Particle filtering for frequency estimation from acoustic time-series in dispersive media

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Acoustic signals propagating in the ocean carry information about geometry and environmental parameters within the propagation medium. Accurately retrieving this information leads us to effectively estimate parameters that are of utmost importance in environmental studies, climate monitoring, and defense. This dissertation focuses on the development of sequential Bayesian filtering methods to obtain accurate estimates of instantaneous frequencies using Short Term Fourier Transforms within the acoustic field measured at an array of hydrophones, which can be used in a subsequent step for the estimation of propagation related parameters. We develop a particle filter to estimate these frequencies along with modal amplitudes, variance, model order. In the first part of our work, we consider a Gaussian model for the error in the data measurements, which has been the standard approach in instantaneous frequency estimation to date. We here design a filter that identifies the true structure of the data errors and implement a $\chi^2$ model to capture this structure appropriately. We demonstrate both with synthetic and real data that our approach is superior to the conventional method, especially for low Signal-to-Noise-Ratios.
PARTICLE FILTERING FOR FREQUENCY ESTIMATION FROM ACOUSTIC TIME-SERIES IN DISPERSIVE MEDIA

by

Nattapol Aunsri

A Dissertation
Submitted to the Faculty of
New Jersey Institute of Technology
Rutgers, The State University of Jersey - Newark
in Partial Fulfillment of the Requirements for the Degree of
Doctor of Philosophy in Mathematical Sciences

Department of Mathematical Sciences, NJIT
Department of Mathematics and Computer Sciences, Rutgers-Newark

January 2014
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Presentations and Publications:

To my family
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CHAPTER 1

INTRODUCTION

1.1 Motivation

The ultimate goal of sequential filtering is to estimate the unobserved states of a dynamic evolving system given noisy observations; terms “states” or “state vectors” refer to parameters that determine the nature of the data and are the results obtained at the output of the filtering process. More specifically, the main issue in such problems is the computation of point estimates and of joint and marginal conditional posterior probability density functions (PDFs) of the current underlying parameters given information from previous states and corresponding PDFs, following a Markovian model. For example, the evolution of the frequency content of an acoustic signal with time, a problem encountered in audio processing [18] and ocean acoustics [59], carries information on the characteristics of the signal and the propagation medium, respectively; it presents us with a suitable problem for developing sequential filtering methods for information extraction.

Sequential filtering, particle filtering, in particular, which will be discussed analytically in subsequent chapters, is a powerful tool in ocean acoustics [42, 56–59], because many features of signals propagating in the ocean and/or parameters of the oceanic medium evolve dynamically with time or space. For example, as mentioned above, the frequency content of an acoustic signal changes with time; the pattern of this change portrays a feature characteristic of the ocean properties, especially when broadband signals with frequencies of a few hundred Hz propagate long distances in dispersive environments. This characteristic feature, that is, the pattern of dispersion of the waveguide, can be extracted from the observed frequency variation with time. Such estimation can be employed in conjunction with global or local optimization
techniques for source localization and environmental parameter estimation [15,37,43,47,51]. Mostly, Short Time Fourier Transforms (STFTs) and wavelet analysis have been used to date for the estimation of time-frequency characteristics for inversion in underwater acoustics, which is the ultimate goal of this work. Now, however, we want to employ sequential filtering as a processing stage that allows quantification of the uncertainty in the inversion process without the need to make simplistic assumptions that may negatively impact inversion results.

Motivation behind this work stems from the importance of environmental parameter estimation and geoacoustic inversion in antisubmarine warfare and environmental monitoring. We focus on acoustic signals propagating in the ocean, which carry significant information on the physics of the environment. Using particle filtering, we wish to obtain accurate estimates of instantaneous frequencies and amplitudes within acoustic time-series measured at an array of hydrophones. As just mentioned, we treat the particle filtering process as a first step in a two-step inversion process. Once estimates are obtained for parameters associated with the received signals (initially frequencies, with amplitudes added later on), these can be used in the second stage of the process in order to find estimates of environmental properties of the medium, water depth, and source location.

1.2 Background

1.2.1 Estimation in Ocean Acoustics

A sound source emits signals and those, after interaction with the waveguide (mostly shallow-water environments in the problems we are investigating), are measured at a set of hydrophones and are then employed in the estimation of physical and geometric parameters. The received signals differ from the emitted signal due to the distortion caused by the medium characteristics, such as sound speed, bottom properties, and currents among other factors. For a simple case, if we model the
ocean acoustic channel as a linear time invariant system, a convolution of the acoustic channel impulse response with the emitted signal with additive noise superimposed is a mathematical model that can adequately describe the received signals. The impulse response contains information on medium properties and source and receiver location. It is the impulse response that is unknown and, as just mentioned, depends on the parameters that we eventually want to estimate. Methods such as Matched-Field Processing [52] have been developed in the past for estimating the impulse response by maximizing a measure of similarity between the received acoustic fields and possible fields (replicas), that are generated for combinations of the unknown medium and source location parameters. The parameter values that maximize this measure of similarity are the estimates that we wish to obtain. Although successful in many cases involving both synthetic and real data, these approaches are computationally intensive when the dimensionality of the problem is large. We wish to bypass the computational requirements of full field matching by focusing on specific features of the acoustic field (dispersion and its effect on modal arrivals), that we can then match to corresponding features of fields that are produced by theoretical sound propagation models.

1.2.2 Sequential Filtering

It has been more than 50 years that nonlinear filtering has been a topic of significant interest in statistics and many engineering disciplines: several problems of interest in such fields require estimation of the state of a system that changes over time or space, utilizing a sequence of noisy measurements.

A probabilistic framework is often considered as the optimal way to perform tracking in order to deal with uncertainty over time or space. Since tracking frequencies is the goal of this work, such a framework is very suitable. The mathematical and statistical foundations of Bayesian filtering have been extensively presented in the
literature. The first filter that was proposed is the well-known Kalman Filter [30], that tracked parameters in cases of additive Gaussian noise present in the observed data, additive and Gaussian perturbations in the evolution of the unknown parameters, and a linear relationship between measurements and state vector parameters. A number of filtering approaches have been developed since then, building on the traditional KF and improving performance for estimation under more complex and realistic circumstances. These filtering methodologies have been used successfully for target tracking and encompass, beyond the simple KF, a number of KF variants and particle filters (PFs), that is, numerical extensions of KFs that can handle problems with non-linear state and observation equations and noise that is not necessarily additive and Gaussian [3, 6, 48]. In summary, sequential Bayesian filtering combines information on parameter evolution, a mathematical model that relates field measurements to unknown quantities, and a statistical model describing the random perturbations/errors in the measurements.

As just mentioned, KFs are the foundation of sequential Bayesian filtering. When (i) data and parameters are linearly related, (ii) consecutive state parameters vary linearly, and (iii) noise both in the state and observation equations is additive and Gaussian, the KF is the optimal estimator for parameters between consecutive steps in terms of minimizing root mean squared (RMS) errors. Thus, the classical KF has been extensively and successfully employed for the solution of many problems under linear and Gaussian problem assumptions. Based on the linearity and Gaussian nature of the noise, the KF provides PDFs of the state parameters by propagating their expectations and covariances from state to state. Some examples where KFs are successfully employed in source tracking in the ocean are presented in [16,20].

When the problem involves nonlinear/non-Gaussian systems as well as non-additive noise, variations or generalizations of the standard filter are necessary. One of the first extensions of the KF is the Extended Kalman Filter (EKF) [31]. The
EKF linearizes both the system and observation equations and approximates both the system and observation noise as Gaussian, with the first two moments fully characterizing their PDFs. The approximated linear Gaussian system can then be approached using the standard Kalman Filter. There is a series of papers [7–12, 50] that investigate the potential of EKFs in ocean acoustics. Although the EKF is computationally efficient, the convergence of the filter is not guaranteed and its performance could be poor depending on the degree of nonlinearity of the problem. Several algorithms were, subsequently, developed to lower the RMS errors between estimates and true values with better convergence properties than those of the EKF. The Unscented KF (UKF) [14, 53, 56] is such an example. Unlike the EKF that approximates nonlinear functions, the UKF selects deterministic points to represent PDFs and these sample points completely capture the true mean and covariance of Gaussian densities up to the second order of nonlinearity. The UKF can still diverge, however, under challenging circumstances [56]. Strongly nonlinear models and complex noise processes require numerical methods for the computation of posterior PDFs and, ultimately, point estimates and uncertainty. A recent and powerful method for nonlinear filtering developed for such circumstances is particle filtering. It is a class of Monte Carlo simulation-based filtering methods for nonlinear/non-Gaussian systems, a system setting where traditional methods often fail to accomplish satisfactory estimation. Particle filters (PFs) have been explored and used with success in ocean acoustics, as demonstrated in [42, 56, 57, 59]. Particle Filters (PFs) will be designed in this thesis for extracting acoustic field features from signals dispersed in oceanic waveguides. Mathematical and statistical models will be integrated in order to, in the future, successfully estimate source location and ocean environment parameters and express the uncertainty in their estimation.
1.3 Contribution of the Dissertation

In [59], a PF was developed for extracting dispersion curves from spectrograms of acoustic signals that propagate in an oceanic dispersive medium. The field was modeled in the frequency domain (in terms of a Fourier transform power spectrum) as a sum of Gaussian pulses, the centers of which gave us the information that is necessary for inversion. In our work, based on a more accurate theoretical foundation, we extend the formulation of [59] by using superposition of sincs rather that Gaussian pulses to model modal propagation. The need for the use of the new wavelets is made clear when normal mode modeling for sound propagation is employed.

We use the concept of mode identification with state-space models [7] and we propose an approach based on particle filtering to accurately extract arrival times and corresponding instantaneous frequencies of distinct modes from time-frequency representations of signals, extending the approach of [59] using the mathematical model just mentioned. Our PF achieves modal amplitude estimation as well as order determination (number of modes), parameters of great significance in inversion in the ocean because of their link to sediment attenuation. The implementation of a noise variance estimation approach is also considered, so that no assumptions are necessary regarding the noise level of the background distortion imposed on the acoustic signals. A key component of this work is that a more advanced statistical model than the Gaussian one is considered. Specifically, noise can indeed be considered Gaussian in the time domain but, transformed through squaring in the frequency domain, it ceases to have a normal behavior. This is a new direction that we have pursued, which appears to be promising, offering improved results over modeling under conventional assumptions.

As also mentioned previously, we are not only interested in point estimates of the above parameters, as those may be misleading. One of the primary goals is the quantification of the uncertainty in the estimation process through PDFs.
1.4 Organization of Dissertation

A review of the basic theoretical concepts used throughout this work is provided in Chapter 2. This chapter discusses sequential filtering and the need for and principles of frequency estimation from acoustic time-series. Once the fundamental concepts are reviewed, the application of particle filtering to frequency estimation with a simple model is described in Chapter 3. This chapter illustrates details for the use of PFs incorporating the proposed mathematical models for the propagating signals, in order to obtain frequency estimates. Simulation results are also presented. A more sophisticated model including amplitude estimation is the focus of Chapter 4: while work discussed in Chapter 3 assumes the amplitudes associated with frequencies to be known, in this chapter a more realistic problem is addressed, where no knowledge is available on the amplitudes. Additionally, the number of observed modes is typically unknown and is a critical element for successful mode detection and parameter estimation; that is, the model order, as this problem is typically referred to [48], is uncertain. We extend our approach to handle problems where the model order varies and needs to be estimated as well; it is, thus, included in the estimation process as a state parameter. This is done with multiple-model PFs (MMPFs) [48], that allow flexibility in regards to the considered dimensionality of the state. In such cases, the filter not only tracks the typical model parameters (frequencies in our problem as well as amplitudes and noise variance later on) but also tracks the best model order. This problem will be discussed in Chapter 5. In Chapter 6, noise variance estimation is addressed and results are presented. More realistic mathematical and statistical models with $\chi^2$ PDFs for modeling the spectrogram data and corresponding noise are incorporated in our filters and results are shown in Chapter 7. Chapter 8 demonstrates how the PFs presented in the preceding sections can be employed for frequency tracking with real data. Conclusions and plans for future work are discussed in Chapter 9.
2.1 Bayesian Filtering

In many science and engineering applications, we need to recover parameters from measured data corrupted by random noise. The data frequently arrive sequentially in time and/or space; sequential estimation is typically preferred to static estimation, because it integrates evolution information. There are several special cases restricted to a narrow linear Gaussian class of models. Filters, based on the foundations of the original Gaussian/linear class of models but with suitable extensions and modifications, have been developed for more complicated scenarios. Until recently, however, there existed no general framework whenever non-Gaussian behavior and/or extensive non-linearity were implicit in the problems. Although significant progress has been made, these problems still present a major challenge to researchers. Fortunately, sequential Monte Carlo (SMC) filtering, also known as particle filtering or sequential Bayesian filtering [48], that has emerged in the field of engineering and statistics relatively recently, is a key tool in addressing non-Gaussian, non-linear problems that involve dynamically arriving data. We discuss the subject in the following section.

2.1.1 State-Space Models

Many problems in dynamical systems require estimation of the state of a system evolving with time or space using a sequence of noisy measurements. As data become available, unknown parameters related to the data, which form a state vector, are estimated sequentially using a collection of data history and prior knowledge on the
evolution of the state. Tracking in time or space presents us with problems that can be formalized in a state-space framework.

Let $y_k$ be the $n_y$-dimensional measurement or observation vector (for example, pressure along a hydrophone array in ocean acoustics) at step $k$ and let $x_k$ represent the $n_x$-dimensional state vector of interest (for example, location coordinates of a source emitting a signal or frequencies and associated amplitudes of the modes, which is our problem of interest), where $k = 1, \ldots, K$. Our goal is to sequentially estimate $x_k$ as data observations $y_k$ become available, employing knowledge from the $(k-1)$th state.

In order to analyze and make any inference about a dynamic system, we need two models, a transition model describing the evolution of the state with time or space depending on the task at hand and an observation model relating noisy data measurements to the state vector. The two models can be written as:

$$x_k = f_{k-1}(x_{k-1}, v_{k-1})$$  \hspace{1cm} (2.1)

$$y_k = g_k(x_k, w_k).$$ \hspace{1cm} (2.2)

Equation 2.1, the state or system equation, describes the evolution or transition of $x_k$ and assumes that states follow a first order Markov process. Function $f_{k-1}$ is a known function relating the state vector at step $k$ to that at step $k-1$. Term $v_k$ is the state noise or perturbation from one state to the next and has a known probability density function.

The observation equation, Eq. 2.2, also referred to as the update equation, relates measurements $y_k$ to state vector $x_k$ through a usually known function $g_k$, which is a mathematical/physical model (a sound propagation model, in our case). Under rare circumstances, function $g_k$ is unknown and is estimated through a preprocessing,
training stage; that complication does not apply to our problems. Term $w_k$ is the measurement noise with a known probability density.

The Bayesian inference and state-space model setting for Kalman and particle filters will be discussed below.

2.1.2 Bayesian Inference

Let $Y_k = [y_1, y_2, \ldots, y_k]$ be the set of data observations measured at the first $k$ steps and $X_k = [x_1, x_2, \ldots, x_k]$ be the sequence of unknown state vectors. Posterior PDF, $p(X_k|Y_k)$, provides all information about state $X_k$ that is embedded in the observation data $Y_k$ and the prior $p(X_0)$, and it can be written in the form:

$$p(X_k|Y_k) = \frac{p(Y_k|X_k)p(X_0)}{p(Y_k)}.$$ (2.3)

Provided that the observations up to state $k$ are independent and given $X_k$, the likelihood $p(Y_k|X_k)$ in Eq. 2.3 can be written as:

$$p(Y_k|X_k) = \prod_{i=1}^{k} p(y_i|X_k).$$ (2.4)

Conditional on $x_k$, measurement $y_k$ is independent of the states at all other times. As a result, we have:

$$p(Y_k|X_k) = \prod_{i=1}^{k} p(y_i|x_i).$$ (2.5)

Since we assume that the system follows a first order Markov process, $p(X_0)$ holds the following form:

$$p(X_0) = p(x_0) \prod_{i=1}^{k} p(x_i|x_{i-1}),$$ (2.6)

and the posterior PDF is given by

$$p(X_k|Y_k) = \frac{p(x_0) \prod_{i=1}^{k} p(y_i|x_i)p(x_i|x_{i-1})}{p(Y_k)}.$$ (2.7)
As mentioned previously, the goal is to estimate state vector $x_k$ based on the observed data $y_k$. Because of the associated computational cost, the problem is often simplified by recursively estimating the marginal PDF $p(x_k|Y_{k-1})$ from $p(x_{k-1}|Y_{k-1})$, rather than estimating the joint PDF $p(X_k|Y_k)$. A recursive formula for the joint PDF can be obtained from Eq. 2.7.

Assuming that $p(x_{k-1}|Y_{k-1})$ is available, we can predict $p(x_k|Y_{k-1})$ through the transition PDF $p(x_k|x_{k-1})$. The state equation and the noise PDF, $p(v_k)$, determine the transition density. As mentioned earlier, a first order Markov chain process is assumed for $x_k$; this suggests that $p(x_k|x_{k-1})$ does not depend on data $Y_{k-1}$. Density $p(x_k|Y_{k-1})$ can then be written as:

$$p(x_k|Y_{k-1}) = \int p(x_k|x_{k-1}, Y_{k-1})p(x_{k-1}|Y_{k-1})dx_{k-1}$$

$$= \int p(x_k|x_{k-1})p(x_{k-1}|Y_{k-1})dx_{k-1}. \quad (2.8)$$

Eq. 2.8 is referred to as the Chapman-Kolmogorov Equation. The new estimate of the state vector at state $k$ is calculated from Bayes theorem:

$$p(x_k|Y_k) = \frac{p(y_k|x_k)p(x_k|Y_{k-1})}{p(y_k|Y_{k-1})}. \quad (2.9)$$

The denominator in Eq. 2.9 is the normalizing constant of $p(x_k|Y_k)$ and can be expressed as:

$$p(y_k|Y_{k-1}) = \int p(y_k|x_k)p(x_k|Y_{k-1})dx_k. \quad (2.10)$$

Note that $p(y_k|x_k)$ represents the PDF of $y_k$ given $x_k$, defined by the measurement model and observation noise. When an observation $y_k$ is available, $p(y_k|x_k)$ is the likelihood of the state vector $x_k$, that is, $l(x_k) = p(y_k|x_k)$.

Using the PDF of Eq. 2.9, estimates of the state at time $k$, conditional upon the measurements up to time $k$, can be made. Optimal estimators [31] include the minimum mean squared error (MMSE) estimator, which calculates the expected value
E[x_k|Y_k] of the state after data have been observed in the following way:

\[
\hat{x}_k = E_{p(x_k|Y_k)}[x_k|Y_k] = \int x_k p(x_k|Y_k) \, dx_k, \tag{2.11}
\]

and the maximum a posteriori (MAP) estimator that provides the most likely value of the state after observing the data:

\[
x_k^{MAP} = \arg \max p(x_k|Y_k). \tag{2.12}
\]

We can easily extend these to estimate functions of the state as

\[
\hat{\psi}(x_k) = E_{p(x_k|Y_k)}[\psi(x_k)|Y_k] = \int \psi(x_k) p(x_k|Y_k) \, dx_k. \tag{2.13}
\]

2.1.3 Kalman Filter

The Kalman Filter assumes that the state and measurement models are linear and that all noise incorporated in the system is additive and Gaussian, fully characterized by a mean and covariance. Assuming that density \(p(x_{k-1}|Y_{k-1})\) is Gaussian, it can be shown that \(p(x_k|Y_k)\) is also Gaussian. Because of that, the mean and covariance of the posterior density of the state parameters can be tracked efficiently from state to state. Under the Gaussian assumption, the KF is the optimal MMSE state estimator [30].

The KF uses the state-space model, formally consisting of a state equation and an observation equation, such as the ones described in the previous section. The complete equations defining the KF are provided below:

\[
x_k = F_{k-1} x_{k-1} + v_{k-1} \tag{2.14}
\]

\[
y_k = G_k x_k + w_k. \tag{2.15}
\]

where \(F_{k-1}\) and \(G_k\) are known matrices that define the linear functions describing state evolution and relation between data and state, respectively. Quantities \(v_{k-1}\)
and \( w_k \) have zero mean and are statistically independent. The covariances of \( v_{k-1} \) and \( w_k \) are \( Q_{k-1} \) and \( R_k \), respectively.

The following recursive system forms the Kalman Filter [48]:

\[
p(x_{k-1}|Y_{k-1}) = \mathcal{N}(x_{k-1}; \hat{x}_{k-1|k-1}, P_{k-1|k-1})
\]  

(2.16)

\[
p(x_k|Y_{k-1}) = \mathcal{N}(x_{k}; \hat{x}_{k|k-1}, P_{k|k-1})
\]  

(2.17)

\[
p(x_k|Y_k) = \mathcal{N}(x_{k}; \hat{x}_{k|k}, P_{k|k})
\]  

(2.18)

where

\[
\hat{x}_{k|k-1} = F_{k-1} \hat{x}_{k-1|k-1}
\]  

(2.19)

\[
P_{k|k-1} = Q_{k-1} + F_{k-1} P_{k-1|k-1} F_k^T
\]  

(2.20)

\[
\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k (y_k - G_k \hat{x}_{k|k-1})
\]  

(2.21)

\[
P_{k|k} = P_{k|k-1} - K_k S_k K_k^T
\]  

(2.22)

Here \( \mathcal{N}(x; m, \Sigma) \) is a Gaussian density with argument \( x \), mean \( m \), and covariance \( \Sigma \).

Matrix \( S_k \),

\[
S_k = G_k P_{k|k-1} G_k^T + R_k,
\]  

(2.23)

is the covariance of innovation term \( y_k - G_k \hat{x}_{k|k-1} \), and

\[
K_k = P_{k|k-1} G_k^T S_k^{-1}
\]  

(2.24)

is the Kalman gain.

This is the optimal solution to the tracking problem, when the normality and linearity assumptions hold: no other algorithms can perform better than a KF in terms of MMSE in this linear Gaussian environment [4].
The disadvantage of the KF is its reliance on the linear/Gaussian forms of the state and observation densities. In many estimation problems, non-linearities may be mild and the noise could be well described as additive and Gaussian. Then a simple KF approximation may suffice for accurate estimation. The idea is to locally linearize the state and observation functions as needed with a first-order Taylor approximation and then apply a KF. This approach is referred to as the Extended Kalman Filter (EKF). For some problems, such an approximation may not be adequate. Considering higher order EKFs that retain further terms in the Taylor expansion may provide better estimates. If the model equations are highly nonlinear, however, and the noise is far from additive and Gaussian, any approximations along these lines may lead to erroneous estimates and divergence. Further details can be found in [1,4].

For highly nonlinear problems, the EKF is known to encounter issues with both accuracy and stability, as just mentioned. In this filter, the state density is propagated via a first order linearization of the nonlinear system; this approximation could result in the posterior mean and covariance being corrupted. The Uncertain Kalman Filter (UKF) [29], which is a derivative-free alternative to the EKF, overcomes this problem by using a deterministic sampling approach, employing a weighted set of deterministically sampled points, which are passed through the nonlinearity and are used to approximate the statistics of the density. The unscented transformation is accurate to at least third-order. However, the UKF can still diverge in complex cases [56]. For detailed information, we refer the reader to Refs. [29,54].

Since our tracking problem involves highly nonlinear and non-Gaussian systems in some aspects, analytical implementations of the KF and its variants are not suitable. Particle filtering is considered as a tool for achieving effective tracking, because of its numerical flexibility and applicability to problems of the nature we are handling in our work.
2.2 Particle Filtering

Particle filtering performs sequential estimation based on representation of probability densities of which main characteristics (mean and variance, for example) are not known. The idea is that the posterior density is represented by a set of particles, and their associated weights which correspond to probabilities, \( \{x^i_k, w^i_k\}_{i=1}^{N_p} \), where \( N_p \) is the number of particles. Particles are expected to populate high probability regions for the unknown parameters and be sparse in lower probability regions. Depending on the nature of the problem, each particle evolves with time/space according to the state equation, with evolving posterior PDFs determined by the observation equation.

To define the tracking problem using PFs and following the previous discussion of the state-space framework, we consider the evolution of the sequence of the state parameters (the unknown parameters that we want to estimate) and the relation between measurements and parameters as follows:

\[
x_k = f_{k-1}(x_{k-1}, v_{k-1}) \quad (2.25)
\]
\[
y_k = g_k(x_k, w_k). \quad (2.26)
\]

Note that functional relations in Eqs. 2.25 and 2.26 are not denoted as linear in contrast to Eqs. 2.14 and 2.15; similarly, noise is not shown as additive. This is because we consider a general case that, although not analytically solvable with a KF, can be approached with numerical techniques.

In this framework, noise \( v_k \) and \( w_k \) can be additive, multiplicative, or incorporated in the state and measurements through complex functions of \( f_k \) and \( g_k \), respectively. In addition to the state vector \( x_k \) and data \( y_k \) and also functions \( f_k \) and \( g_k \), noise components \( v_k \) and \( w_k \) can change with \( k \) as well. An example where \( w_k \) changes with \( k \) in the ocean is when we have time-dependent interference corrupting the data.
Since, in our problems, noise $w_k$ and parameter perturbation $v_k$ are considered to be additive and Gaussian, at least in the beginning, we will be using equations:

$$x_k = f_{k-1}(x_{k-1}) + v_{k-1} \quad (2.27)$$
$$y_k = g_k(x_k) + w_k. \quad (2.28)$$

Eq. 2.28 will later be replaced by Eq. 2.26, after we show that modeling noise behavior in the frequency domain, in which we will be operating, as additive and Gaussian is not accurate.

The state equation, Eq. 2.27, along with the distribution of the system noise induce the state transition density $p(x_k|x_{k-1})$, and the likelihood $p(y_k|x_k)$ is formed based on the observation equation, Eq. 2.28, and the density of the observation noise.

### 2.2.1 Sequential Importance Sampling

Particle filters utilize Monte Carlo simulation and importance sampling techniques [44] to estimate the conditional density of the state given past observations. The principles of sequential importance sampling (SIS) is reviewed in this section.

As previously mentioned, a PF approximates a conditional probability density using a finite number of particles and models the evolution of the conditional density through the propagation of these particles. The approximated density $p(x_k|y_k)$ by a probability mass function (PMF) is given by

$$p(x_k|y_k) = \sum_{i=1}^{N_p} w_k^i \delta(x_k - x_k^i), \quad (2.29)$$

where $\delta$ is the Dirac delta function. The accuracy of the approximation increases as $N_p$ increases, that is, the estimated density approaches the true PDF as $N_p$ approaches infinity [6, 48]. A set $\{x_k^i\}_{i=1}^{N_p}$ is a random sample of values for the state parameters, whose associated weights form the set $\{w_k^i\}_{i=1}^{N_p}, \sum_{i=1}^{N_p} w_k^i = 1$. The particles are generated via sampling from a known density referred to as the proposal
or importance density \( q(x) \) and the weights of the samples are adjusted through the likelihood function to provide an estimate of \( p(x_k|y_k) \). When we choose a \( q(x) \) that is proportional to \( p(x) \), the variance in the estimate is minimized. However, it may not be easy to sample from this density and simpler PDFs such as Gaussian are often used. Using importance sampling (IS) requires the selection of a balanced \( q(x) \): easy to sample from but without compromising accuracy. Bayesian filtering runs IS successively in order to compute the integral in Eq. 2.10 sequentially with \( k \). The output of the previous step forms the prior for the next one. This process is called Sequential Importance Sampling (SIS).

To approximate \( p(X_k|Y_k) \) using \( \{X_k^i\}_{i=1}^{N_p} \), we draw samples from an importance density \( q(X_k|Y_k) \) and their weights can be defined as

\[
w_k^i \propto \frac{p(X_k^i|Y_k)}{q(X_k^i|Y_k)}. \tag{2.30}
\]

If we use the results from the previous IS step, and choose the following importance density

\[
q(X_k|Y_k) = q(x_k|X_{k-1}, Y_k)q(x_{k-1}, Y_{k-1}), \tag{2.31}
\]

we can obtain the posterior PDF as:

\[
p(X_k|Y_k) = \frac{p(y_k|x_k)p(x_k|x_{k-1})}{p(y_k|Y_{k-1})}p(X_{k-1}, Y_{k-1}). \tag{2.32}
\]

Substituting Eqs. 2.31- 2.32 into Eq. 2.30, the weight of the \( i \)th particle at time/space \( k \) is \([48]:\)

\[
w_k^i \propto \frac{p(X_{k-1}^i|Y_{k-1})p(y_k|x_k^i)p(x_k|x_{k-1}^i)}{q(x_k^i|X_{k-1}^i, Y_k)q(x_k^i|X_{k-1}^i, Y_{k})} \tag{2.33}
\]

\[
w_k^i = w_{k-1}^i \frac{p(y_k|x_k^i)p(x_k|x_{k-1}^i)}{q(x_k^i|X_{k-1}^i, Y_k)}. \tag{2.34}
\]
As discussed in [17, 48], if a sampling density of the form of 

\[ q(x_k|x_{k-1}, y_k) = q(x_k|x_{k-1}, y_k) \]

is selected, then the importance density only depends on \( x_{k-1} \) and \( y_k \); data history \( Y_{k-1} \) can be discarded. The new weights are:

\[
w_k^i = \frac{p(y_k|x_k^i)p(x_k^i|x_{k-1}^i)}{q(x_k^i|x_{k-1}^i, y_k)} w_{k-1}^i. \tag{2.35}
\]

In the SIS PF, the importance density \( q(x_k|x_{k-1}, y_k) = p(x_k|x_{k-1}, y_k) \) is chosen to minimize the IS error [17]. A simple variant of the SIS can be obtained by choosing the transition density as:

\[
q(x_k|x_{k-1}, y_k) = p(x_k|x_{k-1}), \tag{2.36}
\]

which is independent of the current observation \( y_k \) [46]. From this choice, Eq. 2.35 is finally reduced to

\[
w_k^i = p(y_k|x_k^i)w_{k-1}^i. \tag{2.37}
\]

The variance of the importance weights increases with time/space [33, 48]. This strongly influences the filter performance since the majority of the normalized weights tends to be zero after a few states. The posterior PDF approximation could be a poor representation for the true PDF because of that; these samples are numerically insignificant and negligible in the PDF approximation. In the extreme case, there is only one particle left with a large \( w_k^i \) (equal to one), resulting in poor accuracy; this complication is referred to as degeneracy. Implementation of suitable methods is essential for resolving this problem; this leads to the resampling scheme that will be described in the following section.

### 2.2.2 Sequential Importance Resampling

To remedy degeneracy, a resampling step is proposed at each state at the end of the integral calculation [21, 49], that is, after prediction and update. Resampling is a key
process for the successful implementation of PFs. The resampling procedure is used to remove those particles with negligible importance weights and produce multiple copies of those particles with significant weights. Thus, samples are resampled with replacement using the importance weights as probabilities. The particles with larger weights may be chosen a number of times and samples with small weights may not be selected at all. The process that incorporates resampling within sequential filtering is referred to as Sequential Importance Resampling (SIR) [17, 21, 48].

Recall the posterior PDF \( p(\mathbf{x}_k | \mathbf{Y}_k) \) that is represented by particle clouds and associated weights \( \{\mathbf{x}^i_k, w^i_k\}_{i=1}^{N_p} \) obtained from the IS step. From the set \( \{\mathbf{x}^i_k, w^i_k\}_{i=1}^{N_p} \), the resampling process removes particles that have small weights; those particles with significant weights are retained and replicated. All new weights of the new particle set after resampling are the same, that is, \( \{\mathbf{x}^j_k, w^j_k = 1/N_p\}_{j=1}^{N_p} \). The concept of resampling is to retain the large probability particles and ignoring others. This provides a larger number of particles in the high likelihood regions than that before the resampling stage.

As mentioned, resampling improves the estimation of future states by concentrating particles into higher probability region. However, it can reduce the accuracy of the current estimate by increasing the variance of the estimate after resampling. As a result, we need to perform resampling with careful consideration. A technique for judging the need for resampling is to use the effective number of particles \( N_p^{\text{eff}} \) needed to prevent the degeneracy problem encountered in SIS, comparing it to a threshold [33]: \( N_p^{\text{eff}} = 1/\sum_{i=1}^{N_p} (w^i_k)^2 \). It has been suggested that when all the importance weights are nearly equal, there is no benefit in performing resampling [35, 36].

Although one can reduce the effect of degeneracy through the resampling step, a problem that could occur is the loss of particle diversity. This is known as sample impoverishment and occurs when all particles are identical. This is especially preva-
lent when the noise level is very low. There are developments of PFs [13, 22, 48] that have been designed for remedying these problems.

2.3 Frequency Approximation of a Signal Propagating in the Ocean via a Fourier Transform

In this section we discuss the mathematical modeling of acoustic signals in the time-frequency domain, which is the foundation for the incorporation of physics within our sequential Bayesian filtering framework. Specifically, the model presented below is the main building block of the observation equation.

Consider a broadband acoustic signal received at a hydrophone in the ocean. The sound pressure as a function of time can be written as:

\[
p(r, z, z_r, t) = \frac{1}{2\pi} \sum_n \int_{-\infty}^{+\infty} \mu(\omega') G_n(r, z, z_r, \omega') \exp\{i(\omega' t - k_n r - \pi/4)\} d\omega'. \tag{2.38}
\]

Here, \( r \) represents the distance between source and receiver, \( z \) and \( z_r \) are the source and receiver depths, respectively, \( k_n \) is the modal wave number, \( \mu \) is the source spectrum, \( \omega = 2\pi f \), where \( f \) is frequency, and

\[
G_n(r, z, z_r, \omega) = \frac{i\sqrt{\pi}}{\rho(z_r)\sqrt{2k_n r}} \Upsilon_n(z) \Upsilon_n(z_r), \tag{2.39}
\]

where \( \Upsilon_n \) are orthogonal, normalized, depth-dependent functions, and \( \rho(z_r) \) is the medium density. Although our signals are multimodal, we initially consider only one mode for simplicity. This restriction will be relaxed later.

The frequency spectrum of a finite time segment of the signal is provided in [39, 55] and can be expressed as follows:

\[
P_n(\omega, t) = \int_{t-\Delta t}^{t+\Delta t} p_n(r, z, z_r, t)e^{-i\omega \tau} d\tau, \tag{2.40}
\]
where the segment starts and ends at \( t - \Delta t \) and \( t + \Delta t \), respectively. Substituting the \( n \)th term of Eq. 2.38 into Eq. 2.40, interchanging the order within the integral, and integrating over time, we obtain:

\[
P_n(\omega, t) = \frac{e^{-i\omega t}}{\pi} \int_{-\infty}^{\infty} \mu(\omega') G_n(r, z, z_r, \omega') \frac{\sin(\omega' - \omega) \Delta t}{\omega' - \omega} \exp\{i(\omega' t - k_n r - \frac{\pi}{4})\} d\omega'.
\]

(2.41)

By applying the stationary phase approximation to Eq. 2.41, we can obtain the instantaneous power spectrum of the field:

\[
|P_n(\omega, t)|^2 = \frac{\pi}{|k_n|^2} |\mu(\omega_n) G_n(r, z, z_r, \omega_n)|^2 \frac{\sin(\omega' - \omega) \Delta t}{\omega' - \omega}^2,
\]

(2.42)

for \(|\omega - \omega_n| < \frac{\pi}{2\Delta t}\).

The spectrum expressed in Eq. 2.42 has a peak at the modal frequency \( \omega_n \). Thus, we can trace the peaks of the instantaneous power spectra to identify the modal frequencies of the acoustic signal. Instead of \( \omega_n \), in order to follow conventional notation for instantaneous frequency estimation, we will be using from now on symbol \( f_n \) for frequency.

From the analysis that led to Eq. 2.42, it appears that the power spectrum can be approximated by a squared sinc pulse, weighted by the squared amplitude of the modal arrival. Moreover, the superposition of these pulses provides an approximation of the power spectrum of a multiple-mode signal. We can write the measurement equation relating data \( y \) to frequency particles \( x_{kj} \) for the STFT approach as follows:

\[
y_k = \sum_{j=1}^{M} a_{kj}[\text{sinc}(f - x_{kj})]^2 + w_k.
\]

(2.43)

Note that \( y_k \) is the FT of the acoustic time series: our approach relies on modeling the signal in the frequency domain at consecutive time slices. The likelihood for unknown frequencies for a particle is:

\[
p(y_k|x_k) \propto \exp\{-\frac{1}{2\xi^2} \|y_k - \sum_{j=1}^{M} a_{kj}[\text{sinc}(f - x_{kj})]^2\|^2\}.
\]

(2.44)
This likelihood will be used in subsequent chapters for the eventual estimation of posterior PDFs within the sequential filtering process.
PARTICLE FILTERING: A SIMPLE MODEL

In this chapter, we develop a PF for the estimation of the frequency content of signals from time-frequency representations calculated via STFT. We first consider a fixed model order, that is, the number of modes present over time is known and constant. We treat signal (modal) amplitudes in a similar fashion.

3.1 Model Description and State Space Model

As described in Chapter 2, the instantaneous power spectrum of a unimodal acoustic signal received at a hydrophone in the ocean can be written as a multiple of a squared sinc function. This leads to the model setting for the PF, which can be extended to multimodal signals in a straightforward manner. The aim of this chapter is the estimation of the multiple modal frequencies within a received acoustic time-series, as these evolve with time. To estimate those quantities, we need to calculate the joint PDF of the frequencies for each time step. To construct the state-space representation, let \( x_k = [x_{k1} \ x_{k2} \ldots \ x_{kM}] \) be the vector containing all unknown frequencies at time \( k \), where \( M \) is the number of modes. These modal frequencies are represented as states in the state equation (Eq. 2.27). Another equation required for our set-up is the observation equation (Eq. 2.28). This equation includes the noise model in the measurements at the receivers; initially, we consider the noise to be Gaussian and additive to the spectra. From these assumptions and as we also explained in Chapter 2, the state and measurement equations for predicting and updating frequencies from acoustic time-series can be written in the simplest case as:

\[
x_k = x_{k-1} + v_{k-1}
\]  

(3.1)
\[ \mathbf{y}_k = \mathbf{g}(\mathbf{x}_k) + \mathbf{w}_k. \]  

(3.2)

Instead of target locations - typically the case in tracking, as previously mentioned, \( \mathbf{x}_k \) is a state vector containing modal frequencies; its dimension is the number of frequencies that we are tracking, which is initially assumed to be known. Vector \( \mathbf{y}_k \) contains data observations at time \( k \). In addition, \( \mathbf{v}_k \) and \( \mathbf{w}_k \) represent state and measurement noise. In this work “perturbation noise” is additive, zero mean, and normally distributed, that is, \( \mathbf{v}_k \sim \mathcal{N}(\mathbf{0}, \Sigma_v) \), where \( \Sigma_v \) is an \( M \times M \) diagonal matrix, with its diagonal elements \( \sigma_v^2 \) having values that are empirically selected; we assume them to be identical, but that restriction can be relaxed in a straightforward manner. Quantity \( M \) is the number of the modal frequencies present in the time series. We also set \( \mathbf{w}_k \sim \mathcal{N}(\mathbf{0}, \xi^2_w) \). The latter is, however, an assumption. In reality, noise can be assumed to be Gaussian and additive to the original time-series but not to the spectra. Tracking probability densities related to the spectrogram for time-series that have been measured in a Gaussian environment, we can identify the PDFs of the spectra slices to be non-central \( \chi^2 \). This topic will be examined later.

We need to sequentially estimate the frequencies evolving with time from the spectrum of measured data as discussed in the previous chapter. The spectrum computed via STFT is modeled as a sum of \( M \) squared, shifted, and scaled squared sinc functions (shown in Chapter 2). The measurement equation can be written as:

\[ \mathbf{y}_k = \sum_{j=1}^{M} a_{kj}[\text{sinc}(f - x_{kj})]^2 + \mathbf{w}_k. \]  

(3.3)

The length of \( \mathbf{y}_k \) is the range of frequencies of interest. The likelihood for the unknown frequencies evaluated for a particle given the Gaussian noise assumption is:

\[ p(\mathbf{y}_k|\mathbf{x}_k) \propto \exp\left\{ -\frac{1}{2\xi^2_w} \| \mathbf{y}_k - \sum_{j=1}^{M} a_{kj}[\text{sinc}(f - x_{kj})]^2 \|^2 \right\}. \]  

(3.4)
3.2 Implementation of the Particle Filter

According to our state space-model for frequency tracking, the propagation of the frequencies in the state vector $x_k$ follows a simple Brownian process resulting in the following transition density:

$$p(x_k|x_{k-1}) \sim N(x_{k-1}, \sigma_k^2).$$

(3.5)

We design an SIR filter for our problem. Initially, the PDF of frequencies at $k = 0$ is unknown. For this reason, we initialize the process by drawing a set of particles $\{x^i_0\}_{i=1}^{N_p}$ from a uniform distribution over the entire support frequency space. The particles are propagated via a normal density dictated by the transition density of Eq. 3.5. The likelihood, and, consequently the weights, can be computed for each particle using a multivariate Gaussian density function. Then, the weights are normalized and a resampling step is implemented, in order to create a new set of particles. We are now ready to make inferences regarding the frequencies; we use a MAP estimator to obtain estimates of the modal frequencies.

3.3 Simulation Results

3.3.1 Tracking Results from a Test Signal

Below, we present an example of frequency tracking from a synthetic signal using a spectrogram. The signal is composed of three modes and Figure 3.1(a) shows the spectrogram. Figure 3.1(b) demonstrates the tracking results for the three modes. Because the problem is very simple, the process does not require a large number of particles. For the results presented here, we only needed 300 particles to achieve successful tracking. The figures demonstrate a good performance throughout, even at the initial states.

Figure 3.2 illustrates the evolution of the distribution of the same mode at times 50, 100, 150, and 200 ms. The frequency is decreasing as the time evolves, which is
Figure 3.1  (a) Spectrogram of a three-mode synthetic signal and (b) tracks estimated by the PF.
Figure 3.2 Distribution of the first mode.

the expected physical behavior of high-order modal arrivals that we imposed on our synthetic signal.

Figure 3.3 shows the evolution of the distributions of all modes at times 50, 100, 150, and 200 ms.

To demonstrate the importance of accurate information on the amplitudes (or suitable modeling of the uncertainty as will be seen later), we set up a filter where the assumed amplitude values are far from the true amplitudes. Tracking results show that the PF delivers poor estimates, as demonstrated in Figure 3.4. Figure 3.4(a) illustrates the same spectrogram of the three-mode signal as the one in the previous example. When the amplitudes are not known correctly, Figure 3.4(b) depicts the tracking results for the three modes. We see that the PF does not track all the modal frequencies correctly: only one mode (top mode) can be tracked correctly. For the other two modes, it provides incorrect estimates right from the beginning and does not reach the correct tracks at any time. From these results, we can see that inaccurate assumptions for the unknown amplitudes result in poor frequency estimation, even
when the signal has a simple form. Unless accurate prior information is available (which is unrealistic), amplitudes need to be estimated concurrently with frequencies. This problem will be discussed in the next chapter.

It was previously mentioned that a multiple model/order particle filter (MMPF) will be developed in this thesis. We here discuss an example where the PF with a fixed order performs estimation for a time-varying number of modes. Figure 3.5(a) shows the spectrogram of the varied order signal and Figure 3.5(b) presents the tracking results for the signal using the fixed order PF. We assume that the amplitudes are correctly known in this example. The PF tracks frequencies correctly in the first 50 ms because the assumed order is three and, indeed, three modes are present in the signal. It should be noted that several time steps are required before the PF can track all frequencies accurately. This is expected, because, during the initial steps, the particle filter has not yet accumulated a significant amount of information from previous states. This information “builds” and enhances the performance of the filter after a few time slices. After 50 ms, the PF erroneously keeps tracking
three frequencies, but there are only two modes between 50 and 90 ms. Thus, the PF produces incorrect estimates in this interval. This is especially evident in times 130 – 170 ms: although there are three modes present during this time, the PF has already lost their tracks from previous slices, resulting in poor estimates. Since the PF is built conditionally on the number of modes, that is, on the assumption of the presence of exactly three modes, it tries to identify these three frequencies within an observation slice. This is incorrect, because the order or number of frequencies within each time “slice” changes over time.

3.3.2 Tracking Results from a Synthetic Signal

A synthetic signal, more similar to the spectrogram of interest than the one presented above, was generated in order to study the efficiency of the algorithm under controlled circumstances. The signal at each time slice is a superposition of squared sinc pulses,
Figure 3.5  (a) Spectrogram of a synthetic signal with a time-varying number of modes and (b) tracks estimated by the PF when a known and constant number of modes is assumed.

with these pulses being weighted reproductions of the source waveform. The signal contains a number of modes varying with time. The generated signal is shown in Figure 3.6.

Just for this test, we set the amplitudes to be fixed and constant at one. White Gaussian noise with a constant variance over time is added to the synthetic signal. Figure 3.7 presents a noisy signal with noise variance equal to 0.2.

We assume that the model order is four and known for all time slices. Figure 3.8 shows the tracking results obtained from the PF for the arriving modal frequencies.

We see in the figure that, because the number of expected frequencies is fixed to four, the PF always produces tracks of four modal frequencies at every time instant, no matter how many modes are truly present. When the model order of the PF matches the true order, the replica and the true signal exhibit a high degree of similarity and the frequency estimates are correct as shown from the snapshot of the signal at 99
Figure 3.6 Synthetic signal consisting of a varying number of modes without noise.

Figure 3.7 A noisy synthetic signal.
Figure 3.8 Frequency estimation with a fixed number of modes set to four.

ms and its estimated modes in Figure 3.9. The red dots represent the replica of the estimated signal and the blue line is the true signal.

The simple PF implemented in this chapter results in suboptimum performance when the order of the filter is not well matched to the true number of modes. We can see from the noise-free signal that there are three modes for the first 40 ms. Because we assume that the order is fixed at four, the estimation results show one additional (nonexistent) mode in addition to the three correct frequency trajectories. A snapshot of the signal at time 20 ms (Figure 3.10) demonstrates this occurrence.

There is a different complication when the true number of modes is greater than the assumed model order. An example is illustrated in Figure 3.11. The true signal consists of five modes, whereas the PF only tracks four modes. In this case, the model order does not adequately capture all the information within the signal.
Figure 3.9  Snapshot of the true signal at time 99 ms and its estimate using frequencies obtained via the PF.

Figure 3.10  Snapshot of the true signal at 20 ms and its estimate using the frequencies estimated via the PF.
3.3.3 Tracking Results from a Synthetic Dispersed Acoustic Signal

We now apply the PF to a synthetic dispersed acoustic signal. The time-series we process is the result of the transmission and propagation of a sound signal in a dispersive ocean for a distance of 20 km. The frequency range is 200-600 Hz. The signal was generated using normal-mode modeling. The time-series is shown in Figure 3.12 and its spectrogram is illustrated in Figure 3.13.

We see in the spectrogram that the beginning of the signal (approximately first 0.1 s) does not contain a significant amount of information. For an interval after that (until approximately 0.25 s) the behavior of the dispersion curves is different than the typical one (downward moving curves) that we want to track. Thus, to test the algorithm, we consider only the portion of the signal after the first 0.25 s. The shorter spectrogram and a noisy spectrogram are shown in Figures 3.14 and 3.15, respectively. The noisy spectrogram is the result of adding white Gaussian noise to
Figure 3.12 Time-series of a received dispersed acoustic signal.

Figure 3.13 Spectrogram of the dispersed acoustic signal generated via STFT (no noise).
the original spectrogram after performing STFT; the SNR is 15 dB. It should be noted that the noise added to the signal changes with time. The reason is that the strength of the signal decays with time. Thus, adding noise that has constant noise variance would not preserve the constant SNR which we want to achieve, since we are investigating how the PF performs at different noise levels. To obtain a constant SNR for all signal slices, for a particular slice we added noise in such a way that its variance is proportional to the squared norm of the signal within that slice. This approach generates a noisy signal that has constant SNR for all slices.

Tracking results from the PF assuming that the amplitudes are known but incorrect are shown in Figures 3.16, 3.17, 3.18 for orders of seven, eight, and nine, respectively. It should be noted that the same set of amplitude values was used for each of these cases.
Figure 3.15  Noisy spectrogram of the shorter version of the spectrogram of the dispersed acoustic signal.

Figure 3.16  Tracks of the dispersed acoustic signal with a fixed order set to seven.
Figure 3.17  Tracks of the dispersed acoustic signal with a fixed order set to eight.

Figure 3.18  Tracks of the dispersed acoustic signal with a fixed order set to nine.
From Figure 3.16, we can see that the assumption that the model order is seven is not adequate for capturing all existing modes; specifically, the filter cannot track the top mode at time after 0.65 s. Our model assumption, thus, leads to underestimation.

When the presence of eight modes is considered, the PF can capture more modes in comparison to the previous case. For example, the PF can identify the top mode after 0.65 s as depicted in Figure 3.17. Figure 3.18 illustrates that the PF can also track the top track after 0.65 s for an assumed order of nine, which was expected. The filter, however, presents us with a different problem, now identifying two very close frequencies. This means that the assumption of the presence of nine modes leads to overestimation.

For the results presented here, we assumed that the order is known and fixed, limiting the filter’s flexibility and its ability to seek the most suitable model order. When we make the assumption of a higher order, for example, the filter is forced to produce two very close frequencies. Erroneous assumptions about the amplitudes also result in accuracy problems.

To test the performance of the filter vs. noise level, another point of interest, we show tracking results for an SNR of 10 dB in Figure 3.19. The model order was set to nine. Results show that the filter performs well under a lower SNR regime.

### 3.4 Conclusions

The focus of the research presented in this chapter is on the design of a simple PF for modal frequency tracking, where it is assumed that the modal amplitudes are known and fixed throughout the observation time; the same applies to the number of modes. Noise variance is known but varies with time to reflect the fact that the relative SNR decreases with time. If the assumptions and the true signal characteristics agree, the PF performs very well. However, when there is a mismatch between assumptions and reality, the simple PF fails to estimate the unknown parameters.
Figure 3.19  Tracks of the dispersed acoustic signal with a fixed number of modes set to nine; the SNR 10 dB.

correctly. The following chapters will resolve these problems by implementing more advanced models, in order to improve the tracking performance of the filter. The next chapter will discuss the estimation of amplitudes of modes in addition to their frequency content.
In previous discussions, the modal amplitudes were assumed to be known at all time slices. This assumption is, however, not realistic. This chapter establishes a more sophisticated scenario by including amplitudes as unknowns. We continue our discussion on filtering by designing a sequential filtering method that handles this problem.

4.1 A Particle Filtering Model for Amplitude Estimation

This section develops an approach for estimating spectral amplitudes corresponding to modal frequencies. The most intuitive idea is to include amplitudes as additional parameters in the state vector: that is, amplitudes can be treated similarly to modal frequencies and can be included as parameters in the state vector and estimated along with modal frequencies.

Consider the state vector containing all parameters (frequencies and corresponding amplitudes), \( x_k = [f_k \ a_k] \). The transition model for the amplitudes can be described as:

\[
a_k = a_{k-1} + w_a(k-1),
\]

(4.1)

where \( w_{ak} \) represents additive white Gaussian noise: \( w_{ak} \sim \mathcal{N}(0, \sigma_{w_a}^2) \). Therefore, each amplitude particle can be sampled from the following density:

\[
a_k^i \sim \mathcal{N}(a_{k-1}^i, \sigma_{w_a}^2).
\]

(4.2)

After sampling is performed, the likelihood can be calculated via the measurement equation and weights can be obtained as described previously.
Adding the amplitudes to the state vector leads to doubling the state dimension and, consequently, the number of particles that are necessary for achieving a specific accuracy. To address the “curse of dimensionality,” we seek more efficient approaches for expressing the amplitudes analytically and estimating them without necessarily drawing particles, a computationally onerous process. We base our amplitude estimation process on the discussion in [2,19,34,40]. For this approach, amplitudes need not be included in the state vector; they are instead estimated via an ML or MAP estimator after determination of their conditional densities given frequency values/particles. Specifically, the estimates of amplitudes are straightforward to calculate based on the fact that the amplitude PDFs, conditional on modal frequencies, are normally distributed when the signal is embedded in white Gaussian noise. The mean and the covariance matrix of these conditional PDFs can be readily obtained. If complete amplitude PDFs are needed for our problem, it is efficient and effective to form these PDFs by sampling from the normal distributions determined by the calculated conditional mean and variance. In addition to forming amplitude PDFs at a particular state, those samples can then be used at the next time slices in the prediction step for frequencies and amplitudes. If complete PDFs are not needed, amplitudes are “marginalized” and only their ML/MAP estimates are used for prediction at the next step; in that case, the PF process after amplitudes have been added as unknowns hardly carries an additional computational burden.

4.2 MAP Estimation of Amplitudes

The MAP estimator of amplitudes is formulated as follows (since the priors are uniform in our case, both MAP and ML methods provide the same amplitude estimates). ML parameter estimates are obtained by maximizing likelihood functions with respect
to the parameters. Recall the measurement equation expressed for time $k$:

$$
y_k = \sum_{j=1}^{M} a_{kj} [\text{sinc}(f - x_{kj})]^2 + w_k. \quad (4.3)
$$

Without loss of generality, let $s(x) \triangleq \text{sinc}^2(x)$. We can rewrite Eq. 4.3 as:

$$
y_k = \sum_{j=1}^{M} a_{kj} s(f - x_{kj}) + w_k. \quad (4.4)
$$

Once we maximize the likelihood (or loglikelihood which is often preferable because of numerical complications), we obtain the estimates $A$ of the unknown amplitudes in vector form as

$$
A = R\Phi. \quad (4.5)
$$

Matrix $R$ is defined as:

$$
R = \begin{bmatrix}
    r_{11} & r_{12} & \cdots & r_{1M} \\
    r_{21} & r_{22} & \cdots & r_{2M} \\
    \vdots & \vdots & \ddots & \vdots \\
    r_{M1} & r_{M2} & \cdots & r_{MM}
\end{bmatrix}, \quad (4.6)
$$

where $r_{ij} = \sum_{f=1}^{L} s(f - x_{ik}) s(f - x_{jk}), i, j = 1, \ldots, M$, and

$$
\Phi = \begin{bmatrix}
    \sum_{f=1}^{L} s(f - x_{1k}) y_k \\
    \sum_{f=1}^{L} s(f - x_{2k}) y_k \\
    \vdots \\
    \sum_{f=1}^{L} s(f - x_{Mk}) y_k
\end{bmatrix}, \quad (4.7)
$$

where $L$ is the length of the Fourier transform that supports the frequency space.

The advantage of this method is that, as indicated above, the required computational effort is significantly reduced compared to the approach where amplitudes
are treated as additional parameters for which particles are drawn. It should be noted that MAP estimation so far is based on the assumption that the number of modes is known. A more complete model integrating both amplitudes and order along with frequency estimation is required will be discussed later.

4.3 Simulation Results

We validate our model by estimating amplitudes from the test signal shown in Figure 3.1(a) and from the synthetic signal shown in Figure 3.6. We also apply the method to the synthetic acoustic signal of Figure 3.14, simulating sound that has propagated through a dispersive waveguide.

4.3.1 Tracking Results from a Test Signal

The test signal is the same as the first one used in the previous chapter and contains three modes. Figure 4.1(a) shows the spectrogram of the three-mode signal. When the amplitudes are not known, they are estimated according to the discussion in the previous section. Figure 4.1(b) shows the tracking results for the three modes. The PF has taken only a few (seven) steps to correctly track all three frequencies. The improvement in the results compared to those of Figure 3.4(b) is evident; note that the results of Figure 3.4(b) were obtained after making erroneous assumptions about the amplitudes.

4.3.2 Tracking Results from a Synthetic Signal

This section provides tracking results from the PF applied to the synthetic signal depicted in Figure 4.2, when amplitudes are included as unknown parameters in the state vector. The true amplitudes for each modal frequency (from left to right) are given in Table 4.1.
Figure 4.1  (a) Spectrogram of a three-mode synthetic signal and (b) track estimates by the PF; amplitudes are estimated in addition to frequencies.

Figure 4.2  Spectrogram of a noise-free synthetic signal.
Table 4.1 Amplitudes for Modal Frequencies

<table>
<thead>
<tr>
<th>Mode</th>
<th>Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.6</td>
</tr>
<tr>
<td>2</td>
<td>0.8</td>
</tr>
<tr>
<td>3</td>
<td>0.7</td>
</tr>
<tr>
<td>4</td>
<td>0.9</td>
</tr>
<tr>
<td>5</td>
<td>0.7</td>
</tr>
<tr>
<td>6</td>
<td>0.5</td>
</tr>
<tr>
<td>7</td>
<td>0.8</td>
</tr>
<tr>
<td>8</td>
<td>0.9</td>
</tr>
<tr>
<td>9</td>
<td>0.7</td>
</tr>
</tbody>
</table>

Figure 4.3 shows tracking results obtained from the PF for the arriving frequencies, when amplitudes are assumed to be known. There is a mismatch, however, between our assumptions and the true amplitude values. The model order was assumed to be known and fixed to three for all time slices. Figure 4.4 shows the tracking results when we assumed that the model order is known and fixed to four. Tracking results for the case where an order of five was assumed are provided in Figure 4.5.

Tracks shown in Figure 4.3 indicate that the PF provides poor estimates in this case. The signal is composed of three modes in the first 40 ms and four modes from 40 ms to 90 ms. However, the PF provides two superimposed tracks (white lines) from the beginning to about 90 ms. These two superimposed tracks appear as the second line from the bottom (the tracks that start at about 500 Hz in the first slices). Although three modes (match to assumption of the model order) are present
Figure 4.3  Frequencies estimated by the PF with a fixed number of modes set to three without amplitude estimation.

in the signal in the first 40 ms, inaccurate information about the amplitudes results in incorrect estimates in this interval.

A similar problem occurs in both cases when orders of four and five were assumed. As seen in the results illustrated in Figure 4.4, the PF produces incorrect estimates; the filter provides two very close tracks in the first 50 ms. Moreover, it is obvious that the PF cannot track at all the mode that has the smallest amplitude (sixth from the left). When an order of five was assumed, the tracking results shown in Figure 4.5 seem superior to the previous two. The mode that has the smallest amplitude (sixth from the left) is now tracked. However, there are two very close frequency estimates that occur when the true signal has a smaller number of modes than considered (during the first 90 ms, for instance).

We now present the results from the new model, where amplitude estimation is included in the process. The tracks obtained from the PF when the model order was
**Figure 4.4** Frequencies estimated by the PF with a fixed number of modes set to four without amplitude estimation.

**Figure 4.5** Frequencies estimated by the PF with a fixed number of modes set to five without amplitude estimation.
assumed known and fixed to three are shown in Figure 4.6. The results illustrate an improvement from the track estimation shown in Figure 4.3, where the amplitudes were assumed known but were incorrect. The PF tracks frequencies correctly even at the beginning (a few slices are adequate for gathering information). Because the number of expected frequencies is fixed to three, the PF produces tracks of three modal frequencies at every time instant. Now that amplitudes are included in the search, all three estimated modal frequencies are accurate. In Figure 4.7, we show estimates when the number of expected frequencies is set to four. Again, compared to the previous results from the PF with fixed and known (but wrong) amplitudes, the new PF produces much better track estimates. Moreover, the sixth mode (from the left) can now be tracked nicely. This emphasizes that having accurate information about amplitudes of modal frequencies in the filtering process is critically important for good tracking performance. We finally show results in Figure 4.8 when the number of modes is set to five. For this case, the PF also performs better than that from the case where we did not estimate amplitudes; the improvement is especially clear in tracking the fourth mode. But there still is a problem, namely, overestimation. The reason behind this is the assumed order of five. Most of the time, there are four modes present at a given slice. This problem (over/underestimation) will be addressed in the next chapter.

4.3.3 Tracking Results from a Synthetic Dispersed Acoustic Signal

The synthetic dispersed acoustic signal of Figure 3.13 was used to test the filter performance. The amplitude estimation process described earlier was included in the filtering process. We present results for an SNR of 15 dB with an assumed order of seven, eight, and nine in Figures 4.9, 4.10, and 4.11, respectively.

Studying the results, we can make the following observations:
Figure 4.6  Frequencies estimated by the PF with amplitude estimation and a fixed number of modes set to three.

Figure 4.7  Frequencies estimated by the PF with amplitude estimation and a fixed number of modes set to four.
Figure 4.8  Frequencies estimated by the PF with amplitude estimation and a fixed number of modes set to five.

Figure 4.9  Frequencies estimated for the synthetic dispersed acoustic signal by the PF with amplitude estimation and a fixed number of modes set to seven; the SNR is 15 dB.
Figure 4.10  Frequencies estimated for the synthetic dispersed acoustic signal by the PF with amplitude estimation and a fixed number of modes set to eight; the SNR 15 dB.

Figure 4.11  Frequencies estimated for the synthetic dispersed acoustic signal by the PF with amplitude estimation and a fixed number of modes set to nine; the SNR is 15 dB.
1. The comparison between the tracking results shown in Figures 3.16 and 4.9 demonstrates an improvement in estimation when the filter incorporates amplitudes as unknowns. This is more prominently evident in interval 0.35-0.55 s. In Figure 3.16, there are two superimposed tracks that occur when we use a simple filter without amplitude estimation. The result shown in Figure 4.9 does not present this problem in the same time interval. Moreover, in Figure 3.18, there are two superimposed tracks that occur for most slices when we use a simple filter. On the other hand, the results shown in Figure 4.11 do not exhibit this problem. We observe that the amplitude estimation process remedies the problem nicely.

2. There is a new mode entering from the top of the spectrogram close to the end of the signal. We have noticed that the filter with a fixed number of modes set to nine (result shown in Figure 4.11) begins to identify that mode. The other two (results in Figures 4.9 and 4.10) cannot do that.

To test the performance of the filter with respect to noise level, we show two more results for an SNR of 10 dB. Figures 4.12 and 4.13 illustrate the tracks, when the assumed number of modes is eight and nine, respectively. Results show that the PF with amplitude estimation performs well in a higher noise level situation.

### 4.4 Conclusions

In this chapter, estimation of amplitudes corresponding to modal frequencies was discussed. A new PF was developed to simultaneously extract frequency content and amplitudes. Tracking results demonstrate that the new model enhances the estimation performance of the filter. Without accurate information about amplitudes, frequency estimates are poor regardless of the model order. Estimating amplitudes at
Figure 4.12  Frequencies estimated for the synthetic dispersed acoustic signal by the PF with amplitude estimation and a fixed number of modes set to eight; the SNR is 10 dB.

Figure 4.13  Frequencies estimated for the synthetic dispersed acoustic signal by the PF with amplitude estimation and a fixed number of modes set to nine; the SNR is 10 dB.
the same time results in a dramatic improvement in frequency estimates as demonstrated by the results.

Although amplitude estimation improves the detected tracks, there is still a concern about under- and overestimation. This problem was already introduced in Chapter 3 and still appears in this chapter as demonstrated in the results of Figures 4.7 and 4.8. To overcome this problem, we need a Multiple Model Particle Filter, in order to select the model order appropriately. In other words, the filter not only has to track frequencies and amplitudes but also the most suitable model in terms of dimensionality. We will discuss this extended filter in the next chapter.
In many tracking problems addressed via Bayesian filtering, often one of the main goals is to estimate the actual number of unknown parameters or model order; for instance, in target tracking applications, we may want to know how many sources are present, in addition to their location and velocity. To address such challenges, a Multiple Model Particle Filter (MMPF) [48] is needed. Such a filter is necessary in our problem as well. In the earlier chapters, the number of modal frequencies was assumed to be known and constant and we saw that this created complications, when it was not actually the case. This chapter treats the number of modal frequencies (and, consequently, amplitudes) as an unknown parameter. This leads to the formulation of a state vector containing modal frequencies, associated amplitudes, and model order, since the dimension of the state vector is unknown at every slice. This addition of a state variable allows us to capture the most suitable model within the filtering process.

Several approaches to the problem of determining the number of signals within a signal have been reported in the signal processing and information theory literature. In ocean acoustics applications, work in [38] estimated the model order together with arrival times, amplitudes, and noise variance of ray paths with Gibbs sampling. An MMPF was subsequently developed in [28] for arrival time tracking in space and source localization. An MMPF was also successfully used for passive fathometer tracking in [42], where the filtering process extracted the depth and the strength of an unknown number of acoustic reflectors. Using a PF within this framework, the dimension of the model is allowed to change at every slice based on information on
the number of modal frequencies available from the preceding slice. The model we use for this method is explained in the following section.

5.1 Model Description

The state equations describing the multiple model problem are the same as before, except that the additional parameter mentioned, \( r_k \), is now included in the state vector, where \( r_k \) represents the model order at time step \( k \) (\( k \)th slice). The new state vector becomes \( X_k = [x_{k,r_k}, a_{k,r_k} r_k] \). Vector \( x_{k,r_k} \) contains frequencies, \( a_{k,r_k} \) contains corresponding amplitudes as described in Chapter 4, and the dimension of the vector, \( 2r(k) + 1 \), is dictated by \( r_k \). A transition probability matrix \([22]\) is needed, which contains the probabilities of order changes (or not). The transition probability matrix is defined as \( P_r = [\pi_{ij}]_{S \times S}, \) where \( S \in \{1, 2, \ldots, s\} \), with transition probabilities

\[
\pi_{ij} \triangleq \text{Prob}[r_{k+1} = j | r_k = i], \quad i, j \in S. \tag{5.1}
\]

Transition probabilities describe the possible movement from a current state to another state in subsequent times. We must have \( \sum_j (\pi_{ij}) = 1 \). To illustrate the process, we begin with the assumption that no more than three modes exist at a given time; in that case, the transition matrix is:

\[
P_r = \begin{bmatrix}
\pi_{11} & \pi_{12} & \pi_{13} \\
\pi_{21} & \pi_{22} & \pi_{23} \\
\pi_{31} & \pi_{32} & \pi_{33}
\end{bmatrix}, \tag{5.2}
\]

which means that, if the current model order is one (a single mode is present), at the next time frame the order may be two with probability \( \pi_{12} \), three with probability \( \pi_{13} \), or one (same order) with probability \( \pi_{11} \). Similarly, if the current order is two, it will remain the same with probability \( \pi_{22} \) and may decrease or increase to one or...
three with probability $\pi_{21}$ or $\pi_{23}$, respectively. It should be noted that the matrix does not necessarily need to be symmetric.

5.2 Formulation of the Multiple Order Tracking Problem for Frequency Estimation

The estimation problem for the multiple model scenario needs careful consideration. In a noisy environment, the PF favors the model with the highest order because there is an inherent bias towards large dimensionality. Let’s consider the situation where we have a time-series with one modal frequency with an amplitude of one. There are infinite ways to generate combinations of multiple modes with the exact same modal frequencies and different amplitudes in such a way that their sum is the true mode. To compensate for this, a penalizing factor is added to the original likelihood for remedying the typical preference of high-order models. This penalty factor comes from the prior density on the order. In this work we select uniform priors:

$$p(a_{kj}) = 1$$ \hspace{1cm} (5.3)

$$p(x_{kj}) = \frac{1}{L}$$ \hspace{1cm} (5.4)

where $L$ is the length of the Fourier transform that supports the frequency space. The likelihood function in this case is given by

$$p(y_k|x_k) \propto \frac{1}{r_k} exp\{-\frac{1}{2\xi^2}\|y_k - \sum_{j=1}^{r_k} a_{kj}[sinc(f - x_{kj})]\|^2\}.$$ \hspace{1cm} (5.5)

The penalizing term $\frac{1}{L}$ impacts the value of the likelihood. For small values of $r_k$, this term has a value associated with a higher probability than that for larger values of $r(k)$. 

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5.3 Kinematic Model: Gradient Components

This section discusses a dynamic model for frequency tracking using “velocities” (gradients) of the modal frequencies. This model, which is improved over the one neglecting gradients, will be incorporated in the MMPF. As seen in the synthetic spectrogram illustrated in Figure 3.6, the modal frequencies move consistently downwards as time increases. In other words, it is the physical nature of our problem that the modal trajectories move from higher to lower frequencies. This observation helps us in establishing appropriate constraints. The work presented earlier did not include the gradient information inherent in the frequency evolution.

To implement a new dynamic model exploiting this information, the state equation and transition function must be updated to include a gradient component for each particle from the previous state to the current one.

We demonstrate below the basic idea behind this approach. For simplicity we do not include amplitudes and order. Let \( X_k = [x_k \ \dot{x}_k] \) be the state vector, where \( \dot{x}_k \) represents the “velocity” (gradient) of a particle at time \( k \). The state transition equation is given by:

\[
X_k = f_k(X_{k-1}, v_{k-1}) \quad (5.6)
\]

For our new problem, the transition equation for consecutive states can be written as:

\[
x_k = x_{k-1} + \dot{x}_{k-1}dt + v_{1k-1} \quad (5.7)
\]

\[
\dot{x}_k = \dot{x}_{k-1} + v_{2k-1}, \quad (5.8)
\]

where \( x_k \) is a vector containing modal frequencies, \( \dot{x}_k \) is a gradient vector of these frequencies, and \( dt \) is the time between segments. In [27,28] a similar technique was successfully developed for estimating spatially evolving arrival times.
5.4 Simulation Results

5.4.1 Tracking Results from a Synthetic Signal
The synthetic signal of the previous chapter (shown in Figure 4.2) was used to study the performance of our filter for an unknown and varying model order. Under the simulation assumptions, the signal has initially three modes for the first 50 slices; then a fourth mode begins. After that the order varies between four and five until the end of the observation length. The gradient information was incorporated in the PF and tracking results for modal frequencies using the transition model are shown in Figure 5.1. The PMF of the model order, \( r \), is given in Figure 5.2. The PMF illustrates the success of the filter in detecting the evolving order of the composite signal. For this example, we assumed that the order can only change by one between two consecutive time instances. We also set the minimum order (number of modes) to two and maximum order to six. For this case, we had \( S \in \{2, 3, 4, 5, 6\} \). The transition matrix \( P_r \) was:

\[
P_r = \begin{bmatrix}
0.6 & 0.4 & 0 & 0 & 0 \\
0.2 & 0.6 & 0.2 & 0 & 0 \\
0 & 0.2 & 0.6 & 0.2 & 0 \\
0 & 0 & 0.2 & 0.6 & 0.2 \\
0 & 0 & 0 & 0.4 & 0.6 \\
\end{bmatrix}.
\]

(5.9)

5.4.2 Tracking Results from Synthetic Dispersed Acoustic Signal
We test the MMPF performance with the synthetic dispersed acoustic signal shown in Figure 3.14. Figure 5.3 demonstrates the estimates of the modal frequencies, when their number is unknown. In Figure 5.4, we show the PMF for the order. Even though uncertainty was introduced because the number of modal frequencies was unknown, the results show that the tracks are well estimated. An assumption of
Figure 5.1 Track estimates of modal frequencies with the MMPF.

Figure 5.2 PMF of the number of modal frequencies.
\( S \in \{6, 7, \ldots, 12\} \) was made and we also assumed that the order can only change by one from a time-frame to the next, that is, the order can decrease by one or increase by one, provided that it is not six or twelve. If the present order is six, it can be either six or seven in the next time frame. Similarly, if the present order is twelve, it can be either twelve or eleven in the next time frame. Following this assumption, the transition matrix was defined as:

\[
P_r = \begin{bmatrix}
0.6 & 0.4 & 0 & 0 & 0 & 0 & 0 \\
0.2 & 0.6 & 0.2 & 0 & 0 & 0 & 0 \\
0 & 0.2 & 0.6 & 0.2 & 0 & 0 & 0 \\
0 & 0 & 0.2 & 0.6 & 0.2 & 0 & 0 \\
0 & 0 & 0 & 0.2 & 0.6 & 0.2 & 0 \\
0 & 0 & 0 & 0 & 0.4 & 0.6 & 0
\end{bmatrix}.
\]
Figure 5.3 Estimates of modal frequencies when their number is unknown.

Figure 5.4 PMF of the number of modal frequencies.
CHAPTER 6
PARTICLE FILTERING: TREATMENT OF NOISE VARIANCE AS AN UNKNOWN PARAMETER

Work in [45] shows how variance estimation is important in reducing the uncertainty of the estimates of arrival times at a set of phones. It was shown that, if an assumed variance value is incorrect, inversion results for environmental and geoacoustic parameters may not be satisfactory in terms of uncertainty that is often increased. Similarly, in our work, correct noise variance estimates should reduce the uncertainty of the estimates of the modal frequencies. This is crucial since environmental parameter estimation and geoacoustic inversion depends on the accuracy of the estimation of modal frequencies and modal amplitudes. The contribution of this chapter is the estimation of the noise variance along with the previously estimated parameters in order to address the concern just mentioned.

6.1 Model and Implementation

The observation equation and the structure of the measurement noise are the same as discussed in the previous chapters except that the noise variance is now added to the state vector. The likelihood function in this case is given by:

\[
p(y_k|x_k, a_k, r_k) \propto \left(\frac{1}{\sigma^2_k}\right)^{L/2} \frac{1}{L^{y_k}} \exp\left\{ -\frac{1}{2\sigma^2_k} \|y_k - \sum_{j=1}^{r_k} a_{kj} [\text{sinc}(f - x_{kj})]^2 \|^2 \right\}.
\]  

(6.1)

The following non-informative prior for the variance of the additive white Gaussian noise is employed [2,5]:

\[
p(\sigma^2) \propto \frac{1}{\sigma^2}.
\]  

(6.2)
By including the above prior density, the joint posterior density of the unknown parameters given the observed data is:

$$
p(x_k, a_k, r_k, \sigma_k^2 | y_k) \propto \frac{1}{\sigma_k^2} \frac{1}{L/k} \exp\left\{-\frac{1}{2\sigma_k^2} \|y_k - \sum_{j=1}^{r_k} a_{kj} [sinc(f - x_{kj})]^2 \|^2 \right\}.
$$

(6.3)

The conditional PDF for the variance conditional on frequencies, amplitudes, and order can be written as:

$$
p(\sigma_k^2 | x_k, a_k, r_k, y_k) = A \frac{1}{\sigma_k^{2L/2+1}} \exp\left\{-\frac{1}{2\sigma_k^2} \|y_k - \sum_{j=1}^{r_k} a_{kj} [sinc(f - x_{kj})]^2 \|^2 \right\},
$$

(6.4)

where $A$ is a constant and $L$ is the length of the Fourier transform. This is recognized as an inverse-$\chi^2$ distribution with $L$ degrees of freedom, $\chi_L^{-2}$.

Let’s consider

$$
\chi^2 = \sum_{i=1}^{L} (X_i - \mu)^2 / \sigma^2,
$$

(6.5)

having the posterior PDF

$$
p(\chi^2 | \mu, X_1, ..., X_L) \propto (\chi^2)^{L/2-1} \exp(-\chi^2/2),
$$

(6.6)

which is a $\chi^2$ distribution with $L$ degrees of freedom. To estimate $\sigma^2$ in Eq. 6.5, we first draw samples from this density and $\sigma^2$ is obtained by dividing $\sum_{i=1}^{L} (X_i - \mu)^2$ with the $\chi^2$ value that we have sampled. Similarly, consider the conditional PDF in Eq. 6.4. Quantity $\|y_k - \sum_{j=1}^{r_k} a_{kj} [sinc(f - x_{kj})]^2 \|^2$ corresponds to $\sum_{i=1}^{L} (X_i - \mu)^2$ in Eq. 6.5. According to Eq. 6.4, we can write:

$$
\chi^2 = \|y_k - \sum_{j=1}^{r_k} a_{kj} [sinc(f - x_{kj})]^2 \|^2 / \sigma_k^2
$$

(6.7)

or

$$
\sigma_k^2 = \|y_k - \sum_{j=1}^{r_k} a_{kj} [sinc(f - x_{kj})]^2 \|^2 / \chi^2.
$$

(6.8)
Therefore, to obtain the noise variance in our problem, $\sigma^2_k$ of Eq. 6.4, we draw samples from a $\chi^2$ density with $L$ degrees of freedom. Then, $\sigma^2_k$ is obtained by dividing $\|y_k - \sum_{j=1}^{r_k} a_{kj} [sinc(f - x_{kj})]\|^2$ by the $\chi^2$ value we have sampled, as indicated in Eq. 6.8. This is the estimate of variance for each particle. To provide a MAP estimate of the variance for a given slice, variances values were first calculated for all particles.

### 6.2 Simulation Results

#### 6.2.1 Results from the Synthetic Signal

White Gaussian noise was added to the synthetic signal constructed in earlier chapters, as shown in Figure 4.2. Noisy signal slices with noise variance equal to 0.05 (SNR $\approx$ 12 dB) are shown in Figure 6.1.

Figure 6.2 illustrates the frequency estimates from the new PF. The estimates of model order and variance are demonstrated in Figures 6.3 and 6.4, respectively. Figure 6.5 presents samples of the posterior densities of the variance at times 100 ms,
Figure 6.2 Frequency estimates for the synthetic signal.

200 ms, 300 ms, and 400 ms. The results show that the PF estimates the variance successfully: the estimates are actually very close to the true value.

6.2.2 Results from Synthetic Dispersed Acoustic Signal

We apply the complete model - where the PF estimates all parameters of interest including frequencies and associated amplitudes, model order, and noise variance - to the synthetic dispersed signal of Figure 3.14. Snapshots of the noisy signal with noise variance 0.0025 (SNR ≈ 5 dB) are shown in Figure 6.6.

In Figure 6.7 we show the frequency estimates from the PF. The estimates of model order and noise variance are shown in Figures 6.8 and 6.9. Figure 6.10 illustrates samples from the posterior densities of the variance at times 0.35 s, 0.45 s, 0.55 s, and 0.65 s.

From the estimation results, we observe that noise variance is slightly overestimated. From the previous, simpler example, we know that the algorithm provides
Figure 6.3 PMF of the number of modal frequencies for the synthetic signal.

Figure 6.4 Noise variance estimates for the synthetic signal: the true variance is 0.05.
Figure 6.5  Densities of the variance for the synthetic signal obtained from the PF: the true variance is 0.05.

Figure 6.6  Noisy synthetic dispersed signal with noise variance 0.0025 at times 0.25, 0.5, and 0.75 s.
Figure 6.7 Frequency estimates for the dispersed acoustic signal.

Figure 6.8 PMF of the number of modal frequencies for the dispersed acoustic signal; the true variance is 0.0025.
Figure 6.9 Noise variance estimates for the dispersed acoustic signal.

Figure 6.10 Densities of the variance for the dispersed acoustic signal obtained from the PF: the true variance is 0.0025.
excellent variance results when we know exactly the structure of the signal that we process. We, thus, attribute the overestimation to the mismatch between the source model ($sinc$) used in the PF and the true source signal. In addition to that, the noise variance estimates for the first 200 ms are not as close to the true variance as later estimates. The reason behind this is that the modes at this interval are not well separated. In other words, modes merge, resulting in ambiguity. At later times, when modes are better separated and more distinct, the estimates of noise variance are very good.
In this chapter, we extend our work by now considering the true nature of the noise and the resulting PDF of the spectrogram observations. The Gaussian model used in earlier chapters was based on the assumption that noise is additive in the frequency domain. This model, although it has been applied with success to instantaneous frequency estimation both in [18] and our work, is inaccurate. When Gaussian noise is added in the time domain, the nature of the noise changes in the frequency domain because of the squaring process entering the calculation of the power of the Fourier transform. We start the chapter by discussing the derivation of a \( \chi^2 \) model for the PDF of the spectrogram components [24–26]. Then, the implementation of a filter relying on the \( \chi^2 \) model is presented. Finally, we provide simulation results from the \( \chi^2 \) model and a comparison of the filters with the two distinct noise settings.

### 7.1 Chi-Squared Distribution and Spectrogram PDF

Consider the sum of the squared and independent random variables \( Z_i \sim \mathcal{N}(0, 1) \), \( i = 1, \ldots n \):

\[
X = \sum_{i=1}^{n} Z_i^2. \tag{7.1}
\]

It can be shown that \( X \) is distributed as a \( \chi_n^2 \) random variable [23,32], where \( n \) is the number of degrees of freedom. To obtain the PDF of \( X \), one can use the characteristic function [23]. It can be shown that a \( \chi^2 \) distribution with \( n \) degrees of freedom can be expressed as:

\[
f_X(x) = \frac{1}{\Gamma(n/2)2^{n/2}} x^{n/2-1} e^{-x/2}, \quad x > 0. \tag{7.2}
\]
We write: $X \sim \chi^2_n$. If $Z_i, i = 1, \ldots, n$, are independent Gaussian random variables with zero mean and common variance $\sigma^2$, the PDF of $X$ is given by

$$f_X(x) = \frac{1}{\Gamma(n/2)(2\sigma^2)^{n/2}} x^{n/2-1} e^{-x/2\sigma^2}, \quad x > 0. \quad (7.3)$$

The mean and variance of the random variable $X$ can be computed using the characteristic function and are:

$$E[X] = n\sigma^2, \quad (7.4)$$

$$Var[X] = 2n\sigma^4. \quad (7.5)$$

Now, we consider

$$X = \sum_{i=1}^{n} (\mu_i + Z_i)^2, \quad (7.6)$$

where $\mu_i$ are constants and $Z_i$ are independent zero mean Gaussian random variables with common variance $\sigma^2$. We have $X_i = \mu_i + Z_i \sim N(\mu_i, \sigma^2)$. The random variable $X$ has a noncentral $\chi^2$ distribution with $n$ degrees of freedom and its PDF is:

$$f_{X:n,\alpha,\theta}(x) = \frac{1}{2\alpha} \left( \frac{x}{\theta} \right)^{(n-2)/4} e^{-(x+\theta)/2\alpha} I_{n/2-1} \left( \frac{\sqrt{x\theta}}{\alpha} \right), \quad (7.7)$$

where $I_k(\cdot)$ represents the $k$-order modified Bessel function of the first kind and $\alpha$ is defined as the coefficient of proportionality: $\alpha = \sigma^2$. Quantity $\theta$ is defined as the noncentrality parameter and $\theta = \sum_{i=1}^{n} \mu_i^2$. The mean and variance associated with a noncentral $\chi^2$ distribution with $n$ degrees of freedom are:

$$E[X] = \theta + n\sigma^2, \quad (7.8)$$

and

$$Var[X] = 4\sigma^2\theta + 2n\sigma^4. \quad (7.9)$$
Consider signal $x(k)$ of length $K$, which is composed of a deterministic discrete sequence $d(k)$ and additive zero mean white Gaussian noise $n(k)$ with variance $\sigma^2$:

$$x(k) = d(k) + n(k), \quad 1 \leq k \leq K. \quad (7.10)$$

The discrete spectrogram $S_x(m, l)$ at time $m$ and frequency $l$ of signal $x(k)$ can be computed by squaring the modulus of the STFT or, equivalently, summing the squares of the real and imaginary parts of the STFT:

$$S_x(m, l) = X^r(m, l)^2 + X^{im}(m, l)^2, \quad (7.11)$$

where

$$X^r(m, l) = \sum_{k=1}^{K} w(k - m)x(k) \cos(-2\pi l \frac{k}{L}), \quad (7.12)$$

and

$$X^{im}(m, l) = \sum_{k=1}^{K} w(k - m)x(k) \sin(-2\pi l \frac{k}{L}). \quad (7.13)$$

Here, $w(k)$ is the Fourier transform analysis window of length $K$ and the length of the transform is denoted by $L$.

Since the time-domain signal in $x(k)$ is normally distributed (Eq.7.10), from Eqs. 7.12 and 7.13 we can see that both real and imaginary parts of the STFT are linear combinations of the signal samples in the time domain. Therefore, both $X^r(m, l)$ and $X^{im}(m, l)$ are also Gaussian random variables. The variance of both real and imaginary parts of the STFT is $\sigma^2/2$. Eq. 7.11 further suggests that $S_x(m, l)$ is a noncentral $\chi^2$ variable with two degrees of freedom. The corresponding PDF of the spectrogram observations is given by:

$$f_{X,\alpha,\theta}(x) = \frac{1}{2\alpha} e^{-(x+\theta)/2\alpha} I_0\left(\frac{\sqrt{x\theta}}{\alpha}\right). \quad (7.14)$$

Its parameters $\alpha$ and $\theta$ are provided in the next section.
7.2 Implementation

The implementation of the method described above for frequency estimation using particle filtering is different from what was developed and presented in previous chapters. The $\chi^2$ behavior of the data now implies a different likelihood that will be used for weight/probability calculation for individual particles. PFs implemented earlier were constructed based on Gaussian data errors, which led to the formulation of the likelihood function. This changes here with the $\chi^2$ behavior of the FT dictating the form of the likelihood as follows.

The real and imaginary parts of the FFT are normally distributed with non-zero means. These means are the corresponding FTs of the acoustic signal. The sum of the squares of the real and imaginary parts follows non-central $\chi^2$ distribution with two degrees of freedom (see Eq. 7.11). From Eq.7.6, we have $X_i, i = 1, 2$ which are the real and imaginary part of the Fourier transform; quantity $n$ is two.

Quantity $\mu_1$ in Eq.7.6 is the mean corresponding to the real part and $\mu_2$ in Eq. 7.6 is the mean corresponding to the imaginary part. The sum of the squares of the means of the real and imaginary parts is the non-central parameter.

The above analysis is for a single point in the frequency domain. For the complete length of the Fourier transform for a slice of the spectrogram, the non-central parameter is a vector of length $L$, which is actually the replica of the squared spectrum of the signal. The $\chi^2$ parameters are as follows: the noncentral parameter is the replica of the signal in our problem and the number of degrees of freedom is two as discussed earlier. The coefficient of proportionality is based on the fact that both real and imaginary parts of the STFT are Gaussian random variables with common variance $\sigma^2/2$ [24]. Therefore, the coefficient of proportionality is $\sigma^2/2$.

In addition to the change of the likelihood calculation, the amplitude estimation process for the $\chi^2$ model is different than the approach used under the frequency domain Gaussian noise assumption. Using the $\chi^2$ model, we could not identify a
tractable model that would provide estimates of the amplitudes by extracting them from the state vector. Thus, to estimate the modal amplitudes when the $\chi^2$ model is used, we include them in the state vector and treat them as parameters to be estimated by generating particles. Although this adds to the computational load of the method, it seemed to be the appropriate method for proceeding.

7.3 Simulation Results

7.3.1 Results from the Synthetic Signal

In order to investigate the effectiveness of the new model, we construct synthetic signal time-series composed of time-varying modes. The spectrogram of the signal is presented in Figure 7.1. White Gaussian noise was added to the signal in the time domain. In Figure 7.2 we show the spectrogram of the noisy synthetic signal that we process; the SNR is 20 dB. Figure 7.3 illustrates the frequency estimates obtained from the PF for this signal. The PMF of the model order is shown in Figure 7.4.
Figure 7.2 Spectrogram of the noisy synthetic signal.

Figure 7.3 Frequency estimates from the synthetic signal using the $\chi^2$ model; the SNR is 20 dB.
Figure 7.4  Order estimates PMF from the synthetic signal using the $\chi^2$ model; the SNR is 20 dB.

Figures 7.5 and 7.6 illustrate the frequency estimates and order estimates, respectively, when the SNR is 15 dB. For the case where the SNR is 10 dB, results are demonstrated in Figures 7.7 and 7.8.

We compare the results from the $\chi^2$ model with estimates obtained with the Gaussian model. When the SNR is 20 dB, Figure 7.9 shows the frequency estimates from the PF and Figure 7.10 shows the PMF of the model order. For SNRs of 15 and 10 dB, frequency tracks are illustrated in Figures 7.11 and 7.13. The order PMFs are provided in Figures 7.12 and 7.14.

The tracking results from the $\chi^2$ scheme are clearly better than those from the Gaussian model for all SNRs. This is expected because the Gaussian model cannot capture correctly the structure of the data errors. The superiority of the $\chi^2$ model is more strongly evident in the lower SNR cases. It must be noted that the PF
Figure 7.5  Frequency estimates from the synthetic signal using the \( \chi^2 \) model; the SNR is 15 dB.

Figure 7.6  Model order PMF from the synthetic signal using the \( \chi^2 \) model; the SNR is 15 dB.
Figure 7.7  Frequency estimates from the synthetic signal using the $\chi^2$ model; the SNR is 10 dB.

Figure 7.8  Model order PMF from the synthetic signal using the $\chi^2$ model; the SNR is 10 dB.
Figure 7.9  Frequency estimates from the synthetic signal using the Gaussian model; the SNR is 20 dB.

Figure 7.10  Model order PMF from the synthetic signal using the Gaussian model; the SNR is 20 dB.
Figure 7.11  Frequency estimates from the synthetic signal using the Gaussian model; the SNR is 15 dB.

Figure 7.12  Model order PMF from the synthetic signal using the Gaussian model; the SNR is 15 dB.
Figure 7.13  Frequency estimates from the synthetic signal using the Gaussian model; the SNR is 10 dB.

Figure 7.14  Model order PMF of the synthetic signal using the Gaussian model; the SNR is 10 dB.
implementations for both models are identical except that the likelihood functions and the amplitude calculations are different and model-dependent.

7.3.2 Results from the Synthetic Dispersed Acoustic Signal

We test the PF with the $\chi^2$ model on the synthetic acoustic signal that has propagated in a dispersive waveguide shown in Figure 3.14. Estimates of modal frequencies and model orders are shown in Figures 7.15 and 7.16; the SNR is 20 dB. We also provide the tracking results using the Gaussian model for the same signal. The results are provided in Figures 7.17 and 7.18. We can observe from the figures that the quality of the estimates for both models is similar. Results for a lower SNR (15 dB) are then shown in Figures 7.19, 7.20, 7.21, and 7.22. We now see that the $\chi^2$ model is superior to the Gaussian one. This is even more noticeable by comparing Figures 7.23 and 7.24 to 7.25 and 7.26), obtained for an SNR of 10 dB. These observations confirm our hypothesis that a $\chi^2$ model more accurately describes the statistics of STFTs, resulting in significantly improved dispersion estimates.
Figure 7.15 Frequency estimates from the synthetic dispersed signal using $\chi^2$ model; the SNR is 20 dB.

Figure 7.16 PMF of the number of modal frequencies for the synthetic dispersed signal using $\chi^2$ model; the SNR is 20 dB.
Figure 7.17  Frequency estimates from the synthetic dispersed signal using Gaussian model; the SNR is 20 dB.

Figure 7.18  PMF of the number of modal frequencies for the synthetic dispersed signal using Gaussian model; the SNR is 20 dB.
Figure 7.19  Frequency estimates from the synthetic dispersed signal using $\chi^2$ model; the SNR is 15 dB.

Figure 7.20  PMF of the number of modal frequencies for the synthetic dispersed signal using $\chi^2$ model; the SNR is 15 dB.
Figure 7.21  Frequency estimates from the synthetic dispersed signal using Gaussian model; the SNR is 15 dB.

Figure 7.22  PMF of the number of modal frequencies for the synthetic dispersed signal using Gaussian model; the SNR is 15 dB.
Figure 7.23  Frequency estimates from the synthetic dispersed signal using the $\chi^2$ model; the SNR is 10 dB.

Figure 7.24  PMF of the number of modal frequencies for the synthetic dispersed signal using the $\chi^2$ model; the SNR is 10 dB.
Figure 7.25  Frequency estimates of the synthetic dispersed signal using the Gaussian model; the SNR is 10 dB.

Figure 7.26  PMF of the number of modal frequencies for the synthetic dispersed signal using the Gaussian model; the SNR is 10 dB.
RESULTS OF DEVELOPED FILTERS APPLIED TO REAL DATA

The particle filtering framework presented in this thesis is applied to real data from the Gulf of Mexico experiment [41]. The signal propagated for around 21.5 km in a shallow water waveguide (the depth was around 116 m) and was received at five phones. In this work we process the time-series recorded at one of the hydrophones. The data were collected at a sampling rate of 1562.5 Hz. The spectrogram of the signal is displayed in Figure 8.1.

In order to be able to study closely the PF results, we first present the estimated frequency tracks for the first half of the signal (around 1.56 s). Using the Gaussian data error model, frequency estimates from the PF are shown in Figure 8.2. The PMF of the model order is given in Figure 8.3. The results from the PF with the \( \chi^2 \) model are presented in Figures 8.4 and 8.3.

We display the tracking results for the second half of the signal from the Gaussian model in Figures 8.6 and 8.9 and those from the \( \chi^2 \) model in Figures 8.8 and 8.9.

As with the simulations, in order to have a fair comparison, the PF implementations for both models are identical except for the error modeling and amplitude estimation process. We observe that the Gaussian model results are not very good. A large number of modes is estimated, with several “estimates” potentially corresponding to noise. Because the Gaussian model is not an accurate model when the noise is additive to the acoustic signal in the time domain but the signal is processed in the frequency domain, the filter cannot clearly distinguish whether spectral peaks correspond to signal or noise. This limitation results in poor tracking performance.
Figure 8.1 Spectrogram of the real signal.

Figure 8.2 Frequency estimates in the first 1.56 s for the real data employing the Gaussian model.
Figure 8.3 PMF of the number of modes in the first 1.56 s for the real data employing the Gaussian model.

Figure 8.4 Frequency estimates in the first 1.56 s for the real data employing the $\chi^2$ model.
Figure 8.5  PMF of the number of modes in the first 1.56 s for the real data employing the $\chi^2$ model.

Figure 8.6  Frequency estimates from the second half of the signal for the real data using the Gaussian model.
Figure 8.7  PMF of the number of modes from the second half of the signal for the real data using the Gaussian model.

Figure 8.8  Frequency estimates from the second half of the signal for the real data using the $\chi^2$ model.
Figure 8.9  PMF of the number of modes from the second half of the signal for the real data using the $\chi^2$ model.

Results from the PF using the $\chi^2$ model are superior to the results just discussed. Although there is still uncertainty, distinct frequency tracks can be identified with the spectral estimates appearing to belong to specific modes (we should note here that, because this is real data, we do not know the exact structure of the signal).
9.1 Conclusions

In this work, we developed an approach for sequentially estimating modal frequencies, amplitudes, number of modes, and noise variance. Our main goal was the identification of a dispersion pattern of a signal propagating in the ocean. Our technique is a Monte Carlo method for drawing inferences from state-space models, where the state of a system evolves with time or space and information about the state is obtained via noisy observations made at each time step. In addition to providing estimates of modal frequencies along with their amplitudes, number of modes, and noise variance, the approach estimates fully joint PDFs of those parameters. Tracking of the unknown parameters was performed in the frequency domain, where data observations were the STFT power spectrum of acoustic time-series.

Our initial assumption was that the signal we processed is embedded in an additive white Gaussian noise environment. Were we to work in the time-domain, that would have been a fairly accurate assumption. This is not the case, however, for frequency domain signals such as the FTs that were the observations for the problem at hand. We started from this assumption, because this has been the standard approach in previous work on instantaneous frequency estimation.

Under the above assumption, we developed a PF to track only the frequency content of the signal, provided all other parameters were known. The very first simulation was performed on a simple signal consisting of three components for all times. The tracking performance was excellent with a small number of particles sufficing for accurate estimation. The method was then applied to more realistic synthetic signals for further performance evaluation. The results suggested that, if the assumptions
on the known parameter values are accurate, frequency (and, thus, dispersion curve) estimates are excellent. When, however, there was mismatch between assumptions and true values, the performance of our method was unsatisfactory.

To resolve the limitations of the simple filter, we built a more elaborate method that estimated modal amplitudes along with frequencies. The new approach, now insensitive to any amplitude assumptions and uncertainty, was an effective estimator. Adding the amplitudes to the state vector increased its dimensionality and, was, thus, expected to roughly double the computational cost of the approach. However, instead of drawing particles for amplitudes at every step, we identified them using a MAP estimator, after observing that their conditional PDFs on frequency particles were normal with easily tractable mean and variance. Although we focused on frequency estimation in this work and did not report amplitude estimates, it is important that modal amplitudes can be recovered via our method. Such amplitudes can be employed in a straightforward manner to estimate sediment attenuation in ocean acoustics.

Although the filter that incorporated amplitude estimation improved on the simple PF, under- and overestimation of the number of modes present in a specific time window remained a concern. We extended our work to develop a filter that allows flexibility in regards to the considered dimensionality of the state vector. We formulated a Multiple Model Particle Filter that tracked the number of modal frequencies (model order) along with their values and corresponding amplitudes. The filter, permitting “births” and “exits” of tracks at every STFT window, estimated successfully these parameters and was tested under different noise levels. Lastly, we considered variance as an unknown parameter as well. The component of the filter relating to variance estimation was based on the derivation of the posterior PDF of the noise variance conditional on frequencies and amplitudes. It was found that, using a Gaussian additive noise model, the density of the noise variance is inverse \( \chi^2 \) with \( L \) degrees of freedom, where \( L \) is the length of the Fourier transform or the length of the
slice of the signal. The results demonstrated that the filter integrating all unknowns has a significant potential in successful mode tracking and uncertainty characterization within the estimation, a feature necessary for interpreting the reliability of our results.

As mentioned earlier, the white Gaussian noise assumption for our FT signals is not accurate. We identified the true error statistics in our data and we constructed a PF for frequency estimation based on a $\chi^2$ model. This model is accurate when the noise is white and Gaussian and also additive to the acoustic signal in the time domain. Under this assumption, a noncentral $\chi^2$ density with two degrees of freedom is the correct model for the spectrogram of the acoustic time-series. Although the filter was more computationally demanding than our first method, it provided results that were significantly superior to those of the latter filter. The potential of the method was also validated by applying it to real data.

9.2 Future Work

There are several aspects of this work that will be addressed and improved by developing more sophisticated algorithms. We expect that modal frequency estimation will become more accurate after the implementation of a smoothing scheme. Specifically, once our filter, which is a “forward” process moving from one time instant to the next, estimates tracks, a smoother will be employed to refine the estimates of the frequency PDFs and, thus, the MAP inference on the modal frequencies.

We will also investigate a way to improve the tracking performance of the filter for the $\chi^2$ model, which is the preferred method in such problems. We see in the real data tracking results when we used the $\chi^2$ model in Chapter 8 that, for some slices, some “modes” were identified for a few slices and then disappeared. These tracks could be real modes or an effect of noise. To resolve this ambiguity, we may be able to add appropriate constraints to the PF to decide if the obtained estimates
are true modes. The smoothing processor mentioned before may assist in resolving this problem.

The ultimate goal of our work is to find estimates for geoacoustic parameters using the results of arrival times and amplitudes obtained by the PF. That is, particle filtering is a first step in a two-step inversion process. Once estimates are obtained for parameters associated with the received signal - frequencies, amplitudes, noise variance - they can be used to find estimates of source location, sediment sound speed, and sediment attenuation.
REFERENCES


