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Analysis and mitigation of carrier frequency offset for uplink of OFDMA

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ABSTRACT

ANALYSIS AND MITIGATION OF CARRIER FREQUENCY OFFSET FOR UPLINK OF OFDMA

by
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Orthogonal Frequency Division Multiplexing (OFDM) is being used in many wireless standards because of its immunity to multipath fading, high spectral efficiency and simple implementation, making it suitable for high data rate multimedia wireless applications. One of the significant drawbacks of the OFDM is its sensitivity to Carrier Frequency Offset (CFO). CFO causes Inter Carrier Interference (ICI) between subcarriers and Multiple User Interference (MUI) at Uplink between different users. ICI and MUI at uplink cause significant degradation in the performance of the receiver, therefore, to improve the receiver performance up to acceptable level, compensation of the CFO becomes necessary.

In this research, Suppression of MUI by Minimum Mean Squared Error (MMSE) Feedback Equalizer in frequency domain which was originally proposed for Single Carrier- Frequency Domain Multiple Access (SC-FDMA) has been studied for Uplink of Orthogonal Frequency Division Multiple Access (OFDMA). However, calculation of MUI power required in this algorithm for all users impose very high computational burden on the receiver. In the proposed Low Complexity MUI Suppression by MMSE Equalization for Uplink of OFDMA approximation to the calculation of MUI power is applied to reduce its complexity. Simulation result & calculated complexity show that proposed method obtains good performance with much lower complexity.

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by
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APPROVAL PAGE

ANALYSIS AND MITIGATION OF CARRIER FREQUENCY OFFSET FOR UPLINK OF OFDMA

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To my beloved mother Mrs. Harshadaben Panchal,
father Mr. Rajendrakumar Panchal and my advisor.

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CHAPTER 1

BASICS OF OFDMA

1.1 Introduction

The demand for multimedia wireless communication has increased tremendously over past few years and it's going to increase rapidly in years to come. Orthogonal Frequency Division Multiplexing (OFDM) has been adopted by many wireless standards for high rate multimedia transmission. OFDM is widely used in modern wireless communication Standards like in Digital Terrestrial Television Broadcast- DVD in Europe, ISDB in Japan, in IEEE802.11d and IEEE802.11e standard.

There is a very strong interest of using OFDM to provide Multiple Access called as Orthogonal Frequency Division Multiple Access (OFDMA). Extensive research has been done is past decade on implementation of OFDMA.OFDMA is a combination of OFDM and Frequency Division Multiplexing (FDM). This idea was first suggested by Sari and Karam for Cable TV Networks [1] and later adopted in the Uplink of Interaction channel for Digital Terrestrial Television (DVB_RCT) [2]. In OFDMA system the total available bandwidth B is divided into N subcarriers, with subcarriers spacing $\Delta f = B / N$. N subcarriers are then divided into P sub-channels, each sub-channel having $R=N/P$ subcarriers. Different Subcarrier Assignment strategies (SAS) are used for allocating these subcarriers to the different sub-channels. Currently available SAS are Sub-band SAS, Interleaved SAS and Dynamic SAS, each having its own advantages and disadvantages. Different SAS in OFDMA provides flexibility in resource allocation to different users, while OFDM due to use of Orthogonal Subcarriers provides protection against the multipath distortion, avoiding the use of complex time domain equalizers at receivers etc. Another important reason for using OFDM as multiple access technique is that OFDM is very spectrally efficient, as compared to single carrier system, making it suitable for high data rate multimedia wireless applications.

1.2 Brief History of OFDM

- A principle for multi-channel transmission over a band limited Channel was proposed in 1966 [3].

- Parallel transmission of data over overlapping channels was studied in [4], where precise phasing of carriers was suggested to reduce crosstalk between the channels. Improvement in performance was observed by using Raised Cosine filter for this system.

- Digital implementation of parallel data transmission system over overlapping channels was studied in [5] using DFT, which avoided the need for separate oscillators for each subcarriers and coherent demodulation at the receiver.

- Cyclic extension of the OFDM symbol by using last few samples of IFFT was suggested in [6], allowing the use of discrete convolution, to simply the processing of synchronization.

- Technique for frequency offset estimation by using repeated symbols was introduced in [7].

- BER sensitivity of OFDM due to frequency offset and Weiner phase was studied in [8].

- Technique for frequency and timing offset correction involving two reference symbols with improved acquisition range was introduced in [9].

- A technique for frequency offset correction and reduction in ICI at uplink known as CLJL was introduced in [10]. It involved circular convolution and diagonal matrix to separate the signals of each user in uplink of OFDMA.

1.3 OFDM System Model

In this section OFDM transmitter and receiver is presented.

1.3.1 OFDM Transmitter

OFDM is an Orthogonal Pulse Amplitude Modulation (OPAM). OFDM signal is generated by combining N orthonormal pulse shapes $\{g_n(t); n = 0,1\ldots N-1\}$ amplitude modulating a different symbol $a_k^{(n)}$ from and alphabet of M symbols and transmitting all the pulses simultaneously. The transmitted pulses $x(t)$ can be given from [11] as follows

$$x(t) = \sum_{k=-\infty}^{k=\infty} \sum_{n=0}^{N-1} a_k^{(n)} g_n(t-kT) \tag{1.1}$$

where $$g_n(t) = \frac{1}{N} \sum_{k=0}^{N-1} e^{j2\pi n\, k/N} p(t-kT/N), \quad \textit{for } n = 0\ldots N-1 \tag{1.2}$$

$p(t)\sum_{i=1}^{n} X_i^2$ represents ideal reconstruction filter, where input is sampled at the rate *N/T*

$$p(t) = \sqrt{\frac{T}{N}} \frac{\sin(\pi Nt/T)}{\pi t} \tag{1.3}$$

for one shot transmitter, Equation (1.1) can be written as

$$x(t) = \sum_{n=0}^{N-1} a^n g_n(t) \tag{1.4}$$

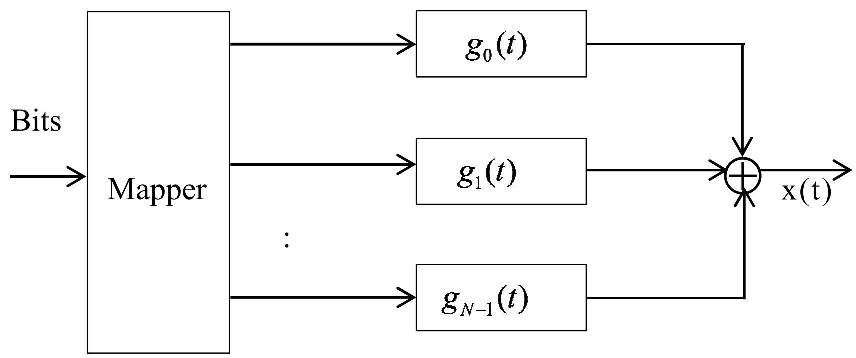

Figure 1.1 An OPAM transmitter.

substituting value of $g_n(t)$ from Equation (1.2) in Equation (1.4), we get

$$x(t) = \sum_{n=0}^{N-1} a^n \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} e^{j2\pi nk/N} p(t - kT / N) \tag{1.5}$$

by changing the order the summation, $x(t)$ can be given as

$$x(t) = \sum_{k=0}^{N-1} \{\frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} a^n e^{j2\pi nk/N} \} p(t - kt / N) \tag{1.6}$$

The equation with the brackets is k^{th} inverse DFT coefficients of $\{a^{(0)}, a^{(1)}, ..., a^{(N-1)}\}$ hence, the transmitted signal can be given as

$$x(t) = \sum_{k=0}^{N-1} x_k p(t - kT / N) \quad (1.7)$$

where x_k is N point IDFT of $x(n)$, *for n=0, 1….N-1*. Thus, equivalent structure of the OFDM transmitter can be given as

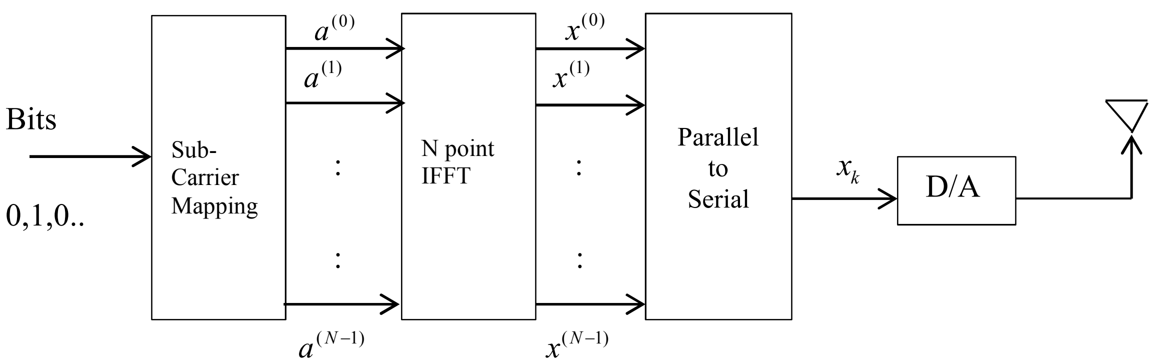

Figure 1.2 OFDM transmitter.

1.3.2 OFDM Receiver

For demodulation of the OFDM symbol minimum distance receiver is used [11]. The received signal is correlated with each of the pulses of $\{g_n(t)\}$. The signal after correlation with the $g_n(t)$ can be given as

$$y(t) = \int_{-\infty}^{\infty} r(t) g_n^*(t) dt \quad (1.8)$$

where $r(t)$ is the received OFDM signal. Substituting the value of $g_n(t)$ in Equation (1.8), $y(t)$ can be given as

$$y(t) = \int_{-\infty}^{\infty} r(t) \sum_{k=0}^{N-1} e^{-j2\pi nk/N} p(t - kT / N) dt \qquad (1.9)$$

$$y(t) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} e^{-j2\pi nk/N} \int r(t) p(t - kT / N) dt \qquad (1.10)$$

The term within integration represents the passing of the received signal through receive filter $p(t)$, which converts Analog signal to Digital signal and sampled at rate kT / N. The k^{th} sample of the ADC is given by r_k. In time period *T*, total *N* samples are collect from ADC. The discrete form of equation can be given below

$$y_n = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} r_k e^{-j2\pi nk/N} \qquad (1.11)$$

Thus, equivalent structure of the OFDM receiver is given by

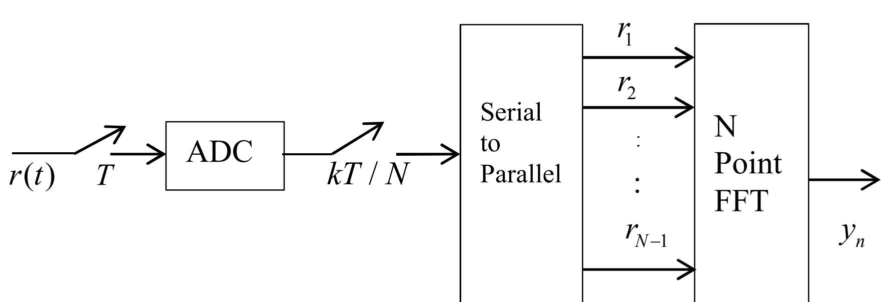

Figure 1.3 OFDM receiver.

1.4 Advantages of OFDM

1.4.1 Advantage of OFDM over FDM

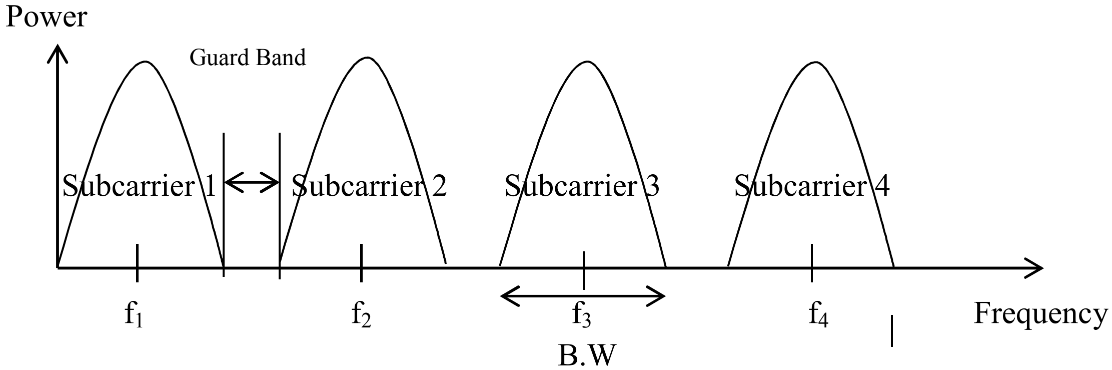

Figure 1.4 Spectrum of frequency division multiplexing.

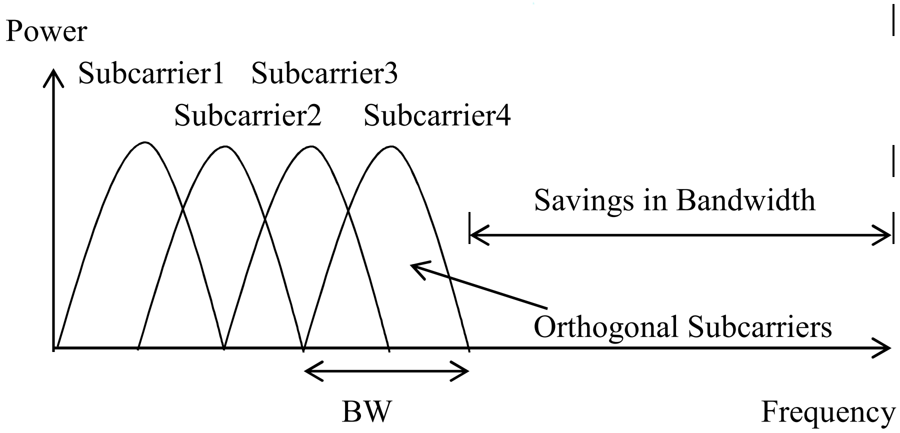

Figure 1.5 Spectrum of orthogonal frequency division multiplexing.

In FDMA the entire bandwidth in divided into several narrow bandwidth channels known as subcarriers. Figure (1.4) shows the spectrum allocation of Frequency Division Multiple Access for four subcarriers along with guard band to avoid ICI. If the bandwidth of each subcarrier is suitably small it can be considered as flat fading channel. FDMA uses guard band between channels to avoid ICI, but with reduction in spectral efficiency. Basic idea of the OFDM is to split the Data into N parallel streams of reduced data rate and transmit each on a separate subcarrier, When these subcarriers are orthogonal and have sufficient subcarrier spacing they overlap each other in frequency domain [5]. Due to the overlapping of the subcarriers in the frequency domain it provides higher spectral efficiency as compared to Frequency Division Multiplexing (FDM) as can be seen in Figure (1.5). Comparing Figure (1.4) and Figure (1.5), it can be concluded that OFDM provides higher spectral efficiency than FDM.

1.4.2 Protection Against Adjacent and Co-channel Interference

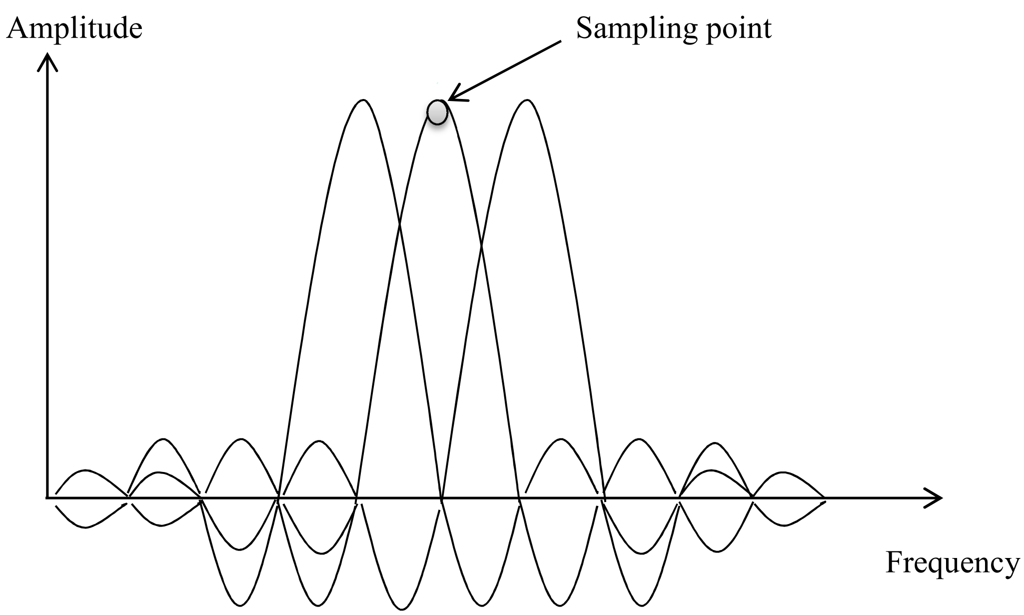

Figure 1.6 OFDM symbol representation using Sinc function.

Another contrasting feature of the OFDM is shaping of the signal in frequency domain by Sinc function, which is shown in Figure (1.6). Due to this shaping in frequency domain, when one subcarrier is sampled at its peak, amplitude of all other subcarriers is zero and there is no Inter Carrier Interference (ICI), as long as a carrier remains orthogonal. Truncating of the spectrum of each subcarrier reduces the demand for the filters, and it allows the symbols to be restricted in time.

There are several other advantages of OFDM like affinity to different transmission arrangement bandwidth, it can be easily generated using IFFT and received using FFT, channel coding and modulation schemes can be independently defined for different subcarriers etc.

1.5 Technical Challenges in Implementation of OFDM

Apart from several advantages offered by OFDM, it has several disadvantages too. OFDM is very sensitive to time offset and frequency offset. In multicarrier systems like OFDM, at each Discrete Fourier Transform (DFT) window samples from only one OFDM symbol should be present. Due to the multipath dispersion, samples from different OFDM symbols overlap at the DFT window. The overlapping of samples from different symbols at DFT window results in Inter Block Interference (IBI). To avoid IBI DFT window of each received symbol needs to be adjusted so that samples from different OFDM symbols do not overlap each other, this method of adjusting DFT window to avoid IBI is called as time offset correction.

Frequency offset is caused due to Doppler Spread and mismatch of transmitter’s and receiver’s oscillator’s frequency. Frequency offset causes apparent shift in the frequency of the received signal and also causes loss in the orthogonality of the subcarriers, these factors causes ICI and MUI at the receiver. Time offset and frequency offset causes degradation performance of the receiver, hence, it needs to be compensated.

Effect of time offset can be minimized by using long Cyclic Prefix (CP). Impact of frequency error cannot be minimized by using simple method like in the case of time offset compensation; hence mechanism to compensate frequency offset at receiver should be made available to avoid any degradation in receiver's performance.

Frequency offset compensation process can be broadly classified as two stage process, first one is frequency offset estimation and second is frequency offset correction. Frequency offset compensation at downlink is quite simple, since it involves estimation and compensation for single user only, while frequency offset compensation at Uplink is difficult process since it involves estimation and compensation for many users at the same time. Frequency offset compensation at Uplink will be the main focus of this research work.

CHAPTER 2
ANALYSIS OF WIRELESS CHANNEL

2.1 Introduction of Wireless Channel

In a mobile communication the signal transmitted through a wireless channel is received with its delayed replicas having different attenuation, phase and delay. This effect is known as Multipath propagation. This effect is caused due to reflection, diffraction and scattering of the transmitted signal from Buildings, trees, mountains, lamp posts etc. The Multipath propagation channel can be broadly be classified as 1) Large Scale Fading Channel and 2) Small Scale Fading Channel.

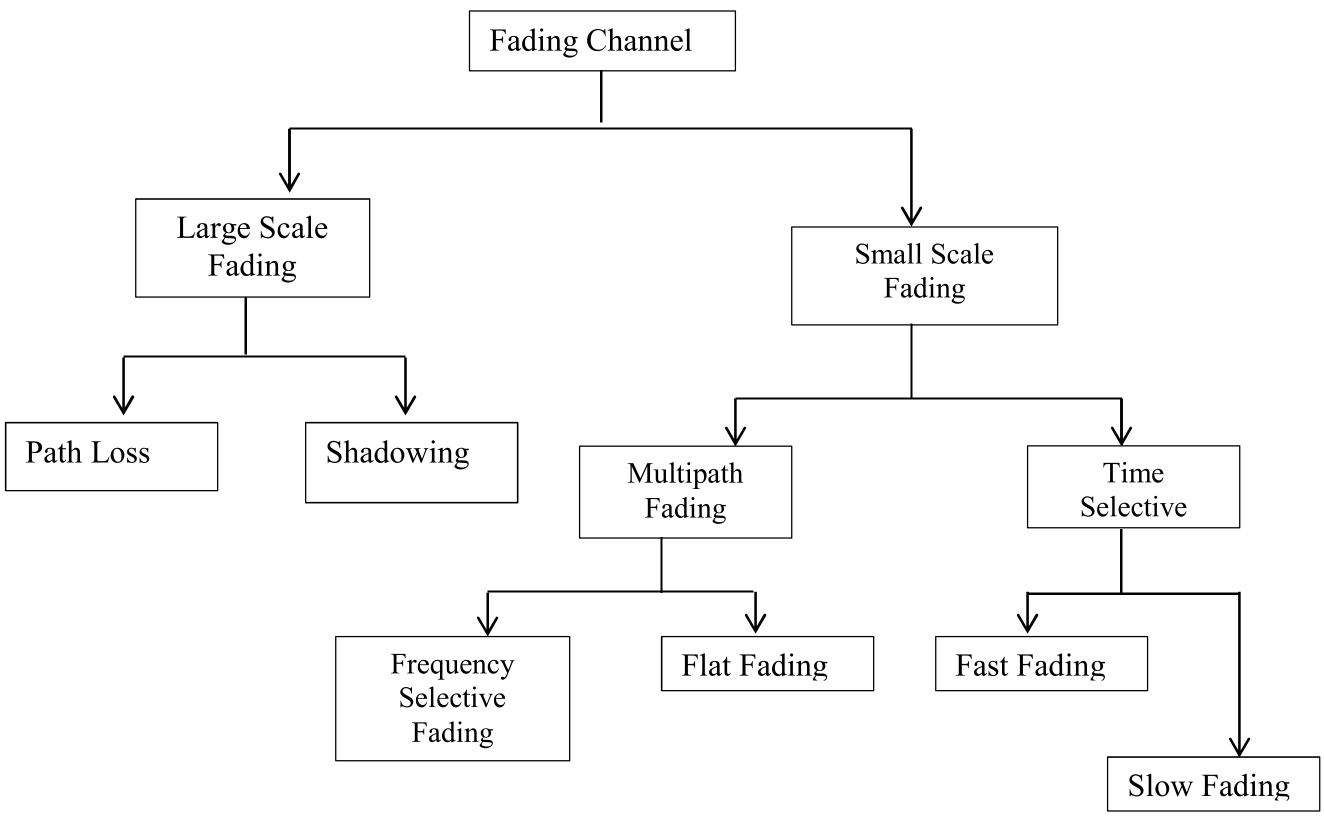

Figure 2.1 Classification of wireless channel [12].

Large Scale fading is function of distance between transmitter and receiver. It is also caused by propagation path loss and shadowing of the waves by hills, mountains and the terrains. Large scale fading occurs when receiver moves over a large distance. Small scale fading is caused by constructive and destructive interference of multipath waves at receiver, when mobile user moves over shorter distances. Depending upon the extent of the multipath the small scale fading can be broadly classified as Multipath Fading and Time Selective fading [12].

2.2 Time Selective Fading

Relative motion between the mobile station and Base Station causes apparent shift in the frequency of received signal at Base Station. Doppler spread causes shift in the frequencies of the transmitted signal. If the angle of n^{th} incident wave at mobile station is θn, the maximum Doppler spread is given by

$$f_d = \frac{v f_c \cos\theta_n}{c} \tag{2.1}$$

where f_d is the maximum Doppler Spread, V Is the velocity of the mobile station and C is the speed of light (3*10^8m/s) and f_c is the carrier frequency. Doppler spread in frequency domain implies time variant nature of the impulse response of the channel. The scattering function $S(\tau, f)$ can be used to characterize the time variant nature of the channel [13]. The scattering function shows Doppler power spectrum for different delays τ and frequency f. When $S(\tau, f)$ is averaged over delay τ, the average power of the channel as the function of the frequency is obtained.

$$S(f) = \int_{-\infty}^{\infty} S(\tau, f) d\tau \tag{2.2}$$

The root mean square (RMS) value of $S(f)$ is called as Doppler Spread f_{rms} whose RMS value is given by [13] as

$$f_{rms} = \sqrt{\frac{\int_{R_f} (f - f_{avg})^2 S(f) df}{\int_{R_f} S(f) df}} \tag{2.3}$$

where R_f is the region where $f_0 - f_{\max} \leq f \leq f_0 + f_{\max}$ and f_{avg} is the average frequency of the Doppler spectrum, given by

$$f_{avg} = \frac{\int_{R_f} f\, S(f) df}{\int_{R_f} S(f) df} \tag{2.4}$$

where f_{rms} is related to coherence time of the channel by relation $T_c \approx 1/f_{rms}$. T_c, is measure of how fast the channel changes with time. Small value of T_c means channel is rapidly changing with time, which is known as Fast Fading Channel, while T_c is large means channel is changing slowly with time and resulting channel is known as Slow Fading Channel. To overcome the effect of the Doppler spread the symbol period should be selected much smaller than the coherence time of the channel i.e. $T_s << T_c$.

In multicarrier modulation the available bandwidth B is divided into N channels, each with subcarrier spacing or bandwidth of B/N. The symbol duration is thus $T_s = N/B$. Therefore, if the number of subcarriers in a given bandwidth increases, the subcarrier spacing reduces and time period increases. To combat the Time selective fading of the channel proper care should be taken that the symbol duration does not exceed the coherence time of the channel.

2.3 Multipath Fading

In multipath fading channel, signal is reflected from many objects like Buildings, trees and ground. Due to the reflection different delayed versions of the transmitted signal is received at receiver with different delays. The time difference between the first arrived wave and significant last replica of the received signal is called as delay spread. The RMS value of the delay spread is given by [13].

Let, $\tau_{\max}, \tau_{avg}$ & τ_{rms} are maximum, average and RMS Delay Spread of the channel respectively. $A(\tau)$ is the average output power of the channel as function of the delay. Depending on the Bandwidth of the transmitted signal and τ_{RMS}, the Multipath channel can be classified either as Flat fading channel or Frequency selective channel. If τ_{rms} is greater than the symbol duration T_s of the transmitted signal then receiver cannot

$$\tau_{rms} = \sqrt{\frac{\int_{0}^{\tau_{\max}} (\tau - \tau_{avg})^2 A(\tau) d\tau}{\int_{0}^{\tau_{\max}} A(\tau) d\tau}} \tag{2.5}$$

where
$$\tau_{avg} = \frac{\int_{0}^{\tau_{\max}} \tau A(\tau) d\tau}{\int_{0}^{\tau_{\max}} A(\tau) d\tau} \tag{2.6}$$

distinguish between the pulses received with different delays for the same transmitted pulse and it results in ISI at the receiver. While if the τ_{rms} is less than the symbol duration T_s, then there is no ISI at the receiver. Hence, in order to avoid ISI the symbol duration should be $T_s >> \tau_{rms}$.

The dual of the delay spread is called as coherence Bandwidth and is given by $B_c \approx 1/\tau_{rms}$. It's defined as the frequency region of the channel over which the two frequency components are highly correlated.

In multicarrier modulation like OFDM since the entire bandwidth is divided into several subcarriers with relatively small bandwidth i.e. with large symbol duration, each subcarrier's bandwidth is chosen such that it is less than the coherence bandwidth of the channel, thus, frequency selective channels acts like a flat fading channel on each subcarrier. This is most important advantage of OFDM. Hence, from the discussion of the multipath channel it can be concluded that to avoid ISI, either $T_s >> \tau_{rms}$ or $\Delta f << B_c$, where Δf is the subcarriers spacing of OFDM or in general bandwidth of the transmitted signal. To artificially increase the symbol duration of the Transmitted OFDM symbols, Cyclic Prefix is used. The last few samples after IDFT are appended before the transmitted symbol as CP and these increases the symbol duration of the OFDM signal and help to avoid ISI.

2.4 Modeling of the Wireless Channel for Simulation

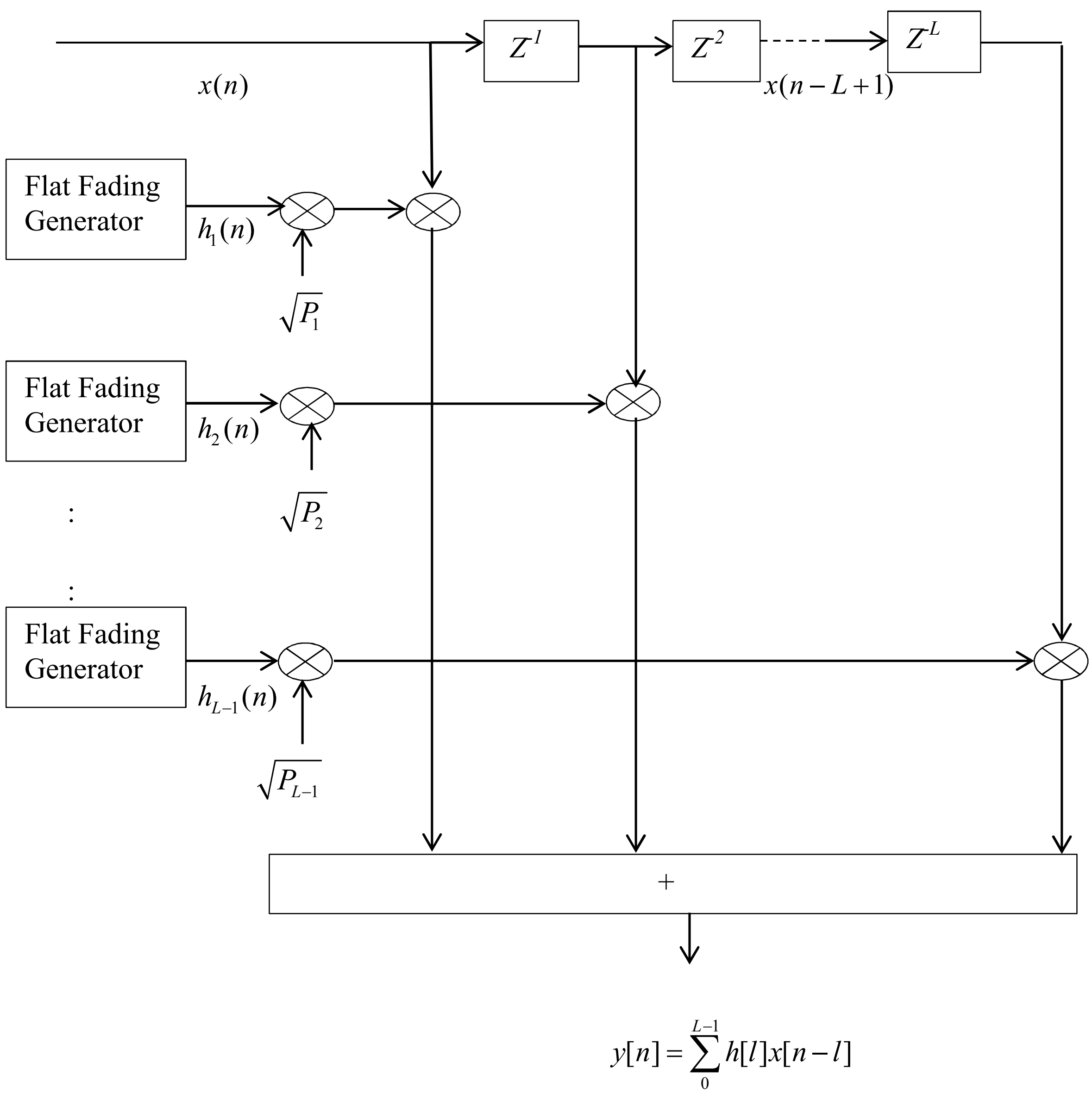

Figure 2.2 Taped delay line model of frequency selective channel.

As seen before in multipath channel, delayed versions of the originally transmitted signal is received with different delays and attenuation. The good approximation of such a channel is tapped delay line represent of the discrete impulse response of the channel. Here the channel acts as the FIR. If discrete impulse response of the channel is given by $h[n]$ of length L or taps then the received $y[n]$ signal can be given as

$$y[n] = \sum_{0}^{L-1} h[l]x[n-l] \tag{2.7}$$

where $h[n] = [h(0), h(1).....h(L-1)$. is the impulse response of the channel. Flat fading generators are frequency non selective flat fading generators and independent of each other. The average power of fading generators is one. Output of each fading generator is then multiplied by the power of tap to get the coefficient of the tap. The tap power is defined by the power delay profile of the channel. Here tap delay is assumed to be integer multiple of the symbol period T_s of OFDM symbol.

CHAPTER 3

ANALYSIS OF OFDM

3.1 Importance of Cyclic Prefix in OFDM

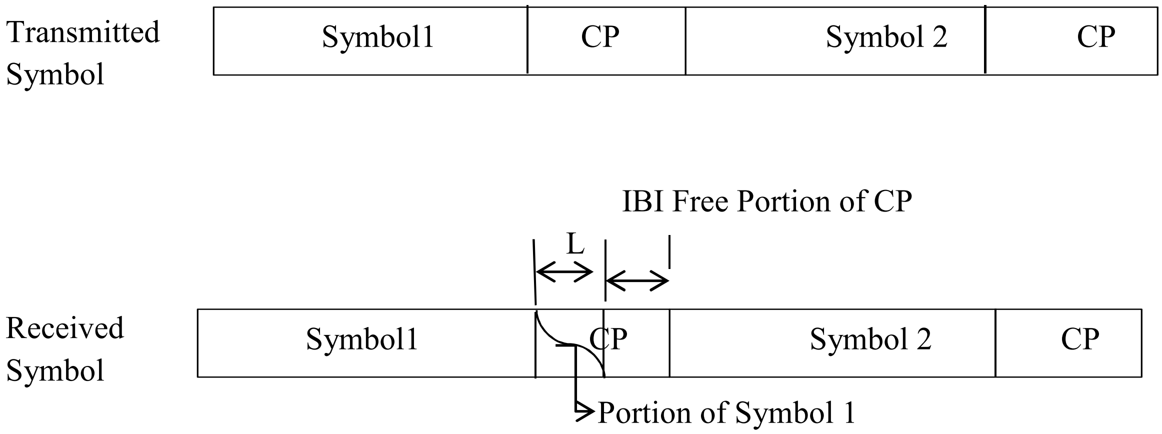

Figure 3.1 Partial overlapping of symbols due to multipath distortion.

In order to avoid Inter Block Interference, Cyclic Prefix is added to each OFDM symbol. Last few samples after IFFT are used as the cyclic prefix. The Figure (3.1) shows how CP can avoid IBI. At the receiver due to the multipath distortion the L samples of the symbol 1 extend into the Cyclic Prefix but doesn't extend into symbol 2, thus avoiding IBI. The Cyclic prefix avoids the need for accurate time synchronization and maintains the orthogonality as long as receiver window is positioned with the tolerance of the cyclic prefix.

Length of the cyclic prefix should be carefully chosen. CP duration should be greater than channel impulse response to avoid IBI. CP shouldn't be selected too long so as to avoid unnecessary overhead. Therefore, there is a tradeoff in selecting the size of the CP, too long cyclic prefix will reduce the data rate while smaller cyclic will induce

IBI. Long Cyclic Prefix avoids the need for the time offset compensation at the receiver, this lead to a quasi-synchronous system [14]. In a quasi–synchronous system the time offset is very small and is incorporated as the part of the unknown channel impulse response and is compensated by the channel equalizer at the receiver, avoiding the need for complex time offset correction at the receiver.

3.1.1 Simulation of BER of OFDM symbol for two different CP length

Total Number of subcarriers=64, Number of subcarriers used for data transmission=16, to reduce the bit error, FEC using half rate convolution code and Hard Decision Viterbi decoding was employed, QPSK modulation was used for channel coding, Rayleigh

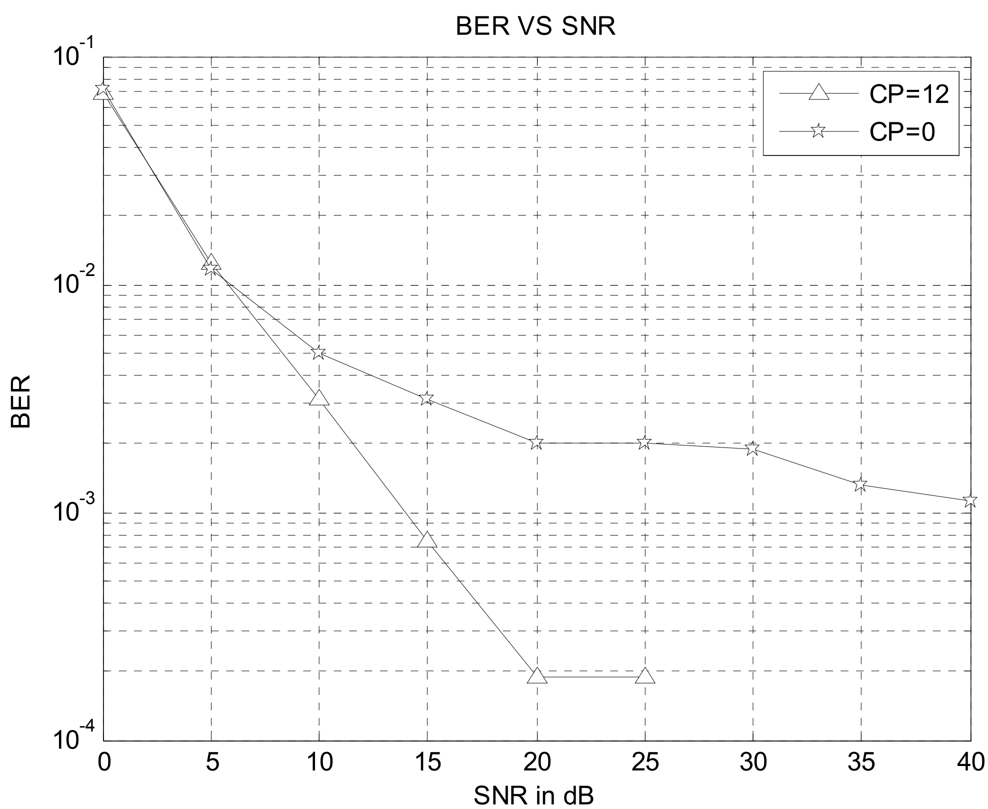

Figure 3.2 BER vs. SNR for varying CP length.

Fading channel with exponential power delay profile and AWGN model of the wireless channel for L=5 taps was used for simulation. To see the effect of change in CP, OFDM symbol with two different CP lengths was transmitted. First OFDM symbol was transmitted without CP and the other OFDM symbol was transmitted with CP of length 12 samples. It can be seen from Figure (3.2) that as SNR increases the difference in the BER between two cases increases. Performance of OFDM symbol with CP is better than that of OFDM symbol without CP. Hence, it can be concluded that CP helps in avoiding ISI.

3.2 Effect of Frequency Offset on OFDM

In communication system employing OFDM as the modulation scheme at the transmitter, first data is modulated by OFDM modulator, converted from Digital to Analog and then up converted by multiplying with the carrier signal f_c. At transmitter the OFDM signal in down converted by multiplying with the carrier signal f'_c . Even though very stable oscillators are used for generation of carrier signal at transmitter and receiver, still there exist difference in the carrier frequencies at transmitter and receiver which can be given by $f_c - f_c^{'}$.

As discussed before, the presence of Doppler spread also causes shift in the frequencies of the received signal. For purpose of analysis usually frequency offset is expressed in terms of normalized frequency offset, which can be given as follows

$$\varepsilon = f_{offset} / \Delta f \tag{3.1}$$

$$f_d = v f_c / c \tag{3.2}$$

$$\varepsilon = \varepsilon_i + \varepsilon_f \tag{3.3}$$

where f_d is the Doppler spread and ε is the normalized frequency offset, which can be divided into two parts, the normalized integer and fractional CFO $\varepsilon_i, \varepsilon_f$ respectively.

3.2.1 Effect of Integer Frequency Offset

Let $x_p[n]$ be the p^{th} transmitted OFDM symbol, $h[n]$ be the impulse response of the Channel in time domain then received signal in time domain from [12] can be given as below, assuming only integer frequency offset is present in the received OFDM symbol.

$$y_p[n] = \frac{1}{N} \sum_{m=0}^{m=N} H[m] X_p[m] e^{j2\pi m(\varepsilon_i)/N} + W[n] \tag{3.4}$$

where $h[n] = \{h(0), h(1), ..., h(L-1)\}$ is discrete impulse response of the channel and $W[n]$ is AWGN noise. Since, multiplication by and exponential in time domain results in a frequency shift in the frequency domain here corresponding signal in the frequency domain can be shown as following

$$Y_p[k] = H[k - \varepsilon_i] X_p[k - \varepsilon_i] + W[k] \tag{3.5}$$

It can be seen from the Equation (3.5) above that each subcarrier of received OFDM symbol $X_p[k]$ is shifted by integral frequency offset by ε_i in frequency domain. Since, the shift in the frequency in an integer there is no loss in orthogonality of the subcarriers, and hence, there is no ICI.

3.2.2 Effect of Fractional Frequency Offset

Consider only fractional frequency offset is present in the p^{th} received OFDM symbol, then by taking N point FFT of $y_p[n]$ in Equation (3.4), $Y_p[k]$ can be given from [12] as

$$\begin{aligned} Y_p[k] = &\sum_{n=o}^{N-1} \frac{1}{N} \sum_{m=0}^{N-1} H\ [m] X_p[m] e^{j2\pi(\varepsilon_f)mn/N} e^{-j2\pi k n/N} \\ &+ \sum_{n=0}^{N-1} W[n] e^{-j2\pi k n/N} \end{aligned} \tag{3.6}$$

$$Y_p[k] = \sum_{n=o}^{N-1} \frac{1}{N} \sum_{m=0}^{N-1} H[m] X_p[m] e^{j2\pi(\varepsilon_f+m-k)n/N} + \sum_{n=0}^{N-1} W[n] e^{-2j\pi k n/N} \tag{3.7}$$

$$\begin{aligned} Y_P[k] = &\frac{1}{N} \sum_{n=0}^{N-1} H[m] X_p[m] e^{j2\pi \varepsilon_f n/N} \\ &+ \frac{1}{N} \sum_{m=0, m\neq k}^{N-1} X_p[m] H[m] \sum_{n=0}^{N-1} e^{j2\pi(\varepsilon_f+m-k)n/N} + W[k] \end{aligned} \tag{3.8}$$

after performing some standard mathematical calculations final expression for received with fractional carrier frequency offset can be given by [12] as

$$\mathrm{Y_p}[k] = \frac{\sin(\pi\,\varepsilon_f)}{N\sin(\pi\,\varepsilon_f\,/\,N)} e^{j2\pi\,\varepsilon_f\,(N-1)/N} H_p[k]X_p[k] + I_p[k] + W[k] \qquad (3.9)$$

where $I_p[k]$ is Interference on k^{th} subcarrier due to other subcarriers and can be given as

$$I_p[k] = e^{j2\pi\varepsilon_f(N-1)/N} \sum_{m=0, m\neq k}^{N-1} \frac{\sin(\pi(m-k-\varepsilon_f))}{N\sin(\pi(m-k+\varepsilon_f)/N)} H[m]X_p[m]e^{j\pi(m-k)(N-1)/N} \qquad (3.10)$$

In Equation (3.9) the first term represents the Amplitude distortion and second term represents the phase distortion, on the received OFDM symbol due to the CFO. Third term represents AWGN noise. Also from Equation (3.10), it can be seen that fractional frequency offset causes interference to a particular subcarrier from other subcarriers resulting in ICI. That is fractional frequency offset causes loss of orthogonality between the subcarriers which results in ICI.

3.2.3 Simulation of the Effect of Fractional CFO on BER

For Simulation following simulation model was used:

IFFT and FFT size=64, Number of subcarriers used=16, Length of CP=12, Modulation Scheme= QPSk, Source Coding= ½ rate convolutional encoder and Hard Decision Decoder, Channel Model=AWGN, fractional frequency offset was introduced in the transmitted signal by multiplying each sample by an exponential $e^{-j2\pi\varepsilon_f n/N}$, where n=1, 2 … N. Graph of BER for different values fractional CFO was plotted at SNR=10dB, it can be seen from the Figure (3.3) that for small change in the fractional CFO creates large change in BER.

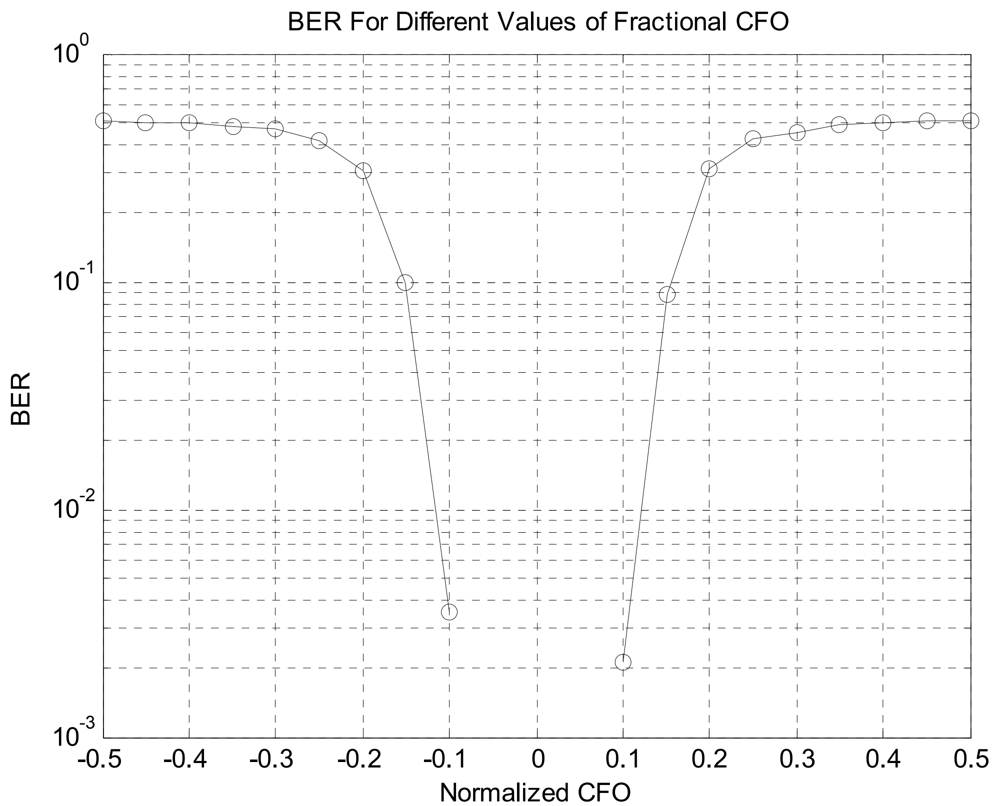

Figure 3.3 Effect of fractional CFO on BER.

CHAPTER 4

ESTIMATION OF CFO IN OFDM

The frequency offset in the received OFDM symbol can be estimated by using either Time domain estimation method or Frequency domain Estimation method.

4.1 Time Domain Estimation of CFO Using Cyclic Prefix

Redundant information of the cyclic prefix can be used for estimation of CFO [12]. With assumption of perfect timing synchronization, the presence of CFO ε results in the phase rotation of the received OFDM signal by $2\pi\varepsilon n / N$, where n is the time domain index of the OFDM signal and N is the IFFT size. In OFDM last N_g samples after IFFT are used as CP and appended to the beginning of the OFDM signal. The phase difference between the samples of CP and last N_g samples of OFDM symbol which are spaced N samples apart can be given by $\frac{2\pi\varepsilon N}{N} = 2\pi\varepsilon$. The CFO ε can be estimated from phase angle of product of the CP and last N_g samples of the OFDM symbol. The CFO can be expressed as

$$\varepsilon = \frac{1}{2\pi}\arg\left\{\sum_{n=1}^{N_g} y^*\left(n\right) y\left(n+N\right)\right\} \tag{4.1}$$

Since, the CFO ε is the argumentation operation performed using $\tan^{-1}()$ the range of ε is from $[-\pi,\pi)/2\pi = [-.5,.05)$

4.2 Frequency Domain Estimation of CFO Using Training Symbol

This technique uses the redundant information in the training symbols for the estimation of the CFO. Two identical training symbols are transmitted with every uplink frame for the purpose of estimation of CFO at the Base Station. Training symbols are composed of some repetitive parts in time domain which remain identical after passing through the channel, except for the phase shift introduced by the channel. This algorithm was introduced by P H Moose [7]. The frequency error can be measured by estimation of the phase shift induced between the two training symbols in frequency domain. Let $Y_1(k)$ & $Y_2(k)$ be the received training symbols in frequency domain when training symbol $X(k)$ was transmitted.

$$Y_1(k) = X(k) + W(k) \tag{4.3}$$

$$Y_2(k) = X(k)e^{-j2\varepsilon N_T/N} + W(k) \tag{4.4}$$

where $W(k)$ is the AWGN noise in frequency domain at k^{th} sub-carrier, N_T is the total time period of one OFDM symbol i.e. $N_T = N + N_g$ and N_g is the length of CP. Then from above two equations we may write the estimated CFO as

$$\hat{\varepsilon} = \frac{1}{\left(\dfrac{2\pi N_T}{N}\right)} \arg\left\{\sum_{k=0}^{N-1} Y_1(k) Y_2^*(k)\right\} \tag{4.7}$$

here $\arg()$ is performed using $\tan^{-1}(\;)$.The range of ε is from $[-\pi,\pi)/2\pi=[-.5,.05)$ or $|\varepsilon|<(\frac{N}{2N_T})$, which is less than half of the subcarrier spacing.

4.2.1 Increasing the Range of CFO Estimation for Moose Method of CFO Estimation

As seen before the range of estimated CFO using Moose Method is less than one half of the subcarrier spacing. This range of estimation can be increased by using training symbol that are repetitive in time domain with shorter period [12]. A training symbol with P repetitive pattern in time domain can be generated by taking IFFT of a comb type signal $X_l(k)$. In frequency domain $X_l(k)$ can be given as

$$X_l\left[k\right]=A_m, \qquad if \quad k=i.P,\ i=1,2\ ,\ldots,\frac{N}{P} \tag{4.7}$$

$$X_l\left[k\right]=0\ , \qquad otherwise \tag{4.8}$$

where, A_m represents *M*-array symbol, and *N/P* is an integer. If $Y\left(k\right)_1\ and\ Y_2\left(k\right)$ is N point DFT of the received first and second training symbols, when two identical symbols obtained from N point IFFT of Equation (4.7) and (4.8) were transmitted, then received OFDM symbol can be given as

$$Y_1\left(k\right)=X_l\left(k\right)+W_1\left(k\right) \tag{4.9}$$

$$Y_2\left(k\right)=X_l\left(k\right)e^{-j2\pi\varepsilon N_T/N}+W_2\left(k\right) \tag{4.10}$$

the estimated CFO can be then given as

$$\hat{\varepsilon} = \frac{P}{2\pi N_T / N} \arg\left\{ \sum_{k=0}^{N/P-1} Y^*_1(k) Y_2(k) \right\} \tag{5.11}$$

The range of estimated CFO is $|\hat{\varepsilon}| < P/2$ using P repetitive pattern in time domain the range of the CFO increases by P times but reduces the number of samples available for estimation by *1/P*, which results in degradation of the MSE performance. Hence, increase in the range of the CFO estimation results in degradation of MSE performance, therefore a compromise is to be made between the range and required MSE performance.

4.3 Comparison of MSE of the CFO estimation Using CP and Training Symbol

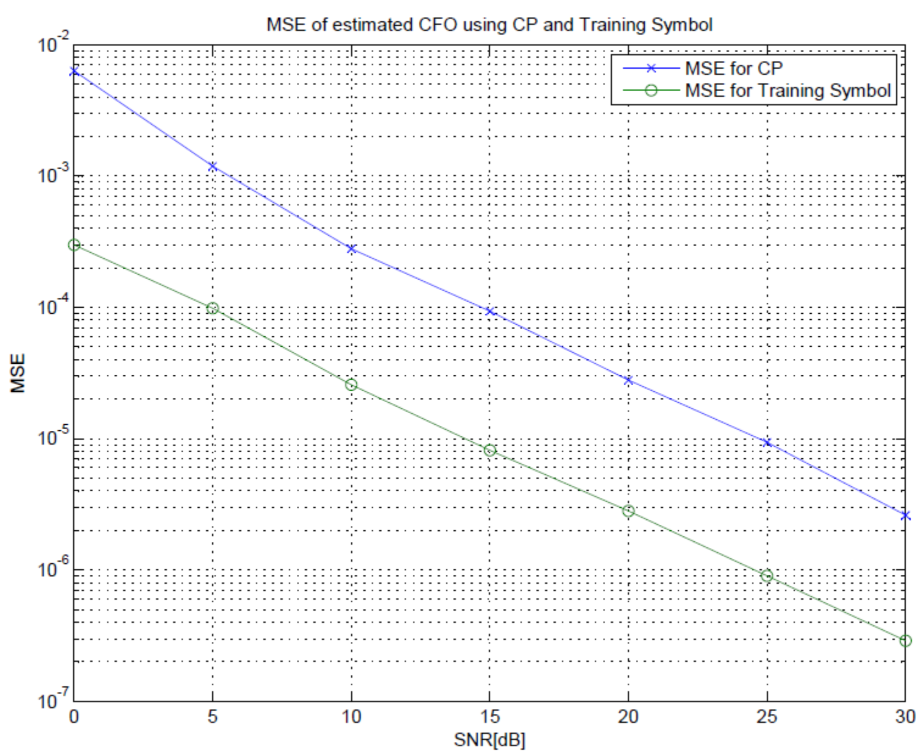

Figure 4.1 Comparison of MSE of estimated CFO by CP and training symbol.

Following assumptions were used to compare the MSE performance of estimated CFO by using CP method and by using Training Symbol (Moose) method:

Total number of subcarriers (N) =64, CP length=12, Number of Training Symbols=2, Rayleigh fading channel with three taps and with AWGN noise was used for simulation.

It can be seen from the Figure (4.1), than MSE of the estimation using Training Symbol (Moose method) is less than the MSE of the estimated CFO using CP. Hence, from the above simulation results it can be concluded that performance of CFO estimation using Training Symbol (Moose Method) is better than CFO estimation using CP.

CHAPTER 5

CFO COMPENSATION AT UPLINK OF OFDMA

5.1 Uplink Model of OFDMA

With a multiuser scenario, in uplink, there are many mobile stations communicating with the Base Station, and available resources are shared by these mobile devices. Base Station receives the signals from all of these devices at the same time. A multiuser

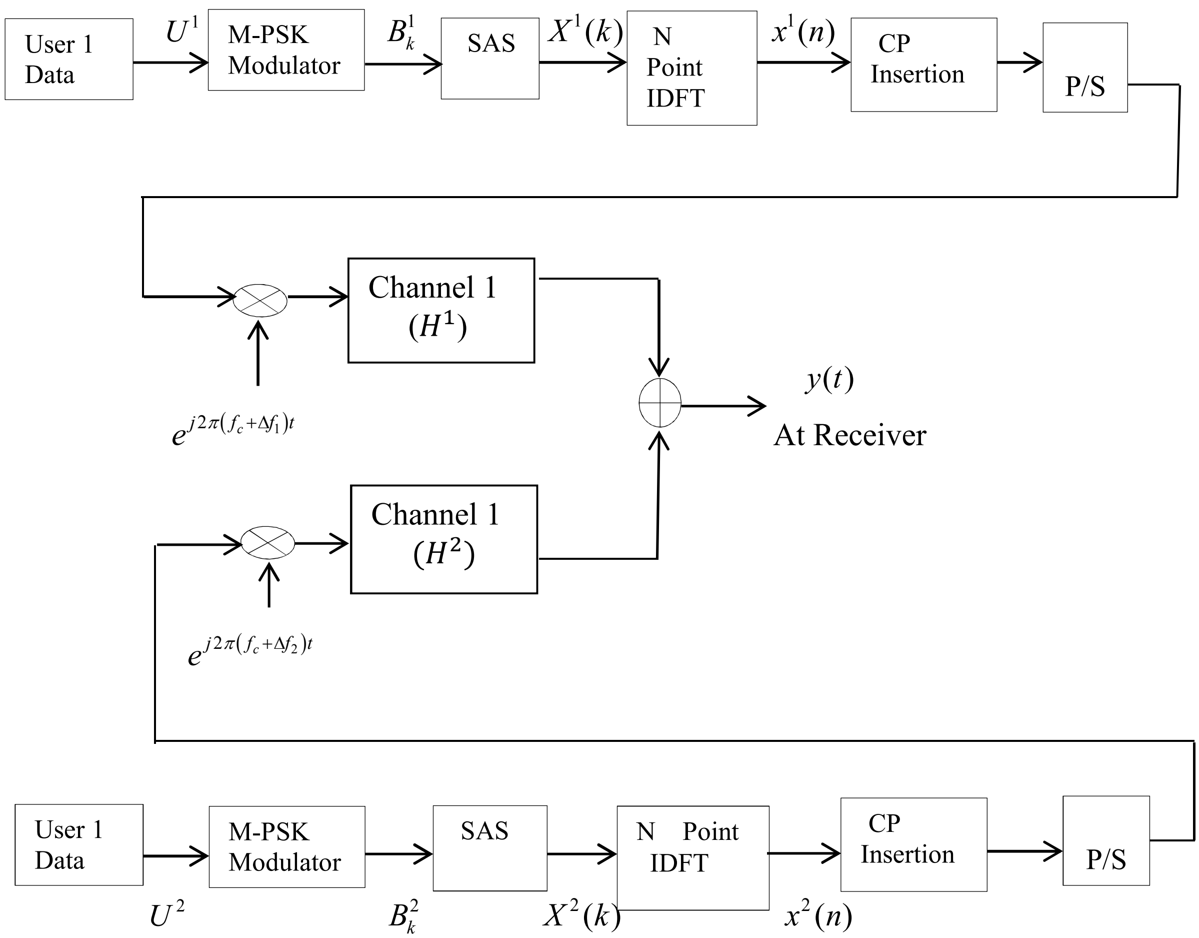

Figure 5.1 Uplink of OFDMA.

scenario with two mobile devices communicating with Base Station is in Figure (5.1). Here, total number of subcarriers are *N* and each user uses *N/2* subcarriers for transmitting data. As seen in the Figure (5.1) the parallel data from m^{th} user U^m is modulated by *M*-PSK modulator to get complex symbols B_k^m. SAS assigns the carriers to each user on which parallel data is to be transmitted. Input to the IDFT for two users can be shown as follows:

$$X^1(k) = B_k^1, \qquad \textit{for } k = 1,2....32 \tag{5.1}$$

$$X^1(k) = 0, \qquad \textit{otherwise} \tag{5.2}$$

$$X^2(k) = B_k^2, \qquad \text{for } k = 33,34,.....N \tag{5.3}$$

$$X^2(k) = 0, \qquad \textit{otherwise} \tag{5.4}$$

The equation of the transmitted OFDM symbol after N point IFFT of $X^m(k)$ for the m^{th} user can be given as

$$x^m(n) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N} X^m(k) e^{j2\pi nk}, \quad \textit{for } n = 1,2\ldots N \tag{5.5}$$

The transmitted OFDM signal passes through wireless channel, where multipath due to the reflected signal from buildings, sign post or mountains is common. Due to multipath multiple copies of the transmitted signal are received at the receiver at different times with different amplitudes. The discrete impulse response of the channel can be given as

$$h(n) = [h(1), h(2), ..., h(L)] \tag{5.6}$$

where L is the length of the impulse response of the channel. The OFDM received symbol $z^m(n)$ for m^{th} user after passing through multipath channel can be given as

$$z^m(n) = x^m(n) * h(l-n), \qquad for\, n = 1,2\ldots L \tag{5.7}$$

where $*$ is time domain linear convolution. The received OFDM symbol $y^m(n)$ for the m^{th} user in presence of multipath and Doppler Spread and AWGN can be given as

$$y^m(n) = z^m(n) e^{\frac{j2\pi\varepsilon_m n}{N}} + w^m(n), \quad for\, n = 1,2\ldots N \tag{5.8}$$

where ε_m is the normalized CFO and $w^m(n)$ is AWGN noise for m^{th} user. The above equation can be written in frequency domain as

$$Y^m(k) = Z^m(k) \otimes C^m(k) + W^m(k), \;\; for\; k = 0,\; 1. .., \; N\text{-}1 \tag{5.9}$$

where
$$c^m(k) = \sum_{n=0}^{N-1} e^{j2\pi\varepsilon_m nk/N} \cdot e^{-j2\pi kn/N} \tag{5.10}$$

$\otimes$ represents N point circular convolution, $Y^m(k), Z^m(k)\, and\; W^m(k)$ are N point DFT's of $y^m(n), z^m(\mathrm{n})\;\; and\;\; w^m(n)$ for m^{th} user respectively.

In uplink, at Base Station, summation of received signal $y^i(n)$ for different users takes place and can be given as

$$r(n)=\sum_{m=1}^{M} z^{m}(n)e^{\frac{j2\pi\varepsilon_m n}{N}}+w^{m}(n) \qquad for\, n=1,2\ldots N \tag{5.11}$$

or in Frequency Domain

$$R(k)=\sum_{m=1}^{M} Z^{m}(k)\otimes C^{m}(k)+W^{m}(k) \tag{5.12}$$

5.2 Frequency Offset Compensation at Uplink of OFDMA by Direct Method

In direct method, at the receiver (Base Station), signal $y^{m}(n)$ of m^{th} user is separated user using filter banks. Frequency offset compensation at uplink for two users is shown in Figure (5.2), In direct method, estimation and correction of the CFO for each user takes place separately. In this method $y^{m}(n)$ for m^{th} user is used to calculate the CFO $\hat{\varepsilon}_m$.

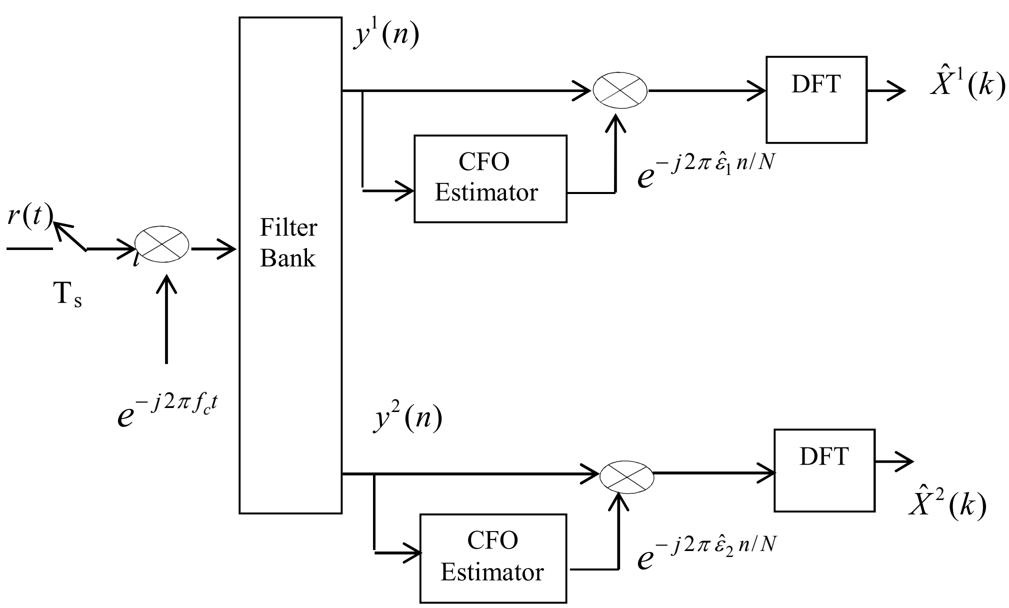

Figure 5.2 CFO compensation at uplink of OFDMA by direct method.

After estimation of CFO, $y^m(n)$ for m^{th} user is multiplied by exponential $e^{-j2\pi\hat{\varepsilon}_m n/N}$ to correct the CFO error. In this method separate DFT is required for each user, thus complexity and computational load increases as number of users increases. Perfect separation of the user's signals is not possible even with ideal brick wall filters because of the frequency leakage among the frequency bands of adjacent users caused by synchronization errors, hence, it results in degradation of receivers performance.

5.3 Frequency Offset Compensation at Uplink of OFDMA by CLJL Algorithm

To reduce the computational complexity in frequency offset correction at uplink in Direct method, new frequency offset correction scheme was introduced by Choi–Lee–Jung–Lee in [10], known as CLJL. This scheme uses only a single N point DFT as compared to '*M*' DFT's required in case of direct method for '*M*' number of users. Base Stations samples the signal at symbols interval T_S. It is then down converted to baseband by multiplying with a carrier signal generated from local oscillator. The block diagram of the Base Station receiver is shown in Figure (5.3). Assuming the same uplink model as shown in Section 5.1, the received signal at Base Station from Equation (5.11) in time domain as

$$r(n) = \sum_{m=1}^{M} z^m(n) e^{\frac{j2\pi\varepsilon_m n}{N}} + w^m(n) \quad for\, n = 1, 2 \ldots N \tag{5.13}$$

or in frequency domain from Equation (5.12)

$$R(k) = \sum_{m=1}^{M} Z^m(k) \otimes C^m(k) + W^m(k) \tag{5.14}$$

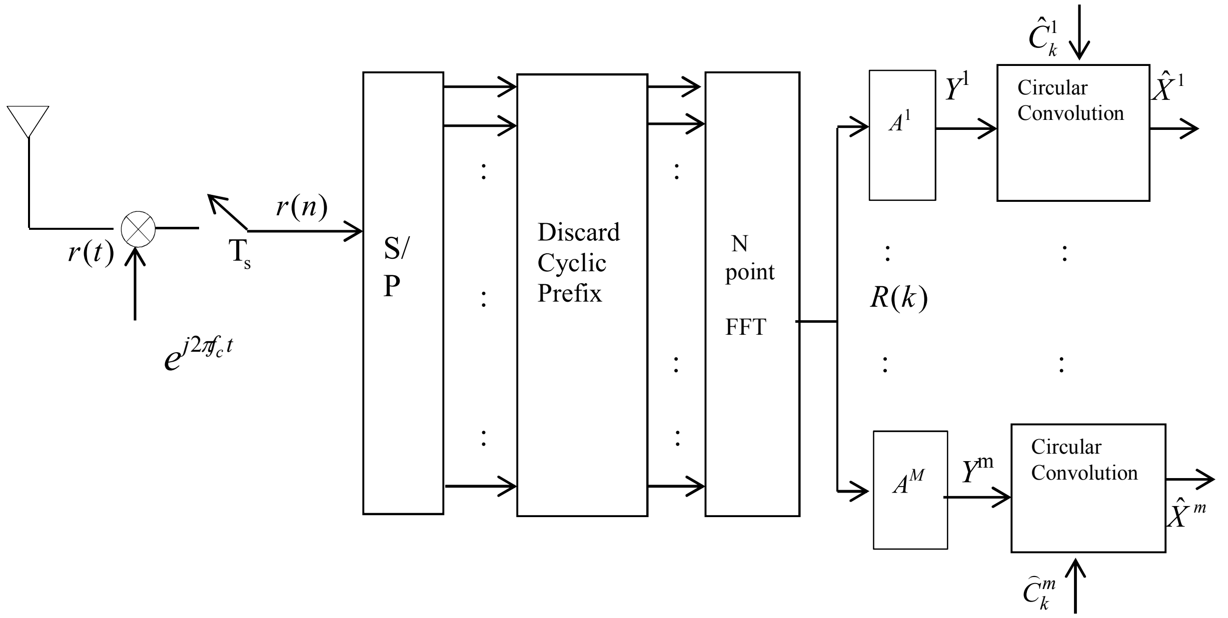

Figure 5.3 Frequency offset compensation at uplink of OFDMA by CLJL algorithm.

In order to separate the signals for different mobile users in frequency domain, filters are used at the output of FFT. A_m is the diagonal matrix of size (*N, N)* where *(n, n)* entry is one for subcarrier belonging to the m^{th} user. Thus, for M number of users, *an* M, A matrices are required. So for the m^{th} user, received OFDM symbol in frequency domain is given by

$$Y^m(k) = A^m R(k) \tag{5.15}$$

Assuming total number of users *M*=2, *N*=64, 32 Sub-carriers assigned to each user using sub-band SAS. In equation (5.14) for 1st user $Y^1(k) = \{Y_0, Y_1, ...Y_{N/2}, 0, 0, ...0\}$, and for 2nd user $Y^2(k) = \{0, 0, ...0, Y_{N/2+1}, ...Y_N\}$ $\quad for\, k = 0, 1, \ldots N-1$. The received OFDM symbol $y^m(n)$ for m^{th} user in time domain can be given as

$$y^m(n) = \left[z(n)^m \right] e^{\frac{j2\pi \varepsilon_m n}{N}} + w^m(n) \tag{5.16}$$

It can be seen from Equation (5.16) that transmitted signal is multiplied by an exponential in time domain which causes shift in the frequencies of the subcarriers. Since time domain multiplication is equal to frequency domain convolution the received signal for m^{th} user can be given from Equation (5.9) as

$$Y^m(k) = Z^m(k) \otimes C^m(k) + W^m(k) \tag{5.17}$$

To compensate for the frequency offset for m^{th} user, first frequency offset $\hat{\varepsilon}_m$ is computed from the training symbols using Moose method or using CP based Method. $Y^m(k)$ for m^{th} user is obtained by filtering the received OFDM symbol $R(k)$ by matrix A^m. The matrix A^m can be given as

$$A^m = diag\left\{ A^m(0,0)\ A^m(1,1) \ldots\ldots A^m(N-1, N\text{-}1) \right\} \tag{5.18}$$

$$A^m(k,k) = 1\ , \quad if\ k \in \{Set\ of\ Subcarriers\ for\ m^{th}\ user\} \tag{5.19}$$

$$A^m(k,k) = 0\ ,\ otherwise \tag{5.20}$$

To get the estimated OFDM symbol $\hat{X}^m(k)$ for m^{th} user, $Y^m(k)$ is circularly convolved with CFO coefficients $\hat{C}_k'^{(m)}$ given by

$$\hat{C}_k'^{(m)} = \sum_{k=0}^{N-1} e^{-j2\pi \hat{\varepsilon}_m n/N} \cdot e^{-j2\pi k n/N} \tag{5.21}$$

the estimated OFDM symbol $\hat{X}^m(k)$ for m^{th} user after frequency offset correction by CLJL is given as follows

$$\hat{X}^m(k) = A^m(A^m R(k)) \otimes \hat{C}_k'^{(m)} \tag{5.22}$$

substituting the value of $R(k)$ from Equation (5.14) in Equation (5.21) we get

$$\hat{X}^m(k) = A^m\left[A^m\left(\sum_{i=1}^{M} Z^i(k) \otimes C^i(k) + W^m(k)\right)\right] \otimes \hat{C}_k'^{(m)} \tag{5.23}$$

$$\begin{aligned}\hat{X}^m(k) = {} & A^m\left(A^m\left(Z^m(k) \otimes C_k^{(m)}\right) \otimes \hat{C}_k'^{(m)}\right) \\ & + A^m\left(\left(A^m \sum_{i=1, i\neq m}^{M} Z^i(k) \otimes C_k^{(i)}\right) \otimes \hat{C}_k'^{(m)}\right) \\ & + A^m\left(\left(A^m W(k)\right) \otimes \hat{C}_k'^{(m)}\right)\end{aligned} \tag{5.24}$$

where $\hat{X}^m(k)$ is the estimated frequency offset corrected received symbol for m^{th} user. Since, perfect user separation using matrix A^m is not possible, there would be some Multiple Access Interference in the estimated received symbol. In Equation (5.24) the first term represents the signal of m^{th} user while second term represents the MUI and the third term is the noise. The estimated OFDM symbol $\hat{X}^m(k)$ is then given to the channel equalization and data detection. Thus, it can be concluded from Equation (5.24) that the presence of MUI from other users results in some degradation of the receiver SNR and SINR performance. CLJL algorithm is well suited for sub-band CAS since, it would have higher MUI with other SAS.

5.3.1 Simulation of CFO correction by CLJL Algorithm

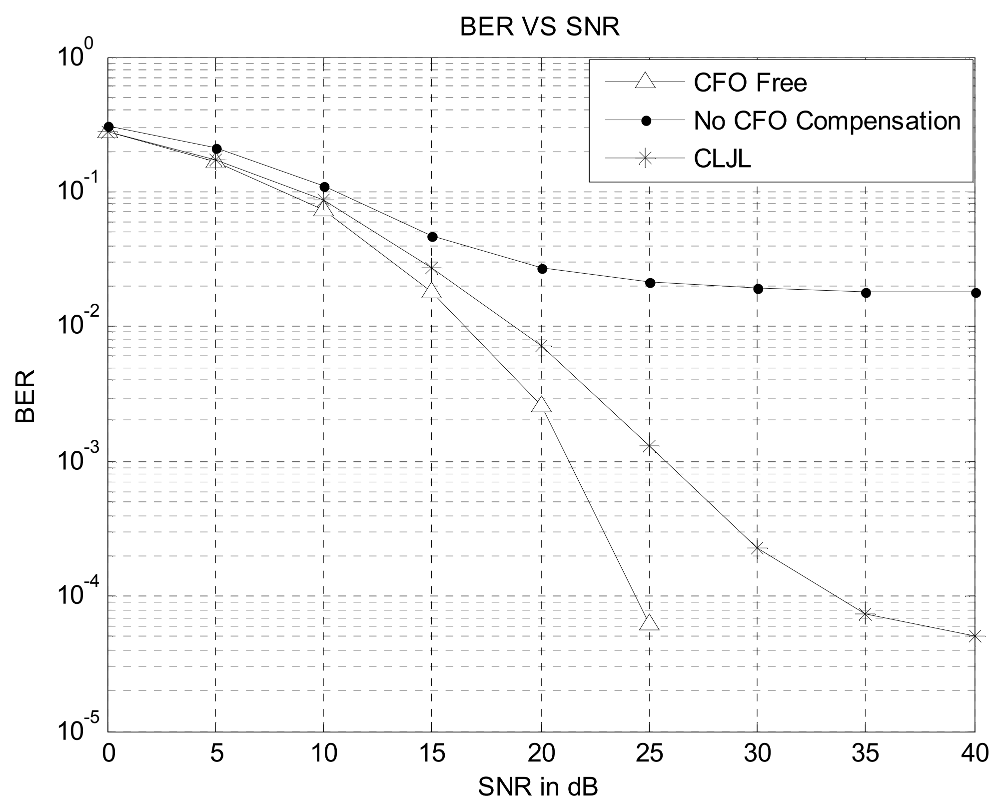

Figure 5.4 BER vs. SNR at Uplink of ODMA by CLJL method.

For the purpose of simulation, the following simulation model is used:

Total number of subcarriers N=64, total numbers of users=4, number of subcarriers allotted to each user=16, ½ rate convolution code and hard decision decoder is used for channel coding, Rayleigh fading channel with three taps and AWGN is used for simulation purpose. CFO values for user 1 to user 4 is [0.1 0.05 -.05 -.1] respectively. CFO values and Channel Impulse Response is completely known to the receiver. Sub-band SAS is used for allocation of resources to different users. Channel Equalization is performed at receiver after detection of the OFDM symbol. To depict the improvement with CLJL algorithm, a simulation was carried for three different cases viz one with no CFO error, with no CFO compensation and the third one CFO compensated using CLJL. It can be seen from Figure (5.4) that CFO compensation using CLJL closely reaches the CFO free condition.

5.4 Frequency Offset Compensation at Uplink of OFDMA by Huang and Letaief (HL) Algorithm

In CLJL the estimated symbol for m^{th} user is given by Equation (5.24), whose second term represents the MUI from other users, which degrades the BER and SINR performance. To compensate MUI induced from other users because of the frequency errors, Huang and Letaeif introduced interference algorithm which cancels MUI in iterative manner [15]. CLJL is particularly suited when sub-band SAS is used, but HL algorithm can be used with any SAS scheme. Let $\hat{X}^m(k)^j$ represent the estimated OFDM symbol for m^{th} user at j^{th} iteration. Estimated $\hat{X}^m(k)$ from CLJL is used as the initial estimate for HL algorithm, hence, at 0^{th} iteration or at initial condition the estimated OFDM symbol for m^{th} user can be given as

$$\hat{X}^m(k)^{j=0} = A^m\left(A^m R(k)\right) \otimes \hat{C}'^{(m)}, \quad \textit{for } m = 1,2 \,..M \tag{5.25}$$

At j^{th} iteration the interference for m^{th} user is regenerated by N point circular convolution of estimated symbol from Equation (5.25) and $\hat{C}^m(k)$, and is canceled from the received signal to get estimated received signal $\tilde{X}^m(k)^j$. The estimated received signal $\tilde{X}^m(k)^j$ after interference cancellation for $j=1, 2, 3 \ldots J$ iterations can be given as

$$\tilde{X}^m(k)^j = R(k) - \sum_{i=1, i\neq m}^{M} \hat{X}^i\left(k\right)^{j-1} \otimes \hat{C}^i\left(k\right), \quad for\ m, i = 1, 2\ ..M \tag{5.26}$$

The $\tilde{X}^m(k)^j$ obtained is free from the MUI caused from other users. After interference cancellation the estimated received OFDM symbol of Equation (5.26) is compensated for CFO similar to CLJL. The estimated OFDM symbol for m^{th} user at j^{th} iteration by HL algorithm can be given as

$$\hat{X}^m(k)^j = A^m\left[\left(A^m \tilde{X}^{m(j)}\right) \otimes \hat{C}'^{(m)}\right] \tag{5.27}$$

the estimate obtained from Equation (5.27) is free from MUI and hence, HL algorithm offers significant advantage over CLJL algorithm in just few iterations.

5.4.1 Simulation of CFO Compensation by HL algorithm

For the purpose of simulation, the following model is used:

Total number of subcarriers N=64, total numbers of users=4, number of subcarriers allotted to each user=16, ½ rate convolution code and hard decision decoder is used for channel coding, Rayleigh fading channel with three taps and AWGN is used for simulation purpose. CFO values for user1 to user 4 is [0.1 0.05 -.05 -.1] respectively. Sub-Band SAS is used for allocation of resources to different users. Channel Equalization is performed after detection of OFDM symbol. To compare the performance improvement of the different algorithms, simulation was carried for three different cases viz one with no CFO error, CFO compensation using CLJL and third one CFO compensated using HL algorithm with an assumption that receiver has complete knowledge about the CFO values and the channel impulse response.

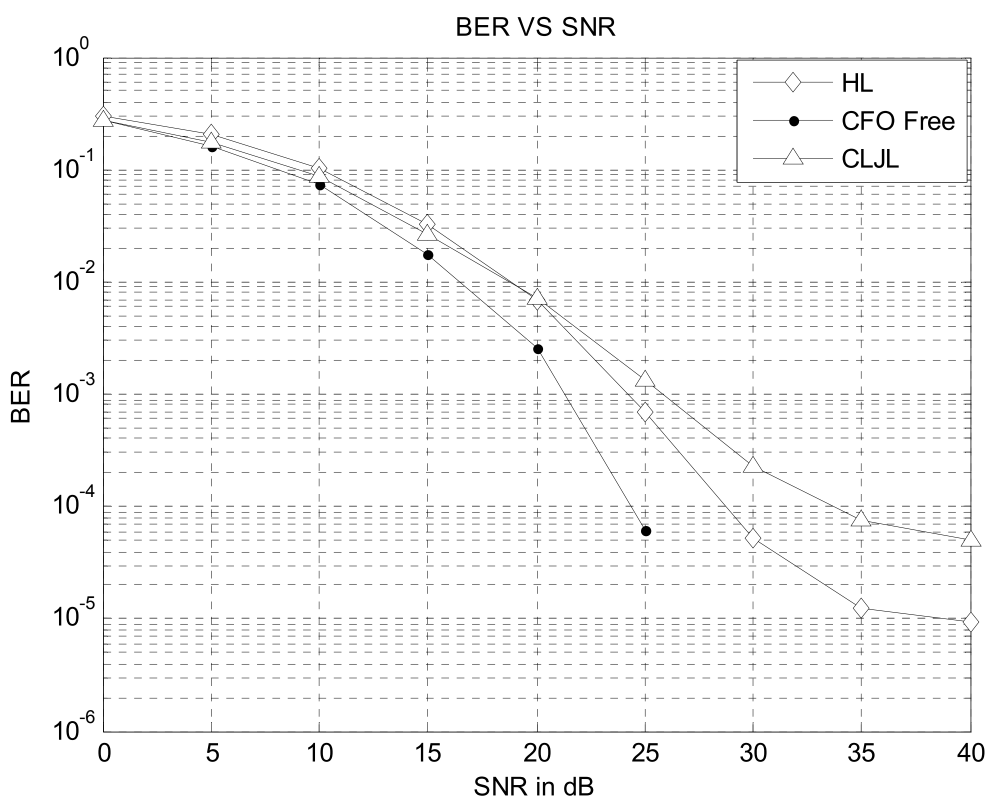

Figure 5.5 BER vs. SNR at uplink of OFDMA by HL algorithm.

It can be seen from Figure (5.5) that substantial improvement in BER performance is obtained by using HL algorithm just with two iterations. More iteration could be used for better accuracy but with an increase in computational load.

5.5 Iterative Interference Cancellation for Uplink of OFDMA

HL algorithm improves the BER performance of the OFDM receiver by canceling the interference in the received OFDM symbol in iterative manner by using multiple circular convolutions. Computing multiple circular convolutions increases the computational complexity in the design of the OFDM receiver. Interference cancellation in received OFDM symbol at Base Station with reduced complexity was introduced in [16]. This algorithm cancels the interference in the received OFDM symbols at uplink in iterative fashion by matrix multiplication. Since, matrix multiplication is easier to compute than circular convolution; the computation complexity of this algorithm is less as compared to the HL algorithm.

Assuming the same uplink model as described in Section (5.1), the received OFDM symbol $y^m(n)$ for m^{th} user can be given by Equation (5.28). Where the CFO for the m^{th} user is ε_m. $X^m(k)$ *and* $H^m(k)$ in Equation (5.29) are the transmitted OFDM symbol and impulse response of the channel for m^{th} user respectively. The equation of the received OFDM symbol in time domain for m^{th} user can be given as

$$y^m(n) = z^m(n)e^{\frac{j2\pi\varepsilon_m n}{N}} + w(n) \ , \ for\, n = 0,1,\ldots,N-1 \qquad (5.28)$$

$$z^m(n) = \frac{1}{N}\sum_{n=0}^{N-1} X^m(k) H^m(k) e^{\frac{j2\pi nk}{N}}, \; for\, n = 0,1,...,N-1 \tag{5.29}$$

At Base Station, superposition of the received OFDM symbols received from different users takes place, hence, the received OFDM symbol at Base Station can be given as

$$r(n) = \sum_{m=1}^{M} y^m(n), \; for\, n = 0,1,...,N-1 \tag{5.30}$$

after removing the CP and taking DFT of the Equation (5.30), the OFDM symbol at k^{th} subcarrier can be given from [16] as

$$R(k) = Z^m(k) I_{k,k} + \sum_{k'=0,k'\neq k}^{N-1} Z^{m'}(k') I_{k,k'} + W(k) \tag{5.31}$$

In Equation (5.31) the first term represents the desired OFDM symbol for m^{th} user, second term represents the MUI induced from other subcarriers due to CFO and third term represents the AWGN noise. $I_{k,k'}$ represents the interference induced from k' subcarrier to k^{th} subcarrier and can be given as

$$I_{k,k'} = \frac{1}{N}\sum_{n=0}^{N-1} e^{j2\pi(k'-k+\varepsilon'_m)n/N} \tag{5.32}$$

where ε_m' *and* k' are the CFO and subcarrier belonging to the set of subcarriers of user m'. When $k = k'$, $I_{k,k}$ represents the fading factor of that subcarrier. To cancel the MUI induced from other users, this algorithm uses MUI cancellation technique in iterative fashion. The Equation (5.31) can be represented in matrix form as follows

$$\mathbf{R} = \mathbf{TX} + \mathbf{W} \tag{5.33}$$

where $\mathbf{R}$ is $[R(0)R(1)\ldots\ldots R(N-1)]^T$, $\mathbf{W}$ $=[W(0)W(1)\ldots\ldots W(N-1)]^T$, $\mathbf{T}$ is $N \times N$ matrix, whose k,k' term is given by $\gamma_{k,k'} = H^{m'}(k')I_{k,k'}$ and $k,k' \in (0,1,\ldots,N-1)$.

$$\mathbf{T} = \begin{bmatrix} 0 & \gamma_{0,1} & \gamma_{0,2} & \cdots & \gamma_{0,N-1} \\ \gamma_{1,0} & 0 & \gamma_{1,2} & \cdots & \gamma_{1,N-1} \\ \gamma_{2,0} & \gamma_{2,1} & 0 & .. & \gamma_{2,N-1} \\ \vdots & \vdots & \vdots & . & \vdots \\ \gamma_{N-1,0} & \gamma_{N-1,1} & \gamma_{N-1,2} & .. & \gamma_{N-1,N-1} \end{bmatrix} \tag{5.34}$$

let $\boldsymbol{V}$ be a diagonal matrix of size $N \times N$ whose k,k entry is $\gamma_{k,k}$

$$\boldsymbol{V} = Diag\left\{\frac{1}{\gamma_{0,0}}\,\frac{1}{\gamma_{1,1}}\ldots\ldots\frac{1}{\gamma_{N-1,N-1}}\right\} \tag{5.35}$$

Let $\hat{X}^{(j)}$ denote the estimated OFDM symbol at j^{th} iteration. At initial condition *j*=0 and estimated OFDM symbols is given as

$$\hat{\mathbf{X}}^{(0)} = \boldsymbol{V}\mathbf{R} \tag{5.36}$$

For j=*1, 2,..,J* iterations the estimated OFDM symbol is given as

$$\hat{\mathbf{X}}^{(j)} = \boldsymbol{V}(\mathbf{R} - \mathbf{T}\hat{\mathbf{X}}^{(j-1)}) \tag{5.37}$$

It can be seen from Equation (5.37) that, to compute the estimated OFDM symbol requires computation of $N \times N$ matrix $\mathbf{T}$, which increases the computational complexity of the OFDMA receiver. In each iteration of Equation (5.37), the numbers of multiplications required are $2(N^2 + N)$. To reduce the computation complexity of this algorithm, modification to this algorithm was proposed in [16]. In the Equation (5.32), the interference fading factor $I_{k,k'}$ decreases as the difference $k' - k$ increases. Hence, when the difference $k' - k$ is greater than certain threshold value effect of fading factor $I_{k,k'}$ on particular subcarrier can be neglected and can be assumed to be zero. This assumption considerably reduces the computational complexity of the receiver at the cost of some degradation in the receiver performance. Let β be the threshold value such that $I_{k,k'} = 0$ for $\beta \geq (k' - k)$. Let T^B be the bounded form of the matrix T such that $I_{k,k'} = 0$, $for\ (k' - k) > \beta$

$$\begin{aligned} [T^B]_{k',k} &= [T]_{k',k} \ , \quad for\ |k' - k| \leq \beta \\ &= 0, \qquad\quad else \end{aligned} \tag{5.38}$$

substituting banded matrix T^B in Equation (5.35) we get,

$$\hat{X}^{(j)} = \vartheta\left(R - T^B \hat{X}^{(j-1)}\right) \tag{5.39}$$

Equation (5.39) represents the estimated value of the OFDM using the approximated matrix T^B. The number of multiplications required by using approximated matrix T^B, in each iteration of Equation (5.39) is $2[(2\beta+1)N - \beta(\beta+1) + N]$ [16] as compared to $2(N^2 + N)$ iterations required in Equation (5.37). Hence, the MUI in the received OFDM symbol can be cancelled with much reduced complexity by using approximated matrix T^B.

5.5.1 Simulation of CFO Compensation for Iterative Interference Cancellation Method

For the purpose of simulation the following model is used:

Total number of subcarriers N=64, total numbers of users=4, number of subcarriers allotted to each user=16, ½ rate convolution code and hard decision decoder is used for channel coding, Rayleigh fading channel with three taps and with AWGN is used for simulation purpose. CFO values for user1 to user 4 are [0.1 0.05 -.05 -.1] respectively. Sub-band SAS is used for allocation resources to different users. CFO values and Channel Impulse Response is completely known to the user. Channel Equalization is performed after OFDM symbol detection. To compute the estimated OFDM symbol and cancel interference from received OFDM symbol total five iterations of Equation (5.37) were performed with an assumption that receiver has complete knowledge about the CFO values and the channel impulse response. Figure (5.6) shows the simulated BER performance of algorithm when bounded matrix T^B is used with threshold of $\beta = 5$ and for the full implementation. From above simulation results it can be concluded that even with threshold $\beta = 5$ algorithm provides acceptable performance with much reduced complexity.

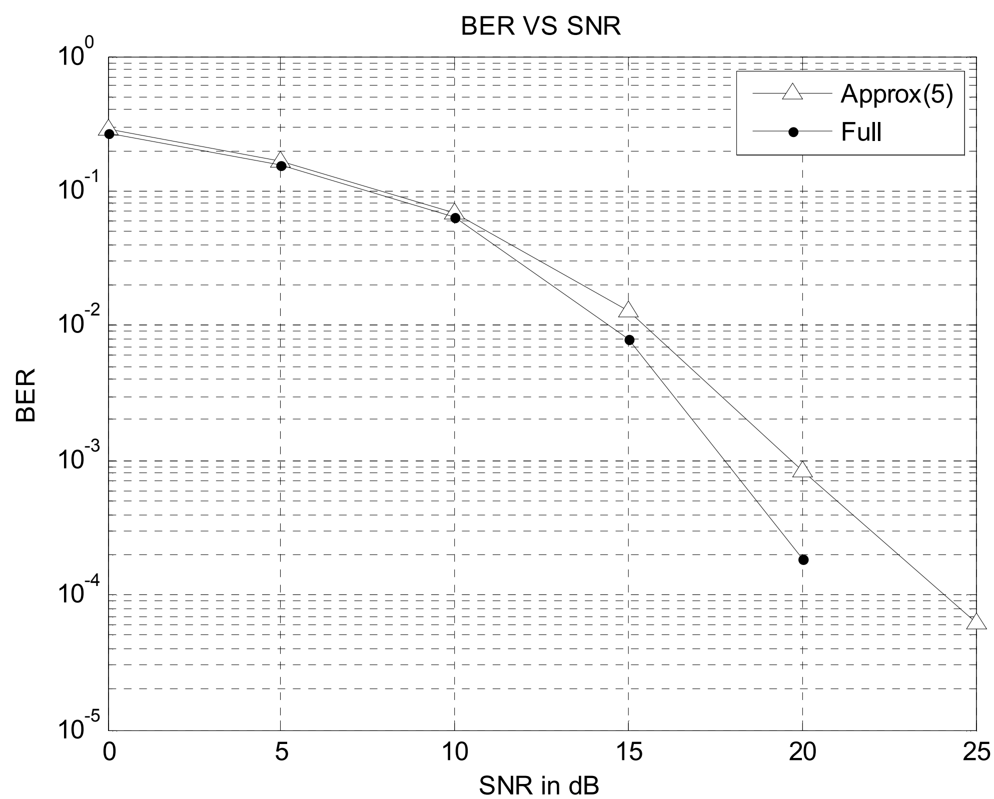

Figure 5.6 BER Vs. SNR for interference cancellation algorithm.

5.6 MUI Suppression by MMSE Equalization for Uplink Of OFDMA

A different approach of suppressing MUI for Uplink of Single Carrier Frequency Division Multiple Access (SC-FDMA) was suggested in [17], by using Minimum Mean Squared Error (MMSE) Feedback Equalizer (FDE) in frequency domain. In [17], CLJL scheme was used for CFO compensation, while MMSE equalization used for MUI suppression. In this Section MUI suppression by MMSE equalization will be presented for OFDM, similar to as suggested in [17] for SC-FDMA.

Assuming the same Uplink model as shown in Section (5.2), then estimated OFDM symbol by CLJL can be given from Equation (5.24) as

$$\hat{X}^m(k) = A^m\left(A^m\left(Z^m(k)\otimes C_k^{(m)}\right)\otimes \hat{C}_k'^{(m)}\right)$$
$$+A^m\left(\left(A^m\sum_{i=1,i\neq m}^{M} Z^i(k)\otimes C_k^{(i)}\right)\otimes \hat{C}_k'^{(m)}\right)$$
$$+A^m\left(\left(A^m W(k)\right)\otimes \hat{C}_k'^{(m)}\right) \tag{5.40}$$

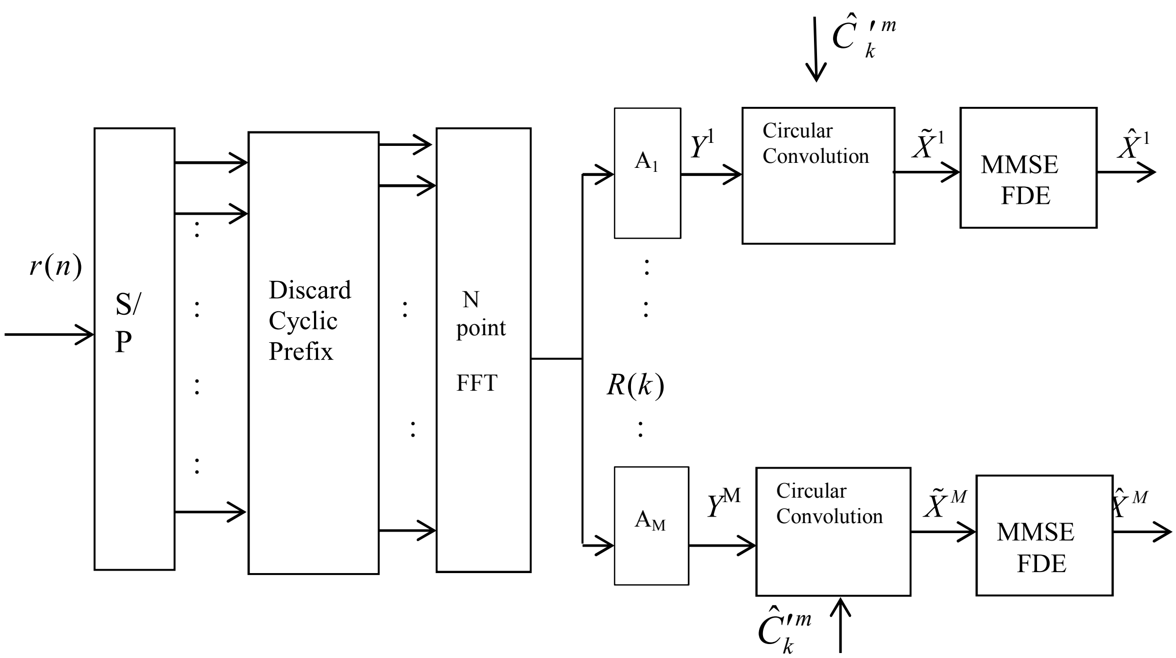

Figure 5.7 MUI suppression by MMSE equalization for uplink of OFDMA.

The first term in the Equation (5.40) is the desired OFDM symbol for m^{th} user, a second term represents the MUI from other users and third term is the AWGN noise. The goal of this scheme is to suppress MUI by MMSE equalization. Figure (5.7) shows model of the Uplink receiver for MUI suppression by MMSE equalization. Based on [17] the effect of Inter Symbol Interference due to multipath is mitigated and MUI is suppressed. It is done by frequency domain equalizer based on MMSE criteria. The k^{th} coefficient of the filter for m^{th} user can be given in [17] by

$$G_k^{(m)} = \frac{H_k^{(m)*}}{\left|H_k^{(m)}\right|^2 + \sum_{i=1, i\neq m}^{M} \sigma^{(m)2}{}_{MUI,i}(k) + \sigma_n^2} \tag{5.41}$$

where σ_n^2 noise is power and $H_k^{(m)}$ is the impulse response of the channel of m^{th} user at k^{th} subcarrier. $\sigma_{MUI,i}^{(m)2}(k)$ is the MUI power from i^{th} user to m^{th} user at k^{th} subcarrier and can be given as

$$\sigma_{MUI,i}^{(m)2}(k) = {}^{1}\!/_{N^4} \sum_{k_i \in \Gamma_i} E\left(\left|Y_{k_i}^{(i)}\right|^2\right) \times \left| \sum_{k_m \in \Gamma_m} \frac{\sin(\pi(k_i + \varepsilon_i - k_m))\sin(\pi(k_m + \varepsilon_m - k)}{\sin(\pi(k_i + \varepsilon_i - k_m)/N)\sin(\pi(k_m + \varepsilon_m - k)/N)} \right| \tag{5.42}$$

where Γ_i, Γ_m represents set of subcarriers belonging to i^{th} and m^{th} user respectively. ε_m represents the CFO of m^{th} user. After MUI suppression**,** estimated OFDM symbol can be given as

$$\hat{X}^m(k) = G^m(k)\tilde{X}^m(k), \quad for\ k \in \Gamma_m \tag{5.43}$$

where $\tilde{X}^m(k)$ is given by Equation (5.40).

5.7 Proposed Low Complexity MUI suppression by MMSE Equalization for Uplink of OFDMA

If N is the total numbers of available subcarriers, M is the total number of users than $P=N/M$ subcarriers are assigned to each users, then for calculating the estimated symbol $\hat{X}^m(k)$ for, $n = 0,1,...N-1$, the number of multiplications required can be approximately given by $N\log_2(N)/2 + N^2/M + 2N(M-1)(P^2+P)+N$. The number of multiplications increases as N increases, increasing the complexity of the system. Computation complexity in computing of matrix can be substantially reduced by applying approximation to calculation of MUI in Equation (5.42). The term $\sigma_{MUI,i}^{(m)2}(k)$ in Equation (5.42) decreases as the difference $k_i - k_m$ or $k_m - k$ increases. Let

$$\gamma_{k_i,k_m} = \frac{\sin(\pi(k_i + \varepsilon_i - k_m))}{\sin(\pi(k_i + \varepsilon_i - k_m)/N)} \tag{5.44}$$

$$\gamma'_{k_m,k} = \frac{\sin(\pi(k_m + \varepsilon_m - k))}{\sin(\pi(k_m + \varepsilon_m - k)/N)} \tag{5.45}$$

substituting value of γ_{k_i,k_m} & $\gamma'_{k_m,k}$ in Equation (5.42) we get

$$\sigma_{MUI,i}^{(m)2}(k) = \frac{1}{N^4} \sum_{k_i \in \Gamma_i} E\left(\left|Y_{k_i}^{(i)}\right|^2\right) \times \left| \sum_{k_m \in \Gamma_m} \gamma_{k_i,k_m} \times \gamma'_{k_m,k} \right|^2 \qquad (5.46)$$

Let η be a design threshold such that γ_{k_i,k_m} can be considered zero for $|k_i - k_m| \geq \eta$ .Let $G_B{}^{(m)}(k)$ be the bounded k^{th} element of MMSE equalizer computed using approximation in calculation of $\sigma_{MUI,i}^{(m)2}(k)$. Hence, the estimated OFDM symbol can be given as

$$\hat{X}^m(k) = G_B{}^m(k)\tilde{X}^m(k), \quad for\ k \in \Gamma_m \qquad (5.47)$$

Approximately, $N\log_2(N)/2 + N^2/M + 2N(M-1)[\tau(\tau+1)+2\tau]/M + N$ number of multiplications is required to compute the estimated symbol $\hat{X}^m(k)$ by Equation (5.47).

For *N*=64, *M*=4, *P*=16 and τ =5, the total number of multiplications required to calculate estimated OFDM symbol by Equation (5.43) are 105728. While, for calculating the estimated symbol by Equation (5.47) are 5120. Thus, to calculate the estimated OFDM symbol by Equation (5.47) are approximately 1/20$^{\text{th}}$ of the number of multiplications required to calculate estimated OFDM symbol by Equation (5.43).

5.7.1 Simulation of MUI Suppression by MMSE Equalization and Proposed Low Complexity MUI Suppression by MMSE Equalization for Uplink of OFDMA

For the purpose of simulation the following model is used:

Total number of subcarriers N=64, total numbers of users=4, number of subcarriers allotted to each user=16, ½ rate convolution code and hard decision decoder is used for channel coding, Rayleigh fading channel with three taps and with AWGN is used for simulation purpose. CFO values for user1 to user 4 are [0.1 0.05 -.05 -.1] respectively. Assuming, receiver has complete knowledge about CFO values and the channel impulse response and sub band SAS is used for subcarrier allocation. The simulation of BER versus SNR for proposed method by assuming $\eta = 5$.

Figure (5.8) shows the simulation results for MMSE equalization using the full implementation of matrix G^m and bounded matrix G_B^m. From the simulation results it can be seen BER curve of bounded matrix G_B^m tends to follow BER curve of matrix G^m expect at higher values of SNR. Thus, proposed scheme reduces the complexity considerably while maintaining affordable system performance.

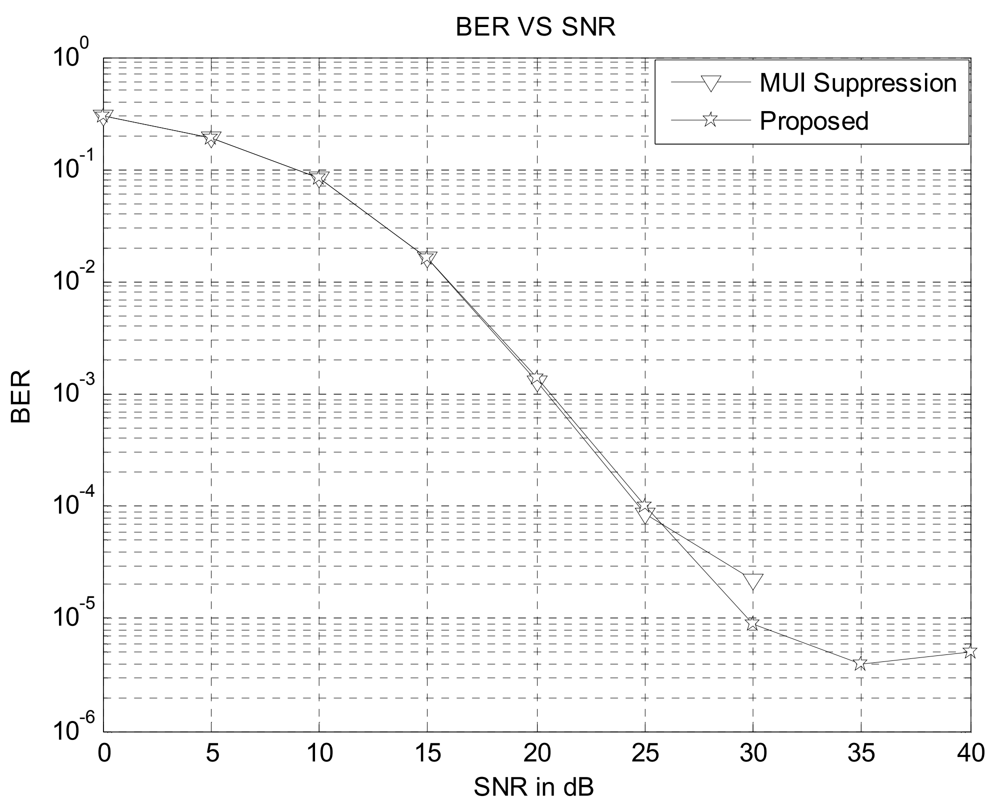

Figure 5.8 BER vs. SNR of MUI suppression by MMSE equalization & proposed method.

5.8 Comparison of Computation Complexities for the Different Methods of CFO Compensation at Uplink of OFDMA

The Table (5.1) below summarizes the number of multiplications required by different methods of CFO compensation for Sub-band SAS. Number of multiplications required for estimating OFDM symbol for all users for Direct Method, CLJL and HL algorithm is given in [18]. Computational complexity of Iterative Interference Cancellation method is given in [17]. Calculated values of required multiplications are based on the assumption $N = 64,\ M = 4,\ P = 16, \tau = \beta = 5, i = 2, j = 5$, where i and j are the number of iterations performed.

Table 5.1 Comparison of complexities for different methods of CFO compensation at uplink of OFDMA

Method for CFO compensation	Number of multiplications required	Calculated Complexity
Direct Method	$\frac{MN\log_2(N)}{2} - \left[\frac{MN\log_2(M)}{2} - \frac{3}{2}(M-1)N\right]$	800
CLJL Algorithm	$N\log_2(N)/2 + N^2/M$	1216
HL Algorithm	$N\log_2(N)/2 + N^2/M + (i-1)(N^2 + N^2/M)$	6336
Iterative Interference Cancellation	$2j[2(\beta+1)N - \beta(\beta+1) + N]$	8020
MUI suppression by MMSE Equalization	$N\log_2(N)/2 + N^2/M$ $+2N(M-1)(P^2+P) + N$	105728
Proposed Method	$N\log_2(N)/2 + N^2/M$ $+2N(M-1)[\tau(\tau+1) + 2\tau]/M + N$	5120

Proposed scheme is the revised version of the MUI suppression scheme presented in [17], however, proposed scheme is more efficient and offers acceptable performance at much reduced complexity. To compare the BER performance of different algorithms presented in this research work simulation was carried out with following assumptions**:** Total number of subcarriers N=64, total numbers of users=4, number of subcarriers allotted to each user=16, ½ rate convolution code and hard decision decoder is used for channel coding, Rayleigh fading channel with three taps and with AWGN is used for simulation purpose. CFO values for user1 to user 4 are [0.1 0.05 -.05 -.1] respectively. Assuming, receiver has complete knowledge about CFO values and the channel impulse response and Sub-band SAS is used for subcarrier allocation. Channel Equalization is performed after OFDM symbol detection for CLJL and HL algorithm.

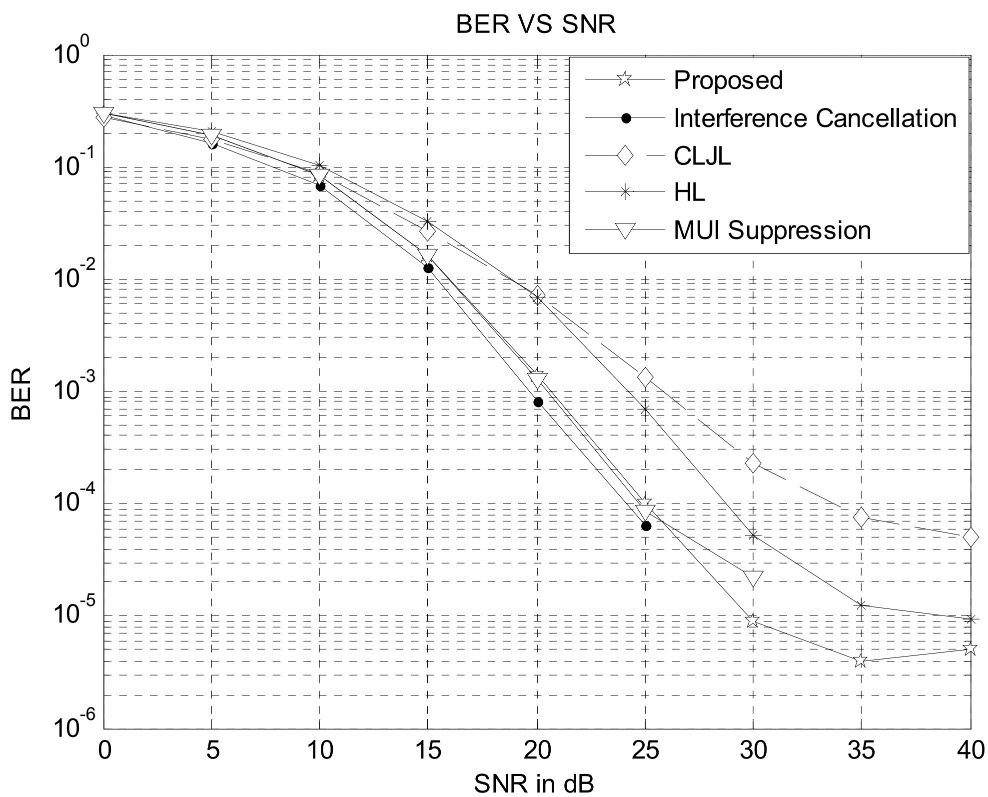

Figure 5.9 BER Vs. SNR for different methods of CFO compensation at uplink of OFDMA.

The simulation of BER versus SNR was done for HL algorithm with two iterations, iterative interference cancellation method for $\beta = 5$ and Proposed Low Complexity MUI Suppression by MMSE Equalization for $\tau = 5$. It can be seen from Figure (5.9) that performance of proposed scheme closely follows MUI suppression in [17]. Performance of proposed method is better than CLJL and HL algorithm, while close to that of iterative interference cancellation approach. Also from the Table (5.1) it can be concluded that computational complexity of proposed scheme is much less than complexity of MUI suppression by MMSE Equalization. Complexity of proposed method is also less than CLJL algorithm, HL algorithm and iterative interference approach to achieve same BER performance. Hence, proposed scheme offers acceptable BER performance without the need to iteratively cancel the interference.

CHAPTER 7

CONCLUSION

OFDM is very attractive for future wireless high data rate multimedia applications because of its high spectral efficiency and robustness against the channels frequency selectivity. However, OFDM is very sensitive to frequency offset and even small values of frequency offset causes severe degradation of the receiver's performance.

Several methods for frequency offset compensation at Uplink of OFDMA were studied in this research, each with its advantages and disadvantages. The CLJL method obtains acceptable performance with simple implementation but it doesn't cancel the MUI from other users. While, HL algorithm gave good performance by iteratively cancelling the MUI from other users, but has high computational complexity due to use multiple circular convolutions. Iterative Interference Cancelation algorithm in [16] canceled the MUI by matrix multiplication in iterative manner, but large number of iterations is required in order to obtain good performance. MUI Suppression by MMSE Equalization for SC-FDMA was presented in [17] which compensates the CFO in frequency domain by using CLJL and suppresses the remaining MUI by MMSE Equalization but it also has high computational complexity. The Proposed Low Complexity MUI Suppression by MMSE Equalization applied approximation to the calculation of MUI power given in [17]. The complexity of the proposed method was shown to be significantly less than that of the CFO compensation method given in [17].

Simulation result confirms that the proposed method obtains good performance, making it suitable for practical implementation.

REFERENCES

[1] H. Sari and G. Karam, “Orthogonal frequency-division multiple access and its application to CATV networks,” *Eur. Trans. Commun.*, vol. 45, pp. 507- 516, Nov.–Dec. 1998.

[2] Interaction Channel for Digital Terrestrial Television (RCT) Incorporating Multiple Access OFDM, *ETSI DVB RCT*, Mar. 2001.

[3] R. W. Chang, “Synthesis of band-limited orthogonal signals for multichannel data transmission”, *Bell System Tech. Journal*., vol. 45, no., pp. 1775-1796, Dec. 1966.

[4] B. R. Saltzberg, “Performance of an efficient parallel data transmission system”, *IEEE Trans. Commun*., vol. 15, no. 6, pp. 805-811, Dec. 1967.

[5] S. B. Weinstein and P. M. Ebert, “Data transmission by frequency division multiplexing using the discrete Fourier transform”, *IEEE Trans. Commun.*, vol. 19, no. 10, pp. 628–634, Oct. 1971.

[6] A. Peled and A. Ruiz , "Frequency domain data transmission using reduced computational complexity algorithms," *IEEE Int. Conf. Acoust., Speech and Signal Processing,*, vol.5, no., pp. 964- 967, Apr. 1980.

[7] P.H. Moose, "A technique for orthogonal frequency division multiplexing frequency offset correction", *IEEE Trans. Commun.*, vol.42, no.10, pp.2908-2914, Oct. 1994.

[8] T. Pollet, M. V. Bladel and M. Moeneclaey, ”BER sensitivity of OFDM to carrier frequency offset and Wiener phase noise,” *IEEE Trans. Commun,* vol. 43, no. 2, pp. 191–193, Feb. 1995.

[9] T. M. Schmidl and D. C. Cox, “Robust frequency and timing synchronization for OFDM,” *IEEE Trans. Commun*., vol. 45, no. 12, pp. 1613–1621, Dec. 1997.

[10] J. Choi, Y. H. Lee, C. Lee and H. W. Jung, "Carrier frequency offset compensation for uplink of OFDM-FDMA systems," *IEEE Int. Conf. on Commun.*, vol.1, no., pp.425-429, 2000.

[11] J. R. Barry, A. E. Lee and D. G. Messerschmitt,” Digital Communication”, *Kluwer Academic Publisher,* third edition, Massachusetts, USA, pp. 230-238, 2004.

[12] Y. S. Cho, J. Kim, W. Y. Yang and C. G. Kang,” MIMO OFDM for Wireless Communication with Matlab”*, John Wiley & Sons (Asia) Pte. Ltd*., Singapore, Chapter 1,4 & 5, 2010.

[13] A. B. Narasimurthy, M. K. Banavar and C. Tependelenlioglu, “OFDM System for Wireless Communication”, *Morgan & Clay Publisher*, *Arizona State University,* pp. 5-10, 2010

[14] M. Morelli, C. C. J. Kuo and M. Pun, “Synchronization Techniques for Orthogonal Frequency Division Multiple Access (OFDMA),” *IEEE Proceed.*, vol. 95, no. 7, Jul. 2007

[15] D. Huang and K. B. Letaief, ”An Interference-Cancellation Scheme for Carrier Frequency Offsets Correction in OFDMA Systems”, *IEEE Trans. on Commun*., vol.53, no.7, pp. 1155- 1165, Jul. 2005.

[16] R. Miao, J. Sun, L. Gui and J. Xiong, "An Iterative Interference Cancellation Approach for OFDMA Uplink System," *Int. Conf. on Commun. and Mobile Compt.*, vol.3, no., pp.541-544, Apr. 2010.

[17] D. Yan, W. Bai, Y. Xiao and S. Li, "Multiuser interference suppression for uplink interleaved FDMA with carrier frequency offset"*, Int. Conf. on Wireless Commun. & Signal Processing*, vol., no., pp.1-5, Nov. 2009

[18] S. Manahor, V. Tikiya, D. Sreedhar and A. Chockalingam, "Cancellation of Multiuser Interference due to Carrier Frequency Offsets in Uplink OFDMA," *IEEE Int.*