Spring 5-31-2010

Power flow in railway electrification power system

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ABSTRACT

POWER FLOW IN
RAILWAY ELELCTRIFICATION POWER SYSTEM

by
Sisi Li

This thesis discusses aspects of the design of the railway electrification system (RES) and in particular the electric distribution system for a 2x25 kV railway power system.

The analysis of Railway Power Supplies (RPS) presents a unique type of load flow problem which is characterized by its constant moving loads, dynamically changing load types (constant current/constant power), and its specialized network structure. The autotransformer power supply system with double track was the system studied in this thesis.

The details of the system studied were provided by Caltrain (California Commuter Rail) for a planned high speed extension to Caltrain system. A more accurate electrical model of such a system than currently available is used. The thesis investigates the use of SimPower toolbox provided by MATLAB/SIMULINK to model and solves the load flow problem in the system under study.

It was found that the RES could be modeled and solved successfully using SimPower. Further, the study paves the way for fault analysis of such a system using SimPower.

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RAILWAY ELECTRIFICATION POWER SYSTEM

by
Sisi Li

A Thesis
Submitted to the Faculty of
New Jersey Institute of Technology
in Partial Fulfillment of the Requirements for the Degree of
Master of Science in Electrical Engineering

Department of Electrical and Computer Engineering

May 2010

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APPROVAL PAGE

POWER FLOW IN
RAILWAY ELECTRIFICATION POWER SYSTEM

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To
My Beloved Grandmother

ACKNOWLEDGMENT

I would like to express my gratitude to all those who helped me during the writing of this thesis. I gratefully acknowledge the help of my supervisor, Dr. Walid Hubbi, who has offered me valuable suggestions. During the preparation of the thesis, he spent much time reading through each draft and provided me with inspiring advice. Without his patient instruction, insightful criticism and expert guidance, the completion of this thesis would not have been possible.

Secondly, special thanks should go to my friends who gave me their help and time in listening to me and helping me work out my problems during the difficult course of the thesis.

Finally, I am indebted to my beloved parents for their continuous support and encouragement.

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LIST OF SYMBOLS

The following list of technical terms and acronyms is used in the industry.

ACSR   Aluminum Conductor Steel Reinforced

AEC   Aerial Earth (Ground) Conductor, (Also
                                    Called Earth Wire or Protective Wire)

AT   Autotransformer

BEC   Buried Earth (Ground) Conductor

CW   Contact Wire

HV   High Voltage

IMP   Impedance Bond

LV   Low Voltage

MW   Messenger Wire

NF   Negative Feeder

OCS   Overhead Contact System (Also Called
                                    Overhead Catenary System)

PS   Paralleling Station (with Autotransformer)

SWS   Switching Station (with Autotransformers)

TPSS   Traction Power Supply System

The following symbols are used in this thesis. Suffix 1 refers to track 1 and suffix 2 refers to track 2. Subscripts are indicated by an underscore, for example I_C is typed as I_C. Superscript are indicated by a "^", for example I^2 is typed as I^2. This notation is commonly used in Matlab programming.

w.r.t	With Respect to
RES	Railway Electrification System
SS	Substation
AT	Autotransformer
M	Messenger Wire
C	Contact Wire
F	Feeder Wire
C	Catenary System Composed of C1, M1,C2,M2
F	Feeder System Composed of F1 and F2
Ri	Inside Rail (Closer to the Center of the System)
Ro	Outside Rail (Farthest from the Center of the System)
S	Static Wire (or Earth Wire)
R	Equivalent Grounded Wire (and Rails) Composed of Ri1, Ro1, Ri2, Ro2, S1, and S2.
I_C	Catenary System Current
I_R	The Ground Current (This is the Sum of the Currents in the Four Rails and the Two Earth Wires
I_F	The Negative Feeder Current

V_o	The Utility Voltage
V_1	The Voltage across the Primary Winding of the Substation Transformer
V_C	The Catenary Voltage w.r.t Remote Earth
V_R	Rail Voltage w.r.t. Remote Earth
V_F	Negative Feeder Voltage w.r.t. Remote Earth
Z_o	The Short Circuit Impedance of the Grid
Z_1	The Primary Winding Leakage Impedance
Z_2	The Secondary Winding Leakage Impedance
A	Transformation Ratio
GMR	Geometric Mean Radius

CHAPTER 1

INTRODUCTION

The objective of this thesis is to solve the load flow problem in an electric railway traction power system using the power system toolbox of Matlab and to verify the obtained solution.

In this introductory chapter, general background information about railway transportation is given in Section 1.1; this is followed by overview of the electrification systems of railways in Section 1.2. An overview of the design process of railway electrification systems (RES) is given in Section 1.3 in order to see where the problem addressed in this thesis fits in. The outline of the thesis is presented in Section 1.4.

1.1   Background

In this section, the advantages and importance of railway transportation will be presented briefly. This is followed by some characteristics of railway electrification systems (RES).

Railway transportation of people and cargo has many advantages compared with other modes of land/air transportation. Among these advantages: better control on schedule and travel time, lower exposure to risk factors such as heavy traffic and road accidents, more comfortable seating, contribution to environment and economy. Another advantage of electrified railway system is that the needed energy is delivered to the train when needed, unlike the other modes of transportation (land, air, sea) where the source of energy must be carried by the vehicle for the duration of the journey.

Recently, the federal government allocated $8 billion of the American Recovery and Reinvestment funding for high-speed intercity rail. Part of this amount is awarded to

California which is planning a high-speed train system capable of speeds of more than 200 miles per hour. When the full system is built it will stretch close to 800 miles, from Sacramento to San Diego. This is an indication of the increased awareness of the efficiency of railway transportation. A recent CSX advertisement claimed that their trains are able to move 1 ton of freight 423 miles on 1 gallon of fuel. This is 3 to 4 times more efficient than trucks. A further increase in efficiency is provided if electric traction is used. Because of that, over 50% of all rail transport was carried by electric traction in 2006. Railway tracks that were not electrified were electrified in many countries. For example, in France, at the end of 1992, 12,869 km of track had been electrified [1].

The supply voltage of RES can be DC or AC. The standard DC voltages vary between 600 V to 3000 V. The standard AC voltages vary between 11 kV and 25 kV, although older systems may have a voltage below that range. The frequencies found are 16 2/3, 25, 50, and 60 Hz. Although other types of tractions are possible (steam, diesel...), electric railway traction appears superior in many respects.

A great deal of interesting information can be found in Wikipedia.org under "railway Electrification Systems" [2].

1.2   Railway Transportation Systems

The two most common Electrification Power Supply Systems for High Speed Rail and Commuter Rail Lines are:

1. 1 x 25 kV Systems

2. 2 x 25 kV Systems

Both Systems receive power from High Voltage, 60 Hertz (in the USA), 3 Phase Utility Circuits and both Systems provide 25kV, 60 Hertz, Single Phase Power to the Overhead Catenary System. 1 x 25 kV Systems utilizes Traction Power Supply Stations with Standard Power Supply Transformers which provide Power at 25kV to the Overhead Contact System as shown in Figure 1.1.
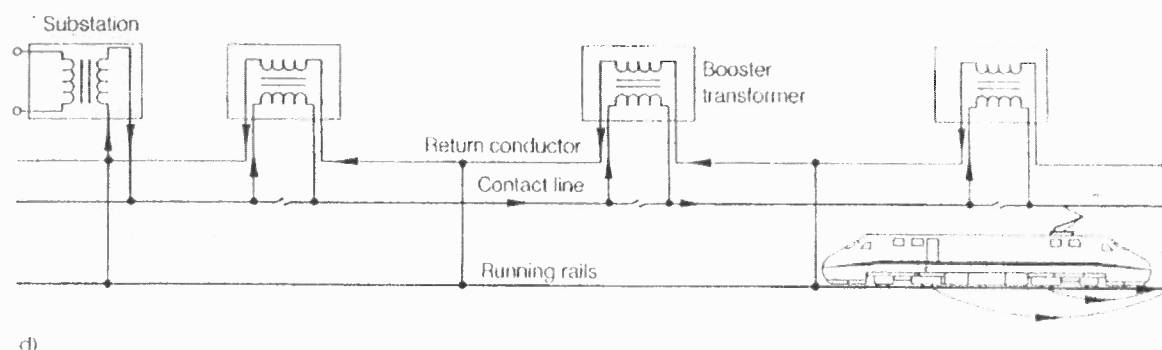
Figure 1.1 Diagram of a 1x25 kV system with Booster transformers.

Source: F. Kiessling, R. Piff, A. Schmieder, and E. Scheneider, "AC 25kV 50 Hz traction power supply of the Madrid-Seville line" in *Contact Lines for Electrical Railways: Planning-Design-Implementation-Maintenance*, SIEMENS, August 2009.

The 2 x 25kV Systems utilize Traction Power Supply Stations, Switching Stations and Paralleling Stations with Autotransformers, which provide 25kV to the Overhead Contact System and -25kV along Track Negative Feeders as shown in Figure 1.2.

To ensure that power is not lost to an entire Traction Power Supply System (TPSS) as a result of a single contingency occurrence both Systems utilize redundant Utility Power Supply Circuits and redundant HV Transformers.
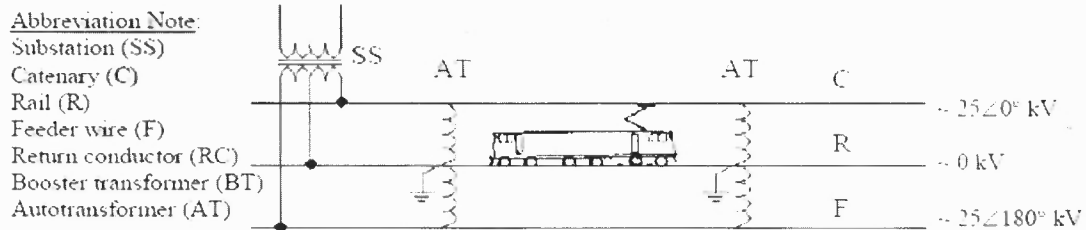
Figure 1.2 Schematic of a 2x25 kV RES.

Source: T. Kulworawanichpong, "Optimizing AC Electric Railway Power Flows with Power Electronics Control," Ph.D thesis, University of Birmingham, November 2003.

The advantage of the 2x25 system over the 1x25 system, especially for high speed trains is well documented. Among other, the New Haven-Boston line is electrified with a 2x25 kV system. Both systems use single phase AC provided by the three-phase power grid. In order to keep the power grid as balanced as possible, different sections of the RES are supplied from different phases of the grid.

1.3   Designing a Railway Electrification System (RES)

The RES is a subsystem of the overall railway system and cannot be designed in vacuum. Therefore, to specify the requirements of the RES, the starting point is the physical layout of the route, the location of the passenger stations, the topography of the track route including the curves to be encountered and their physical characteristics. This information together with the required acceleration and speed of the train as a function of position along the track can be used, (using Newtonian mechanic), to determine the tractive effort requirement of the train as a function of position. Depending on the type of motor used, the motor drive, and efficiency, the electric power demand by the train as a function of position can be calculated using the tractive effort and speed (power = tractive effort x speed). Next, different operation scenarios are assumed starting with the number of trains per hour to run,

in what station a train should stop, what are the speed limits at various lengths of the route, the required acceleration and deceleration, etc. All this is to determine where the trains are on the tracks at any instant of time. Having those snapshots, the engineer determines the power requirements at the different points on the tracks (where the trains are). This information provides the interface between the overall system and the RES (the power system providing electric energy to the trains).The specifications of the electrification system constitute a subset of the overall specifications, for example, voltage specifications are specified by international standards such as BS EN 50163 and IEC 60850. These take into account the number of trains drawing current and their distance from the substation. For example: Nominal System Voltage (25.0 kV), Maximum Long Term Voltage (27.5 kV), Maximum Short Term Voltage (29.0 kV), Minimum Long Term Voltage (19.0 kV), Minimum Short Term Voltage (17.5 kV), Minimum Supply Voltage for Utility Circuits (115 kV at SS) among many others. The numbers in parenthesis are taken from the European Standard. However, this thesis deal with a sub-problem of the electrification system and the following paragraph is pertinent to the LF problem.

Designing an RES is done in several stages. In the first stage the following information is generated:

1. Locations and speeds of trains: This is based on design specification (speed, headway...), topography, and type of trains.

2. Power requirements: The power requirement of each train on each track and the location of each train is calculated using the speed and the traction effort. This calculation is usually done using software that takes the slope of the track, the acceleration of the train, etc., into consideration when using Newton's laws of motion.

3. Certain power system configuration and wire sizes are selected. The wires are: The messenger, the catenary, the static, and the feeder wires; one for each track. In some literature, the messenger wire and the catenary wire are called the catenary system or the Overhead Catenary System (OCS). The rails are used as return

conductors also. The configuration of the system specifies the coordinates of each conductor (horizontal and vertical distances, usually from the center of the track) and also what type of conductors are to be used.

The second stage receives the train locations, the power requirements of each train and the system data. The output of this stage is the catenary voltage at the location of every train, the currents in the different conductors, and the rail voltage. This last item is important for public safety consideration and usually the acceptable limits are specified by regulation.

Having the output of the second stage, the designer verifies that no line is overloaded, the rail voltage is within specifications, and the train's powers requirements are met. If any of these requirements is found to be violated, the design is modified and the calculations in the second stage repeated.

The second stage, therefore, deals with power network calculations and it is the subject of this thesis.

1.4   Thesis Outline

In Chapter 2 an overview of the load-flow problem in RES is presented. A typical system configuration is given followed by the problem statement and a survey of methods presented in the literature to solve the problem. In Chapter 3, models of the different system components are presented using typical data. These components are: 1-the substation transformer, 2- the Autotransformer, 3- the line model 4- and the train. The model of the lines is based on Carson's model and the theory is complicated and important. Part of the thesis contribution is related to the line model; therefore, Chapter 4 is devoted to

this model. Chapter 5 presents the developed methodology to solve the problem. Chapter 6 presents results and conclusions.

CHAPTER 2

THE LOAD FLOW PROBLEM IN RAILWAY TRACTION SYSTEMS

In this chapter a typical RES configuration that will be used in the rest of the thesis is given. This followed by the statement of the load-flow problem in RES. Finally, reviews of current methods to solve the problem are summarized.

2.1   System Configuration

A typical cross section of a two tracks system is shown in Figure 2.1.
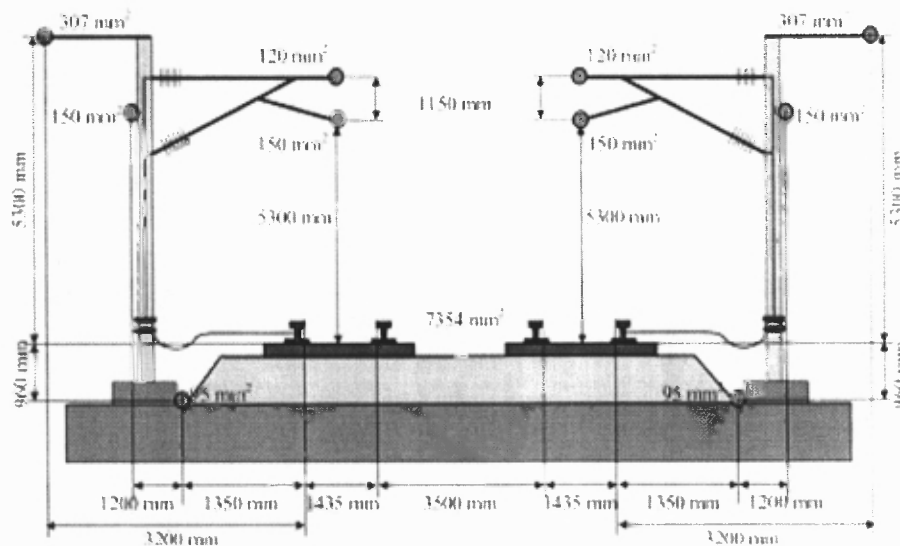
Figure 2.1 Typical arrangement of AT double track European system.

Figure 2.2 shows a similar arrangement of the wires in an RES with dimensions in British system of units. The wires and the rails of one track only are shown. The other track is symmetrical. Typical x and y coordinates of the different wires are shown in the diagram

and in Table 2.1. A convenient origin of the coordinates is the center point between the two tracks. This system will be used in this thesis to test the proposed method.
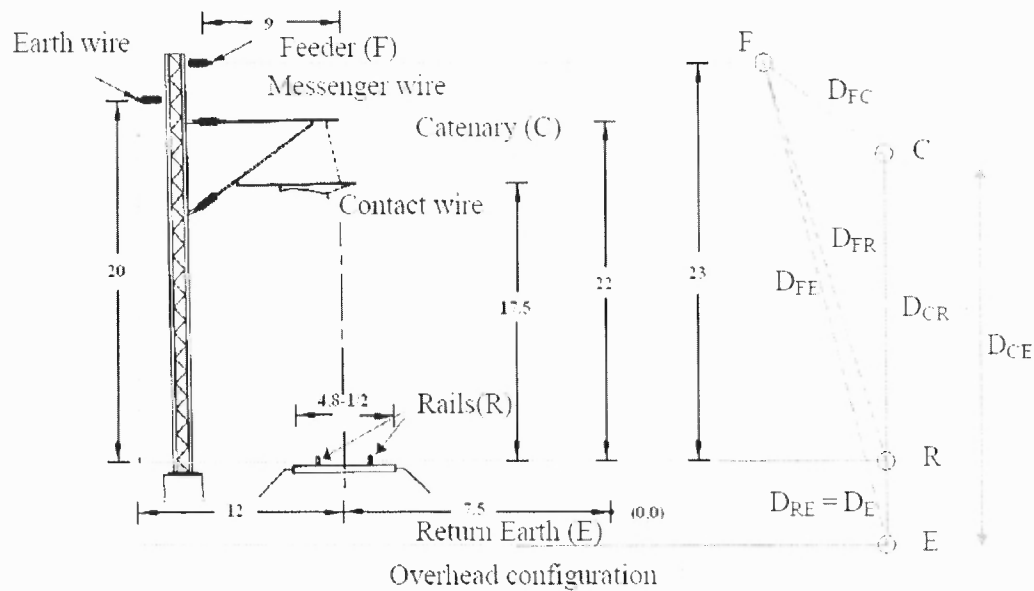
Figure 2.2 Wire configuration in the system under study.

Source: T. Kulworawanichpong, "Optimizing AC Electric Railway Power Flows with Power Electronics Control," Ph.D thesis, University of Birmingham, November 2003.

The distance between the ATs is usually around 4 miles. The catenary system (C) in the Figure 2.2 represents the *messenger* wire and the *contact* wire. The contact wire (the wire actually touching the pantograph) should be as horizontal as possible so that the contact pressure between the contact wire and the pantograph is as uniform as possible. However, a wire supported at the two ends on poles will assume a catenary (hyperpolic function) shape. Hence, a wire, called the messenger wire is supported at the two ends at the poles and the contact wire is attached to the messenger wire by hangers of varying lengths to keep the contact wire as horizontal as possible.

In each cross section there are twelve wires including the four rails (R), messenger wires (M), feeder wires (F) and Static wires (S). These are connected at the ATs to form 3 sets of wires as follows (the suffix 1 refers to track 1 and 2 refer to track 2):

C: composed of C1, M1, C2, and M2.

F: composed of F1 and F2.

R: composed of Ri1, Ro1, Ri2, Ro2, S1, and S2.

In a real system, there are many ATs and some sections between ATs do not have any trains running in them.

The following data will be used in this thesis (the British System of Units will be adhered to as is common in the USA in the railroad industry):

Average distance between centerline of two tracks = 15 feet

Gauge = 4 feet 8-1/2 inches (standard gauge)

Contact wire height above rail level at supports = 17.5 feet

Messenger wire height above rail level at support = 22 feet

Both contact and messenger wires are assumed to be located vertically above the track center.

Negative feeder height above rail level = 23 feet

Negative feeder offset with respect to track center (towards outside) = 9 feet

Static (Aerial) ground wire height above rail level = 20 feet

Static (Aerial) ground wire offset with respect to track center (towards outside) = 12 feet.

Table 2.1 Coordinates of the Different Conductors in the RES of Figure 2.2

Points	(x,y)
Earth wire	(-19.5,20)
Feeder	(-16.5,23)
The center of track	(-7.5,0)
Contact wire	(-7.5,17.5)
Messenger wire	(-7.5,22)
Outside of track	(-9.8542,0)
Inside of track	(-5.1468,0)

In addition to the infrastructure required for power transmission, electrical signals must be transmitted to convey information in real time about the position of the trains (in what section) to avoid collisions and to control traffic light at rail crossings. To accomplish this task and to distinguish between sections of the rails, the rail sections must be isolated for signaling purposes but must be continuous for power purposes. Therefore, circuits for signals' transmission must be considered when designing the power circuits. In order to distribute the rail currents more evenly so that the rail voltage (step voltage and touch voltage) is reduced to safe levels (as required by regulations) the tracks are bonded at regular intervals. This cross-bonding, commonly called "impedance bonding" may interfere with signal circuits and must be done in consultation with the signal engineers. Impedance bonds are earthed connections of two earth wires and two rails. Rails can be connected also to the static wires (earth wires) to absorb the current flowing in the rail. Figure 2.3 shows the earth wires bonded to the rails.
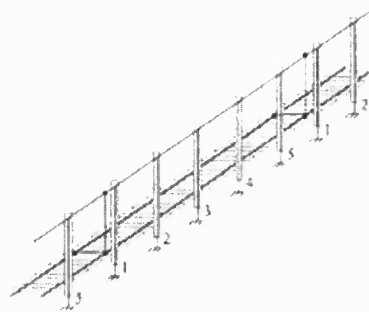
Figure 2.3 Connection of the earth wire to the rails.

Source: B. Hemmer, A. Mariscotti, and D. Wuergler, "Recommendations for the Calculation of the Total Disturbing Return Current From Electric Traction Vehicles," *IEEE Transactions on Power Delivery,* vol. 19, no. 3, July 2004.
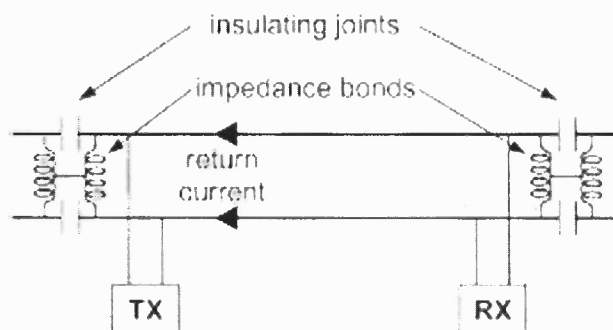
Figure 2.4 Impedance bonds provide continuity for the rail currents while insulating joints isolate track signals between sections.

Source: A. Mariscotti, P. Pozzobon, and M. Vanti, "Distribution of the Traction Return Current in AT Electric Railway Systems," *IEEE Transactions on Power Delivery,* vol. 20, no. 3, July 2005.

2.2   The Load Flow Problem in RES

The Load-Flow (LF) problem in *power systems* is well defined and many successful methods to solve it have been proposed and are in use today by the utility industry. Solving this problem is very important because it must be solved almost continuously by the power systems engineers for secure and economic operation of the power system. A similar problem in RES exists, but it is solved mostly at the design stage of the railway power

system. Unlike the power system load, the load in the RES is predictable and controllable. This load consists of the power requirements of the trains on the tracks. The number of trains and their locations can be controlled. Hence the RES is designed for maximum load and solving the LF problem in an RES is done thoroughly at the design stage before the system is built. Solving the LF problem in a power system is necessary for design and operational purposes; while solving the similar problem in RES is done for design purposes. The goal is to ensure electrical compatibility of the whole electrical system with the signaling systems with respect to the normative standards and operator's regulations [4]. Another difference between the LF in a power system and the LF in an RES is the layout of the two systems. The power system consists of a large number of nodes connected by a large number of lines to form many loops. The RES usually consists of two tracks in parallel supplied by the equivalent of one power line. The number and locations of the train is variable. Therefore, when designing an RES, a worst condition has to be studied.

The input data for an RES load-flow study are:

The system configuration, this includes the conductors' arrangement and the type of conductors to obtain the GMR and the resistance as explained in section 2.1.

The trains positions (on what track and the distance on the track from a reference point) and power requirements.

The cross-bonding locations and if they are earthed or not.

Earth resistivity

The output data are:

The voltage profile, this would include: the catenary voltage applied to every train on the tracks, the rail voltage as a function of distance, the voltage variations with distance for every conductor.

The current distribution in every conductor.

The output can be used for sizing an adequate traction power distribution system and for predicting how the overall system will perform.

2.3   Review of Current Methods

Methods have been developed with emphasis on different aspects of the design process. Some of the papers reviewed, although not totally devoted to the problem at hand, are alluded to in this section to present related problems. In [5] the AT is assumed ideal with "infinite transformer magnetizing inductance, no iron loss, and zero leakage and winding impedance." It is further assumed that there is no admittance between any two components of the RES. With these assumptions, the rail currents will be confined to the occupied section. Studying one section and only one train in that section, simplifies the study greatly. Once the currents are found, the variation of the voltages along the three sets of conductors (catenary, rail, feeder) is a linear function of the distance from the point where the voltage is known. The authors simulated operation under constant current and under constant power motor control. In [6] Bernhard Hemmer, Andrea Mariscotti, and Dieter Wuergler] the interference problem is studied in detail "where the locomotive is the source of the disturbances, the signaling equipment the potential victim and line and track act as the mean of propagation." The whole line model was validated "by means of a measurement campaign on a double track line section (5 km long) of the Italian railway network." The

fact that measurements were done on a 5 km track to validate a model with an estimated cost in the hundreds of k$, illustrates the importance of the problem.

Similar to the LF problem in power systems, the RES load flow problem is not linear. After modeling all the components in the system, the problem can be linearized by converting the power requirements of the trains into currents requirements. That can be done if the train voltage is known or assumed. The flow chart in Figure 2.5 summarizes the method.
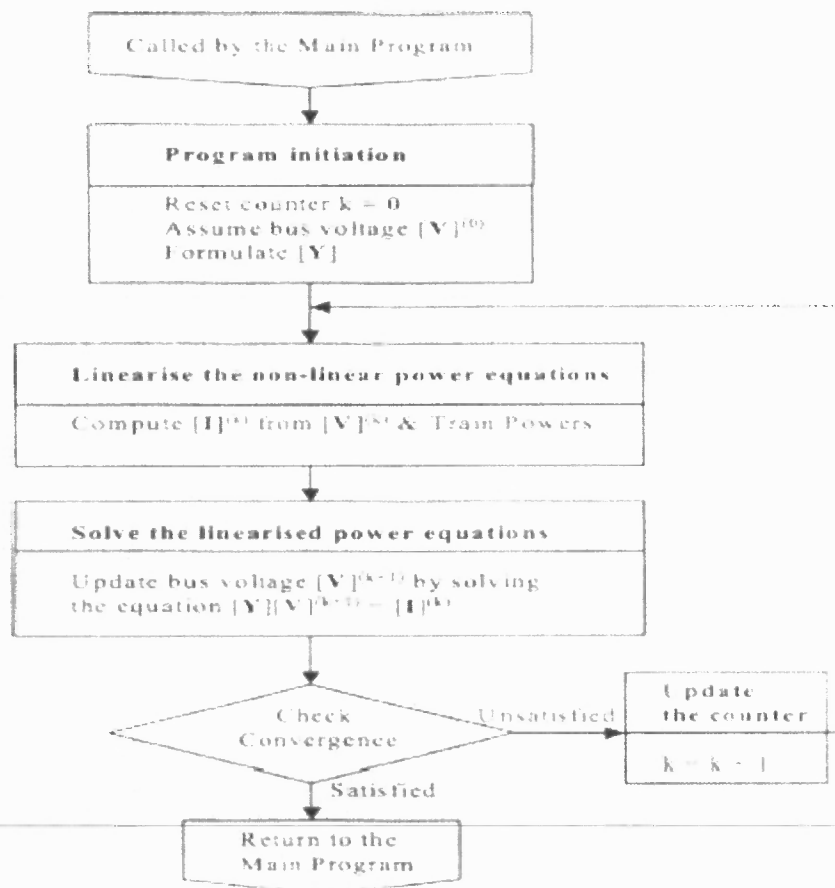
Figure 2.5 Flow chart for solving the LF problem in RES.

Source: C. J. Goodman "Modeling and Simulation," Professional Development Course on *Railway Electrification Infrastructure and Systems*, 2009.

Industry relies on transportation computer models, an example is the "Transportation System Energy Management Model (EMM)" computer program developed at Carnegie Mellon University in 1985 with support from the FTA (Federal Transportation Agency). A description of this software can be found at http://home.comcast.net/~rail_systems_center/tom.htm. Most of these models have two major components: 1- the Train Performance Simulator (TPS) simulates the operation of a single vehicle on a given route, and 2- the Electrical Network Simulator (ENS) simulates the electrical network with the operation of several vehicles on the entire system. Some of these programs give results that differ from actual field tests by about 10% [11]. The EMM is widely used in the USA and it uses FORTRAN as the programming language. The code is proprietary.

Ho, Chi, Wang, and Leung [13] proposed a probabilistic load flow analysis on AC electrified railways. The authors used the train position as the probabilistic variable that defined a pdf of the power demand of the trains.

CHAPTER 3

MODELLING OF COMPONENTS

In this chapter, the modelings of components are analyzed.

3.1 Substation Model

3.1.1 Description

A typical AC railway power feeding system receives the electric energy at the substation. For technical reasons, like feeding reliability, protection, rotation of phases, and so on, any feeding section is isolated from the others and supplied with only one power substation. A feeding section is about 25 miles long. Conventionally, the power substation is modeled by a combination of an infinite busbar (ideal voltage source) in series with equivalent high-voltage grid impedance connected to a feeder transformer with on-load tap changer. The tap changer is assumed set to give the nominal voltage ratio.

In a 1x25 kV system, the model of the feeding transformer is straightforward and is as shown in Figure 3.1. The excitation admittance is usually neglected.
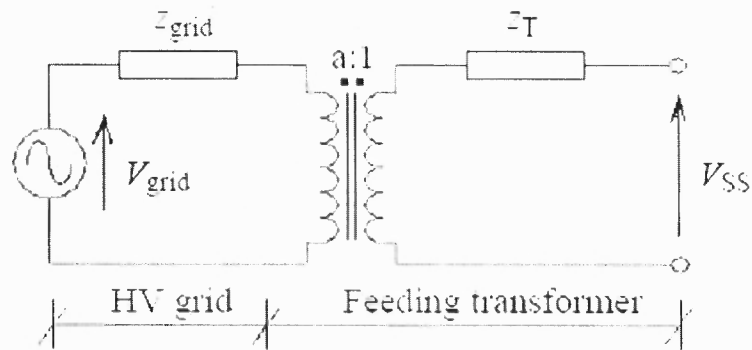
Figure 3.1 Usual model of a feeding transformer used in a single phase RES.

Source: T. Kulworawanichpong, "Optimizing AC Electric Railway Power Flows with Power Electronics Control," Ph.D thesis, University of Birmingham, November 2003.

The model of the feeding transformer in the 2x25 kV systems under study is as shown in Figure 3.2. The secondary is center-tapped, with the center solidly grounded. An earthing resistance is included in the model to simulate the effectiveness of the earthing system. The voltage of the catenary is about 25 kV, and that of the feeder is about -25 kV with respect to the center tap (w.r.t.). The reference direction for the currents is positive if the currents are leaving the wire on the secondary side, and entering the positive terminal on the primary side. The three currents leaving the secondary wires are: the catenary current, the rail current, and the feeder current, I_C, I_R, and I_F respectively.

a) Power substation   b) Circuit model
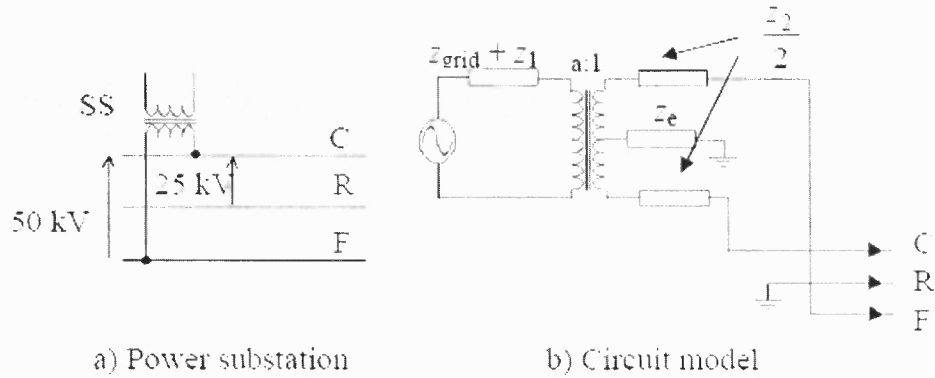
Figure 3.2 Model of a 2x25 kV feeding transformer, Ze is the earthing resistance, Z2 is the total leakage impedance of the secondary, Z1 is the primary leakage impedance.

The parameters of the equivalent circuit can be calculated from the nameplate data of the transformer. Obtaining a model of the substation transformer means finding the relationship between V_C, V_R, and V_F on the one hand and I_C, I_R, and I_F on the other. The key point in this derivation is that since the excitation admittance is neglected, the total magentomotive force (mmf) is equal to zero which gives equation (3.1).

$$I_1 = \frac{1}{2a}(I_F - I_C) \tag{3.1}$$

$$I_F + I_C + I_R = 0 \tag{3.2}$$

Let E be the voltage across one half of the secondary winding with the secondary open circuited. Then:

$$E = V_1 / (2a) \tag{3.3}$$

$$V_1 = V_C - I_1(z_1) = V_C - \frac{1}{2a}(I_F - I_C)(z_1) \tag{3.4}$$

$$V_R = -Z_e(I_C + I_R + I_F) \tag{3.5}$$

$$V_C = -V_R + E - I_C \times Z_2 / 2 \tag{3.6}$$

$$V_F = -V_R - E - I_F \times Z_2 / 2 \tag{3.7}$$

Let $Z_1' = Z_1 /(2a)^2$, the following can be written from equations (3.1) to (3.7)

$$\begin{bmatrix} V_C \\ V_R \\ V_F \end{bmatrix} = \frac{V_0}{2a} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + \begin{bmatrix} -\dfrac{Z_2}{2} + Z_e + Z_1' & Z_e & Z_e - Z_1' \\ -Z_e & -Z_e & -Z_e \\ Z_e - Z_1' & Z_e & -\dfrac{Z_2}{2} + Z_e + Z_1' \end{bmatrix} \begin{bmatrix} I_C \\ I_R \\ I_F \end{bmatrix} \tag{3.8}$$

This can be written as:

$$\begin{bmatrix} V_C \\ V_R \\ V_F \end{bmatrix} = \frac{V_0}{2a} \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + T \begin{bmatrix} I_C \\ I_R \\ I_F \end{bmatrix} \tag{3.9}$$

The inverse of T can be found using the Matlab "syms" command:

$$T^{-1} = \frac{-2}{z_2} \begin{bmatrix} \dfrac{Z_1' - Z_2/2}{2Z_1' - Z_2/2} & 1 & \dfrac{Z_1'}{2Z_1' - Z_2/2} \\ -1 & -2 + \dfrac{Z_2}{2Z_e} & -1 \\ \dfrac{Z_1'}{2Z_1' - Z_2/2} & 1 & \dfrac{Z_1' - Z_2/2}{2Z_1' - Z_2/2} \end{bmatrix} \tag{3.10}$$

Premultiplying equation (3.8) by T^{-1}, the current vector can be obtained in terms of the voltage vector.

3.1.2 Results

The short circuit impedance of the grid is assumed zero, this is equivalent to assuming the primary voltage of the substation transformer an infinite bus with a voltage of 115 kV. The no-load secondary voltage between the OCS and feeder is 52.5 with a grounded center tap.

The nameplate data of the substation transformer are assumed to be: 115/52.5 kV, 60 MVA, X/R ratio = 10. The base impedance in the secondary side is taken as 52.5^2/60=45.94 Ohms and therefore, the TPSS impedance is 0.1x45.94 = 4.6 Ohm with an X/R ratio of 10, this gives Zeq = 0.457 + j4.57 Ohms referred to the secondary. Half or Zeq will be assumed primary impedance and half will be assumed the impedance of the secondary, Z_2. Referring the primary impedance to the primary side and assuming an earthing resistance of 0.1Ohms, gives the following values (in Matlab format):

z2=0.228 + 2.286i ; z1 = 0.914 + 9.140i; z0=0;Ze=0.1; a = 2.1905; v0 = 115000.

A Matlab script is written to give i_source and y_station such that:

$$\begin{bmatrix} I_C \\ I_R \\ I_F \end{bmatrix} = i_{source} + y_{station} \begin{bmatrix} V_C \\ V_R \\ V_F \end{bmatrix} \tag{3.11}$$

[i_source,y_station]=station_model_re(a,z0,z1,z2,ze,Ze,v0)

```
z1prime = (z0+z1)/(4*a^2);Re=-Re;
Z_matrix=[(Re-z2/2+z1prime)  Re  Re-z1prime
Re          Re-ze      Re
Re-z1prime      Re      (Re-z2/2+z1prime)];
i_source=[v0/(2*a) 0  -v0/(2*a)]';
i_source=-inv(Z_matrix)*i_source;
y_station=-inv(Z_matrix);
```

The obtained results are:
i_source =

1.0e+005 *
0.1343 - 1.3642i

0.0000
-0.1343 + 1.3642i

>> y_station

y_station =

0.2990 - 3.0316i -0.0864 + 0.8663i -0.2126 + 2.1653i
-0.0864 + 0.8663i -9.8272 - 1.7325i -0.0864 + 0.8663i
-0.2126 + 2.1653i -0.0864 + 0.8663i 0.2990 - 3.0316i

With i_source and y_station determined, eqn (3.11) can be used to specify the relationship between [I_C   I_R   I_F]' and [V_C   V_R   V_F]'. The apostrophe (') indicates transposition of vectors or matrices.

3.2   Autotransformer Model

3.2.1 Description

The autotransformer has a single winding connected between the catenary and the feeder

wires. The rail system (rails and grounded wires) is connected to a center point on the

winding. The usual voltage rating is 50 kV supply between the catenary and feeder with a

transformation ratio of 1:1 to obtain a 25 kV catenary to rail and from rail to the return

feeder. The station where the autotransformer is located is also called paralleling station

because the two tracks (C wire and F wire) are connected in parallel as shown in Figure 3.3.
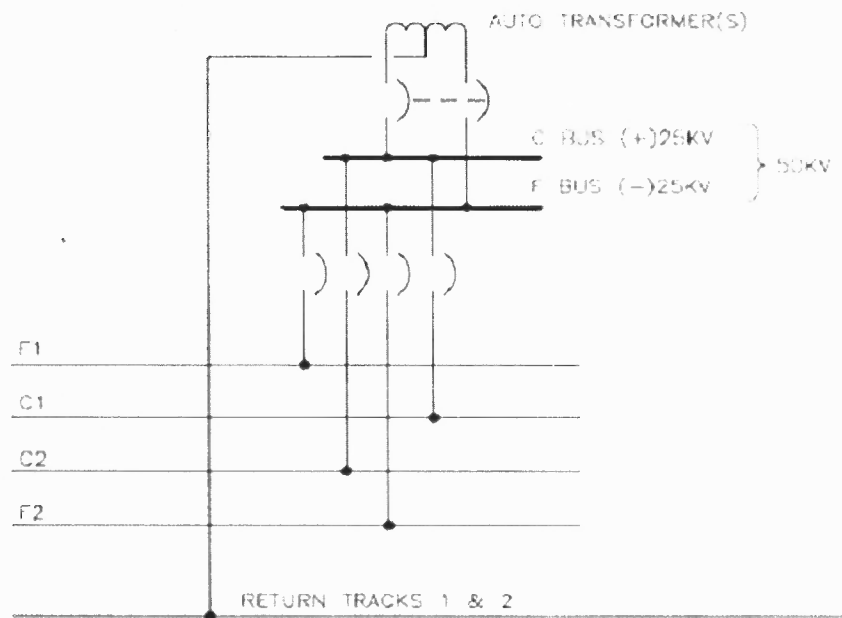
Figure 3.3 Electrical diagram of a paralleling station autotransformer model.

The autotransformer equivalent circuit is as shown in Figure 3.4.
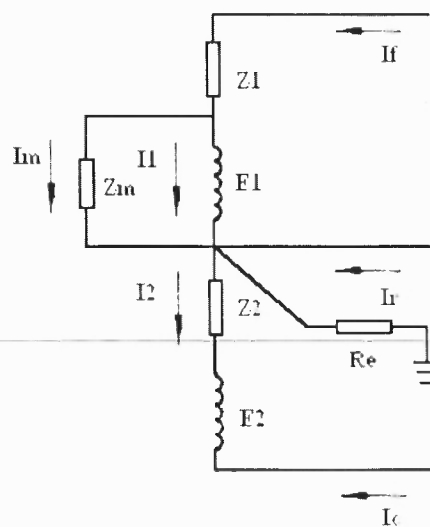
Figure 3.4 Equivalent circuit of an autotransformer.

Symbols are explained below.

$Z1$ and $Z2$ are AT's leakage impedances

Zm is AT's magnetizing impedance

Im is AT's magnetizing current

E_1, E_2 are electromotive forces of AT's primary and secondary windings

N_1, N_2 are turns of two windings

Referring to the Figure 3.4, the magnetizing impedance, Zm, is assumed infinite. $Z1 = Z2$ because the two windings are similar. An earthing resistance, Re, is assumed form the center-tap to remote earth.

The assumed ratings of the AT are: 50/25 kV, 10 MVA, with 1.2 % impedance and an X/R ratio of 6. Therefore, z_base=25^2/10 = 62.5 Ohms, zeq = .012*62.5 = 0.75 ohms and with X/R = 6, zeq = 0.062 + 0.37i. Hence z1 in the AT model is (0.062 + 0.37i)/2 = 0.031 + 0.185i ohms.

The relationship between [vc vr vf]' and [ic ir if]' is derived and it is of the form: [I]=[Y][V]. Where [I] is [Ic Ir If]' and V=[Vc Vr Vf]'. A Matlab function to calculate the Y_matrix of the AT is written: [psYmat]=ps_Y_mat_with_Re(zm,z1,Re). If zm entered is greater than 1000 ohm, the magnetizing current is neglected.

The model of the autotransformer is derived as follows:

Assume $N_1 = N_2$, $z_1 = z_2 = z_g$, therefore

$$N_1 I_1 + N_2 I_2 = 0 \tag{3.12}$$

$$E = E_1 = E_2 = z_m I_m \tag{3.13}$$

$$I_1 = -I_2 \tag{3.14}$$

$$I_R + I_F - I_C = 0 \tag{3.15}$$

$$I_F = I_1 + I_m \tag{3.16}$$

$$I_C = -I_2 \tag{3.17}$$

$$V_R - V_C = E - z_g I_C \tag{3.18}$$

$$V_F - V_R = E - z_g I_F \tag{3.19}$$

From equation (3.16) and (3.17)

$$I_m = I_F - I_C \tag{3.20}$$

$$I_1 = \frac{1}{2}(I_F - I_C - I_m) = -\frac{1}{2}I_R - \frac{1}{2}I_m \tag{3.21}$$

From equation (3.18) and (3.19),

$$V_F - V_C = 2E + z_g I_F - z_g I_C = 2z_m I_m + z_g I_m = I_m(2z_m - z_g) \tag{3.22}$$

$$V_F + V_C - 2V_R = z_g I_F - z_g I_C = -z_g I_R \tag{3.23}$$

From equation (3.20) and (3.21),

$$I_m = \frac{V_F - V_C}{2z_m - z_g} \tag{3.24}$$

$$I_R = -\frac{V_C + V_F - 2V_R}{z_g} = -\frac{V_C}{z_g} + \frac{2V_R}{z_g} - \frac{V_F}{z_g} \tag{3.25}$$

Substitute equation (3.24) and (3.25) into equation (3.16) and (3.17),

$$I_F = I_1 - I_m = -\frac{1}{2}I_R + \frac{1}{2}I_m = \frac{V_C}{2z_g} - \frac{V_R}{z_g} + \frac{V_F}{2z_g} + \frac{V_F}{4z_m + 2z_g} - \frac{V_C}{4z_m - 2z_g}$$

$$= \frac{V_C}{2z_g} - \frac{V_C}{4z_m + 2z_g} - \frac{V_R}{z_g} + \frac{V_F}{2z_g} + \frac{V_F}{4z_m - 2z_g}$$

$$= \left(\frac{1}{2z_g} - \frac{1}{4z_m - 2z_g}\right)V_C - \frac{1}{z_g}V_R + \left(\frac{1}{2z_g} + \frac{1}{4z_m + 2z_g}\right)V_F \tag{3.26}$$

$$I_C = -I_2 = -\frac{1}{2}I_R - \frac{1}{2}I_m = \frac{V_C}{2z_g} - \frac{V_R}{z_g} - \frac{V_F}{2z_g} - \frac{V_F}{2z_m - 4z_g} - \frac{V_C}{2z_m + 4z_g}$$

$$= \frac{V_C}{2z_g} + \frac{V_C}{4z_m - 2z_g} - \frac{V_R}{z_g} + \frac{V_F}{2z_g} - \frac{V_F}{4z_m - 2z_g}$$

$$= \left(\frac{1}{2z_g} + \frac{1}{4z_m - 2z_g}\right)V_C - \frac{1}{z_g}V_R - \left(\frac{1}{2z_g} - \frac{1}{4z_m - 2z_g}\right)V_F \tag{3.27}$$

Thus

$$
Y_{AT} = \begin{bmatrix}
\dfrac{1}{2z_g} - \dfrac{1}{4z_m + 2z_g} & -\dfrac{1}{z_g} & \dfrac{1}{2z_g} - \dfrac{1}{4z_m - 2z_g} \\[2ex]
-\dfrac{1}{z_g} & \dfrac{2}{z_g} & -\dfrac{1}{z_g} \\[2ex]
\dfrac{1}{2z_g} - \dfrac{1}{4z_m + 2z_g} & -\dfrac{1}{z_g} & \dfrac{1}{2z_g} + \dfrac{1}{4z_m - 2z_g}
\end{bmatrix}
$$

$$
= \begin{bmatrix}
\dfrac{z_m - z_g}{z_g\left(2z_m - z_g\right)} & -\dfrac{1}{z_g} & \dfrac{z_m}{z_g\left(2z_m - z_g\right)} \\[2ex]
-\dfrac{1}{z_g} & \dfrac{2}{z_g} & -\dfrac{1}{z_g} \\[2ex]
\dfrac{z_m}{z_g\left(2z_m - z_g\right)} & -\dfrac{1}{z_g} & \dfrac{z_m + z_g}{z_g\left(2z_m - z_g\right)}
\end{bmatrix}
\tag{3.28}
$$

$$
\begin{bmatrix} I_C \\ I_R \\ I_F \end{bmatrix} = Y_{AT} \begin{bmatrix} V_C \\ V_R \\ V_F \end{bmatrix}
\tag{3.29}
$$

Especially, when is it open circuit, that means $z_m \rightarrow \infty$, meanwhile the matrix

becomes:

$$
Y_{AT} = \begin{bmatrix}
\dfrac{1}{2z_g} & -\dfrac{1}{z_g} & \dfrac{1}{2z_g} \\[2ex]
-\dfrac{1}{z_g} & \dfrac{2}{z_g} & -\dfrac{1}{z_g} \\[2ex]
\dfrac{1}{2z_g} & -\dfrac{1}{z_g} & \dfrac{1}{2z_g}
\end{bmatrix}
\tag{3.30}
$$

3.2.2 Results

A Matlab script was written:

```
function [psYmat]=ps_Y_mat_with_Re(zm,zs,Re)
%this function generates the 3x3 Y matrix to model the autotransformer.
%I=Y.V.  The I vector is [Ic Ir If]' and the V_vector is [vc vr vf]'
%zm is the magneitzing impedance assumed between the C and the F wires.
%the equivalent impedance of the c-r and the f-r winding are assumed equal
```

%to zs. If zm is 1000 or greater, it will be ignored (assumed open and the
%magnetising current neglected).
psYmat=zeros(3,3) ;
zm2zs=zm+2*zs;
if(zm<1000)
    psYmat=[1/(2*zs)+1/zm2zs   -1/zs   1/(2*zs)-1/zm2zs
            -1/zs   1/Re+2/zs   -1/zs
           1/(2*zs)-1/zm2zs   -1/zs   1/(2*zs)+1/zm2zs]
else
   psYmat=[1/(2*zs)   -1/zs   1/(2*zs)
          -1/zs   1/Re+2/zs   -1/zs
          1/(2*zs)   -1/zs   1/(2*zs)];

End

The following Y matrix representing the autotransformer is obtained:

0.4405 - 2.6289i -0.8810 + 5.2578i 0.4405 - 2.6289i
-0.8810 + 5.2578i 11.7621 -10.5155i -0.8810 + 5.2578i
0.4405 - 2.6289i -0.8810 + 5.2578i 0.4405 - 2.6289i

3.3   Train Model

The train represents the load in the RES. Usually the MVA requirement of the train is

determined for a specific speed, acceleration, and position of the train as explained earlier.

The type of motor and drive on the train is usually considered also. Having a fixed MVA at

a certain node or nodes, renders the problem nonlinear, Iterative techniques are resorted to.

Usually one of two methods are used: (1) Admittance model: a load is represented by an

admittance $Y=S^*/V^2$ and the voltage V is the voltage between the catenary and the rail

and is calculated every iteration, Y is adjusted, etc. (2) Current Injection model: a load is

represented by a current source (or sink) $I = S^*V^*$ (where V is the voltage between the

catenary and the rail at the location of the train). This current is calculated during every

iteration when the voltage is updated. This train current will be out of the catenary and into

the rails at the train location. For some systems, the load could be more conveniently

expressed as a function of voltage. This is done in a study for the Hong Kong MTR system [Tse]. In this thesis, the train is modeled by its power requirements.

CHAPTER 4

INDUCTANCE CALCULATION

4.1   Inductance Calculations in Power Systems

To calculate the inductances of an overhead wire placed among n conductors, it is usually assumed that the conductors (overhead lines) are infinite in length and parallel to each other. It is known that if the sum of all of the currents in a group of conductors is zero, then the flux linkages for the kth conductor, is

$$\lambda_k = \frac{\mu_0}{2\pi}\left\{I_1\ln\frac{1}{D_{k1}} + I_2\ln\frac{1}{D_{k2}} + ... + I_k\ln\frac{1}{r'_k} + ... + I_n\ln\frac{1}{D_{kn}}\right\} \tag{4.1}$$

Where D_{kj} is the distance between conductors k and j and r'_k is the GMR for conductor k.   This is equation (4.1) in [Glover and Sarma]

Equation (4.1) can be written as:

$$\lambda_k = \lambda_{k1} + \lambda_{k2} + ... + \lambda_{kk} + ... + \lambda_{kn} \tag{4.2}$$

Where λ_{km} is the flux linking conductor k due to a current in conductor m.

The self-inductance is:

$$L_{kk} = \frac{\lambda_{kk}}{I_k} = \frac{\mu_0}{2\pi}\ln\frac{1}{r'_k} \tag{4.3}$$

And the mutual inductance between conductors k and j is defined as:

$$L_{kj} = \frac{\lambda_{kj}}{I_j} = \frac{\mu_0}{2\pi}\ln\frac{1}{D_{kj}} \tag{4.4}$$

Both self and mutual inductances are given in units of H/m.

Where

$$\lambda_{kk} = I_k L_{kk} = I_k \frac{\mu_0}{2\pi} \ln \frac{1}{r_k'} \tag{4.5}$$

$$\lambda_{kj} = I_j L_{kj} = I_j \frac{\mu_0}{2\pi} \ln \frac{1}{D_{kj}} \tag{4.6}$$

Using Faraday's Law of induction, and from equations (4.5) and (4.6), the voltage drop along conductor k due to the current in conductor k is:

$$V_{kk} = I_k(j\omega)L_{kk} = I_k(j\omega)\frac{\mu_0}{2\pi} \ln \frac{1}{r_k'} \tag{4.7}$$

Similarly, the voltage drop in conductor k due to the current in conductor j is:

$$V_{kj} = I_j(j\omega)L_{kj} = I_j(j\omega)\frac{\mu_0}{2\pi} \ln \frac{1}{D_{kj}} \tag{4.8}$$

Defining

$$\overline{Z}_{kk} = R_k + j\omega \frac{\mu_0}{2\pi} \ln \frac{1}{r_k'} \tag{4.9}$$

$$\overline{Z}_{kj} = j\omega \frac{\mu_0}{2\pi} \ln \frac{1}{D_{kj}} \tag{4.10}$$

equations (4.7) and (4.8) can be written as:

$$V_{kk} = I_k \overline{Z}_{kk} \tag{4.11}$$

$$V_{kj} = I_j \overline{Z}_{kj} \tag{4.12}$$

For a 60 Hz system, equations (4.9) and (4.10) are:

$$\overline{Z}_{kk} = R_k + j0.12134 \ln \frac{1}{r_k'} \Omega / mile \tag{4.13}$$

$$\overline{Z}_{kj} = j0.12134 \ln \frac{1}{D_{kj}} \Omega / mile \tag{4.14}$$

4.2 The Geometric Mean Radius

In three-phase power systems, HV transmission lines are some times composed of more than one conductor per phase. This is called bundling because a phase is composed of a bundle (2, 3, or 4) of conductors. Usually the distances between the conductors of a bundle are much smaller than the distance between the centers of the phases. With this assumption, it is shown in power systems textbooks that the inductance per phase is

$L = 2 \times 10^{-7} \ln \frac{GMD}{GMR}$, where GMD is the equivalent geometric mean distance between the

phases and GMR is the Geometric Mean Distance of a bundle. It should be remembered that the above expression of inductance is valid only if the distances between the conductors of a bundle, d, is much less than the GMD between phases. This assumption is not valid in the RES and the error introduced in the calculations of L could be a subject of further research. The grounded conductors are composed of the four rails and the two earth wires and the distances between those six wires could be greater than the distances between C1 and C2. The conductors that are at the same potential and, therefore, can be considered as a bundle, are shown in Figure 4.1.
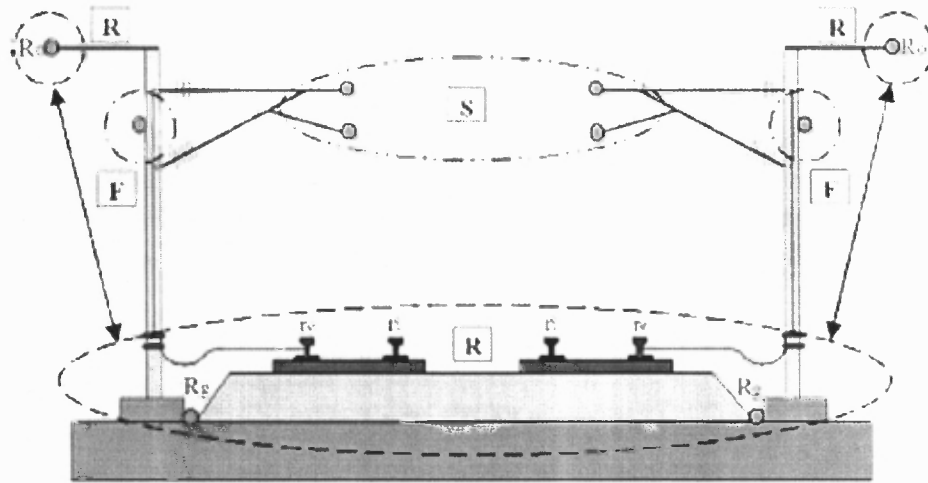
Figure 4.1 Grouping of conductors in a double track RES. S is the catenary group, F is the feeder group, R is the rails and the earthed conductors group.

Since the assumption of d << GMD is not valid in RES, each conductor is modeled separately using Carson's theory introduced in the next section.

4.3   Carson's Line

If some conductors are earthed or the system is not balanced, the earth will carry a return current. This earth current will spread in the earth following the path of least resistance and obeying Maxwell's equations. To consider the effect of earth currents on inductance calculations, Carson [Carson] proposed to replace the earth by a set of "earth return" conductors located directly under the overhead conductors as shown in Figure 4.2. The theory of Carson's line is well established and can be found in many resources. This section is included for completeness and to explain the line modeling used in this thesis.
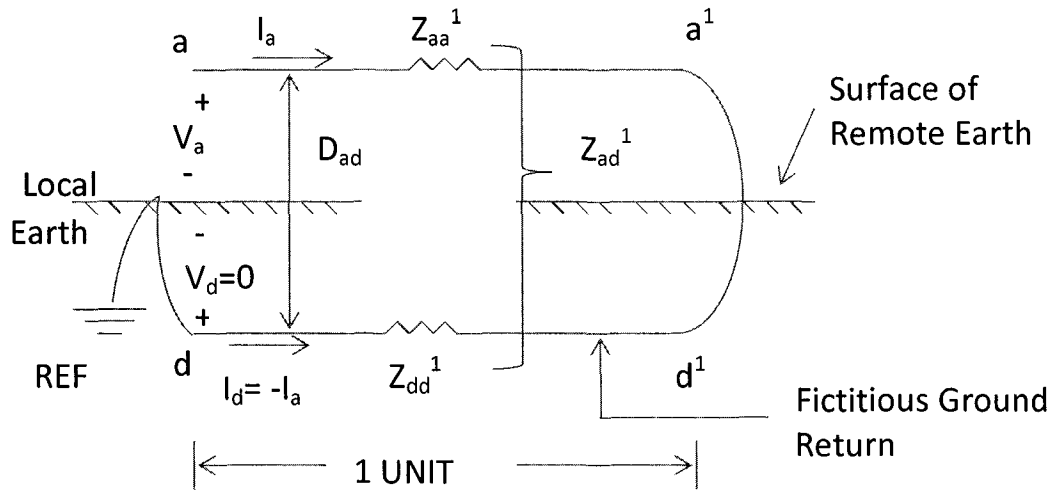
Figure 4.2 Carson's line.

The conductor aa¹ shown in Figure 4.1 carries a current I_a with a return through circuit d-d¹ beneath the surface of the earth. The earth is considered to have a uniform resistivity and to be semi-infinite. The current I_a in the ground spreads out over a large area. Wagner and Evans [9] show that Carson's line can be thought of as having a single return conductor, **d-d¹**, with self GMD equal to that of **a-a¹** and located at a distance $\mathbf{D_{ad}}$ below the overhead line, where $\mathbf{D_{ad}}$ is adjusted so that the inductance calculated with this configuration is equal to that measured by test.

The equations for Carson's line are,

$$\begin{bmatrix} V_{aa}{}^1 \\ V_{dd}{}^1 \end{bmatrix} = \begin{bmatrix} V_a & - & V_a{}^1 \\ V_d & - & V_d{}^1 \end{bmatrix} = \begin{bmatrix} Z_{aa}{}^1 & Z_{ad}{}^1 \\ Z_{ad}{}^1 & Z_{dd}{}^1 \end{bmatrix} \begin{bmatrix} I_a \\ -I_a \end{bmatrix}$$

$$(4.15)$$

Where $\mathbf{V_a}, \mathbf{V_a}{}^1, \mathbf{V_d}$ and $\mathbf{V_d}{}^1$ are measured with respect to the same reference, $\mathbf{V_d}$. Since $\mathbf{V_d = 0}$ and $\mathbf{V_a}{}^1 - \mathbf{V_d}{}^1 = 0$, we solve for $\mathbf{V_a}$ by subtracting the two equations to find:

$$V_a = (Z_{aa}^1 + Z_{dd}^1 - 2Z_{ad}^1)I_a = Z_{aa} \, I_a \tag{4.16}$$

This last equation defines Z_{aa}, where we clearly distinguish between z^1 and z. Thus

$$Z_{aa} \approx Z_{aa}^1 + Z_{dd}^1 - 2Z_{ad}^1 \text{ ohm/unit length} \tag{4.17}$$

Using the above equations and ignoring skin effect, we may write the self and mutual impedances of equation (4.33) as follows. The self impedance of line *a* is

$$Z_{aa}^1 = r_a + jwl_a = r_a + jwk \, (\ln \, (1/D_{sa})) \text{ ohm/unit length} \tag{4.18}$$

Similarly,

$$Z_{dd}^1 = r_d + jwk \, (\ln \, 1/D_{sd})) \text{ ohm/unit length} \tag{4.19}$$

Where $k = 2 * 10^{-7}$.

Carson found that the earth resistance r_d is a function of frequency and he derived the empirical formula

$$r_d = 1.588 * 10^{-3} \, f \text{ ohm/mi} = 9.869 * 10^{-4} \, f \text{ ohm/km} \tag{4.20}$$

Which has a value of 0.09528 ohms/mile at 60 Hz. Finally, the mutual impedance is

$$Z_{ad}^1 = jwm_{ad} = jwk \, (\ln \, (1/D_{ad})) \text{ ohm/unit length} \tag{4.21}$$

Combining (4.36), (4.37) and (4.39) as specified by formula (), we compute the impedance of wire a with earth return as

$$Z_{aa} = Z_{aa}^1 + Z_{dd}' - 2Z_{ad}^1 = (r_a + r_d) + j2wk \, \ln \, D_{ad}/D_{sa} \text{ ohm/unit length} \tag{4.22}$$

The self impedance of a circuit with earth return depends upon the impedance of the earth which in turn fixes the value of D_{ad}. Wagner and Evans [10] discuss this problem in some detail and offer a physical explanation of Carson's original work.

The quantity D_{ad} is a function of both the earth resistivity ℓ and the frequency f and is defined by the relation

$$D_{ad} = 2160\sqrt{(\ell/f)} \text{ ft} \qquad (4.23)$$

If no actual earth resistivity data is available, it is not uncommon to assume ℓ to be 100 ohm-meter. The earth resistivity, ℓ, depends on the nature of the soil and is commonly found in the literature.

The significance of this derivation is that the grounded configuration of Figure 4.1 can be replaced with a line above an earth with infinite conductivity.

4.4   Line Model

Whether analytical techniques or commercial software are to be used, It is necessary to model the different components of the RES and perform simulation. Each of the 12 lines is modeled separately using Carson's theory. Mutual inductances between the different wires including those of the grounded conductors can be considered. Not being able to model mutual inductances is a major limitation in the commercially available software.

4.4.1 Results

A matrix showing the various distances in feet between the conductors of the system based on the configuration described previously follows:

Table 4.1 Various Distances in Feet between the Conductors of the System Based on the Configuration

C1	M1	F1	C2	M2	F2
0.0186	4.5	10.5475	15	15.6605	24.6221
4.5	0.0187	9.0554	15.6605	15	24.0208
10.5475	9.0554	0.0309	24.6221	24.0208	33
15	15.6605	24.6221	0.0186	4.5	10.5475
15.6605	15	24.0208	4.5	0.0187	9.0554
24.6221	24.0208	33	10.5475	9.0554	0.0309
17.6576	22.1256	25.6499	21.5909	25.3755	31.5839
17.6576	22.1256	23.9409	24.6458	28.0208	34.9792
12.2577	12.1655	4.2426	27.1155	27.074	36.1248
21.5909	25.3755	31.5839	17.6576	22.1256	25.6499
24.6458	28.0208	34.9792	17.6576	22.1256	23.9409
27.1155	27.074	36.1248	12.2577	12.1655	4.2426

Ri1	Ro1	S1	Ri2	Ro2	S2
17.6576	17.6576	12.2577	21.5909	24.6458	27.1155
22.1256	22.1256	12.1655	25.3755	28.0208	27.074
25.6499	23.9409	4.2426	31.5839	34.9792	36.1248
21.5909	24.6458	27.1155	17.6576	17.6576	12.2577
25.3755	28.0208	27.074	22.1256	22.1256	12.1655
31.5839	34.9792	36.1248	25.6499	23.9409	4.2426
0.266	4.7083	24.6179	10.2917	15	31.7398
4.7083	0.266	22.2046	15	19.7083	35.52
24.6179	22.2046	0.0105	31.7398	35.52	39
10.2917	15	31.7398	0.266	4.7083	24.6179
15	19.7083	35.52	4.7083	0.266	22.2046
31.7398	35.52	39	24.6179	22.2046	0.0105

The 12x12 Z-matrix is shown below:

Table 4.2 Twelve by Twelve Z-matrix

C1	M1	F1	C2	M2	F2
0.3324 + 1.4458i	0.0953 + 0.7800i	0.0953 + 0.6766i	0.0953 + 0.6339i	0.0953 + 0.6287i	0.0953 + 0.5738i
0.0953 + 0.7800i	0.3276 + 1.4452i	0.0953 + 0.6951i	0.0953 + 0.6287i	0.0953 + 0.6339i	0.0953 + 0.5768i
0.0953 + 0.6766i	0.0953 + 0.6951i	0.2555 + 1.3842i	0.0953 + 0.5738i	0.0953 + 0.5768i	0.0953 + 0.5382i
0.0953 + 0.6339i	0.0953 + 0.6287i	0.0953 + 0.5738i	0.3324 + 1.4458i	0.0953 + 0.7800i	0.0953 + 0.6766i
0.0953 + 0.6287i	0.0953 + 0.6339i	0.0953 + 0.5768i	0.0953 + 0.7800i	0.3276 + 1.4452i	0.0953 + 0.6951i
0.0953 + 0.5738i	0.0953 + 0.5768i	0.0953 + 0.5382i	0.0953 + 0.6766i	0.0953 + 0.6951i	0.2555 + 1.3842i
0.0953 + 0.6141i	0.0953 + 0.5867i	0.0953 + 0.5688i	0.0953 + 0.5897i	0.0953 + 0.5701i	0.0953 + 0.5436i
0.0953 + 0.6141i	0.0953 + 0.5867i	0.0953 + 0.5772i	0.0953 + 0.5737i	0.0953 + 0.5581i	0.0953 + 0.5312i
0.0953 + 0.6584i	0.0953 + 0.6593i	0.0953 + 0.7871i	0.0953 + 0.5621i	0.0953 + 0.5623i	0.0953 + 0.5273i
0.0953 + 0.5897i	0.0953 + 0.5701i	0.0953 + 0.5436i	0.0953 + 0.6141i	0.0953 + 0.5867i	0.0953 + 0.5688i
0.0953 + 0.5737i	0.0953 + 0.5581i	0.0953 + 0.5312i	0.0953 + 0.6141i	0.0953 + 0.5867i	0.0953 + 0.5772i
0.0953 + 0.5621i	0.0953 + 0.5623i	0.0953 + 0.5273i	0.0953 + 0.6584i	0.0953 + 0.6593i	0.0953 + 0.7871i

Ri1	Ro1	S1	Ri2	Ro2	S2
0.0953 + 0.6141i	0.0953 + 0.6141i	0.0953 + 0.6584i	0.0953 + 0.5897i	0.0953 + 0.5737i	0.0953 + 0.5621i
0.0953 + 0.5867i	0.0953 + 0.5867i	0.0953 + 0.6593i	0.0953 + 0.5701i	0.0953 + 0.5581i	0.0953 + 0.5623i
0.0953 + 0.5688i	0.0953 + 0.5772i	0.0953 + 0.7871i	0.0953 + 0.5436i	0.0953 + 0.5312i	0.0953 + 0.5273i
0.0953 + 0.5897i	0.0953 + 0.5737i	0.0953 + 0.5621i	0.0953 + 0.6141i	0.0953 + 0.6141i	0.0953 + 0.6584i
0.0953 + 0.5701i	0.0953 + 0.5581i	0.0953 + 0.5623i	0.0953 + 0.5867i	0.0953 + 0.5867i	0.0953 + 0.6593i
0.0953 + 0.5436i	0.0953 + 0.5312i	0.0953 + 0.5273i	0.0953 + 0.5688i	0.0953 + 0.5772i	0.0953 + 0.7871i
0.1373 + 1.1231i	0.0953 + 0.7745i	0.0953 + 0.5738i	0.0953 + 0.6796i	0.0953 + 0.6339i	0.0953 + 0.5430i
0.0953 + 0.7745i	0.1373 + 1.1231i	0.0953 + 0.5863i	0.0953 + 0.6339i	0.0953 + 0.6008i	0.0953 + 0.5293i
0.0953 + 0.5738i	0.0953 + 0.5863i	0.7062 + 1.5152i	0.0953 + 0.5430i	0.0953 + 0.5293i	0.0953 + 0.5180i
0.0953 + 0.6796i	0.0953 + 0.6339i	0.0953 + 0.5430i	0.1373 + 1.1231i	0.0953 + 0.7745i	0.0953 + 0.5738i
0.0953 + 0.6339i	0.0953 + 0.6008i	0.0953 + 0.5293i	0.0953 + 0.7745i	0.1373 + 1.1231i	0.0953 + 0.5863i
0.0953 + 0.5430i	0.0953 + 0.5293i	0.0953 + 0.5180i	0.0953 + 0.5738i	0.0953 + 0.5863i	0.7062 + 1.5152i

CHAPTER 5

SYSTEM SIMULATION USING SIMPOWER

SimPower is a Matlab/Simulink toolbox. Although this toolbox appears suited to transients studies of power networks, it seems that is can be used also for steady state analysis. This approach is attempted in [23] but the 12 lines were bundled in three groups in [23] as is usually done in the literature. This limitation is removed in the present study.

The system to be studied is shown in Figure 5.1 and it is typical of a dual track system. This system consists of a substation, a line section 4 miles long, an AT, a line section 4.6 miles long, an AT, followed by the last line section of 4.6 miles long and the last AT. The worst case as far as rail voltage is concerned is if the train is in the middle of the section farthest from the substation.

The toolbox used is an older version of SimPower. More recent versions are available in the market but this is the version used because of its availability. This chapter describes the SimPower models of the various components of RES.
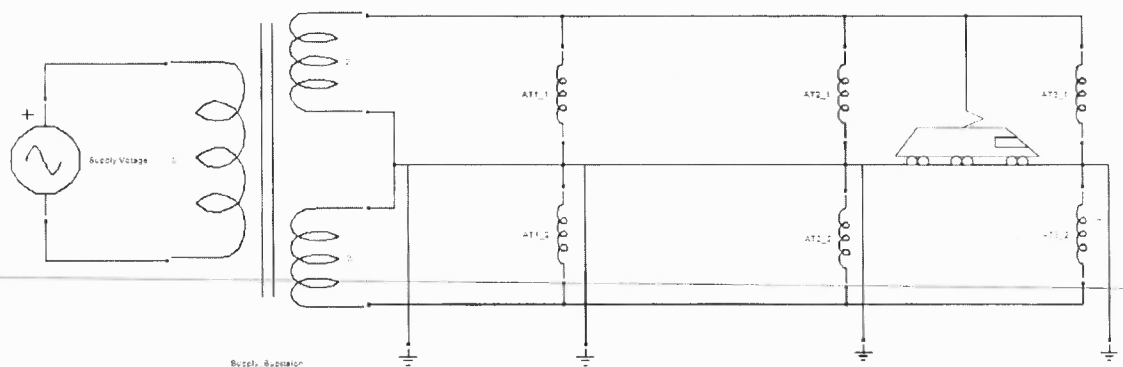

Figure 5.1 The 2x25 kV system under study with one train in the middle of section three.

5.1   Simulation of the Supply Substation

SimPower has models for different types of transformers including Zigzag phase shifting, 3-phase, and Linear Transformer. The linear transformer appears closest to the substation transformer. It has three windings, a primary winding and two secondary windings. The two secondary windings can be connected at one end to form the center tap. Care should be taken when creating the center tap by connecting the LV end of one winding to the HV end of the other. The connected SimPower transformer to simulate the substation transformer is shown in Figure 5.2.

Typical data of the substation transformer are: primary voltage 115 kV; The no-load secondary voltage between the OCS and feeder is 52.5 kV with a grounded center tap; the MVA rating is 60 MVA, with 10% impedance and an X/R ratio of 10. The calculated turn ratio is 115/52.5 = 2.1905. The impedance of the primary will be assumed equal to that of the two secondary.
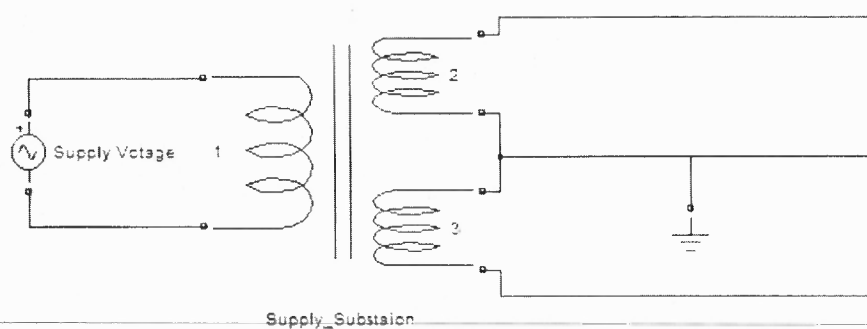

Figure 5.2 SimPower linear transformer connected to simulate a substation transformer.

The following parameters must be entered in the description of this component, the values that are actually entered based on the previous paragraph are also shown in parenthesis:

Nominal power and frequency: (60e3 60hz)

Winding 1 parameters [V1(rms) R1(pu) L1(pu)]: (115e3 0.004975 0.04975)

Winding 2 parameters [V1(rms) R1(pu) L1(pu)]:(26.25e3 0.00124 0.0124)

Winding 3 parameters [V1(rms) R1(pu) L1(pu)]: (26.25e3 0.00124 0.0124)

Magnetizing resistance and reactance[Rm(pu) Lm(pu)]: (500 500)

The magnetizing impedance is usually neglected and arbitrary large values are entered in the last row.

The parameters of the other two windings are entered similarly.

5.2   The Line Model

For greater accuracy, each wire is modeled separately as explained earlier and no bundling of conductors was performed. The mutual impedances have been calculated as presented in Chapter 4. To model the line in SimPower, the mutual inductance element is used. Since there are 12 wires, there are 66 mutual impedances. These are connected as shown in Figure 5.3. For clarity, portions of the line model are shown enlarged in Figures 5.4 and 5.5. The parameters of the mutual impedance components are entered individually and they are (the numbers in parenthesis are examples):

Winding 1 self impedance, R and L: (2 x 0.3325/11,   2x1.4459/377/11)

Winding 2 self impedance, R and L: (2 x 0.1375/11,   2 x 1.1232/377/11)

Mutual impedance, R and L: (2 x 0.0954,   2 x 0.6142/377).

The values in parenthesis are multiplied by 2 because these values pertain to a section 2 miles long. The self impedances are divided by 377 and 11 because the given values (1.4459 for example) are the impedances per mile and converted to inductance by dividing by 377. Because each line is coupled to 11 other lines, there are 11 mutual impedances for each line. The parameters of each individual mutual impedance require a "self impedance" parameter; hence, the total self impedance of the line is divided by 11.
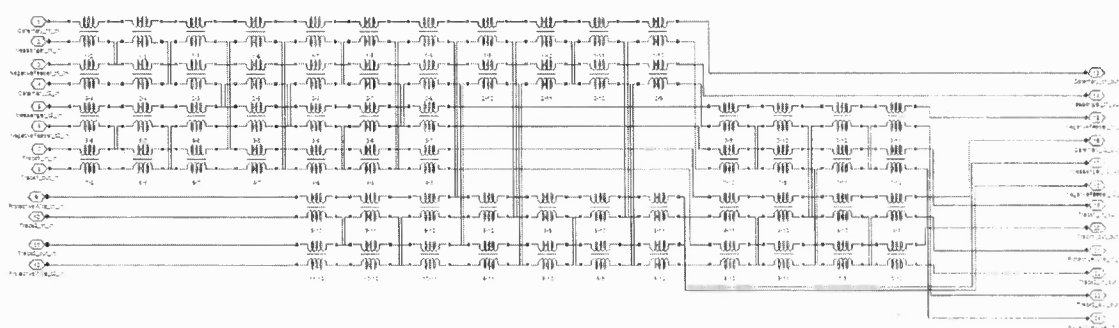

Figure 5.3 The line model created using the mutual impedance element in SimPower.

The four 25 kV (nominal) wires are to be connected to the catenary terminal of the AT, the six grounded wires are to be connected to the center tap of the AT, and the two feeder wires are to be connected to the -25 kV (nominal) of the AT. Therefore, the twelve input wires to the line diagram of Fig 5.3 can be combined and form a set of three wires; similarly the twelve output wires. These sets are appropriately called Catenary, Rail, and Feeder. If there are no trains in the section, the twelve wires can be represented by a line module having three inputs and three outputs as shown in Figure 5.6. It should be emphasized that the lines in the module (within the box) are individually modeled and no bundling took place.
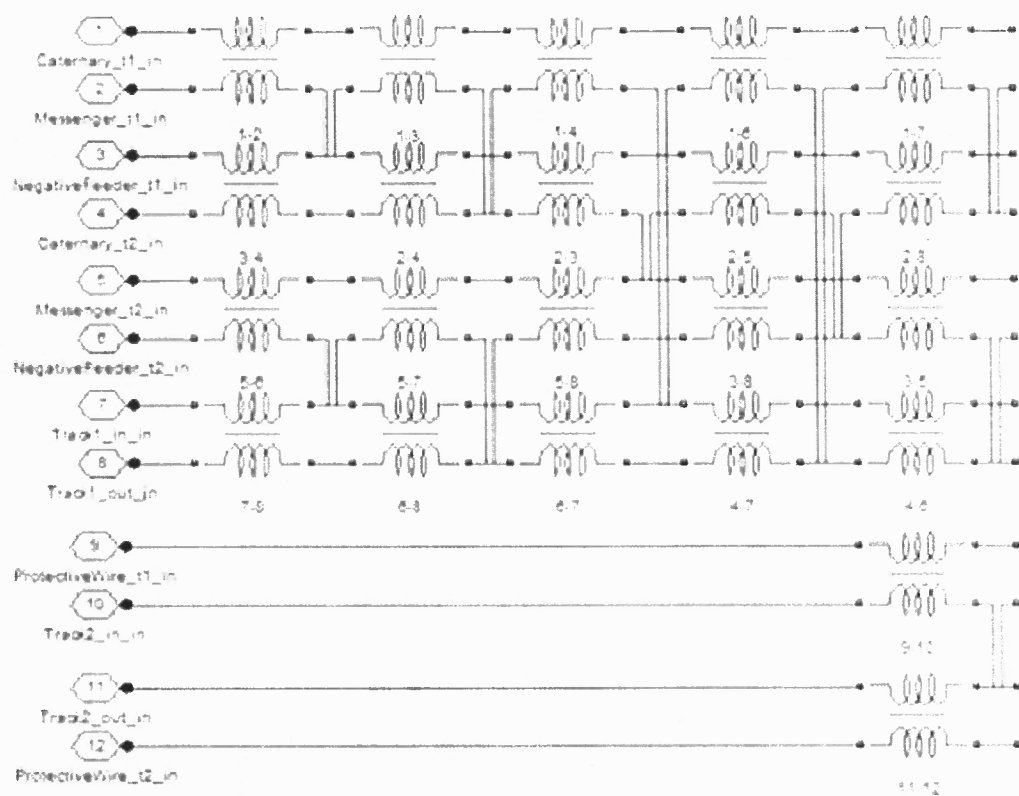
Figure 5.4 The input part of the line model.
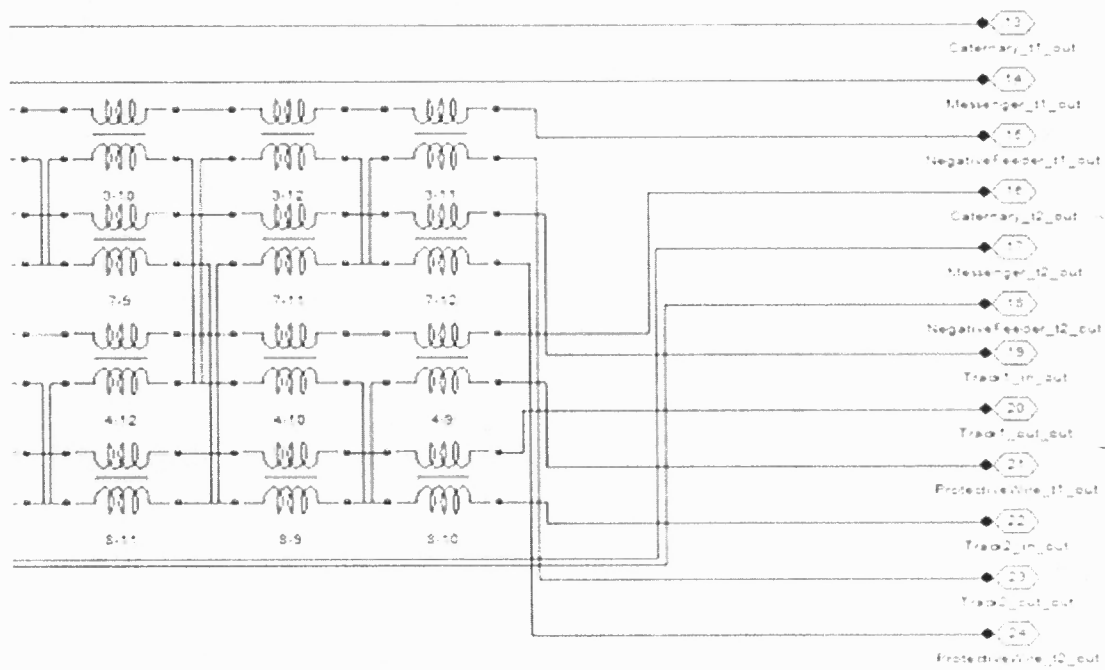
Figure 5.5 The output part of the line model.
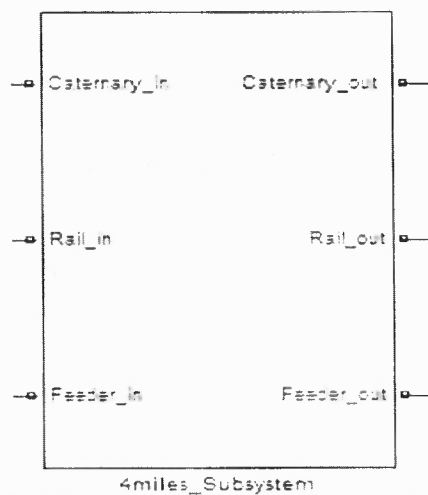
Figure 5.6 The line module with the twelve wires forming three sets of wires.

5.3 The Autotransformer Model

There is no autotransformer model in SimPower, therefore, a two-winding transformer is used and connected as an autotransformer. This is shown in Figure 5.7.
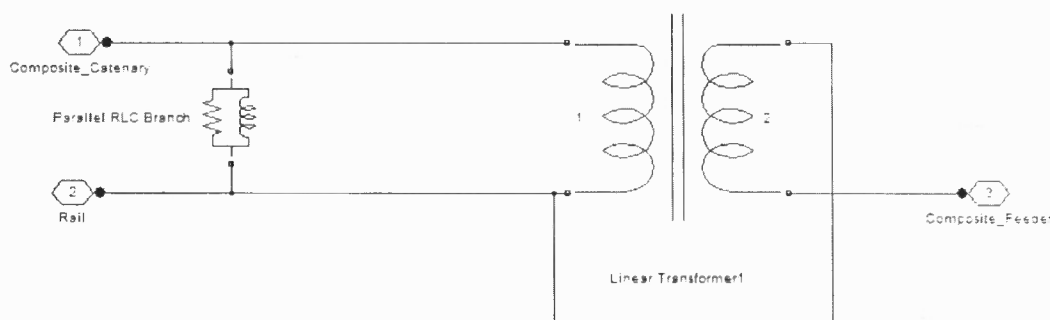
Figure 5.7 A SimPower two-windings transformer connected as an AT.

The parameters requested for the two-winding transformer are (the actual values entered are in parenthesis):

Resistance (1e5 ohms)

Inductance (1e5 H)

Winding 1 parameters: (25e3 0.031 0.185)

Winding 2 parameters: (25e3 0.031 0.185).

The shunt RL that appears in the transformer module is not used in our model and, therefore, arbitrary large values are entered.

5.4 The Train Model

The train is modeled by a series RLC load. The parameters to be entered are shown in Figure 5.8.
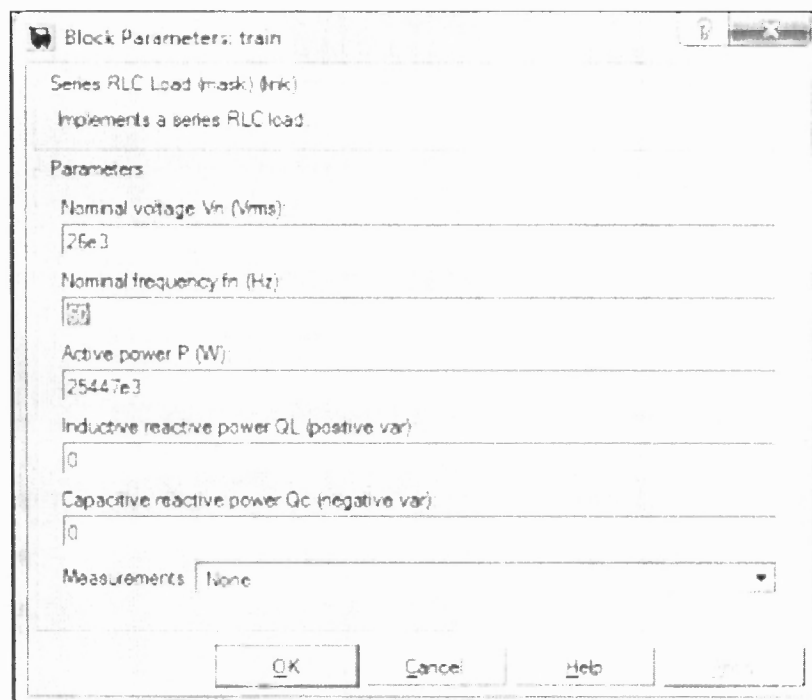
Figure 5.8 The parameter window for the SimPower RLC load representing the train model.

5.5 The Measurements Module

Any current and any voltage can be measured in a SimPower by simply inserting a current measurement block in series with the line. Any number of these current blocks can be combined in one module as shown in Figure 5.9 which includes three current measurements blocks and shown in detail in Figure 5.10. Voltages can be measured similarly by voltage measurement blocks to be inserted across the two points across which the voltage is to be measured.
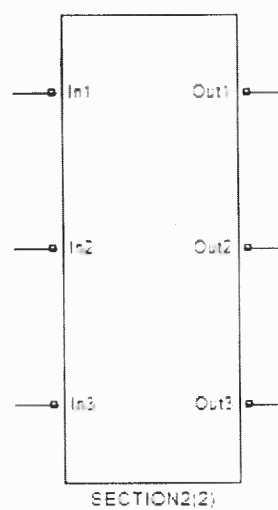
Figure 5.9 SimPower current measurement block to measure three currents.
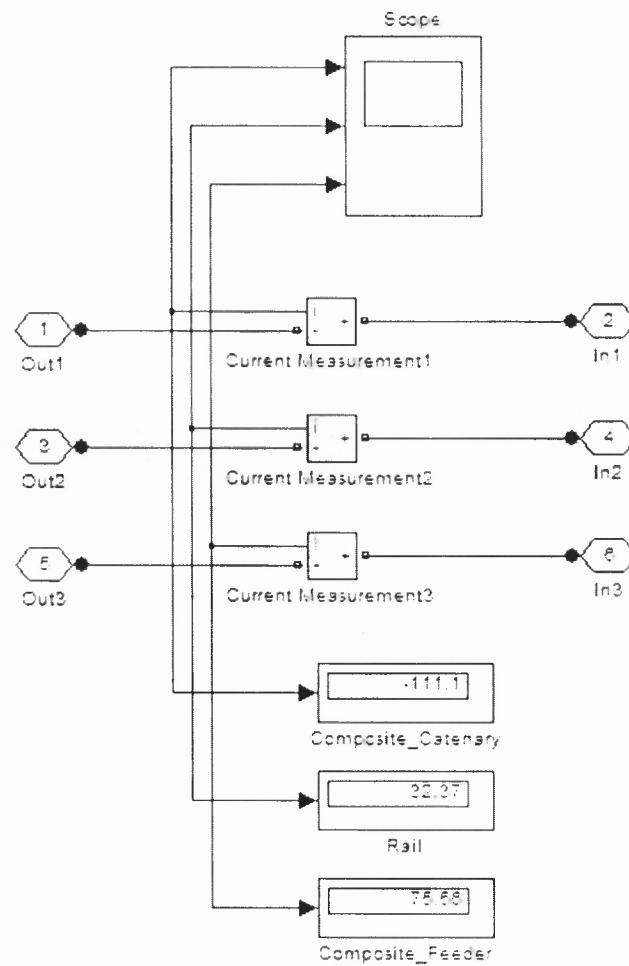
Figure 5.10 Three current measurement blocks included in one module and connected to scope to show instantaneous values.

5.6   The Complete System Model

The model of the complete system is shown in a block diagram in three sections as shown

in Figures 5.11, 5.12, and 5.13.
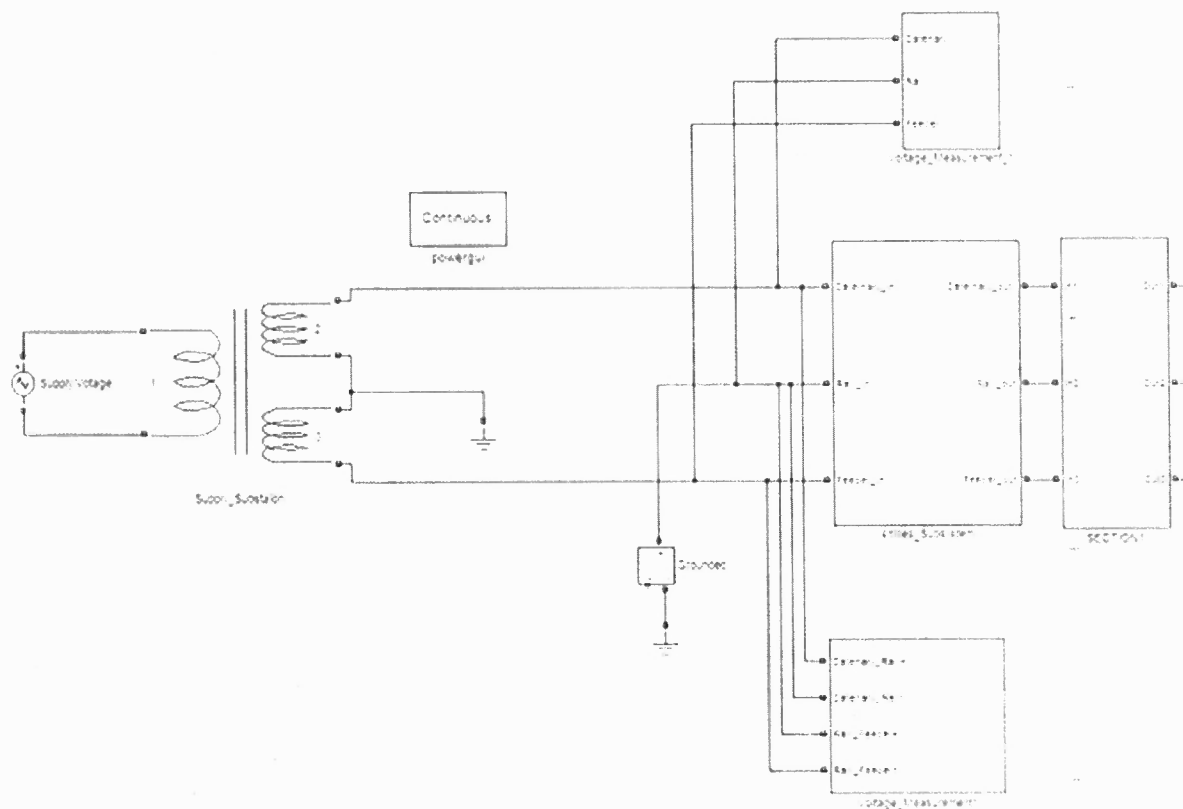
Figure 5.11 The model of one part of the complete system that includes the substation transformer and a 4 mile long line.
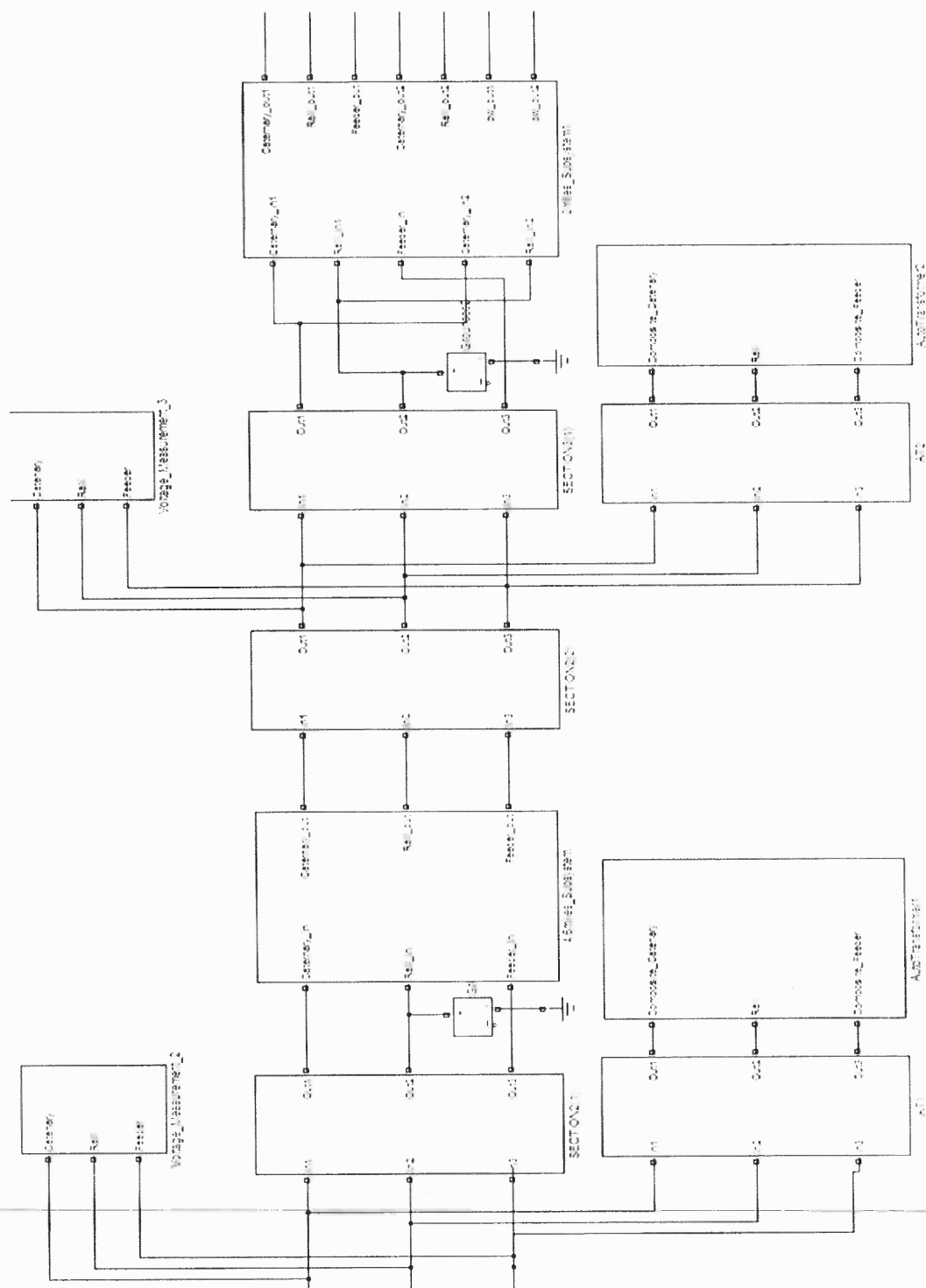
Figure 5.12 The model of the middle part of the complete system that includes AT1, a 4.6 mile long line, AT2, and a 2 mile long line.

Figure 5.13 The model of last part of the complete system that includes the train model, a 2 mile long line, and AT3.
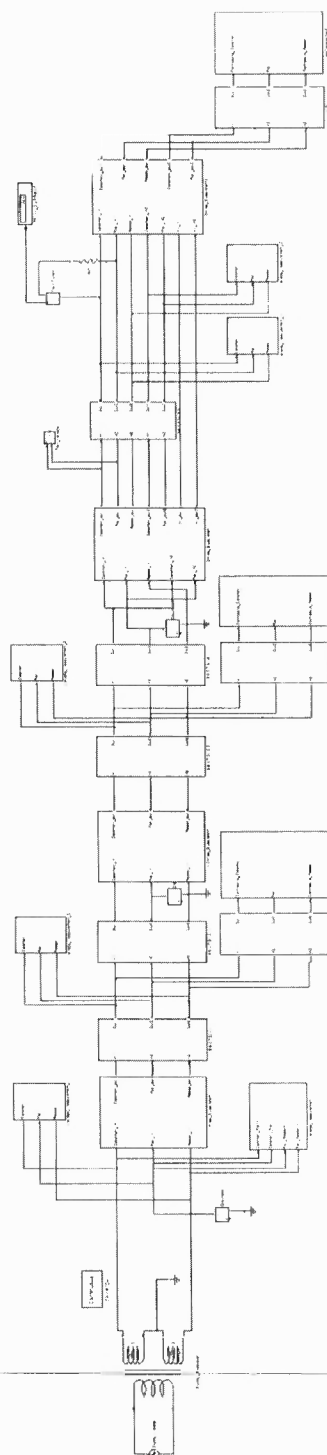
Figure 5.14 A block diagram of the complete system including the measurement modules.

CHAPTER 6

RESULTS AND CONCLUSIONS

The reduced admittance matrix for the line per unit length is shown below:

$$0.6454 - 2.8038i \quad -0.3864 + 1.5844i \quad -0.2091 + 0.8707i$$

$$-0.3864 + 1.5844i \quad 0.9027 - 3.4635i \quad -0.3586 + 1.0571i$$

$$-0.2091 + 0.8707i \quad -0.3586 + 1.0571i \quad 0.6010 - 2.2194i$$

It is clear that the mutual admittances are significant and could be around 1/3 to 1/2 the self admittance. Hence, it is essential that the mutual inductances be considered as is done in this thesis.

Table 6.1 Complex Currents in the Individual Conductors

Result of Simulating the three Sections Between TPS-1 to SWS-2 with One Train in the middle of Section 3
Power Requirement of 25447 kW at Unity PF

	Section_1 4 miles		Section_2 4.6 miles		Section_3 2.0 miles		Section_4 2.0 miles	
	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase
Cat1	138.30	-9.29	141.94	-7.90	316.56	-5.24	181.00	176.99
M1	142.44	-9.05	144.88	-7.83	302.30	-6.60	162.79	174.60
F1	234.08	172.17	216.35	172.91	134.53	175.06	134.53	175.06
Cat2	138.30	-9.29	141.94	-7.90	73.664	-6.37	62.301	-10.37
M2	142.44	-9.05	144.88	-7.83	64.142	-10.10	75.522	-6.24
F2	234.08	172.17	216.35	172.91	93.240	170.35	93.240	170.35
Ri1	26.053	172.18	32.469	172.09	255.39	174.68	227.63	-5.08
Ro1	23.061	171.85	31.383	172.72	252.38	174.54	227.02	-5.21
Pw1	10.198	33.03	4.355	34.71	3.7979	38.40	3.7979	38.40
Ri2	26.053	172.18	32.469	172.09	14.203	172.76	14.203	172.76
Ro2	23.061	171.85	31.383	172.72	12.384	172.21	12.384	172.21
Pw2	10.198	33.03	4.355	34.71	3.7979	38.40	3.7979	38.40

	Section_1 4 miles	Section_2 4.6 miles	Section_3 2.0 miles	Section_4 2.0 miles
Cat1	136.49-22.33i	140.59-19.51i	315.24-28.91i	-180.75+9.50i
M1	140.67-22.41i	143.53-19.74i	300.30-34.75i	-162.07+15.32i
F1	-231.90+31.89i	-214.70+26.70i	-134.03+11.58i	-134.03+11.58i
Cat2	136.49-22.33i	140.59-19.51i	73.21-8.17i	61.28-11.21i
M2	140.67-22.41i	143.53-19.74i	63.15-11.25i	75.07-8.21i
F2	-231.90+31.89i	-214.70+26.70i	-91.92+15.63i	-91.92+15.63i
Ri1	-25.81+3.54i	-32.16+4.47i	-254.29+23.68i	226.74-20.16i
Ro1	-22.83+3.27i	-31.13+3.98i	-251.23+24.01i	226.08-20.61i
Pw1	8.55+5.56i	3.58+2.48i	2.98+2.36i	2.98+2.36i
Ri2	-25.81+3.54i	-32.16+4.47i	-14.09+1.79i	-14.09+1.79i
Ro2	-22.83+3.27i	-31.13+3.98i	-12.27+1.68i	-12.27+1.68i
Pw2	8.55+5.56i	3.58+2.48i	2.98+2.36i	2.98+2.36i

	Section_1 4 miles		Section_2 4.6 miles		Section_3 2.0 miles		Section_4 2.0 miles	
	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase	Magnitude	Phase
C	561.49	-9.17	573.65	-7.87	756.46	-6.31	206.53	178.50
R	83.911	162.85	121.44	169.14	507.78	174.61	433.63	-4.31
					26.585	172.51		
F	468.17	172.17	432.69	172.91	227.58	173.14	227.58	173.14
G	83.911	-17.15	9.3494	172.19	19.720	-6.45	19.720	-6.45

	Section_1	Section_2	Section_3	Section_4
C	554.31-89.48i	568.25-78.55i	751.88-83.14i	-206.46+5.41i
R	-80.18+24.74i	-119.27+22.88i	-505.53+47.70i	432.40-32.59i
			-26.36+3.47i	
F	-463.81+63.78i	-429.38+53.41i	-225.95+27.18i	-225.95+27.18i
G	83.13-11.43i	-9.26+1.27i		19.60-2.22i

Table 6.2 Complex Transformer Currents

		AT_1		AT_2		AT_3	
		Section 1		Section 2		Section 3	
		Magnitude	Phase	Magnitude	Phase	Magnitude	Phase
C		17.737	-141.79	183.70	178.58	206.53	178.50
R		48.355	0.69	388.30	-4.55	433.63	-4.31
F		35.947	163.21	205.12	172.65	227.58	173.14

	AT_1	AT_2	AT_3
	Section 1	Section 2	Section 3
C	-13.94-10.97i	-183.64+4.55i	-206.46+5.41i
R	48.35+0.58i	387.08-30.80i	432.40-32.59i
F	-34.42+1.37i	-203.43+26.24i	-225.95+27.18i

Table 6.3 Complex Autotransformer Voltages

		AT_1		AT_2		AT_3	
		Section 1		Section 2		Section 3	
		Magnitude	Phase	Magnitude	Phase	Magnitude	Phase
C		25991	-1.57	25760	-3.12	25672	-3.72
R		0.0000	0.00	0.0000	0.00	16.223	75.62
F		25994	178.48	25795	177.27	25603	176.65

	AT_1	AT_2	AT_3
	Section 1	Section 2	Section 3
C	25981-712i	25722-1420i	25618-1666i
R	0.00+0.00i	0.00+0.00i	4.03+15.72i
F	-25985+690i	-25766+1229i	-25559+1496i

Table 6.4 Train Voltage and Current

	Train		
	Magnitude	Phase	Complex
Current(A)	962.42	-5.28	958.33-88.56i
Voltage(kV)	25.567	-5.28	25.459-2.358i

The obtained results are verified by checking that Kirchhoff voltage and current laws are satisfied at every node.

The work done for this study shows that SimPower can be used to solve the Load-Flow problem in RES. Entering the large number of mutual inductances to model the

line is tedious and prone to error. It seems that recent versions of the Simpower allow the inputting of the 66 mutual inductances in matrix form. In this work, each line section is modeled separately because the sections are of different lengths. If this work is to be done again, a section of say, 0.1 mile long, is modeled, and then as many of this primitive section as needed are cascaded to create a model of a line of any length (multiple of 0.1 mile).

Having gained experience with SimPower, it is realized that transient analysis (fault studies) can be performed using SimPower. Therefore, the model of a snapshot of the system can be used with little modifications to perform the more complex fault analysis and more detailed modeling of the system. For example, impedance bonding can be easily modeled simply by connecting the two rails of one track, or the four rails of the two tracks if cross bonding is implemented.

Voltage variations with distance on the different conductors can not be obtained efficiently using the discrete model used. Although variation of voltage with distance is desired, the critical points can be easily identified and the voltage at those points obtained.

In conclusion, using SimPower toolbox to solve the load flow problem has been successful, and certainly save time compared with solving the problem using equation modeling as is done in [3]. Once the model is built, modifications can be easily implemented; this is a very useful feature especially in design projects.

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