A statistical study of plasmawaves and energetic particles in the outer magnetosphere

Kyungguk Min
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ABSTRACT
A STATISTICAL STUDY OF PLASMA WAVES AND ENERGETIC PARTICLES IN THE OUTER MAGNETOSPHERE

by
Kyungguk Min

The Earth magnetosphere contains energetic particles undergoing specific motions around Earth’s magnetic field, and interacting with a variety of waves. The dynamics of energetic particles are often described in terms of three kinds of adiabatic invariants. Energetic electrons are often unstable to the whistler-mode chorus waves, and ions, to the electromagnetic ion cyclotron (EMIC) instability. These waves play an important role in the dynamics of the magnetosphere by energizing electrons to form a radiation belt, extracting energy from the hot, anisotropic ions and causing pitch angle scattering of energetic ions and relativistic electrons into the loss cone. EMIC waves correspond to the highest frequency waves in the ultra-low frequency (ULF) spectral regime, and field line resonances at the lower frequency may serve as diagnostics for the plasma distribution in the magnetosphere. This dissertation investigates (1) a rapid, efficient way of specifying particle’s adiabatic motion in the magnetosphere, (2) source of the whistler-mode chorus waves, (3) physical properties and coherent spatial dimensions of the EMIC waves and (4) a diagnostic use of the toroidal mode Alfvén waves on the plasma density distribution in the Earth magnetosphere.

The studies presented in this dissertation have significantly been benefited from the comprehensive data obtained by several space missions, including the Time History of Events and Macroscale Interactions during Substorms (THEMIS) spacecraft, Cluster mission, the Geostationary Operational Environment Satellites (GOES), Los Alamos National Laboratory (LANL) satellites, the Polar spacecraft and the Active Magnetospheric Particle Tracer Explorers (AMPTE)/Charge Composition Explorer (CCE), and from ground-based Automatic Geophysical Observatories (AGO).

The main findings and achievements in this dissertation are as follows: (1) A method of rapidly and efficiently computing the magnetic drift invariant ($L^*$) was devel-
oped. This new method is not only fast enough for near real-time calculation of $L^*$, enabling spacecraft tracking in this coordinates, but scalable to a large number of $L^*$ values that are often required for inter-comparison between simulation results and observations.

(2) The relationship between the electron injection and the chorus waves was studied from the simultaneous observations of a substorm event on 23 March 2007 made in space and on ground. Timing analysis and a test particle simulation indicated that the electrons injected during the substorm could form a pitch-angle distribution suitable for the whistler-mode instability when they arrive near the dawn-side magnetopause. (3) The EMIC waves are found to occur ubiquitously throughout the outer magnetosphere and their properties distribute asymmetrically in local time. The asymmetry in the wave properties seems to be correlated with the electron density distribution and ion temperature anisotropy, as supported by a linear EMIC instability model. (4) The size of coherent activity of the EMIC waves was estimated using the multi-spacecraft observations made by the THEMIS spacecraft and cross correlation analysis. It is found that the characteristic dimension in the direction transverse to the local magnetic field is 2–3 times the local EMIC wavelength. (5) The global distribution of the equatorial mass density was derived from the toroidal mode standing Alfvén waves in an unprecedented spatial scale. The equatorial mass density is distributed asymmetrically with a bulge at the dusk sector and the magnitude falls logarithmically with increasing radial distance. It is confirmed that the variation in the derived mass density is only weakly related to the geomagnetic activity, but has strong correlation with the solar activity.

The major contribution of this dissertation is the extension of the scope of previous understanding of various plasma wave properties and energetic particle dynamics in the inner magnetosphere to outer magnetosphere by new, in-depth analyses of the data from the THEMIS, GOES and AMPTE/CCE missions.
A STATISTICAL STUDY OF PLASMA WAVES AND ENERGETIC PARTICLES IN THE OUTER MAGNETOSPHERE

by

Kyungguk Min

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A STATISTICAL STUDY OF PLASMA WAVES AND ENERGETIC PARTICLES IN THE OUTER MAGNETOSPHERE

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Min, K., J. Bortnik, and J. Lee, A Novel Technique for Rapid $L^*$ Calculation, GEM Summer Workshop, Snowmass, CO, June 2012
Min, K., J. Lee, K. Keika, and W. Li, Properties of EMIC waves measured by the THEMIS mission, Space Weather Workshop, Boulder, CO, April 2012
Min, K., J. Lee, K. Keika, and W. Li, Global distribution of EMIC waves derived from THEMIS observations, Chapman Conference on Dynamics of the Earth’s Radiation Belts and Inner Magnetosphere, St. John’s, Newfoundland and Labrador, Canada, June 2011
Min, K., J. Lee, K. Keika, and W. Li, Global distribution of EMIC waves derived from THEMIS observations, GEM Summer Workshop, Santa Fe, NM, June 2011
Min, K., J. Lee, and K. Keika, Chorus wave generation near the dawnside magnetopause due to drift shell splitting of substorm electrons, 10th International Conference of Substorm (ICS10), San Luis Obispo, CA, March 2010
Min, K., K. Keika, and J. Lee, A Three Dimensional Test Particle Simulation for the Electron Injections associated with the 23 March 2007 Substorm, AGU Fall Meeting, San Francisco, CA, December 2009
To My Beloved Parents
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7.2 Contour lines of (a) the derived equatorial mass density and (b) the approximated mass density shown in the $x$-$y$ plane. The shaded area in (b) indicates the region where the relative error is greater than 10%.
1.1 General Description of the Terrestrial Magnetosphere

The terrestrial magnetosphere comprises the region of space where the properties of naturally occurring ionized gases are controlled by the presence of Earth’s magnetic field. This means that the terrestrial magnetosphere extends from the bottom of the ionosphere to more than ten Earth radii ($R_E$) in the sunward direction and to several hundred $R_E$ in the anti-sunward direction. The magnetosphere is formed as a result of the interaction of the supersonic, super-Alfvénic, magnetized solar wind with the intrinsic magnetic field of the Earth. The content of this section was mainly referenced from Baumjohann (1997) and Gombosi (1998), and partly from Kivelson and Russell (1995).

1.1.1 Intrinsic Magnetic Field

In general, the near Earth magnetic field can be expressed as the gradient of a scalar potential

$$\mathbf{B} = -\nabla \Phi_{\text{tot}} = -\nabla (\Phi_{\text{int}} + \Phi_{\text{ext}}),$$  \hspace{1cm} (1.1)

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$^aR_E$ denotes Earth radius, 6,378 km.
where $\Phi_{\text{int}}$ and $\Phi_{\text{ext}}$ represent scalar potentials due to intrinsic and external sources, and $\Phi_{\text{tot}}$ is the sum of these two potentials. These magnetic potentials can be expressed as an infinite series using associated Legendre polynomials and the coefficients of each basis are determined with the help of ground-based and space-borne measurement. These coefficients are regularly updated by the collaborative work between the modelers and the institutes involved in collecting and disseminating magnetic field data obtained around the world and space, and provided as the International Geomagnetic Reference Field (IGRF) model.

Clearly, the first order term in the series describes the magnetic dipole field which in the case of Earth, dominates the Earth’s intrinsic magnetic field. The terrestrial dipole field parameters are summarized in Table 1.1. In addition to the magnetic dipole tilt angle, the inclination of the Earth’s rotation axis to the ecliptic plane and the Earth’s daily rotation and its orbiting around the Sun has important consequences for the configuration of the terrestrial magnetosphere, since the large-scale morphology is controlled to a large extent by the interaction of the magnetized solar wind with the terrestrial magnetic field.

1.1.2 Interaction of the Solar Wind with the Terrestrial Magnetic Field

**Solar Wind** The solar wind is the high-speed conducting plasma continuously blowing out from the solar corona in the interplanetary space. At around the Earth’s orbit, the solar wind speed typically ranges between 300–1400 km/s. Because of the high conductivity, the solar magnetic field is frozen in the plasma and drawn outward by the expanding solar wind. Once the solar wind has been formed, its plasma expands into the interplanetary space. This expansion is a radial propagation of the solar wind plasma fluid away from the Sun during which it must become diluted and at the same time cool down.

Close to the Earth, the solar wind is fully ionized plasma consisting mainly of electrons, protons, and $\alpha$ particles. Typical values for the electron density and temperature in the solar wind near the Earth are $n_e \approx 5 \text{ cm}^{-3}$ and $T_e \approx 10^5 \text{ K}$. The interplanetary magnetic
field is of the order of 5 nT. The Alfvén velocity is the fundamental speed at which magnetic signals in a plasma can be transported by waves. The typical Alfvén velocity of the solar wind is only about 30–50 km/s so that the solar wind is a supersonic and super-Alfvénic flow.

**Bow Shock, Magnetosheath and Magnetopause** Because of its supersonic and super-Alfvénic velocity, the solar wind flow is subject to shocking if there is an, especially magnetized, obstacle in its path. When the solar wind hits on the Earth’s dipolar magnetic field, it cannot simply penetrate, it but rather is slowed down and, to a large extent, deflected around it. Since the solar wind hits the obstacle with supersonic speed, a bow shock wave is generated, where the plasma is slowed down and a substantial fraction of the particles’ kinetic energy is converted into thermal energy. The region of thermalized subsonic plasma behind the bow shock is called the magnetosheath. Its plasma is denser and hotter than the solar wind plasma and the magnetic field strength has higher values in this region.

The shocked, fully ionized and magnetized solar wind plasma in the magnetosheath cannot easily penetrate the terrestrial magnetic flux tubes. Instead, it will deviate from its original direction and will, by its dynamical pressure, compress the terrestrial field into a small region of space, the magnetosphere. The interaction boundary between two regions is called the magnetopause. This is a consequence of the fact that the interplanetary magnetic field lines cannot penetrate the terrestrial field lines and that the solar wind particles cannot leave the interplanetary field lines due to the frozen-in characteristic of a highly conducting plasma. As a tangential discontinuity, the magnetopause is a surface of total pressure equilibrium between the solar wind-magnetosheath plasma and the geomagnetic field confined in the magnetosphere. The typical value of the stand-off distance from the Earth’s center to the magnetopause is \( R_{MP} \approx 10 R_E \). The kinetic pressure of the solar wind plasma stretches the nightside magnetic field out into a long magnetotail which reaches far beyond the lunar orbit.
Magnetospheric Cavity  

As a result of the shielding, the magnetospheric cavity is formed within the magnetopause. Figure 1.1 shows a schematic representation of the magnetosphere in the noon-midnight meridian. In contrast to the compressed configuration of the dayside magnetosphere, the magnetic field lines on the nightside are highly stretched along the solar wind flow direction and they form the magnetotail. The reversal of the magnetic field between the northern and southern geomagnetic lobes implies a tail current sheet called the neutral sheet with electric current flowing from dawn to dusk across the tail.

The plasma in the magnetosphere consists mainly of electrons and protons. The source of these particles are the solar wind and the terrestrial ionosphere. In addition, there are small fractions of He\(^+\) and O\(^+\) ions of ionospheric origin and some He\(^{++}\) ions originating from the solar wind. However, the plasma inside the magnetosphere is not evenly distributed, but is grouped into different regions with quite different densities and temperatures.
The radiation belt lies on dipolar field lines between about 2 and 6 $R_E$. It consists of energetic electrons and ions which move along the field lines and oscillate back and forth between the two hemispheres. Typical electron densities and temperatures in the radiation belt are $n_e \approx 1 \text{ cm}^{-3}$ and $T_e \approx 5 \times 10^7 \text{ K}$. The magnetic field strength ranges between about 100 and 1000 nT.

Most of the magnetotail plasma is concentrated around the tail mid-plane in an about 10 $R_E$ thick plasma sheet. Geomagnetic field lines within the plasma sheet are highly stretched, and the plasma there is hot and dense. Near the Earth, it reaches down to the high-latitude auroral ionosphere along the field lines. Average electron densities and temperatures in the plasma sheet are $n_e \approx 0.5 \text{ cm}^{-3}$ and $T_e \approx 5 \times 10^6 \text{ K}$, with $B \approx 10 \text{ nT}$.

The outer part of the magnetotail is called the magnetotail lobe. It contains a highly rarified plasma with typical values for the electron density and temperature and the magnetic field strength of $n_e \approx 10^{-2} \text{ cm}^{-3}$, $T_e \approx 5 \times 10^5 \text{ K}$, and $B \approx 30 \text{ nT}$, respectively.

1.1.3 Magnetospheric Currents

The plasmas are usually not stationary, but they move under the influence of external forces. Sometimes ions and electrons move together, like in the solar wind. The movement of the charged particles produces currents. Such currents are very important for the dynamics of the Earth’s plasma environment. They transport charge, mass, momentum and energy. Moreover, the currents create magnetic fields which may severely alter or distort any pre-existing fields. There are five principal current systems that play an important role in the formation of the magnetosphere: the magnetopause current, the tail current, the ring current, field-aligned currents, and ionospheric currents.

**The Magnetopause Current** The magnetopause current (also known as Chapman-Ferraro current) is the current system flowing around the magnetopause. This current system generates a magnetic field that prevents the terrestrial dipole field from penetrating into the solar wind (shielding). For the typical values of the solar wind, the magnetopause current
\( j_{\text{MP}} \approx 10^{-7} \text{ A/m}^2 \).

**The Tail Current**  The tail-like field of the nightside magnetosphere is accompanied by the tail current flowing on the tail surface and the neutral sheet current in the central plasma sheet, both of which are connected and form a \( \Theta \)-like current system, if seen from along the Earth-Sun line. For the nominal tail magnetic field strength \( B_T = 20 \text{ nT} \), the tail current \( I_T \approx 30 \text{ mA/m} \).

**The Ring Current**  Another large-scale current system, which influences the configuration of the inner magnetosphere, is the ring current. The ring current is primarily composed of geomagnetically trapped 10 to 200 keV particles (\( \text{H}^+ \) and \( \text{O}^+ \)) bouncing along closed geomagnetic field lines and is typically located between 3 and 6 \( R_E \). In addition to their bounce motion, these particles drift slowly around the Earth, which will be introduced in section 1.2. Since the protons drift westward while the electrons move in the eastward direction, this constitutes a net charge transport that flows around the Earth in a westward direction.

The westward current flow generates the negative magnetic field perturbation at the center of the Earth and weakens the geomagnetic field. This perturbation is a good measure of the total energy of the ring current particles which can be estimated by the \( D_{st} \) index.

**Ionospheric Currents**  A number of current systems exist in the conducting layers of the Earth’s ionosphere, at altitudes of 100–150 km. The ions and, to a lesser degree, the electrons in the E-region are coupled to atmosphere winds and tidal oscillations of the atmosphere and follow their dynamics. The relative movement constitutes an electric current and the separation of charge produces an electric field. Most notable are the auroral electrojets inside the auroral oval, the \( S_q \) currents in the dayside mid-latitude ionosphere, and the equatorial electrojet near the magnetic equator.

**Field-Aligned Currents**  In addition to these perpendicular currents, currents also flow along magnetic field lines. The field-aligned currents, also called Birkeland currents named
after Kristian Birkeland, connect the magnetospheric current systems in the magnetosphere to those flowing in the polar ionosphere. The field-aligned currents are mainly carried by electrons and are essential for the exchange of energy and momentum between these regions. Recent observations suggest that these field-aligned currents connect the tail current, the ionospheric currents and the partial ring current, completing the closed circuit.

1.1.4 Geomagnetic Activity

Magnetic Storms Magnetospheric storms are large, prolonged disturbances of the magnetosphere caused by variations in the solar wind. Many magnetic storms follow solar flares or coronal mass ejections. The impulse from the interplanetary disturbance impulsively compresses the magnetosphere, causing rapid increase of the magnetopause current.

Most storms are related to long period (several hours) southward interplanetary magnetic field (IMF) which is the most favorable configuration for magnetic reconnection at the dayside magnetopause. The increased dayside reconnection increases the penetration of the solar wind electric field and, consequently, increases the magnetospheric convection. The enhanced convection leads to injection of the energetic ring current ions into the ring current. The enhanced ring current reduced the magnetic field strength on the surface of the Earth’s equator, which can be measured by the $Dst$ index drop. This time period is called the main phase of the magnetic storm and lasts from a few hours to about a day.

As the southward IMF component weakens, the reconnection rate decreases, the convection decreases and in turn the injection of new particles into the ring current decreases. At this period, the ionospheric cold plasma starts to fill the depleted flux tubes due to the reduced injection. The interaction between the cold plasma and the energetic ring current ion population enhances the ring current loss rate via wave-particle interaction or direct charge exchange. This phase is called recovery phase.

Substorms A magnetospheric substorm is a time period of enhanced energy input into the magnetosphere from the solar wind and its subsequent dissipation in the magnetosphere-
ionosphere system. It is termed substorm because the main phase of large magnetic storms often appear to be the superposition of many smaller storms, each of which contributes to the growth of the ring current.

Individual substorms usually follow periods of southward IMF configuration. When the IMF turns negative, the reconnection rate increases considerably at the front of the magnetopause. This leads to a subsequent increase in the magnetic flux density transferred to the geomagnetic tail. The increasing flux density is accompanied by a growing neutral sheet current, which stretch the field lines threading the plasma sheet into a more tail-like configuration. The period of enhanced convection and loading of the tail with magnetic flux is called substorm growth phase.

As the magnetic flux and magnetic energy is accumulated in the tail, the tail becomes unstable and the surplus energy is released through the reconnection at the tail. This is the time of substorm onset and the beginning of the substorm expansion phase. The release of the energy and the particles greatly enhances the auroral activity and the ionospheric current flow, which are seen in the variation of the Kp and AE indices. About an hour after the substorm onset, the ionospheric current flow and the bright aurora start to decrease and the substorm recovery phase begins.

**Geomagnetic Indices** Geomagnetic indices are widely used to characterize the dynamic state of various aspects of the magnetosphere-ionosphere system.

The K and Kp indices are a measure of the irregular short-term variations of standard magnetograms and characterizes the general level of disturbance. The Kp index, which is most widely used geomagnetic index, was intended to characterize the worldwide geomagnetic activity level, but it is most sensitive to auroral zone activity.

The AE (auroral electrojet) indices are defined to obtain a measure of the strength of the auroral electrojet and calculated from the horizontal component of the magnetic field measured by a worldwide chain of auroral zone magnetometers.

The Dst index is a measure of the strength of the ring current. It is average around
Figure 1.2 An example of a typical magnetic storm. From the top, IMF $B_y$ and $B_z$, solar wind dynamic pressure $P_{dyn}$, Kp, Dst and AE indices measured during 15 November, 2001 are shown.
the world of the adjusted residuals of the horizontal components of the magnetic field measured by low magnetic latitude observatories. The adjustment is made for the quiet day levels and for the station’s magnetic latitude.

Figure 1.2 shows an example of the solar wind parameters and recorded geomagnetic indices for a typical geomagnetic storm during 15 November, 2001. Before storm onset (first vertical bar in the Dst panel), the disturbance in the solar wind suddenly compresses the magnetopause. The sudden compression, as seen from the large increase in \( P_{\text{dyn}} \), enhances the magnetopause current which increases the horizontal component of the surface magnetic field. As a result, the Dst slightly increases at this period. The main phase, period between two vertical dashed bars in the Dst panel, begins during the prolonged negative IMF \( B_z \) which enhances the global convection and supplies ring current ions. During this stage, Kp and AE indices typically increase. As soon as the IMF \( B_z \) recovers, the convection reduces and the ring current starts to decay, causing slow recovery of the Dst as shown after the dashed bar.

1.2 Trapped Particles

In Earth’s magnetosphere, a particle motion trapped in the magnetic field is well separated into three types each of which has the adiabatic invariant and the associated periodicity. Due to the invariant nature of these motions in a slowly varying electric and magnetic fields, the adiabatic invariants are preferentially used to describe the trapped particles in the magnetosphere. The content of this section was mainly referenced from Roederer (1970).

1.2.1 Gyration and the First Adiabatic Invariant

A motion of a particle of mass, \( m \), and charge, \( q \), in the magnetic field, \( \mathbf{B} \), can be fully described by

\[
m \frac{d\mathbf{v}}{dt} = q(\mathbf{v} \times \mathbf{B}),
\] (1.2)
Figure 1.3 Schematics of charged particle motions in the Earth’s magnetosphere: (a) gyromotion and the first adiabatic invariant, (b) bounce motion and the second adiabatic invariant, and (c) drift motion and the third adiabatic invariant. Filled dots indicate a charged particle and light gray curves in (b) and (c) indicate dipole field lines.

where \( \mathbf{v} \) is the velocity of the charged particle. The equation states that the particle moved freely along the magnetic field direction but executes the circular motion around the magnetic field line, as shown in Figure 1.3, with the period given by the gyrofrequency

\[
\Omega_g = \frac{qB}{m},
\]  

(1.3)

and the radius of the circle that the particle makes is given by the gyroradius

\[
r_g = \frac{v_\perp}{|\Omega_g|} = \frac{mv_\perp}{|q|B},
\]  

(1.4)

where \( v_\perp \) is the velocity perpendicular to the magnetic field line. If the spatio-temporal variation of the magnetic field remains smaller than \( r_g \) and \( \Omega_g \), then the magnetic moment of the current loop due to the circular motion of the charged particle

\[
\mu = \frac{mv^2 \sin^2 \alpha}{2B},
\]  

(1.5)

where \( \alpha \) is the pitch angle between \( \mathbf{v} \) and \( \mathbf{B} \), remains invariant of motion.
1.2.2 Bounce Motion and the Second Adiabatic Invariant

Assuming a particle moving along an inhomogeneous magnetic field line, only the pitch angle can change when the magnetic field increases or decreases along the guiding center trajectory (Figure 1.3) if the spatio-temporal gradients is weak and the total energy is a constant of motion. The pitch angles of the particle at different locations are then directly related to the magnetic field strengths according to

\[ \frac{\sin^2 \alpha_2}{\sin^2 \alpha_1} = \frac{B_2}{B_1}. \]  

(1.6)

In a converging magnetic field geometry, a particle moving into regions of stronger fields will have its pitch angle increase. If \( B_m \) is a point along the field line where the pitch angle reaches \( \alpha = 90^\circ \), the particle is reflected from this mirror point. The force that reduces the velocity parallel to the magnetic field is called mirror force, \( -\mu \nabla \parallel B \).

In a symmetric magnetic field geometry with a minimum field in the middle and converging magnetic field lines on both sides, like in a dipole field, a particle bounces back and forth between its two mirror points with the bounce frequency, \( \omega_b \), and becomes trapped. The second adiabatic invariant is defined by

\[ J = \oint m_\parallel ds, \]  

(1.7)

where \( v_\parallel \) is the parallel velocity, \( ds \) is an element of the guiding center path and the integral is taken over a full oscillation between the mirror points.

1.2.3 Drift Motion and the Third Adiabatic Invariant

If there is an external force applied to a particle in the magnetic field, the guiding center of the particle drift across magnetic field lines with the velocity

\[ v_D = \frac{1}{\Omega_g} \left( \frac{F_{\text{ext}}}{m} \times \frac{B}{B} \right). \]  

(1.8)
The first order contributions in the terrestrial magnetosphere are usually the inhomogeneous magnetic field and the electric field that result in the gradient/curvature drift and $\mathbf{E} \times \mathbf{B}$ drift, respectively.

In an axisymmetric magnetic field configuration, the drift of the guiding center forms a closed drift shell around the magnetic field axis (Figure 1.3). The third invariant, $\Phi$, is simply the magnetic flux encircled by the drift shell defined as

$$\Phi = \oint \mathbf{A} \cdot d\mathbf{l}, \quad (1.9)$$

where $\mathbf{A}$ is the vector potential and $d\mathbf{l}$ is an element of the closed drift orbit.

Unlike the previous two adiabatic invariants, the integration of the third adiabatic invariant, Eq. (1.9), requires the complete drift shell, computing which has been challenging (McCollough et al. 2008). Chapter 2 discusses the algorithm and implementation, the performance of the new method.

### 1.3 Waves in Space Plasma

Any plasma will react to external source of violent distortion. If the state of the plasma is extremely distorted, it may lead to a chaotic turbulence. If the distortion is moderate, the resulting disturbances may be thought of as a superposition of linear waves onto the quiescent plasma state which propagate across the plasma in order to transport the energy of the distortion and to communicate it to the entire plasma volume (Baumjohann 1997). Such plasma waves have been measured in many different frequency ranges which are subdivided into ultra-low (ULF), extremely-low (ELF) and very-low frequency (VLF) waves.

Plasma waves are not generated at random and must satisfy the appropriate equation of the plasma. The amplitude of a wave must exceed the level of the thermal fluctuations always present in a plasma, which sets a limit on the initial disturbance causing the waves. If there is no mechanism acting to amplify the small disturbance in the plasma, there is no
wave.

![Cold Plasma Dispersion Relation](image)

**Figure 1.4** Low-frequency whistler-mode dispersion relation. Electron density $n_e = 10^2$, magnetic field magnitude $\Omega_{H^+} = 4\pi$ and proton to electron mass ratio $m_p/m_e = 4$ are set, and 10% helium and oxygen were assumed.

### 1.3.1 Low-frequency Whistler-mode Dispersion Relation

The dispersion relations for R-mode ($s = 1$) or L-mode ($s = -1$) waves propagating parallel or antiparallel to a uniform magnetic field in a cold, multi-ion ($H^+, He^+, O^+$) plasma are
\[
\frac{y^2}{x^2} = 1 + \frac{1}{\alpha^* x} \left( \frac{1}{s - x} \frac{\epsilon \eta_{H^+}}{s + s \epsilon} - \frac{\epsilon \eta_{He^+}}{x + s \epsilon} - \frac{\epsilon \eta_{O^+}}{16x + s \epsilon} \right), \tag{1.10}
\]

where \( \alpha^* = \Omega_{ge}^2 / \omega_{pe}^2 \) is a cold-plasma parameter; \( \omega_{pe} \) and \( \Omega_{ge} \) are the electron plasma frequency and gyrofrequency, respectively; \( \eta \) is the ion composition; \( \epsilon = m_e / m_p \) is the ratio of the electron mass to the proton mass; and the dimensionless variables

\[
x = \frac{\omega}{\Omega_{ge}}, \quad y = \frac{k c}{\Omega_{ge}},
\]

where \( \omega \) is the wave frequency; \( k \) is the wave number and \( c \) is the speed of light.

### 1.3.2 Whistler-mode Waves

Whistler-mode waves are parallel propagating, right-hand polarized electromagnetic waves in the VLF range generated by the electron cyclotron instability which comes from the anisotropic distribution of the electrons (e.g., Kennel and Petschek 1966; Stix 1992). Assuming that the wave frequencies are above the ion gyrofrequency but well below the electron gyrofrequency \( (\Omega_{gi} < \omega \ll \Omega_{ge} \text{ or } \epsilon < x \ll 1) \), Eq. (1.10) can be reduced to

\[
\frac{k^2 c^2}{\omega^2} \approx \frac{\omega_{pe}^2}{\omega \Omega_{ge}} \quad \text{or} \quad \frac{y^2}{x^2} \approx \frac{1}{\alpha^* x}, \tag{1.12}
\]

and the group and phase velocities of these waves are

\[
\frac{v_{ph}}{c} = \frac{\sqrt{\Omega_{ge}}}{\omega_{pe}} \sqrt{\omega}
\]
\[\frac{v_g}{c} = 2 \frac{\sqrt{\Omega_{ge}}}{\omega_{pe}} \sqrt{\omega} = 2v_{ph}. \tag{1.13}\]

If one generates a pulse containing many frequencies, the higher frequency components of the pulse propagate faster along the magnetic field line than the lower frequency components.
ones. A broadband receiver placed far enough from the origin of the pulse will detect the high frequencies first and the lower frequencies later. This phenomenon was first observed by field radio operators in World War I using radios operating around 10 kHz. The explanation of these observations is that lightning generates a pulse of electromagnetic waves containing many frequencies. Some of the waves propagate along the geomagnetic field lines to the opposite hemisphere of the Earth, where it is observed as whistling noise and thus called whistlers.

In the Earth magnetospheric environment, the whistlers are commonly generated from the anisotropic energetic electron distribution which is usually formed during the particle transport or by the fluctuations of the magnetic field. These waves are known to play an important role in accelerating and scattering the relativistic electrons in the radiation belts (e.g., Millan and Thorne 2007).

1.3.3 Electromagnetic Ion Cyclotron Waves

The ion counterpart of the whistler-mode waves is the Electromagnetic Ion Cyclotron (EMIC) waves that are parallel propagating, left-hand polarized \((s = -1)\) electromagnetic waves generated by the ion cyclotron instability. As the previous case, the EMIC waves are unstable to the anisotropic distribution of energetic (mostly) \(\text{H}^+\) ions (e.g. Cornwall 1965; Young et al. 1981; Roux et al. 1982; Rauch and Roux 1982; Anderson et al. 1992a, 1996), and cause the scattering of the radiation belt ions and electrons during the geomagnetically disturbed time (e.g. Cornwall et al. 1970; Thorne and Kennel 1971) and thermal plasma heating (e.g. Thorne and Horne 1992, 1997; Zhang et al. 2010, 2011) through wave-particle interaction. Unlike the whistler-mode waves, the EMIC waves are significantly damped through the wave-particle interaction and often causes the low-energy ion heating while propagating along the field line.

The strong EMIC waves are often observed in the region where the dense cold plasma from the plasmasphere and the energetic ion population from the ring current dur-
ing the geomagnetic storm (e.g. Halford et al. 2010), and play a critical role in the ion heating and scattering of the relativistic electrons (e.g. Ukhorskiy et al. 2010). Generally, the EMIC waves seems to be ubiquitous in the entire magnetosphere (Anderson et al. 1992a; Fraser and Nguyen 2001; Fraser et al. 2010) and the sources of generation include the variation in the magnitude of the magnetic field by the variation in the solar wind and the continuous stream of the energetic ions from the plasma sheet (Anderson and Hamilton 1993; Engebretson et al. 2002; Usanova et al. 2008; McCollough et al. 2010). The present dissertation investigates the statistical distribution of the occurrence and the wave parameters in the outer magnetosphere based on the satellite observation in chapter 4.

Figure 1.5 Lowest six toroidal mode harmonic oscillations in the magnetic field. The solution was obtained using the Orbit 920 event in Denton et al. (2001) assuming the Tsyganenko 89 (T89) field model (Tsyganenko 1987, 1989).
1.3.4 ULF waves and Field Line Resonance

The fast fluctuations in the lower frequency range from a few Millihertz to a few Hertz are ubiquitous in the terrestrial magnetosphere and classified as geomagnetic pulsations, and can be measured on the ground. Since any location on ground is magnetically linked to anywhere in space, the ground magnetometer array all over the world can serve as tools for global monitoring of the ULF waves.

The simplest wave mode is the oscillations of the magnetic field line that are tied to the two reflection points on both hemispheres. A field line resonance (FLR) is the resonant Alfvénic oscillation of a closed geomagnetic field line, whose foot points lie in the ionosphere. Dungey (1955) suggested that ground based magnetometer observations of discrete frequency fluctuations were the signature of MHD standing waves on closed geomagnetic field lines (Claudepierre et al. 2008). The discrete frequency oscillations arise from the restriction to an integral number of half wavelengths along the field line fixed to the conducting ionospheric boundary.

The FLRs are ubiquitous in the magnetosphere and known to be driven externally by the solar wind (e.g., Takahashi and Anderson 1992). Broadband shock waves and pressure pulses propagate in a fast mode through the magnetosphere during which particular oscillations at resonating frequency are picked up (Takahashi et al. 1984), which is supported by MHD simulation (Claudepierre et al. 2008, 2010). They are also internally excited through drift-bounce resonances (e.g., Southwood 1976) and drift mirror instability (e.g., Hasegawa 1969) of energetic ions injected from the plasma sheet. Figure 1.5 illustrates the azimuthal magnetic field perturbations for the lowest six harmonic modes calculated from the wave equation given by Singer et al. (1997).

The discrete frequency oscillations embed the magnetic field magnitude and the plasma mass distribution along the field line (through Alfvén speed). Since the direct measurement of the mass density is difficult owing mainly to instrumental limitations, the indirect estimation from inversion of the observed discrete frequencies can be an alternative to
the mass density (e.g. Denton 2006). In addition to the local estimation, this technique can provide the field line distribution of plasma mass, provided that there are enough frequency measurements and the underlying field model is more or less accurate (Denton et al. 2001, 2004a). The present dissertation investigate the global distribution of the equatorial plasma mass density derived from the toroidal mode Alfvén waves observed from space in chapter 6.
Computing the magnetic drift invariant, $L^*$, rapidly and accurately has always been a challenge to magnetospheric modelers, especially given the importance of this quantity in the radiation belt community. In this chapter, a new, efficient method of calculating $L^*$ is proposed and developed, which is based on the principle of energy conservation. While the new method can be regarded as an extension of the technique of Roederer (1970), it has a major difference from the latter technique in that the new technique of tracing in a finite grid is free from error accumulation and ensures the existence of a closed drift orbit. A key feature of the new method is the dynamic computation of the field line parameters that are only necessary for the drift shell tracing, which reduces the computational cost significantly. The systematic analysis of accuracy and speed of the new method shows that the relative error is on the order of $10^{-3}$ when $\sim 0.1 \, R_E$ grid resolution is used and the calculation speed is about two seconds per particle in the popular Tsyganenko and Sitnov 05 model (TS05). Based on the application examples, it is suggested that this method could be an added resource for the radiation belt community.

2.1 Introduction

Constrained by the large-scale electric and magnetic fields of the inner magnetosphere, the dynamics of energetic particles in the Earth’s radiation belts are most conveniently expressed and visualized with the aid of three adiabatic invariants (e.g., Roederer 1970; Schulz and Lanzerotti 1974). In the terrestrial magnetosphere, these adiabatic invariants are associated with three periodic motions: gyration around the magnetic field, bounce motion along the magnetic field line between magnetic mirror points in the northern and southern hemispheres, and gradient/curvature drift across the magnetic field, leading to motion around the Earth. These adiabatic motions are well separated by their adiabatic
time scales and the adiabatic invariants are conserved as long as the external field remains quasi-static on each invariant time scale (e.g., Cary and Brizard 2009).

Computation of the first two invariants requires only the local magnetic field (single point) or its magnitude along the field line (one-dimensional line) where the guiding center of the particle is located. Computation of the third invariant ($L^*$) is, however, computationally more demanding because the particle drift shell in the global magnetic field (and electric field in general) must be known. Because the global magnetic field is difficult to measure, if not impossible, one must rely on magnetic field models to obtain this drift shell. As McCollough et al. (2008) demonstrated, computation of the drift shell and $L^*$ is evidently dependent upon the complexity of the model field used (exceeding well over ten days to compute one day worth of $L^*$ values in the Tsyganenko 02 (T02) model (Tsyganenko 2002a,b), Table 5 in McCollough et al. (2008)). These authors also demonstrated that using a simpler model (i.e., a faster model) with 2% difference in the model field can produce as much as a 10% difference in the resulting $L^*$.

Several libraries and methods that compute $L^*$ values are currently available. International Radiation Belt Environment Modeling (IRBEM) (Boscher et al. 2012) implements the technique introduced by Roederer (1970), in which adjacent field lines with the same magnetic field magnitude at a mirror point and the same second invariant as the initial value are iteratively searched until the drift shell is completed. Recently developed, LANLGeoMag (M. Henderson, http://www.rbsp-ect.lanl.gov), currently providing the orbits of the Van Allen Probes (Mauk et al. 2012) in $L^*$ coordinates, can compute various magnetic ephemerides including $L^*$ in real-time. LANLGeoMag numerically solves the particle’s guiding center equation of motion to obtain the drift shell and can achieve an accuracy as high as nine decimal places. LANL* (Koller et al. 2009; Koller and Zaharia 2011; Yu et al. 2012), unlike the previous two methods that are based on first principles (referred to as physics-based methods), uses a Neural Network that predicts $L^*$ values at a given parameter set based on the statistical $L^*$ values that were calculated at many independent
parameter sets. The calculation speed of this Neural Network is on the order of a tenth of a second and the relative error is about $3 \times 10^{-2}$ ($\Delta L^* < 0.2$) at geosynchronous orbit (Koller and Zaharia 2011).

The new method proposed and developed here is similar to the technique introduced by Roederer (1970) in that it uses conserved quantities to obtain the drift trajectory. The main differences are, however, as follows: first, unlike the technique of Roederer (1970), the trajectory is obtained by drawing the iso-energy contour from the discrete 2D energy space defined by constant first and second adiabatic invariants. This approach ensures that the calculated trajectory is closed if the particle’s drift shell is so, regardless of the grid resolution. Second, the functional relationship between $B_m$ and $K$ is approximated before the $L^*$ calculation (Sheldon and Gaffey 1993) used a polynomial expression and Min et al. (2013) used spline interpolation), where $B_m$ is the magnitude of the magnetic field at a mirror point $s_m$ and $K$ is the modified invariant (Roederer 1970)

\[
K = \int_{s_m}^{s_m'} \sqrt{B_m - B(s)} ds.
\] (2.1)

In this expression, $s_m'$ denotes the conjugate mirror point to $s_m$ so that $B(s_m') = B(s_m) \equiv B_m$, and the integration is made along the guiding field line $s$. Once the model coefficients are known, $B_m(K)$ or $K(B_m)$ is approximated from the functional form whenever they are needed, avoiding the repeated expensive field line tracing.

Following a short description of underlying physics, section 2.2 describes the algorithm and implementation in detail, while section 2.3 analyzes performance. sections 2.4 and 2.5 demonstrate a simple application and section 2.6 summarizes the chapter.
2.2 Algorithm and Implementation

2.2.1 Formulation

In a static electric and magnetic field, the dynamics of charged particles are completely described by three conserved quantities in the guiding center approximation: \( \mu, J \) (or \( K \)) and \( W \), which are linked by

\[
W = qU + mc^2 = qU + mc^2 \sqrt{1 + 2\frac{\mu B_m}{mc^2}},
\]  

(2.2)

where \( W \) is the total energy of the particle, \( U \) is the electric potential, \( \mu = p^2 / 2mB(s) \) is the first adiabatic invariant, \( q \) and \( m \) are charge and rest mass of the particle, respectively, \( c \) is the speed of light, \( \gamma = \sqrt{1 + 2\frac{\mu B(s)}{mc^2} + \frac{p^2}{(mc)^2}} \) is the Lorentz factor and \( p_{\parallel(\perp)} \) is the momentum parallel (perpendicular) to the magnetic field. The second adiabatic invariant \( J = \oint p_{\parallel} ds \) is related to \( \mu \) and \( B_m \) through Eq. (2.2). Eq. (2.2) can be approximated by \( W - mc^2 \simeq mv_{\parallel}^2 / 2 + qU + \mu B(s) = qU + \mu B_m \) for non-relativistic particles, which is analogous to the motion of a particle in a one-dimensional potential well defined by \( qU + \mu B(s) \) (e.g., Roederer 1970). Since \( W \) remains constant while a particle drifts for the given \( \mu \) and \( K \), the trajectory is conveniently obtained from the coordinates of the iso-energy contour. Note that all the variables in Eq. (2.2) are implicitly a function of the position. In general, \( U \) can be an arbitrary function as long as the guiding approximation is valid, but in the Earth’s magnetospheric environment, the electric field along the field line is usually assumed to be zero, in which case \( J \) can be replaced with a more convenient form \( K \sqrt{2m\mu} \) which only depends on the field line geometry.

Once the drift orbit of a particle is defined, evaluation of the magnetic flux, \( \Phi \) contained within the drift orbit of the particle is then obtained by integrating

\[
\Phi = \oint A \cdot dl = \int_A B \cdot da,
\]  

(2.3)

where \( A \) is the vector potential of the magnetic field \( B \). Note that the integration of the
vector potential form is taken along the closed contour defined by the drift orbit. In other words, Φ is defined only if the drift orbit of the particle is closed. The second form of the integration is usually preferred because the magnetic field is well defined from the model. The areal integration can be easily performed over the area A on the polar cap defined by the footpoints of the guiding field lines (Roederer 1970). \( L^* \) is then the ratio of \( \Phi_0 \) to \( \Phi \), where \( \Phi_0 = -2\pi k_0 \) and \( k_0 \) is the Earth’s magnetic moment.

Violation of the adiabatic invariants can occur due to Coulomb collisions between particles, due to wave-particle interaction, or due to significant variations in the fields on short time scales. It can also occur on the dayside where a bifurcation of the drift trajectory due to dayside compression breaks the second adiabatic invariant and hence the third adiabatic invariant is not defined (e.g. Ukhorskiy et al. 2011). The bifurcating can be easily tested by comparing the equatorial magnetic field magnitude with the magnitude at the mirror point. For instance, Figure 2.1 shows \( B_m \) as a function of \( s \) (top panel) and as a function of \( K \) (bottom panel) calculated at two field lines. The blue curve represents the magnetic field with single minimum at midnight and the red one, with two minima at noon. The symbols represent the numerical grid space. Each symbol in the top panel corresponds to the symbol at the same position in the bottom panel. At midnight, \( B_m \) monotonically increases with \( K \), but at noon (where \( K \) is just greater than 0), \( B_m \) falls below the equatorial magnetic field strength. If \( B_m(K) < B_m(0) \), the particles do not cross the magnetic equator but stay at the potential well defined by one of two magnetic minima.

### 2.2.2 Functional Dependence of \( B_m \) on \( K \): \( B_m(K) \)

In order to evaluate Eq. (2.1) with constant \( \mu \) and \( K \) at a given field line, \( B_m \) and \( U \) should be a function of those parameters. With the equipotential field line, \( U \) is only a function of 2D spatial coordinates (in this chapter, \( x\)-\( y \) coordinates with \( z \) set to zero), and \( B_m \) is a function of \( x\)-\( y \) coordinates and \( K \) (hereinafter the dependence on the spatial coordinates is implicitly assumed). In general, \( K(B_m) \) given in Eq. (2.1) is neither integrable nor
Figure 2.1 The top panel shows the magnetic field strength as a function of field line length for single (blue) and double (red) minima and the bottom panel shows $B_m$ as a function of $K$ for single (blue) and double (red) minima within the dashed-rectangular box. The symbols on both panels represent the numerical grid points used to calculate both quantities.
invertible. Indeed, the numerical evaluation of $K$ is the most computationally expensive part of the technique of Roederer (1970) due to the necessity of field line tracing.

To approximate the functional dependence of $B_m$ on $K$ (i.e., $B_m(K)$) from Eq. (2.1), Sheldon and Gaffey (1993) used a 10th order polynomial approximation which results in only 3% maximum deviation whereas Min et al. (2013) used for the same purpose spline interpolation, resulting in 0.1% deviation at maximum (with approximately 1° pitch angle resolution). In this study, $B_m(K)$ is approximated using linear interpolation as Min et al. found that the resulting error in $L^*$ has the same order of magnitude. For evaluation of Eq. (2.1), only the real part is taken into account (Northrop and Teller 1960; Sheldon and Gaffey 1993). The field line is integrated using a Runge-Kutta 4th order scheme. Fixed step size rather than adaptive step size (cf. Min et al. (2013)) is used to measure accuracy and calculation speed in a more consistent fashion. During the field line integration, various auxiliary parameters such as loss cone angles at each field line, footpoints on the Earth’s surface and magnetic field magnitudes at the magnetic equator are also obtained.

### 2.2.3 Drift Orbit from Iso-energy Contour

The basic concept follows the algorithm of Sheldon and Gaffey (1993). The $x$-$y$ space is discretized and the total energy $W(x_i,y_j,\mu,K)$ in Eq. (2.2) are calculated in this grid space. $(x,y)$ is used to denote the coordinate of an arbitrary point and $(x_i,y_j)$ is used to denote the coordinate of the grid node at $(i,j)$th index. Any quantities located at $(x,y)$ are approximated by areal interpolation using the quantities at four nodes adjacent to $(x,y)$. For the purpose of description, a “cell” $C_{i,j}$ is defined as a square box that connects vertices $(i,j) - (i+1,j) - (i+1,j+1) - (i,j+1) - (i,j)$ in the index space, and Edges 1–4 as edges of the cell connecting the vertices in that order. In this definition, $C_{i,j}$ shares Edge 2 with $C_{i+1,j}$ which shares Edge 4 with $C_{i,j}$. The same analogy is applied to other edges. Figure 2.2a is an example of mapping between the index space and rectangular grid space. Using this general index space frees the dependence on the underlying coordinate system.
Figure 2.2 Schematic diagram of the tracing algorithm. (a) $x$-$y$ coordinate space and index space. Index coordinates are on the bottom and left axes and the corresponding $x$-$y$ coordinates are on the top and right axes. The cells are labeled following the definition in the text. The values of the example energy at the grid nodes are labeled with red color. (b–d) show a demonstration of the iteration process. The heavy dot locates the initial location of the test particle, the dashed circle is the analytic contour line and the solid curve is the estimated contour line by the algorithm. The intersections that the contour line passes are marked with asterisk symbols.
and thus enables modular implementation.

Figure 2.2 shows a schematic of the tracing algorithm (implementation of contour function in MATLAB has been referenced). For demonstration, a simple energy function, \( W(x,y) = \sqrt{x^2 + y^2} \) was assumed in the system (\( \mu \) and \( K \) dependence are dropped) and a particle was initially located at \( r_0 = 1.5 \) and \( \phi_0 = 20^\circ \) (i.e., \( x_0 \approx 1.41 \) and \( y_0 \approx 0.513 \)). Figure 2.2a shows the index space and values of \( W \) at grid nodes. The corresponding \( (x,y) \) coordinates are shown at the top and right axes. Each cell is labeled following the above definition. The steps of tracing are as follows: First, for the given initial location of the particle in Figure 2.2b, the initial cell location is found (\( C_{4,3} \) in this case) and \( W_0 = W(x_0, y_0) \) is linearly interpolated from the neighboring values at the cell vertices. In this simple example, one can immediately realize that \( W_0 = 1.5 \) is the initial radial distance of the particle location, and that the trajectory (or contour line) should be circular and go through Edge 1 and 3 of \( C_{4,3} \). Mathematically, if the contour line crosses Edge 1 of \( C_{i,j} \), then the condition

\[
(W_0 - W_{i,j}) \times (W_0 - W_{i+1,j}) < 0
\]  

must be satisfied (conditions for other edges are straightforward). In this method, the search starts from Edge 1 through Edge 4. In Figure 2.2b, the search stops after Edge 1 has been tested because it is the first one that satisfies the condition. Once an edge is found, the coordinate of the intersection (asterisk symbol) determined using linear interpolation is recorded and the algorithm moves to the neighboring cell that shares the edge that has been found (\( C_{4,2} \) in this example. Figure 2.2c). Next, since one edge has already been found from the previous cell, there is only one edge to be found (Edge 1 of \( C_{4,2} \)). From this stage, the visited cells are also flagged. Last, this process continues until the contour line is closed (if the tracing algorithm comes back to the initial cell) or the boundary is encountered. Special care is needed when one of two terms in Eq. (4) becomes zero because where the contour line would go after passing a node is unpredictable (there are three cells that the contour
can enter). In this implementation, an infinitesimal number to the term that results in zero is added so that Eq. (2.4) does not ever become zero.

It is clear from the above example that not all $W$ values would be used to trace a contour line and successive traces are most likely to use the $W$ values that were used during the previous traces. In this implementation, the algorithm allocates a memory block for $W$ but defer the evaluation until actual tracing. While tracing a contour line by the above procedure, the tracing algorithm checks for the existence of the $W$ value at a node that the tracing module asked for. If not present, the algorithm dynamically evaluates and caches it before handing it over to the tracing module. Otherwise it returns the cached $W$ value. Overall, only those $W$ values that are needed for tracing are evaluated at most once and reused as much as possible during the entire calculation. In this way, calls to the expensive evaluation of Eq. (2.1) are minimized.

### 2.2.4 Magnetic Flux Integration

The scheme of magnetic flux integration in Eq. (2.3) is described next. For the integration, the drift orbit is mapped on the surface of the Earth. The footpoints of the orbit can be interpolated from the footpoints obtained during the tracing. For numerical integration of Eq. (2.3), Earth’s surface is discretized and the radial component of the magnetic field $B_r$ is calculated (only this component is needed) at grid nodes, as shown in Figure 2.3a. The grid spacing is on the order of a degree in $\phi$ and less than a degree in $\theta$ coordinates where $\phi$ and $\theta$ are the usual spherical coordinates. $\Delta\theta$ is kept small because of the steep change in $B_r$ with $\theta$. Then Eq. (2.3) may be written as

$$\Phi = \oint \int_{\theta_0}^{\theta} |B_r| r^2 \sin \theta d\theta d\phi = \oint d\Phi(\phi, \theta(\phi)),$$

(2.5)

Note that the absolute value of $B_r$ is used. The integrand $d\Phi(\phi, \theta)$ represents the approximated magnetic flux through the area defined by the vertical white stripe in Figure 2.3a.
Figure 2.3 (a) Magnitude of the radial component of the magnetic field $|B_r|$ on the surface of the Earth is shown as arrows on the spherical grid. The magnitude of $d\Phi(\phi, \theta)$ is represented as the background color on the sphere. Open circles represent the footpoints of the drift orbit (an equatorially mirroring particle at approximately $2R_E$). (b) Projection of the grid space of the northern hemisphere in (a) on the equator. The open circles represent the footpoints of the drift orbit. The dashed lines define the area through which the magnetic flux passes. The magnetic flux at each piece of wedge is represented with color scale. $(i, j)$ index represents the spherical grid and $k$ the footpoints.
At initialization, the algorithm calculates \( d\Phi(\phi_i, \theta_j) \) using trapezoidal rule. This approach frees some computational overhead for evaluation of Eq. (2.5) because the last term in that equation is approximated by a one-dimensional summation

\[
\Phi \simeq \sum_k d\Phi(\phi_k, \theta_k),
\]

once \( \theta(\phi) \) is given from the tracing. The index \( k \) denotes the \( k \)th footpoint of the closed drift orbit. Note that the magnetic flux of the Earth's magnetic field \( \Phi_0 \) can be obtained from Eq. (2.5) using \( \theta(\phi) = \pi \), i.e., \( 2\Phi_0 = -\oint d\Phi(\phi, \pi) \). This general approach is taken to get \( \Phi_0 \) because \( k_0 \) (Earth's magnetic moment) is not known in general. One needs to calculate \( d\Phi \) only once whenever the magnetic field configuration changes.

The footpoints of the drift orbit are irregularly spaced and thus evaluation of Eq. (2.6) needs one more step. Figure 2.3b shows the grid space of the northern hemisphere projected onto the equator. Let us assume that one grid boundary at \( \phi_i \) is located between the two footpoints, as in Figure 2.3b. In order to evaluate the magnetic flux through the stripe defined by \( (\phi_k, \theta_k) \) and \( (\phi_{k+1}, \theta_{k+1}) \) footpoints, the area should be separated at \( \phi_i \) because the fluxes on both sides are not the same. Then the flux through the stripe is sum of the fluxes through two stripes between \( \phi_k \) and \( \phi_i \) and between \( \phi_i \) and \( \phi_{k+1} \). Taking into account the fact that \( \theta_k \) is not on the integral grid, the flux can be approximated by interpolation as follows:

\[
d\Phi(\phi_{k+1}, \theta_{k+1}; \phi_k, \theta_k) = \left[ d\Phi(\phi_i, \theta_j) + \Delta\theta_k (d\Phi(\phi_i, \theta_{j+1}) - d\Phi(\phi_i, \theta_j)) \right] \Delta\phi_k
\]

\[+ \left[ d\Phi(\phi_{i+1}, \theta_j) + \Delta\theta_{k+1} (d\Phi(\phi_{i+1}, \theta_{j+1}) - d\Phi(\phi_{i+1}, \theta_j)) \right] \Delta\phi_{k+1}, \tag{2.7}\]

where \( \Delta\phi_k = \phi_{i+1} - \phi_k, \Delta\phi_{k+1} = \phi_{k+1} - \phi_{i+1} \), \( \Delta\theta_k = (\Delta\phi_k (\theta_{k+1} - \theta_k)/(\phi_{k+1} - \phi_k) + \theta_k)/2, \) and \( \theta_{k+1} = (\Delta\phi_k (\theta_{k+1} - \theta_k)/(\phi_{k+1} - \phi_k) + \theta_{k+1})/2. \) Generalization of the cases in which there are more than one \( \phi \) boundary or none between two footpoints is straightforward.
Finally the total magnetic flux through the polar cap is evaluated using equations (2.6) and (2.7).

### 2.2.5 \( L^* \) in a Magnetic Island

Storm-time distortion of the magnetic field can cause a localized extremum in the total energy distribution (Eq. (2.2)), especially for high pitch angle particles, and transition of a particle in and out of this localized region can cause discontinuous jump with sign change in the magnetic flux (Ukhorskiy et al. 2006). Although this effect is not the main interest in this chapter, the following describes how to deal with the drift orbit around the island that needs a special care for generality. Figure 2.4 shows the drift orbit of a trapped particle in the island mapped on the polar cap (solid curve, shown in the same format as Figure 2.3b). Integration of Eq. (2.3) is taken over the hatched area. From the divergence free condition, the flux integrated over the hatched area has the same magnitude as the flux integrated within the area (shaded area in the figure) defined by the drift orbit and an opposite sign. If the drift orbit does not encircle the Earth, then \( \oint d\phi = 0 \). If it is the case, the resulting \( L^* \) will have negative sign.

Figure 2.4 Schematic of the trapped drift orbit and its magnetic flux. The magnitude of the flux through the hatched area is equal to the magnitude of the flux through the shaded area.
2.3 Performance Analysis

A performance analysis for the speed and accuracy of calculating $L^*$ is presented in this section. While the formulation itself is applicable to a general equipotential field line assumption, a constant electric potential in all space (zero electric field) was assumed for the analysis, which is generally assumed for $L^*$ calculation (e.g., Roederer 1970).

2.3.1 Analytic Magnetic Field

For validation, a compressed dipole field (Elkington et al. 2003; Kabin et al. 2007)

\[
\mathbf{B} = e_r \left( \frac{2k_0}{r^3} - b_1 (1 + b_2 \cos \phi) \right) \cos \theta + e_\theta \left( \frac{k_0}{r^3} + b_1 (1 + b_2 \cos \phi) \right) \sin \theta, \\
\mathbf{A} = e_\phi \left( \frac{k_0}{r^2} - \frac{r}{2} b_1 (1 + b_2 \cos \phi) \right)
\]

(2.8)
is used, where $k_0$ is the Earth’s magnetic moment, and parameters $b_1$ and $b_2$ describe the distortion of the dipole field: $b_1$ can be interpreted as a quantity related to the IMF strength and $b_2$ is a non-dimensional parameter responsible for the azimuthal asymmetry of the resulting field (influenced, to a large degree, by the solar wind dynamic pressure). When $b_1 = 0$, the magnetic field becomes a symmetric dipole field and $L^*$ reduces to the dipole $L$ value. Despite the simple analytic form, Eq. (2.3) may not be solved analytically if $b_1 \neq 0$. One special case is when the pitch angle is 90°. Then the drift path is analytically given from the initial location $(r_0, \phi_0)$ by

\[
r(\phi; r_0, \phi_0) = r_0 \sqrt{\frac{k_0}{k_0 - b_1 b_2 r_0^3 (\cos \phi_0 - \cos \phi)}}. \quad (2.9)
\]

The magnetic flux, $\Phi$ can then be numerically evaluated as accurate as the machine precision using the first form of Eq. (2.3).

In the following analysis, both the dipole field for all pitch angles and the asymmetric magnetic field with parameters $b_1 = 5$ nT and $b_2 = 4$ (referred to as the asymmetric magnetic field hereafter) for 90° pitch angle are used. $k_0$ is set to 31,200 nT $R_E^3$. 
2.3.2 Accuracy of Magnetic Flux Integration

The accuracy of the numerical evaluation of the magnetic flux integration was estimated using the asymmetric magnetic field. $d\Phi(\phi, \theta)$ in Eq. (2.5) with the asymmetric magnetic field (Eq. (2.8)) is $d\Phi_{\text{analytic}}(\phi, \theta) = \frac{1}{2} \sin^2 \theta (2k_0 - b_1(1 + b_2 \cos \phi)) d\phi$. Shown in Figure 2.5 is the relative error of the numerically calculated magnetic flux, $\text{Rel Err}(\phi_i, \theta_j) = \langle |\Delta\Phi_{\text{cal},ij} - \Delta\Phi_{\text{analytic},ij}| / \Delta\Phi_{\text{analytic},ij} \rangle_j$. Notation $\langle \rangle_j$ denotes the averaging operation along $j$ index. The result shows that when the resolution of $\theta$ coordinate is on the order of $1^\circ$, the numerical error is on the order of $10^{-3}$ and the error only weakly depends on the resolution of $\phi$ coordinate (not shown) because the magnitude of the magnetic field on the Earth’s surface is nearly azimuthally symmetric.

2.3.3 Accuracy of $L^*$

The relative error of the calculated $L^*$ to the exact $L^*$ is analyzed with the dipole field and asymmetric magnetic field. The appropriate grid resolution is sought which balances accuracy and computational cost. Both cartesian and cylindrical grids were used for comparison. Spatial resolution ($\Delta r, \Delta \phi$) and field line integration step size ($\Delta s$, corresponding to the resolution in $K$ space) were set as follows: $\Delta r = (0.1, 0.2, 0.5) R_E$, $\Delta \phi = (1, 2, 5)^\circ$ and $\Delta s = (0.05, 0.1, 0.5) R_E$. For the rectangular grid, $\Delta x = \Delta y = \Delta r$. As discussed earlier, the resolution in $\Delta \phi$ is not a major source of inaccuracy of the magnetic flux integration. Results shown below are only the cases where $\Delta \phi = 2^\circ$. $\Delta \theta$ was set to $0.25^\circ$.

Figure 2.6 shows the relative error of the calculated $L^*$ values in the dipole field model. $\Delta r$ and $\Delta s$ increase in moving towards the bottom and the right, respectively. Initial locations were regularly spaced from 3 to $9 R_E$ separated by $0.1 R_E$ in radial distance ($r_0$) and from 10 to $90^\circ$ separated by $1^\circ$ in pitch angle ($\alpha_0$). The azimuthal angles ($\phi_0$) were randomly chosen in order to avoid the bias at grid nodes. The relative error is defined as $\text{Rel Err}(r_0) = \langle |L^*_{\text{cal}}(r_0, \alpha_0) - L^*_{\text{exact}}(r_0, \alpha_0)| / L^*_{\text{exact}}(r_0, \alpha_0) \rangle_{\alpha_0}$ and one standard deviation is also calculated in a similar manner. A few key features of these results are noted: first, the
**Figure 2.5** Relative error of the numerically evaluated magnetic flux to the analytic solution using the asymmetric magnetic field. $\Delta \theta$ is set to $2^\circ$ (red) and $0.5^\circ$ (blue) and $\Delta \phi$ is fixed to $2^\circ$.

**Figure 2.6** Relative error of $L^*$ computed in the dipole field using both rectangular (red) and cylindrical (blue) grids as a function of Dipole $L$. $\Delta r$ and $\Delta s$ increase towards the bottom and right panels, respectively.
relative errors, especially in the first two columns, in the rectangular grid show a decreasing trend as $L^*$ increases. If the absolute errors were shown, dependence of the error on $L^*$ would be minimized. Second, due to the cylindrical symmetry of the dipole field, $L^*$ values calculated at the radial grid boundary in the cylindrical grid are very accurate (order of $10^{-5}$) as in Figure 2.6a and b. On the other hand, the error at the non-integral grid is comparable to that of the rectangular grid, as in Figure 2.6d and e (note peaks between the radial grid boundaries). This caused the oscillatory behavior in the relative error for $\Delta r \geq 0.2$. Weighted interpolation based on the dipole geometry were tried but did not show significant improvement. The oscillatory behavior for the rectangular grid is less prominent because of the pitch angle dependence of the calculated $L^*$ (as will be discussed shortly) and the random choice of the initial azimuthal coordinates (the cylindrical grid will not be affected by the azimuthal coordinates in the dipole field). Third, the standard deviation in the rectangular grid is noticeable compared to that in the cylindrical grid (essentially zero). Although not heavily investigated, it is suspected that the areal interpolation of the field line quantities may cause the pitch angle dependence. On the other hand, the areal interpolation in the cylindrical grid essentially becomes 1D linear interpolation in the azimuthal direction on which the dipole field has symmetry. Last, a finer grid spacing was tested but there was no significant improvement in accuracy (not shown), which may indicate an inherent upper limit ($\sim 10^{-5}$) of the achievable accuracy.

Figure 2.7 shows the relative error of the calculated $L^*$ values in the asymmetric magnetic field. $\Delta r$ increases moving down the panels. The initial locations were distributed in radial distance from 3 to $9\, R_E$ separated by $0.1\, R_E$. $\phi_0$ and $\theta_0$ coordinates were set to $0^\circ$ (noon) and $90^\circ$, respectively. Note that $\Delta s$ is fixed to $0.1\, R_E$ because the dependence on $\Delta s$ is meaningless for the equatorial particles. The result indicates that the relative error behaves as expected for the symmetric case. Due to the asymmetry, the errors for both grids behave similarly and thus the absolute errors would be less dependent on the $L^*$ as well. The oscillatory behavior in the relative error for the coarser grids is also clear due
Figure 2.7 Relative error of $L^*$ computed in the asymmetric magnetic field using both rectangular (red) and cylindrical (blue) grids as a function of Dipole $L$. $\Delta r$ increases towards the bottom panel and $\Delta s$ is set to $0.1 \, R_E$. 
to the same reason as the previous result. The clear oscillation for the rectangular grid is because the particles were traced from the same local time (noon meridian).

2.3.4 Calculation Speed

The calculation speed is estimated next for several Tsyganenko magnetic field models (Tsyganenko 1987, 1989, 1995; Tsyganenko and Sitnov 2005, 2007; Sitnov et al. 2008) with the dipole field as control. Since the dipole field is simplest, the algorithm is the major contributing factor to the calculation speed in this field. In other words, the calculation speed in the dipole field forms the base line and the speed above this line is solely due to the model field. A standard laptop with 2.8 GHz Intel Core 2 Duo processor was used for all the tests. The calculation speed is defined as total elapsed time per particle and core. $\Delta s = 0.1 R_E$ and $\Delta r = 0.2 R_E$ were used for the grid resolutions (which has a reasonable balance between accuracy and speed). Figure 2.8 shows the speed as a function of model. The tests were done using two-day Van Allen Probe orbit (Figure 2.9). In the dipole field, the speed is about 6 ms which is the base line. For the T96 model, it takes about half a second and for the T02 and TS05 models about a second. For the TS07 model, it takes an order of magnitude longer than the TS05 model. Based on this result, the new $L^*$ calculation method may be faster than any other physics-based methods currently available.

2.4 Scientific Application: $L^*$ Coordinate of Satellite Orbit

Two major uses of the $L^*$ coordinate are (1) real-time satellite tracking in the $L^*$ coordinate and (2) mapping between invariant space and phase space. The application of the former is shown in this section and that of the latter is briefly discussed in the next section. For the purpose of demonstrating the capability of a real-time computation, the $L^*$ coordinate of the two-day long Van Allen Probe-A orbit is calculated. For the performance benchmark, the calculated $L^*$ values were compared to those of LANLGeoMag. LANLGeoMag uses the T89Q field model (T89 field model with Kp=2) among other various field models. In
Figure 2.8 Calculation speed of a single trajectory as a function of model for rectangular (square symbol) and cylindrical (open circle) grids. The exact speed at the symbols is labeled within the plot.

This test, T89Q model was used (for other models, model parameters were not explicitly given). $\Delta s = 0.1 R_E$ and $\Delta r = 0.2 R_E$ are used for the grid resolution in this method.

Figure 2.9a shows the calculated $L^*$ coordinates and Figure 2.9b and c show the magnitude of $L^*$ difference between two methods for the cylindrical and rectangular grids, respectively. The deviation is about 0.1 at maximum (around $L^* \sim 3$) and is largest at lower pitch angles for both coordinate systems. It took about 1,350 and 1,413 seconds for the cylindrical and rectangular grids, respectively, to produce the results.

2.5 Discussion

2.5.1 A Comparison between New Method and the Technique of Roederer (1970)

Even though the development of the method was motivated by $(U, B, K)$ coordinates (Whipple 1978), one can realize that this method is essentially an extension of the technique of Roederer (1970). Both methods are based on the same underlying physics and assumptions. One major difference is from the drift orbit tracing. Figure 2.10 demonstrates the tracing technique of Roederer (1970), stating that the field lines that have the same initial invariants and the magnitude of the magnetic field at mirror points are iteratively searched
Figure 2.9 (a) Calculated $L^*$ coordinates of the spacecraft orbit using the cylindrical grid as a function of time and pitch angle. (b) The magnitude of the difference between two methods with the cylindrical grid. (c) The magnitude of the difference between two methods with the rectangular grid.
**Figure 2.10** Schematic showing the technique of Roederer (1970). Dots are probing points and asterisks are approximated equatorial intersection of the shell field lines. Figure format is similar to Figure 2.2.
by more or less arbitrarily probing adjacent field lines separated by a small distance (step size). The advantage of the present tracing algorithm is that a fully closed drift orbit is always guaranteed to be found regardless of the grid resolution. This is somewhat difficult in the technique of *Roederer* (1970) because of the error accumulation at each iteration.

### 2.5.2 Mapping between $L^*$ and $x$-$y$ Coordinates

In diffusion simulation, it is often required to convert phase space density in invariant space to differential flux in phase space for comparison with observations. In this section, the performance gain of the present technique is further demonstrated when calculating the mapping between $L^*$ and $x$-$y$ coordinates with a complex magnetic field model.

When the number of calculations become large, the speed ultimately scales solely by the complexity of the model field. In practice, the magnetic fields are often approximated from the tabulated field vectors in a cubic grid space (e.g., field interpolation in MHD simulation). Due to the dimensionality, pre-allocating a chunk of memory block and pre-calculating the vector field in advance may be too costly. As shown in section 2.2, most of the magnetic fields at grid nodes would not be consumed. By taking the similar approach for the drift shell tracing, the speed and accuracy are shown without describing the implementation.

Figure 2.11a shows the error as a function of spatial location. The error is defined as the averaged magnitude (in pitch angle) of difference of the calculated $L^*$ values between the analytic field and interpolated field. Since the purpose of this test is to examine the deviation resulted from using the interpolated field, the accuracy of $L^*$ values of the control is not an issue here as long as the same tracing program is used. The asymmetric magnetic field was used for the analytic field. $\Delta x = \Delta y = \Delta z = 0.1 R_E$ were used for the cubic grid resolution and initial locations were evenly distributed from 2 to 9 $R_E$ separated by 0.2 $R_E$ in radial direction and from 0 to 360° separated by 4° in azimuthal direction. Pitch angles from 10 to 90° was evenly divided into 17 bins. The order of error is $10^{-3}$ and the large
Figure 2.11  (a) Absolute error in calculated $L^*$ using the interpolated field. Undefined $L^*$ values (due to losses through the magnetopause and ionosphere) were excluded from the average. (b) Calculated $L^*$ of 88° pitch angle using the interpolated TS07 field model as a function of $x$-$y$ coordinates. The dashed circles are at 6.6 $R_E$.

error occurs at the tail where the field lines are stretched.

Figure 2.11b shows the mapping from $x$-$y$ coordinates to $L^*$ values under the latest TS07 field model using parameters during the pre-storm period (for model parameters, http://geomag_field.jhuapl.edu/model/). Initial locations were the same as before. The result of 90° (88° to be precise) pitch angle particles is only shown. The 90° pitch angle particles often fail the bifurcation test due to the scratchy field line from the interpolation. The elapsed time was 1,312 seconds, when both CPU cores were fully loaded. According to the result in Figure 2.8, it would have taken 19 days to produce the result. The calculation speed demonstrates the significant performance gain of the technique described in section 2.2.3, especially for a complex magnetic field model. Such a scalability is an added advantage of this method and enables us to calculate a large number of $L^*$ values as often required for intercomparison between the simulation result and the observation.
2.6 Summary

The algorithm and implementation of a new, efficient method for calculating $L^*$ have been thoroughly described. While this method may be regarded as an extension of the technique of Roederer (1970), it has a major difference from the latter technique in that the present technique of tracing in a finite grid is free from error accumulation and ensures the existence of a closed drift orbit. Realizing that not all $W$ values are needed for tracing a drift trajectory, the new method evaluates $K(B_m)$ only at necessary grid points at the time of tracing, which results in a significant performance gain.

From the results of performance analyses, it is believed that the new method is able to calculate $L^*$ values faster than any other physics-based methods currently available. The method can transform the satellite orbit from configuration space to $L^*$ space in near real-time and is highly scalable to a large number of $L^*$ values with reducing incremental cost. Despite a possible inherent limit due to the use of a finite grid, this study suggest that this method could be an added resource for the radiation belt community.
In this chapter, the relationship between the electron injection and the chorus waves during a substorm event on 23 March 2007 is studied. The chorus waves were detected at high geomagnetic latitude (≈ 70° S) Antarctic observatories in the range of 0600–0900 h in magnetic local time (MLT). Electrons drifting from the injection event were measured by two LANL spacecraft at 0300 and 0900 MLT. The mapping of auroral brightening areas to the magnetic equator shows that the injection occurred in an MLT range of 2200–2400. This estimate is consistent with observations by the Time History of Events and Macroscale Interactions during Substorms (THEMIS) A, B, and D spacecraft (which were located at 2100 MLT and did not observe electron injections).

The backward model tracing from the magnetic equator near the dawnside magnetopause (which magnetically connects to the Antarctic observatories) also supports the deduced injection region. Since chorus waves are believed to be generated through the electron cyclotron instability by an anisotropic temperature distribution, the forward model tracing was performed to examine whether the electrons injected during this substorm form a pancake-like pitch angle distribution when they arrive near the dawn-side magnetopause. It is found that the onset time of the modeled pitch angle anisotropy is consistent with that of the observed chorus waves and that the development of the anisotropy is due to particle drift shell splitting.

3.1 Introduction

The whistler-mode intensifications coincident with substorms are often observed by ground-based stations and their relation to substorm particle injections has been of interest (e.g., Sazhin and Hayakawa 1992). Tsurutani and Smith (1974) were the first to show that post
midnight whistler-mode chorus waves are a substorm phenomenon, although dayside chorus events, in some cases, occur without substorm injections. Smith et al. (1996) coined the term substorm chorus events (SCEs) under the interpretation that the SCEs occur as the electrons are injected near midnight and drift toward dawn to generate whistler-mode waves, and the waves travel to the ground in the ducted mode. Such an interpretation has been more firmly established after Collier and Hughes (2004a,b) carried out a more quantitative modeling of SCEs in terms of the drift motion of injected electrons and the associated growth rate of whistler-mode waves for agreement with chorus wave observations. Further analysis of satellite observations of substorm enhanced whistler-mode waves, however, revealed the importance of the inward motion of whistler ducts under the influence of a substorm-enhanced electric field (Abel et al. 2004) and the dominance of drift echoes rather than the initial substorm injection (Abel et al. 2006).

This chapter studied the relation of chorus waves with substorm-injected electrons for a specific substorm event that occurred on 23 March 2007. There are several factors that make this event suitable for such a study. First, the Time History of Events and Macroscale Interactions during Substorms (THEMIS) and Los Alamos National Lab (LANL) satellites were located on the night side at the time of the substorm onset, and the position of the substorm injection could be easily inferred to some extent (Angelopoulos 2008; Liu et al. 2009). Auroral images from the Ultra Violet Imager on board the Polar spacecraft (Polar/UVI) (Torr et al. 1995) are also available with which one can estimate the injection region more straightforwardly. These sets of data permit a reasonable estimate for the substorm injection condition. In addition, the ground-based detection of the very low frequency (VLF) signal was made at multiple Antarctic stations supported by the Polar Experiment Network for Geospace Upper-Atmosphere Investigations (PENGUIN) (Lessard et al. 2009). Second, at the time of the substorm, the ground-based stations were magnetically connected to high L shell near the dawnside magnetopause, whereas previous studies have been concentrated on the chorus events in the middle magnetosphere ($L \approx 4$ to 8). Sev-
eral studies have shown that chorus intensities are statistically found to be the most intense at postdawn side outer zone of magnetosphere (e.g., Tsurutani and Smith 1977; Tsurutani et al. 2009), but the clear understanding of the dependence on the substorm is yet to be resolved. It is thus of new interest how substorm injected electrons can reach the high $L$ shell morning sector where chorus intensities are statistically found to be the most intense (e.g., Tsurutani et al. 2009) and generate the chorus waves which could potentially lead to wave-particle interaction with resonant electrons. Third, the VLF signal intensification occurred in the period of low and stable solar wind dynamic pressure. This event is therefore ideal for studying the causal relation of the VLF signal to the substorm electron injection with little other influence on local adiabatic acceleration. Chorus waves are believed to be generated through the electron cyclotron instability by anisotropic ($T_\perp > T_\parallel$) distributions of energetic electrons in the range of 5 to 150 keV (e.g., Thorne et al. 1977). The goal is therefore to investigate whether such electron distributions can result from substorm particle injections as they drift from the injection region to the dawnside magnetosphere. For this goal, tracing the electron drift motion in a realistic magnetic field model is employed with emphasis on the evolution of their pitch angle distribution. There are two modeling works that one would consider the most relevant to the present study, Liu et al. (2009) and Collier and Hughes (2004a,b). Liu et al. (2009) performed a test particle simulation to reproduce injection signatures of this substorm measured from spacecraft with emphasis on the role of the dipolarization-associated inductive electric field in the acceleration and drift of electrons. While their particle tracing was made on the two-dimensional (2-D) equatorial magnetic field, the investigation requires tracing of test electrons in all pitch angle ranges and the drift motion should be traced in a three-dimensional (3-D) magnetic field. Collier and Hughes (2004a,b) traced electron drift motion at all pitch angles in 3-D dipole magnetic field. The tracing in this study, on the other hand, uses a more realistic 3-D magnetic field configuration provided by the Tsyganenko 2005 (TS05 hereafter) storm model (Tsyganenko and Sitnov 2005) which may reveal any pitch angle dependent effect on elec-
tron drift motion (Fairfield 1964; Roederer 1967) that is not obvious under the symmetric dipolar magnetic field model.

The plan of this chapter is as follows: sections 3.2 and 3.3 present observational data of the event and the test particle simulation results, respectively. Section 3.4 discusses and summarizes the principal results of this study.

3.2 Observational Evidence

Observations of the 23 March 2007 substorm at 1119 UT have been well documented (Angelopoulos 2008; Liu et al. 2009; Lessard et al. 2009) and this study focuses on evidence for the substorm injection and its relation to the VLF signal detected at the ground-based stations. The VLF data are obtained from automatic geophysical observatories (AGO) with time resolution of 0.5 s in three frequency channels: 0.5–1 kHz, 1–2 kHz, and 2–4 kHz (Spasojevic and Inan 2010). Energetic electron flux data are from Synchronous Orbit Particle Analyzer (SOPA) (Belian et al. 1992) on board the LANL geosynchronous spacecraft in the energy range from 50 keV to 1.5 MeV. The spin-averaged electron flux with 6 s time resolution is used in this study. Auroral images are obtained from Polar/UVI, equipped with four filters (1300 to 1900 Å) at the 37 s image frame rate. Solar wind data are from Cluster Ion Spectrometry (CIS) experiment Hot Ion Analyzer (HIA) (Réme et al. 2001) and the magnetic field obtained from the fluxgate magnetometer (FGM) (Balogh et al. 2001) on board Cluster 1 and the magnetometer (MAG) on board the NOAA GOES geosynchronous satellites. Both Cluster CIS and FGM have 4 s and GOES MAG has 60 s time resolution.

3.2.1 Substorm Onset and Expansion

Figure 3.1 shows the VLF signals measured from the VLF receiver for use on the AGO P2 station (Figure 3.1, top) and the energetic electron fluxes from the SOPA instrument on board LANL 1989-046 and 1994-084 (Figure 3.1, middle and bottom). P2 was located at
Figure 3.1 (top) Observations of VLF signals from P2 and (middle and bottom) energetic electron fluxes from SOP A on board LANL 1989-046 and LANL 1994-084.

geomagnetic (100 km reference) latitude of $-69.81^\circ$ and longitude of $19.21^\circ$. The figure only shows the lowest four energy channels (50–75 keV, 75–105 keV, 105–150 keV, and 150–225 keV) of SOP A because these energies are more important for chorus wave generation (Tsurutani and Smith 1974; Thorne et al. 1977). The substorm onset occurred at 1119 UT (Angelopoulos 2008). The VLF signal started to rise at 1126 UT and reached peaks at 1150 UT and 1204 UT. LANL 1989-046 at $\sim$ 01 MLT detected an increase of electron flux at 1121 UT, which was nearly dispersionless in energy. Later at 1132 UT, LANL 1994-084 at $\sim$ 08 MLT detected the electron flux increase. The flux increase occurred in the four channels presented at both spacecraft. On the other hand, THEMIS-A, B, and D detected dispersionless increases of ion flux at 1119 UT, while no significant increases in electron flux were observed (not shown here; see Liu et al. (2009)). Four auroral images obtained around the time of the substorm onset from Polar/UVI are shown in Figure 3.2. After the time of the substorm onset ($\sim$1121 UT), the aurora suddenly brightened up in the region.
surrounded by a white contour in Figure 3.2. Since the UV emission detected by Polar/UVI is due to precipitation of energetic electrons into the atmosphere, one may figure out the injection region by mapping this auroral brightening region into the magnetosphere along magnetic field lines, as will be shown in the next section.

3.2.2 Electron Injection and Drift Motion

Figure 3.3 shows the spatial locations of the spacecraft used in this study (except for Cluster and Polar), the auroral mapping region, and the field of view of P2 station (P2 FOV hereafter) on the $X$-$Y$ plane of GSM coordinates. The filled circles and asterisks denote locations of the spacecraft at 1119 UT and 1300 UT, respectively. The dotted line indicates 2200 MLT as reference. At the time of the substorm onset (1119 UT), LANL 1989-046 and 1994-084 were located at 0100 MLT and 0800 MLT, respectively. GOES 11 and GOES 12 were at 0200 MLT and 0600 MLT, respectively. The THEMIS spacecraft were orbiting inbound with THEMIS-C leading the others. The red contour around midnight is the equatorial region magnetically connected to the auroral brightening region at 1119 UT; the mapping was made with the TS05 model. The green contour on the dawn side represents the region magnetically connected to P2. In calculating P2 FOV, it is assumed that chorus waves reaching within 400 km of P2 can be detected by the VLF receiver at P2.

In Figure 3.3, a rough estimate of the injection region is readily available from the locations and observations of the LANL and THEMIS satellites. Since LANL 1989-046 detected dispersionless increase of electron flux at 1121 UT and THEMIS-A, B, and D detected no significant increases in electron flux, the electron injection must have occurred between 2100 MLT (where THEMIS A, B, and D located) and 0100 MLT (where LANL 1989-046 located). As mentioned above, the magnetic mapping of the auroral brightening into the magnetosphere leads to a more accurate estimate for the injection region, and one can identify the red contour in Figure 3.3 with the electron injection region. The injection region therefore lies between 2200 MLT and 2400 MLT and between 5 $R_E$ and 10 $R_E$ in
Figure 3.2 Auroral images from Polar/UVI at (a) 1117:13 UT, (b) 1118:26 UT, (c) 1120:40 UT, and (d) 1121:30 UT. The white contour in (c) denotes the area of sudden auroral brightening.
Figure 3.3 Locations of the spacecraft and other regions of interest on the equatorial plane in GSM coordinates. The solid circles represent the locations of spacecraft at 1119 UT and asterisks, at 1300 UT. The red contour near midnight encloses the region that is mapped out from the auroral brightening area (white contour in Figure 3.2) using TS05 model. The contour around 0700 MLT is P2 FOV (see text). The circle represents the Earth.
radial directions. This estimate is also consistent with the modeling result by Liu et al. (2009). It is also noted that the onset of the VLF signal occurred at 1126 UT (Figure 3.1) between the time of the electron flux increase detected at LANL 1989-046 (1121 UT) and at LANL 1994-084 (1132 UT). It is therefore quite plausible that the chorus waves detected on the ground at high latitudes are generated when energetic electrons injected during the substorm drift into P2 FOV. It is noted that the VLF signal is long lasting (> 1.5 h) compared with the electron injection (≤ 1 h, see Figure 3.1). It is likely that the first peak of the VLF signal at 1152 UT is due to the substorm injection and the second smooth rise peaked at 1255 UT is associated with echoes (Lessard et al. 2009).

3.2.3 Solar Wind and Geosynchronous Magnetic Field

Figures 3.4a and b show the solar wind condition during this substorm with the data from Cluster satellites. Cluster 1 was located at (16.7, −4.7, −10.3) \( R_E \) in GSM at 1120 UT. One can presume it was outside the bowshock because it detected the same solar wind profile as Wind spacecraft did, except for only time shift. The dynamic pressure calculated using the data from the CIS instrument and the magnetic field obtained from FGM on board Cluster 1 are shown in Figures 3.4a and b. Figure 3.4a shows that the solar wind dynamic pressure remained almost constant at \( \sim 3 \) nPa. Figure 3.4b shows that there was a sudden northward turning of IMF \( B_z \) with a corresponding variation of IMF \( B_x \). It is thus likely that this substorm was triggered by the northward turning of IMF \( B_z \).

The magnetic field at the geosynchronous orbit is shown in Figures 3.4c and d. These measurements are from MAG on board the NOAA GOES geosynchronous satellites, GOES 11 and GOES 12 located at 0200 and 0600 MLT at the substorm onset, respectively. GOES 11 data show a gradual decrease and then increase of magnetic field \( B_z \) around 1116 UT. This change of the magnetic field component might be due to the dipolarization front associated with the substorm. However, it is not so clear because the change of \( B_z \) occurred over 40 min, rather a long period compared with a typical dipolarization timescale.
Figure 3.4 (a) Solar wind dynamic pressure, (b) IMF observed by Cluster 1, magnetic field measurements from the MAG instrument on board the (c) GOES 11 and (d) GOES 12 satellites, respectively. Cluster 1 was located at around \((16, -7, -9)\) \(R_E\) (outside the bow-shock). At the geosynchronous orbit, the magnetic field suddenly changed at the position of GOES 11, whereas GOES 12 detected little change of the magnetic field.
(order of a few minutes (Moore et al. 1981)). The LANL 1989-046 was located close to GOES 11 and detected no clear enhancement of electron flux at 1116 UT. It is thus ambiguous whether any significant dipolarization front reached the location of GOES 11. No corresponding change in any component of magnetic field was detected by GOES 12, and the magnetic field variation measured by GOES 12 can be merely due to the diurnal variation associated with the spacecraft motion, which may indicate that the dipolarization front did not arrive or no local dipolarization occurred at the location of GOES 12, even though the sudden change of $B_z$ detected by GOES 11 was indeed associated with the dipolarization. Since GOES 12 was close to P2 FOV, one can further speculate that the dipolarization effect on pitch angle anisotropy would be insignificant at the location of P2 FOV.

### 3.2.4 Possible Causes of the VLF Signals

On the basis of these observations shown above, there may be several possible causes of the VLF signal observed in this event. Since these whistler-mode waves are amplified in the presence of energetic (5–150 keV) electrons with a pancake-like pitch angle distribution, there is a possibility of the local processes by which electrons preferentially acquire the perpendicular momentum. An increase in the perpendicular momentum can occur when ambient magnetic field suddenly increases and electron motion conserves the first adiabatic invariant (Roederer 1967). In the present case, a sudden increase of $B_z$ near the magnetopause might occur if solar wind exerts an enhanced dynamic pressure on the magnetopause. However, GOES 12 (located near P2 FOV) did not detect such a change in $B_z$ (Figure 3.4). The dipolarization front associated with the substorm may also lead to an increase of the local magnetic field. However, no significant signature for the dipolarization field was found at both GOES 11 and GOES 12 at 1116 UT (section 3.2.3). It is therefore unlikely that such local adiabatic acceleration processes occurred at the time of the VLF signal around the location of P2 FOV. Another possibility is the anisotropic pitch angle
distribution as electrons drift from the injection region to P2 FOV. This is quite plausible in view of the timing of the VLF signal onset and the electron flux increase detected in the LANL satellites. The later part of this study focuses on this drift effect.

### 3.3 Test Particle Simulation

A test particle simulation is performed to explore a possible relation between the chorus wave intensification and the substorm injected electrons. As \textit{Lessard et al.} (2009) pointed out, the generation mechanism of the chorus waves should be a combination of the initial substorm injection and the drift motion. Unfortunately, no spacecraft was near the midnight sector at the time of the injection, and thus no direct information on the energy and pitch angle distribution of the injected electrons was available. It is therefore assumed for simplicity that the electrons initially have isotropic pitch angle distribution and uniform energy distribution. However, the location of the injected electrons could be inferred from the auroral images (section 3.2.1) and spacecraft locations (section 3.2.2). This inferred injection region is further confirmed based on a backward tracing of electrons from the observation point, P2 FOV in section 3.3.1. Finally the forward tracing of the electrons starting from the injection region is performed under the simplifying assumptions on the initial injection properties (section 3.3.2).

For the simulation, the guiding center approximation with the equations of motion given by \textit{Brizard and Chan} (1999) is adopted, which are valid even for relativistic electrons. For simplicity, a static magnetic field obtained from TS05 model at the time of interest (1120 UT) is used throughout the particle tracing because the observation shows little variation of solar wind dynamic pressure around the substorm. As input parameters to the TS05 model, the real solar wind parameters at 1120 UT (IMF $B_y \approx -4$ nT, $B_z \approx 0$ nT, solar wind velocity $V_x \approx -310$ km/s, and solar wind dynamic pressure $P_{\text{dyn}} \approx 3$ nPa) were used as inputs to the TS05 model. The simulation box is set within $-30 \, R_E \leq X_{\text{GSM}} \leq 10 \, R_E$, $-20 \, R_E \leq Y_{\text{GSM}} \leq 20 \, R_E$, and $-10 \, R_E \leq Z_{\text{GSM}} \leq 10 \, R_E$, which encompasses the magne-
topause and midtail. When particles move out of the magnetopause or through the tail, they are removed from the simulation. Particles that precipitate into the ionosphere \((1.1 \, R_E)\) represent particle loss through the loss cone thus are also removed. Neither convective nor inductive electric field is taken into account. One can expect that the convection electric field is small \((\leq 5 \, \text{mV m}^{-1})\) and its contribution to electron drift is weaker compared to that of gradient-curvature drift at the energies considered here \((\geq 50 \, \text{keV})\) because IMF \(B_z\) remained northward after the injection onset (section 3.2.3) and the global convection weakened. The inductive electric field at the dipolarization front might be important in the early expansion phase. However, in the speculation given in sections 2.3 and 2.4, the dipolarization at the location of P2 FOV would not be significant. Note also that the simulation is mainly devoted to the period after the dipolarization. Therefore test electrons drifting in the static magnetic field configuration experience no electric field. Finally, it is checked whether the simulated pitch angle distribution of the electrons can explain the rising phase of the VLF signal (Figure 3.1).

### 3.3.1 Backward Tracing of Electrons

A backward tracing starting from P2 FOV is first performed in order to estimate the injection region. Even though injection region has already been estimated based on the auroral brightening region (Figure 3.3), one can expect this backward tracing to give a cross-check for the result. Figure 3.5 shows a snapshot of the backward tracing on the equator of GSM coordinates. Initially, all test electrons lie inside P2 FOV on the magnetic equator; a nominal energy is \(150 \, \text{keV}\) and the initial pitch angle distribution is uniform ranging from \(5^\circ\) to \(175^\circ\) with \(5^\circ\) apart. A single electron per unit pitch angle bin is assigned, so that initially isotropic pitch angle distribution is assumed. The dots in Figure 3.5 represent locations of the test electrons on the magnetic equator at 530 s after the start of the backward tracing. Note that these electrons undergo bouncing motions along the north-south direction and that their locations are only shown when they pass through the equatorial plane. The colors
Figure 3.5 A snapshot of the backward tracing after 530 s from the start of electrons at P2 FOV. Dots denote the location of the 150 keV electrons on the equatorial plane with colors representing their initial pitch angles. The solid contours represent iso-gauss surface on the equator, and the dashed contour represents P2 FOV.
of the dots denote their initial pitch angles at P2 FOV. Even at a common energy, the positions that the electrons reached after drift and bounce motions vary significantly depending on their initial pitch angles. The electrons with any pitch angles mostly accessed only the region inside $X \approx -13R_E$. Some of the low pitch angle (say $\leq 25^\circ$) electrons access regions outside $-13 \, R_E$. This means that the high pitch angle electrons detected at P2 FOV would have come from the near-Earth plasma sheet and that only a portion of low pitch angle electrons at P2 FOV originated from the distant plasma sheet.

![Figure 3.6](image)

**Figure 3.6** Magnetic field line configuration on $X$-$Z$ plane of GSM coordinates along the midnight meridian at 1119 UT. The cross symbol at $X \approx -13R_E$ is the magnetic X point. The red ellipse indicates the estimated injection region based on the auroral mapping, and the blue ellipse indicates the estimated injection region based on the backward tracing.

Figure 3.6 shows the magnetic field line configuration as viewed on the $X$-$Z$ plane at $Y = 0$. The cross symbol at $X \approx -13R_E$ denotes the magnetic X point, which is not exactly on the GSM coordinate equator. The red ellipse is the region where aurora is mapped to, and the blue ellipse is the region from which most electrons can reach P2 FOV. Although some electrons may cross the X-line to arrive at P2 FOV, most electrons reaching P2 FOV are initially residing inside the X-line in the inner magnetosphere.
3.3.2 Forward Tracing of Electrons

Test electrons are allocated uniformly in the estimated injection region and traced forward, in order to investigate their pitch angle distribution when they arrive at P2 FOV. As mentioned above, no direct information on the pitch angle distribution of the injected electrons is available at the time of the injection, and it is assumed for simplicity that the electrons initially have isotropic pitch angle distribution.

Figure 3.7 shows a snapshot of the forward tracing of 150 keV electrons at $t = 10$ min after the injection, when most electrons reach inside the P2 FOV. Initially, the test electrons are distributed uniformly in space with spacing $0.5 R_E$ inside the estimated injection region lying in 2200–2400 MLT and at the radial distance of 5–13 $R_E$ (orange dots in Figure 3.7). At each injection grid point, a single electron per unit pitch angle bin is assigned ranging from $5^\circ$ to $175^\circ$ with $5^\circ$ apart and the initial pitch angle distribution is isotropic. The total number of particles is 5880. Their positions are shown on the $X$-$Y$ plane with colors representing their equatorial pitch angles detected at P2 FOV under the same convention used in Figure 3.5. The contours are magnetic field strength in the $X$-$Y$ plane. One can find that the high pitch angle electrons (red dots) are following the iso-gauss contours, which is expected under the conservation law of the first adiabatic invariance. However, the low pitch angle electrons (blue dots) are drifting more inwardly. As a result, high equatorial pitch angle electrons (red dots) form a V-like shape and low equatorial pitch angle electrons (blue dots) forms a C-like shape. An interesting finding is that the V shape distribution of high pitch angle electrons well overlap with the area of P2 FOV. There are more high pitch angle electrons inside P2 FOV than low pitch angle electrons.

Figure 3.7b shows the pitch angle distribution of electrons in P2 FOV as a histogram. The number of the electrons were counted in each pitch angle bin with an interval of $5^\circ$. Note that as the Earth rotates, the P2 FOV also changes in size and shape, and thus the changing boundary was taken into account for counting the number of electrons arriving within the boundary. The histogram shows that more high pitch angle electrons are
Figure 3.7 A snapshot of the forward tracing at 10 min after the injection. (a) Electron locations on the GSM equator. Orange dots lying between 2200 MLT and 2400 MLT represent the initial distribution of electrons. The other colored dots indicate the locations of the electrons at 1130 UT when they cross the equator of GSM coordinates. The solid contours represent iso-gauss surface on the equator, and the dashed contour represents P2 FOV. (b) Electron number as a function of pitch angle as measured within P2 FOV at 1130 UT.
found in P2 FOV, that is, a pancake-like pitch angle distribution of electrons develops.

3.3.3 Drift Shell Splitting of Electrons

The above particle tracing result showed that the drift path of electrons diverges according to the initial equatorial pitch angle. The radial distance of the drift shell of low pitch angle electrons tends to decrease. This phenomenon is known as drift-shell splitting (Fairfield 1964; Roederer 1967; Takahashi et al. 1997). The drift-shell splitting occurs as electrons conserve the first and second adiabatic invariants while drifting in an asymmetric magnetic field. In the present case, the low pitch angle electrons tend to drift more inwardly so that they form a C-shape distribution, which is different from the V-shape distribution that high pitch angle electrons form. As a result, the number of the low pitch angle electrons accessing the high $L$ shell is more limited than that of the high pitch angle electrons. The electrons arriving at P2 FOV are then dominated by high pitch angle electrons, which is favorable for the generation of the chorus waves there. Note that this drift-shell splitting effect on the formation of the pancake-like pitch angle distribution could not be found in earlier studies based on either 2-D magnetic field (Liu et al. 2009) or symmetric 3-D magnetic field (Collier and Hughes 2004a,b).

3.3.4 Anisotropic Electron Pitch Angle Distribution

The simulation result is used to determine the pitch angle distribution of those electrons (in all pitch angle ranges) that arrive P2 FOV as a function of time, and check whether the time profile of the anisotropy parameter can reproduce the observed time profile of the VLF signal. As mentioned in section 3.2.2, the first peak of the VLF signal at 1152 UT can be considered to be associated with the substorm injection and the second smooth rise peaked at 1255 UT is due to echoes. Therefore the onset and the rising phase of the chorus waves, i.e., the time profile between 1126 UT and 1152 UT, is further examined.

The calculation is made at four energy steps comparable to the LANL energy ranges. While the pitch angle distribution shown in Figure 3.8b was obtained by an in-
Figure 3.8 (a) Electron fluxes at four different energies: 150, 130, 110 and 80 keV. (b–e) Simulated pitch angle distributions in P2 FOV at the four energies. The color represents the number of electrons in P2 FOV with scales shown in the right. (f) Simulated anisotropy factor. The first and the second vertical dashed lines denote the observed chorus onset time and peak time, respectively.
stantaneous injection, the fact that the actual injection continued for a finite time is taken into account. Therefore the above result is convolved with the actual injection time profile. The LANL 1989 flux observation is used as a proxy for the injection time profile, noting that LANL 1989 was near the injection region and recorded nearly the dispersionless injection feature (section 3.2). Only four energy bands (80, 110, 130, and 150 keV) that are most relevant to generating chorus waves are available from the LANL observation and are used to simulate the time profiles of the flux. The pitch angle anisotropy is estimated using a quantity defined by Chen et al. (1999) as

\[
A(E) = \frac{\int_0^{\pi/2} f(E, \alpha_0) \sin^3 \alpha_0 d\alpha_0}{2 \int_0^{\pi/2} f(E, \alpha_0) \cos^2 \alpha_0 \sin \alpha_0 d\alpha_0} - 1, \tag{3.1}
\]

where \( f \) is the electron phase space density, \( \alpha_0 \) is the equatorial pitch angle, and \( E \) is the kinetic energy. \( A = 0 \) corresponds to the isotropic pitch angle distribution and \( A \) increases as the distribution becomes more pancake-like. The initial condition for the isotropic pitch angle distribution and uniform energy distribution at a given point of the injection region is expressed as \( f(E, m; t = 0) = 1 \). As electrons drift, the local number density \( f(E, m, t) \) changes, and the electrons found within the area of P2 FOV are counted to get \( f(E, m, t) \).

Figure 3.8 shows the result of electron pitch angle distribution as a function of time. From top to bottom, the simulated flux at P2 FOV, pitch angle spectra at four energies (80, 110, 130, and 150 keV), and the anisotropic parameter, \( A \), are shown. The VLF signal itself is not shown; instead the observed onset time and the peak time of the VLF signal are shown as the vertical dashed lines. It is noted that the first dashed line coincides with the arrival of the 150 keV electrons. This suggests that the chorus onset starts by the arrival of the 150 keV electrons. Although these higher-energy electrons pass by P2 FOV earlier than the end of the whistler wave activity, the anisotropy is kept for a longer period. This is likely due to the arrival of lower-energy electrons with the pitch angle anisotropy. As a whole effect, the electron anisotropy can persist as long as the observed period of the enhanced VLF
signal. It thus appears that the anisotropy responsible for the chorus waves is generated due to the pitch angle dependent drift motion, i.e., drift-shell splitting. The maintenance of the anisotropy for a sufficiently long time is attributed to the energy dependent drift velocity and the finite extent of the injection region.

### 3.4 Conclusion

Studied in this chapter was a relationship between the high-latitude ground chorus event and the substorm electron injection during the 23 March 2007 substorm, based on both the observations and the test particle simulation. Observationally, the mapping of the auroral brightening (identified from Polar/UVI) was used together with the electron flux measurements (by LANL and THEMIS) to locate the substorm injection region at 2100–2400 MLT and 5–13 $R_E$ in radial direction.

In the test particle simulation using the 3-D magnetic field model, the development of the pancake-like pitch angle anisotropy was confirmed, which is favorable for the chorus wave generation. It was clear, based on the solar wind measurements (by Cluster 1) and geosynchronous magnetic field (by GOES 11 and GOES 12), that the development of the anisotropic electron pitch angle distribution at P2 FOV was not due to the local adiabatic acceleration associated with the dynamic pressure change or the dipolarization front.

The results of this study show that even though the electrons were injected isotropically, they would tend to form a pancake-like pitch angle distribution as they drift near the magnetopause due to the drift-shell splitting effect. Supporting evidence is that the simulated chorus onset time (in terms of the arrival of 150 keV electrons in a pancake pitch angle distribution) is consistent with the observed VLF onset time. The observational and simulation results suggest that the examined high-latitude chorus waves were caused by energetic electrons injected during the substorm (cf. Abel et al. 2006) and that the ground-based observations at high-latitude and the proper modeling are important for the study of the evolution of the substorm injected electrons.
CHAPTER 4

GLOBAL DISTRIBUTION OF EMIC WAVES DERIVED FROM THEMIS OBSERVATIONS

Electromagnetic ion cyclotron (EMIC) waves play an important role in magnetospheric dynamics and their global distribution has been of great interest. This chapter presents the distribution of EMIC waves over a broader range than ever before, as enabled by observations with the Time History of Events and Macroscale Interactions during Substorms (THEMIS) spacecraft from 2007 to 2010.

The major findings are: (1) There are two major peaks in the EMIC wave occurrence probability. One is at dusk and 8–12 \( R_E \) where the helium band waves dominate the hydrogen band waves. The other is at dawn and 10–12 \( R_E \) where the hydrogen band waves dominate the helium band waves. (2) In terms of wave spectral power, the dusk events are stronger (\( \approx 10 \, \text{nT}^2/\text{Hz} \)) than the dawn events (\( \approx 3 \, \text{nT}^2/\text{Hz} \)). (3) The dawn waves have large normal angles (\( > 45^\circ \)) in the hydrogen band and even larger normal angles (\( > 60^\circ \)) in the helium band. The dusk waves have small normal angles (\( \leq 30^\circ \)) in both the hydrogen and helium bands. (4) The hydrogen band waves at dawn are weakly left-hand polarized near the equator, become linearly polarized with increasing latitude and eventually weakly right-hand polarized at high latitudes, whereas the helium band waves at dawn are linearly polarized at all latitudes. Dusk waves in both bands are strongly left-hand polarized over a wide range of latitude.

Based on the linear EMIC instability model presented by Horne and Thorne (1994), these results suggest that the main underlying factor for the observed spatial variations of these wave properties would be local density of cold plasma and chemical abundance. In addition, the distinct properties of H and He band waves found in this study would deserve a new attention in relation to EMIC wave generation mechanisms.
4.1 Introduction

Electromagnetic ion cyclotron (EMIC) waves are important in magnetospheric dynamics since they are able to cause thermal plasma heating (Thorne and Horne 1992, 1997), and pitch angle scattering and loss of both ring current ions (Cornwall et al. 1970) and relativistic electrons (Thorne and Kennel 1971). EMIC waves can be excited by an anisotropic temperature distribution \( T_\perp > T_\parallel \) of energetic ions with energies of a few tens keV and are left-hand polarized at generation (Cornwall 1965; Young et al. 1981; Roux et al. 1982; Rauch and Roux 1982; Anderson et al. 1992a, 1996). In a plasma consisting mostly of hydrogen (H) and helium (He), EMIC waves are emitted in two frequency bands: H band lying between the H and He gyrofrequencies, and He band lying below the He gyrofrequency. Under the convective instability, the greatest amplification of EMIC waves occurs where the group velocity of the waves is lowest, making the equator the most preferable region for EMIC wave generation (Cornwall 1965, 1966; Kennel and Petschek 1966). It has been suggested that storms provide injections of hot ring current particles into the inner magnetosphere which may then lead to a condition favorable for EMIC wave generation (Cornwall 1965; Criswell 1969). The compression of the magnetopause is suggested as another possible source of EMIC waves (Anderson and Hamilton 1993; Engebretson et al. 2002; McCollough et al. 2010).

The majority of EMIC waves are, however, observed during the quiet time of the geomagnetic activity and occur beyond the geosynchronous orbit. Young et al. (1981) and Roux et al. (1982) used observations by GOES 1 and 2 to suggest that EMIC waves near geosynchronous orbit are often not associated with plasma density enhancements and that the wave characteristics can significantly vary depending on the density and concentration of helium ions. Erlandson et al. (1990) and Anderson et al. (1990, 1992a,b), analyzed AMPTE/CCE magnetic field data to find that EMIC wave events occur primarily for \( L > 7 \) in the afternoon sector, and the wave frequency and polarization of EMIC waves depend on the magnetic local time (MLT) and \( L \) value. In the afternoon sector, EMIC waves occur
below the helium gyrofrequency and are left-hand polarized, whereas in the morning sector, they occur at much higher frequencies and are linearly polarized. Fraser and Nguyen (2001) analyzed observations by CRRES and concluded that the plasmapause is a region of wave generation and propagation with significant wave power seen in the plasmatrough, but is not necessarily the preferred region. They also found MLT dependent wave characteristics similar to those reported by Anderson et al. (1992a,b). Recently, Fraser et al. (2010) investigated the association of EMIC waves with storm phases using GOES observations and showed that EMIC waves are mostly H band at dawnside while H and He band events are mixed at the duskside geosynchronous orbit. Anderson et al. (1996) examined the ion observations with and without EMIC wave events at noon and dawn to conclude that the difference in frequency between noon and dawn is attributable to the combined effects of the different hot proton temperature anisotropies and the cold plasma density. They also found that the dawn events had significant growth for highly oblique waves and this may suggest that the linear polarization of the dawn events is due to waves generated with oblique wave vectors.

A comprehensive theoretical study of the results of Anderson et al. (1992a,b) has been presented by Horne and Thorne (1994) in which the path-integrated wave gain was calculated for two thermal plasma density conditions and used to explain the wave properties observed in the morning and afternoon sectors, respectively. They also argued that the left-hand polarization dominant in the afternoon sector within $|\text{MLAT}| < 10^\circ$ is a generation effect and the linear polarization at dawn is due to a propagation effect associated with wave reflection. Recent hybrid simulation by Hu and Denton (2009) and Hu et al. (2010) further showed that the waves can be generated at the equator with strong left-hand polarization and the left-hand polarization can change to the linearly polarized waves as the wave normal angle increases during propagation. They also pointed out that the waves can be generated with large wave normal angle when the He composition is small.

In this chapter, EMIC waves are studied using the magnetic field observations by
the THEMIS mission for a four year operational period. Compared with the previous statistical studies based on either CRRES, AMPTE/CCE or GOES satellites, the THEMIS observations have a major advantage in that they cover a broader radial range. This data set is used to investigate the spatial distribution of EMIC wave occurrence, wave frequency, polarization and wave normal angle on both the equatorial and the meridional planes. This chapter is organized as follows. Section 4.2 describes the instrumentation and analysis procedure, and section 4.3 shows the results. Section 4.4 is devoted to the discussion and section 4.5 concludes this study.

4.2 Data Set and Analysis Procedure

4.2.1 Instruments

The THEMIS spacecraft, comprising five identical probes in near-equatorial orbits with apogees above 10 \( R_E \) and perigees below 2 \( R_E \) (Angelopoulos 2008), are ideal for detecting EMIC waves in a wide range of \( L \) and MLT in Earth’s magnetosphere. The THEMIS Fluxgate Magnetometer (FGM) measures the background magnetic field and low frequency fluctuations (up to 64 Hz) in near-Earth space (Auster et al. 2008).

The Electro-Static Analyzer (ESA) measures the ion and electron distribution functions over the energy range from a few eV up to 30 keV for electrons and up to 25 keV for ions (McFadden et al. 2008). The total electron density is inferred from the spacecraft potential and the electron thermal speed measured by the Electric Field Instrument (EFI, which measures three components of the ambient vector electric field (Bonnell et al. 2008)) and ESA respectively, including both the cold plasma population and the hot plasma component. The uncertainty for the electron density determination is usually less than a factor of two (see Li et al. (2010) and references therein).
4.2.2 Data Processing

The primary data set is magnetic field data obtained with the FGM instrument in all five spacecraft from April 1, 2007 to December 31, 2010. The low-resolution (sampling rate \(\approx 4\) Hz) FGM data are utilized to construct the dynamic spectra of the EMIC waves. Use of the high resolution FGM data is more desirable but available only for a limited time and thus not suitable for the purpose of investigating global wave distribution in a statistical purpose.

The magnetic field data sometimes include unrealistic dropouts and outliers on the order of seconds, which can occur when the sun pulse needed for accurately determining the spin period is unavailable for short periods (K. H. Glassmeier, private communication, 2011). These are removed using the phase-space method of Goring and Nikora (2002) in which the magnetic field measurement, and its first and second time derivatives are used to define those outliers. The background magnetic field is determined by taking a 1000 data point (250 s) sliding average on the magnetic field measured in the GSM coordinates. The background magnetic field is used to define the mean-field aligned (MFA) coordinate system and to transform the original magnetic fields measured in the GSM coordinates into MFA coordinates. Waveforms are obtained by subtracting the background magnetic field from the total magnetic field in MFA coordinates.

The waveforms in the time domain are Fourier-transformed to the frequency domain where the EMIC waves can be identified. Since the investigation covers a broad range of \(L\) shells, EMIC waves should also be distributed in a correspondingly wide frequency range. The Morlet wavelet transform (Grossmann and Morlet 1984) is applied to the daily data set which returns the spectrum in a logarithmic frequency scale and thus helps identification of EMIC waves over a wide frequency range. In the dynamic spectrum the EMIC waves can be located by setting a threshold for minimum power based on an adequate level of background noise. Any signals above this threshold within the H or He bands are identified as EMIC waves. This critical value was chosen after testing a range of trial values of the
threshold from 0.01 to 0.1 nT²/Hz settling on 0.03 nT²/Hz as the total power threshold. As a comparison, Anderson et al. (1992a) identified EMIC waves by counting signals with peak amplitudes greater than 0.8 nT. Although the criterion in this study differs from theirs, the threshold is chosen to reproduce an overall EMIC wave distribution similar to theirs.

The three-axis magnetic field data in the time domain were Fourier transformed to three components of complex magnetic fields in the frequency domain which are then casted to a covariance matrix (Means 1972). The wave normal angle, θₖ, was then calculated using covariance matrix elements following the method presented by Means (1972). Ellipticity, ε, is determined from the complex representation of the waveform following Kodera et al. (1977). The normalized frequency is also calculated using \( X \equiv f/f_{H^+} \) where \( f \) is the wave frequency and \( f_{H^+} \) is the local proton gyrofrequency. At this stage, all these quantities: the covariance matrix elements, ellipticity, the normalized frequency, and the wave power spectral density (PSD), are functions of frequency. As the last step, these quantities are averaged over frequency as done by Anderson et al. (1996). Namely, at every time segment, the average quantities are calculated as:

\[
A_{\text{avg}} = \frac{\int_{f_{\text{min}}}^{f_{\text{max}}} A(f)PSD(f)df}{\int_{f_{\text{min}}}^{f_{\text{max}}} PSD(f)df},
\]

where \( A \) can be either an element of the covariance matrix, ε, X, or PSD. Here \( f_{\text{min}} \) and \( f_{\text{max}} \) are the minimum and maximum wave frequencies, respectively, beyond which the wave power is below the preset threshold.

An example of this analysis is shown in Figure 4.1. The data was obtained on August 31, 2007 by THEMIS-A (P5) when it was located in the pre-noon southern hemisphere in GSM coordinates at 0050 UT. The two black lines in each of the top three panels denote the local helium and oxygen gyrofrequencies. The top two panels show EMIC wave power spectra parallel and perpendicular to the background magnetic field, respectively. At
Figure 4.1 An example of EMIC wave analysis. From top to bottom, (a) and (b) show wave power spectral density parallel and perpendicular to the background magnetic field, and (c) shows ellipticity as functions of time and frequency. The rest panels show frequency-averaged quantities: (d) normalized wave frequency, (e) ellipticity and (f) wave normal angle, and (g) average wave power, respectively. The thick solid lines in the top three panels indicate local helium and oxygen gyrofrequencies, respectively. The color bars in (a) and (b) are logarithmically scaled. In (c) the right (left) handed polarization is indicated by the red (blue) color table. The dashed line in (d) indicates helium gyrofrequency. These EMIC waves were detected by THEMIS-A (P5) on August 31, 2007 when THEMIS-A was inbound from the southern hemisphere in GSM coordinates between 0050 and 0350 UT and was passing noon meridian at 0305 UT.
a glance, one can see that the transverse oscillation dominates over the parallel oscillation in power, and the wave frequency changes from the H band to He band when THEMIS-A passes 7.4 $R_E$. The third panel from the top shows that the waves were mostly left-hand polarized. The lower four panels show the frequency-averaged quantities: normalized frequency, ellipticity, wave normal angle, and $PSD$ as functions of time. They show more clearly the change of frequency from the H to He band, the low degree of polarization, and small normal angles of the waves. The EMIC wave activity lasted for about four hours.

The criterion for identifying EMIC waves in H and He bands is simply whether the wave power-spectral density appears between the hydrogen and oxygen gyrofrequencies. There must be EMIC waves below oxygen gyrofrequency too, which the current study, however, does not investigate due to difficulties in spectral identification. It should be noted that other types of waves may appear in this spectral range, and possibly add uncertainty to the present analysis. They are Pc3–5 and Pi2 waves (Takahashi 1998) and their characteristics are discussed below compared with EMIC waves. First, Pc4–5 waves driven by solar wind usually occur in the dayside, outer magnetosphere ($L > 6$) including those waves generated by Kelvin-Helmholtz instability (e.g. Claudepierre et al. 2008). These waves have lower frequencies than the minimum frequency of the investigation, which is still as high as 20 mHz even at 12 $R_E$. Second, Pi2 waves are mostly found in the nightside. As will be shown, the wave activity in the nightside is very weak, and will not be a major target of this investigation. Third, Pc3–4 waves associated with the field line resonance are observed at low latitude and connected to the inner magnetosphere ($L < 6$) (Takahashi 1998). At $L = 6$, the minimum frequency of the investigation is 0.15 Hz and still lies above the maximum frequency of Pc3–4 waves. There is, however, another source of Pc3 waves, namely, those originating in the bow shock and propagating near the Earth’s cusp (Yeoman et al. 2012). At this moment, how much contribution these waves make to the wave activity under study cannot be quantitatively estimated. However, if these Pc3 waves propagate near the Earth’s cusp, it should not affect the result of this study. This study uses the THEMIS observations
that were made near the equatorial magnetosphere, mostly less than 20° of the magnetic latitude. One can also expect that this type of Pc3 waves will appear in a narrow region near local noon, which will thus be discussed further in section 4.4.2 based on the wave properties found from the analysis.

4.2.3 Determination of the Spatial Distribution

Since the goal is to determine the global distribution of EMIC waves, the present study attempted to use the data obtained in a radial distance range as wide as possible within the magnetosphere. However, the spatial range of the investigation is limited by a couple of factors. Figure 4.2 is a scatter plot of spacecraft locations in x-y (a) and x-z (b) planes in GSM coordinates marked only when the EMIC wave activity was observed by the probes. As shown in Figure 4.2a, most of the events were detected when the probes were in the noon-dusk sector (1200 ≤ MLT ≤ 1800) and the dawn sector (0400 ≤ MLT ≤ 0900) while relatively fewer events were found at midnight. There is also a narrow region around 1000 MLT where fewer events were detected. This distribution is very similar to the distribution shown in Figure 8 of Anderson et al. (1992a) although the AMPTE spacecraft orbit was limited to within 9.25 \( R_E \). Figure 4.2b also indicates that the EMIC waves were concentrated on the dayside and distributed in a wide range of latitude.

Figure 4.3 shows the number of hours for which the FGM observations were available within the magnetopause. To present the distribution of the quantity on the equatorial plane in this form, off-equatorial observations are projected onto the equator along the field lines that are assumed to be the dipole magnetic field. It can be seen that most of the observations were made between 4 \( R_E \) and 12 \( R_E \) with a high concentration between 10 \( R_E \) and 12 \( R_E \), where the apogees of inner probes (THEMIS-A, D and E) are located. Relatively few observations have been made inside 4 \( R_E \) and beyond 12 \( R_E \) and also at 1800 < MLT < 0700 beyond 12 \( R_E \).

Another constraint comes from the data sampling rate because it should be at least
Figure 4.2 Positions of spacecraft during the EMIC wave observations shown in (a) $x$-$y$ and (b) $x$-$z$ planes in GSM coordinates. The sun is on the right. The inner circle in (a) indicates $4 \, R_E$. 
Figure 4.3 Total hours of the magnetic field observations from all spacecraft displayed on the magnetic equator. The hours are shown in the logarithmic scale. The spacecraft positions were projected along the dipole magnetic field line to the magnetic equator. The sun is on the right and the concentric dashed circles represent the radial distances as labeled at midnight. Each bin has size of 0.5 by 0.5 $R_E$. Regions where total observing time is less than 0.01 hour are uncolored.
twice higher than local EMIC wave frequency in order to detect the waves properly. Since EMIC wave frequency is related to the ion gyrofrequency and thus increases closer to the Earth, this sets another limit on the inner boundary of the investigation. Specifically, the hydrogen gyrofrequency reaches the Nyquist frequency (2 Hz) at $6.2R_E$ and the helium gyrofrequency at $3.9 R_E$ under the dipole magnetic field model. The outer boundary is set to the magnetopause for both bands. In order to exclude the data obtained outside of the magnetopause, whether magnetic field data show sudden change as the spacecraft pass through the magnetopause is checked. The encounter of the magnetopause could also be detected by the power spectrum: a broadband spectral feature appears in the power spectrum whenever the satellite passes by the magnetopause. Although the observations are available up to a larger distance, the observing time at regions between $12 R_E$ and $14 R_E$ is too short to provide any statistically meaningful result. The effective range of the investigation is thus considered to be limited to $3.9–12 R_E$ for He band waves and $6.2–12 R_E$ for H band waves, respectively.

The use of the dipole magnetic field model in projecting the quantities in three-dimensional space onto the magnetic equator is only due to practical limitations as it is very difficult to use a more realistic model for every FGM observation. Although the magnetic field in the outer magnetosphere may occasionally deviate from a dipole, this approximation should not significantly affect understanding of the essential physics. Note also that even though the dipole magnetic field is used, the magnetopause is detected based on the real data, which should give correct spatial information on the magnetosphere. Another issue is the presence of so-called Shabansky orbits on the dayside at the $L$-shells examined ($L > 7$) (Shabansky 1971). Shabansky orbits could play a role in providing the source anisotropy in which case the waves may be generated at higher latitudes on the dayside (McCollough et al. 2012). In this study, however, the wave activities are projected to the magnetic equator, since the majority of EMIC waves measured by THEMIS are from the magnetic latitude less than $15^\circ$. 
To determine the equatorial distribution, the equatorial plane is partitioned into square bins with $\Delta x = 0.5 \, R_E$ by $\Delta y = 0.5 \, R_E$ covering all MLT and $4 \, R_E < R < 14 \, R_E$, and average all quantities within each bin. For the meridional distribution, the local time is divided into four sectors: dawn, ranging from 0300 to 0900 MLT; noon, from 0900 to 1500 MLT; dusk, from 1500 to 2100 MLT; and night, from 2100 to 0300 MLT. Each sector is partitioned as above.

For comparison, Anderson et al. (1992a) used the AMPTE/CCE which measured the magnetic field with a sampling rate ($\approx 8 \, Hz$) is about twice as high as that used here. Therefore they could determine the distribution of EMIC waves further inside of the magnetosphere. However, the present study can investigate the outermost magnetosphere to $12 \, R_E$, more distant than $9 \, R_E$ investigated by Anderson et al. (1992a).

4.3 Results

4.3.1 Occurrence Probability

In order to quantify how frequently EMIC waves occur at each location, the ratio of the total time of EMIC wave activity to the total observing time within each spatial bin is calculated. This normalized quantity is called the occurrence probability (Anderson et al. 1992a). Figure 4.4 shows the equatorial distribution of the EMIC wave occurrence probability in two separate frequency bands: H band (Figure 4.4a) and He band (Figure 4.4b). The regions with the probability less than 0.1% are colored gray, and regions with no observation are uncolored.

For the H band waves, the occurrence probability distribution shows a maximum ($\approx 20\%$) in the morning sector with a secondary peak at noon-dusk ($\approx 10\%$). The He band EMIC waves mostly occur at dusk ($1300 < \text{MLT} < 1800$) with a primary peak (probability $\approx 20\%$) at 1500–1800 MLT and a secondary peak ($\approx 10\%$) at 0600 MLT. Therefore, the maximum occurrence probability of H band waves at dawn is as high as that of the He band waves at dusk. The H band peak distribution at dawn ($0500 < \text{MLT} < 0800$) is relatively
Figure 4.4  EMIC wave occurrence probability for (a) H and (b) He band waves projected on the magnetic equatorial plane along the dipole magnetic field. The figure format is the same as Figure 4.3. The regions with occurrence probability less than 0.1% are colored gray.
confined in the outer magnetosphere ($10R_E < R < 12R_E$) while the He band has a wider radial coverage ($8R_E < R < 12R_E$). For both wave bands, the occurrence probability at night ($2100 < \text{MLT} < 0300$) is very low. Between the two peaks at dawn and dusk, there is a narrow gap of low occurrence probability at around 1000 MLT.

In comparison with Anderson et al. (1992a), the results show both similarities and subtle differences: first, they found that for $L > 7$, EMIC waves occur with 10–20% probability at post-noon (1200–1500 MLT) and $\approx 3\%$ probability at dawn (0300–0900 MLT). In this result, the maximum occurrence probability of the H band waves at dawn is located beyond $10R_E$, reaching 20%, which is as high as that of the He band in the dusk. Second, they found that the occurrence probability tends to increase with $L$ at all MLT ranges. It was confirmed that this trend sustains up to the radial distance, $12R_E$. This could also be the reason why the H band peak could not be seen in Anderson et al. (1992a), i.e., the peak of the H band waves is located beyond the radius limit at $9.25R_E$ in their study. Third, Anderson et al. (1992a) pointed out that the occurrence probability decreases significantly for $R < 8R_E$ to reach a probability of 1% for $L < 5$. This is also seen in this result too, which is, however, less obvious for the radial range set by the low sampling rate. Fourth, Anderson et al. (1992a) reported a radial gap of the occurrence probability at dawn around $L = 6$. No feature of such a radial gap is found in this result, possibly due to either the sampling rate limit or lack of spacecraft access to the area. In this result, a region of relatively low occurrence is located at 0930 MLT for $R > 8R_E$, which was, however, not shown in Anderson et al. (1992a). The gap in MLT becomes prominent in the outer magnetosphere where the probability in regions adjacent to the gap increases.

Zhang et al. (2011) studied EMIC waves in the outer magnetosphere using Cluster data, and found that wave-associated He heating events frequently occur in the dusk sector. This is consistent with the finding of the high occurrence probability of He band EMIC waves at dusk.
Figure 4.5 Average normalized wave frequency, $X = f/f_{H^+}$, for (a) H and (b) He band waves. The figure format is the same as Figure 4.3.
4.3.2 Wave Frequency

The equatorial distribution of the normalized frequency is shown in Figure 4.5, where the H band frequencies \((X > 0.25, \text{Figure 4.5a})\) and He band frequencies \((X < 0.25, \text{Figure 4.5b})\) are separately displayed. In this figure, the frequencies within each band at a given spatial bin were separately averaged. Without such a distinction, the result would be similar to Anderson et al. (1992b) where the average normalized frequency decreases from 0.5 at 1000 MLT to 0.3 at 1800 MLT and increase from 0.28 at \(L = 3–5\) to 0.39 at \(L = 8–9\) for 1200–1500 MLT. The decrease of the average frequency with MLT generally implies the dominance of H band emission at dawn and that of He band at dusk, respectively. For H band waves, the frequency is relatively high \((X \approx 0.5)\) at dawn, and low \((X \approx 0.35)\) at noon and dusk. For He band waves, on the other hand, the wave frequency lies just below the He gyrofrequency \((X \approx 0.17)\) for most MLT values.

4.3.3 Polarization and Normal Angle

The ellipticity distribution is shown in Figure 4.6a (H band) and b (He band). The H band waves are mostly linearly polarized \((\varepsilon \approx 0)\) at dawn. At noon and dusk, they are left-hand polarized for 8–12 \(R_E\). However, even at noon and dusk, linearly polarized H band waves are found in the outer magnetosphere \((> 10 R_E)\). The He band waves are also linearly polarized at dawn and left-hand polarized at noon and dusk in the range of 8–12 \(R_E\), with some linear polarization in the outermost magnetosphere. The general trend is that noon-dusk waves are predominantly left hand polarized \((\varepsilon < -0.3)\) while dawn waves are linearly polarized on average. Therefore, whether it is left-hand or linearly polarized depends on MLT rather than on the wave emission band.

Meridional distributions of polarization are shown in Figure 4.7. The left (right) column shows the distribution of H (He) band waves. From top to bottom, the results at dusk, noon, and dawn sectors are shown, respectively. The top four panels show that at noon and dusk, the left handed polarization dominates regardless of the emission band,
Figure 4.6 Equatorial distribution of average wave ellipticity, $\varepsilon$, for (a) H and (b) He band waves. Negative (positive) values of $\varepsilon$ indicate left (right) hand polarization. $\varepsilon = 0$ corresponds to linear polarization and is colored white. The figure format is the same as Figure 4.3.
Figure 4.7 Meridional distribution of ellipticity. The same color scale as Figure 4.6 is adopted. The left (right) column shows the distribution of H (He) band waves. From top to bottom, the ellipticity at dusk, noon and dawn sectors are shown, respectively.
although there is some variation at the outermost magnetosphere. The bottom two panels show that at dawn, there is an interesting change in polarization depending on the emission band. For H band (Figure 4.7e), the waves are weakly left-hand polarized at the equator and the polarization changes to weakly right-handed at high magnetic latitude ($> 15^\circ$). For He band (Figure 4.7f), there is a trend that the waves are linearly polarized for the majority of latitudes (within $\approx 20^\circ$) although statistical fluctuations exist. At large radial distances ($> 12R_E$), however, both H and He band waves are linearly polarized at all magnetic latitudes. At noon (Figure 4.7c-d) and dusk (Figure 4.7a-b), the waves are left-hand polarized at all magnetic latitudes except at the region of high latitude and large radial distance, where the polarization changes to the right-hand sense.

Anderson et al. (1992b) also reported the dominance of the linear polarization at dawn and left-hand polarization at dusk. In this result, the He (H) band waves appear to be left-hand (linearly) polarized waves except at noon where left-hand polarization is dominant for both bands. Another new result in this study is that the meridional distribution at dawn shows a change of polarization with latitude for H band waves whereas Anderson et al. (1992b) found the linear polarization dominates at all magnetic latitudes.

Wave normal angles are shown in Figure 4.8, again with distinction for H and He bands. The color table in which the blue to red colors correspond to the angles varying from 10 to 80$^\circ$ centered at white (corresponding to 45$^\circ$) is used. At dawn, H band waves have large normal angles ($\theta_k > 45^\circ$), and He band waves have even larger normal angles ($\approx 60^\circ$). At noon and dusk, small normal angles ($\theta_k \approx 30^\circ$) are dominant for both bands. When compared to the ellipticity distribution shown in Figure 4.7, it is obvious that the wave normal angle is small where left hand polarized waves dominate, and large in the region of linearly polarized waves.

The meridional distribution of wave normal angle is shown in Figure 4.9 for both H (He) bands. At noon and dusk (Figure 4.9a–d), the wave normal angle is generally less than 30$^\circ$ at all latitudes for both H and He band waves. Only at large radial distances does
Figure 4.8  Equatorial distribution of average wave normal angle, $\theta_k$, for (a) H and (b) He band waves measured from the local magnetic field direction. $45^\circ$ is colored white. The figure format is the same as Figure 4.3.
Figure 4.9 Meridional distribution of wave normal angle. The same color scale as Figure 4.8 is adopted. The figure format is the same as Figure 4.7.
the wave normal angle increase, which must be related to the linear polarization found in that region (Figure 4.7a–d). At dawn, however, the wave normal angle of H band waves tends to change from below 40° at the equator to 45° at high latitude just as the ellipticity of H band waves changes with latitude. The He band waves are even more oblique (> 60°) independent of the latitude in the outer magnetosphere.

4.3.4 Wave Power and Plasma Parameters

The wave power distribution is shown in Figure 4.10a and b for H and He bands, respectively. Also shown are the total electron density (Figure 4.10c), ion temperature anisotropy (Figure 4.10d) and ion temperature perpendicular to the magnetic field (Figure 4.10e), as they are considered important factors in EMIC wave generation (Gary and Lee 1994). Note that there is a great similarity between the wave power distribution and the occurrence probability distribution (Figure 4.4). Thus strong events are found in the regions of high occurrence probability, e.g., the dusk and dawn sector, and weaker events near midnight and at 1000 MLT. The dusk events have relatively high amplitudes often exceeding 10nT^2/Hz, higher than those of the dawn events (3 nT^2/Hz). In comparison with Figure 4.4 and 4.5, the high amplitude is related to He band waves at dusk and radially inner part at noon while relatively low amplitude is related to the H band waves at dawn and radially outer part at noon.

The instability condition for EMIC waves (Gary and Lee 1994) is more likely to be met in regions of higher thermal density, higher ion temperature anisotropy and weaker field strength. Figure 4.10c-e shows that the dusk region of strong wave power has high density and high ion perpendicular temperature ideal for the growth of EMIC instability. On the other hand, the dawn region has low density (≤ 1 cm^-3), but has higher ion temperature anisotropy (> 1), and low magnetic field strength, which are also important factors for EMIC wave instability.
Figure 4.10 The equatorial distribution of average wave power spectral density of (a) H and (b) He band, (c) total electron density, (d) ion temperature anisotropy \((T_{\perp}/T_{\parallel} - 1)\) and (e) ion temperature perpendicular to the magnetic field \((T_{\perp})\). The figure formats are same as Figure 4.3. The electron density was derived from the spacecraft potential and the data was only available after June 2008. The ion temperature was derived from ESA data during each event. In (a) and (b), the regions with wave power less than 0.1 nT\(^2/\)Hz are colored gray. Despite the presence of EMIC waves, some parts in (c), (d) and (e) are missing due to the absence of data.
4.4 Discussion

The results of EMIC wave properties in section 4.3 can be interpreted mainly based on the theoretical model of Horne and Thorne (1994). The model provides a linear convective instability analysis of EMIC waves designed for a comprehensive understanding of the result of Anderson et al. (1992a,b) and is considered appropriate for the present results. Horne and Thorne (1994) discussed wave properties in three frequency regimes: guided mode below the helium gyrofrequency ($X < 0.25$), an unguided mode between the helium gyrofrequency and the crossover frequency, $X_{cr}$ ($0.24 < X < X_{cr}$), and a guided mode above $X_{cr}$. At the crossover frequency, the dispersion curves for the L- and R-modes are coupled to each other and therefore a mode may change to the other depending on inhomogeneities (Smith and Brice 1964). The prime interest in this discussion is whether the observed interrelations among frequency, ellipticity, and power can be explained based on the physical properties of these three modes predicted by the theoretical model. Since the wave properties at the H and He bands were separately investigated, the result of this study may be better for comparison with the theoretical model.

4.4.1 Dusk Waves

Most of the dusk waves have low frequency ($X < 0.25$) corresponding to the He band, are strongly left-hand polarized, and field-aligned with wave normal angles less than 30°. These results mostly agree to the previous results (Anderson et al. 1992a,b; Fraser and Nguyen 2001). However, the fact that average frequency lies at the He band should not suggest that there is no H band at all. In Figure 4.5, both H and He band waves coexist at dusk with distinct properties. The frequency of the He band waves changes little with MLT, but in the H band, the frequencies at dusk are noticeably lower than those at dawn.

This result is compare with the high density ($n_e \geq 10$ cm$^{-3}$) of the Horne and Thorne (1994) model which they intended for the noon-dusk waves without distinction of the noon from the dusk sector. Through path-integrated wave gain calculation, they
showed that the intense convective growth occurs for the guided mode below the helium gyrofrequency \((X \leq 0.25)\) and the unguided mode waves at \(0.25 \leq X \leq X_{cr}\). The low frequency \((X < 0.25)\) waves that dominate in the region should be the He band, but the waves with higher frequencies \((X > 0.25)\) may be attributed to the unguided mode. The latter waves have frequencies in the H band, but relatively lower than the frequencies of the H band waves found at dawn. At dawn the low density condition applies, and the H band waves are probably the guided mode above \(X_{cr}\).

In this interpretation, the high power of dusk waves with low frequency (Figure 4.10) can be understood as due to the guided mode in the He band having a stronger growth rate than the unguided mode in the H band. The strong growth rate of the guided waves also comes with strong left-hand polarization, consistent with the observation of ellipticity \(\varepsilon < -0.2\) (Figure 4.6). The observation of small wave normal angles \((< 30^\circ)\) in this region (Figure 4.8) is also consistent with the model prediction that the low group velocity in high density regions allows waves to become predominantly field-aligned.

**4.4.2 Noon Waves**

In the noon sector, a high density region at 6–9 \(R_E\) extending from the dusk sector and a low density region at 9–12 \(R_E\) extending from the dawn sector coexist. Such a mixed density distribution is reflected in the normalized frequency distribution in Figure 4.5. As the wave frequency increases with radial distance, the total wave power decreases. Quantitatively, the normalized frequency of the waves in the inner region is lower than 0.25, and the frequency of the waves in the outer region of the noon sector is higher \((X > 0.25)\) than the dusk waves but lower than for dawn waves \((X \leq 0.4)\).

Again, the low frequency waves at the inner region can be related to the guided mode He band waves, and the relatively higher frequency \((0.25 < X < 0.4)\) waves at the outer region to the unguided waves generated above \(X = 0.25\) and below \(X_{cr}\) as discussed above. For a cold H-He plasma, the crossover frequency is given by \(X_{cr} = 0.25(1 + 15\eta)^{1/2}\)
where $\eta = N_{\text{He}}/N_e$ is the He abundance (Smith and Brice 1964). If one tentatively take the observed upper bound frequency ($X \approx 0.4$) near noon as the crossover frequency, then the He abundance at noon should be as high as 10%, consistent with previous observations (Anderson et al. 1996).

The relatively weaker wave power in the noon sector than the dusk sector may again be attributed to nature of the unguided mode in the noon sector. The increasing wave frequency and decreasing wave power with radial distance at noon should then indicate that the unguided mode becomes dominant at large radial distances.

Other types of waves could be present at noon. For instance, the shock-generated Pc3 ULF waves (section 4.2.2) move along the field lines close to the open-close boundary and therefore reach the outer magnetosphere ($\sim 12 R_E$) to appear in the spectral range that was identified as He band. These waves are basically compressional waves (Engebretson et al. 2006) and may have the high normal angle and linear polarization as detected in the area. With no further information, the current study leaves this possibility open. However, except for the He band waves around noon in the outer region ($10 < L < 12$), the rest of waves have left hand polarizations and low wave normal angles, which are likely to be typical features of EMIC waves.

4.4.3 Dawn Waves

The high occurrence probability and strong wave power of EMIC waves in the dawn sector are two of the new findings of this study with the inclusion of the outer magnetosphere beyond $9 R_E$ enabled by the THEMIS spacecraft. According to the Horne and Thorne (1994) model, the instability at $X < 0.25$ is suppressed at a low density ($N_e \leq 2 \text{ cm}^{-3}$), and instead significant instabilities at high frequencies $X \geq 0.25$ are possible if the energetic ions have sufficient anisotropy. Figure 4.10c and d show that the dawn sector has such a condition of low density and high anisotropy as required to generate high frequency ($X = 0.4–0.5$) and high wave power ($\geq 1 \text{ nT}^2/\text{Hz}$) EMIC waves observed at dawn (Figure
The linear polarization of EMIC waves observed at dawn has been of great interest for theoretical studies. Either the waves are generated with large normal angles and are nearly linear polarized (Denton et al. 1992; Hu and Denton 2009; Hu et al. 2010), or the linear polarization of EMIC waves could be produced merely under a propagation effect when waves are reflected in the magnetosphere (Horne and Thorne 1994). Mode conversion from L- to R-mode could be another simple explanation for the linear polarization, but was not favored for the following reasons. The frequency of dawn waves is too high compared with the presumable crossover frequency in the region for the mode coupling process to take place (Anderson et al. 1992b). It could also be that the results of Anderson et al. (1992b) showed only the linear polarization at all latitudes and there was no need to further consider the mode conversion theory.

With the present results where the polarization distribution at H and He bands were separately shown, however, both explanations based on mode conversion and generation of linearly polarized waves are attractive. The latitudinal distribution of the H band polarization shows that left hand polarization is seen near the equator and then changes to right-hand polarization at high magnetic latitudes, although both left- and right-hand polarizations are very weak. Such a polarization distribution may indicate that the mode conversion from left to right hand polarization is going on while waves are generated near the equator and travel away from it. This mode conversion can occur in the H band only since the crossover frequency is above the helium gyrofrequency. The large wave normal angle at all magnetic latitude is also a factor favorable for mode conversion (Young et al. 1981). For the He band, linear polarization is seen at all latitudes, and it is likely that the He band emissions are linearly polarized at generation. The He band waves have even larger normal angles at all latitudes (Figure 4.9e). As a supporting model, Hu et al. (2010) showed that the waves can be generated with large wave normal angle when the He abundance is small. Since He abundance is expected to be low at dawn, the observation in this study is
consistent with the model of *Hu et al.* (2010).

### 4.5 Summary

This chapter investigated the global distribution of EMIC waves using four years of THEMIS observations. The results for wave occurrence rate, polarization and wave power are summarized as follows:

1. There are two major peaks in the EMIC wave occurrence probability. One is at dusk and 8–12 $R_E$ where the helium band dominates the hydrogen band waves. The other is at dawn and 10–12 $R_E$ where the hydrogen band dominates the helium band waves. The latter was not seen in *Anderson et al.* (1992a) because their investigation was limited to within 9.5 $R_E$.

2. In terms of wave power, dusk events are stronger ($\approx 10\text{nT}^2/\text{Hz}$) than dawn events ($\approx 3\text{nT}^2/\text{Hz}$). Noon events are the weakest. Based on the modelling result by *Horne and Thorne* (1994), this result confirms the dominant dusk waves with He band waves, and the dawn and noon waves with guided and unguided H band waves, respectively.

3. At dawn, the waves emitted in the H band have large normal angles ($> 45^\circ$) and the waves in the He band have even larger normal angles ($> 60^\circ$) than the H band waves. At dusk, waves are propagating with small normal angles ($\leq 30^\circ$) and dominated by the He band emission.

4. The dusk waves are strongly left-hand polarized, as has been shown elsewhere. The dawn waves, however, have different polarization characteristics in the H and He bands. For the H band waves, the polarization changes with latitude from weakly left hand polarization near the equator to right hand polarization at high latitudes, suggesting mode coupling. The He band waves at dawn are linearly polarized at all
latitudes, which is consistent with the model for EMIC wave generation by *Hu et al.* (2010).
CHAPTER 5

CHARACTERISTIC DIMENSION OF ELECTROMAGNETIC ION CYCLOTRON WAVE ACTIVITY IN THE MAGNETOSPHERE

In this chapter, the size of coherent activity of Electromagnetic ion cyclotron (EMIC) waves is estimated using the multi-spacecraft observations made during the Time History of Events and Macroscale Interactions during Substorms (THEMIS) mission. The cross correlations between EMIC wave powers measured by different THEMIS spacecraft over the separation distances between pairs of observing spacecraft is calculated and the \(1/e\) folding distance of the correlations is determined as the characteristic dimension of the coherent wave activity. The characteristic radius in the direction transverse to the local magnetic field is found to lie in rather a wide range of 1,500–8,600 km varying from the morning (AM) to afternoon (PM) sectors and also from hydrogen to helium bands. However, the characteristic dimensions normalized by either gyroradius or wavelength fall into narrower ranges almost independent of the emission band and event location. Specifically, the coherent dimension is found to be 10–16 times gyroradius of 100 keV protons and 2–3 times local wavelength. The former may give a useful scale for the source dimension and the latter suggests that the EMIC wave activity maintains coherency only up to a couple of wavelengths.

5.1 Introduction

Electromagnetic ion cyclotron (EMIC) waves are the highest frequency waves in the ultra-low frequency (ULF) spectral regime, and are generated in the magnetosphere by interacting with energetic ions (Fraser et al. 2006). EMIC waves play an important role in extracting energy from the hot, anisotropic ions and causing pitch angle scattering of energetic ions and relativistic electrons into the loss cone (Cornwall et al. 1970; Jordanova et al. 2007). In order to better understand the role of EMIC waves in the dynamics of the magne-
tosphere, spatial dimensions of the EMIC wave activity need to be known. The overall size of EMIC wave sources would be an important factor for the global impact of the EMIC waves on the particles in the magnetosphere. Min et al. (2012) presented a global distribution of EMIC waves statistically reconstructed from synthesis of three-year observations with the THEMIS spacecraft. The global distribution of EMIC waves may, however, be non-uniform in time and space, and there can be a smaller scale within which the wave activity occurs simultaneously. While the global scale is important for assessing total effect of EMIC waves, the small scale must also be important for understanding wave-particle interactions. This chapter focuses on the latter scale of coherent EMIC wave activity in the outer magnetosphere, which is, for simplicity, referred to as a coherent source dimension, while regions of EMIC generation could encompass multiple coherent sources.

At present writing, there has yet been no attempt to measure a typical size of coherent EMIC wave activity in the outer magnetosphere using multi-spacecraft observations of EMIC waves. Such works have, however, been made for the whistler-mode chorus. For instance, Santolík and Gurnett (2003) determined the characteristic dimension for the whistler-mode chorus wave activity using observations with the Cluster spacecraft. They selected time intervals of distinct chorus elements, and calculated the correlation coefficients for the six spacecraft pairs. It is found that the correlation systematically decreases with separation distance, approximately following a gaussian function with $1/e$-width of 100 km, although the resulting correlation values tend to be scattered. In a follow-up paper, Santolík et al. (2004) found that the above mentioned characteristic scale is, in fact, varying between 60 and 200 km for different data intervals inside the source region.

In this chapter, the spatial scale of EMIC wave source is estimated using the magnetic field observations of the Time History of Events and Macroscale Interactions during Substorms (THEMIS) spacecraft (Angelopoulos 2008), and using the essentially same correlation analysis as that of Santolík and Gurnett (2003) to the present data. The method assumes that the correlation should decrease with increasing separation between a pair of
satellites from which a characteristic size can be inferred. Whether this assumption is valid in the case of EMIC waves measured by the THEMIS satellite is also checked in this study. Another noticeable difference between two studies would be the spectral-time structure of EMIC waves versus that of whistler-mode waves. Chorus waves are characterized by a sequence of discrete elements appearing as intense short duration (typically 0.1 s) rising, or less often, falling tones in the frequency range from a few hundreds of Hz to several kHz (e.g. Omura et al. 1991; Sazhin and Hayakawa 1992). EMIC waves also appear as repeating discrete elements, but not necessarily spaced as regularly as whistler-mode waves (see, review by Fraser et al. (2006)). As a result, the time interval and fitting function most appropriate for the correlation analysis of EMIC waves may differ from those for whistler-mode waves. After presenting an example of observation in section 5.2, the correlation analysis is presented in section 5.3. Section 5.4 summarizes and briefly discusses the result of the study.

5.2 THEMIS Observations of EMIC Waves

The EMIC waves was obtained from the magnetic field data from the Fluxgate Magnetometer (FGM) onboard THEMIS that measures the background magnetic field and its low frequency fluctuations (up to 64 Hz) in the near-Earth space (Auster et al. 2008). The data were collected from April 2007 to December 2010. Such a wide spatial coverage was essential because EMIC waves are found to occur mostly in the outer magnetosphere at radius greater than 8 $R_E$ (Min et al. 2012). Such spatial coverage is useful for determining the spatial properties of EMIC waves in the outer magnetosphere.

Constructing the power-spectral density consists of following steps: (1) First, the observed magnetic field is transformed to magnetic field aligned coordinate system where z axis directs to the main magnetic field and x axis directs toward the Earth. (2) Then, the mean field is subtracted from the magnetic field to get waveforms. (3) Finally, the waveforms are fast-fourier transformed with Hamming window to the power-spectral den-
Additionally, the local gyrofrequencies of hydrogen and helium were also computed at each time (only hydrogen and helium band waves were used in the investigation). The power spectral density was counted as EMIC waves if the wave power of a spectral element is greater than 0.03 nT^2/Hz and the spectral elements lies between hydrogen and oxygen gyrofrequencies. It is also required that EMIC wave activity be detected by at least two spacecraft and last longer than 10 minutes. Typical duration of the events is between 30 minutes to 3 hours; in some cases they can sustain up to 6 hours.

![Spectrogram](image1.png)

**Figure 5.1** Two sets of simultaneously observed EMIC wave activities. Left column is an event on September 22, 2010 and right column on June 18, 2007. First two rows show EMIC wave power-spectral densities from each spacecraft on an equal logarithmic color scale. The white curves indicate hydrogen, helium and oxygen gyrofrequencies from the top curve to bottom. The third panel shows averaged power-spectral densities and the bottom one shows separation distance perpendicular and parallel to the local magnetic field. At the bottom of the last panel is the ephemeris information of the two spacecraft: X, Y and Z (R_E) in the GSM coordinate system.

Figure 5.1 shows two examples of the hydrogen band EMIC waves. The left column shows an event detected on September 22, 2010 and the right column, another on June 18, 2007. Top two panels show the power-spectral density from two spacecraft. In the first event observed by THEMIS A and E for about three hours, both spectra show three large consecutive patches nearly simultaneous with each other and look almost identical. In the
second event observed by THEMIS A and C for about the same duration, the bright spectral elements do not match well to each other in time. Since it is hard to compare two spectra with time, the frequency integrated power from the two spacecraft, as shown in the third panel, was used in the calculation.

The cross correlation of powers measured by a pair of spacecraft denoted by indices, \( i \) and \( j \), is then calculated from these frequency-averaged time profiles.

\[
 r_{ij} = \frac{\sum_k (P_{ik} - \bar{P}_i) \sum_k (P_{jk} - \bar{P}_j)}{\left[ \sum_k (P_{ik} - \bar{P}_i)^2 \right]^{1/2} \left[ \sum_k (P_{jk} - \bar{P}_j)^2 \right]^{1/2}}
\]  

(5.1)

where \( P \) denotes the frequency-averaged power spectral density and \( k \) is the sample index in the time series. The range for \( k \) is not fixed but determined for each event based on the duration of the EMIC wave activity as identified by visual inspection. Two correlation methods are used: the linear method known as Pearson’s product-moment correlation coefficient and the statistic method known as Spearman’s rank correlation coefficient (Press et al. 1992). For the first event, both linear and rank correlation coefficients are as high as 0.9 and for the second event, the average correlation coefficients between two observed signals are about 0.25. The bottom panel shows a plot of the distance between two spacecraft as function of time, both parallel and perpendicular to the local magnetic field (denoted as \( d_\parallel \) and \( d_\perp \), respectively in the figure). Since THEMIS spacecraft orbits are always close to the equator, the parallel separation always remains small. During the first event, THEMIS A and E were moving together within 0.22 \( R_E \), and the observed EMIC wave signals are almost identical with each other. During the second event, THEMIS A and C were separated by up to 2 \( R_E \), and the correlation comes out as low. This suggests that a statistical distribution of the correlation as a function of separation distance should yield information on the characteristic scale of the EMIC wave activity.
5.3 Correlation Analysis

The above correlation analysis was applied to all events and all possible pairs of the THEMIS spacecraft. Although both the rank and linear correlations were calculated, they were found to be similar to each other for almost all events, and only the result of the linear correlation analysis will be used in this section. The linear cross correlations, \( r_{ij} \), are then plotted versus their separation distances, \( d_{(\perp),ij} \) and a least-square fit of them to a model function is made. Note that Santolík and Gurnett (2003) adopted gaussian for the model function. In the present case, an exponential approximation was found to better agree to the data, and thus the model function was chosen in the form:

\[
 r_{(\perp)} = A \exp \left( -\frac{d_{(\perp)}}{D_{(\perp)}} \right). 
\]  

The maximum correlation, \( A \), may not be exactly unity for the reasons given in Santolík and Gurnett (2003). The goal of this fit is to determine the \( e \)-folding distance, \( D_{(\perp)} \), which is regarded as the characteristic dimension of coherent EMIC wave activity in this study. Note that \( r_{ij} \) is calculated by integrating the signals over a finite time interval for which \( d_{(\perp),ij} \) keeps changing with time. Nonetheless, \( d_{(\perp)} \) in the above expression is treated as a single value (as we represent by the median separation distance within the time interval), which may lead to a biased relation between \( r \) and \( d_{(\perp)} \) for long-duration events. The time interval for correlation is thus limited so that the difference of maximum and minimum separations within the time interval can always be less than 0.5 \( R_E \). By visual inspection, 124 and 148 events were found in PM sector (1200 < MLT < 1800) and AM sector (0600 < MLT < 1200), respectively.

Figure 5.2 shows the correlation versus spatial separation where no distinction is made for hydrogen and helium bands including all events in both PM and AM sectors. Upper two panels show the correlation coefficient versus total and perpendicular separations, respectively. In bottom panels, we plot the data for hydrogen and helium bands separately.
Figure 5.2 Correlation coefficients as functions of total and perpendicular separation distances. Gray curves represent the exponential function fit to all data points. White curves and shaded areas indicate the mean and one standard deviation of data points in each sub-range. The parameters of the fitting function are denoted in each panel. Top two panels show data including both emission bands and bottom two panels show hydrogen and helium band data separately.
The best fit function for all data points is drawn as a gray curve in each panel and the derived parameters of the function are denoted. The same analysis was also carried out for the parallel separation distance, but the inferred scale was much larger than the most available parallel separation distances due to the near-equatorial orbits of the THEMIS. No further attempt was made to determine the characteristic scale along the magnetic field. The correlation coefficient generally decreases with separation distance approximately following an exponential function rather than a gaussian, although the data points are scattered to large extent from the model function. To show how much the data points are scattered, the display convention of Santolík and Gurnett (2003) was followed to divide the whole distance range into several subranges and the mean and standard deviation of the correlation coefficients in each subrange are shown. In this case, the subrange is taken with $\Delta d = 0.5R_\text{E}$ and the mean is plotted as white solid curves. The upper and lower boundaries of shaded area correspond to one standard deviation in each subrange. The fits shown in this figure yield $1.2\ R_\text{E}$ for $D$ and $0.96\ R_\text{E}$ or $D_\perp$ when hydrogen and helium bands are not distinguished from each other. When two bands are distinguished, $D_\perp = (1.03 \pm 0.32)R_\text{E}$ and $(0.55 \pm 0.31)R_\text{E}$ are obtained for hydrogen and helium bands, respectively.

While the different sizes found for the hydrogen versus helium bands may worth further attention, it is noted that these emissions dominate in different regions in the magnetosphere; the hydrogen band emission peaks at $10 < L < 12$ in AM sector and the helium band, at $8 < L < 12$ in PM sector, respectively (Min et al. 2012). It is therefore necessary to somehow normalize the distances by relevant local parameters to remove the location dependence. As the first attempt, the separation distances were divided by ion gyroradius in the analysis. For this, the representative energy of ions involved with the EMIC wave generation should be known. We refer to the earlier result from AMPTE/CCE in which the bulk of the total measured ion energy density is found to be in the energy range: 10-100 keV of hydrogen ions in quiet times (Daglis et al. 1993). It should be noted that their result pertains to the relatively inner range compared to the region under current investi-
gation, and that the range 10–100 keV was used simply as a trial value for the purpose of presenting a scale.

Figure 5.3 Correlation coefficients as functions of perpendicular separation distances normalized by gyro-radius of 100 keV proton. The data for AM and PM sectors are separately analyzed with distinction of emission bands.

The correlations as functions of the normalized distances are shown in Figure 5.3. Here the data obtained from AM and PM sectors are separately shown because wave properties are found to vary significantly from AM to PM sectors (Min et al. 2012). Upper panels show that without the normalization, $D_\perp$ again varies from hydrogen to helium bands, and from AM to PM sectors. In the lower panels where the distances are normalized by gyroradius of 100 keV protons, $\rho_p$, the characteristic dimensions of sources in two emission bands, however, agree to each other within the uncertainty range. The difference between AM and PM events also reduces in the plots over the normalized distances. The normalized source dimension is found to be $D_\perp/\rho_p \approx 5.1–7.7$ for all data points. If 10 keV were used for the proton energy, the number would increase to 16–24. It thus appears that
EMIC waves can maintain coherence over multiple factors of gyroradius of 10–100 keV protons.

**Figure 5.4** Correlation coefficients as functions of perpendicular separation distances. The data for an inner region ($6 < L < 9$) of the magnetosphere and their model fit results are plotted with open circles and white curves, respectively; those for an outer region ($9 < L < 12$) are plotted as filled circles and black curves, respectively. Distance scales in upper two panels are in units of $R_E$ and those in the bottom panels are normalized by the gyroradius of 100 keV proton. The fit parameters are: $D_{\perp}/R_E = 0.39 \pm 0.28$ (PM), $0.34 \pm 0.29$ (AM) for $6 < L < 9$, and $0.80 \pm 0.37$ (PM), $0.89 \pm 0.40$ (AM) for $9 < L < 12$. In terms of the normalized distances, $D_{\perp}/\rho_p = 6.54 \pm 4.58$ (PM), $5.85 \pm 4.81$ (AM) for $6 < L < 9$, and $6.32 \pm 2.88$ (PM), $6.64 \pm 2.63$ (AM) for $9 < L < 12$.

Figure 5.4 shows the $L$-dependence of the correlation coefficients by separating the data into two groups: those measured in a relatively inner region ($6 \leq L \leq 9$) and the other region ($9 < L < 12$) of the magnetosphere with no distinction between hydrogen and helium bands. Data in the former group and their model fit results are plotted with open circles and white curves, respectively, and those in the latter group are plotted with filled circles and black curves, respectively. The data obtained from AM and PM sectors are again plotted.
separately. In top panels where the separation distances are given in real units without normalization, it is found that the characteristic scale for EMIC wave source increases with increasing $L$ value. There is also MLT dependence. On the other hand, the bottom panels which shows the correlations as functions of the separation distances normalized by the gyroradius of 100 keV protons show that the dependence of the inferred source dimension on both $L$, and that MLT has been drastically reduced. By this plot, it becomes obvious how the distance normalization works. In the inner (outer) region, the source size is smaller (larger) but the gyroradius is also smaller because of stronger (weaker) magnetic field strength. The uniformity from AM to PM sectors can also be understood in the same way because strong AM events tend to occur at higher $L$ shells and have larger source dimension compared with strong PM events (Min et al. 2012). When the model fit was repeated to all data points including both AM and PM events, $D_\perp/\rho_p = 6.28 \pm 1.57$ was found, namely, about 5–8 times larger than gyroradius of 100 keV protons.

As the second attempt, the separation distances were divided by the wavelength of EMIC waves. The wavelength, $\lambda$, is related to the phase velocity, $V_\phi$, as $\lambda = V_\phi/f$, and $V_\phi$ in this case depends on frequency, density, magnetic field and ion abundance even in the simplified linear theory (Summers and Thorne 2003). Since there is no information on the ion abundance among these parameters, an approximate expression for $V_\phi$ was sought as a first try. Recently, Chen et al. (2001) showed that growth of EMIC waves is most likely to occur near the middle of the EMIC bands in which case the Alfvén velocity, $V_A$, is a good approximation for the characteristic phase speed of EMIC waves (Mace et al. 2011). On this basis, the quantity, $V_A/f$, was adopted as a proxy for $\lambda$. $V_A$ could be calculated using the magnetic field measurements and electron density derived from spacecraft potential. The use of electron density for the ion density means that a pure proton-electron plasma is assumed. Note that the spacecraft potential measurement is not always available for all events, and the number of events shown in top panels of Figure 5.5 is much less than that of events in Figure 5.4. This reduced sample still maintains the
Figure 5.5 Correlation coefficients plotted in the same format as Figure 5.4. Only those events with spacecraft potential measurements are included, and the distances in the bottom panels are normalized by local wavelength of the EMIC waves. The fit parameters are: $D_⊥/R_E = 0.31 \pm 0.13$ (PM), $0.26 \pm 0.15$ (AM) for $6 < L < 9$, and $0.75 \pm 0.23$ (PM), $0.97 \pm 0.22$ (AM) for $9 < L < 12$. In terms of the normalized distances, $D_⊥/\lambda = 1.08 \pm 0.46$ (PM), $1.26 \pm 0.75$ (AM) for $6 < L < 9$, and $1.14 \pm 0.36$ (PM), $1.49 \pm 0.37$ (AM) for $9 < L < 12$. 
same trend as shown in Figure 5.4, namely, the source dimension inferred for a relatively inner region \((6 \leq L \leq 9)\) is smaller than the outer region \((9 < L \leq 12)\). Now the bottom panels show the correlations as functions of the separation distances normalized by the local wavelength, where the dependence of the inferred source dimension on both \(L\) and MLT is again drastically reduced, along with little difference between AM and PM events. Thus the ratio of EMIC wave source size to local wavelength also tends to be nearly uniform everywhere in the magnetosphere except the AM sector where the number of events in the inner region is rather small. By redoing the model fit to all data points from both the AM and PM sectors, \(D_{\perp}/\lambda = 1.28 \pm 0.22\) was found (roughly 1.0–1.5). Since this \(e\)-folding distance refers to radius, the source size corresponding to diameter is 2–3 times local wavelength proxy.

**Figure 5.6** Correlation coefficients plotted as functions of separation distances normalized by local EMIC wavelengths. The wavelengths are calculated for three different values of helium abundance with oxygen abundance set to zero. The fits are made to all data points from both the AM and PM sectors.

As the final attempt, the uncertainty due to the ion abundance that has not been fully considered in the above calculation was estimated. Spatial distribution of ion abundance is a key parameter in understanding physical properties of the magnetospheric EMIC waves (Horne and Thorne 1994), but remains as an observational challenge to date. As a means to check how much the fitting result depends on ion abundances, a plausible range of ion abundances was assumed to recalculate the wavelength. Once ion abundances are set,
one can proceed to calculate the phase speed of EMIC waves under the linear theory (e.g., equation (2) of Summers and Thorne (2003)) instead of relying on the above approximation, $V_\phi \approx V_A$. Figure 5.6 shows the results obtained with three different values of helium abundance. The oxygen abundance was still ignored because the current analysis did not resolve the oxygen band. The fit here is made to all data points from both the AM and PM sectors. The first case of no helium component differs from the above approximation in that observed wave frequency relative to local gyrofrequency is counted (rather than assumed to be in the middle of the emission band) in the calculation of the phase speed, which resulted in $D_\perp/\lambda = 1.55 \pm 0.26$. This result obtained using a full expression for the phase velocity but with no helium abundance is not necessarily more adequate than that from the approximate phase speed, $V_\phi \approx V_A$, because in reality helium band exists and the latter approximation applies to all emission bands (Mace et al. 2011). The other two cases where helium abundance exists by 5% and 15% resulted in $D_\perp/\lambda = 1.46 \pm 0.25$ and $1.32 \pm 0.27$, respectively. Of course, each individual result could be unrealistic because ion abundance will be nonuniform in space. There is, at least, a trend that the derived dimension systematically varies with the assumed helium abundance. Assuming that helium exists anywhere lying between 5% and 15%, the latter two results are combined to derive $1.05 \leq D_\perp/\lambda \leq 1.71$. The corresponding diameter of the source is again 2–3 times local wavelength, consistent with the range found in the above.

### 5.4 Discussion

In this chapter, relatively straightforward correlation analysis of EMIC wave powers measured by THEMIS spacecraft was used to determine the characteristic size of coherent EMIC wave activity in the direction perpendicular to local magnetic field at the magnetic equator. The resulting dimension was found to lie in a wide range from 1,500 km to 8,600 km as there was not only large scatter of data around the assumed correlation function but large variations of the result from AM to PM sectors as well as from hydrogen to he-
lium bands. How and why the source dimension varies with MLT and frequency was not immediately clear, except a tendency of the source dimension increasing with $L$. Two possibilities that the source dimension may be affected by physical quantities were explored. As a result, it is found that the characteristic dimensions normalized to either local gyrofrequency or wavelength indeed converge to narrower ranges and are quite uniform over frequency and location. The universality of the normalized size itself may not necessarily be a proof of a certain underlying physics but could, at least, motivate possible interpretations given below.

The first result that source dimension is a constant multiple of gyroradius of protons everywhere in the magnetosphere might imply that gyroradius should be a dominant factor in EMIC wave source size. If it does, and if, by analogy, the size of whistler-mode chorus wave source is also determined by electron gyroradius, one can predict that the size of whistler-mode wave source be smaller than that of EMIC waves by the square root of the electron-to-proton mass ratio. It should also be assumed that EMIC (whistler) waves are emitted by protons (electrons) of similar energy at regions with comparable field strength. In this case, the 1,500–8,600 km for EMIC wave source would be translated to 35–200 km for whistler-mode waves, which actually agrees to the observed range of whistler-mode chorus wave activity, 60–200 km (Santolík et al. 2004). An issue is, however, that the gyroradii were calculated for a specific energy, 100 keV, without evidence. It is not sure whether EMIC waves are predominantly generated by protons in a specific energy range, or what is the energy range, if so. It is rather likely that ions in the PM sector will be more energetic than in the AM sector considering the energy dependent drift of ions from the magnetotail. In this case, the source dimension in the PM sector source will be systematically larger than that in the AM sector, rather than being as uniform as found in this study. With no further information on such proton energy, it may be suggested that the factor of 5–24 for the source size to 10–100 keV proton gyroradius can, at least, be a useful guide. For instance, one may take this coherent scale relative to proton gyroradius for various energies in the
design consideration of any models for EMIC wave emission process.

The second result obtained with local EMIC wavelength as a normalization scale is free from the unknown particle energy, but subject to the unknown heavy ion abundance. For this, the wavelength was estimated either by adopting the approximation $V_\phi \approx V_A$ (Mace et al. 2011) or by calculating $V_\phi$ under the linear theory with assumed helium abundance. Both attempts yielded a similar result that EMIC waves maintain coherency over 2–3 wavelengths. In both calculations, $V_A/f$ is the central parameter for $\lambda$ and an additional minor correction comes from the heavy ion abundance. Since $V_A$ is proportional to magnetic field magnitude and the EMIC wave frequency is also related to the local gyrofrequency, the dependence of $V_A/f$ on magnetic field greatly weakens. This leaves the dependence of $V_A$ on ambient density as an important factor for the spatial variation of local EMIC wavelength. Min et al. (2012) argued that the preferred emission bands in the AM and PM sectors relate to the background plasma density based on the physical model by Horne and Thorne (1994). It is interesting to point out that the density variation also affects the local wavelength and the coherent dimension of EMIC waves, and that their ratio turns out to be quite uniform in space. As a comparison, Santolík and Gurnett (2003) also claimed that their result for the size of chorus wave activity is in the same order of wavelength of the whistler-mode waves.

Finally, it is noted that the present results pertain to the relatively high $L$ shells (say, beyond the geosynchronous orbit). Why the EMIC waves in this region lose coherency in 2–3 wavelengths and whether this property is shared by the EMIC waves in the inner magnetosphere are yet to be explored. Also recall that the correlations of EMIC sources are more closely fit by the exponential functions while Santolík and Gurnett (2003) made gaussian fits to the whistler sources. The resulting coherence dimension does not change much with either the gaussian or the exponential fitting. However, the pattern that the correlation decreases with separation distance differs and whether the difference originates from the nature of these waves could be an important issue.
CHAPTER 6

PLASMA MASS VARIATION ALONG THE FIELD LINE DERIVED FROM TOROIDAL MODE ALFVÉN WAVE FREQUENCY: EQUATORIAL MASS DENSITY DISTRIBUTION

An inversion technique for estimating the properties of the magnetospheric plasma from the harmonic frequencies of the toroidal standing Alfvén waves has been used to derive the global equatorial mass density covering radial distances from 4 to 9 Earth radii ($R_E$), within the local time sector spanning from 0300 to 1900 hours. The toroidal Alfvén waves were detected with magnetometers onboard the Active Magnetospheric Particle Tracer Explorers (AMPTE)/Charge Composition Explorer (CCE) for almost 5 years from August 1984 to January 1989 and Geostationary Operational Environmental Satellites (GOES) (10, 11 and 12) for 2 years from 2007 to 2008, both of which were operational during the solar minimum. The derived equatorial mass density, $\rho_{eq}$, at geosynchronous orbit (GEO) monotonically increases at low magnetic local time (MLT) toward a local maximum at about 1700 MLT for CCE and 2100 MLT for GOES, and then decreases at larger MLT. At other radial distance, $\rho_{eq}$ has the same local time variation as that of GEO, while the magnitude logarithmically falls with increasing $L$ value. The logarithmic variation can be represented by the second order polynomial within 10% deviation in the daytime sector. The investigation of $Dst$ and $Kp$ dependences shows that the median value of $\rho_{eq}$ varies little in the daytime sector during moderately disturbed times, which agrees with previous studies. $\rho_{eq}$ calculated from the $F_{10.7}$ dependent empirical model shows good agreement with that of CCE but overestimates that of GOES probably due to the extreme solar cycle minimum in years 2007–2008. The current study may suggest the possibility of the global monitoring of $\rho_{eq}$ using, e.g., the toroidal waves measured at GEO, which warrants further investigation.
6.1 Introduction

Toroidal mode standing Alfvén waves (referred to as toroidal waves hereafter) can be used to infer the physical properties of the magnetospheric plasma that sustains the oscillation. The inversion technique, termed “normal mode magnetospheric seismology,” has been used by numerous authors to derive the mass variation along the field line from space (e.g., review and references in Denton (2006), and recent studies by Denton et al. (2009), Takahashi and Denton (2007), Takahashi et al. (2008) and Takahashi et al. (2010)) to the ground (e.g., review in Waters et al. (2006) and references therein). Despite the limited number of toroidal frequencies observable from instruments, the strong dependence of the inversion on the accuracy of the measured frequencies and the simplified magnetic field models that made the technique difficult to deploy (Denton et al. 2001, 2004a; Takahashi et al. 2006), the unique advantage of remotely diagnosing the mass variation along the field line has led to several decades worth of improvement.

Summarized below is only a few results from recent studies in the context of space observation. First, a more general polynomial density model revealed small scale structures in the mass variation along the field line, which the simple power law variation used traditionally could not discern otherwise. Takahashi et al. (2004), Takahashi et al. (2006) and Denton et al. (2006) used the toroidal frequencies observed by the Combined Release and Radiation Effects Satellite (CRRES) to show that there could exist a local density enhancement at the magnetic equator and the enhancement increases with increasing radial distance. Similarly, Takahashi and Denton (2007) used the toroidal frequencies observed by Geostationary Operational Environmental Satellites (GOES) and found that the local enhancement is also dependent on the local time of observation. The power law model $\rho = \rho_{eq}(LR_E/R)^\alpha$ used in previous studies (e.g., Cummings et al. 1969) fails to match the local maximum. Several physical mechanisms have been suggested for this local enhancement, including an enhanced potential well at the higher radial distances due to the centrifugal force (Denton et al. 2006), the formation of a localized partial ring current related
to the heavy ion (or oxygen) torus (Roberts et al. 1987; Takahashi and Denton 2007), or an anisotropic distribution (through local adiabatic processes, global transportation or perpendicular heating by ion cyclotron waves) of the ions, or any combination thereof (Takahashi and Denton 2007).

The second point highlighted is the fact that the derived mass density imposes a constraint on the ion composition which is otherwise difficult to measure due mainly to the limited instrument capability at low thermal speeds. Takahashi et al. (2006) investigated the average ion mass, a ratio of the mass density to the electron number density, to narrow down the bounds of the ion composition and Takahashi et al. (2008) inferred the dominance of heavy ions in the plasma trough between the plasmasphere and drainage plume through a similar technique. Berube et al. (2005), on the other hand, used ground-based ULF wave diagnostics to show the enhancement of the heavy ion concentrations and the presence of a heavy ion torus during disturbed times.

As a final note, Takahashi et al. (2010) recently reported the long-term mass density modulation by $F_{10.7}$ and sunspot number as well as short-term variation with respect to the geomagnetic indices. They showed that $F_{10.7}$ and 27-day averaged mass density are highly correlated, implying that the solar UV/EUV control of ion production at ionospheric heights is strongly reflected in mass density variations, whereas the correlation with the geomagnetic indices are somewhat weaker, although the magnetospheric convection system still controls the transport.

Despite a few decades of accumulation of space borne observations, our current understanding of the mass variation on a global scale is yet to be quantified. To this end, the current study attempts to investigate the mass variation on a larger spatial scale than in previous studies using a large number of samples of the toroidal frequencies observed by the magnetometer on board Active Magnetospheric Particle Tracer Explorers/Charge Composition Explorer (CCE) spacecraft (referred to as CCE hereafter). Along similar lines, Takahashi et al. (2002) used the fundamental mode frequency, $f_{T1}$, determined using
the energetic particle as well as magnetometer data, but mainly focused on the frequency
identification technique and indirect derivation of the mass density using the Global Core
Plasma Model (GCPM) (Gallagher et al. 2000). Unlike their study, this study only uses
the magnetometer data and find the third harmonic frequency, \( f_{T3} \), to derive the equatorial
mass density assuming a power law variation, following similar analyses by Takahashi and
Denton (2007) and Takahashi et al. (2010). This study also uses the GOES magnetometer
data to enhance the dataset and for the added purpose of cross-comparison and validation
of the analysis.

The objective of the present study is twofold. First, this study wishes to construct a
equatorial mass density distribution derived solely from the toroidal waves and demonstrate
the potential of the magnetospheric seismology technique. Second, this study examines the
idea of the global monitoring of the equatorial mass density using only limited resources
(which can be either direct measurements, those derived from the toroidal waves or model
prediction) by establishing functional relationship between the governing parameters, the
result of which can be used in a number of related applications including electron/ion cy-
clootron wave modeling.

The present study extensively uses the results of Takahashi et al. (2010) for vali-
dation of the present analysis procedure and inter-comparison of the present results. After
a short description of the inversion technique in section 6.2, section 6.3 demonstrates the
identification process of the toroidal waves and the harmonics with a sample orbit. Sec-
tions 6.4 and 6.5 show the statistical results in detail. Sections 6.6 and 6.7 discuss and
summarize the results of this study, respectively.

### 6.2 Normal Mode Magnetospheric Seismology

The frequencies of the toroidal standing Alfvén waves depend on the field line distribution
of mass, analogous to the oscillation of a string whose ends are fixed. Under the assumption
of a realistic magnetic field model and axisymmetry of the equilibrium and wave perturba-
tion in the inner magnetosphere (Denton et al. 2001), the MHD shear Alfvén waves can be described by the wave equation (Singer et al. 1997) (refer to corrections in Denton et al. (2004a))

\[
\frac{\partial^2}{\partial s^2} \xi' + \frac{1}{h_\alpha^2 B_0} \frac{\partial}{\partial s} (h_\alpha^2 B_0) \frac{\partial}{\partial s} \xi' + \frac{\omega^2}{V_A^2} \xi' = 0, \tag{6.1}
\]

where \(\xi'\) is the linear displacement in the direction of the oscillation \(\alpha\) (assumed to be in the azimuthal direction for the toroidal mode at the magnetic equator) divided by scale factor \(h_\alpha\) (proportional to the distance to an adjacent field line in the direction of oscillation), \(V_A\) is the Alfvén speed, \(B_0\) is the magnitude of the equilibrium magnetic field, and \(s\) is the distance along the field line. This wave equation can be solved in a non-dipolar magnetic field geometry to obtain harmonic frequencies, \(\omega\). This study assumes that the ionosphere is a perfect conductor located at radial distance 1.015 \(R_E\) (\(\sim 100\) km in altitude) (Denton et al. 2006). The Tsyganenko 89 (T89) (Tsyganenko 1987, 1989) magnetic field model was used because this model only requires the Kp index as an input parameter. The dipole magnetic field was also used in parallel for the purpose of verification although the result is not shown in this study.

In order to derive the equatorial mass distribution from the observed toroidal harmonics, this study assumes the power law mass density model (Denton et al. 2001, 2004a)

\[
\rho = \rho_{eq} \left( \frac{L_R}{R} \right)^\alpha, \tag{6.2}
\]

where \(L\) is the farthest radial distance on the field line, \(R\) is the distance at any given point on the field line and \(\alpha\) is the power law index. Given this density model, the free parameters, \(\rho_{eq}\) and \(\alpha\) can be found by minimizing the difference between the observed toroidal frequencies and the solutions of Eq. (6.1) in a least-squares sense (Denton et al. 2001, 2004a).

The power law index, \(\alpha\) is usually assumed to vary between 0 and 6 depending on
the location (Cummings et al. 1969). Theoretical studies suggest for $L < 6$ (plasmasphere), $\alpha = 0–1$ based on diffusive equilibrium, and for $L > 6$ (plasmatrough) $\alpha \sim 3–4$ based on a collision distribution (Lemaire and Gringauz 1998). As discussed by Takahashi et al. (2006), Eq. (6.2) does not represent the mass variation along the field line for $L > 6$ because of the local maximum around the equator. Nevertheless, $\alpha \approx 1$ represents the mass density near the magnetic equator better than $\alpha \approx 4$ (Takahashi et al. 2004; Denton et al. 2006). Following the statistical study by Takahashi et al. (2010), this study chose $\alpha = 1$ and calculated $\rho_{eq}$ to derive the global equatorial mass density distribution.

### 6.3 Data Set and Analysis

#### 6.3.1 CCE

CCE acquired scientific data from August 1984 to January 1989. It had a $1.2 \times 8.8 \, R_E$ elliptical orbit with an inclination of 5°. The spacecraft was spin stabilized with a spin period of 5.9 s, and a spin axis that remained within 15° of the Sun-Earth line. As shown in Figure 6.1a, CCE covered the near-equatorial plane over a wide range of $L$ values although the number of samples are biased towards the prenoon sector. The large number of samples in the 8–9 $R_E$ bin is evident due to the spacecraft apogee location.

The fluxgate magnetometer (Potemra et al. 1985) on board CCE acquired data at a sampling rate of 8 vectors s$^{-1}$, with one component along the satellite spin axis and two components in the spin plane. The vector samples were rotated from the spacecraft coordinates into the geocentric solar ecliptic (GSE) coordinate system and averaged down to a 5.9 s resolution to match the spin period of the spacecraft (Takahashi and Strangeway 1990; Takahashi et al. 2002) (http://sd-www.jhuapl.edu/AMPTE/MAG/).

The spin-averaged field data provided in a daily format were reorganized into orbital format (the data were sliced at each perigee pass). Since the toroidal frequencies of Eq. (6.1) are found to be proportional to the magnitude of the magnetic field ($f_n \sim B / \sqrt{\rho}$) in the Wentzel-Kramers-Brillouin (WKB) approximation, the frequency variation between
Figure 6.1 Spatial coverage of (a) CCE and (b) GOES during the entire operation. The bin size is $1 \, R_E$ in radial distance (CCE only) and 1 hour in local time. The number of samples at each bin is normalized to the maximum number of samples.
inbound and outbound orbits will be roughly symmetric with respect to the apogee and thus organizing the data in an orbital format makes it easier to extract the toroidal frequencies. During this process, data for $L < 4$ were discarded due to the sampling rate of the magnetometer (the second or third harmonic at $L = 4$ is at about the Nyquest frequency) and the increased orbital speed of the spacecraft.

The reorganized data were then resampled with a sampling frequency, $f_{\text{sample}}(t)$ that varied with time. The resampling process is done as follows. Letting $t_{\text{start}}$ and $t_{\text{end}}$ be the start and end of each orbital data, one first sets a time axis, $t_i$, such that $0 \leq t_i < t_{\text{end}} - t_{\text{start}}$ and $\Delta t = t_{i+1} - t_i$ (sliding stride) is 5 minutes. At each time segment, $t_i$, one resamples the entire data at a sampling rate given by $f_{\text{sample}}(t_i)$ and then rotate the resampled data into mean-field-aligned (MFA) coordinates where $e_z$ is along the mean field defined by the running averages of the resampled data, $e_x$ is radially outward in the plane containing the mean field and the spacecraft position vector, and $e_y$ is eastward completing the right hand rule. Practically, one only needs to resample window-length wide (plus extra padding for the running average) data segment centered at $t_i$. Finally, the mean field is subtracted from the rotated field to get waveform.

Figure 6.2a shows the azimuthal component of fast-fourier transformed (FFT) power spectral density with $f_{\text{sample}}(t)$ fixed to $1/6 \text{ s}^{-1}$. A 128 point hamming window was multiplied with the waveform before fourier transform. Five harmonic frequencies can readily be inferred, even though the spectral power at the fourth frequency, $f_{T4}$, is faint. Starting at 80 mHz at the beginning of the spectrogram, the third harmonic frequency, $f_{T3}$, which is most prominent, decreases to about 25 mHz at apogee, shown near the center of the spectrogram.

Due to the large variation in the frequencies as a function of $L$ value, it is difficult, especially for the automatic frequency identification, to extract the toroidal frequencies consistently across the $L$ values. For example in Figure 6.2a, the frequency separation at apogee is too small to resolve discrete harmonics (even though this example shows rela-
Figure 6.2  Toroidal Alfvén waves observed on 15th September 1984. Panels (a) and (b) show FFT power spectra and MEM power spectra, and panel (c) shows the identified toroidal frequencies superimposed on top of the FFT spectra. The red dots shown on panel (c) indicate the third harmonic frequencies. The orbital median power has been subtracted at each time segment to equalize the intensity with respect to the frequency domain. The normalized power less than $10^{-2}$ is masked. The dashed curves at panels (a) and (c) represent $f_{\text{sample}}(t)$, and the dotted curves in panel (b) represent constant frequency lines.
tively clean harmonic structure which is not always the case). For the purposes of automatic
detection of the toroidal waves and frequencies, this study used the power spectra obtained
using the maximum entropy method (MEM) (Ulrich and Bishop 1975; Press et al. 1992).
This technique was used in Takahashi et al. (2010) and their related previous papers, and
proven to be more practical than previous methods in that: (1) the spectra is given in an an-
alytical form, and (2) the spectral peaks are sharply defined. In order for the MEM method
to discriminate between the closely spaced spectral peaks like those at the apogee in Figure
6.2a, a higher MEM order is preferable. The higher order, however, results in additional
peaks that may not necessary be real. To have the automatic detection work reliably, the
automatic detection program scaled the Nyquest frequency by choosing a simple parabolic
function for $f_{\text{sample}}$, given by $1/f_{\text{sample}}(t) = 12 + (6 - 12) \times (2t/(t_{\text{end}} - t_{\text{start}}) - 1)^2$. The
maximum and minimum sampling frequencies were set to $1/6$ and $1/12 \text{ s}^{-1}$, respectively.
$f_{\text{sample}}(t_i)/2$, being the Nyquest frequency at each time segment $t_i$, is shown as the dashed
line in Figure 6.2a. With MEM order 15, Figure 6.2b shows the azimuthal component of
the MEM power spectra with window lengths of 128 points. One can immediately notice
that five harmonic frequencies are more easily separable because of the scaling. The dotted
curves represent constant frequency lines which are not straight because of the varying
Nyquest frequency. The scaling function, $f_{\text{sample}}$, based on the WKB approximation was
tried but the result was not satisfactory. The reason may be attributed to the steep variation
of the magnetic field magnitude, the limited data sampling rate and intermediate data cor-
rupption. The simple parabolic function for $f_{\text{sample}}$ that was determined by examining the
pattern of the frequency variations, however, turned out to be a reasonable choice.

After several experiments, it was found that the third harmonic frequencies, $f_{T3}$,
were most common in the data set. Similar to Takahashi and Denton (2007) and Taka-
hashi et al. (2010), this study selected the $f_{T3}$ samples to which all other identified fre-
quencies were normalized. The selection of $f_{T3}$ is different from Takahashi et al. (2002)
who used the fundamental frequency, $f_{T1}$, identified mainly from the energetic particle flux
anisotropy (proxy of transverse electric field) obtained from the Medium-Energy Particle Analyzer (MEPA) data.

For the automatic detection, this study applied the same constraints as Takahashi et al. (2010) to distinguish the frequencies at real peaks due to noise. That is, the frequency, $f_{\text{peak}}$, at the peak of $y$ component of the spectral power $P_y$ as a toroidal mode spectral peak was counted only if $P_y > P_z$ and $P_y > 3P_x$, and $\Delta f_{\text{peak}} \leq 3$ mHz, where $\Delta f$ is the full width at half maximum. After the automatic algorithm filtered out the frequency at the toroidal mode spectral peak, the $f_{T3}$'s were then manually selected by visually comparing the filtered toroidal frequencies to the harmonic structure in the power spectra shown in Figure 6.2a and b. Shown in Figure 6.2c are the automatically detected $f_{\text{peak}}$'s among which the manually selected $f_{T3}$'s are colored in red. As is clear from the figure, the automatic algorithm was able to detect all the frequency structure in this sample.

The aforementioned automatic procedure with a window length of 200 points for comparison was also followed. The resulting $f_{\text{peak}}$'s from both results were almost identical. Both results were used for manually choosing $f_{T3}$ because (1) one result can be cross checked by another and (2) there are often cases where superimposing two results seamlessly connect the continuous $f_{T3}$ line as well as other $f_{\text{peak}}$ lines by filling each other’s missing gaps. In the above example, these peaks were identical because the spectral peaks were well defined.

### 6.3.2 GOES

Alongside the CCE data, GOES vector magnetic field samples from the GOES 10, 11, and 12 magnetometer data (Singer et al. 1996) were also used. The primary purpose for using this data set is for (1) validation of the harmonic identification process by comparing the GOES statistics of this study with those of Takahashi and Denton (2007) and Takahashi et al. (2010), and for (2) cross-checking the derived equatorial density from the CCE data set at the geosynchronous orbit, again for verification purpose. Since both missions were
operational during solar minimum (Figure 6.3), the GOES data set should be a suitable choice for this purpose.

Figure 6.3 Solar and geomagnetic activity during the CCE and GOES mission periods. One day-averaged 1-hour OMNI data were used to produce the plot. The top panel shows Kp and bottom shows $F_{10.7}$ (red), sunspot number (gray) and $Dst$ index (black). The shaded boxes indicate the periods that data were acquired.

The vector magnetic field data were sampled by the fluxgate magnetometer onboard these three satellites for about 2 years during 2007 and 2008. Figure 6.1b shows the spatial coverage of GOES 10, 11 and 12 as a function of local time. The local time is binned in steps of one hour and the number of samples at each bin and each spacecraft is normalized to the maximum number of samples of all bins and all spacecraft. Except that GOES 10 sampled the magnetic field less frequently than the other two spacecraft, the magnetic field samples are evenly distributed in MLT ($\leq 5\%$ variation).

The data were provided with 0.512 s resolution in the Geocentric Solar Magnetospheric (GSM) coordinate system. The each data set was combined and sliced at 0000 MLT to have the magnetic field samples organized in magnetic local time, and processed
similarly to the CCE data. The $f_{\text{sample}}$ was fixed to $1/8\ s^{-1}$ in this case because the radial
distance of the satellites does not change much. The GOES data usually have very well
defined spectral peaks and thus the window of length 256 points for both FFT and MEM
spectra was used.

### 6.3.3 Solar and Geomagnetic Activities

Following the results obtained by Takahashi et al. (2010), the underlying mass density is
highly correlated with the long-term solar cycle variation and somewhat weakly correlated
with the short-term geomagnetic activity. In contrast to their study though, the data set in
this study were obtained during solar minimum throughout most of the mission period as
shown in Figure 6.3. Most of the time that CCE was operational was during solar minimum;
near the end of its operation, the solar cycle was in the rising phase of solar activity. This
study excluded the CCE data after February 1988 for the statistical analysis to focus on
the mass density variation during the solar cycle minimum. The GOES satellites were also
operational during the solar minimum, as indicated by the shaded area on the right half of
the figure. Comparing the geomagnetic and solar activity side by side, the period of the
GOES operation was quieter than that of the CCE operation: $F_{10.7}$, $Dst$ and the sunspot
numbers were more stable during the GOES operation.

### Table 6.1 Fitting Coefficients of Logarithmic $L$ Dependence

<table>
<thead>
<tr>
<th>MLT</th>
<th>$p_1^a$</th>
<th>$p_2$</th>
<th>$p_3$</th>
<th>$\Delta^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0600</td>
<td>0.0328</td>
<td>-0.6995</td>
<td>3.8379</td>
<td>1.0388</td>
</tr>
<tr>
<td>0900</td>
<td>0.0193</td>
<td>-0.4746</td>
<td>3.0459</td>
<td>1.0767</td>
</tr>
<tr>
<td>1200</td>
<td>0.0111</td>
<td>-0.3749</td>
<td>2.8651</td>
<td>1.0259</td>
</tr>
<tr>
<td>1500</td>
<td>0.0103</td>
<td>-0.3543</td>
<td>2.8446</td>
<td>1.0390</td>
</tr>
</tbody>
</table>

$^a$The polynomial model is given by $\log_{10}\rho_{\text{eq,fit}}(L) = p_1L^2 + p_2L + p_3$.

$^b$ $\rho_{\text{eq,fit}}\times1/\Delta$ contains at least 50% of the predictions of future observations at $L$. 
Figure 6.4 The distribution of the CCE $f_{T3}$ samples within the range $L = 4–10$. The dotted concentric circles are drawn at every $2 \, R_E$ and the dashed circle is at $6.6 \, R_E$. The gray area at midnight indicates the region of low detection rate of $f_{T3}$. 
6.4 Statistical Characteristics of $f_{T3}$

Figure 6.4 shows the radial and local time distribution of the identified $f_{T3}$ samples from the CCE data set. As in Takahashi et al. (2002), the distribution of the $f_{T3}$ samples in the data set is biased towards the dawn and pre-noon sectors and is bound within $L \sim 10$. Unlike their distribution of the $f_{T1}$ samples, a substantial number of the measurements of $f_{T3}$ in the present data set are located within geosynchronous orbit (GEO), but very few are found between 1800 and 0300 MLT. In fact, the toroidal waves were rarely found around midnight in the present data set (shaded area). This may be one weakness of using the magnetic field data in isolation (Denton et al. 2001, 2004a).

It is important to evaluate the detection rate of $f_{T3}$ because the intensity of toroidal waves depends on the local time and the $L$ values (Takahashi et al. 2002, 2010). The local time was divided into 24 equally spaced, non-overlapping bins and the $L$ shells were divided into three equally spaced, non-overlapping bins: $4 < L < 6$, $6 < L < 8$ and $8 < L < 10$. At each bin, the ratio of the number of the $f_{T3}$ samples to the available magnetic field data segments was calculated.

Figure 6.5a shows the $f_{T3}$ detection rate for the data set from the three GOES satellites. The three curves almost coincide each other, indicating the consistent frequency detection among three satellites. The subtle difference can be attributed to the discrete GOES satellite location in magnetic latitude (MLAT) (Takahashi and Anderson 1992; Takahashi et al. 2010). Figure 6.5b shows the MLAT histogram of three GOES satellites. MLAT was binned in steps of $2^\circ$ with no overlap. The MLAT distribution of GOES 11 makes a sharp peak at $5^\circ$, while that of GOES 10 and 12 spans from 9 to $11^\circ$ and from 7 and $13^\circ$, respectively. Considering that the toroidal mode has a $B_y$ node at the magnetic equator and its amplitude increases monotonically towards an antinode located at MLAT $> 10^\circ$ (Cummings et al. 1969), the order of the detection rates, GOES 10 $\geq$ GOES 12 $> \text{GOES 11}$, may be explained by their latitudinal location. Compared to Figure 5b of Takahashi et al. (2010), the result in this study reproduces all the essential features of their analysis.
Figure 6.5  $f_{T3}$ detection rate as a function of the local time calculated from the GOES (a) and CCE (c) magnetometer data and $f_{T3}$ samples, and (b) the GOES magnetic latitude histogram. (a and b) The rates and the magnetic latitude histogram for GOES 10, 11 and 12 satellites are colored in black, red and blue, respectively. (c) The rates for $4 < L < 6$, $6 < L < 8$ and $8 < L < 10$ bins are colored in black, red and blue, respectively. The local times where the detection rate is roughly below 10% are shaded.
Namely, the detection rate is high in the prenoon sector, exceeds 10% between 0300 and 1800 MLT and tapers off towards midnight. These authors suspected that the bias of the detection rate toward the morning side is probably related to the upstream ULF waves in the foreshock region and the orientation of the interplanetary magnetic field (IMF) which statistically follows Parker’s spiral. Another likely cause of the bias is a region of high-density plasma at the postnoon sector, which lowers not only the toroidal frequencies but also the interval of the two adjacent frequencies. In fact, there were quite a few occurrences of the tight harmonic structure in the data set. The MEM spectra with the current settings did not separate them clearly, nor did visual inspection. These spectra were excluded to avoid false identification of $f_{T3}$.

Figure 6.5c shows the $f_{T3}$ detection rate for the CCE data set. The detection rate for $L > 6$ is very similar to that of the GOES data set and that for $L < 6$ is slightly lower in magnitude, but has essentially the same variation with local time. The fast transition of the spacecraft in the inner magnetosphere can cause broadening of spectral peaks of the processed spectra and these peaks do not satisfy the $\Delta f$ constraint. Compared to Takahashi et al. (2002), the number of $f_{T3}$ samples at the nighttime sector is much lower than the number of their $f_{T1}$ samples. As mentioned earlier, the processing issue and use of the magnetometer data in isolation rather than the existence of the toroidal waves could result in the lower detection rate. The region where the detection rate for CCE is less than roughly 10% is indicated by a shaded box and the result in this region may not be statistically significant in this study.

As an example, Figure 6.6 shows samples of the identified frequencies and the normalized frequencies to $f_{T3}$ as a function of local time. For GOES (Figure 6.6a), a quarter of GOES 10 samples were used, and for CCE (Figure 6.6b), samples from $6 < L < 7$ for the first year were used to produce the figure. In the first panel of Figure 6.6a, the data points are mixed and scattered although the lower three harmonic frequencies are separable. When normalized to $f_{T3}$ in the second panel of Figure 6.6a, up to six harmonic frequencies are
Figure 6.6 The automatically detected toroidal frequencies and the manually chosen $f_{T3}$'s as a function of local time. A quarter of the GOES 10 samples (a) and the first year of the CCE samples within $6 < L < 7$ (b) were used to produce the plots. The first and second panels of each group show the unnormalized and normalized toroidal frequencies. The $f_{T3}$ samples are indicated by red dots.
cleanly organized and the distribution is sharp around each toroidal frequency line. This result can be compared to Figure 5 of Takahashi and Denton (2007). The CCE samples in Figure 6.6b have a more disordered distribution for both unnormalized and normalized frequencies. In particular, there are quite a few data points between the regularly spaced frequency lines in the normalized frequencies. There may be two facts that come into play: noise involved in the spectral peaks and overextended $L$ bin size. For the latter case, the $L$ bin size was reduced by half and compared the result (not shown) to Figure 6.6b and there was a slight improvement. Given that, the false identification of noise as the toroidal frequencies and/or the uncertainty in determining the location of the spectral peaks may contribute to the dispersion to a larger degree than those of the GOES samples. Although the improvement of the latter can be difficult, the false identification may be significantly reduced by visual inspection. In the current data processing, only the $f_{T3}$ samples were thoroughly checked.

Despite all the technical difficulties, the toroidal frequencies of the CCE data set are clearly identifiable so the majority of the $f_{T3}$ samples are probably correctly identified as the third harmonic toroidal frequency. As Denton et al. (2001, 2004a) point out, using the accurate toroidal frequencies is the most crucial factor for the reliable mass density derivation. The statistical results in this section may indicate that the detection process of the toroidal waves solely from magnetometer data can be applied to the data set observed over a wide range of $L$ shells.

The present study only uses the $f_{T3}$ samples to investigate the equatorial mass density distribution. The $f_{\text{peaks}}$ samples can, however, give us the field line variation as well, which is under investigation currently.

**6.5 Equatorial Mass Density, $\rho_{\text{eq}}$, During Solar Minimum**

Using the detected $f_{T3}$ samples, the equatorial mass density was calculated using the power law model with the power law index $\alpha$ set to 1 (Takahashi et al. 2010). Both dipole and
To investigate the statistical characteristics of the equatorial mass density, $\rho_{eq}$, and $f_{T3}$, the median values and the interquartile ranges (IQR, the range between the first quartile and the third quartile) were calculated at each local time and $L$ bin. The choice of the median value and IQR is based on the study of Takahashi et al. (2010) and for direct comparison. Throughout the analysis, the local time was binned in steps of two hours from 1 to 24 hours with one hour overlap, and the $L$ shells were binned in steps of $\Delta L = 1$ with $L$
varying from 4 to 9 with no overlap. Additionally, a GEO bin at each MLT bin was defined such that the center of the GEO bin is chosen to be the median value of $L$ of the three GOES satellites at the MLT bin. As shown in Figure 6.7, the $L$ values of GOES range between 6.8 and 7.1.

### 6.5.1 Local Time Variation

Figure 6.8a shows the variation of $f_{T3}$ (top), $\rho_{eq}$ (middle) and the number of samples (bottom) as a function of local time at GEO. The median values and IQR are indicated by the open circles and the error bars. The number of the samples for CCE was more than 10 for most of the local times. The local times at which the number of samples is $\sim 10$ could be from only one or two orbital passes during which the solar and geomagnetic conditions change little. Due to the statistical significance, this section only focuses on the daytime sector, i.e., 0300–1800 MLT.

The CCE $f_{T3}$ median value, starting from about 40 mHz at 0300 MLT, decreases monotonically down to $< 25$ mHz towards 1800 MLT at which it makes a local minimum. The similar study based on more nightside samples by Takahashi et al. (2002) also showed the frequency local minimum approximately at the similar local time. Additionally, the result therein also showed the increasing toroidal frequency towards midnight after the local minimum. In comparison, the GOES $f_{T3}$ median value has the similar behavior to that of CCE except for the slightly larger magnitude and the local time of the local minimum.

What caused the discrepancy in the local minimum MLT is unclear at the time of writing, but it is only suspected that the small number of samples of the CCE data set might have caused the discrepancy.

The derived $\rho_{eq}$ for both data sets varies inversely to $f_{T3}$. That is, the median value monotonically increases starting below 3 amu/cm$^3$ at 0300 MLT towards the local time at which $\rho_{eq}$ has its maximum value up to 10 amu/cm$^3$. Since the field line geometries are more or less symmetric in local time at GEO, one can intuitively expect this inverse
Figure 6.8  Median values of the $f_{T3}$ samples (top row) and the equatorial mass density, $\rho_{eq}$, derived from the $f_{T3}$ samples (middle row), and the number of the $f_{T3}$ samples (bottom row) as a function of MLT. The error bars represent the inter quartile range (IQR). Group (a) compares the results between GOES and CCE for which the $6.1 < L < 7.1$ bin represents geosynchronous orbit. Group (b) compares the CCE results between three different $L$ bins: $4 < L < 5$, $6 < L < 7$ and $8 < L < 9$. The equatorial densities were calculated using the T89 field model. The dashed curves indicate 27 day averaged equatorial mass density, $\rho_{eq,27d}$, calculated from the $F_{10.7}$ dependent empirical model (refer to section 6.6.2) (Takahashi et al. 2010).
behavior. When the dipole field is used, the daytime $\rho_{eq}$ slightly increases owing to the shorter field line than that of the T89 field model. The slight increase of the IQR in $\rho_{eq}$ towards the afternoon can be attributed to the choice of the linear scale comparing with Figure 6.8b. Since there are only a few CCE obits in the 1900–0300 MLT (Figure 6.4), the short IQR bars of $\rho_{eq}$ in the 2200–0100 MLT is likely due to all data coming from an orbit or two during which the geomagnetic condition remained constant. The noticeable difference between the median $\rho_{eq}$ values of CCE and GOES is the magnitude of $\rho_{eq}$ at the daytime sector. The maximum difference is over 2 amu/cm$^3$ at 1300 MLT. Brief discussion about the possible cause is given in section 6.6.2.

The local time variation can be compared to Figure 6 in Takahashi et al. (2010). Note that their result includes the $f_{T3}$ samples obtained during one full solar cycle. As a result, the median value and the IQR of their $\rho_{eq}$ are larger than those in this study.

Figure 6.8b shows the local time variation of $f_{T3}$ and $\rho_{eq}$ for inner and outer $L$ bins. The figure format is same except for the logarithmic scale in the $\rho_{eq}$ panel. The number of samples were greater than 10 between 0400–1900 MLT for all $L$ bins. One thing to immediately note is the same local time behavior of the median value and IQR at all $L$ bins except for the difference in magnitude, which may imply the fact that the statistical behavior of $\rho_{eq}$ can be modeled with a simple functional relationship. The $L$ dependence will be discussed further later in this section.

The $L$ and MLT dependence shown in the CCE data set have good agreement with Takahashi et al. (2002) whose results are based on the $f_{T1}$ samples. Namely, the $f_{T1}$ statistics in Takahashi et al. (2002) also shows a monotonic variation in local time as well as a logarithmic variation in radial distance with the local $f_{T1}$ minimum at approximately the same local time (Figure 6 therein).
6.5.2 Dependence on Geomagnetic Activity

Since the geomagnetic activity can play a role in controlling $\rho_{eq}$, the dependence on the $Dst$ and Kp indices was explored, similar to the study of Takahashi et al. (2010).

**Kp Variation**  Figure 6.9a shows the dependence of $f_{T3}$ and $\rho_{eq}$ from GOES on Kp$_{3d}$ which is the representative Kp value at the current time, $t$, obtained by averaging over earlier times, $t'$, using the weighting factor $\exp(- (t - t')/t_0)$, where $t_0$ is 3 days. It is known that organizing $\rho_{eq}$ by the Kp$_{3d}$ shows better correlation than the instantaneous Kp value (e.g., Denton et al. 2004b; Takahashi et al. 2010). $t_0 = 0$ and 1.5 days were used for the average, but Kp$_{3d}$ seems to show the trend more clearly than Kp$_{1.5d}$ and Kp$_{0d}$. In addition to the spatial binning, the Kp$_{3d}$ was binned in steps of 1 from 0 to 4 with no overlap, which are represented by four colored curves. Additionally the median values of $f_{T3}$ and $\rho_{eq}$ and the number of samples at GEO in Figure 6.8 are superimposed with the thick dashed curves.

The median values of both $f_{T3}$ and $\rho_{eq}$ vary little for Kp$_{3d} > 1$ at all local time. Compared to the dashed curve, the deviations are well contained within the IQR. The median values increase by 2 amu/cm$^3$ at maximum with the significantly larger IQR when Kp$_{3d} < 1$ at the daytime sector. This may be attributed to the refilling of the plasmasphere during extreme quiet times as discussed by Takahashi et al. (2010). The usual plasma trough density observed by GOES and CCE is under 10 amu/cm$^3$ at the prenoon sector, which is well below than 20 amu/cm$^3$ that these authors set as the plasma trough density. It should be noted that compared to Figure 9 of Takahashi et al. (2010), the result in this study show good consistency to theirs.

Figure 6.9b shows the same Kp$_{3d}$ dependence for CCE at GEO. The deviations of $f_{T3}$ and $\rho_{eq}$ for Kp$_{3d} > 1$ are well contained within the IQR as for the GOES case and the local time of the local $\rho_{eq}$ maximum varied slightly. In the CCE case, however, the number of samples for Kp$_{3d} < 1$ was extremely small to derive meaningful behavior.

Figure 6.10a and b show the Kp$_{3d}$ dependence in the inner ($L = 4.5$ bin) and outer
Figure 6.9  Kp dependent statistics of $f_{T3}$ (top row), $\rho_{eq}$ (middle row) and the number of the $f_{T3}$ samples (bottom row) as a function of MLT at geosynchronous orbit. The open circles and the error bars represent the median values and the IQR. Panels on the left column (a) shows the GOES results and panels on the right column shows the CCE results at $6.1 < L < 7.1$. The samples are grouped into four Kp$_{3d}$ ranges: [0,1), [1,2), [2,3) and [3,4) that are indicated by colors. The bold dashed curves are the median values of $\rho_{eq}$ obtained in Figure 6.8.
Figure 6.10  Kp dependent statistics at inner and outer $L$ shell regions: (a) $4 < L < 5$ and (b) $8 < L < 9$. The figure format is the same as Figure 6.9. The dashed curves are the median values of $\rho_{eq}$ at $6.1 < L < 7.1$ and the dot-dashed curves are the median values of $\rho_{eq}$ at either $4 < L < 5$ (a) or $8 < L < 9$ (b). The significant outlier samples are indicated by the shaded box.
\( L = 8.5 \text{ bin} \) \( L \) shells, respectively. The figure format is the same as the previous figure and, additionally, the dot-dashed curves in the left and right columns represent the median values of the inner and outer \( L \) shells from Figure 6.8, respectively. As can be anticipated, both results are only weakly correlated with the \( K_p \) index. The exceptions can be found for \( K_{p3d} < 1 \) in the outer \( L \) shell and the outliers in the inner \( L \) shell. The outliers with in the shaded box reaches up to 150 amu/cm\(^3\) and will be discussed in section 6.6.1.

**Dst Variation** Figure 6.11a and b show the \( Dst \) dependence for GOES and CCE at GEO in the same manner as Figure 6.9. In addition to the spatial binning, the \( Dst \) was divided into three groups: \(-500 < Dst < -50\), \(-50 < Dst < -10\) and \(-10 < Dst < 100\) nT at each of which the median values and IQR were calculated as before. For GOES, the median values of \( f_{T3} \) and \( \rho_{eq} \) are almost identical for quiet or moderately disturbed times. It is when \( Dst < -50 \) nT that the deviation becomes substantial. Takahashi et al. (2010) speculated that this deviation most likely results from storm injection of heavy ions. Unlike the GOES case, \( \rho_{eq} \) for \( Dst > -10 \) nT at the local time of the local maximum is slightly larger than that for \( Dst < -10 \) nT.

Figure 6.12a and b show the \( Dst \) dependence in the same manner as Figure 6.12. In the inner magnetosphere, the median values are weakly correlated to the \( Dst \) except for strongly disturbed times. The largest \( \rho_{eq} \) in the data set was about 180 amu/cm\(^3\) and the associated \( Dst \) and \( K_{p3d} \) were \( \sim -300 \) nT and 5, respectively, which contributed to the huge outliers in the shaded box. In the outer magnetosphere, the deviations for \( Dst > -50 \) nT are somewhat larger than those at GEO even though they are within the IQR.

**6.5.3 \( L \) Variation**

Since the CCE data set provide a unique opportunity to investigate radial variation of \( \rho_{eq} \), the radial dependence of \( \rho_{eq} \) has been further investigated at four selected local time bins. The dependence at other local time bins can be interpolated or extrapolated based on the result in Figure 6.8b.
Figure 6.11  Same as Figure 6.9 but for $Dst$ dependence. The samples are grouped into three $Dst$ ranges: $[-500, -50)$, $[-50, -10)$, and $[-10, 100)$ nT.
Figure 6.12 Same as Figure 6.10 but for $Dst$ dependence. The samples are grouped into three $Dst$ ranges: $[-500, -50)$, $[-50, -10)$, and $[-10, 100)$ nT. The shaded box represents significant outliers that are discussed in section 6.6.1. The upper error bar at 0800 MLT in the box reaches up to 150 amu/cm$^3$. 
Figure 6.13  $L$ dependent statistics at four selected local time ranges: (a) 0500–0700, (b) 0800–1000, (c) 1100–1300 and (d) 1400–1600 MLT that are indicated four light colors. The format is the same as Figure 6.8. The GOES results are superimposed at 6.6 $R_E$. The dashed curves on the $\rho_{eq}$ panels are the quadratic polynomial fit whose coefficients are tabulated in Table 6.1.
Figure 6.13 shows the variation of $f_{T3}$ and $\rho_{eq}$ in $L$ value in the same format as the previous figures. The figure is divided into four groups corresponding to four local time bins and to reduce confusion, the panels in each group have the same background color. At all local time bins, it is clear that $f_{T3}$ and $\rho_{eq}$ monotonically decreases in the linear space and logarithmic space, respectively. The statistical studies using the ground-based ULF observation also show that the mass density falls logarithmically with $L$ value in the inner magnetosphere (Berube et al. 2005; Waters et al. 2006). The $\rho_{eq}$ variation in $L$ value was approximated with the second order polynomial at each local time. The fitting coefficients tabulated in Table 6.1 suggest the strong linear correlation, as was expected. The dashed curves in the figure show the approximated $\rho_{eq}$ and the errors are bound within 10%.

The radial dependence can be compared to Figure 12 in Takahashi and Anderson (1992). Similar to the present analysis, these authors derived the mass density as a function of $L$ value using Eq. (6.2) and the toroidal frequencies from the CCE magnetometer data. The $f_{T3}$ and $\rho_{eq}$ in the 0900–1200 MLT are clearly matched to their result. The connection to $\rho_{eq}$ in the inner magnetosphere is discussed more in section 6.6.3. $\rho_{eq}$ of GOES is also superimposed for a reference at $L = 6.6$, which is lower at the daytime sector.

6.6 Discussion

6.6.1 Short-lived $\rho_{eq}$ Enhancement and Solar Cycle Variation

Since CCE detected the toroidal waves during the rising phase of the solar cycle as well, the solar cycle variation of $\rho_{eq}$ is briefly discussed. Figure 6.14a shows all $\rho_{eq}$ samples, including those during rising phase of solar cycle at the end of the mission period, of CCE for $L < 6$ at the bottom panel and the related Kp, Dst and $F_{10.7}$ on the top two panels. The units of $F_{10.7}$ are $10^{-22}$ Wm/m$^2$Hz, referred to as the solar flux units (sfu). During the solar cycle minimum, the most samples were contained within 50 amu/cm$^3$ for the $L = 4.5$ bin and within 30 amu/cm$^3$ for the $L = 5.5$ bin. The median value of all $\rho_{eq}$ samples shown in the figure is 20 amu/cm$^3$ indicated by the horizontal dashed line. During the rising phase of
Figure 6.14 The $\rho_{eq}$ samples that caused the significant outliers in Figures 6.10 and 6.12. Shown in (a) are $\rho_{eq}$ samples from CCE for $4 < L < 6$ and the corresponding $Kp_{3d}$, $Dst$ and $F_{10.7, 27d}$ ($27$ day averaged $F_{10.7}$ (Takahashi et al. 2010)) parameters for whole years. The shaded boxes indicate the period that has substantial $\rho_{eq}$ jumps. (b) is the same as (a) but the time axes are magnified to fit to the shaded boxes.
the solar cycle after January 1988, it is clear that $\rho_{eq}$ in average increases compared to the median value. One can also notice the annual variation of $\rho_{eq}$. This can be attributed to the precession of the CCE orbit. Overall, this figure clearly shows the strong correlation of $\rho_{eq}$ with the long-term solar cycle variation as shown by Vellante et al. (2007) and Takahashi et al. (2010).

Sporadic density enhancements that are found beyond 50 amu/cm$^3$ in the figure are believed to be related to the plasmaspheric expansion and/or geomagnetic storms. Takahashi et al. (2010) isolated these two effects by carefully inspecting the baseline of $\rho_{eq}$ in relation to the $Kp$ and $Dst$ and concluded that these enhancements are short-lived and usually occur during very quiet or severely disturbed times. They claimed that the enhancements due to the former are related to the refilling of the plasmasphere and the enhancements due to the latter are related to heavy ion ($O^+$) injection. Figure 6.14b shows the samples of these enhancements occurred for about five months indicated by the shaded box including several strong storms. Two enhancements, one in the beginning of January 1886 (the first dashed line) and the other in the beginning of February 1886 (the second dashed line), accompanied significant $Dst$ drop. For the latter event, $\rho_{eq}$ increased up to about 180 amu/cm$^3$ prior to the huge negative $Dst \sim -300$ nT and the $Kp_{3d}$ increase about 5. As Takahashi et al. pointed out, these enhancements occurred during the main phase. Note that no event for other storms does not necessary mean absolute no enhancement of $\rho_{eq}$, but the magnetometer of CCE might have not detected the toroidal mode waves nor was it located at the right position during those storms. Since the plasmapause certainly moves inward during the main phase of storm, these enhancements are likely associated with the heavy ions injected into the inner magnetosphere during the main phase of geomagnetic storms, as also confirmed by the CRESS observation of the toroidal waves (Takahashi et al. 2006).

There are minor enhancements that are found between 50 and 100 amu/cm$^3$ which do not appear to be related to storms but could result from the plasmaspheric expansion.
6.6.2 Comparison with $F_{10.7}$-based Empirical Model

The strong correlation of $\rho_{eq}$ with the solar activity has been investigated in the previous studies (Vellante et al. 1996, 2007; Takahashi et al. 2010). It is known that the solar irradiation (UV/EUV) is a major controlling factor of ionospheric outflows and thus ion production (Strangeway et al. 2005). Based on the result of the strong correlation with long-term variations of $F_{10.7}$, Takahashi et al. (2010) established a simple empirical model of $f_{T3}$ and $\rho_{eq}$ that can be written as

$$f_{T3,27d} = 37.5 - 0.0972F_{10.7,27d},$$  \hspace{1cm} (6.3)

$$\log_{10}(\rho_{eq,27d}) = 0.421 + 0.00390F_{10.7,27d},$$  \hspace{1cm} (6.4)

where units of $f_{T3,27d}$, $F_{10.7,27d}$ and $\rho_{eq,27d}$ are given by mHz, sfu and amu/cm$^3$, respectively, and the variables are all 27-day averaged as indicated by the subscript. The calculated $f_{T3,27d}$ and $\rho_{eq,27d}$ represent the nominal values at the dawn-noon sector.

The thick dashed curves in Figure 6.8a shows the median values of $f_{T3,27d}$ and $\rho_{eq,27d}$ that were calculated from $F_{10.7,27d}$ at each sample in the dawn-noon sector. The model well estimates the derived mass density at 1000 MLT for CCE, but overestimates the density at all dawn-noon local times for GOES. The reason for the first case is clear because their data set (1980–1992) includes the solar cycle minimum of CCE. On the other hand, the present GOES data set was taken during the recent solar cycle minimum. When comparing these two solar cycle minima in Figure 6.3, average $F_{10.7}$ for GOES is only slightly lower than that for CCE although $F_{10.7}$ for CCE has more spikes. The sunspot number and $Dst$ indicate that the solar activity during the recent solar minimum was much quieter than the one for CCE, which likely resulted in lower $\rho_{eq}$ for GOES.

6.6.3 Comparison with $\rho_{eq}$ Derived from Ground-based ULF

Several references related to the mass density derivation using the ground-based ULF (Waters et al. 1996; Menk et al. 1999; Dent et al. 2003, 2006; Berube et al. 2005) showed a
common result: (1) the mass density falls logarithmically with $L$ and (2) there exists an order or more of magnitude difference in the density at the plasmapause that is usually located between $L = 4–6$. They also showed that the density outside the plasmapause is on the order of 10 amu/cm$^3$. Since $\rho_{eq}$ derived from CCE in the innermost $L$ shell is within that order, one may assume that CCE was located outside the plasmapause for most of time when $f_{T3}$ was sampled. Figure 12 in Takahashi and Anderson (1992) clearly shows that the plasmapause was located at $L \sim 4.2$ at the prenoon sector.

6.7 Summary

This study has determined the global variation of the equatorial mass density derived from the toroidal Alfvén frequencies acquired by GOES satellites during solar minimum and CCE during solar minimum and rising phase of solar cycle. These frequencies were normalized to the third harmonic frequency, $f_{T3}$, which were used to derive the equatorial mass density, $\rho_{eq}$, assuming the power law variation along the field line with $\alpha = 1$ using the inversion technique. The analysis procedure and frequency identification algorithm were thoroughly cross-checked with the result of Takahashi et al. (2010), and between the CCE and GOES samples at GEO. Due to the statistical significance, this study only focused on the 0300–1800 MLT.

The main findings are summarized as follows:

1. The local time variations of $f_{T3}$ and $\rho_{eq}$ at GEO observed by two different missions operated during two solar cycle minima are consistent each other although the magnitude is slightly different. These variations have good agreement with the previous studies, confirming the applicability of the inversion technique and the analysis procedure described in this study to remote sensing.

2. The current study extended our understanding of $\rho_{eq}$ distribution by including the samples from a wider $L$ range, and showed that the magnitude of $\rho_{eq}$ falls logarith-
mically with increasing $L$ value while the local time variations remain consistent to those at GEO. This suggests the possibility of the global monitoring of $\rho_{eq}$ with only limited resources.

3. The current study confirmed that $\rho_{eq}$ is only weakly correlated with the geomagnetic activities that were represented by the Kp and $Dst$ and the weak correlation is maintained at other $L$ shells. That is, $\rho_{eq}$ varies little for moderately disturbed times and the short-lived enhancements usually occur during either quiet times or geomagnetic storms.

4. The current study confirmed the strong correlation of $\rho_{eq}$ with the solar cycle represented by $F_{10.7}$ and the sunspot number, and the empirical model based solely on $F_{10.7}$ may not be enough to represent the solar activity.
CHAPTER 7

PLASMA MASS VARIATION ALONG THE FIELD LINE DERIVED FROM TOROIDAL MODE ALFVÉN WAVE FREQUENCY: QUIET-TIME EQUATORIAL MASS DENSITY MODEL

Continuing on the effort of the previous chapter, this chapter is devoted to the derivation of a simple analytical representation of the obtained equatorial density distribution. Using a combination of Legendre polynomials in radial distance and sinusoidal functions in local time as a model function, the amplitudes of each mode were sought for analysis of the dominant term(s). It is found that there is indeed a strong linear trend in the logarithmic equatorial mass density with respect to the radial distance, as discussed in the previous chapter. The equatorial mass density distribution can be represented with the three lowest order terms within 10% relative errors in the 0500–1800 MLT, which captures the plasmaspheric bulge in the afternoon and a local minimum in $\rho_{\text{eq}}$ at $\sim 1000$ MLT.

7.1 Introduction

Magnetospheric mass density serves as a medium for plasma waves and controls the time response of the magnetosphere to external and internal forces (Denton 2006): (1) It controls loss rate of ring current particles through the coulomb collision, (2) the rate of propagation of shock waves and pressure pulses through the magnetosphere, (3) the frequency and field line structure of ultra-low frequency (ULF) waves, and (4) stability and growth of electromagnetic ion cyclotron (EMIC) waves. These processes controlled by the magnetospheric mass density are associated with energization and scattering of radiation belt particles and thus important for space weather.

Direct measurement of the mass density is difficult because it not only requires particle detectors to discriminate ion species but also to count low temperature ions that are obscured by spacecraft charging. Alternatively, the mass density can be obtained from the
toroidal mode Alfvén waves (azimuthally oscillating field line resonance. Referred to as toroidal waves hereafter). A field line resonance (FLR) is the resonant Alfvénic oscillation of a closed geomagnetic field line, whose foot points lie in the ionosphere (Dungey 1955). The discrete frequency oscillations arise from the restriction to an integral number of half wavelengths along the field line fixed to the conducting ionospheric boundary. The field line distribution is embedded in these frequency oscillations from which the mass density inversion is possible.

The FLRs are ubiquitous in the magnetosphere and known to be driven externally by the solar wind (e.g., Takahashi and Anderson 1992). Broadband shock waves and pressure pulses propagate in a fast mode through the magnetosphere during which particular oscillations at resonating frequency are picked up (Takahashi et al. 1984), which is supported by MHD simulation (Claudepierre et al. 2008, 2010). They are also internally excited through drift-bounce resonances (e.g., Southwood 1976) and drift mirror instability (e.g., Hasegawa 1969) of energetic ions injected from the plasma sheet.

Satellites mostly carry the magnetometer instrument, and some are also equipped with the electric field and particle instruments. Although global monitoring of the mass density may not be possible, the space-borne measurements provide some advantages over the ground-based observation, as Denton (2006) summarizes. First, the field line mapping from the satellite is more accurate than the mapping from the ground. Since the field line near the magnetic equator most strongly determines the toroidal wave frequency, any error in mapping to the equator can lead to large errors in the inferred mass density. Second, multiple harmonics are more frequently observed in space than on the ground. Finally, the electron density measurements can be used to determine the average ion mass (ratio of mass density to electron density) which can impose a constraint on ion species discrimination.

The study in chapter 6 used a large number of samples of the toroidal wave frequencies observed by the magnetometer on board Active Magnetospheric Particle Tracer Explorers/Charge Composition Explorer (CCE) spacecraft (referred to as CCE hereafter)
and derived the equatorial mass density distribution with $L$ values 4–10 and the magnetic local time (MLT) 0300–2000 MLT. Continuing with the goals outlined therein, the study in this chapter attempts to derive a simple analytical representation of the constructed equatorial density distribution obtained from the previous chapter. Because of the small number of samples of the CCE observation, the study in this chapter only takes into account $L$ and MLT dependences, but the methodology itself can be extended to include more dependences (for example, the Kp and $Dst$ to represent the geomagnetic activity) as database grows. The samples that satisfy $1 \leq Kp_{3d} \leq 3$ and $Dst > -50$ nT from CCE were taken into account for derivation of the analytical representation.

### 7.2 Equatorial Mass Density Distribution and Its Simple Functional Representation

Based on the observations presented in our previous chapter, the equatorial mass density, $\rho_{eq}$, during the solar cycle minimum as a function of $L$ and MLT statistically varies in a predictable way: $\rho_{eq}$ falls logarithmically with $L$ value and must be periodic in local time. A simple analytical representation may, therefore, be found with only a few lowest order terms of bases.

In order to systematically analyze the dominant functional bases other than the linear trend in the logarithmic scale, the equatorial mass density distribution $\rho_{eq}(L, \text{MLT})$ is

<table>
<thead>
<tr>
<th>$l$</th>
<th>$m^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-2$</td>
</tr>
<tr>
<td>0</td>
<td>0.0573</td>
</tr>
<tr>
<td>1</td>
<td>0.0020</td>
</tr>
<tr>
<td>2</td>
<td>-0.0269</td>
</tr>
</tbody>
</table>

$^a m \leq 0$ ($> 0$) corresponds to $A_{lm}$ ($B_{lm}$).
decomposed into

$$\log_{10} \rho_{eq}(L(x), \phi) = \sum_{l,m} P_l(x) [A_{lm} \cos m\phi + B_{lm} \sin m\phi], \quad (7.1)$$

where $L = 2.5(1-x) + 4.5$ and $4.5 \leq L \leq 9.5$, $P_l$ is the normalized Legendre polynomial of degree $l \geq 0$, $m \geq 0$ is the azimuthal mode number and $\phi = \pi \text{MLT}/12$. Since the distribution is not continuous, a matrix form

$$\log_{10} \rho_{eq}^{ij} = \sum_{l,m} P_{il}(x_i) [A_{lm} \xi_{mj} + B_{lm} \zeta_{mj}] \quad (7.2)$$

is solved for $A_{lm}$ and $B_{lm}$, where $\rho_{eq}^{ij} = \rho(x_i, \phi_j)$, $P_{il}(x_i) = P_l(x_i)$, $\xi_{mj} = \cos m\phi_j$ and $\zeta_{mj} = \sin m\phi_j$.

For the analysis, the median density samples in the range of $\text{Dst} > -50 \text{ nT}$ and $1 \leq \text{Kp}_{3d} < 3$ were only used in order to minimize bias from the outlier samples during extremely quiet and disturbed times. Unlike the previous chapter, the local time was binned in steps of three hours from 1 to 24 hours with one and a half hour overlap, and the $L$ shells were binned in steps of $\Delta L = 1$ with $L$ varying from 4 to 10 with no overlap. The increase in the local time bin is to increase the number of samples and make the distribution smoother.

The result of the analysis is shown in Figure 7.1. Figure 7.1a shows line plots of the median values of $\rho_{eq}$ as a function of the local time. Since there are several gaps in the nighttime sector due to no samples, these gaps are first interpolated using a cubic spline interpolation and the periodic condition in local time, which is shown with the dashed curves. The interpolation at $L = 4.5$ and $L = 5.5$ bins seamlessly filled the gaps smoothly connecting the adjacent values, whereas the interpolated values at $L = 9.5$ bin did not fall as quickly as those from other bins. Since only one or two local time bins are missing in the nighttime sector where the statistical significance is questionable, this study assumes that the interpolation to fill these gaps is acceptable. Figure 7.1 shows $\rho_{eq}$ distribution in a form of spectrogram.
Figure 7.1 Steps of the functional approximation of the equatorial mass density for $Dst > -50$ nT and $1 \leq Kp_{3d} < 3$. (a) The solid curves represent median mass density profile as a function of MLT and the dashed curve represents the interpolated density profile. (b) $L$-MLT spectrogram of the interpolated mass density obtained from (a). (c) Reconstructed mass density using all functional bases in Eq. (7.1). (d) Approximated mass density using functional bases in Eq. (7.1) with only $l \leq 2$ and $|m| \leq 2$ modes. (e) The relative error of the approximated mass density in %. (f) The distribution of the mode power, $A_{lm}^2$ ($m \leq 0$ part) and $B_{lm}^2$ ($m > 0$ part) in the basis function space. The power is normalized to the total power. More than 99% of the power are concentrated within the dashed box. The dashed vertical lines in Panels (a–e) are at 0500 and 1800 MLT, respectively.
Using the interpolated density, Eq. (7.2) can be solved for $A_{lm}$ and $B_{lm}$. Figure 7.1 shows the reconstructed $\rho_{eq}$ distribution using the calculated $A_{lm}$ and $B_{lm}$ with finer grid resolutions. By definition, the reconstructed distribution exactly represents Figure 7.1b. The square of the coefficients, $A_{lm}$ and $B_{lm}$, can be considered as the power of a mode, $(l, m)$, and when normalized to the total power, as a measure of the dominance of a function basis, $\psi_{lm}$. Figure 7.1f shows the percentage of the mode power given by $(A_{lm} \text{ or } B_{lm})^2 / \sum_{l,m} (A_{lm} \text{ or } B_{lm})^2 \times 100$. In the figure, the sine and cosine components are combined together so that the negative $m$ numbers indicate the cosine components. The upper and lower color scales are set to $10^{-2}$ and 70%, respectively. The pixels with percentage below $10^{-2}\%$ are masked with white. The result shows that more than 98% are concentrated in $l \leq 1$ and $-1 \leq m \leq 1$ modes with the constant term dominating followed by the linear term ($l = 1, m = 0$), confirming the strong linear variation with $L$ value in the previous chapter.

To find a simple analytic representation of the $\rho_{eq}$ distribution, the dominant modes were only chosen, $l \leq 2$ and $-2 \leq m \leq 2$ as indicated by the dashed box in the figure. This choice of $l \leq 2$ is equivalent to using a quadratic polynomial for the radial dependence as done in the previous chapter. Figures 7.1d and c show the approximated distribution using only the lowest order terms and the relative error from Figure 7.1c, respectively. Pixels with the relative error below 10% are masked with white. Not surprisingly, the approximated density represents the major features in Figure 7.1b (or equivalently c) remarkably well, although the small scale structures are missing as a result of omitting the higher order terms. The approximation at the daytime sector in the 0500–1800 MLT (between two dashed vertical lines) only resulted in less than or equal to 10% relative error. The relative error increases up to about 70% at the nighttime sector where more cosine/sine terms are needed to represent the small scale structures. Table 7.1 summarizes the coefficients of the $l \leq 2$ and $-2 \leq m \leq 2$ modes.

A few features are worth to be mentioned: First, the azimuthal asymmetry in the
distribution is clear for all $L$ shells, as also shown in the previous chapter. $\rho_{eq}$ at around 0500 MLT is on an order of unity at geosynchronous orbit and increases towards 1600 MLT at which a bulge is formed. Second, there is a local minimum in $\rho_{eq}$ at $\sim 1000$ MLT, which is most clear in the inner magnetosphere. Finally, the samples were obtained during the nominally moderate geomagnetic condition and the $\rho_{eq}$ distribution during the geomagnetically disturbed or extremely quiet times may significantly deviate. The plasmaspheric expansion during quiet times or storm injections during disturbed times can change the morphology of the distribution.

### 7.3 Summary

The functional dependence of the obtained equatorial mass density from chapter 6 on the local time and radial distance was analyzed assuming a Legendre-sinusoidal model function. The samples obtained during the nominally moderate geomagnetic condition were only used in order to avoid bias. It is found that the equatorial mass density distribution can be represented with the three lowest order terms within 10% relative error in the 0500–1800 MLT. The approximated mass density captures the plasmaspheric bulge in the afternoon and a local minimum in $\rho_{eq}$ at $\sim 1000$ MLT, which is most clear in the inner magnetosphere. As a final note, Figure 7.2 shows a comparison between the obtained equatorial mass density and the approximated equatorial mass density. The approximated density distribution captures the smooth variation with the local time and the plasmaspheric bulge at around 1600 MLT although improvement is needed at the nighttime sector.
Figure 7.2 Contour lines of (a) the derived equatorial mass density and (b) the approximated mass density shown in the $x$-$y$ plane. The shaded area in (b) indicates the region where the relative error is greater than 10%.
CHAPTER 8
SUMMARY AND DISCUSSION

The present dissertation is devoted to a statistical study of plasma waves and energetic particles in the outer magnetosphere as enabled with the data obtained from several space missions, including the Time History of Events and Macroscale Interactions during Substorms (THEMIS) spacecraft, Cluster mission, the Geostationary Operational Environment Satellites (GOES), Los Alamos National Laboratory (LANL) satellites, the Polar spacecraft and the Active Magnetospheric Particle Tracer Explorers (AMPTE)/Charge Composition Explorer (CCE), and from ground-based Automatic Geophysical Observatories (AGO).

Scientific topics that were addressed in this dissertation include: (1) a technique for accurately and rapidly calculating particle motion in the magnetosphere, (2) source of the whistler-mode chorus waves, (3) physical properties of the electromagnetic ion cyclotron (EMIC) waves, (4) spatial dimension of EMIC wave sources, and (5) a diagnostic use of the Alfvén waves on the plasma density distribution in the Earth’s magnetosphere.

The primary achievements are summarized as follows:

1. A rapid, efficient method of calculating the magnetic drift invariant ($L^*$) was developed. Use of generalized grid space and deferred field line evaluation scheme reduces the computational cost significantly, while calculated $L^*$ maintains reasonable accuracy. The new method evaluates $K(B_m)$, which involves expensive field model calls, only at necessary grid points at the time of tracing, which results in a significant performance gain. The new method is fast enough for near real-time calculation for the satellite tracking in the $L^*$ coordinate and maintains scalability to a large number of $L^*$ values.

2. Using the test particle simulation, the relationship between the electron injection and the chorus waves was studied from the simultaneous observations of a substorm
event on 23 March 2007 made in space and on ground. The electron injection and
the drift of these electrons toward the morning sector were detected from the multiple
spacecraft, and the chorus waves were detected at AGO mapped to the equator in the
range of 0600–0900 magnetic local time (MLT). Performing the backward particle
tracing from the region where AGO is mapped, the injection region was inferred
to occur in a range of 2200–2400 MLT. Further examination by the forward model
tracing indicated that the electrons injected during the substorm could form a pitch-
angle distribution suitable for the whistler-mode instability when they arrive near the
dawn-side magnetopause. The development of this anisotropy is due to the particle
drift-shell splitting effect.

3. The global distribution of EMIC waves was studied over a broader range than ever
before, as enabled by observations with the THEMIS spacecraft from 2007 to 2010.
The main finding are: (1) There are two major peaks in the EMIC wave occurrence
probability. One is at dusk and 8–12 $R_E$ where the helium band dominates the hy-
drogen band waves. The other is at dawn and 10–12 $R_E$ where the hydrogen band
dominates the helium band waves. (2) In terms of wave spectral power, the dusk
events are stronger ($\approx 10 \text{nT}^2/\text{Hz}$) than the dawn events ($\approx 3 \text{nT}^2/\text{Hz}$). (3) The dawn
waves have large normal angles ($> 45^\circ$) in the hydrogen band and even larger normal
angles ($> 60^\circ$) in the helium band. The dusk waves have small normal angles ($\leq 30^\circ$)
in both the hydrogen and helium bands. (4) The hydrogen band waves at dawn are
weakly left-hand polarized near the equator, become linearly polarized with increas-
ing latitude and eventually weakly right-hand polarized at high latitudes, whereas
the helium band waves at dawn are linearly polarized at all latitudes. Dusk waves
in both bands are strongly left-hand polarized over a wide range of latitude. Based
on the linear EMIC instability model presented by Horne and Thorne (1994), it is
suggested that the main underlying factor for the observed spatial variations of these
wave properties would be local density of cold plasma and chemical abundance.
4. The size of coherent activity of EMIC waves was estimated using the multi-spacecraft observations made by the THEMIS spacecraft. The cross correlation between EMIC wave powers measured by different THEMIS spacecraft was used to determine the characteristic dimension of the coherent wave activity as represented by the $1/e$ folding distance of an exponential fit to the calculated correlation samples. The characteristic dimension in the direction transverse to the local magnetic field is found to lie in rather a wide range of 1,500–8,600 km and vary in local time and wave band. However, the characteristic dimensions normalized either by local gyroradius or by local wavelength fall into narrower ranges almost independent of the emission band and event location. Specifically, the coherent dimension is found to be 10–16 times local gyroradius of 100 keV protons and 2–3 times local EMIC wavelength.

5. The statistical distribution of the equatorial mass density was derived assuming the power law density model and the Tsyganenko 89 (T89) (Tsyganenko 1987, 1989) field model in an unprecedented spatial scale. The equatorial mass density falls logarithmically with increasing radial distance and monotonically increases towards the dusk meridian where a local maximum is formed. The overall structure (shifted density contour to the dusk sector) conforms to what is expected from zero-energy plasma dynamics. It also has been found that the magnitude of the equatorial mass density is only weakly correlated to the geomagnetic activity, but has strong variation in the solar activity.

6. A simple analytical representation of the equatorial mass density distribution was found. The systematic analysis showed that the distribution can be represented by combination of a quadratic function in radial variation and the lowest order sinusoidal functions in local time with 90% confidence. This analysis method can be extended to include more parameters as database grows.

Based on the results of these studies, we would like to suggest additional scien-
tific questions worthy of future studies. First, the newly found characteristics of the EMIC waves in the dawn sector need to be further investigated. Although the properties of EMIC waves at the dusk sector could be understood from the ion dynamics and the high plasma density detached from the plasmasphere, the properties of the waves at the dawn sector, especially at large radial distance are yet to be understood. Especially, the polarization characteristics at the dawn sector still remain as a challenge. Second, unlike the total plasma number density, the mass density of plasma in the magnetosphere has been poorly understood because of the difficulty in the direct measurement and the limited spatial coverage. A refined remote sensing technique is presented in this dissertation. With data from the new space mission, the Van Allen Probes (Mauk et al. 2012), this technique can be used to monitor the field line distribution of mass density more accurately than ever. Third, the plasma mass density can impose a constraint on ion composition. Especially, during storm times, it is known that \( \text{H}^+ \) and \( \text{O}^+ \) ions are dominant at the plasma trough, and several studies attempted to pinpoint the exact composition. Since the ion mass and composition control the time response of low frequency waves to the external and internal forces, accurate amount of the ion composition along with the mass density is needed. The ion abundance distribution may also provide a clue to the polarization characteristics of the dawnside EMIC waves. Addressing these questions will further enhance our understanding of the physics of the magnetosphere.
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