, (f) block 2 right.
G.5.3 Plate Over First Heat Source

Figure G.59 Local dimensionless heat transfer coefficient as a function of dimensionless time: \( d=0.20 \), \( V=0.40\pi \), \( \omega=2\pi \), \( c=0.30 \): (a) block 1, (b) block 2.
Figure G.60  Local dimensionless heat transfer coefficient as a function of dimensionless time: \( d=0.20 \) \( V=0.8\pi \) \( \omega=4\pi \) \( c=0.30 \): (a) block 1, (b) block 2.
Figure G.61 Time averaged local dimensionless heat transfer coefficient at Point 1: (a) block 1, (b) block 2.
Figure G.62 Time averaged local dimensionless heat transfer coefficient at Point 2: (a) block 1, (b) block 2.
Figure G.63  Time averaged local dimensionless heat transfer coefficient at Point 3: (a) block 1, (b) block 2.
Figure G.64 Time averaged local dimensionless heat transfer coefficient at Point 4: (a) block 1, (b) block 2.
Figure G.65 Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: $d=0.20$ $V=0.8\pi$ $\omega=4\pi$ $C=0.30$: (a) block 1, (b) block 2.
Figure G.66  Variation of surface average heat transfer coefficient with time: \( d=0.20 \)
\( V=0.8\pi \) \( \omega=4\pi \) \( c=0.30 \): (a) block 1 left, (b) block 1 top (cont.).
Figure G.66 Variation of surface average heat transfer coefficient with time: \(d=0.20\) \(V=0.8\pi\) \(\omega=4\pi\) \(c=0.30\) (c) block 1 right (d) block 2 left (cont.).
Figure G.66  Variation of surface average heat transfer coefficient with time:  d=0.20
V=0.8\pi \ \omega=4\pi \ c=0.30:  (e) block 2 top, (f) block 2 right.
G.5.4 Plate Between Heat Sources – Hole Level

Figure G.67 Local dimensionless heat transfer coefficient as a function of dimensionless time: \( d=0.20 \), \( V=0.40\pi \), \( \omega=2\pi \), \( c=0.30 \): (a) block 1, (b) block 2.
Figure G.68  Local dimensionless heat transfer coefficient as a function of dimensionless time: $d=0.20 \ V=0.80\pi\ \omega=4\pi\ c=0.30$: (a) block 1, (b) block 2.
Figure G.69  Time averaged local dimensionless heat transfer coefficient at Point 1: (a) block 1, (b) block 2.
Figure G.70  Time averaged local dimensionless heat transfer coefficient at Point 2: (a) block 1, (b) block 2.
Figure G.71 Time averaged local dimensionless heat transfer coefficient at Point 3: (a) block 1, (b) block 2.
Figure G.72 Time averaged local dimensionless heat transfer coefficient at Point 4: (a) block 1, (b) block 2.
Figure G.73  Variation of local heat transfer coefficient over the top surface of the element at dimensionless times indicated: $d=0.20$, $V=0.80\pi$, $\omega=4\pi$, $c=0.30$: (a) block 1, (b) block 2.
Figure G.74 Variation of surface average heat transfer coefficient with time: $d=0.20 \ V=0.80\pi \ \omega=4\pi \ c=0.30$: (a) block 1 left, (b) block 1 top (cont.).
Figure G.74  Variation of surface average heat transfer coefficient with time: d=0.20  V=0.80\pi  \omega=4\pi  c=0.30: (c) block 1 right, (d) block 2 left (cont.).
Figure G.74 Variation of surface average heat transfer coefficient with time: 
d=0.20 V=0.80\pi \omega=4\pi, c=0.30: (e) block 2 top, (f) block 2 right.
REFERENCES


