Distributed spectrum leasing via cooperation

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“Cognitive radio” networks enable the coexistence of primary (licensed) and secondary (unlicensed) terminals. Conventional frameworks, namely commons and property-rights models, while being promising in certain aspects, appear to have significant drawbacks for implementation of large-scale distributed cognitive radio networks, due to the technological and theoretical limits on the ability of secondary activity to perform effective spectrum sensing and on the stringent constraints on protocols and architectures.

To address the problems highlighted above, the framework of distributed spectrum leasing via cross-layer cooperation (DiSC) has been recently proposed as a basic mechanism to guide the design of decentralized cognitive radio networks. According to this framework, each primary terminal can "lease" a transmission opportunity to a local secondary terminal in exchange for cooperation (relaying) as long as secondary quality-of-service (QoS) requirements are satisfied.

The dissertation starts by investigating the performance bounds from an information-theoretical standpoint by focusing on the scenario of a single primary user and multiple secondary users with private messages. Achievable rate regions are derived for discrete memoryless and Gaussian models by considering Decode-and-Forward (DF), with both standard and parity-forwarding techniques, and Compress-and-Forward (CF), along with superposition coding at the secondary nodes. Then a framework is proposed that extends the analysis to multiple primary users and multiple secondary users by leveraging the concept of Generalized Nash Equilibrium. Accordingly, multiple primary users, each owning its own spectral resource, compete for the cooperation of the available secondary users under a shared constraint on
all spectrum leasing decisions set by the secondary QoS requirements. A general formulation of the problem is given and solutions are proposed with different signaling requirements among the primary users.

The novel idea of interference forwarding as a mechanism to enable DiSC is proposed, whereby primary users lease part of their spectrum to the secondary users if the latter assist by forwarding information about the interference to enable interference mitigation at the primary receivers. Finally, an application of DiSC in multi-tier wireless networks such as femtocells overlaid by macrocells whereby the femtocell base station acts as a relay for the macrocell users is presented. The performance advantages of the proposed application are evaluated by studying the transmission reliability of macro and femto users for a quasi-static fading channel in terms of outage probability and diversity-multiplexing trade-off for uplink and, more briefly, for downlink.
DISTRIBUTED SPECTRUM LEASING VIA COOPERATION

by

Tariq Elkourdi

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To my parents, Sana and Hesham and my wife, Somoud
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CHAPTER 1

INTRODUCTION

1.1 Overview

Current research on mobile wireless networks places substantial emphasis on scenarios in which primary (or licensed) and secondary (or unlicensed) terminals utilize the same spectral resource. Such wireless systems are typically referred to as “cognitive radio” networks and hold the promise to improve the overall spectral efficiency via the coexistence of incumbent primary users and “opportunistic” secondary terminals. Irrespective of the traffic class (multihop, uplink/ downlink, etc.), the main conventional approaches proposed to enable such a coexistence are:

- Underlay/ overlay/ interweave strategies: This family of strategies assumes the primary transmitters to be oblivious to the secondary activity, and enforces strict constraints on the secondary behavior in order to avoid interference to the primary communications (see, e.g., [1]);

- System-wide dynamic spectrum allocation/ sharing: This framework prescribes the optimization of a given system-wide utility function with the aim of dynamically allocating resources among distributed nodes, in the presence of a given hierarchy of spectrum providers and (primary/ secondary) users. Decentralized implementation is in principle possible via iterative pricing schemes, which typically require communication to and from a central (system-wide) authority (see, e.g., [2]).

Both frameworks mentioned above, while promising in some aspects, appear to have relevant drawbacks for deployment of large-scale distributed cognitive radio networks. Briefly, underlay/ overlay/ interweave strategies pose a number of major
(and as of yet unresolved) challenges at a system level, due mainly to the technological and theoretical limits on the ability of secondary activity to perform effective spectrum sensing and thus avoiding excessive interference to the primary. By centralizing the decision-making process (e.g., in the form of pricing schemes), the second approach of system-wide spectrum allocation solves the problem of secondary interference management, but at the expense of imposing stringent constraints on protocols and architecture in a decentralized setting.

To address the problems highlighted above, the novel framework of distributed spectrum leasing via cooperation (DiSC) has been introduced in [3] as a basic mechanism to inform the design of Medium Access Control/ Data Link (MAC/DL) - physical (PHY) layer protocols in decentralized cognitive radio wireless networks. The main tenet of the approach is the introduction of a novel functionality at the MAC/DL - PHY layers that enables decentralized spectrum leasing based on cooperation (see Figure 1.1): “Leasing” of a transmission opportunity (e.g., a timeslot) from a primary node to a secondary terminal is performed locally as driven by primary needs in terms of given Quality-of-Service (QoS) measures (e.g., probability of outage or of MAC buffer overflow). Specifically, distributed spectrum leasing via cooperation enables each primary terminal to “lease” a transmission opportunity to a local secondary terminal with granularity of MAC Protocol Data Units (MPDUs), in exchange for cooperation (relaying). Upon leasing of a transmission opportunity, the secondary user guarantees cooperation on the given MPDUs to the agreed QoS and, under this condition, is allowed to exploit the opportunity to transmit its own data as well. Negotiation between primary and secondary users needs to account for the fact that secondary users accept the given transmission opportunity only if advantageous in terms of the trade-off between resources used for cooperation and the amount of
spectrum (time, frequency) leased\(^1\). An elementary example illustrating the potential advantages of this approach follows.

**Example:** Consider a scenario in which a primary link operates over a distance \(d\) with path loss exponent \(\gamma\). Assume that a local secondary user is available that is half-way between primary transmitter and receiver, and has data for a secondary receiver at a distance \(d_S\) (refer to Figure 1.1-(a)). Should the primary decide to transmit without resorting to spectrum leasing, it would be able to achieve a rate \(R_{noDiSC}\) (bit/s/Hz) approximately proportional to the primary link signal-to-noise ratio (SNR) \(R_{noDiSC} \propto 1/d^\gamma\) (in the low-SNR regime\(^2\)). Instead, if the secondary is willing to fully cooperate, the rate can be improved to \(R_{noDiSC} \cdot 2^{\gamma-1}\) by simply using two-hop transmission\(^3\). However, with distributed spectrum leasing via cooperation, the secondary accepts to cooperate only if able to satisfy its own rate requirements.

Considering a simple time-sharing scheme for transmission in the second hop, the secondary can then guarantee a rate \(R_{SL} = \alpha \cdot R_{noDiSC} \cdot 2^{\gamma-1}\) by transmitting primary traffic for a fraction \(0 \leq \alpha \leq 1\) of the time and \(R_S = (1 - \alpha) \cdot 1/2 \cdot P/d\gamma\) for its own traffic by exploiting the remaining part of the second slot (assuming equal transmit powers \(P\)). Now, for instance, if the secondary requires a rate \(R_S = R_{noDiSC}/4\) and \(d = 2d_S\) and the path loss \(\gamma = 3\), it can be easily seen that, via distributed spectrum leasing via cooperation, the primary link can obtain a rate gain with respect

\(^1\)It is noted that distributed spectrum leasing via cooperation is proposed as a framework that enables spectrum coexistence via cooperative transmission. However, it can also be seen, conversely, as an enabler of cooperation through a hierarchical partitioning of the users into primary/secondary nodes. In this sense, it complements systems in which cooperation (relaying) is enforced in networks of peer nodes via reputation-based mechanisms or repeated-games approaches.

\(^2\)In fact, \(\log(1 + SNR) \simeq SNR\) for low \(SNR\).

\(^3\)The two-hop cooperative rate is in fact proportional to \(1/2 \cdot 1/(d/2)^\gamma = R_{noDiSC} \cdot 2^{\gamma-1}\), due to the fact that the secondary is half-way between primary transmitter and receiver (the first 1/2 term follows from the fact that half time is spent on the first hop and half on the second).
to the non-cooperative case of a factor of $15/4 \simeq 4$ (i.e., $R_{SL} \simeq 4R_{noDiSC}$), while the secondary is simultaneously fulfilling its own rate requirement$^4$. 

![Diagram of primary and secondary transmitters](image)

**Figure 1.1** The main feature of the distributed spectrum leasing via cooperation framework is the introduction of a novel functionality at the MAC/DL - PHY layers that enables decentralized spectrum leasing based on cooperation. According to this mechanism, a primary transmitter can decide to lease a retransmission slot for a given MPDU to a local secondary transmitter in case the latter happens to be in a more advantageous path loss (a) or fading/ shadowing (b) channel conditions towards the primary receiver. The secondary accepts the lease only if the allocated spectral resource is sufficient to convey, along with the primary data, secondary MPDUs with sufficient QoS to either the same (b) or a different (a) receiver.

From the basic definitions and considerations given above, distributed spectrum leasing via cooperation has the potential to solve the drawbacks of underlay/ overlay/ interweave and system-wide spectrum allocation strategies in that: (i) Being initiated and driven by the primary users, spectrum sharing in spectrum leasing via cooperation does not rely, as the underlay/ overlay/ interweave approaches, on the ability of secondary users to manage interference to the primary activity; (ii) distributed

---

$^4$This follows by noticing that the secondary rate is $R_S = (1 - \alpha) \cdot 1/2 \cdot R_{noDiSC} \cdot (d/d_S)^\gamma$. 
spectrum leasing via cooperation is fully distributed and thus does not require global coordination (e.g., pricing) to operate.

1.2 Background and State of the Art

In this section, a brief review the current state of the art of the research activity on cognitive radio networks is presented in order to appropriately frame the scope of this dissertation. Two main families of approaches to the design of cognitive radio-based system can be identified, one comprising underlay/ overlay/ interweave strategies (also referred to as the “commons model”) and the other exploiting spectrum leasing (also referred to as the “property-rights model”) [4]-[5]. The main difference between the two families lies in the fact that: (i) In the commons model, primary users are assumed to be oblivious to the presence of secondary nodes, and the latter are in turn constrained to avoid excessive interference to the primary; (ii) With spectrum leasing, the decision of initiating secondary transmissions is made at the primary nodes, and an appropriate “price” is paid back by the secondary nodes that benefit from the leased transmission opportunity. While the review of this section covers mostly academic studies, it is worth mentioning that recent standardization efforts within the working groups IEEE 802.22, 802.21, 802.11, 802.16 and 15.4, among the others, are already leveraging the ideas of cognitive radio and networks, dynamic spectrum access and coexistence (see [6] for a thorough discussion, and [7] for some considerations on the expected market for such technology).

1.2.1 Commons Model (Underlay/ Overlay/ Interweave)

Within the basic framework of the commons model, a number of strategies have been proposed: (i) Interweave/ Underlay: Secondary nodes attempt to access the channel only in spectral resources (frequency, space or time) where primary activity is not present (“interweave”) [8] [9] or, more generally, control their power so as to satisfy
given interference requirements at the primary receivers\(^5\) \cite{10}-\cite{11} ("underlay"). It is noted that this is the framework that has been mostly adopted by the standardization efforts mentioned above; (\(ii\)) \textit{Overlay}: Secondary users transmit over the same spectral resource as the primary nodes and, to avoid interference, leverage \textit{a priori} side information on the primary transmission (e.g., about the transmitted waveform \cite{12} or messages \cite{13}-\cite{14}) and sophisticated transmission strategies such as interference pre-cancellation.

With the interweave/underlay approaches, the main problem is that of designing effective \textit{spectrum sensing} (or \textit{primary activity detection}) and \textit{transmission opportunity exploitation} strategies at the secondary nodes (see, e.g., \cite{15}). Spectrum sensing can be carried out via simple energy detection or more sophisticated cyclostationarity-based detectors matched to the primary transmissions \cite{16}. However, given the receiver-centric nature of interference, interference avoidance would in principle require detecting or locating primary \textit{receivers}, which is even more challenging, if feasible at all \cite{17}. As a result, the very mechanism of “sense-before-talk” at the core of the interweave/underlay models has been shown to have inherent limitations in identifying the transmission opportunities \cite{18}. Yet another issue that further complicates things is the coordination between the spectrum sensing processes at the secondary transmitter and receiver \cite{19}. Among the currently investigated solutions to these problems, cooperative distributed detection at the secondary nodes \cite{20} and the deployment of sensor networks \cite{21} appear to be among the most promising. Once a spectral hole has been identified, due to the inherent uncertainty on the detection outcome, transmission opportunity exploitation amounts to trading missed transmission opportunities for an increased interference level on the primary activity \cite{22} \cite{23}. Moreover, competitive or cooperative approaches may be devised, see, e.g., \cite{10}-\cite{24} \cite{15} \cite{25}. At the MAC layer, proposals for protocols applicable to

\(^5\)The concept of interference temperature is sometimes advocated to quantify such constraints \cite{8}.
multi-hop interweave/underlay cognitive radio networks include purely time-slotted access methods and hybrid strategies based on synchronized control slots and random access for data transmission [23], possibly with multiple radio interfaces per node. MAC protocols that use random access for both control and data packet transmission have also been studied for single-interface radios and for multiple-interface radios.

As discussed above, most works focus on either the physical or MAC layer in isolation. However, a cross-layer view of the system has also been recently advocated by a number of authors. References [22] [26] [23] thoroughly study the joint design of sensing and transmission opportunity exploitation at a single secondary user in the framework of Markov decision processes with no channel sensing errors, while [27] tackles the problem in the presence of imperfect state information. A MAC/DL-PHY analysis of a simple cognitive interference channel that accounts for queuing can be found in [28] [29]. It is relevant to remark that a few preliminary testbed trials have been reported to validate the feasibility of the principle of exploiting spectral holes for the commons model of cognitive radio (see, e.g., [9]).

Layers higher than MAC/DL-PHY have been investigated only recently for multihop cognitive networks. The general and most widely used approach resorts to some type of on-demand routing protocol, leveraging different metrics to assess the quality of a given path [30, 31] [32]-[33]. Similar on-demand based routing protocols are also proposed in [34, 35, 36], where secondary throughput is maximized, and in [37] where the quality of the end-to-end path is weighted also by the probability of a secondary user causing interference to the primary. Traffic characteristics are considered in [38] to allocate spectrum along a given route. Learning-based route and frequency selection algorithms are explored in [39] and [40]. In [41] [42], non-linear optimization tools are exploited to address the problem of achieving throughput optimal routing and scheduling for secondary users. A centralized heuristic based of a graph-layering approach is introduced in [43].
With overlay, the basic assumption is that secondary nodes have access to (typically non-causal) information regarding the primary transmissions, e.g., through an auxiliary channel\textsuperscript{6}. The limitations of this approach have been reported in [44] and [45] where it is show that in the presence of imperfect Channel State Information (CSI), e.g., of an unknown fading channel with small mean (i.e., small Ricean factor) or even an unknown "phase-fading" channel [45], remarkable performance degradations are measured (see also [46]). Another evidence in the same direction has been provided in [47]: therein, based on a simple on/off memoryless model for the primary traffic dynamics and on perfect channel sensing, the "dirty-paper"/cooperative overlay strategy of [48] is generalized to a scenario where the secondary can also "interweave" their transmission and side information on the primary message is not known a priori (but has to be decoded). The results in [47] support the conclusion that, in practice, simultaneous primary and secondary transmitter transmission (that is, overlay) is of little help, especially in the high and low SNR regimes.

To sum up the discussion on the commons model: (i) Practical limitations in spectrum sensing have been shown to pose the major challenge to the implementation of secondary spectrum access based on the commons model; (ii) Cross-layer approaches to system design have been proved to be most effective.

\subsection{1.2.2 Property-Rights Model (Spectrum Leasing)}

Spectrum leasing solves the implementation issues of the commons model (and specifically point (i) identified above) by demanding the primary users to directly manage the possible allocation of spectrum to secondary users. Most research activity in this

\textsuperscript{6}A crucial condition that must be satisfied for this approach to be within the scope of the commons model is that the primary users should be oblivious to the presence of the secondary nodes, and therefore employing the same transmission/reception strategies that would use in the absence of secondary activity. This condition is sometimes violated in related works (see, e.g., [13] [14]), that are more properly classified within the property-rights model, to be discussed in the next section.
field has focused on the centralized optimization of a system-wide utility function in order to appropriately allocate spectrum resources, with possible decentralized implementation based on iterative pricing (see [2] [49] and references therein) or auction-based (see, e.g., [50]) strategies. This line of work generally: (i) assumes specific network architectures consisting of different tiers of providers, regulators and users, thus requiring a thorough redesign of the network; (ii) proposes dynamic spectrum leasing on a large time-scale (e.g., session-by-session). As such, while this approach provides an optimal clean-slate system design, it may be hardly implementable in networks with legacy constraints and in complex dynamic scenarios.

Overall, the discussion of this section on both commons and property-rights models motivates the proposed approach that follows the property-rights model, but seeks an implementation of spectrum leasing that is both local and dynamic, and based on a cross-layer view of cooperation. This is elaborated upon in the next section.

1.3 Contributions

In this dissertation, the framework of DiSC proposed in [3] is further analyzed and extended. Specifically, in Chapter 2, the principle of spectrum leasing via cooperation is studied from an information-theoretic standpoint by focusing on a scenario with one primary node and multiple secondary nodes, which may act as relays for the primary, communicating to a common receiver. The scenario is modelled as a multirelay channel where each relay (secondary user) has a private message for the destination. Achievable rate regions are derived for discrete memoryless and Gaussian models by considering Decode-and-Forward (DF), with both standard and parity-forwarding techniques, and Compress-and-Forward (CF), along with superposition coding at the secondary nodes. Numerical results for the Gaussian channel confirm that spectrum
leasing via secondary cooperation is a promising framework to enable secondary spectrum access.

Chapter 3 provides an extension to the multiple primary and multiple secondary users scenario. Within the paradigm of spectrum leasing via cooperation, primary (licensed) nodes can lease some of the owned spectral resources to secondary (unlicensed) users in exchange for cooperation. Secondary users in turn set a minimal Quality-of-Service (QoS) requirement on the spectrum leased as a pre-condition for cooperation. Previous work assumed that a single primary user makes spectrum leasing decisions in the presence of possibly multiple secondary users. In this Chapter, the analysis is extended to accommodate multiple primary users, by adopting the framework of Generalized Nash Equilibrium (GNE) problems. Accordingly, multiple primary users, each owning its own spectral resource, compete for the cooperation of the available secondary users under a shared constraint on all spectrum leasing decisions set by the secondary QoS requirements.

A general formulation of the problem is given and solutions are proposed with different signaling requirements among the primary users. Then, application of the framework is discussed for a practical example that includes communication over fading channels with retransmissions. Numerical results bring insight into the advantages of spectrum leasing and of the effectiveness of the proposed solutions.

Chapter 4 studies interference forwarding as an incentive for cooperation. A primary (licensed) link communicates in the presence of an interferer. A secondary (unlicensed) link is also present that can operate only if leased spectrum by the primary link. The Chapter investigates the possibility that the secondary link gain credit to access the channel by forwarding information about the interference to the primary receiver so as to enable interference mitigation. In particular, the primary link leases part of the spectrum to the secondary link if the advantage accrued from interference mitigation overcomes the loss of spectral resources due
to spectrum leasing. This form of primary-secondary cooperation contrasts with previously proposed approaches whereby the secondary user gains credit by forwarding the primary packet, and not the interference. Numerical results demonstrate conditions under which the proposed approach based on interference forwarding can outperform the said known approach.

Finally, Chapter 5 focuses on multi-tier wireless networks such as femtocells overlaid by macrocells. Femtocells promise to increase the number of users served in a given macrocell by creating indoor hotspots through the deployment of home base stations (HBSs) connected to the mobile operator network via cheap backhaul links (i.e., the Internet). However, the interference created by femtocell transmissions may critically impair the performance of the macrocell users. In this Chapter, a novel approach to the operation of HBSs is proposed, whereby the HBSs act as relays with the aim of improving transmission reliability for femtocell users and, possibly, also macrocell users. The proposed approach enables cooperative strategies between HBS and macrocell base stations (BSs), and is unlike the conventional deployment of femtocells where HBSs operate as isolated encoders and decoders.

The performance advantages of the proposed approach are evaluated by studying the transmission reliability of macro and femto users for a quasi-static fading channel in terms of outage probability and diversity-multiplexing trade-off for uplink and, more briefly, for downlink. Overall, the analytical and numerical results lend evidence to the fact that operating femtocells as relays may potentially offset the performance losses associated with the presence of additional active users in the cell due to femtocells and even provide overall performance gains.
CHAPTER 2

INFORMATION-THEORETICAL VIEW OF SPECTRUM LEASING

2.1 Introduction

Spectrum leasing is a spectrum management technique that enables regulated coexistence between licensed (primary) and unlicensed (secondary) users in "cognitive radio" networks. With spectrum leasing, unlike the commons model of cognitive radio, primary users actively lease spectrum for secondary access in exchange for some remuneration (see, e.g., [51]). As proposed in [3], spectrum leasing may be effectively implemented by allowing secondary users to pay back the primary for the leased spectrum via cooperation, i.e., by relaying primary packets. With this solution, rather than attempting to fill so-called primary "white spaces", as for the commons model, secondary nodes can capitalize on good channel conditions from primary transmitters and to primary receivers to successfully relay primary packets and, through this, gain access to the primary-owned spectrum. Generally, the secondary nodes are interested in acquiring spectrum access through this mechanism, but only if a minimum quality-of-service (e.g., rate) constraint is guaranteed on their traffic.

While [3] focuses on simple transmission strategies that orthogonalize primary and secondary transmission (e.g., TDMA), in this chapter, the idea of spectrum leasing via cooperation is more thoroughly studied from an information-theoretic standpoint. This chapter concentrates on a scenario with one primary node and multiple secondary nodes communicating to a common receiver. The spectrum leasing problem (run at the primary nodes) consists of maximizing the primary rate over a set of strategies that allow for secondary cooperation, but only under the constraint that minimum secondary private rates (i.e., quality-of-service constraints) are guaranteed. Specifically, the scenario of interest is modelled as a multirelay channel where each
relay (secondary user) has a private message for the destination (see Figure 5.1). To address the spectrum leasing problem, achievable rate regions are proposed for discrete memoryless and Gaussian models via Decode-and-Forward (DF) and Compress-and-Forward (CF). The model of Figure 5.1 and the proposed achievable schemes extend the analysis for the multirelay channel (without private relay messages) of [52]-[53] and the single-relay channel with private messages of [54]. Related works are [55], which studies the two-way relay channel with "piggybacking" of a private relay message, papers [56], which address a two-receiver broadcast channel where the relay is also the recipient of a private message, and [57], where "pairwise" cooperation is analyzed.

2.2 System Model

In this section, the discrete memoryless multirelay channel with private messages (MCPM) of Figure 5.1 is described. While the proposed strategies are designed so that they can be easily extended to an arbitrary number $K$ of secondary nodes (or relays), the chapter focuses for simplicity on the case of two secondary nodes, i.e., $K = 2$. Nodes indexed as 0, 1, 2, and 3 identify the primary transmitter, secondary node 1, secondary node 2, and the common destination, respectively. Notice that node 0 will be referred to as either primary or source, and nodes 1 and 2 as either secondary users or relays, to emphasize that the considered model applies more generally than only to the spectrum leasing scenario.

The channel of Figure 5.3 is memoryless and used without feedback, and is defined by the conditional probability distribution $p(y_1, y_2, y_3|x_0, x_1, x_2)$, where $x_0, x_1$ and $x_2$ represent the inputs of the primary source, secondary node 1, and secondary node 2, respectively, which are chosen from the corresponding finite input alphabets $\mathcal{X}_0, \mathcal{X}_1, \text{and} \mathcal{X}_2$, whereas, $y_1, y_2$ and $y_3$ represent the outputs of secondary node 1,
secondary node 2, and the destination, respectively, which belong to the corresponding finite output alphabets $Y_1, Y_2,$ and $Y_3$.

\[ W_0 \rightarrow \text{Primary TX} \xrightarrow{X_0^n} p(y_1, y_2, y_3 | x_0, x_1, x_2) \rightarrow \text{Common RX} \xrightarrow{W_0, \hat{W}_1, \hat{W}_2} \]

**Figure 2.1** Spectrum leasing via cooperation modelled as a multirelay channel where relays (secondary users) have private messages (MCPM) for the common destination: $K$ relays are willing to collaborate with the primary transmitter on the condition that minimum individual secondary rates are guaranteed.

A $(n, R_0, R_1, R_2)$ code for the MCPM is defined by: (i) Three message sets $W_0 = \{1, 2, ..., 2^{nR_0}\}$, $W_1 = \{1, 2, ..., 2^{nR_1}\}$, and $W_2 = \{1, 2, ..., 2^{nR_2}\}$; (ii) An encoding function at the primary node $f_0^{(n)}: W_0 \rightarrow X_0^n$ that maps the primary message into a codeword $x_0^n = f_0^{(n)}(w_0)$; (iii) $2n$ relay functions $f_{j,i}^{(n)}: Y_j^{n-1} \times W_j \rightarrow X_j$ for relay index $j = 1, 2$ and $i = 1, ..., n$, that map previously received samples and private message into the symbol sent at each time $i$ as $x_{1,i} = f_{1,i}^{(n)}(y_1, y_2, ..., y_1, i-1, w_1)$ and $x_{2,i} = f_{2,i}^{(n)}(y_2, y_2, ..., y_2, i-1, w_2)$; (iv) a decoding function at the destination $d_3: Y_3^n \rightarrow W_0 \times W_1 \times W_2$ as $(\hat{W}_0, \hat{W}_1, \hat{W}_2) = d(y_3^n)$.

Once an achievable rate region $\mathcal{R}$ of rates $(R_0, R_1, R_2)$ has been obtained for a given transmission strategy, the spectrum leasing problem for the primary user (node 0) consists in selecting rates such that

\[
\max R_0 \text{ s.t. } \left\{ \begin{array}{l}
(R_0, R_1, R_2) \in \mathcal{R} \\
R_j \geq R_{j,\text{min}} \text{ for } j = 1, 2
\end{array} \right., \quad (2.1)
\]
so that the primary rate $R_0$ is maximized and the minimum quality-of-service constraints (rates) $R_{j,\text{min}}$ requested by the cooperating secondary (relay) nodes are guaranteed. The definition can be immediately extended to more than two secondary nodes. Notice that the primary could also select only a subset of secondary nodes, say $\mathcal{S}$ (where possibly $\mathcal{S} = \emptyset$) for relaying, in which case problem (2.1) is easily adapted by considering the corresponding achievable rate region where only the selected relays are active and including only the rate constraints $R_j \geq R_{j,\text{min}}$ for $j \in \mathcal{S}$. The final spectrum leasing decision will be obtained by choosing the subset $\mathcal{S}$ that maximizes the primary rate $R_0$. In the following, the derivation of achievable rate regions $\mathcal{R}$ will be focused on and problem (2.1) will be revisited in Sec. V.

### 2.3 Achievable Rates

In this section, achievable rate regions for MCPM are derived using DF and CF.

#### 2.3.1 Decode-and-Forward (DF)

Within the basic DF approach, a number of possible strategies will be considered to be employed at source and relays. In particular, at first an extension of the "multihop" technique of [58] will be considered and then of the parity-forwarding (PF) approach of [53] from the multirelay channel (without relay messages) to the MCPM. As discussed below, the proposed techniques also extend some of the results of [54] from the single-relay channel to the MCPM.

##### 2.3.1.1 DF-MultiHop (DF-MH)

In the MH strategy of [58] "downstream" nodes decode the source message (sent in a previous block) based on the signals received from the "upstream" nodes using sliding-window decoding. The destination may either use backward or sliding-window decoding (see, e.g., discussion in [59]). Here an extension of this technique that allows
transmission of the private secondary messages is proposed. In the proposed method, successive interference cancellation at the destination is performed, by decoding the primary message first (essentially treating the secondary private messages as noise) and then decoding the secondary message after having cancelled the primary signal.

**Proposition:** *(DF-MH Achievable Rate Region)* The convex hull of the union of all sets of rates \((R_0, R_1, R_2)\) that satisfy

\[
R_0 < \min \{I(X_0; Y_1|X_1, U_1, U_2), \]
\[
I(X_0, U_1; Y_2|X_2, U_2), \] \hspace{1cm} (2.2a)
\[
I(X_0, U_1, U_2; Y_3)\} \] \hspace{1cm} (2.2b)

and

\[
R_1 < I(X_1; Y_3|X_0, X_2, U_1, U_2) \] \hspace{1cm} (2.3a)
\[
R_2 < I(X_2; Y_3|X_0, X_1, U_1, U_2) \] \hspace{1cm} (2.3b)
\[
R_1 + R_2 < I(X_1, X_2; Y_3|X_0, U_1, U_2) \] \hspace{1cm} (2.3c)

for some joint distribution

\[
p(u_1, u_2)p(x_0|u_1, u_2)p(x_1|u_1, u_2)p(x_2|u_2) \]
\[
p(y_1, y_2, y_3|x_0, x_1, x_2) \] \hspace{1cm} (2.4)

is achievable for the MCPM via DF-MH.

**Proof:** See [60].

**Remark:** The two auxiliary random variables \(U_1\) and \(U_2\) represent the contribution of relay 1 and 2, respectively, to the primary transmission. Private relay messages of rates \(R_1\) and \(R_2\) are sent superimposed on such cooperative signals.

**Remark:** In the absence of secondary private messages \((R_j = 0 \text{ for } j \in \{1, 2\})\), the achievable rate region reduces to the one derived in [58, Theorem 3.1] for the
two-level relay channel (by setting $X_1 = U_1$ and $X_2 = U_2$). Also, with null primary rate ($R_0 = 0$), the result in the above Proposition coincides with the MAC capacity region by setting the auxiliary random variables $U_j$ to a constant. Finally, a special case of the achievable rates in Theorem 1 of [54] (with $R_{12} = 0$ and $V = X_1$ in the notation of [54]) can be recovered by letting $X_1 = U_1 =$constant (i.e., shutting down the first relay) and neglecting condition (2.2a).

2.3.1.2 DF-Parity Forwarding (PF)

With the PF scheme proposed in [53], the first relay does not cooperate by forwarding the entire primary message sent in the previous block, as in DF-MH of [58], but only parity bits (this is also referred to as irregular encoding [59]). This design choice eases decoding requirements at the second relay that can now decode only the parity bits forwarded by the first relay. In particular, in the absence of relay private messages, reference [53] shows that the PF strategy improves over the MH approach of [58] if the channel from source to relay 2 is poor, and it achieves capacity for doubly degraded channels as defined in [53].

**Proposition:** (DF-PF Achievable Rate Region) The convex hull of the union of all sets of rates $(R_0, R_1, R_2)$ that satisfy

\[
R_0 < \min \{ I(X_0; Y_1|X_1, U_1, U_2), \quad \text{(2.5a)} \\
I(X_0; Y_3|U_1, U_2) + I(U_1; Y_2|X_2, U_2), \quad \text{(2.5b)} \\
I(X_0, U_1, U_2; Y_3) \} \quad \text{(2.5c)}
\]

and (2.3) for some joint distribution (2.4) is achievable for the MPCM via DF-PF.

**Proof:** See [60].

**Remark:** The difference between the achievable rate regions of DF-MH and DF-PF is only in the inequalities (2.2b) and (2.5b). In fact, inequality (2.2b) accounts for the fact that DF-MH requires relay 2 to fully decode the source message, whereas
(2.5b) reflects the fact that relay 2 only decodes the parity information transmitted by relay 1 (of rate $I(U_1; Y_2|X_2, U_2)$).

### 2.3.2 Compress-and-Forward (CF)

In this section, the basic idea is that each relay compresses its received signals and performs random binning before transmitting the quantization index (binning exploits the correlation with the received signal of the destination, which serves as side information), see, e.g., [59]. The bin index is sent superimposed on the codeword that carries the relay private message. The destination first decodes the secondary private messages, then decompresses the compressed relay observations and finally decodes the primary message.

**Proposition: (CF Achievable Rate Region)** The convex hull of the union of all sets of rates $(R_0, R_1, R_2)$ that satisfy

$$R_0 < I(X_0; \hat{Y}_1, \hat{Y}_2, Y_3|U_1, U_2, X_1, X_2)$$  \hspace{1cm} (2.6)

and

$$R_1 < I(U_1; Y_3|U_2)$$  \hspace{1cm} (2.7a)

$$R_2 < I(U_2; Y_3|U_1)$$  \hspace{1cm} (2.7b)

$$R_1 + R_2 < I(U_1, U_2; Y_3)$$  \hspace{1cm} (2.7c)

for some joint distribution that factorizes as

$$p(x_0)p(u_1)p(u_2)p(x_1|u_1)p(x_2|u_2)p(\hat{y}_1|y_1, x_1, u_1)$$

$$p(\hat{y}_2|y_2, x_2, u_2)p(y_1, y_2, y_3|x_0, x_1, x_2)$$  \hspace{1cm} (2.8)

and satisfies the inequalities

$$I(\hat{Y}_1; Y_1|X_1) \leq I(X_1, \hat{Y}_1; Y_3|U_1, U_2, X_2)$$  \hspace{1cm} (2.9)
\begin{align}
    I(\hat{Y}_2; Y_2 | X_2) &\leq I(X_2, \hat{Y}_2; Y_3 | U_1, U_2, X_1) \\
    I(\hat{Y}_1; Y_1 | X_1) + I(\hat{Y}_2; Y_2 | X_2) &\leq I(X_1, X_2, \hat{Y}_1, \hat{Y}_2; Y_3 | U_1, U_2)
\end{align} (2.10) (2.11)

is achievable for the MCPM via CF.

**Proof:** See [60].

**Remark:** The CF strategy does not require the secondary nodes to be aware of the primary codebooks, which may reduce the signalling burden between primary and secondary nodes in the spectrum leasing application. Moreover, unlike the previous Propositions, here, random variables \(U_1\) and \(U_2\) represent the codebooks used to convey the private messages of secondary node 1 and secondary node 2, respectively (and not the cooperative signals).

**Remark:** The above achievable rates reduce to a special case of Theorem 2 of [54] (with \(R_{12} = 0\), \(U_1 = X_1\) in the notation of [54]) by removing the second relay (i.e., setting \(X_2, U_2, \) and \(\hat{Y}_2\) to constants).

**Remark:** For all schemes developed in this chapter, the destination may either use backward decoding or sliding window obtaining the same performance.

### 2.4 Gaussian Model

In this section, the Gaussian MCPM, which is defined by the following received signals for the secondary nodes and the destination, is considered:

\[
    \begin{align*}
    Y_1 &= g_{01}X_0 + Z_1 \\
    Y_2 &= g_{02}X_0 + g_{12}X_1 + Z_2 \\
    Y_3 &= g_{03}X_0 + g_{13}X_1 + g_{23}X_2 + Z_3,
    \end{align*}
\]
Figure 2.2 The Gaussian MCPM. Channel gains $g_{tr}$ depend on the distances $d_{tr}$ and the path loss exponent $\nu$.

where $Z_1$, $Z_2$ and $Z_3$ are independent, zero-mean, Gaussian noise with variances $N_1$, $N_2$ and $N_3$, respectively and $g_{tr}^2$ represent the channel power gains, as discussed below. Consider the geometry of Figure 2.2 in which $d_{tr}$ is the distance between transmitting node $t$ and receiving node $r$. Assume that the distance between the source and the common destination is normalized ($d_{03} = 1$). This is equivalent to the simple collinear geometry in which all nodes lay on a straight line. The corresponding channel gains are then defined as $g_{tr}^2 = \frac{1}{d_{tr}^\nu}$ $\forall t \in \{0, 1, 2\}$, $r \in \{1, 2, 3\}$ and $t < r$, where $\nu$ is the path loss exponent. The following power constraints $\frac{1}{n} \sum_{i=1}^{n} E[X_i^2] \leq P_t$ for $t = 0, 1, 2$ are imposed. In the following, the transmission strategies studied in the previous sections will be adapted to the Gaussian model at hand.
2.4.1 Decode-and-Forward (DF)

Define \( C(x) = \frac{1}{2} \log_2 (1 + x) \), and \( \bar{x} = 1 - x \).

**Proposition:** (DF-MH Achievable Rate Region) The convex hull of the union of all sets of rates \((R_0, R_1, R_2)\) that satisfy (2.12) and

\[
R_1 < C\left( \frac{g_{13}^2 \gamma_1 P_1}{N_3} \right) \tag{2.13a}
\]

\[
R_2 < C\left( \frac{g_{23}^2 \gamma_2 P_2}{N_3} \right) \tag{2.13b}
\]

\[
R_1 + R_2 < C\left( \frac{g_{13}^2 \gamma_1 P_1 + g_{23}^2 \gamma_2 P_2}{N_3} \right) \tag{2.13c}
\]

for some value of the parameters \( 0 \leq \beta_j \leq 1 \) and \( 0 \leq \gamma_j \leq 1, j = 1, 2 \), is achievable via DF-MH.

**Remark:** As seen in the proof below, parameters \( \beta_1 \) and \( \beta_2 \) control the power allocated by the source (node 0) to the source-to-destination message first transmitted in the previous blocks \( b - 1 \) and \( b - 2 \) respectively (according to the block-Markov strategy). Parameters \( \gamma_1 \) and \( \gamma_2 \) instead rule the power portions allocated by relay 1 and relay 2 respectively to convey private messages.

**Proof:** Follows from the first Proposition by fixing the following auxiliary variables: \( U_2 \sim N(0, \beta_2 P_1) \), \( U_1' \sim N(0, \beta_1 P_1) \), \( U_1 = U_2 + U_1' \), \( X_0' \sim N(0, (\beta_1 + \beta_2) P_0) \), \( X_1' \sim N(0, \gamma_1 P_1) \), \( X_2' \sim N(0, \gamma_2 P_2) \), \( X_0 = U_1 + X_0' \), \( X_1 = \sqrt{\frac{\gamma_1 P_1}{(\beta_1 + \beta_2) P_0}} U_1 + X_1' \) and \( X_2 = \sqrt{\frac{\gamma_2 P_2}{\beta_2 P_1}} U_2 + X_2' \).

**Proposition:** (DF-PF Achievable Rate Region) The convex hull of the union of all sets of rates \((R_0, R_1, R_2)\) that satisfy

\[
R_0 < \min \left\{ \left( (2.12a), C\left( \frac{g_{03}^2 \beta_1 + \beta_2 P_0}{g_{13}^2 \gamma_1 P_1 + g_{23}^2 \gamma_2 P_2 + N_3} \right) \right) \right. \right.
\]

\[
\left. + C\left( \frac{2g_{03}^2 \beta_1 P_1}{g_{02}^2 \beta_1 + \beta_2 P_0 + g_{12}^2 \gamma_1 P_1 + N_2} \right) \right\} \tag{2.14}
\]
\[ R_0 < \min \left\{ C \left( \frac{g^{2\beta_1+\beta_2}P_0}{N_1} \right), \right. \]
\[ \left. C \left( \frac{g^{2\beta_2}P_0+g^{2\gamma_1}P_1+2g^{2}\sqrt{\beta_1\beta_2\gamma_1}P_0P_1}{g^{2\gamma_1}P_1+N_2} \right), \right. \]
\[ \left. C \left( \frac{\left( g^{2}P_0+g^{2}\gamma_1P_1+g^{2}\gamma_2P_2+2g^{2}\sqrt{(\beta_1+\beta_2)\gamma_1}P_0P_1 \right)}{g^{2}\gamma_1P_1+2g^{2}\sqrt{\beta_2\gamma_1\gamma_2}P_1P_2} \right) \left( g^{2}P_1+g^{2}\gamma_2P_2+N_3 \right)^{-1} \right\} \]

and (2.13) for some value of the parameters 0 \leq \beta_j \leq 1 and 0 \leq \gamma_j \leq 1, j = 1, 2, is achievable for the MCPM via DF-PF.

**Proof:** Follows from the second Proposition by fixing the same auxiliary variables used in DF-MH.

**Remark:** A special case of the achievable rate in Corollary 1 of [54] (with \( R_{12} = 0 \) and \( \alpha = 0 \) in the notation of [54]) can be recovered from the Proposition above by setting all channel gains equal to 1, \( P_1 = 0, \beta_1 = 0 \) and neglecting condition (2.12a).

### 2.4.2 Compress-and-Forward (CF)

**Proposition:** (CF Achievable Rate Region) The convex hull of the union of all sets of rates \((R_0, R_1, R_2)\) that satisfy (4.14)

\[ R_1 < C \left( \frac{g^{2}\gamma_1P_1}{g^{2}P_0+g^{2}\gamma_1P_1+g^{2}\gamma_2P_2+N_3} \right), \]
\[ R_2 < C \left( \frac{g^{2}\gamma_2P_2}{g^{2}P_0+g^{2}\gamma_1P_1+g^{2}\gamma_2P_2+N_3} \right), \]
\[ R_1+R_2 < C \left( \frac{g^{2}\gamma_1P_1+g^{2}\gamma_2P_2}{g^{2}P_0+g^{2}\gamma_1P_1+g^{2}\gamma_2P_2+N_3} \right), \]
\[ N_{c_1} \geq \frac{g_{01}^2 P_0 N_3 + N_1 (g_{03}^2 P_0 + N_3)}{g_{13}^2 \gamma_1 P_1}, \]  
\[ N_{c_2} \geq \frac{g_{02}^2 P_0 N_3 + (g_{12}^2 P_1 + N_2) (g_{03}^2 P_0 + N_3)}{g_{23}^2 \gamma_2 P_2}, \]  
and (2.15) for some parameters \( 0 \leq \gamma_j \leq 1, j = 1, 2, \) is achievable via CF.

**Remark:** Parameters \( \gamma_j \) rule the power portions allocated by relay 1 and relay 2 respectively to convey their respective private messages.

**Proof:** Follows from the third Proposition by fixing the following auxiliary variables \( X_0 \sim \mathcal{N}(0, P_0), U_1 \sim \mathcal{N}(0, \gamma_1 P_1), U_2 \sim \mathcal{N}(0, \gamma_2 P_2), X'_1 \sim \mathcal{N}(0, \bar{\gamma}_1 P_1), X'_2 \sim \mathcal{N}(0, \bar{\gamma}_2 P_2), X_1 = U_1 + X'_1, X_2 = U_2 + X'_2, \check{Y}_1 = Y_1 + Z_{c_1} \) and \( \check{Y}_2 = Y_2 + Z_{c_2}, \) where \( Z_{c_1} \sim \mathcal{N}(0, N_{c_1}) \) and \( Z_{c_2} \sim \mathcal{N}(0, N_{c_2}) \) are the independent compression noise of secondary node 1 and secondary node 2, respectively.

**Remark:** A special case of Corollary 2 in [54] (with \( R_{12} = 0 \) \( P_{U_1} = P_0 \) and \( P_{U_2} = 0 \) in the notation of [54]) can be recovered from the Proposition above by nulling \( g_{02} \) and \( P_2 \) and setting all other gains equal to 1.

### 2.5 Numerical Results

In light of the spectrum leasing problem (2.1), this section starts by assessing feasible secondary rate requirements \( R_{j,\text{min}} \) for given target primary rates \( R_0 \), by showing cross section plots of the achievable regions with DF-MH and DF-PF in the \( R_1, R_2 \) plane for different primary rates \( R_0 \) and fixed source-to-secondary-node distances \( d_{01} = 0.3 \), and \( d_{02} = 0.4 \) as shown in Figure 2.3. In general, as the primary rate \( R_0 \) decreases, the set of feasible private rates \( R_1 \) and \( R_2 \) becomes larger, but the increase in secondary sum-rate becomes less relevant as the primary rate decreases. Moreover, DF-PF outperforms DF-MH for sufficiently low primary rates. Finally, notice that nulling the primary rate retrieves the MAC capacity region.

Figure 2.4 shows the maximum primary rate \( R_0 \) achievable via the proposed DF schemes for an equal secondary rate requirement \( R_{1,\text{min}} = R_{2,\text{min}} \) (i.e., solution of
\[ R_0 < C \left( \frac{g_{01}^2 P_0 N_3 (N_2 + N_{c_2}) + g_{03}^2 P_0 (N_1 + N_{c_1}) (N_2 + N_{c_2}) - g_{01}^2 g_{02}^2 P_0^2 N_3}{(N_1 + N_{c_1}) (N_2 + N_{c_2}) N_3} \right) \]
\[ N_{c_1}, N_{c_2} \geq \left( \frac{(g_{03}^2 P_0 + g_{13}^2 \gamma_1 P_1 + N_3) (g_{03}^2 P_0 + g_{23}^2 \gamma_2 P_2 + N_3)}{(g_{03}^2 P_0 + g_{13}^2 \gamma_1 P_1 + g_{23}^2 \gamma_2 P_2 + N_3)^2} \right) \cdot \left( g_{01}^2 P_0 N_3 + (N_1 + N_{c_1}) (g_{03}^2 P_0 + N_3) \right) \]
\[ (g_{02}^2 P_0 N_3 + (g_{12}^2 P_1 + N_2 + N_{c_2}) (g_{03}^2 P_0 + N_3)) (g_{03}^2 P_0 + N_3)^{-2} \quad (2.15) \]

problem (2.1)). The relays positions are represented as pairs of distances \((d_{01}, d_{02})\) (see Figure 2.2). For comparison between spectrum leasing and a non-cooperative scenario, the rate achievable in case none of the secondary nodes is selected is also shown, i.e., \(P_1 = P_2 = 0\). Starting with DF-MH, Figure 2.4 shows that when both secondary nodes are closer to the destination, i.e. \((0.8, 0.9)\), spectrum leasing gains are possible even for large secondary rates. Moving one or both of the secondary nodes closer to the source, i.e. \((0.1, 0.6)\) or \((0.1, 0.15)\), generally decreases the maximum supported secondary rates, but increases the potential primary rate gains. Comparing with DF-PF, it is seen that the latter may outperform DF-MH when the two relays are sufficiently close to one another (e.g., for \((0.1, 0.15)\)). This is due the fact that with PF secondary node 2 decodes only the signal received from secondary node 1.

Finally, the possibility of selecting a subset of one of the relays for spectrum leasing, rather than selecting either both or none as done above (recall also discussion in Sec. 2.2) is investigated. To this end, Fig. 2.5 shows the maximum primary rate \(R_0\) achievable via DF-MH and CF for the case where the relays are at positions \((0.8, 0.9)\), when the source leases spectrum to either the second relay (at position \(d_{02} = 0.9\)) or both relays. In general, selecting only one relay allows primary rate gains from spectrum leasing even for large secondary rate requirements \(R_{j,\text{min}}\). Conversely,
for smaller secondary rate requirements $R_{j,\text{min}}$ it is more advantageous to select both relays when using DF-MH, whereby coherent power gains are accrued, but not necessarily for CF, where the two compression indices are sent as independent codewords to the destination. Finally, when selecting only one relay, it can be seen that the proposed CF scheme attains some gain over DF-MH when the secondary node is closer to the destination.
Concluding Remarks

In this work, an information-theoretic view on the spectrum leasing approach has been provided, by deriving achievable rates for a multirelay channel with private relay message, whereby relay nodes are interpreted as secondary users. The derived achievable rates extend a number of previous works. Moreover, they enable the study of the rate gains achievable by a primary node through spectrum leasing for given secondary rate requirements. Numerical results have shown that these gains can be significant, especially if the primary node is able to optimize the employed cooperation strategy based on the geometry of available secondary users.
Figure 2.5  Spectrum leasing via cooperation using DF-MH and CF for fixed and equal secondary rate requirements $R_{j,\text{min}}$ and selection of either both relays or only the second ($d_{01} = 0.8$, $d_{02} = 0.9$, $P_0 = P_1 = P_2 = 1$, $N_1 = N_2 = N_3 = 0.1$, $\nu = 4$).
CHAPTER 3
GAME-THEORETICAL VIEW OF SPECTRUM LEASING

3.1 Objective

Within the paradigm of spectrum leasing via cooperation, primary (licensed) nodes can lease some of the owned spectral resources to secondary (unlicensed) users in exchange for cooperation. Secondary users in turn set a minimal Quality-of-Service (QoS) requirement on the spectrum leased as a pre-condition for cooperation. Previous work assumed that a single primary user makes spectrum leasing decisions in the presence of possibly multiple secondary users. In this Chapter, the analysis is extended to accommodate multiple primary users, by adopting the framework of Generalized Nash Equilibrium (GNE) problems. Accordingly, multiple primary users, each owning its own spectral resource, compete for the cooperation of the available secondary users under a shared constraint on all spectrum leasing decisions set by the secondary QoS requirements.

A general formulation of the problem is given and solutions are proposed with different signaling requirements among the primary users. Then, application of the framework is discussed for a practical example that includes communication over fading channels with retransmissions. Numerical results bring insight into the advantages of spectrum leasing and of the effectiveness of the proposed solutions.

3.2 Introduction

Consider the scenario where multiple primary (licensed) users $PT_m, m \in \{1, \ldots, M\}$, communicating over orthogonal spectral resources, coexist with multiple secondary (unlicensed) users $ST_n, n \in \{1, \ldots, N\}$, as depicted in Figure 3.1-(a) (for $M = 2$ and $N = 1$). Within the paradigm of spectrum leasing via cooperation [61] (see
also related ideas in [62][63][64][65]), the primary nodes can lease some of the owned spectral resources to secondary nodes in exchange for cooperation. A secondary node $ST_n$ accepts to cooperate in forwarding primary traffic only if offered enough spectral resources to satisfy its own Quality-of-Service (QoS) requirement. For instance, in Figure 3.1, user $ST_1$ accepts to forward data for $PT_1$ and $PT_2$ (see Figure 3.1-(c)) upon being offered fractions, say $\alpha_1$ and $\alpha_2$, of the time-slots (or bandwidths) owned by $PT_1$ and $PT_2$, respectively, for its own transmission (see Figure 3.1-(d)).

Previous work [61] assumed that a single primary user is present that makes spectrum leasing decisions in the presence of possibly multiple secondary users (i.e., $M = 1$, $N > 1$). In this Chapter, the analysis is extended to accommodate multiple primary users, i.e., $M > 1$, by adopting the framework of Generalized Nash Equilibrium (GNE) problems. With multiple primary users, the QoS requirements of the secondary nodes impose a shared constraint on the spectrum leasing decisions of the primary nodes. For instance, in the example of Figure 3.1, $ST_1$ may request a QoS corresponding to a fraction $q \geq 0$ of a time-slot (or bandwidth). Therefore, as long as enough spectrum is collectively leased by $PT_1$ and $PT_2$, i.e., $\alpha_1 + \alpha_2 \geq q$, $ST_1$ will be willing to cooperate with the primary users.

The presence of shared secondary QoS constraints ties the decisions of different primary users, despite the fact that their operation is in orthogonal spectral resources. This scenario is modelled as a GNE problem, which is a generalization of standard Nash Equilibrium (NE) problems (see, e.g. [66]) that includes joint constraints on the actions of the players (Sec. II). GNE solutions are discussed in Sec. III that have different trade-offs between performance and signaling requirements among the primary users. Moreover, an application of the framework is proposed for a scenario such as in Figure 3.1 that includes communication over fading channels with retransmissions is discussed in Sec. IV. Numerical results are also provided in Sec. IV to obtain insight into the performance of the proposed solutions.
3.3 System Model

Consider $M$ primary users $PT_m, m \in \{1, ..., M\}$, each active on a separate spectral resource, and $N = 1$ secondary user $ST$, with a minimum QoS requirement $q$. Note that the subscript has been removed for the secondary user for simplicity of notation. Moreover, the analysis here can be extended to the general case of $N > 1$ secondary users under assumptions that will be discussed below. Each primary user $PT_m$ optimizes a spectrum leasing parameter $\alpha_m$, belonging to a non-empty, compact and convex set $A_m$, thus deciding the amount of spectrum to be leased to ST. Assume $A_m = [0,1]$ for simplicity, so that $\alpha_m$ accounts for the fraction of the spectral resources that $PT_m$ leases to ST. The idea is that the remaining fraction $(1 - \alpha_m)$ of the spectral resources will be used by ST to cooperate with the primary user $PT_m$ as illustrated in Figure 3.1-(c),(d). Therefore, in general, each $PT_m$ is interested in minimizing a cost function $f_m(\alpha_m)$ which is strictly monotonically increasing in $\alpha_m$, i.e., the amount of leased spectrum, and is independent of all other $\alpha_i$ with $i \in \{1, ..., M\}, i \neq m$. Independence of the other $\alpha_i$ follows from the orthogonality of the spectral resources of all primary users. Assuming that $f_m(\alpha_m)$ is continuously differentiable and quasi-convex on the set $A_m$. Specific examples will be given in Sec. III-C and IV.

The secondary user $ST$ accepts to cooperate with the primary users as long as it receives enough leased spectrum. The QoS requirement is parametrized by a value $q > 0$ and imposes a joint constraint on all the spectrum decisions $\alpha = (\alpha_1, ..., \alpha_M)$ as

$$g(\alpha, q) \leq 0,$$

(3.1)

where $g(\alpha, q)$ is a convex and continuously differentiable function in $\alpha \in A_1 \times ... \times A_M$ for each $q > 0$. For instance, following the example given in Sec. I, a possible choice for $g(\alpha, q)$ is $g(\alpha, q) = -\sum_{m=1}^{M} \alpha_m + q$, which guarantees from (3.1) that the sum of all
fractions of leased spectra, \( \sum_{m=1}^{M} \alpha_m \), is larger than the QoS target \( q \). Overall, each PT\(_m\) attempts to solve the problem

\[
\begin{align*}
\text{minimize} & \quad f_m(\alpha_m) \\
\text{subject to} & \quad g(\alpha, q) \leq 0
\end{align*}
\]  (3.2)

The problems (3.2) for \( m \in \{1, ..., M\} \) are coupled by the shared secondary QoS constraint \( g(\alpha, q) \), and their collection constitutes a GNE problem [67]. A GNE problem generalizes the classical notion of NE problems due to the presence of the joint constraint (3.1).
A practical application of this framework is discussed in Sec. IV. It is noted that generalization of (3.2) to the case of multiple secondary users is straightforward as long as each secondary user ST$_n$ has a fixed QoS requirement, say $q_m$. In fact, the extension requires to include a spectrum leasing decision, say $\alpha_{mn}$, for each pair PT$_m$-ST$_n$, and one QoS constraint for each ST$_n$ in (3.2). In the more general case where QoS requirements $q_m$ can be optimized by the secondary users in a strategic fashion, the problem is more complex. Related scenarios, with $M = 1$, were addressed in [61]. In the next section, a discuss on how the primary users can perform the desired minimization in (3.2) is represented.

3.4 Spectrum Leasing Strategies

As discussed above, each primary user is interested in solving problem (3.2) in order to maximize the advantages accrued from spectrum leasing. In performing this optimization, there is clearly a conflict among the primary users due to the shared QoS constraint (3.1). In the following, two classes of solutions are discussed, namely GNE and Variational Inequality (VI) solutions. As it will be shown, these two classes strike different trade-offs between signaling requirements among the primary users and overall system performance.

3.4.1 Generalized Nash Equilibrium (GNE) Strategies

Define as $\alpha_{-m}$ the vector obtained from $\alpha$ by removing $\alpha_m$. For a fixed $\alpha_{-m}$, let $S_m(\alpha_{-m}) \subseteq A_m$ be the set of solutions, possibly empty, of problem (3.2). A GNE $\alpha$ is any vector such that

$$\alpha_m \in S_m(\alpha_{-m}) \text{ for all } m \in \{1, \ldots, M\}.$$  \hspace{1cm} (3.3)

In other words, a GNE $\alpha$ is such that any $m$th entry $\alpha_m$ solves problem (3.2) given the other entries $\alpha_{-m}$. In other words, a GNE $\alpha$, generalizing the concept of NE,
corresponds to a solution that discourages unilateral deviations by the players (here, primary users) under the joint constraint (3.1). Given the assumptions made, it can be proved that the GNE problem at hand admits at least one GNE (Theorem 4.1 of [67]). As it will be discussed below, in fact, there are typically many GNEs.

The question arises as to how the primary users can select spectrum leasing decisions \( \alpha_m \) that are GNEs. The following distributed algorithm (with the signaling requirements discussed below) can be proved to have the property that, if a limit point \( \alpha \) is reached, that point \( \alpha \) is a GNE [67].

**Algorithm GNE:**

1. **Choose a feasible initial vector** \( \alpha^0 = [\alpha^0_1 \cdots \alpha^0_M] \in A_1 \times \cdots \times A_M \), with iteration index \( j = 0 \);
2. **If** \( \alpha^j \) **satisfies a suitable termination criterion, stop and take** \( \alpha = \alpha^j \);
3. **For** \( m \in \{1, \ldots, M\} \), **find a solution** \( \alpha^j_{m+1} \in S_m(\alpha^j_{-m}) \);
4. **Set** \( \alpha^{j+1} = [\alpha^{j+1}_1 \cdots \alpha^{j+1}_M] \), and \( j \leftarrow j + 1 \), then go to (s.1).

The algorithm above, which is an instance of Gauss-Seidel iterations, requires the secondary to communicate the QoS value \( q \) to all primary users at the beginning of the decision process. Then, each iteration requires the primary users to exchange their previous decisions \( \alpha^j \).

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3.5.1 Generalized Nash Equilibrium (GNE) Strategies

Define as $\alpha_{-m}$ the vector obtained from $\alpha$ by removing $\alpha_m$. For a fixed $\alpha_{-m}$, let $S_m(\alpha_{-m}) \subseteq A_m$ be the set of solutions, possibly empty, of problem (3.2). A GNE $\alpha$ is any vector such that

$$\alpha_m \in S_m(\alpha_{-m}) \text{ for all } m \in \{1, ..., M\}. \quad (3.4)$$

In other words, a GNE $\alpha$ is such that any $m$th entry $\alpha_m$ solves problem (3.2) given the other entries $\alpha_{-m}$. In other words, a GNE $\alpha$, generalizing the concept of NE, corresponds to a solution that discourages unilateral deviations by the players (here, primary users) under the joint constraint (3.1). Given the assumptions made, it can be proved that the GNE problem at hand admits at least one GNE (Theorem 4.1 of [67]). As it will be discussed below, in fact, there are typically many GNEs.

The question arises as to how the primary users can select spectrum leasing decisions $\alpha_m$ that are GNEs. The following distributed algorithm (with the signaling requirements discussed below) can be proved to have the property that, if a limit point $\alpha$ is reached, that point $\alpha$ is a GNE [67].

**Algorithm GNE:**

(s.0) Choose a feasible initial vector $\alpha^0 = [\alpha_1^0 \cdots \alpha_M^0] \in A_1 \times \cdots \times A_M$, with iteration index $j = 0$;

(s.1) If $\alpha^j$ satisfies a suitable termination criterion, stop and take $\alpha = \alpha^j$;

(s.2) For $m \in \{1, ..., M\}$, find a solution $\alpha_{m}^{j+1} \in S_m(\alpha_{-m}^j)$;

(s.3) Set $\alpha^{j+1} = [\alpha_1^{j+1} \cdots \alpha_M^{j+1}]$, and $j \leftarrow j + 1$, then go to (s.1).

The algorithm above, which is an instance of Gauss-Seidel iterations, requires the secondary to communicate the QoS value $q$ to all primary users at the beginning of the decision process. Then, each iteration requires the primary users to exchange their previous decisions $\alpha^j$. 

3.5.2 Variational Inequality (VI) Strategies

GNE solutions are simple to obtain using "Algorithm GNE" discussed above (although convergence in principle is not guaranteed). However, there are typically many GNEs and most GNEs typically correspond to solutions that are rather inefficient from the standpoint of the system performance [68]. A subclass of GNEs that turns out to have desirable performance is given by the so called VI solutions. A point $\alpha$ is a VI solution if there exists a parameter $\mu \geq 0$ such that $\alpha$ is a NE of the strategic game set by the simultaneous solution of the problems

$$\min_{\alpha_m \in A_m} f_m(\alpha_m) + \mu g(\alpha, q)$$

for $m \in \{1, ..., M\}$, and the conditions $\mu g(\alpha, q) = 0$ and $g(\alpha, q) \leq 0$ are satisfied.

Parameter $\mu$ can be interpreted as a "price" variable that inflicts an additional cost to the primary users in case the constraint $g(\alpha, q) \leq 0$ is not satisfied. It can be seen that, in general, VI solutions are also GNEs, but the converse is not true. Moreover, given the aforementioned assumptions, at least one VI solution always exists [68]. In important cases, such as when the cost functions $f_m(\alpha_m)$ are strongly convex\(^1\), the VI solution can also be proved to be unique [68].

It will be seen below that a VI solution can be attained by a distributed algorithm with stronger signaling requirements than "Algorithm GNE". Now, however, it will be shown that VI solutions have very desirable performance in the model at hand. In fact, since the cost function $f_m(\alpha_m)$ of each primary user is independent of all other primary users’ decision variables, it can be seen that a VI solution is also a solution of the following centralized problem

$$\min_{\alpha \in A_1 \times \ldots \times A_M} \sum_{m=1}^{M} f_m(\alpha_m)$$

subject to $g(\alpha, q) \leq 0$.

\(^1\)A continuously differentiable function $f_m$ is said to be strongly convex on $A_m$ if $(\nabla f_m(u) - \nabla f_m(v))^T (u - v) \geq m \|u - v\|^2$, $m > 0$, $\forall u, v \in A_m$
This can be proved by noting that the KKT optimality conditions \[69\] of problem (3.6) coincide with the collection of the KKT optimality conditions of problems (3.5) along with the additional conditions mentioned above, \( \mu \geq 0, \mu g(\alpha, q) = 0 \) and \( g(\alpha, q) \leq 0 \) (see also [70])\(^2\). Problem (3.6) corresponds to the centralized optimization of spectrum leasing that minimizes the sum of all costs. The fact that a VI solution also solves (3.6) implies that VI solutions are efficient in the sense of minimizing the sum-cost.

The following algorithm is known to converge to a VI solution.

**Algorithm VI:**

\((s.0)\) Choose an initial price \( \mu^j \geq 0 \), with iteration index \( j = 0 \), and a step size \( \tau > 0 \);

\((s.1)\) If \( \alpha^j \) satisfies a suitable termination criterion, stop and take \( \alpha = \alpha^j \);

\((s.2)\) Find a NE \( \alpha^j = [\alpha^j_1, \ldots, \alpha^j_M] \) of the game defined by (3.5) for \( m \in \{1, \ldots, M\} \) where \( \mu = \mu^j \). This can be done using Gauss-Seidel iterations as per "Algorithm GNE" with (3.5) in lieu of (3.2);

\((s.3)\) Update the price according to the subgradient rule

\[ \mu^{j+1} = [\mu^j - \tau (g(\alpha^j, q))]^+; \]    \hspace{1cm} (3.7)

\((s.4)\) Set \( j \leftarrow j + 1 \), then go to (s.1).

"Algorithm VI" described above requires two nested loops, instead of a single loop as for "Algorithm GNE". The outer loop updates the price \( \mu \), while the inner loop calculates the required NE for a fixed price. This double loop requires more signaling among the primary users in order to converge, in a proportion that depends on the number of iterations of the outer loop necessary for convergence. In fact, the inner loop of "Algorithm VI" requires approximately the same number of iterations of "Algorithm GNE". Note that the price \( \mu^j \) can be either set by the secondary user at

\(^2\)In other words, \( \sum_{m=1}^{M} f_m(\alpha_m) \) is a potential function for the game at hand (see, e.g., [66])
the beginning of each outer iteration, or, more practically, calculated by each primary user based on the knowledge of $\alpha^j$ and $q$.

### 3.5.3 Linear QoS Secondary Constraints and $M = 2$

Here the results above are specialized to the special case where $M = 2$ primary users and the QoS function is linear as in $g(\alpha_1, \alpha_2, b) = -b_1\alpha_1 - b_2\alpha_2 + q$, where $\alpha_1, \alpha_2 \in \mathcal{A}_1 = \mathcal{A}_2 = [0, 1]$, $b_1, b_2 \geq 1$, $0 \leq q \leq 1$. The QoS constraint (3.1) is thus

$$b_1\alpha_1 + b_2\alpha_2 \geq q, \quad (3.8)$$

so that parameters $b_1, b_2$ weight the usefulness of spectrum leased by PT$_1$ and PT$_2$, respectively, for ST. Moreover, spectrum leasing parameters $\alpha_1, \alpha_2$ represent the fraction of spectral resources leased to ST by PT$_1$ and PT$_2$, respectively. As an example, if the channel from ST towards its destination is in better conditions on the spectrum owned by PT$_1$, then $b_1 \geq b_2$. A linear QoS constraint is meaningful since many metrics such as achievable rates are indeed linear in the fraction of time the ST is allowed to transmit.

Now the GNE and VI solutions are characterized for this example for generic cost functions satisfying the general assumptions. The set of all feasible solutions $\alpha$ satisfying the QoS constraint and $\alpha_1, \alpha_2 \in [0, 1]$ is the shaded region in Figure 3.2. If $q = 0$, due to the assumption of strict monotonicity of the cost functions, the only GNE and VI solution is easily seen to be $\alpha_1 = \alpha_2 = 0$. With $q > 0$, there are an infinite number of GNE, which are given by all the points on the segment shown in Figure 3.2, or equivalently all points of the form $(\alpha_1, \alpha_2) = \left(\alpha_1, \frac{q - b_1\alpha_1}{b_2}\right)$ for $\alpha_1 \in [0, q/b_1]$. This is because, for any point $\alpha$ on this segment, no primary user can further reduce its cost while still satisfying the secondary QoS constraint (recall the the cost function $f_m(\alpha_m)$ is strictly increasing in $\alpha_m$). VIs solutions are given
instead by the point or points on this segment that minimize (3.6), that is, the sum cost function \( f_1(\alpha_1) + f_2(\alpha_2) \).

\[
\alpha_1 = \max \left\{ 0, \min \left\{ \left( 1 - \frac{b_2 c_1}{b_1 c_2} + q \right) / \left( 1 + \frac{b_1}{b_2} \right), \frac{q}{b_1} \right\} \right\} \tag{3.9}
\]

and \( \alpha_2 = (q - b_1 \alpha_1) / b_2 \), and is shown in Figure 3.3 for different values of \( c_1 \) and fixed \( b_1 = 1, c_2 = 1, q = 0.5 \) and \( b_2 = 1 \) in Figure 3.3-(a), while \( b_2 = 2 \) in Figure 3.3-(b).

**Figure 3.2** Illustration of the feasible set and the GNE for the case of linear QoS secondary constraints and \( M = 2 \) primary users.

For a more specific example, assume that the rate that the secondary is able to provide for \( \text{PT}_m \) via cooperation is proportional to the fraction of time that is not leased, namely to \((1 - \alpha_m)\). A fairly standard cost function is \( f_m(\alpha_i) = -c_m \log(1 - \alpha_m) \), where \( c_m \geq 0 \) are constants, which for \( c_m = 1 \) leads to the so called proportional fairness solution when solving (3.6). Since this cost is strongly convex, as mentioned above, there is only one VI solution. Using the KKT conditions of problem (3.6), the VI solution is easily found as

\[
\alpha_1 = \max \left\{ 0, \min \left\{ \left( 1 - \frac{b_2 c_1}{b_1 c_2} + q \right) / \left( 1 + \frac{b_1}{b_2} \right), \frac{q}{b_1} \right\} \right\} \tag{3.9}
\]
Figure 3.3 VI solutions for the example of linear QoS secondary constraints and $M = 2$ primary users different values of $c_1$ and (a) $b_2 = 1$; and (b) $b_2 = 2$ ($b_1 = 1, c_2 = 1, q = 0.5$).

It can be seen that as $c_1$ increases the VI solution moves from $(q/b_1, 0)$, where only PT$_1$ leases the spectrum, to $(0, q/b_2)$, where only PT$_2$ leases spectrum.

3.6 Spectrum Leasing via HARQ

In this section, an application of the framework discussed so far is provided.

3.6.1 Setting

Consider the system in Figure 1 with $N = 1$ secondary user, ST, and $M = 2$ primary users, PT$_1$ and PT$_2$. Primary users PT$_1$ and PT$_2$ are active in their dedicated channels (e.g., different frequencies) and their transmission to their respective destinations
PR₁ and PR₂ is slotted and not necessarily synchronous. Nevertheless, the time-slots for the primary users are numbered so that time-slots with the same index take place sufficiently close in time so as to enable the protocol discussed below. In the first slot, primary transmitters PT₁ and PT₂ communicate at fixed rates R₁ and R₂ (bit /channel use) and with powers P₁ and P₂, respectively, directly to their intended primary receivers as in Figure 3.1-(a). If either direct link is in outage, a retransmission takes place and the corresponding primary transmitter may decide to grant the retransmission slot to ST in exchange for cooperation. Specifically, with spectrum leasing, any primary link PTₘ that was in outage in the previous slot offers a fraction of duration 0 ≤ αₘ ≤ 1 to the secondary link for secondary transmission as an incentive for cooperation (Figure 3.1-(d)). The remaining fraction 1 − αₘ is utilized by the secondary user for relaying primary traffic (Figure 3.1-(c)). The aggregate of all primary offers must satisfy a linear secondary QoS requirement q as in (3.8) for given b₁, b₂ = 1 in order for the secondary link to accept cooperation. Cooperation takes place via decode-and-forward, as explained below.

Fading channels are assumed and the power gain for the link between a transmitter i and a receiver j is gᵢⱼ = |hᵢⱼ|² dᵢⱼ⁻η, where hᵢⱼ is the complex Rayleigh fading channel gain between nodes i and j, dᵢⱼ is the distance between the nodes i and j and η is the path loss exponent. Fading channels are constant during the transmission slot, but change independently from one slot to the other. If any packet is not decoded successfully, a Not-Acknowledge (NACK) message is broadcast requesting retransmission. As introduced above, primary links employ type-I HARQ, whereby copies of the same packet are retransmitted and decoded without leveraging previous transmissions, with a maximum number of retransmissions of one. Define

\[ P_{out,ij} = 1 - \exp \left( -\frac{2R_c - 1}{g_{ij}P_i} \right), \]  

\( (3.10) \)
as the outage probability of link $i - j$ when transmission takes place with rate $R_i$ (bit/channel use) and power $P_i$. $m$ will be used as the label to identify the $m$th primary link, $PT_m$ or $PR_m$, and $s$ to identify ST, so that $P_{out,ms}$ is the outage probability on the link between $PT_m$ and ST and $P_{out,mm}$ is the outage probability of the direct link $PT_m$-$PR_m$. Note that the outage probability $P_{out,sm}$ between ST and $PR_m$ depends on the fraction $\alpha_m$ leased to ST. This will be denote explicitly as $P_{out,sm}(\alpha_m) = 1 - \exp\left(-\frac{R_m}{g_{ij} P_i} \alpha_m \right)$.

Four cases are distinguished. (i) Both primary packets are received correctly in the first slot. In this case, no retransmission is required and a new transmissions begin in the next slot. (ii) Only primary $PT_2$ is in outage in the first slot. If the ST was able to decode $PT_2$’s packet in the first slot, in the second slot of $PT_2$ the ST is assigned a fraction $\alpha_2 = q/b_2$ of the spectrum for its own transmission (so as to satisfy (3.8)) and retransmits the primary packet in the remaining fraction $1 - \alpha_2$. If instead ST was not able to decode $PT_2$’s packet, $PT_2$ performs retransmission. $PT_1$, which was not in outage in the first slot, is allowed to send a new packet in its second slot; (iii) Only $PT_2$ is in outage in the first slot. The protocol proceeds as for case (ii) but with the roles of $PT_1$ and $PT_2$ reversed; (iv) Both primary users are in outage in the first slot. If ST was able to decode both packets from the signal received in the first slot, ST can help both primary users in the second slot provided that its QoS constraint (3.8) is satisfied. The fractions $(\alpha_1, \alpha_2)$ leased by $PT_1$ and $PT_2$ in the second slot are decided based on either a GNE or VI solution, as further discussed below. If instead ST was able to decode only one packet, say that of $PT_m$, operation is conducted as in cases (ii) and (iii) above, except that the other primary will perform retransmission. If instead the ST did not decode any packet, retransmissions are performed by the primary users.
3.6.2 GNE Problem Formulation

As explained above, it remains to be discussed how the leased fractions \((\alpha_1, \alpha_2)\) are calculated for the case where both primary users are in outage (case \((iv)\)) and the secondary is able to decode both packets in the first slot. Since the QoS constraint (3.8) must be satisfied, \(\text{PT}_1\) and \(\text{PT}_2\) face a GNE problem (3.2), where the cost functions \(f_m(\alpha_m)\) need to be specified. It will be shown below that the following choice,

\[
f_m(\alpha_m) = (1 - P_{\text{out,ms}}) P_{\text{out,sm}} (\alpha_m),
\]

is well justified and leads to desirable performance. Note that \(f_m(\alpha_m)\) satisfies aforementioned general assumptions, and is, in particular, quasi-convex [69, page 95]. With this choice, parameters \((\alpha_1, \alpha_2)\) at hand will be chosen as either a GNE or a VI solution of the problem (3.2) with QoS constraint (3.8). Note that since (3.11) only depends on the channel statistics, primary and secondary nodes can agree on \((\alpha_1, \alpha_2)\) in advance, by running either ”Algorithm GNE” or ”Algorithm VI” and keep the same decision for as long as the channel statistics remain the same. This protocol provides a generalization of the HARQ-based protocol studied in [61] to the setting with multiple primary users.

To further discuss (3.11), let us calculate the average primary system throughput, \(T_P = E [\text{Packets}] / E [S]\), where \(E [\text{Packets}]\) is the average number of packets received successfully by the primary receivers and \(E [S]\) is the average number of slots. This
can be easily calculated from the description of the system given above as

\[
\frac{E [\text{Packets}]}{E [S]} = 2P_1 + P_2 \left( 1 + (1 - P_{out,11}) + (1 - P_{out,22}) \right) \\
\left( 1 - P_{out,s2} \left( \frac{q}{b_2} \right) \right) + P_{out,2s} (1 - P_{out,22}) \\
+ P_3 (1 + (1 - P_{out,22}) + (1 - P_{out,1s}) \\
\left( 1 - P_{out,s1} \left( \frac{q}{b_1} \right) \right) + P_{out,1s} (1 - P_{out,11}) \\
+ P_4 P_{out,1s} (1 - P_{out,11}) \\
+ (1 - P_{out,2s}) (1 - P_{out,s2} (\alpha_2)) + (1 - P_{out,1s}) (1 - P_{out,s1} (\alpha_1)) \\
+ P_{out,2s} (1 - P_{out,22}) (1 + P_2 + P_3 + P_4)^{-1},
\]

where

\[
P_1 = (1 - P_{out,11}) (1 - P_{out,22}),
\]

\[
P_2 = (1 - P_{out,11}) P_{out,22},
\]

\[
P_3 = P_{out,11} (1 - P_{out,22}),
\]

\[
P_4 = P_{out,11} P_{out,22},
\]

are the probabilities of the events (i), (ii), (iii) and (iv), respectively, discussed above. For instance, \(P_1\) is the probability that packets of \(\text{PT}_1\) and \(\text{PT}_2\) are received successfully, and \(P_2\) is the probability \(\text{PT}_1\)’s packet is successfully received while \(\text{PT}_2\)’s packet was not. To interpret (3.12), note that, for instance, the second term accounts for the average number of packets successfully delivered conditioned on case (ii) above taking place. In fact, the four terms in the sum multiplying \(P_2\) are, respectively, the probability that \(\text{PT}_1\)’s packet was successfully decoded in the first slot, which is equal to 1, the probability of successful transmission of \(\text{PT}_1\)’s packet on the \(\text{PT}_1\)-\(\text{PR}_1\) link in the second slot, which is equal to \((1 - P_{out,11})\), the probability that \(\text{PT}_2\)’s packet was decoded and is being relayed by \(\text{ST}\), which happens with probability
\[(1 - P_{\text{out,2s}}) \left(1 - P_{\text{out,s2}} \left(\frac{q}{k_2}\right)\right)\]

and the probability that PT_2’s packet was not decoded by ST but is being retransmitted by PT_2, which is equal to \(P_{\text{out,2s}} (1 - P_{\text{out,22}})\). Moreover, parameters \((\alpha_1, \alpha_2)\) are obtained as GNE or VI solutions as explained above.

From (3.12), it can be seen that solving the centralized problem (3.6) with the cost functions (3.11) leads to the maximization of the throughput (3.12). This is because the only term that depends on \(\alpha\) in (3.12) is \((1 - P_{\text{out,2s}} (1 - P_{\text{out,s2}} (\alpha_2)) + (1 - P_{\text{out,1s}}) (1 - P_{\text{out,s1}} (\alpha_1))\), whose minimization equals problem (3.6) with cost functions (3.11). This implies that VI solutions, given the discussion in Sec. 3.5.2 maximize the throughput. The same cannot be in general said about general GNE solutions, which as instead given by all points on the segment in Figure 2, as explained in Sec. 3.5.3.

**Remark**: Recall that the secondary QoS requirement \(q\) entails that any time spectrum is leased, and thus secondary cooperation takes place, ST is guaranteed a QoS of \(q\).

### 3.6.3 Numerical Results

In order to provide some numerical insight, assume that PT_1, PR_1, PT_2 and PR_2 have fixed locations in an \(x-y\) plane at \((0, 0.25), (0, -0.25), (0.5, 0.25)\) and \((0.5, -0.25)\), respectively. Let \(d_s\) be the \(x\)-coordinate of ST and assume that it moves on the \(x\)-axis along with SR with a fixed distance between them. Assume fixed transmit powers \(P_1 = P_2 = P_s = 1\), fixed rates \(R_1 = R_2 = 2\) and path loss exponent \(\eta = 3\). The GNE and VI solutions are obtained from "Algorithm GNE" and "Algorithm VI" respectively, by choosing random initialization and averaging over the outcomes.

Figure 3.4 plots the average primary system throughput \(T_P\) (3.12) versus ST’s location \(d_s\) for GNE and VI for \(b_1 = b_2 = 1\) and different values of the secondary QoS \(q = 0, 0.25, 0.5, 1\). The performance is compared with that obtained with no spectrum
leasing (NSL), which correspond to using only direct (re)transmissions (i.e., setting $P_{\text{out},1s} = P_{\text{out},2s} = 1$). First, it is observed that for QoS $q$ sufficiently small (say $q = 0.25$), spectrum leasing provides very relevant performance gains for the primary users, but only as long as the location of ST is in the vicinity of the two primary users (say $-0.25 \leq d_s \leq 0.75$) so that ST is able to cooperate with both users. This way, both users can share the burden of satisfying the secondary QoS constraints and secondary cooperation is still advantageous despite the fact that ST is not in the best position for neither PT_1 or PT_2. If instead the QoS constraint $q$ is large (say $q \geq 0.5$), then spectrum leasing is generally not advantageous for the primary. Moreover, in

Figure 3.4  Average primary system throughput $T_P$ versus distance $d_s$ for spectrum leasing based on GNE and VI solutions and for no spectrum leasing (NSL) for $(b_1, b_2) = (1, 1)$ ($R_1 = R_2 = 2$, $P = 1$, $\eta = 3$).
Figure 3.5  Average primary system throughput $T_P$ versus distance $d_s$ for spectrum leasing based on GNE and VI solutions and for no spectrum leasing (NSL) for $(b_1, b_2) = (1, 2)$ $(R_1 = R_2 = 2, P = 1, \eta = 3)$.

In this case, the largest primary throughput under spectrum leasing is obtained with the ST being closer to either PT$_1$ or PT$_2$, i.e., $d_s \simeq 0$ or $d_s \simeq 0.5$, since, otherwise, the benefits of cooperation are outweighed by the amount of spectrum leased. Note also that when ST is further away from PT$_1$ and PT$_2$, i.e., $d_s < -1.5$ or $d_s > 2$, decoding at the ST is not possible anymore, and the GNE and VI average throughput converge to the NSL one. Finally, it is noted that VI solutions perform better, as expected from the analysis, but the gains are not extremely large. This implies that, if complexity of signaling for calculation of $(\alpha_1, \alpha_2)$ is an issue, one should resort to GNE solutions.
Figure 3.5 plots the average primary system throughput $T_P$ versus $d_S$ for GNE and VI solutions with the same parameters as in the previous plot, with the difference that $b_1 = 1, b_2 = 2$. In other words, in this setting, the spectrum leased by PT$_2$ is worth double to the ST. As discussed, this could be the case if the channel quality experienced by the secondary user is higher on the spectral resource of PT$_2$. It can be seen that in this set-up even for large QoS $q$, say $q = 0.5$, as long as ST is sufficiently close to PT$_2$, say $d_s \simeq 0.5$, spectrum leasing is still advantageous with respect to NSL. This is because, when ST is close to PT$_2$, the opportunity for PT$_2$ to lease spectrum will be more frequent and spectrum leasing by PT$_2$ is more efficient as discussed.

### 3.7 Concluding Remarks

In this Chapter, the framework of spectrum leasing via cooperation has been extended by accounting for the presence of multiple primary users. With spectrum leasing via cooperation, secondary users gain access to the channel by cooperating with the primary users, but under the QoS constraint that they receive enough spectral resources for transmission of their own data. The approach proposed in this chapter is based on the observation that such QoS secondary requirement imposes a shared constraints on the spectrum leasing decisions of the primary users. This is under the reasonable assumption that secondary users are interested in the overall amount of spectral resources they receive. The spectrum leasing problem to be solved at the primary users is then formulated as a Generalized Nash Equilibrium problem, which, unlike conventional strategic games, enables to impose a shared constraints on the players’ actions. Two classes of solutions that have different signaling requirements have been studied. An application of the framework that includes retransmissions has been also studied, leading to insight into the performance of spectrum leasing and of the proposed solutions.
CHAPTER 4

SPECTRUM LEASING VIA INTERFERENCE FORWARDING

4.1 Introduction

Spectrum leasing via cooperation, proposed in [3] (see also [62] for a similar independent idea) rules the local coexistence of primary (licensed) and secondary (unlicensed) users through the following mechanism. Secondary users gain credit to access the channel by cooperating with the primary users, and primary users lease spectrum to the secondary nodes under two conditions: (i) That the advantage on the primary performance accrued from secondary cooperation overcomes the loss of spectral resources for the primary system due to spectrum leasing; and (ii) that secondary nodes are leased enough spectrum to satisfy their Quality of Service (QoS) constraints (which are made known to the primary system to enable spectrum leasing decisions).

Previous work [3][62] has investigated the principle of spectrum leasing via cooperation by assuming that secondary-to-primary cooperation takes place, conventionally, by having the secondary users relay packets for the primary nodes. This conventional approach is referred to as "Cooperative Transmission" (CT). Recent research has demonstrated that, from an information-theoretic standpoint, in interference-limited scenarios, CT can be outperformed by a different form of cooperation, which is referred to as "Cooperative Interference Management" (CIM) [71]. In CIM, the relay node forwards to the destination information about the interference, and not about the useful signal. The rationale of this approach is that, boosting reception of the interference at the receiver can allow the latter to decode the interfering signal jointly with the useful signal, and thus enhance performance via interference mitigation [71][72].
In this chapter, CIM is proposed to enable spectrum leasing via cooperation. In other words, unlike previous work [3][62], the secondary user gains credit to access the channel by forwarding interference, rather than primary signal, information to the primary receiver. Specifically, a spectrum leasing mechanism is proposed based on CIM that leverages the Hybrid ARQ processes at the primary and interfering link. Accordingly, similar to [73], spectrum leasing is prompted by a channel outage on a primary transmission and involves retransmissions of the same packet.

4.2 System Model

Consider the system in Figure 4.1, in which a primary link, between a primary transmitter (PT) and a primary receiver (PR), coexists with a secondary link between a secondary transmitter (ST) and a secondary receiver (SR). An interfering link between interfering transmitter (IT) and interfering receiver (IR) is also present that affects both PR and SR. To fix the ideas, think of PR and SR as the base stations of two neighboring femtocells, with IT being the base station of the macrocell that encompasses the femtocells. In this setting, PT and ST are user of the respective femtocells and IR is a macrocell user. The femtocell containing the ST-SR link has lower priority with respect to the femtocell containing the PT-PR link, and its transmission is ruled via spectrum leasing. The reason for considering IT to be a macro base station is that the proposed strategy based on CIM becomes especially relevant, as will be seen, when the disturbance caused by the interference sets the main bottleneck in the performance of the PT-PR link.

The wireless channel between a pair $a$ of nodes is characterized by a small-scale fading coefficient $h_a$ and by a path loss $d_a^{-\gamma}$, where $d_a$ is the distance between the two nodes and $\gamma$ is the path loss exponent. The power gain for link $a$ is thus $g_a = |h_a|^2 d_a^{-\gamma}$. For instance, the power gain, fading coefficient and distance for the PT-ST link are $g_{PS}, h_{PS}$ and $d_{PS}$, respectively. Figure 4.1 illustrates all channel gains. Time is
Figure 4.1 System model: A primary link PT-PR and a secondary link ST-SR communicate in the presence of an interfering link IT-IR. To fix the ideas, IT may be the macrobase station of a a macrocell, and the PT and ST are two base stations of two neighboring femtocells.

slotted. A block Rayleigh fading model is assumed, in which all fading channels stay constant during each transmission slot, but change independently from slot to slot. No link Channel State Information (CSI) is assumed at the transmitters, but full CSI is available at the receivers. A primary packet carries $R_P$ bits/s/Hz, which is referred to as primary rate, while the secondary rate is $R_S$ and the interferer rate is $R_I$. Assume that the codebook used by the interferer is known at PR and SR.

4.3 Transmission Strategies

Both links PT-PR and IT-IR employ type-I HARQ with a maximum number of attempts (original and retransmissions) of $K \geq 2$ and $K_I \geq 2$, respectively. Recall that with type-I HARQ, the transmitter retransmits a copy of the same packet at
every new attempt, and the receiver discards previously received packets and decodes based only on the last received signal. If the packet is unsuccessfully decoded at the last attempt, i.e., the $K$th for the primary link and the $K_I$th for the interfering link, the packet is dropped and a new packet is transmitted in the next slot. Type-I HARQ is selected for simplicity of analysis, but the proposed principle can be applied also to more complex forms of HARQ.

![Diagrams](image)

**Figure 4.2** The secondary transmitter (ST) gains access to the spectrum either by cooperating with the primary for transmission of the primary packet (spectrum leasing via cooperative transmission, SL-CT) or by forwarding interference information (spectrum leasing via cooperative interference management, SL-CIM): (a) primary transmission; (b) cooperation slot and leased slot.

The process can be described by following the transmission of a primary packet and denoting the first transmission slot of a primary packet as slot $i = 1$, the second transmission (or first retransmission) slot as $i = 2$, and so forth until the $K$th primary transmission. The state of the HARQ process of the interferer at time slot $i \in \{1, ..., K\}$ is described by a variable $U_{I,i} \in \{1, ..., K_I\}$, so that $U_{I,i} = k$ if in slot $i$ the
interferer (re)transmits the current packet for the $k$th time (i.e., $k = 1$ corresponds to the first transmission, etc.). For simplicity of analysis, assume that $U_{i,1}$ is uniformly distributed in $\{1, ..., K\}$.

In the proposed approach, spectrum leasing is enabled by errors on the PT-PR link. Specifically, PT can follow three different policies on how to handle retransmissions. The first option is not to perform spectrum leasing. In this case, the retransmissions are performed directly by PT. With the last two options, instead, part of the retransmission slots, under given conditions to be discussed, are leased to ST. The three policies are detailed below.

**No Spectrum Leasing (NSL):** The primary link does not lease spectrum to ST at any time. If reception of the primary packet fails in the first slot, PT performs up to $(K - 1)$ retransmissions until the packet is successfully received or the maximum number $K - 1$ of retransmissions is carried out.

**Spectrum Leasing via Cooperative Transmission (SL-CT):** If ST decodes PT’s packet in the first slot or any slot during the following $(K - 2)$ retransmissions, it informs PT and/or PR. Part of the next retransmission slot is then leased to ST, along with all possible subsequent retransmissions. Specifically, the spectral resources in the slots at hand are divided into two parts, e.g., in time or frequency, as shown in Figure 4.2-(b). In the first part of the slot, termed *cooperation slot*, of relative size $0 \leq \alpha \leq 1$, ST cooperates with PT in forwarding a fraction $\alpha$ of symbols of the primary packet to the PR. The second part of the slot, termed *leased slot*, of relative size $\bar{\alpha} = 1 - \alpha$, is instead leased to secondary transmission for communication between ST and SR. This scheme is akin to the strategy proposed in [73].

**Spectrum Leasing via Cooperative Interference Management (SL-CIM):** If ST decodes IT’s packet in the first slot or any slot during the following $(K - 2)$ retransmissions, it informs PT and/or PR. Assume that at least one node among ST, PT or PR is able to overhear the ACK or NACK messages fed back
by IR regarding the previous transmission of IT. Since these messages are typically transmitted with powerful error correcting codes, the assumption appears reasonable in practical systems. In a typical scenario, PR might be for instance able to decode the ACK/NACK message sent by IR (see Figure 4.1). If a NACK message from IR is observed, and ST has signalled its decoding of PT’s packet, part of the retransmission slot is leased to ST, along with all subsequent retransmission slots in which IT retransmits the same packet. The rationale for this is that, unlike SL-CT, in the cooperation slot shown in Figure 4.2-(b), ST forwards a fraction $\alpha$ of symbols of IT’s packet, rather than the primary packet, to PR. This way, ST boosts the reception of the interfering signal with the aim of enabling more effective interference mitigation by joint decoding at PR.

Parameter $\alpha$ in both SL strategies described above is set by the secondary link so as to satisfy its QoS requirements. Secondary QoS requirements are defined by a maximum probability of outage $P_{S,\text{out}}^{\text{max}}$ that must be supported on the ST-SR link in case SL is granted. Note that the ST-SR link is best-effort and does not employ HARQ.

### 4.4 Performance Analysis

In this section, the performance of NSL, SL-CT and SL-CIM is analyzed. To this end, the following definitions are useful. Let the Shannon capacity of a Gaussian channel be $C(x) = \log_2 (1 + x)$ and $(x)^+ = \max \{x, 0\}$. Consider a scenario with two transmitters and one receiver, i.e., a multiple access channel (MAC), in which transmitter 1 is received with power $\rho_1$ and transmitter 2 with power $\rho_2$. The maximum rate achievable by user 1 if user 2 transmits at rate $r_2$ is well known to be given by $C_1(\rho_1, \rho_2, r_2) = \max \left( R_N(\rho_1, \rho_2), R_J(\rho_1, \rho_2, r_2) \right)$ where [74, Lecture note 4]

\begin{align*}
R_N(\rho_1, \rho_2) &= C \left( \rho_1 (1 + \rho_2)^{-1} \right), \quad \text{(4.1)} \\
R_J(\rho_1, \rho_2, r_2) &= \min \left\{ C(\rho_1), (C(\rho_1 + \rho_2) - r_2)^+ \right\}. \quad \text{(4.2)}
\end{align*}
Rate $R_N(\rho_1, \rho_2)$ is achieved if the receiver treats the signal of transmitter 2 as noise (subscript "N" stands for "Noise"), whereas rate $R_J(\rho_1, \rho_2, r)$ is achieved if the receiver jointly decodes the two users (subscript "J" stands for "Joint"). By optimally choosing between the two decoders, rate $C_1(\rho_1, \rho_2, r_2)$ is achieved.

By using the definitions above, the primary outage probability $P_{out,P}$ for all (re)transmissions in which PT transmits directly to PR is given by

$$P_{out,P} = \Pr [R_P \geq C_1(g_{PP}P_P, g_{IP}P_I, R_I)]. \quad (4.3)$$

This is because, when PT transmits, the PR is the receiver in a MAC with the two transmitters being PT, which plays the role of user 1 in the discussion above, and IT, which plays the role of user 2. Recall that with type-I HARQ decoding in different slots takes place independently. Similar calculations apply also for the other links as explained in the next section.

The throughput, i.e., the average number of primary packets that is successfully delivered per slot, is considered as the performance metric of interest, which can be calculated as

$$T_P = \frac{P^{(K)}_{suc}}{E[N_P]}, \quad (4.4)$$

where $P^{(K)}_{suc}$ is the probability of successful primary packet delivery within the maximum number of transmissions of $K$ slots and $E[N_P]$ is the average number of time-slots used by the primary HARQ process. The random variable $N_P \in \{1, ..., K\}$ denotes the (random) number of transmission attempts spent by the primary HARQ process, accounting also for the possibly leased time slots, and its probability distribution is given by

$$\Pr [N_P = k] = \begin{cases} (1 - P^{(k)}_{out,P})^{k-1} P^{(j)}_{out,P}, & \text{for } k = 1, ..., K - 1, \\ K^{-1} P^{(j)}_{out,P}, & \text{for } k = K. \end{cases} \quad (4.5)$$
where $P_{\text{out},P}^{(k)}$ is the probability of outage at the PR in slot $k$ given that all previous transmission attempts up to the $(k-1)$th one were unsuccessful. Note that $N_P = K$ only entails that the first $K-1$ transmissions were unsuccessful, which explains the second line in (4.5). The probability $P_{\text{succ}}^{(K)}$ is then given by

$$P_{\text{succ}}^{(K)} = \sum_{k=1}^{K-1} \Pr [N_P = k] + \Pr [N_P = K] \left(1 - P_{\text{out},P}^{(K)}\right),$$

(4.6)

whereas the average number of retransmissions is evaluated as $E[N_P] = \sum_{k=1}^{K} k \Pr [N_P = k]$.

The evaluation of $P_{\text{out},P}^{(k)}$ for the different schemes is now detailed.

### 4.4.1 No Spectrum Leasing (NSL)

With NSL, the probability of outage at the $k$th retransmission is simply given by $P_{\text{out},P}^{(k)} = (P_{\text{out},P})^k$, since with HARQ type-I, all transmission attempts are independent. Note that the HARQ processes of the PT-PR and IT-IR links evolve independently with NSL.

### 4.4.2 Spectrum Leasing via Cooperative Transmission (SL-CT)

In this section, the performance of SL-CT is derived. The derivation does not follow from [73] due to the presence of the interferer. Consider first the calculation of the SL parameter $\alpha$ based on the secondary QoS as defined by outage probability $P_{S,\text{out}}^{\text{max}}$. Assuming for simplicity that SR decodes based only on the signal received in the leased slot, the SL parameter $\alpha$ is calculated by imposing the condition

$$\Pr \left[ R_S \geq \bar{\alpha}C_1(g_{SS}P_S, g_{ISR}P_I, R_I\bar{\alpha}^{-1}) \right] \leq P_{S,\text{out}}^{\text{max}},$$

(4.7)

where the left-hand side of (4.7) is the secondary outage probability. This is because in the leased slot SR acts as the receiver in a MAC with the two transmitters being ST and IT (recall the discussion around (4.1)-(4.2)). Note that the effective interferer’s rate observed by SR in the leased part of the slot is $R_I\bar{\alpha}^{-1}$ due to the fraction $\bar{\alpha}$ of
channel uses allocated to the leased slot. If (4.7), taken with equality, has a solutions in \(0 \leq \alpha \leq 1\), this choice of \(\alpha\) guarantees the secondary QoS constraint. If it does not have a solution, then it is said that SL is not feasible for the given secondary QoS constraints.

Assuming that SL is feasible, the primary outage probability \(P_{out,P}^{(k)}\) in the \(k\)th slot given that all previous transmissions were unsuccessful can be calculated by definition as

\[
P_{out,P}^{(k)} = \Pr [O_k | O_1, ..., O_{k-1}] = \frac{\Pr [O_1, ..., O_k]}{\Pr [O_1, ..., O_{k-1}]},
\]

(4.8)

where \(O_j\) is the outage event at the PR in time slot \(j\). The joint probability \(\Pr [O_1, ..., O_k]\) can be calculated using the law of total probability as

\[
\Pr [O_1, ..., O_k] = \sum_{j=1}^{k-1} \Pr [N_{ps} = j] \left( P_{out,P}^{SL-CT} \right)^{j-1} + \left( 1 - \sum_{j=1}^{k-1} \Pr [N_{ps} = j] \right) (P_{out,P})^k,
\]

(4.9)

where the random variable \(N_{ps}\) measures the number of primary transmission attempts needed for ST to decode the primary packet. Specifically, \(N_{ps} = j\) if ST decodes PT’s packet at the \(j\)th slot (i.e., at the \(j\)th primary transmission attempt). Moreover, \(P_{out,P}^{SL-CT}\) is the probability of outage at PR given that ST transmits the primary packet in the cooperation slot, which is given by

\[
P_{out,P}^{SL-CT} = \Pr \left[ R_P \geq \alpha C_1 \left( \left| h_{PP} \sqrt{d_{PP}^{-\gamma} P_P + h_{SP} \sqrt{d_{SP}^{-\gamma} P_S}} \right|^2, g_{IP} P_I, R_I \alpha^{-1} \right) \right].
\]

(4.10)

This is because ST does not know the channel to SR and thus cooperation with PT takes place by forwarding PT’s packet non-coherently. The probability of \(N_{ps} = j\) is given by \(\Pr [N_{ps} = j] = \left( P_{out,S}^{SL-CT} \right)^{j-1} (1 - P_{out,S}^{SL-CT})\), where \(P_{out,S}^{SL-CT}\) is the probability that ST is not able to decode PT’s packet in a slot, which is easily seen to be given by

\[
P_{out,S}^{SL-CT} = \Pr [R_P \geq C_1(g_{PS} P_P, g_{IS} P_I, R_I)].
\]

It is remarked that equation (4.9)
reflects the fact that, upon decoding at the \( j \)th retransmission, all the following possible primary retransmissions are leased to ST. Furthermore, the probability (4.9) is calculated using the fact that, conditioned on the event \( \{ N_{ps} = j \} \) for \( j = 1, \ldots, k-1 \), or on the complement of event \( \bigcup_{j=1}^{k-1} \{ N_{ps} = j \} \) (i.e., on the event the ST does not decode PT’s packet during the first \( k-1 \) transmissions) the decoding attempts at different slots by PR are independent.

### 4.4.3 Spectrum Leasing via Cooperative Interference Management (SL-CIM)

With SL-CIM, calculation of parameter \( \alpha \) is done in the same way as for SL-CT, i.e., through condition (4.7). Assume that SL is feasible, i.e., that (4.7), taken with equality, has a solution. Calculation of the outage probability \( P_{out,P}^{(k)} \) of PT in the \( k \)th slot with SL-CIM is complicated by the fact that \( P_{out,P}^{(k)} \) depends not only on whether ST successfully decoded IT’s packet in some previous slot, but also on the current state of IT’s HARQ process. This is because, as described above, spectrum leasing is performed only if IT retransmits a previously transmitted packet in the current slot so as to enable interference boosting at PR. Note that, based on the above, the HARQ process of IT and ST correlated with SL-CIM.

To elaborate, for each \( k = 1, \ldots, K \), two random vectors, namely \( U_{I}^k = [U_{I,1}, \ldots, U_{I,k}] \) and \( U_{IS}^k = [U_{IS,1}, \ldots, U_{IS,k}] \), are defined where it is recalled that \( U_{I,j} \in \{1, \ldots, K_I\} \) is the index of IT’s transmission attempt during the \( j \)th transmission slot of PT, while random variable \( U_{IS,j} \in \{0,1\} \) indicates whether the ST has decoded in some prior slot the packet currently being transmitted by IT \( (U_{IS,j} = 1) \) or not \( (U_{IS,j} = 0) \). Therefore, \( U_{IS,j} = 1 \) if, at the beginning of slot \( j \), ST has available the packet that the IT transmits in slot \( j \), and \( U_{IS,j} = 0 \) otherwise. With these definitions, the probability
Figure 4.3  Diagram of the Markov chain \((U^k_I, U^k_{IS})\), where \(U^k_I = (U_{I,1}, ..., U_{I,k})\), with \(U_{I,j}\) being the index of the IT’s transmission attempt during the \(j\)th transmission slot of PT, \(U^k_{IS} = (U_{IS,1}, ..., U_{IS,k})\), with \(U_{IS,j}\) indicating whether the ST has decoded in some prior slot the packet currently being transmitted by IT or not. States are represented by \((U_{I,j} = a, U_{IS,j} = b)\) with \(a \in \{1, ..., K_I\}\) and \(b \in \{0, 1\}\). Only non-zero transition probabilities are illustrated as edges.

\[ P^{(k)}_{out,P} \] can be calculated as follows

\[
P^{(k)}_{out,P} = \sum_{a \in \{1, ..., K_I\}} \Pr[U^k_I = a, U^k_{IS} = b] (P_{out,P}^{(nI(a,b))})(P_{out,P}^{SL-CIM})^{k-N_I(a,b)}, \quad (4.11)
\]

where the sum in (4.11) is taken with respect to all possible pairs of sequences \(U^k_I\) and \(U^k_{IS}\). Moreover, for given sequences \(U^k_I = a\) and \(U^k_{IS} = b\), \(N_I(a,b)\) is the number of slots \(j\) at which ST does not have available the currently transmitted IT packet, i.e., at which either IT starts a new transmission (i.e., \(U_{I,j} = 1\)), or IT retransmits but ST was not able to decode IT’s packet in any of the previous slots.
(i.e., \( U_{I,j} \neq 1, U_{IS,j} = 0 \)). Finally, \( P_{out,P}^{SL-CIM} \) is the probability of outage at PR given that ST forwards interference in the cooperation slot. This is given by

\[
P_{out,P}^{SL-CIM} = \text{Pr} \left[ R_P \geq \alpha C_1 \left( g_{PP} P_P, \left| h_{IP} \sqrt{d_{IP}^{-\gamma} P_I} + h_{SP} \sqrt{d_{SP}^{-\gamma} P_S} \right|^2, R_I \alpha^{-1} \right) \right],
\]

(4.12)
since for a fraction \( \alpha \) of the time (the cooperation slot) IT’s signal is received by PR boosted by the transmission of ST. Notice that the signals from IT and ST add incoherently at PR due to the lack of CSI. Probability (4.12) follows since the PR in the cooperation slot acts as the receiver in a MAC with the transmission to be decoded being PT’s packet in the presence of IT’s transmission. It is finally remarked that (4.11) reflects the fact that conditioned on sequences \( (U_{I,k}^k, U_{IS,k}^k) \), the decoding error events at PR in each slot are independent.

Now calculation of the probability \( \text{Pr} \left[ U_{I}^k = a, U_{IS}^k = b \right] \) in (4.11) is explained. Recalling that the state \( U_{I,1} \) of the HARQ process of the IT-IR link at slot \( j = 1 \) is assumed to have a uniform probability distribution on the set \( \{1, \ldots, K_I\} \) and that \( U_{IS,1} = 0 \) with probability 1, using the chain rule for probability distributions

\[
\text{Pr} \left[ U_{I}^k = a, U_{IS}^k = b \right] = \text{Pr} \left[ U_{I,1} = a_1, U_{IS,1} = b_1 \right] \prod_{j=1}^{k-1} \text{Pr} \left[ U_{I,j+1} = a_{j+1}, U_{IS,j+1} = b_{j+1} | U_{I,j} = a_j, U_{IS,j} = b_j \right]
\]

\[
= \frac{1}{K_I} \delta (b_1) \prod_{j=1}^{k-1} \text{Pr} \left[ U_{IS,j+1} = b_{j+1} | U_{I,j} = a_j, U_{IS,j} = b_j \right] \text{Pr} \left[ U_{I,j+1} = a_{j+1} | U_{I,j} = a_j, U_{IS,j} = b_j, U_{IS,j+1} = b_{j+1} \right],
\]

(4.13)

where \( \delta (\cdot) \) is the Kronecker delta function, i.e., \( \delta (x) = 1 \) if \( x = 0 \) and \( \delta (x) = 1 \) otherwise. Equation (4.13) follows since the joint process \( (U_{I}^k, U_{IS}^k) \) is easily seen to be Markovian. The probability terms in (4.13) are obtained by evaluating the transition probabilities of this Markov chain, which is illustrated in Figure 4.3. From
the description of the system model, it is not difficult to see that

\[
\begin{align*}
\Pr [U_{IS,j+1} = b_{j+1} | U_{I,j} = a_j, U_{IS,j} = b_j] &= \begin{cases} \\
\delta (b_{j+1}) & \text{if } a_j = K_I \\
\frac{p_{SL-CIM}^{SL-CIM}}{1 - p_{out,I}^{SL-CIM}} & \text{if } a_j \neq K_I, b_j = 1, b_{j+1} = 1 \\
\frac{p_{out,I}^{SL-CIM}}{1 - p_{out,I}^{SL-CIM}} & \text{if } a_j \neq K_I, b_j = 1, b_{j+1} = 0 \\
\frac{p_{out,I}^{SL-CIM}}{1 - p_{out,I}^{SL-CIM}} & \text{if } a_j \neq K_I, b_j = 0, b_{j+1} = 1 \\
\frac{p_{out,I}^{SL-CIM}}{1 - p_{out,I}^{SL-CIM}} & \text{if } a_j \neq K_I, b_j = 0, b_{j+1} = 0 \\
\end{cases}
\end{align*}
\]  

(4.14)

where \( p_{SL-CIM}^{SL-CIM} \) is the probability of outage at IR in a leased slot, while \( p_{out,I}^{SL-CIM} \) is the probability that ST does not successfully decode IT’s packet, which are calculated below. To interpret (4.14), note that the first line reflects the fact that, if the previous slot, the \( j \)th, contained the last transmission of an IT packet (i.e., \( U_{I,j} = K_I \)), then necessarily the IT sends a new packet in the current slot, the \((j+1)\)th, and therefore this packet is not available at ST (i.e., \( U_{IS,j+1} = 0 \)). The following lines account for the cases in which the previous slot was not the last transmission of an IT packet. Specifically, the second and the third line follow since, when ST had IT’s packet available in the previous slot (i.e., \( U_{IS,j} = 1 \)), then in the current slot \( j + 1 \) it can be seen that \( U_{IS,j+1} = 1 \) if IT’s transmission was in outage in the previous slot, and \( U_{IS,j+1} = 0 \) otherwise. Finally the fourth and fifth line reflect the fact that if ST does not have the current IT packet in slot \( j \) (i.e., \( U_{IS,j} = 0 \)), it will have it in the next slot if IT suffers outage and, at the same time, ST successfully decodes IT’s packet in the \( j \)th slot.

The probability that the link IT-IR is in outage in a leased slot can be calculated as

\[
P_{out,I}^{SL-CIM} = \Pr \left[ R_I \geq \alpha C \left( \left| h_{II} \sqrt{d_{II}^{\gamma} P_I} + h_{SI} \sqrt{d_{SI}^{\gamma} P_S} \right| \right)^2 + \alpha R_N (g_{II} P_I, g_{SI} P_S) \right].
\]  

(4.15)
This follows from simple information-theoretical considerations since for a fraction \( \alpha \) of the time (the cooperation slot) IT’s signal is received by IR boosted by the transmission of ST (first term in (4.15)), while for the remaining fraction of time ST transmits the secondary packet, which is treated for simplicity as noise (second term in (4.15)). Instead, the probability that ST does not successfully decode IT’s packet is given by \( P_{\text{out,IS}}^{\text{SL-CIM}} = \Pr \left[ R_I \geq C_I (g_{\text{IST}} P_I, g_{\text{PST}} P_P, R_P) \right] \), since ST acts as the receiver in a MAC with two transmitters being IT and PT.

Finally, following similar reasoning as for (4.14), it can be seen that

\[
\Pr \left[ U_{I,j+1} = a_{j+1} | U_{I,j} = a_j, U_{IS,j} = b_j, U_{IS,j+1} = b_{j+1} \right] = \begin{cases} 
\delta(a_{j+1} - 1) & \text{if } a_j = K_I \\
\delta(a_{j+1} - (a_j + 1)) & \text{if } a_j \neq K_I, b_j = 1, b_{j+1} = 1 \\
\delta(a_{j+1} - 1) & \text{if } a_j \neq K_I, b_j = 1, b_{j+1} = 0 \\
\delta(a_{j+1} - (a_j + 1)) & \text{if } a_j \neq K_I, b_j = 0, b_{j+1} = 1 \\
(1 - P_{\text{out,I}}) \left( P_{\text{out,I}} P_{\text{out,IS}}^{\text{SL-CIM}} + (1 - P_{\text{out,I}}) \right)^{-1} & \text{if } a_j \neq K_I, b_j = 0, b_{j+1} = 0, \\
a_{j+1} = 1 & \text{if } a_j \neq K_I, b_j = 0, b_{j+1} = 0, \\
a_{j+1} = a_j + 1 & \text{if } a_j \neq K_I, b_j = 0, b_{j+1} = 0, 
\end{cases}
\]

where \( P_{\text{out,I}} \) is the probability of outage at IT in a slot in which ST does not transmit (i.e., a slot that is not leased) given by \( P_{\text{out,I}} = \Pr [R_I \geq R_N (g_{II} P_I, g_{PI} P_P)] \), where it is assumed for simplicity that since PT’s signal is treated as noise at IR. To interpret (4.16), note that the first line reflects the fact that, if the previous slot contained the last transmission of a certain IT packet (i.e., \( U_{I,j} = K_I \)), then necessarily IT sends a new packet in the current slot (i.e., \( U_{I,j+1} = 1 \)). The following lines account for the cases in which the previous slot was not the last transmission of an IT packet (i.e., \( U_{I,j} \neq K_I \)). Specifically, the second line follows since, if ST has IT’s packet available in the previous slot (i.e., \( U_{IS,j} = 1 \)), and also in the current slot \( j + 1 \) (i.e.,
$U_{IS,j+1} = 1$), then IT’s transmission was in outage in the previous slot and hence $U_{I,j+1} = U_{I,j} + 1$. The third and forth lines have similar interpretations. Finally, the fifth and sixth line indicate that, if ST does not have IT’s packet in slot $j$ and in slot $j+1$, then IT transmits a new packet in slot $j+1$ if IR was able to successfully decode IT’s packet in slot $j$, and otherwise it retransmits.

4.5 Numerical Results

In this section, some insights into the performance comparison of NSL, SL-CT and SL-CIM are provided. Assume that PT, PR and SR are located at the positions $(x = 0, y = 0)$, $(x = 1, y = 0)$ and $(x = 0.5, y = 0.5)$ of the $x$-$y$ plane, respectively, as shown in Figure 4.4. ST is located on the $x$-axis aligned with PT and PR at a PT-ST distance $d_{PS}$. The primary, secondary and interferer transmit powers are $P_P = P_S = 4$, $P_I = 10$, respectively, and the path loss exponent is $\gamma = 3$. The larger power sent by the interferer is typical of scenarios such as the one discussed in Sec. II, in which IT is a high-power node such as a macro-base station.

Figure 4.4 Geometry of nodes on the $x$-$y$ plane.
Figure 5.10 plots the primary throughput $T_P$ versus the PT-ST distance $d_{ps}$ for NSL, SL-CT and SL-CIM with interference rate $R_I = 4 \text{ bits/s/Hz}$ and for different interferer locations, namely $(x = 0.5, y = 0.2)$ and $(x = 1.8, y = 0.2)$. Note that exclusion zones (boxes on the x-axis) around the points are introduced where PT and PR are located in order to avoid divergence of the received power. A first observation is that SL techniques can widely outperform NSL, while allowing both primary and secondary transmissions, as also pointed out in [3][62]. In this regard, it is noted that the primary throughput of SL-CT and SL-CIM reduces to the corresponding NSL throughput only as ST moves sufficiently far away from PT and IT, respectively. This is because SL-CT requires ST to be able to decode PT’s packets, while ST-CIM is enabled by ST’s decoding of IT’s packets.

![Graph showing primary throughput $T_P$ versus PT-ST distance $d_{ps}$ for NSL, SL-CT and SL-CIM for $K = 5$ and fixed IT location $(x = 1.8, y = 0.2)$ and $(x = 0.5, y = 0.2)$.]
Figure 4.6 Primary throughput $T_p$ versus the interferer’s rate $R_I$ for NSL, SL-CT and SL-CIM for fixed IT location ($x = 1.8, y = 0.2$).

Regarding the performance comparison of SL-CT and SL-CIM, it is seen that SL-CIM outperforms SL-CT whenever the ST is in the vicinity of IT so that it is more capable of decoding the interference rather than the primary signal. Such performance gains increase as the maximum number of interferer retransmissions $K_I$ is increased, since a larger $K_I$ implies that IT’s packets are dropped due to exceeding the maximum number of retransmissions, and hence more opportunities for SL arise. It is also seen that moving IT closer to the primary link, i.e., to position ($x = 0.5, y = 0.2$), reduces the primary throughput gain of SL-CIM as compared to SL-CT since PT has a better observation of IT’s transmission and can thus perform effective interference management even without the help of ST.
Finally, Figure 4.6 plots the primary throughput \( T_P \) versus IT’s rate \( R_I \) for a fixed position \( d_{ps} = 1.5, K_I = 5 \) and \( K = 2, 4 \). It is shown that SL-CIM is the best performing strategy unless the rate \( R_I \) is either too small, in which case interference forwarding is not necessary for effective interference management, or too large, in which case SL-CIM is not feasible.

4.6 Conclusions

In this chapter, the novel scheme of spectrum leasing based on cooperative interference management was proposed. For fixed secondary QoS requirement, the scheme was shown to provide substantial performance gains in terms of primary throughput with respect to conventional techniques based on primary packet relaying in case interference sets the main performance bottleneck on the primary system. The proposed scheme leverages the retransmission (HARQ) processes of the primary and interfering links. The performance gains are shown via numerical results to depend on the network topology, and in particular on the relative position of the primary, secondary and interfering terminals, and to increase as the maximum delay allowed by the HARQ processes grows larger.
CHAPTER 5

FEMTOCELL AS A RELAY

5.1 Introduction

Femtocells are often seen as an easy fix to the problem of increasing network coverage in cellular systems. This is mostly due to the availability of cheap backhaul connections between the home base stations (HBSs), installed by the subscribers in their premises, and the operator’s network, in the form of last-mile links followed by the Internet. The two basic operating modes of HBS are open-access (OA), whereby all users have the same privileges in accessing the HBS, and closed-access (CA), for which only the subscriber’s devices are allowed to access the HBS[75].

Current femtocell deployments dictate that the HBS acts essentially as an independent base station (BS). In particular, focusing on the uplink (mobile-to-BS), each HBS is required to decode the intended users and to pass the decoded (hard) information, along with necessary control signalling, to the mobile operator networks via the backhaul links. It is noted that the intended users are the subscriber’s devices, typically located indoors, and possibly also macrocell users, typically located outdoors. Two classes of users will be referred to as indoor and outdoor users for simplicity. The advantage of deploying conventional femtocells stems from the fact that the presence of a decoder, the HBS, in the vicinity of the users to be decoded, allows devices to transmit at a reduced power. However, on the negative side, allowing more indoor users to transmit, femtocells may possibly affect the quality of service of the existing macrocell users communicating directly to the macrocell BS, due to the additional interference. There is clearly a trade-off between the additional interference created by femtocell transmissions and the increased system capacity due to the
larger number of users served. This has been explored in a number of works such as [76][77][78] (see below).

### 5.1.1 Contributions

This chapter explores the possibility to operate the femtocells in a different way than merely as additional BSs or equivalently, focusing on the uplink, as decoders. In particular, the performance advantages of operating HBSs as relays will be investigated. To elaborate, consider the scenario in Figure 5.1, which depicts a single cell with a single femtocell and two users, one indoor and one outdoor. The standard deployment discussed above dictates that the HBS decodes the signal from the indoor user and, in case of OA femtocells, possibly also the outdoor user. The decoded information is sent to the mobile operator network via the backhaul link. Instead this chapter proposes to implement the decoder of both indoor and outdoor users at the mobile operator network. This way, the decoder has access to both the signal received by the macrocell BS and the bits sent by the HBS on the backhaul link. Moreover, the HBS can be used as a relay, that is, it can be used to provide "soft" information regarding the received signal to the decoder in order to facilitate decoding. Notice that the HBS communicates to the decoder via the backhaul link. This is illustrated in Figure 5.1, where for simplicity of representation, the decoder is depicted as the macrocell BS.

The above novel framework of "HBSs as relays" is in practice enabled by two main modifications of the traditional femtocell architecture, which seem within the reach of current technology. One is the routing of the information sent on the backhaul link by the HBS towards a decoder that performs joint decoding of indoor and outdoor users. The second is the possibility to send potentially soft information from the HBS to the mobile operator network. This is unlike the current deployment where HBSs
are required to format the transmitted information as the decoded (hard) information from the intended users [75].

To assess the performance advantages of operating the HBSs as relays, the transmission reliability in the system of Figure 5.1 over quasi-static fading channels will be analyzed. Specifically, analytical expressions for the outage probability of uplink transmission will be derived first with relaying techniques, inspired by both CA and OA modes, for fixed transmission rates and SNR. Then the diversity-multiplexing trade-off (DMT) [79] in both scenarios will be addressed, thereby considering the regime of high SNR and of different transmission rate scalings (multiplexing gains). It will be demonstrated that the framework of femtocells as relays has the potentiality to solve the issue identified above with the standard deployment of femtocells. In particular, it allows more users to be served with potentially no performance disadvantage,
in terms of reliability, for the existing macrocell (outdoor) users. In fact, by choosing appropriate relaying techniques, performance advantages can be accrued for both indoor and outdoor users. Finally, the conclusions above will be extended to the downlink scenario.

5.1.2 Related Work

Related analyses of the performance of cellular systems in the presence of femtocells can be found in [76] and references therein. Especially related are [77][78]. In [77], the DMT analysis of a single-macrocell single-femtocell system is presented, by modelling the latter as a "Z-interference channel" so that no interference is assumed between femtocell user and BS. In [78] a performance comparison of OA and CA femtocells is provided in terms of achievable throughputs, accounting for the random location of the femtocell with a cell, but not for fading. In both works, one key assumption is that the HBS decodes the signal from the femtocell user and, for OA (in [78]), also the signal of the macrocell users assigned to the HBS. In this work, instead, this conventional restriction is not imposed and the HBS is allowed to operate, more generally, as a relay for the macro-BS, which is the intended decoder for both femtocell and macrocell users. It is remarked that, using this standpoint, performance of CA and OA femtocells in multicell systems in the absence of fading is studied in [80].

Notation: The notation $\hat{=} \equiv$ is the exponential equality $f(\rho) \hat{=} \rho^d$ if $\lim_{\rho \to \infty} f(\rho)/\rho = d$, and $\hat{\leq}, \hat{\geq}$ are similarly defined. $(x)^+ \text{ denotes max } \{x, 0\}$ and $C(x) = \log_2 (1 + x)$.

5.2 System Model

Consider a macrocell, served by single BS which is overlaid with a femtocell served by a HBS, as depicted in Figure 5.1. For simplicity, the discussion will be focused on the case of a single active indoor (i.e., femtocell) user and a single active outdoor (i.e., macrocell) user per cell. First consider uplink transmission, where indoor and
outdoor users transmit one message per transmission block with rates $R_I$ and $R_O$ (bits/ channel use), respectively. Downlink will be discussed in Sec. 5.6. Assuming time synchronization, the discrete-time received signals for the BS and HBS at time $t = 1, ..., n$ are

$$y_{B,t} = \sqrt{\alpha_I} h_{IB} x_{I,t} + \sqrt{\alpha_O} h_{OB} x_{O,t} + z_{B,t}, \quad (5.1)$$

$$y_{H,t} = \sqrt{\beta_I} h_{IH} x_{I,t} + \sqrt{\beta_O} h_{OH} x_{O,t} + z_{H,t}, \quad (5.2)$$

where subscripts distinguish indoor ("$I$") user, outdoor ("$O$") user, HBS ("$H$") and BS ("$B$"); $\alpha_i, \beta_i$ are the user $i$-to-BS and user $i$-to-HBS average channel power gains respectively, $i \in \{O, I\}$; $h_{iB}, h_{iH}$ model independent quasi-static Rayleigh fading unit-power channels (i.e., $h_{iB}, h_{iH}$ are complex Gaussian with unit power); $x_{i,t}$ represents user $i$’s transmitted symbol, which is assumed to satisfy the block power constraint

$$\frac{1}{n} \sum_{t=1}^{n} E[|x_{i,t}|^2] \leq \rho_i, \quad (5.3)$$

where $\rho_i = \rho$ since any difference in power can be captured by the average channel gains $\alpha_i, \beta_i$; and, finally, $z_{B,t}, z_{H,t}$ are the independent unit-power complex Gaussian noise sequences at the BS and HBS respectively. Channel state information is assumed only at the receivers. The HBS is connected to the BS via a last-mile link (e.g., DSL or cable) followed by the Internet, which is modelled here as an out-of-band (i.e., orthogonal) link of capacity $C$ bits/ channel use. The HBS receives (5.2) for $t = 1, ..., n$ and, based on this, decides the $nC$ bits to be sent to the BS. The BS decodes both messages of indoor and outdoor users based on the signal (5.1) for $t = 1, ..., n$ and the bits received from the HBS.

It is noted that the considered model can be classified as a multiple access channel with an out-of-band relay following standard nomenclature (see, e.g., [81]). For comparison, an outage analysis of the corresponding scenario with in-band relaying
can be found in [82] and references therein. An out-of-band relay channel with a single user and without fading is instead studied in [83].

5.2.1 Transmission Strategies

Inspired by the classification of HBS operation modes into OA and CA, the following transmissions schemes are considered.

Closed Access (CA): The femtocell attempts to decode the indoor user’s signal and treats the outdoor user’s signal as noise. Upon successful decoding of the indoor user’s message, the femtocell dedicates a rate up to the total backhaul capacity $C$ for transmission of such message towards the BS. If decoding is not successful, the backhaul link is not used. Notice that the scheme is based on Decode-and-Forward (DF) [84].

Open Access (OA): The femtocell attempts to decode both the indoor and the outdoor users’ signals. If decoding is successful on both messages, the femtocell transmits up to $\gamma C$ bits/ channel use for the indoor user’s message and up to $(1 - \gamma) C$ bits/ channel use for the outdoor user’s signal, where $0 \leq \gamma \leq 1$ determines the fraction of the capacity allocated for each message. If decoding is successful only on one message, the HBS dedicates rate up to the total backhaul capacity $C$ for transmission of such message towards the BS. If decoding is not successful, the backhaul link is not used. This scheme is also based on DF.

Compress-and-Forward (CF): The HBS compresses the received signal from the indoor and outdoor users to $C$ bits/ channel use using the scheme proposed in [85], which improves on the standard compress-and-forward (CF) scheme [84]. It is noted that, with CF, the HBS implicitly serves both indoor and outdoor users in a similar fashion for the OA scheme.
The performance of CA, OA and CF femtocells will be analyzed in the uplink in terms of outage probability (for fixed transmission rates) in Sec. 5.3 and DMT in Sec. IV, respectively.

5.3 Outage Analysis

In this section, the probability of outage under the assumption of fixed rates $R_I$ and $R_O$, channel power gains $\alpha_i, \beta_i$ and power $\rho$ is analyzed. The outage probability is defined as the probability that at least one of the messages from the indoor and/or outdoor users is not successfully decoded at the BS (i.e., common outage event).

Using the law of total probability, the outage probability for OA can be computed as follows

$$P_{out}^{OA} = P_{H,O|I}P_{out|O} + P_{H,O}P_{out|O} + P_{H,I}P_{out|I} + P_{H,\text{none}}P_{out|\text{none}}, \quad (5.4)$$

where $P_{H,O|I}, P_{H,O}, P_{H,I}$ are the probabilities of successful decoding at the HBS of both outdoor and indoor messages ($P_{H,O|I}$), of the outdoor message only ($P_{H,O}$), and of the indoor message only ($P_{H,I}$), respectively; $P_{H,\text{none}}$ is the probability of decoding no message at the HBS; and, finally, $P_{out|O}, P_{out|I}, P_{out,O}, P_{out|\text{none}}$ denote the outage probability (at the BS) conditioned on the corresponding decoding events at the HBS (e.g., $P_{out|O}$ is the outage probability conditioned on the HBS decoding both messages).

The outage probability for CA can be similarly found as

$$P_{out}^{CA} = P_{H,I}P_{out|I} + P_{H,\text{none}}P_{out|\text{none}}, \quad (5.5)$$

where $P_{H,I}$ and $P_{H,\text{none}}$ are similarly redefined for CA (notice that $P_{H,O|I} = P_{H,O} = 0$ for CA). Calculation of the decoding probability at the HBS will be detailed below for CA and OA, while outage probability for CF will be detailed in Sec. 5.3-C.
For the evaluation of the conditional outage probabilities at the BS, definition of the following quantity turns out to be useful. Denote as $P_{out}(R_I, R_O)$, the probability of outage for a BS decoder based only on the received signal (5.1) for $t = 1, \ldots, n$, i.e., without accounting for the bits received from the HBS. The set of rates that can be reliably decoded by such decoder is given by the capacity region of the multiple access channel (5.1), which is

$$\mathcal{R}_B = \{(R_O, R_I) : R_O \leq C(\alpha_O g_{OB} \rho), \quad R_I \leq C(\alpha_I g_{IB} \rho), \quad R_O + R_I \leq C((\alpha_I g_{IB} + \alpha_O g_{OB}) \rho)\},$$

with $g_{ij} = |h_{ij}|^2$, and is sketched in Figure 5.2-(a). Accordingly, by extending the analysis in [86] to multiple access channels with unequal channel gains, it can be seen that $P_{out}(R_I, R_O) = \Pr[(R_O, R_I) \notin \mathcal{R}_B]$ as

$$P_{out}(R_I, R_O) = P_{B,I}(R_I, R_O) + P_{B,O}(R_I, R_O) + P_{B,none}(R_I, R_O),$$

(5.6)

where $P_{B,I}(R_I, R_O)$, $P_{B,O}(R_I, R_O)$, and $P_{B,none}(R_I, R_O)$ are the outage probabilities at the BS due to not decoding the outdoor message, not decoding indoor message,
and not decoding any of the messages, respectively, and can be calculated as

\[
P_{B,I}(R_I, R_O) = G_{IB} \exp \left[ (-K_{B,I}) - \exp \left( - \left( \frac{K_{B,O}}{G_{IB}} + K_{B,I} \right) \right) \right],
\]

(5.7)

and

\[
P_{B,\text{none}}(R_I, R_O) = 1 - \exp \left( -K_{B,O} \right) \\
+ \frac{\exp \left( -2^{R_O + R_I - 1} \right) \left( \exp \left( -K_{B,O} \left( \alpha_O - 1 \right) \right) - \exp \left( -2^{R_I}K_{B,O} \alpha_O \left( \alpha_O - 1 \right) \right) \right)}{a_O - 1} \\
+ \frac{\exp \left( \frac{1}{a_O \rho} \right) \left( \exp \left( - \left( \frac{\zeta_B}{a_O \rho} - K_{B,O} \right) \right) - \exp \left( -2^{R_O} \left( \frac{\zeta_B}{\rho} - K_{B,O} \alpha_O \right) \right) \right)}{\left( 2^{R_O} - 1 \right)^{-1} \left( 2^{R_O} - 1 + \zeta_B \right)} \\
+ \exp \left( -K_{B,I} \right) \left( \exp \left( \frac{K_{B,O}}{G_{IB}} \right) G_{IB} \right),
\]

(5.8)

with definitions \(G_{ij} = \left( \left( 2^{R_i} - 1 \right) \zeta_j + 1 \right)^{-1} \), \(K_{B,i} = \left( 2^{R_i} - 1 \right) / (\alpha_i \rho)\) with \(i \in \{ I, O \}\), \(j \in \{ B, H \}\) and \(\zeta_B = \alpha_O / \alpha_I\). \(P_{\text{out,}O}(R_I, R_O)\) is the same as \(P_{\text{out,}I}(R_I, R_O)\) with switched subscripts "I" and "O".

**Remark:** For the special case of the symmetric channel gains, i.e., \(\zeta_B = 1\), the above probabilities reduce to eq. (15)-(17) of [86].

### 5.3.1 Closed Access (CA)

In this section the outage probability (5.5) for CA is evaluated. Recall that, with CA, the HBS decodes the indoor user’s message and treats the outdoor user’s message as (Gaussian) noise of power \(\beta_O \rho\). Moreover, upon decoding, the HBS provides up to \(C\) bits of the message of the indoor user to the BS. This can be seen to reduce the effective rate of the indoor message to be decoded at the BS to \((R_I - C)^+\) (see, e.g., [83]). This leads to the following.
**Proposition:** The outage probability with a CA femtocell is given by (5.5), where

\[
P_{H,\text{none}} = 1 - G_{IH} \exp(-K_{H,I}),
\]

(5.9)

\[
P_{\text{out}|I} = P_{\text{out}}((R_I - C)^+, R_O),
\]

(5.10)

and

\[
P_{\text{out}|\text{none}} = P_{\text{out}}(R_I, R_O),
\]

(5.11)

with \(G_{IH} = (2^{R_I} - 1) \zeta_H + 1)^{-1}\), \(K_{H,i} = (2^{R_i} - 1) / (\beta_i \rho)\), \(\zeta_H = \beta_O / \beta_I\) and \(P_{H,I} = 1 - P_{H,\text{none}}\).

**Proof:** Probability \(P_{H,\text{none}}\) is given by \(P_{H,\text{none}} = \Pr[R_I > C \left( \frac{\beta_I \rho}{1 + \rho_0 \rho} \right)]\), since HBS treats the outdoor user as noise, which can be easily calculated using the fact that \(g_{OH}\) and \(g_{IH}\) are exponentially distributed. Moreover, the outage probability \(P_{\text{out}|I}\) at the BS, conditioned on the HBS decoding the indoor user message, is given by \(P_{\text{out}}((R_I - C)^+, R_O)\) based on the discussion above. □

### 5.3.2 Open Access (OA)

With OA, the HBS attempts to decode both the messages of indoor and outdoor users. Therefore, with OA, the performance is not adversely affected by the outdoor’s user interference on the HBS, unlike for CA. Moreover, following the discussion above, the HBS is able to reduce the effective rates to be decoded at the BS to \((R_I - (1 - \gamma) C)^+\) and \((R_O - \gamma C)^+\) if both messages are decoded at the HBS, while \(\gamma = 0\) or \(\gamma = 1\) if only the indoor or only the outdoor messages, respectively, are decoded at the HBS.

**Proposition:** The outage probability with a OA femtocell is given by (5.4), where \(P_{H,I}, P_{H,O}\) and \(P_{H,\text{none}}\) are the same as \(P_{B,I}(R_I, R_O), P_{B,O}(R_I, R_O),\) and \(P_{B,\text{none}}(R_I, R_O)\) with \(\alpha_O, \alpha_I, \zeta_B\) and \(K_{B,i}\) replaced with \(\beta_O, \beta_I, \zeta_H,\) and \(K_{H,i}\) respectively, and
\( P_{H,OI} = 1 - P_{H,I} - P_{H,O} - P_{H,\text{none}} \). Moreover,

\[
P_{\text{out}|OI} = P_{\text{out}}((R_I - (1 - \gamma)C)^+, (R_O - \gamma C)^+),
\]

(5.12a)

\[
P_{\text{out}|O} = P_{\text{out}}(R_I, (R_O - C)^+),
\]

(5.12b)

\[
P_{\text{out}|I} = P_{\text{out}}((R_I - C)^+, R_O),
\]

(5.12c)

\[
P_{\text{out}|\text{none}} = P_{\text{out}}(R_I, R_O).
\]

(5.12d)

for some \( 0 \leq \gamma \leq 1 \).

**Proof:** The probabilities of successful decoding at the HBS can be evaluated from Figure 5.2-(b) similarly to [86]. For instance, the probability of decoding only the indoor user is given by

\[
P_{H,I} = \Pr \left[ R_O > C (\beta_O g_{OH} \rho), \ R_I \leq C \left( \frac{\beta_I g_{IH} \rho}{1 + \beta_O g_{OH} \rho} \right) \right].
\]

(5.13)

The other terms follow from the discussion above. \( \Box \)

### 5.3.3 Compress-and-Forward (CF)

With CF, the HBS does not decode, but merely forwards soft information about the received signal to the BS. Consider a CF technique based on the noisy network coding strategy of [85]. This choice is dictated by the fact that this scheme is known to perform better in terms of outage probability with respect to standard CF techniques [81]. From [85], the following rate region is achievable for given channel gains by such scheme can be derived:

\[
\mathcal{R}_B^{CF} = \{(R_I, R_O) : \begin{align*}
R_O & \leq \min \left\{ C \left( \left( \frac{\beta_O g_{OH}}{2} + \alpha_O g_{OB} \right) \rho \right), \ C \left( \alpha_O g_{OB} \rho + [C - 1]^+ \right) \right\}, \quad (5.14a) \\
R_I & \leq \min \left\{ C \left( \left( \frac{\beta_I g_{IH}}{2} + \alpha_I g_{IB} \right) \rho \right), \ C \left( \alpha_I g_{IB} \rho + [C - 1]^+ \right) \right\}, \quad (5.14b) \\
R_O + R_I & \leq \min \left\{ C \left( \rho \mathbb{H}^\dagger \right), \ C \left( (\alpha_O g_{OB} + \alpha_I g_{IB}) \rho \right) + [C - 1]^+ \right\}. \quad (5.14c)
\end{align*}
\]
where $H = [h_O \ h_I]$, $h_O = [\sqrt{\alpha_O} h_{OB} \ \sqrt{\beta_O} h_{OH}]^T$, $h_I = [\sqrt{\alpha_I} h_{IB} \ \sqrt{\beta_I} h_{IH}]^T$. A brief derivation is in [87].

Therefore, the corresponding outage probability is $P_{out}^{CF} = \Pr [(R_O, R_I) \notin \mathcal{R}_B^{CF}]$. This probability is obtained using Monte Carlo simulations since a closed-form solution appears to be mathematically intractable.

### 5.4 DMT Analysis

Here, the DMT analysis of OA, CA and CF is addressed. When evaluating the DMT, one assumes asymptotically large power (or signal-to-noise ratio, SNR) $\rho$ and a collection of transmission schemes, one for each $\rho$, with rates $R_O = r_O \log_2 \rho$ and $R_I = r_I \log_2 \rho$, with $(r_O, r_I)$ being the corresponding multiplexing gains. It is set that $r_O = r_I = r$, with $r$ being the per-user multiplexing gain and assume that the capacity of the HBS-BS link scales as $C = c \log_2 \rho$ for some $c \geq 0$. Notice that the scaling of the backhaul capacity is necessary to obtain meaningful results given the scaling of the transmission rates. Moreover, the channel power gains $\alpha_i, \beta_i, i \in \{O, I\}$ can be written as

$$
\alpha_i = \rho^{\bar{\alpha}_i - 1} \quad \text{and} \quad \beta_i = \rho^{\bar{\beta}_i - 1}, \quad (5.15)
$$

so that $\bar{\alpha}_i$ and $\bar{\beta}_i$ define the scaling of $\alpha_i \rho$ and $\beta_i \rho$ in $dB$ versus the power $\rho$ (see, e.g., [88]). Notice that varying $\bar{\alpha}_i, \bar{\beta}_i$ allows to account for differences in the power gains as measured in $dB$. This is especially important in the scenario at hand, where indoor and outdoor channels may have significantly different powers. Given the system parameters above, a diversity gain $d(r)$ is achievable if the probability of outage satisfies $P_{out} \leq \rho^{-d(r)}$. The following result will be useful.

**Lemma:** Setting $R_O = r_O \log \rho$ and $R_I = r_I \log \rho$, it can be obtained that $P_{out} (R_O, R_I) \leq \rho^{-d_{out}(r_O, r_I)}$, with

$$
d_{out} (r_O, r_I) = \min \left( (\bar{\alpha}_I + \bar{\alpha}_O - 2 (r_O + r_I))^+, (\bar{\alpha}_O - r_O)^+, (\bar{\alpha}_I - r_I)^+ \right). \quad (5.16)
$$
Proof: Using the conventional definition $g_{iB} = \rho^{-a_i}$ and $g_{iH} = \rho^{-b_i}$, where $a_i$ and $b_i$ are random variables representing the exponential order of $g_{iB}$ and $g_{iH}$, it can be proved that the probability density function of $a_i, b_i$ can be written as [79]

$$f_{a_i}(x) = f_{b_i}(x) = \begin{cases} \rho^{-\infty} = 0, & \text{for } x < 0 \\ \rho^{-x}, & \text{for } x \geq 0. \end{cases} \quad (5.17)$$

Using the union bound, it can be easily obtained

$$P_{out}(R_O, R_I) \leq Pr \left[ (r_O + r_I) > \max \left( (\bar{\alpha}_I - a_I)^+, (\bar{\alpha}_O - a_O)^+ \right) \right]$$

$$+ Pr \left[ r_O > (\bar{\alpha}_O - a_O)^+ \right] + Pr \left[ r_I > (\bar{\alpha}_I - a_I)^+ \right], \quad (5.18)$$

and the result follows from the standard application of Laplace’s principle using (5.17) [79]. □

5.4.1 Closed Access (CA)

Proposition: The following DMT is achievable for a femtocell with CA

$$d^{CA}(r) = \min \{ d_{out|I}, d_{H,none} + d_{out|none} \}, \quad (5.19)$$

where

$$d_{out|I} = d_{out}(r, (r - c))^+, \quad (5.20)$$

$$d_{H,none} = (\bar{\beta}_I - \bar{\beta}_O - r)^+, \text{ and } d_{out|none} = d_{out}(r, r). \quad (5.21)$$

with definition (5.16).

Proof: the evaluation of (5.5) is need in the given setting. To this end, the bound $P_{out}^{CA} \leq P_{out|I} + P_{H,none}P_{out|none}$ (using $P_{H,I} \leq 1$) can be used and then exponential inequalities for the three terms at hand can be found. For instance, the probability of outage at the HBS $P_{H,none}$ satisfies the exponential inequality

$$P_{H,none} \leq Pr \left[ r > ((\bar{\beta}_I - b_I)^+ - (\bar{\beta}_O - b_O)^+)^+ \right] \quad (5.22)$$
which leads to $P_{H,\text{none}} \leq \rho^{-d_{H,\text{none}}}$ with (5.21) using the Laplace principle and (5.17). The other terms $P_{\text{out}|I}$ and $P_{\text{out}|\text{none}}$ can be treated similarly by using the above Lemma and recalling that, upon detection of the indoor user, the HBS communicates (up to) $C = c \log \rho$ bits/ channel regarding the indoor message. □

5.4.2 Open Access (OA)

Proposition: The following DMT is achievable for a femtocell with OA

$$d^{\text{OA}}(r) = \max_{0 \leq \gamma \leq 1} \min \{ d_{\text{out}|OI}, d_{H,O} + d_{\text{out}|O}, d_{H,I} + d_{\text{out}|I}, d_{H,\text{none}} + d_{\text{out}|\text{none}} \} , \quad (5.23)$$

where

$$d_{\text{out}|OI} = d_{\text{out}} ((r - \gamma c)^+ , (r - (1 - \gamma) c)^+ ) , \quad (5.24a)$$

$$d_{\text{out}|O} = d_{\text{out}} ((r - c)^+ , r) , \quad (5.24b)$$

$$d_{\text{out}|I} = d_{\text{out}} (r, (r - c)^+) , \quad (5.24c)$$

and

$$d_{\text{out}|\text{none}} = d_{\text{out}} (r, r) , \quad (5.24d)$$

and

$$d_{H,O} = (\bar{\beta}_I - r)^+ , \quad (5.25a)$$

$$d_{H,I} = (\bar{\beta}_O - r)^+ , \quad (5.25b)$$

$$d_{H,\text{none}} = \max \left\{ (\bar{\beta}_I + \bar{\beta}_O - 4r)^+ , (\bar{\beta}_I + \bar{\beta}_O - r)^+ , (\bar{\beta}_O - \bar{\beta}_I - r)^+ \right\} \quad (5.25c)$$

Proof: The outage probability (5.4) can be bounded as

$$P_{\text{out}}^{OA} \leq \rho^{-d_{\text{out}|OI}} + \rho^{-d_{H,O} + d_{\text{out}|O}} + \rho^{-d_{H,I} + d_{\text{out}|I}} + \rho^{-d_{H,\text{none}} + d_{\text{out}|\text{none}}} \quad (5.26)$$

where $P_{H,OI} \leq 1$ have been used and achievable diversity orders for the remaining individual probabilities in (5.4) are defined as $P_{\text{out}|OI} \leq \rho^{-d_{\text{out}|OI}}$ and similarly for
These diversity orders can be obtained by using the above Lemma and the Laplace principle.

5.4.3 Compress-and-Forward (CF)

In this section, for simplicity, this chapter restricts the results to a scenario with average channel gains characterized by \( \bar{\alpha}_O = \bar{\alpha}_I = \bar{\beta}_O = \bar{\beta}_I = 1 \). Extension to a more general case turns out to pose some analytical challenges that is not tackled here.

**Proposition:** The following DMT is achievable for a femtocell with CF

\[
d^{CF}(r) = \min \left\{ 2(1 - r)^+, (1 - r + c)^+, \right. \\
\left. \max \left\{ (4 - 6r)^+, (2 - 2r)^+ \right\}, (2 - 4r + c)^+ \right\}.
\]

(5.27)

**Proof:** The union bound is used on the probability that any of the inequalities in (5.14) is not satisfied, and find exponential inequalities for the corresponding three probabilities. For instance, the probability \( P_I \) that the first inequality is not satisfied is upper bounded by

\[
P_I \leq \Pr \left[ r > \max \left( (1 - b_I)^+, (1 - a_I)^+ \right), (1 - a_I + c)^+ \right].
\]

(5.28)

The second inequality can be treated in the same way. For the third, the well-known DMT of a \( 2 \times 2 \) MIMO system [79] is exploited. Finally, using the Laplace’s principle concludes the proof.

5.5 Numerical Results and Discussion

In this section, some numerical results are presented to substantiate the analysis above. Considering the probability of outage \( P_{out} \) optimized over \( \gamma \) for a system with fixed rates \( R_O = R_I = 1 \) (bits/channel use) and different link capacity \( C \) (bits/channel use) versus the SNR \( \rho \) is plotted first in Figure 5.3. We set channel power gains as \( \alpha_O = -10dB, \alpha_I = -20dB, \beta_O = 10dB \) and \( \beta_I = 20dB \), so that the indoor user-HBS
channel is $40dB$ better than the indoor user-BS channel [75]. Performance as a function of the location of the users is discussed below around Figure 5.7 and 5.8.

![Figure 5.3](image_url)

**Figure 5.3** Uplink probability of outage $P_{out}$ versus $\rho$ for fixed user rates $R_O = R_I = 1$ and different values of backhaul link capacity $C$ for CA, OA and CF femtocells ($\alpha_o = -10dB, \alpha_I = -20dB, \beta_o = 10dB, \beta_I = 20dB$).

Throughout, the outage performance of the proposed schemes is compared with the performance of a scenario, referred to as "No Femtocell" (NF), where the femtocell is not present so that neither the HBS nor the indoor users are in the system. In other words, with NF, the outdoor user communicates directly to the BS and no additional user is active. A further reference scenario of interest is obtained by setting $C = 0$ in the model. In this case, both outdoor and indoor users are active, and the HBS
is disabled (e.g., malfunctioning) since it cannot communicate to the BS. It is noted that for $C = 0$, clearly, CA, OA and CF have the same performance.

From Figure 5.3, it is seen that allowing the indoor user to transmit with the HBS disabled ($C = 0$) increases the outage probability with respect to the NF case\(^4\). However, exploiting the HBS-BS backhaul link ($C > 0$), with either CA or OA, enables a significant performance improvement. In fact, for $C \geq 1$, CA performs as well as NF due to the possibility to cancel the indoor user’s interference at the BS thanks to relaying by the HBS. Moreover, for OA the performance can even be improved with respect to the NF case, since both indoor and outdoor users benefit from the presence of the HBS. Most notably, for $C > R_I + R_O = 2$, an increased diversity order with respect to NF is obtained, since outage is in this case prevented as long as either HBS or BS decodes.

Finally, from Figure 5.3, it is seen that if the backhaul capacity $C$ is larger than $R_I + R_O = 2$, while the DF-based technique OA does not further improve its performance, this is not the case with CF. Indeed, CF provides the receiver with information about the received signal at the HBS whose accuracy can be increased as $C$ gets large, while OA cannot further exploit the excess backhaul capacity $C - (R_I + R_O)$. Therefore, as $C$ increases, CF enables the outage probability to decrease down to the performance of an ideal system in which the signal received by the HBS is available at the BS (shown as “CF (C → ∞)” in the figure). This is further discussed below in the terms of DMT. Note that the performance of the ideal system is in practice achieved with reasonably small values of $C$ (here, $C = 3$).

Then the DMT analysis is considered. Figure 5.4 shows the DMT for the OA femtocell for $\bar{\alpha}_O = \bar{\alpha}_I = \bar{\beta}_O = \bar{\beta}_I = 1$, and $c = 1$, compared to the case where the femtocell is turned off ($c = 0$) and to the NF case. Similar to the discussion above,\(^4\) Recall that the common outage probability is considered. The individual outage probability for the outdoor user increases as well, albeit less than the common outage probability (not shown here).
allowing transmission of the indoor user with $c = 0$ is seen to reduce the achievable diversity for sufficiently large multiplexing gain $r$ with respect to the NF case. In particular, no multiplexing gains $r \geq 0.5$ are achievable at non-zero diversity if $c = 0$. This is well known from the analysis in [79], since when $c = 0$ the scenario at hand boils down to a multiple access channel. The analysis reveals that OA with $c = 1$ enables a diversity gain of 1 to be achieved for all multiplexing gains $r \leq 3/8$. This confirms that, with OA, the overall performance of the system, including outdoor users, can be improved.

Figure 5.4 also shows the impact of the different error events on the DMT (5.23) for OA femtocells. In particular, it is seen that for small multiplexing gains $r$ the dominating error event corresponds to the case where the HBS decodes both
messages, whereas for larger $r$ the dominating error events is when the HBS decodes no message.

Figure 5.5 compares the performance of OA and CA in terms of DMT for $\bar{\alpha}_O = \bar{\alpha}_I = \bar{\beta}_O = 1$, $c = 1$ and different indoor user-to-HBS gain $\bar{\beta}_I$. It is seen that OA outperforms CA unless the multiplexing gain and $\bar{\beta}_I$ are large. In this case, the dominating error event corresponds to decoding no messages at the HBS, which, due to large $\bar{\beta}_I$, turns out to have the same asymptotic probability for both CA and OA. Also notice that with $\bar{\beta}_I = 2$, CA has the same DMT as NF, since correct decoding of the indoor user at the HBS happens with high probability.

Figure 5.6 plots the DMT for CF femtocell and $\bar{\alpha}_O = \bar{\alpha}_I = \bar{\beta}_O = \bar{\beta}_I = 1$. It is shown that for $c$ sufficiently large, the performance tends to that of a two user
Figure 5.6 Comparison between the DMT of OA and CF schemes.

multiple access channel with two receiving antennas, which was derived in [89]. This is consistent with the discussion around Figure 5.3. Comparing the performance with the OA scheme, it is clear that CF has the ability to exploit large backhaul capacities to improve the system performance as compared to DF-based schemes in terms of both diversity and multiplexing gains.

The discussion above focuses mostly on the impact of the additional interference created by the indoor user on the system performance. Instead, some further remarks on the role of the interference from the outdoor user to the HBS are provided, and on the near-far effect that may result from power control at the outdoor user. To this aim, a scenario where the BS, HBS, indoor user and outdoor user are on a straight line is considered. Defining as $d_{HBS}$ the BS-HBS distance, the indoor
user is placed at a normalized distance $\bar{d}_I = d_I/d_{HBS} = 0.8$ so that the normalized indoor user-HBS distance is 0.2. See Figure 5.7 and 5.8 for an illustration. Then the normalized outdoor user-BS distance $\bar{d}_O = d_O/d_{HBS}$ is varied in order to study the effect of outdoor-to-femtocell interference.

![Diagram](image)

**Figure 5.7** $P_{out}$ versus the normalized outdoor user-BS distance $\bar{d}_O$ for user rates $R_O = R_I = 1$ and backhaul link capacity $C = 2$ for NF, CA and OA schemes with no power control at the outdoor user ($\rho = 20dB, d_{HBS} = 10$).

Figure 5.7 and 5.8 plot $P_{out}$ versus $\bar{d}_O$ for fixed user rates $R_O = R_I = 1$ and backhaul link capacity $C = 2$ for NF, CA and OA with no power control and with power control at the outdoor user, respectively. When no power control is performed, average channel power gains are given as $\alpha_O = 1/d_O^{\eta}, \alpha_I = 1/d_I^{\eta}, \beta_O = 1/|d_O - d_{HBS}|^{\eta}$ and $\beta_I = 1/(d_{HBS} - d_I)^{\eta}$ with path loss exponent $\eta$, exclusion zones (dark areas in
the figures) are introduced around BS and HBS in order to avoid divergence of the received power. Instead, when power control is performed at the outdoor user, the latter is assumed to scale its transmit power so as to enable the BS to receive at an average constant power. This is obtained by setting the channel gains as $\alpha_O = 1$ and $\beta_O = d_O^\eta / |d_O - d_{HBS}|^{\eta}$, while $\alpha_I$ and $\beta_I$ are as above. It is set $\eta = 3$, $\rho = 20\, dB$ and $d_{HBS} = 10$.

From Figure 5.7, it is seen that without power control, the performance of the outdoor user when the femtocell is not present (NF) clearly decreases as the distance to the BS $\bar{d}_O$ increases. Moreover, CA is able to accommodate also the indoor user with no performance degradation with respect to NF except when the outdoor user creates excessive interference to the HBS, that is, for $\bar{d}_O$ close to 1. OA is able instead to improve the system performance especially when the outdoor user is either close to the HBS, so that decoding of the outdoor user at the HBS is extremely likely, or close to the BS, which boosts reception at the BS.

With power control at the outdoor user, from Figure 5.7, the performance with NF is clearly independent of the distance $d_O$. Moreover, CA shows similar gains as discussed above. The effect of power control on OA, instead, depends on $\bar{d}_O$. In particular, if $\bar{d}_O$ is small, the performance of OA is degraded with respect to no power control due to the fact that the HBS receives at a smaller power and is thus not able to effectively decode the outdoor user. Instead if $\bar{d}_O$ is large, the OA performance is improved due to the larger power transmitted by the outdoor user.

5.6 Downlink

In this section, the considerations made above for the uplink will be extended to the corresponding downlink model, which is shown in Figure 5.9. The BS communicates with indoor and outdoor users with rates $R_I$ and $R_O$, respectively, using the HBS as a possible relay. Power constraints for BS and HBS are both given by $\rho$ and channel
Figure 5.8 \( P_{out} \) versus the normalized outdoor user-BS distance \( \bar{d}_O \) for user rates \( R_O = R_I = 1 \) and backhaul link capacity \( C = 2 \) for NF, CA and OA schemes with power control at the outdoor user \( (\rho = 20dB, d_{HBS} = 10) \).

Gains are defined as for the uplink. Communication between BS and HBS takes place over a backhaul link of capacity \( C \) (bits/ channel use). Channel state information is available at both transmitters and receivers, idealizing standard cellular scenarios in which some form of channel state information is typically known at the base stations. Notice that the messages of both users are provided by the network to an encoder that is assumed to be located at the BS for both outdoor and indoor users, unlike the conventional femtocell design in which messages for the indoor users are directly sent to the HBS (recall discussion in Sec. I, which was focused on the uplink).
Figure 5.9 Downlink of a macrocell overlaid with a femtocell with one indoor (femtocell) and outdoor (macrocell) user. An out-of-band link (e.g., last-mile link) connects the HBS to the BS.

5.6.1 Transmission Strategies

For downlink, two DF-based strategies similar to CA and OA for the uplink are considered. The techniques are based on standard maximum-SNR beamforming and time-division multiplexing.

Closed Access (CA): The encoder at the BS divides the bits (message) intended for the indoor user in two parts, of respective rates $R_{I1}$ and $R_{I2}$ (with $R_I = R_{I1} + R_{I2}$). These, along with the outdoor message, are transmitted in three separate time-slots with appropriate time allocation. The first part of the indoor message, of rate $R_{I1}$, is sent by the BS directly to the indoor user without the help of the HBS, while the second, of rate $R_{I2}$, is sent in cooperation with the HBS using beamforming. To enable cooperation, the second part is conveyed to the HBS over
the backhaul link prior to transmission to the users, so that it must be that \( R_{I2} \leq C \).

It is taken that \( R_{I2} = \min(C, R_I) \) to maximize the amount of cooperation.

**Open Access (OA):** While in the CA scheme the HBS operates as a relay only for the indoor user, with the OA scheme, as for the uplink, the relay assists both users. In order to enable cooperation, the BS divides the indoor message as above and performs a similar operation on the outdoor message, producing two submessages of rates \( R_{O1} \) and \( R_{O2} \) (with \( R_O = R_{O1} + R_{O2} \)). The first is transmitted only by the BS, while the second is sent cooperatively using beamforming by BS and HBS. The resulting four submessages are transmitted in four separate time-slots with appropriate time allocation. Beamforming by the BS and HBS is performed over the ”cooperative” messages of rates \( R_{I2} \) and \( R_{O2} \) in the corresponding time-slots. As for CA, both ”cooperative” messages have to be conveyed to the HBS via the backhaul link. For this purpose, a parameter \( 0 \leq \gamma \leq 1 \) is defined so that \( R_{I2} = \min(R_I, \gamma C) \) and \( R_{O2} = \min(R_O, (1 - \gamma)C) \).

### 5.7 Outage Analysis

As for the uplink, the outage probability is defined as the probability that *at least one* of the broadcast messages from the BS is not successfully decoded at the indoor user and outdoor users. The outage probability can then be computed as \( P_{out} = P_{out,O} + P_{out,I} - P_{out,O} \cdot P_{out,I} \), where \( P_{out,I} \) and \( P_{out,O} \) are the probabilities of outage at the indoor user and outdoor user, respectively. Defining as \( \lambda_{I1}, \lambda_{I2}, \lambda_{O1}, \) and \( \lambda_{O2} \) the time fractions allocated to the messages of rates \( R_{I1}, R_{I2}, R_{O1} \) and \( R_{O2} \), respectively, where \( \lambda_{I1} + \lambda_{I2} + \lambda_{O1} + \lambda_{O2} = 1 \), calculations of \( P_{out,I} \) for the OA scheme can be elaborated. Other calculations follow in a similar fashion. We have

\[
P_{out,I} = 1 - \Pr \left[ R_{I1} \leq \lambda_{I1} C \left( \alpha_{IgIB} \rho \right) , \right.
\]

\[
R_{I2} \leq \lambda_{I2} C \left( \left( \alpha_{IgIB} + \beta_{IgIH} + 2 \sqrt{\alpha_I} \sqrt{\beta_I} |h_{IB}| |h_{IH}| \right) \rho \right) .
\]
Notice that probability (5.29b) is the probability of decoding correctly the cooperative message of rate $R_{I2}$ which benefits from beamforming by the BS and HBS.

5.7.1 Numerical Results

Figure 5.10 plots the probability of outage $P_{out}$ optimized over the backhaul allocation parameter $\gamma$ and the time allocation parameters versus the SNR $\rho$ for a system with fixed rates $R_O = R_I = 1$ (bits/ channel use), link capacity $C = 0.5$ and $C = 2$ and channel power gains $\alpha_o = -10dB, \alpha_I = -20dB, \beta_o = 10dB$, and $\beta_I = 20dB$. The outage performance of OA and CA is compared with the performance of NF and $C = 0$, as discussed in Sec. 5.5. It is observed that similar performance gains and conclusions can be attained in the downlink as in the uplink when exploiting the HBS-BS backhaul link ($C > 0$) with either CA or OA. In particular, it is seen that a CA scheme enables reduction of the performance loss with respect to a NF scenario, while an OA approach may improve the overall system performance in terms of outage.

5.8 Concluding Remarks

This chapter elaborates on the premise that home base stations (HBSs) may be used as relays for both the subscriber’s devices and macrocell users, rather than being used merely as isolated encoders and decoders as in the standard deployment of femtocells. The advantages of this approach have been studied by performing an outage analysis over quasi-static fading channels for specific relaying strategies based on both decode-and-forward and compress-and-forward techniques. The analysis, mostly focused on the uplink, shows that using a “closed-access” approach the overall performance loss due to the presence of additional indoor users in the femtocell can be overcome if the HBS is used as relay. Moreover, leveraging an “open-access” approach, especially in regimes of low multiplexing gains or sufficiently large outdoor
Figure 5.10  Downlink probability of outage $P_{out}$ versus $\rho$ for user rates $R_O = R_I = 1$ and different values of link capacity $C$ for CA and OA schemes.

user-HBS channel power gains, operating HBSs as relays is able to even improve the overall system transmission reliability while accommodating also indoor users. The results also lend evidence to the advantages of communicating soft information via compress-and-forward techniques from the home base station when the backhaul capacity is sufficiently larger than the users’ aggregate rate. This is a fairly common scenario, especially if the backhaul link is a fiber optic cable.

Overall, while further analysis in more complex scenarios with multiple cells and users is required for a more thorough assessment, the analysis suggests that the proposed approach is viable and has the potentiality to greatly improve system performance.
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